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# IMPROVED CASCADE CONTROL STRUCTURE FOR ENHANCED PERFORMANCE

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**Abstract:** In conventional single feedback control, the corrective action for disturbances does not begin until the controlled variable deviates from the set point. In this case, a cascade control strategy can be used to improve the performance of a control system particularly in the presence of disturbances. In this paper, an improved cascade control structure and controller design based on Standard forms, which was initially given by authors, is suggested to improve the performance of cascade control. Examples are given to illustrate the use of the proposed method and its superiority over some existing design methods.

## I. INTRODUCTION

The standard feedback control loop sometimes does not provide a performance good enough for processes with long time delays and strong disturbances. Cascade control loops can be used and are a common feature in the process control industries for the control of temperature, flow and pressure loops.

Cascade control (CC), which was first introduced many years ago by Franks and Worley [1], is one of the strategies that can be used to improve the system performance particularly in the presence of disturbances. In conventional single feedback control, the corrective action for disturbances does not begin until the controlled variable deviates from the set point. A secondary measurement point and a secondary controller,  $G_{c2}$ , in cascade to the main controller,  $G_{c1}$ , as shown in Fig. 1, can be used to improve the response of the system to load changes.

A typical example is the natural draft furnace temperature control problem [2], shown in Fig. 2. When there is a change in hot oil temperature, which may occur due to a change in oil flow rate, the conventional single feedback control system, Fig. 2, will immediately take corrective action. However, if there is a disturbance in fuel gas flow no correction will be made until its effect reaches the temperature-measuring element. Thus, there is a considerable lag in correcting for a fuel gas flow change, which subsequently results in a sluggish response. With the cascade control strategy shown in Fig. 3, an improved performance can be achieved, since any change in the fuel gas flow is immediately detected by the flow-measuring element and the flow controller takes corrective action.

Recent contributions on the tuning of PID controllers in cascade loops include [3]-[5]. More recently, Lee *et al.* [6] suggested using Internal Model Controller (IMC) principles [7, 8] for tuning both the inner and outer loop controllers in a cascade control system.

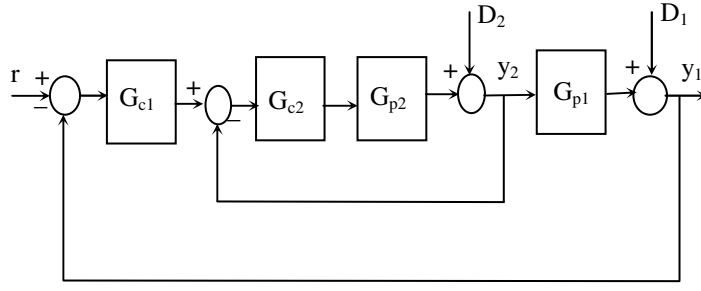


Fig. 1: Cascade Control System

A cascade control strategy can be used to achieve better disturbance rejections. However, if a long time delay exists in the outer loop the cascade control may not give satisfactory closed loop responses for set point changes. In this case, a Smith predictor scheme can be used for a satisfactory set point response. Therefore, Kaya [9] suggested using a Smith predictor configuration in the outer loop of a cascade control system to bring together the best merits of the cascade control and Smith predictor scheme. In this paper, a modified form of cascade control structure given in reference [9] is proposed. The modified form was first suggested by the authors [10]. In this modified form, the inner loop incorporates Internal Model Controller (IMC) principles and the outer loop the Smith predictor scheme. In addition, in the modified cascade control scheme [10] a PI-PD Smith predictor is used in the outer loop while in reference [9] the standard Smith predictor was used. Using IMC principles for the inner loop simplifies the design procedure. Using a PI-PD structure, which is proved to give better closed loop performances for process transfer functions with unstable poles or an integrator [11]-[13], and large time constants or complex poles [14], improves the performance of the closed loop. The outer loop PI-PD controllers' parameters are identified by the use of standard forms, which is a simple algebraic approach to controller design. Another advantage of the standard forms is that one can predict how good will be the performance of closed loop system. The inner loop controller is designed based on IMC principles, as stated above. Different than reference [10], where the standard forms only with ISTE criterion were given, results are also provided for ISE and  $IST^2E$  criteria. Simulation results

are extended for comparison with conventional cascade control configuration. Furthermore, simulations are carried out in deep to illustrate the value of proposed cascade control structure.

The paper is organized as follows: The next section gives a brief review of standard forms for a closed loop system with a zero to minimize the integral performance criteria, as it is used to design the outer loop controllers. Section 3 provides the design procedure for both the inner and outer loop controllers. Section 4 gives simulation results to illustrate the use of the proposed cascade control structure and design method. Conclusions are provided in section 5.

