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Do higher solvency ratios reduce the costs of bailing out insured banks?

by

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Abstract: The relationship between solvency constraints and bank behaviour in the presence of fixed rate deposit insurance is investigated. A rise in the minimum solvency ratio does not necessarily reduce the adverse consequences of moral hazard: bank efficiency may fall and expected bailout costs may rise. Such outcomes are possible even if credit risk is purely systemic. Similar results obtain in respect of level increases in bank capital, tangible or intangible, although in this case purely systemic risk excludes perverse outcomes.

Keywords: bank regulation, capital adequacy, deposit insurance

JEL classification: G28

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1. Introduction

It is well known that in the presence of free or fixed-premium deposit insurance banks may be subject to significant moral hazard. The extent of moral hazard depends on the health of the bank. If the bank has no chance of failing, the deposit guarantee is irrelevant; if, however, the bank will fail in bad states of the world, its owners may have an incentive to take on negative expected value loans at the margin, since the default-state losses associated with these loans will fall on the insuring agency. Such induced inefficiency in portfolio selection can magnify the sensitivity of deposit-taking institutions to negative net worth shocks, especially if supervision by the insuring agency is inadequate. The extreme severity of the U.S. Savings and Loans crisis of the 1980s, for instance, can be best understood in these terms.¹

The objective of containing the losses of deposit insurance agencies through reform of the deposit insurance contract has led in the past decade to widespread adoption of minimum risk-adjusted solvency ratios as the centrepiece of regulation in advanced countries.² Forcing a bank to hold more capital than it might otherwise choose is doubly attractive to a regulator concerned to limit deposit insurance claims. Losses have to be greater before the bank fails, with a greater share borne by the stockholders in the event of failure; it seems, moreover, that reduced incentives to make inefficient portfolio choices must further reduce both the likelihood and the magnitude of insurance claims. The policy literature on solvency ratios takes such benefits for granted, stressing a tradeoff with a possible weakening of the competitive position of banks vis-a-vis other financial institutions. For example, Berlin *et al*[1991] write (p.740): 'Clearly, stockholders have more to lose on closure when their bank has more

¹ Berlin *et al*[1991] discuss these shocks in this case and review microeconomic evidence of the effects of moral hazard on behaviour. See also Akerlof and Romer[1993], who interpret the Savings and Loans crisis in terms of the incentives for fraud offered by the deposit insurance contract when net worth is low.

² The régime agreed by the membership of the Bank for International Settlements in 1988 came into force in the EC, Japan and the US in January 1993. See Committee on Bank Regulations and Supervisory Practices[1988].

capital, and their incentives to undertake risky investment strategies will be correspondingly weaker. Indeed, safe banking implies high capital ratios.'

What matters to a deposit insurer is, clearly, the expected bailout cost associated with a given bank. The main new result in this paper is that a perverse relationship between expected bailout cost and minimum solvency ratio is quite possible. A rise in the ratio may *raise* the expected bailout cost, at the same time inducing the bank to choose a less efficient portfolio. Such an outcome is not excluded even if the risks associated with bank loans are restricted to be systemic (i.e. rank correlated). It is now common regulatory practice to impose higher solvency ratios than the 8% minimum on banks deemed especially risky³ and this is recommended in the draft New Capital Adequacy Framework (Bank for International Settlements 1999). My main result implies that, while this practice may be effective in any given case, it cannot be guaranteed to produce the desired effect.

The model of the paper is also used to consider the effect of intangible capital on expected bailout costs. Here, too, there are perverse cases where higher expected bailout costs and lower efficiency result from a higher level of intangible capital but now *not* if risks are purely systemic. Since, as will be argued, intangible capital can be viewed as equivalent to a solvency constraint in level rather than ratio form, a possible role for level solvency constraints is implied. Before setting out these results, I give a brief review of the literature.