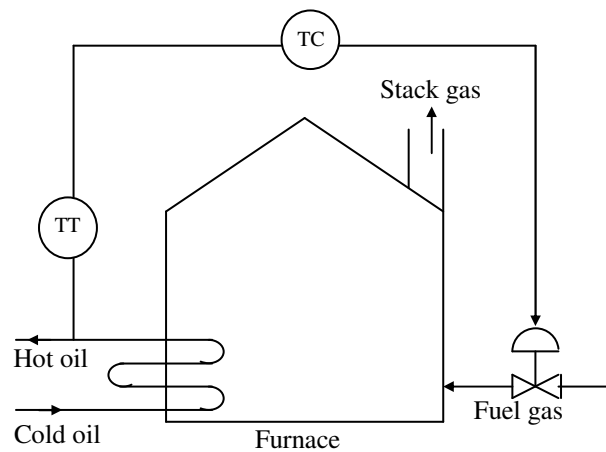


Fig. 2: The natural draft furnace temperature control with single feedback control

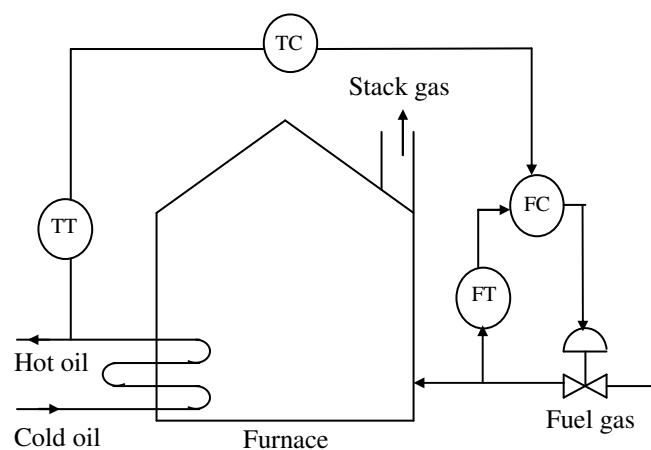


Fig. 3: The natural draft furnace temperature control with cascade feedback control

## II. STANDARD FORMS

The use of integral performance indices for control system design is well known. Many text books, such as [15]-[16], include short sections devoted to the procedure. When integral performance criteria were first suggested in the early 1950s, digital computers were in their infancy and evaluations could take a long computation time.

For linear systems, the ISE can be evaluated efficiently on digital computers using the s-domain approach with Åström's recursive algorithm [17]. Thus for

$$J_0 = \int_0^{\infty} e^2(t) dt \quad (1)$$

the s-domain solution is given by

$$J_0 = \frac{1}{2\pi j} \int_0^{\infty} E(s)E(-s)ds \quad (2)$$

where  $E(s)=B(s)/A(s)$ , and  $A(s)$  and  $B(s)$  are polynomials with real coefficients, given by

$$A(s) = a_0 s^m + a_1 s^{m-1} + \dots + a_{m-1} s + a_m$$

$$B(s) = b_1 s^{m-1} + \dots + b_{m-1} s + b_m$$

Criteria of the form  $J_n = \int_0^{\infty} [t^n e(t)]^2 dt$  can also be evaluated using this approach, since

$L[tf(t)] = (-d/ds)F(s)$ , where  $L$  denotes the Laplace transform and  $L[f(t)] = F(s)$ . Minimizing a control system using  $J_0$ , that is the ISE criterion, is well known to result in a response with relatively high overshoot for a step change. However, it is possible to decrease the overshoot by using a higher value of  $n$  and responses for  $n = 1$ . Typically, for  $n=0, 1, 2$  overshoots of 20-30%, around 5-10% and <5% can be expected, with possibly very small increases in settling time as  $n$  increases. Responses for

$n=1$ , that is the ISTE criterion, are often quite similar to those for the ITAE (integral of time weighted absolute error) criterion.

Another approach to optimization which has been little discussed for many years is the direct synthesis approach where the closed loop transfer function is synthesized to a standard form. Using this approach, it is possible to obtain the optimal parameters of a closed loop transfer function, which will provide a minimum value of the ISE. Tables of such transfer functions with poles only were given many years ago [18] but are of little use in design, because even with an all pole plant transfer function the addition of a typical controller produces a closed loop transfer function with a zero. Results with a single zero were also given so that the feedback loop would follow a ramp input with zero steady-state error but these expressions are not appropriate for step response design. For a closed loop transfer function with one zero it is easy to present results for these optimum transfer functions as the position of the zero varies and this is done in reference [11].

Consider an open loop plant transfer function with no zero,  $G(s)$ , and a controller with a zero,  $G_c(s)$ , then a closed loop transfer function,  $T_{1j} = G(s)G_c(s)/[1 + G(s)G_c(s)]$ , of the form

$$T_{1j} = \frac{c_1 s + 1}{s^j + d_{j-1} s^{j-1} + \dots + d_1 s + 1} \quad (3)$$

can be obtained, where the subscript '1' in  $T_{1j}$  indicates a zero in the numerator of the standard form and the subscript 'j' indicates the order of the denominator. Also, for a unit step set point, the error is obtained as

$$E_{1j} = \frac{s^{j-1} + d_{j-1} s^{j-2} + \dots + (d_1 - c_1)}{s^j + d_{j-1} s^{j-1} + \dots + d_1 s + 1} \quad (4)$$

Minimizing  $E_{1j}$  for different performance indices, namely the ISE, ISTE and  $IST^2E$ , the optimum values of the  $d$ 's as functions of  $c_1$  are shown in Fig. 4 for  $T_{13}(s)$  and in Fig. 5 for  $T_{14}(s)$ . Fig. 6



shows how  $J_0$ ,  $J_1$  and  $J_2$ , the minimum value for the ISE, ISTE and  $IST^2E$  criteria respectively, varies as  $c_1$  increases for  $T_{13}(s)$ . Similar results for  $T_{14}(s)$  are given in Fig. 7. Both figures illustrate that as  $c_1$  increases the step response of the closed loop improves. However, it is also seen from the figure that any further increase in  $c_1$  above the value of 4 or 5 has a negligible improvement in the response. Also, the step responses for the  $J_1$  criterion for a few different  $c_1$  values are shown in Fig. 8 for  $T_{13}(s)$ . It is seen that as  $c_1$  increases the step responses are faster. It should be noted that a similar result can be obtained for  $T_{14}(s)$  as well.

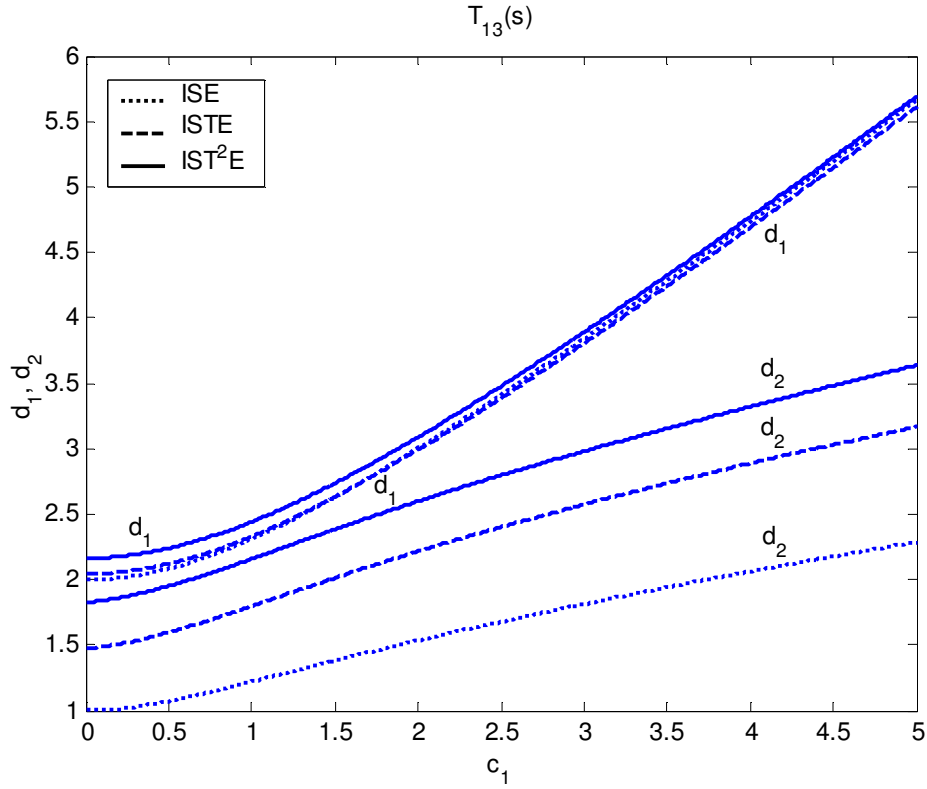


Fig. 4: Optimum values of  $d_1$  and  $d_2$  for varying  $c_1$  values.