The equivalence between fixed rate deposit insurance and a European put option on the assets of a bank was noted by Merton[1977]. This equivalence is exact in a static context; if, at the end of the period, the assets of the bank are worth less than the debt to depositors, the assets are put to the insurer at a strike price equal to the debt. The option is valuable to the owners if failure is possible, since deposits can be attracted at

³ The UK is one example; see Richardson and Stephenson[2000]

the risk-free rate.⁴ The early literature which sought to analyse the effects of capital regulation on bank behaviour (Kahane[1977], Koehn and Santomero[1980]) failed to take proper account of the option value of deposit insurance and was decisively criticised by Keeley and Furlong[1990]. Their own analysis (Furlong and Keeley[1989]) employs a very simple static model of bank behaviour. Bank operating costs are ignored and the deposit insurance premium is set to zero; banks are restricted to choosing among zero expected net present value (NPV) assets, thought of as purchased in competitive markets. Bank expected profit maximisation is then equivalent to maximisation of the option value of the deposit guarantee, equal to the expected bailout cost. Therefore, in a two-state two-asset world, the bank will always invest exclusively in the riskier asset, but the scale of operation of the bank is not determinate. A rise in the minimum solvency ratio has effects which depend on whether the bank responds by shrinking its assets or raising its capital, a question on which the model cannot pronounce. The bank's incentive per dollar of capital to switch from the safer to the riskier asset falls, as does the expected insurer cost per dollar of assets, from which the authors conclude that minimum solvency ratios are useful, but only in conjunction with prudential oversight of asset risk.

Gennotte and Pyle[1991] address the most obvious weakness of Furlong and Keeley's paper, namely its rudimentary modelling of banks as, effectively, traders of zero-expected-NPV assets. Banks here choose among assets whose expected returns and risks differ, although returns across assets are restricted to be perfectly linearly correlated. Any given asset is available in unlimited quantity, the size of the bank in equilibrium being limited by operating costs which are assumed to be (eventually) convex in both the scale and the risk of the chosen portfolio. It is shown that a bank may increase or reduce asset risk in response to a tightening of the solvency constraint, depending on the cost function in a rather obscure way. If asset risk does

⁴ In a dynamic context, matters are more complicated, since the insurer may exercise forbearance over closure, either because the bank is worth more as a going concern than if it is liquidated, or for systemic reasons. See, for example, Pennacchi[1987].

increase, however, the bank will simultaneously scale down its portfolio. Numerical analysis of special cases of their model shows asset risk rising with tightening of the capital constraint; in some cases the probability of bankruptcy also rises, but in all cases expected bailout costs fall and the efficiency of lending increases. As we will see, the latter results do not survive in my model, in which bank size is constrained via finiteness of the set of loans that the bank can make rather than via convexity of its costs. The merits of this choice are open to debate, but it leads to different analytical methods which, crucially, allow the unduly restrictive assumption of perfectly linearly correlated asset returns to be discarded.

2. Solvency constraints and portfolio choice in a simple bank model

2.1 Basic framework

I start from the presumption that banks can be viewed as entities that have an advantage in the screening and the monitoring of loans, or 'projects', as they will be called.⁵ The advantage may partly be a matter of human capital, but also has an informational component, associated with the bank's experience of loan applicants as previous borrowers or as depositors.⁶ In this spirit, once the restriction of a static framework is accepted, it seems reasonable to define a bank in terms of a *finite* set of projects available to it, with each project characterised by an investment level and a state-dependent return. Given that banks have a degree of market power, attention is not restricted to projects with zero expected net present value. Nor is any general restriction placed on the extent to which project returns are correlated. Bank operating costs are assumed to be project-specific and so do not require explicit analysis: project returns are to be understood as net of such costs.

⁵ See Hellwig[1991] for a review of modern ideas on financial intermediation.

⁶ Discussion of the notion of 'relationship' banking may be found in Davis[1992], ch.2, and Diamond and Dybvig[1986].

The bank is thus assumed to have available to it a fixed, finite, set of projects represented by the index set I . Project i costs $v(i)$ dollars to undertake and yields a gross return to the bank of $x(i,j) \geq 0$ in state of the world j . Projects mature immediately and are financed by a mix of bank capital and non interest bearing deposits, which are in unlimited supply.⁷ J denotes the index set of states of the world. The net return to project i in state j is written $y(i,j)$, where:

$$y(i,j) = x(i,j) - v(i) \quad (1)$$

The price of a dollar in state j is denoted $p(j)$. It may be that the prices $\{p(j)\}$ equal the probabilities of the various states occurring, implying that these insurance markets are dominated by risk-neutral behaviour. In any case, agents in a position to diversify risks away at prices $\{p(j)\}$ will evaluate risky projects using these prices, whatever their attitudes to risk. The $p(j)$ may be interpreted as 'risk-neutral probabilities'; all probability statements in what follows are with respect to these 'probabilities'. The bank's owners, but not its loan clients, are assumed to be able to diversify risk in the manner described. The insurer is assumed to fully indemnify depositors against risk, and the insurance premium is assumed to be zero, without loss of generality. The minimum solvency ratio imposed by the insurer is denoted by c .