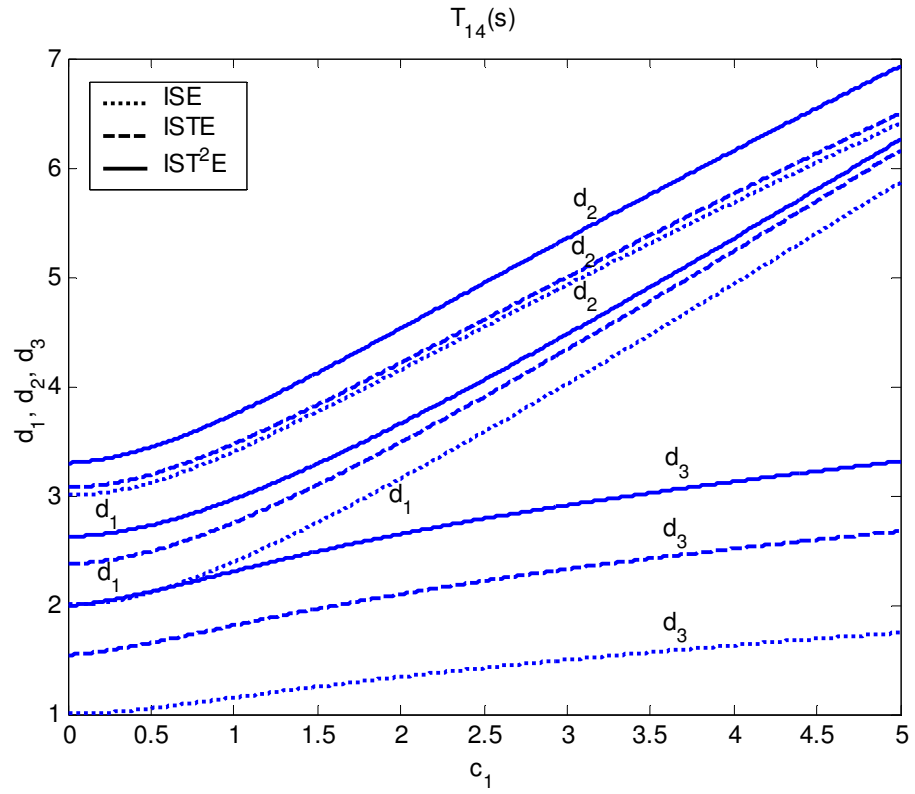


Fig. 5: Optimum values of  $d_1$ ,  $d_2$  and  $d_3$  for varying  $c_1$  values.

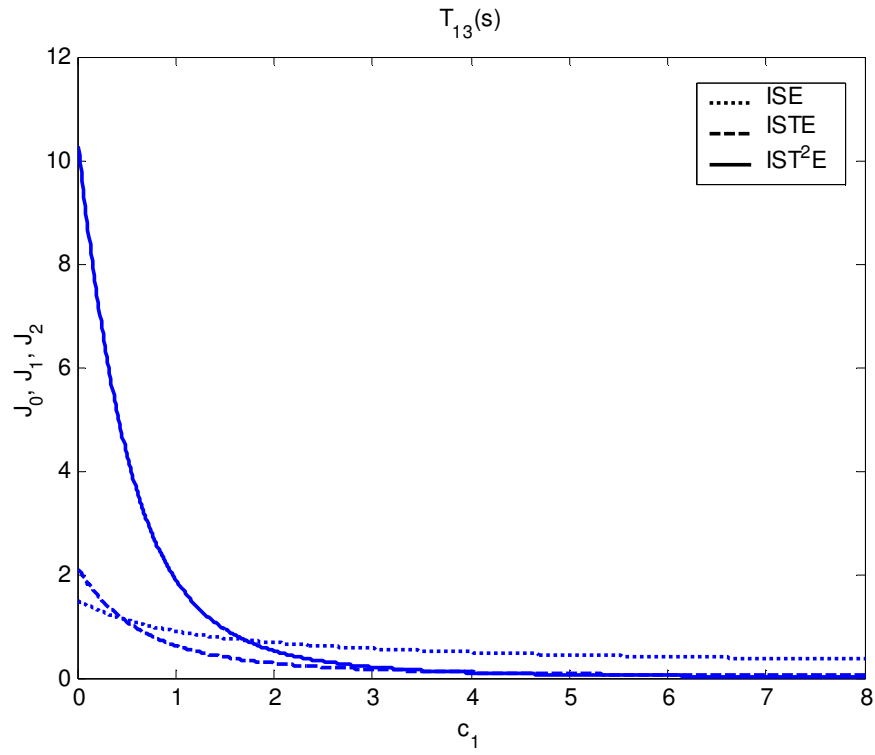


Fig. 6:  $J_0$ ,  $J_1$  and  $J_2$  integral values for  $T_{13}(s)$

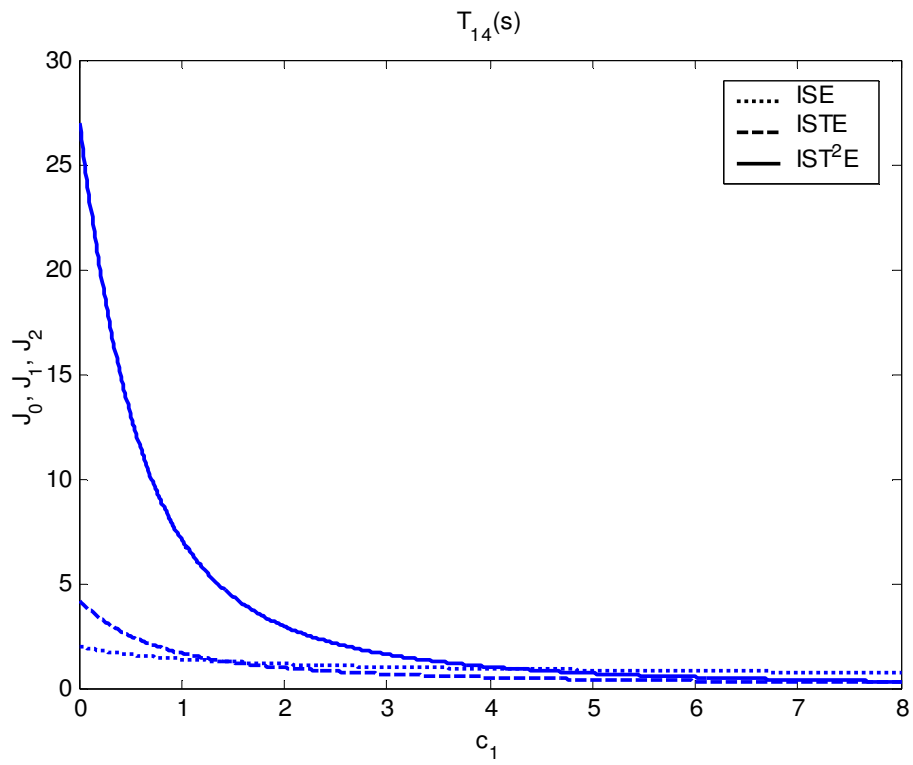


Fig. 7:  $J_0$ ,  $J_1$  and  $J_2$  integral values for  $T_{14}(s)$

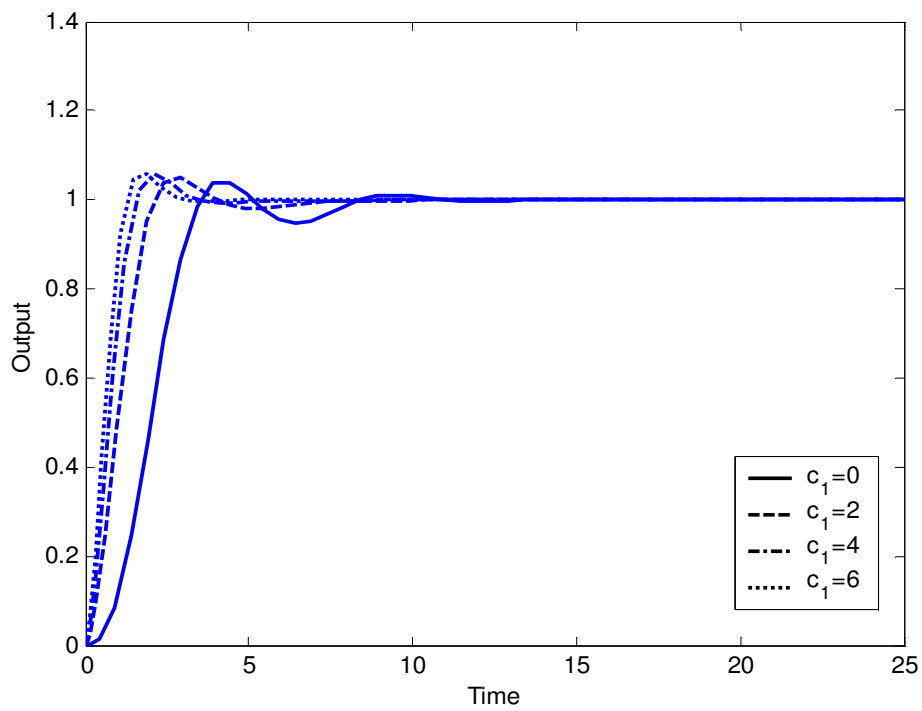


Fig. 8: Step responses for  $T_{13}(s)$  and  $J_1$  criterion

### III. THE NEW CASCADE CONTROL STRUCTURE AND DESIGN METHOD

The proposed cascade control structure is shown in Fig. 9.  $G_{c2}$  is used for stabilization of the inner loop while  $G_{c1}$  and  $G_{c3}$  are used for the outer loop stabilization.  $G_{p2m}$  and  $G_{pm}$  are the model transfer functions of the inner and outer loops, respectively. Assuming that the plant transfer functions are known, then two loops can be tuned simultaneously.