The set of projects chosen by the bank is denoted by X , a subset of I . We want to explain the choice of X , and especially how this choice depends on c . X and c together determine a partition of J into a set of default states, denoted by D , and a set of solvent states, denoted by S . Since, in default states, the net losses summed over projects exceed the capital of the bank, D is made up of those states for which:

$$\sum_{i \in X} \{y(i,j) + c \cdot v(i)\} < 0 \quad (2)$$

⁷ To introduce a time dimension by allowing projects to mature over some fixed interval (and by letting deposits earn a riskless interest rate) would complicate the algebra of the paper without adding anything of substance.

The expected return to the bank (its maximand), the expected bailout cost (the option value of the deposit guarantee), and the value (efficiency) of the portfolio are denoted R , C and V respectively and satisfy:

$$R = V + C \quad (3)$$

$$V = \sum_{i \in X} \sum_{j \in J} p(j) \cdot y(i, j) \quad (4)$$

$$R = \sum_{i \in X} \left\{ \sum_{j \in S} p(j) \cdot y(i, j) - \sum_{j \in D} c \cdot v(i) \cdot p(j) \right\} \quad (5)$$

$$C = - \sum_{i \in X} \left\{ \sum_{j \in D} \{ p(j) [y(i, j) + c \cdot v(i)] \} \right\} \quad (6)$$

Thus (equation(3)), the expected net return to the bank's owners is made up of the expected net return on its asset portfolio, V in equation (4), plus the expected bailout cost, C . Equation (5) expresses the expected return to the owners as the expected net return in solvency states minus the expected capital loss in default states. Equation (6), derivable from (3)-(5), expresses the expected bailout cost as the total expected loss in default states net of the contribution of the owners. It may be seen from equation (5) that, except when c equals unity, the problem of project selection facing the bank is rather a complex one. Whether an individual project should be added to the bank's portfolio is independent of its return in default states, so that its assessment requires knowledge of which the default states are. Even a 'bad' project may be worth doing if it performs worst in states in which the bank is planning to default anyway. However, the partition of states into default states and solvency states itself depends on the totality of projects selected, so that there is an unavoidable interdependence in project selection. Despite the consequent difficulties in characterising optimal bank behaviour, we will see that it is possible to reach conclusions about the effects of changes in the minimum solvency ratio.

The absolute value of the right hand side of this equation is the probability of default multiplied by the book value of bank assets, which may be termed the expected scale of default. It is clear from (2) that, as c is raised, there will be a series of switch points at which default states become solvency states. Hence for given X_0 , R is a piecewise linear, convex, function of c , as illustrated by ABCD in Figure 1. For solvency ratios above c_1 the bank never fails, so that the expected scale of default is zero and R is equal to V . Between c_0 and c_1 there is one default state, and below c_0 there are two. Clearly we may superimpose on Figure 1 the $R(c)$ functions for all possible portfolios X , and the unrestricted dependence of R on c is represented by the upper envelope of these functions. Consider the effect of adding just one more portfolio, X_1 , illustrated by EFGH in Figure 1. When c is equal to c_2 , the bank is indifferent between the two portfolios; below c_2 , X_0 is selected and, above c_2 , the bank switches to X_1 . Note that, at the switch, the expected scale of default must fall, since the $R(c)$ function must be flatter for X_1 than for X_0 . Thus:

Proposition 1: (a) Maximised bank expected return is a non-increasing, piecewise linear, convex function of the solvency ratio; (b) The expected scale of default is non-increasing in the solvency ratio.

It follows from (b) that if a rise in the solvency ratio causes a rise in the probability of default, it must also cause a more than proportionate decline in bank assets.⁸ As the solvency ratio is increased, the expected scale of default experiences a series of step declines, of two types. There are steps associated with the conversion of default states into solvency states, but with no portfolio change (points B and G in Figure 1), and

⁸ Gennotte and Pyle prove a somewhat weaker result, and only for the case of perfectly linearly correlated project returns (ibid. Proposition 2).

steps involving a portfolio change together with what may be a wholesale respecification of the set of default states (point Z in Figure 1).

What matters to the deposit insurer, however, is not the expected scale of default but the expected bailout cost, C . How then does C depend on c ? Consider Figure 1 again. The value, V , of any portfolio is equal to its value of R for $c=1$ and is thus shown by the height of the right-hand vertical intercept (LD for portfolio ABCD). This allows C to be read off for any point in the figure: at point B, for example, C is equal to BJ. In Figure 1, C falls throughout as c rises, continuously except at the portfolio switch point Z, where it falls discretely by amount HD. The geometry (or propositions 1(a) and 3) make it clear that if C is ever to rise with c , this can only happen at a portfolio switch point, and then if and only if V simultaneously *falls*. This in turn can only happen if the $R(c)$ functions of the portfolios intersect more than once – if there is portfolio reswitching, in other words. Under what circumstances might this happen? Equation (6) can be rewritten in the following form:

$$C = (1-c) \sum_{i \in X} \sum_{j \in D} p(j) \cdot v(i) - \sum_{i \in X} \sum_{j \in D} p(j) \cdot x(i,j) \quad (8)$$

The first term on the RHS of (8) is what the expected bailout cost would be if nothing at all were recovered in default states, while the second term is the expected recovery in default states. Now, the first term is just $(1-c)$ multiplied by the expected scale of default, and Proposition 1(b) tells us that this cannot increase with c at a portfolio switch point. It follows that necessary for an increase in C is that the expected default state recovery falls at the switch point. Summarising:

Proposition 2: (a) If the bank is restricted to zero NPV projects, the expected bailout cost is non-increasing in the solvency ratio; (b) For general NPV projects, the expected bailout cost can rise with the solvency ratio only at a portfolio switch point, and will do so if, and only if, the bank switches to a less efficient portfolio; (c)

Necessary for such an outcome is that the switch lowers expected default state recovery.

While easily stated, (c) is not a very tight condition, given that the expected scale of default itself falls with the switch. A little more can be said by rewriting the equation for C once more:

$$C = \sum_{i \in X} \sum_{j \in D} p(j) \cdot v(i) \{1 - c - x(i,j)/v(i)\} \quad (9)$$

Here the bracketed term is the bailout cost per dollar of investment on project i in default state j ; this falls short of a dollar by the owners' contribution, c , plus the amount recovered per dollar from the project - equal to one plus the rate of return. Equation (9) gives C as a weighted average of such bracketed terms, and since we know from Proposition 1(b) that the sum of the weights, $[p(j) \cdot v(i)]$, necessarily falls at a portfolio switch point, C can only rise if (loosely speaking) the bracketed terms rise on average, that is, if default state rates of return fall on average. To illustrate the perverse case I give two examples:

Example 1: Suppose that the bank has two projects in which it can invest, and that there are two states of the world. Table I gives the costs of each project, the net returns in each state, the probabilities of each state occurring and the NPVs of the projects. Clearly the bank has four options: (a) do project 1; (b) do project 2; (c) do both projects; (d) do nothing. Figure 2 shows the returns from each of these options as a function of the capital-asset ratio, illustrated by ABC, DEF, GHF and OC respectively. Taking them in turn:

(a) For $0 \leq c \leq 3/4$, the expected return to the bank, $R(c)$, is given by:

$$R(c) = 2/3 \cdot 9/8 + 1/3 \cdot (-3c) = 3/4 - c; \text{ for } c \geq 3/4, R(c) = 0.$$

(b) For $0 \leq c \leq 11/12$, $R(c) = 2/3 \cdot (-c) + 1/3 \cdot 7/4 = 7/12 - 2c/3$; for $c \geq 11/12$, $R(c) = -1/36$.

(c) For $0 \leq c \leq 1/8$, $R(c) = 5/36 - 4c/3$; for $c \geq 1/8$, $R(c) = -1/36$.

(d) For all c , $R(c) = 0$.

Option (c) is weakly dominated by each of (a) and (b). The disadvantage of doing both projects is that, because their returns are inversely correlated, the scale of default in the bad state (state B, as it turns out) is much reduced, and, with it, the expected subsidy.