There are several issues of practical industrial application that must be addressed before carrying on finding tuning rules for both loops. First, as the integral action is used in the two forward loop controllers, namely  $G_{c1}$  and  $G_{c2}$ , a scheme to avoid integral anti-windup may be necessary. Especially, the inner loop control signal may saturate, which may also be the case for a conventional cascade control where the integral action is used in both loops, for instance, a PI-PI or PI-PID cascade control structure. Secondly, when the system is in the manual mode, the control algorithm will yield a control signal which may be different from the value specified manually. Hence, it is necessary to make sure that the two outputs match each other. This is called bumpless transfer, which must be considered in practice. Finally, tracking of the outer loop controllers when the inner loop controller is not in automatic mode with external set-point is another issue that must be considered. All of these issues related to the proposed structure can be solved as in conventional cascade control scheme for the inner loop with internal model control, because the output of  $G_{c2}$  is directly fed into the process  $G_{p2}$ , and the model  $G_{p2m}$  can be running with the same input value independent of the operating mode of  $G_{c2}$ . In many cases, the principle of internal model control can be used as a design tool to find parameters of controller  $G_{c2}$ , and the model  $G_{p2m}$  is not necessary as a separate function block for online control. However, the outer loop needs special attention: the addition of the signal coming from  $G_{c3}$  must be inside the controller  $G_{c1}$ , in front of the output limitation, to make sure that the sum of the outputs from  $G_{c1}$  and  $G_{c3}$  is limited correctly, and the anti-windup strategies of  $G_{c1}$  are triggered

as appropriate. If  $G_{c1}$  is in manual or tracking mode, the internal integrator inside  $G_{c1}$  must be set such that the sum of the outputs from  $G_{c1}$  and  $G_{c3}$  is tracking the desired value. In some distributed process control systems, this can be achieved by using a special input of the PID function block  $G_{c1}$  that is normally intended for an additional disturbance variable compensation. Detailed explanations to these issues can be found in the book given in reference [20], which is completely dedicated to PID controllers.

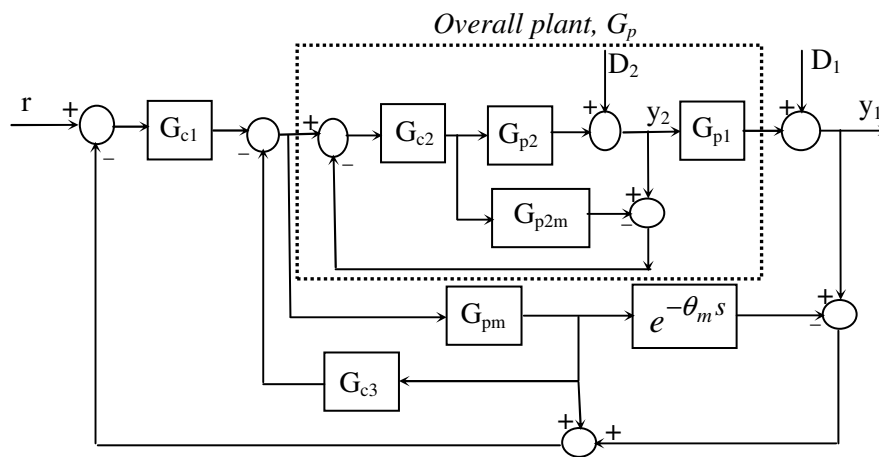


Fig. 9: Improved Cascade Control Structure

### 3.1 Designing inner loop controller ( $G_{c2}$ ):

As stated in the introduction, the inner loop controller is designed based on IMC principles [8]-[9]. The details of design procedure are not given here, since, one can easily obtain them from the abovementioned references.

The inner loop plant transfer function is assumed to be a FOPDT

$$G_{p2}(s) = \frac{K_2 e^{-\theta_1 s}}{T_2 s + 1} \quad (5)$$

Note that in real case the process can have higher order transfer functions. However, it is assumed that it can satisfactorily be approximated by the above FOPDT model transfer function. One of the modeling approaches existing in the literature, such as [19], can be used for this purpose.

It can easily be shown that the inner loop controller, using IMC principles, is given by

$$G_{c2}(s) = \frac{(T_2s + 1)}{K_2(\lambda s + 1)} \quad (6)$$

where,  $\lambda$  is the only tuning parameter to be found.

The inner loop should be faster than the outer loop; the bigger the speed difference, the better the performance of a cascade control system. That is, the smaller the values of  $\lambda$  the better the performance of the cascade control system. Hence, as a rule of thumb  $\lambda$  can be chosen equal to inner loop time delay. If a faster response is requested,  $\lambda$  can be chosen as half time delay of the inner loop, namely,  $\lambda = \theta_2 / 2$ , which is the value used throughout the paper.