It can be seen from Figure 2 that the portfolios represented by projects 1 and 2 exhibit reswitching: the two switch points are labelled X and Y. For $c \leq 1/2$, project 1 is chosen; for $1/2 \leq c \leq 7/8$, project 2 is chosen; for $c \geq 7/8$, the bank reswitches to project 1.⁹

The first switch point, X, is the perverse one: as c is raised through the critical value of $1/2$, the bank switches from project 1 to project 2, and chooses to default in state A rather than state B. Efficiency falls and the probability of default rises from $1/3$ to $2/3$. Remembering equation (2) we see that expected bailout costs rise from $1/4$ to $5/18$. As required by Proposition 1(b), however, the expected scale of default falls - from 1 to $2/3$; the bank is undertaking a smaller project. At Y, the bank reswitches to project 1, and the variables of interest behave better: efficiency rises, and both the probability of default and expected bailout costs fall.

⁹ For $c \geq 7/8$, the bank may choose to do nothing; to shorten the discussion, this possibility is neglected in the text.

TABLE 1

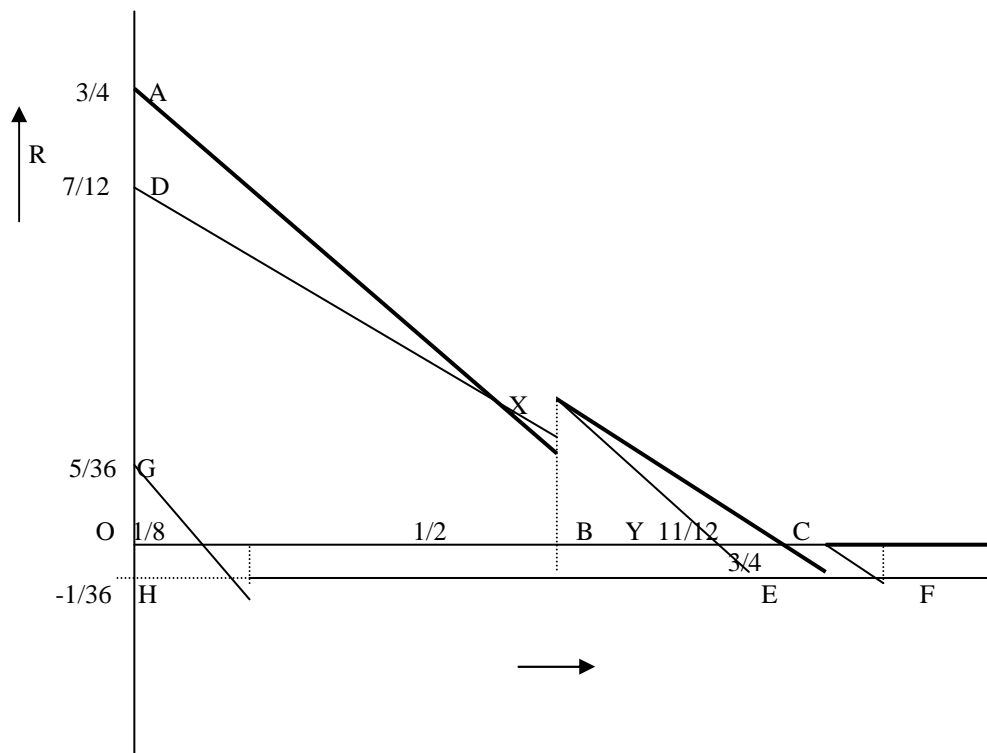
		NET RETURNS		NPV
		STATE A prob. = $\frac{2}{3}$	STATE B prob. = $\frac{1}{3}$	
Project 1	Cost = 3	$\frac{9}{8}$	$-\frac{9}{4}$	0
Project 2	Cost = 1	$-\frac{11}{12}$	$\frac{7}{4}$	$-\frac{1}{36}$

TABLE 2

		NET RETURNS			NPV
		STATE A prob. = $\frac{1}{3}$	STATE B prob. = $\frac{1}{3}$	STATE C prob. = $\frac{1}{3}$	
Project 1	Cost = 1	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{7}{10}$	$-\frac{1}{10}$
Project 2	Cost = 1	-1	$-\frac{1}{10}$	$+\frac{1}{2}$	$-\frac{1}{5}$