### 3.2 Designing outer loop controllers ( $G_{c1}$ and $G_{c3}$ ):

It is easy to show that the inner closed loop transfer function is given by

$$G_{pi}(s) = \frac{e^{-\theta_2 s}}{\lambda s + 1} \quad (7)$$

Then, the overall plant transfer function for the outer loop is

$$G_p(s) = G_{pi}(s)G_{p1}(s) = G_{pm}(s)e^{-\theta_m s} \quad (8)$$

where,  $G_{pm}(s)$  is the delay free part of the overall plant transfer function and  $\theta_m = \theta_1 + \theta_2$ .

Since the Smith predictor scheme is used in the outer loop, it can easily be shown that the closed loop transfer function between  $y_1$  and  $r$ , assuming a perfect matching, that is  $G_p = G_{pm}(s)e^{-\theta_m s}$ , is given by

$$T(s) = \frac{G_{pm}(s)G_{c1}(s)e^{-\theta_m s}}{1 + G_{pm}(s)[G_{c1}(s) + G_{c3}(s)]} \quad (9)$$

Eqn. (9) reveals that the parameters of two controllers  $G_{c1}(s)$  and  $G_{c3}(s)$  can be determined using delay free part of the overall plant transfer function.

The outer loop controllers  $G_{c1}(s)$  and  $G_{c3}(s)$  are assumed to be ideal PI and PD controllers, respectively, which have the following forms

$$G_{c1}(s) = K_c \left(1 + \frac{1}{T_i s}\right) \quad (10)$$

$$G_{c3}(s) = K_f + T_f s \quad (11)$$

Next sections consider controller designs for two different cases.

Case 1 (Design for a FOPDT): It is assumed that the outer loop plant transfer function is stable and can be modeled by

$$G_{p1}(s) = \frac{K_1 e^{-\theta_2 s}}{T_1 s + 1} \quad (12)$$

Therefore, from eqn. (8), the overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1 + \theta_2)s}}{(\lambda s + 1)(T_1 s + 1)} \quad (13)$$

Eqn. (13) can be rearranged as

$$G_p(s) = \frac{k e^{-(\theta_1 + \theta_2)s}}{s^2 + as + b} \quad (14)$$

where,

$$k = \frac{K_1}{T_1 \lambda} \quad (15a)$$

$$a = \frac{1}{T_1} + \frac{1}{\lambda} \quad (15b)$$

$$b = \frac{1}{T_1 \lambda} \quad (15c)$$

Taking delay free part of eqn. (14) equal to  $G_{pm}(s)$  and using delay free part eqn. (9) gives the closed loop transfer function

$$T_{13}(s) = \frac{kK_c(T_i s + 1)}{T_i s^3 + (a + kT_f)T_i s^2 + (b + kK_f + kK_c)T_i s + kK_c} \quad (16)$$

Normalization of the eqn. (16), assuming

$$s_n = s \left( \frac{T_i}{kK_c} \right)^{1/3} = \frac{s}{\alpha} \quad (17)$$

which means the response of the system will be faster than the normalized response by a factor of  $\alpha$ , results in the standard closed loop transfer function

$$T_{13}(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (18)$$

where,

$$c_1 = \alpha T_i \quad (19a)$$

$$d_2 = (a + kT_f) / \alpha \quad (19b)$$

$$d_1 = (b + kK_f + kK_c) / \alpha^2 \quad (19c)$$

In principle  $\alpha$  can be selected by the choice of  $K_c$  and  $c_1$  by the choice of  $T_i$ . Based on the value of  $c_1$ , the coefficients  $d_2$  and  $d_1$  can be found from Fig. 4 and then the value of  $T_f$  and  $K_f$  can be computed from eqns. (19b) and (19c) respectively.

Note that for a selected  $T_i$ , choosing larger  $K_c$  values result in larger  $\alpha$  and  $c_1$  values. This implies faster closed loop system response. In practice,  $K_c$  will be constrained, possibly to limit the initial



value of the control effort, so that the choice of  $K_c$  and  $T_i$  may involve a trade off between the values chosen for  $\alpha$  and  $c_1$ .