FIGURE 2

Expected bank return, R , and the solvency ratio, c : reswitching case



$c=0$

c

$c=1$

This example conforms to the requirements of Proposition 2(c) concerning expected default state recovery. For project 1 this equals $1/4$ - there is a recovery of $3/4$ with probability $1/3$; for project 2 the corresponding figure is only $1/18$ (a recovery of $1/12$ with probability $2/3$). Furthermore, the default state rates of return on capital are $1/4$ and $1/12$ for the two projects. This underlines the point that perverse switches are associated with switches towards portfolios which are particularly bad in default states. It is, however, not the case that such switches require project returns to be negatively correlated. In the next example the states of the world are rankable from 'bad' to 'good' in the sense that the returns on all projects are non-decreasing as one moves to a 'better' state.

Example 2: There are now three states of the world, A, B, and C, with A being the worst and C the best. The costs, returns, probabilities and net present values are given in Table 2.

There turns out to be a portfolio switch point at a solvency ratio of $3/10$ with the bank indifferent between two actions, (a) and (b):

(a) Do project 1, and default in states A and B. Since the bank only loses $3/10$ in each of the default states, this action yields an expected return to the bank of $1/30$.

(b) Do project 2, and default only in state A. This, similarly, can be seen to yield an expected return to the bank of $1/30$.

Doing both projects is worse than either (a) or (b); the return to the bank is zero. The expected scale of default is $2/3$ for action (a) and $1/3$ for (b). In accordance with Proposition 1(b), it can be seen that, as c is raised through $3/10$, the switch is from (a) to (b). This is a perverse switch: V falls and C rises, each by $1/10$.¹⁰

¹⁰ The full solution to this example is that the bank does both projects for $c \leq 1/4$, does project 1 for $1/4 \leq c \leq 3/10$, does project 2 for $3/10 \leq c \leq 2/5$, and does nothing for $c \geq 2/5$.

This example shows that, even with projects with returns exhibiting perfect rank correlation across states (the 'tie' in Table 2 obviously plays no role), and even in a case where default probability declines as the solvency ratio is increased, perverse responses of efficiency and expected bailout cost are possible. To eliminate such cases plainly requires quantitative rather than qualitative association between the vectors of project returns.

2.3 Intangible capital and level solvency constraints

In this section I modify the analysis to allow for changes in the initial conditions facing the bank. Until now it has been supposed that the bank comes into existence endowed with advantages in the loan market and that it operates for only one period. Ideally one would want to explain how such advantages were built up and how preservation of them for the future might affect current period behaviour. The first of these issues would require the construction of a dynamic model and this is not attempted here. The second issue can be addressed in a simple way by adding intangible capital to the model. If the bank fails, it will lose the opportunity to exploit its market position in the future. Future rents may derive from reputation, informational or geographical monopoly, the bank's charter, or merely the organisational structure of the bank: current behaviour must take the potential losses into account.¹¹ I shall assume that the present value of the future rents that are put at risk in the current period has a fixed value, and shall refer to this as intangible capital, denoted K . If there are current period losses which fall short of K , I assume that the bank survives, the owners bearing the costs¹²; if the losses exceed K , the bank closes and a deposit insurer indemnifies the depositors. At issue is how far intangible capital

¹¹ Whether failure would extinguish such rents or just transfer them elsewhere does not matter as far as the owners are concerned, but could be relevant to the behaviour of the insurer, who may be more inclined to close the bank rather than exercise 'forbearance' if the rents are transferable. See Allen and Saunders[1993].

¹² If uninsured (see the discussion in section 2.1).

can substitute for an imposed minimum solvency ratio – specifically, whether a rise in intangible capital can be relied on to reduce expected bailout costs.

Note that, as far as the bank's owners and depositors are concerned, this case is identical to one in which there is no intangible capital, but a *level* solvency constraint in amount K , instead. We may now apply an analysis like that in section 2.1.

Denote the state-dependent net returns from a given portfolio X by $\{Y(j)\}$, so that:

$$Y(j) = \sum_{i \in X} \{y(i,j)\} \quad (10)$$

The default states are those for which $Y(j)$ is less than $(-K)$. The bank's maximand is, by analogy with (5):

$$R = \sum_{j \in S} p(j).y(i,j) - \sum_{j \in D} p(j).K \quad (11)$$

Differentiating w.r.t. K (where (11) is differentiable), we obtain:

$$\partial R / \partial K = - \sum_{j \in D} p(j) \quad (12)$$

So R is a piecewise linear convex function of K , with the slopes of the linear sections now equalling minus the probabilities of default. As before we may superimpose the graphs for all possible portfolios, concluding that the unrestricted dependence of R on K is itself non-increasing and convex; in this case the slope at any point is equal to minus the default probability. So we have a companion result to Proposition 1:

Proposition 3: (a) Maximised bank expected return is a non-increasing piecewise linear convex function of K , (b) the probability of default is non-increasing in K .

Although a bank with more intangible capital (or a higher level solvency constraint) cannot now be more likely to default, it remains possible that expected bailout costs are higher. As before, equation (3) implies that this can only happen if the bank

chooses a less efficient portfolio. This can only be ruled out if reswitching of portfolios (now in (R,K) space rather than (R,c) space) is ruled out, which is not possible in general. An example to demonstrate this is given in the Appendix. It turns out now, however, that if project returns are perfectly rank correlated the perverse outcome is excluded. Summarising:

Proposition 4: (a) Efficiency may rise or fall with K at a portfolio switch point, and expected insurer cost will fall or rise accordingly; (b) If project returns are perfectly rank correlated, efficiency is non-decreasing and expected insurer cost non-increasing in K .

Proof of part (b): see Appendix. The implication of this proposition, taken together with Example 2, is that when risks are systemic, the possibility that expected insurer cost might be perversely related to the tightness of the solvency constraint is linked to the expression of that constraint in the form of a ratio.¹³

3. Concluding observations

Is the discovery that the expected liability of a deposit insurance agency to a given bank will not necessarily fall if its minimum solvency ratio is raised of more than academic interest? If the results mean that confidence in solvency ratios is weakened, the case for buttressing or replacing them with other measures to strengthen stockholder or debtholder discipline must be correspondingly strengthened. Examples of such measures are, respectively, risk-adjusted insurance premia and a requirement for a certain fraction of assets to be backed by subordinated debt (Calomiris 1997).

The finding that, when risk is systemic, both intangible capital and level solvency constraints *are* necessarily well-behaved as regards bailout costs perhaps underlines the importance of the protection provided by intangible capital and, therefore, the

¹³ Example 2 is well-behaved with a level solvency constraint: if K is less than $3/5$ the bank does both projects, and otherwise it does neither.

potential severity of the adverse consequences that can follow its erosion through - for instance - changed market structure in banking¹⁴. A possible role for level solvency constraints, perhaps in conjunction with ratio constraints, is also implied. Nevertheless a vital difference between level and ratio constraints should be noted. For the long run, and if banks do not have market power, the assumptions used here and elsewhere in the literature to constrain equilibrium bank size are hard to defend: in circumstances where banks can increase scale without technological or market impediment, *level* solvency constraints may lose much of their significance.

¹⁴ Keeley[1990] finds a link from falling intangible capital to higher default risk for US banks over 1970-86.

Appendix

Proof of Proposition 4(a): To show that a perverse outcome is possible with a minimum capital constraint, consider the setup of Table A1. Note that the capital costs of each project are here immaterial, so are not specified. Doing both projects is never worthwhile, since returns in both states are then non-positive. It is easily seen that for $0 \leq K \leq 1/3$, project 1 is chosen; for $1/3 \leq K \leq 2/3$, project 2 is chosen, and for $2/3 \leq K$, the bank switches back to project 1 (or does nothing). The first switch point is the perverse one: default probability falls, but expected value falls and expected bailout cost rises - each by $1/9$.

Proof of Proposition 4(b): The proof that perfect rank correlation of returns implies that V cannot decline with K may be approached via two lemmas.

Perfect rank correlation is defined here to mean that we can rank the states of the world from best to worst, so that, as j increases for any project i , $y(i,j)$ is non-increasing.

Lemma 1. (a) For any K and X , the J states of the world will be partitioned into solvency states for $1 \leq j \leq j(X,K)$, say, and default states for $j > j(X,K)$. $j(X,K)$ is non-decreasing in K . (b) Let $X(K)$ denote the optimal portfolio and $J(K)$ the highest solvency state for a given K . $J(K)$ is non-decreasing in K .

Proof: (a) Since for all i , $y(i,j)$ is non-increasing in j , so is $Y(j)$ for any chosen portfolio. Since default is in states for which $Y(j) < K$, (a) is established. (b) This is a direct consequence of Proposition 3(b).

Lemma 2. $X(K)$ contains project i if and only if project i has non-negative expected return evaluated over the states $(1, \dots, J(K))$.

Proof: Only if. Suppose false, so that there is a project i in $X(K)$ with a negative expected value, $-v$, over those states. By monotonicity, it necessarily has a negative value for state $J(K)$. Therefore, if the project is deleted and capital left unchanged, the

number of solvency states must remain the same or increase. If it remains the same, the expected return to the bank rises by v ; if it increases, the expected return rises by at least v . This establishes the required contradiction. If. Suppose false, so that there is a project i excluded from $X(K)$ with positive expected value over the states up to $J(K)$. Adding the project may or may not change the number of solvency states. If it does not, R rises by v . If the number of solvency states is raised, then the project must have contributed a positive return in all (the expanded set of) solvency states, so must raise R . If the number of solvency states is reduced, then the extra loss in the new default states cannot exceed $(-y(i,j))$, so again R rises by at least v .

Now let portfolios X_0 and X_1 be optimal for capital values K_0 and K_1 , with $K_0 < K_1$. Lemmas 1 and 2 imply directly that X_0 and X_1 are related as follows: (a) All projects in X_1 are in X_0 ; (b) projects in X_0 but not X_1 have non-positive expected values. Proposition 4(b) follows at once.

TABLE A1

	NET RETURNS		NPV
	STATE A	STATE B	
	prob. = $1/3$	prob. = $2/3$	
Project 1	1	$-1/2$	0
Project 2	-1	$1/3$	$-1/9$

References

- Akerlof, G.A. and P. Romer. Looting: the economic underworld of bankruptcy for profit. *Brookings Papers on Economic Activity* **2** (1993) 1-73
- Allen, L. and A. Saunders. Forbearance and valuation of deposit insurance as a callable put, *Journal of Banking and Finance* **17** (1993) 629-643
- Bank for International Settlements 1999 *A new capital adequacy framework* Basel Committee Publications, 50
- Berlin, M., A. Saunders, and G.F. Udell. Deposit insurance reform: what are the issues and what needs to be fixed, *Journal of Banking and Finance* **15** (1991) 735-752
- Calomiris, C.W. 1997 *The postmodern bank safety net*, American Enterprise Institute
- Committee on Bank Regulations and Supervisory Practices 1988 *International Convergence of Capital Measurements and Standards* Basle, Bank for International Settlements
- Davis, E.P. 1992 *Debt, financial fragility, and systemic risk* Clarendon Press, Oxford
- Diamond, D.W. and P.H. Dybvig. Banking theory, deposit insurance, and bank regulation. *Journal of Business* **59** (1986) 55-68
- Furlong, F. and M. Keeley. Capital regulation and bank risk-taking: a note. *Journal of Banking and Finance* **13** (1989) 883-891
- Gennotte, G. and G. Pyle. Capital controls and bank risk. *Journal of Banking and Finance* **15** (1991) 805-824
- Hellwig, M. 1991 Banking, financial intermediation and corporate finance, in C.P. Mayer and A. Giovannini(eds), European financial integration, Cambridge University Press, 35-63
- Kahane, Y. Capital adequacy and the regulation of financial intermediaries. *Journal of Banking and Finance* **2** (1977) 207-217
- Keeley, M.C. Deposit insurance, risk, and market power in banking. *American Economic Review* **80** (1990) 1183-1199
- Keeley, M.C. and F.T. Furlong. A reexamination of mean-variance analysis of bank capital regulation. *Journal of Banking and Finance* **14** (1990) 69-84
- Koehn, M. and A.M. Santomero. Regulation of bank capital and portfolio risk, *Journal of Finance* **35** (1980) 1235-1250
- Merton, R.C. An analytic derivation of the cost of deposit insurance and loan guarantees. *Journal of Banking and Finance* **1** (1977) 3-11

Pennacchi, G. A reexamination of the over- (or under-) pricing of deposit insurance.

Journal of Money, Credit and Banking **19** (1987) 340-360

Richardson, J. and M. Stephenson. 2000 *Some aspects of regulatory capital*.

Occasional Paper 7, Financial Services Authority, London