Case 2 (Design for a SOPDT): In this case, it is assumed that the outer loop plant transfer function is stable and can be modeled by

$$G_{p1}(s) = \frac{K_1 e^{-\theta_1 s}}{(T_0 s + 1)(T_1 s + 1)} \quad (20)$$

Hence, the overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1 + \theta_2)s}}{(\lambda s + 1)(T_0 s + 1)(T_1 s + 1)} \quad (21)$$

Rearranging eqn. (21) gives

$$G_p(s) = \frac{k e^{-(\theta_1 + \theta_2)s}}{s^3 + a s^2 + b s + c} \quad (22)$$

where,

$$k = \frac{K_1}{T_0 T_1 \lambda} \quad (23a)$$

$$a = \frac{1}{T_0} + \frac{1}{T_1} + \frac{1}{\lambda} \quad (23b)$$

$$b = \frac{1}{T_0 \lambda} + \frac{1}{T_0 T_1} + \frac{1}{T_1 \lambda} \quad (23c)$$

$$c = \frac{1}{T_0 T_1 \lambda} \quad (23d)$$

Taking delay free part of eqn. (22) equal to  $G_{pm}(s)$  and using delay free part eqn. (9) results in closed loop transfer function

$$T_{14}(s) = \frac{k K_c (T_i s + 1)}{T_i s^4 + T_i a s^3 + (b + k T_f) T_i s^2 + (c + k K_f + k K_c) T_i s + k K_c} \quad (24)$$

Normalization of the eqn. (24), assuming

$$s_n = s \left( \frac{T_i}{kK_c} \right)^{1/4} = \frac{s}{\alpha} \quad (25)$$

results in

$$T_{14}(s_n) = \frac{c_1 s_n + 1}{s_n^4 + d_3 s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (26)$$

where,

$$c_1 = \alpha T_i \quad (27a)$$

$$d_3 = a / \alpha \quad (27b)$$

$$d_2 = (b + kT_f) / \alpha^2 \quad (27c)$$

$$d_1 = (c + kK_f + kK_c) / \alpha^3 \quad (27d)$$

In this case, the time scale  $\alpha$  and the four coefficients cannot be selected independently using the four controller parameters, namely,  $K_c$ ,  $T_i$ ,  $K_f$  and  $T_f$ . Achieving independency would require feedback of an additional state but often a satisfactory response, provided that  $a$  has a reasonable value, is possible. The compromise is between  $\alpha$ ,  $c_1$  and  $d_3$  as  $d_2$  and  $d_1$  can be chosen independently using  $K_f$  and  $T_f$ . As in case 1, the larger values of  $\alpha$  implies faster closed loop system for a fixed value of  $T_i$ . However, the choice of the value of  $\alpha$  depends on the value of  $a$  if a suitable value of  $d_3$  is to be obtained. Therefore, the larger the value of  $a$ , the faster the possible response satisfying the integral performance criteria which can be obtained. The procedure for calculating controller parameters can thus be summarized as: For a chosen value of  $\alpha$ ,  $d_3$  is determined from eqn. (27b). Once  $d_3$  is calculated,  $c_1$ ,  $d_2$  and  $d_1$  coefficients can be identified from Fig. 5 for a chosen criteria form corresponding to the calculated value of  $d_3$  to obtain an optimum overall closed loop performance. Note that  $a$  must be positive, as seen from Fig. 5, in order to use standard forms in this case.

#### IV. SIMULATION EXAMPLES

Two examples are given to illustrate the use of the proposed cascade control structure and design procedure. The first example assumes FOPDT plant transfer functions in both loops. The second example assumes a FOPDT plant transfer function in the inner loop and a SOPDT plant transfer function with poorly located poles in the outer loop.

**Example 1:** Here, the case studied by Lee *et al.* [6] is considered, where  $G_{p1}(s) = \frac{e^{-10s}}{100s+1}$ ,  $G_{p2}(s) = \frac{2e^{-2s}}{20s+1}$ . Taking  $\lambda=1$ , which is the half inner loop time delay, gives  $G_{c2}(s) = (20s+1)/(2s+2)$ . The resulting overall plant transfer function is given by eqn. (13). Hence, using the above  $\lambda$  value in eqns. (15a)-(15c), results in  $k=0.01$ ,  $a=1.01$  and  $b=0.01$ . Limiting  $K_c$  to 0.5 and choosing  $T_i=0.25$  gives  $\alpha=0.27$  and  $c_1=0.068$ . The standard form  $T_{13}(s)$  to minimize  $J_2$  for  $c_1=0.068$  has  $d_2=1.84$  and  $d_1=2.16$ , which gives  $T_f=-51.59$  and  $K_f=14.25$ . Alternatively, choosing  $K_c=1.0$  and keeping  $T_i$  as before results in  $T_f=-38.12$  and  $K_f=23.24$  for  $J_2$ . Similarly, limiting  $K_c=1.0$  gives  $T_f=-21.58$  and  $K_f=37.20$  for  $J_2$ . Responses of the proposed design method with calculated controllers' parameters for  $y_1$  and  $y_2$  are shown in Fig. 10 for a unit magnitude of set-point change and disturbance  $D_2$  introduced at  $t=70$  s. As expected, increasing  $K_c$  results in slightly faster output responses  $y_1$ . On the other hand, increasing  $K_c$  causes higher overshoots in the inner loop responses  $y_2$ . The disturbance rejection capabilities of all criteria are almost the same.

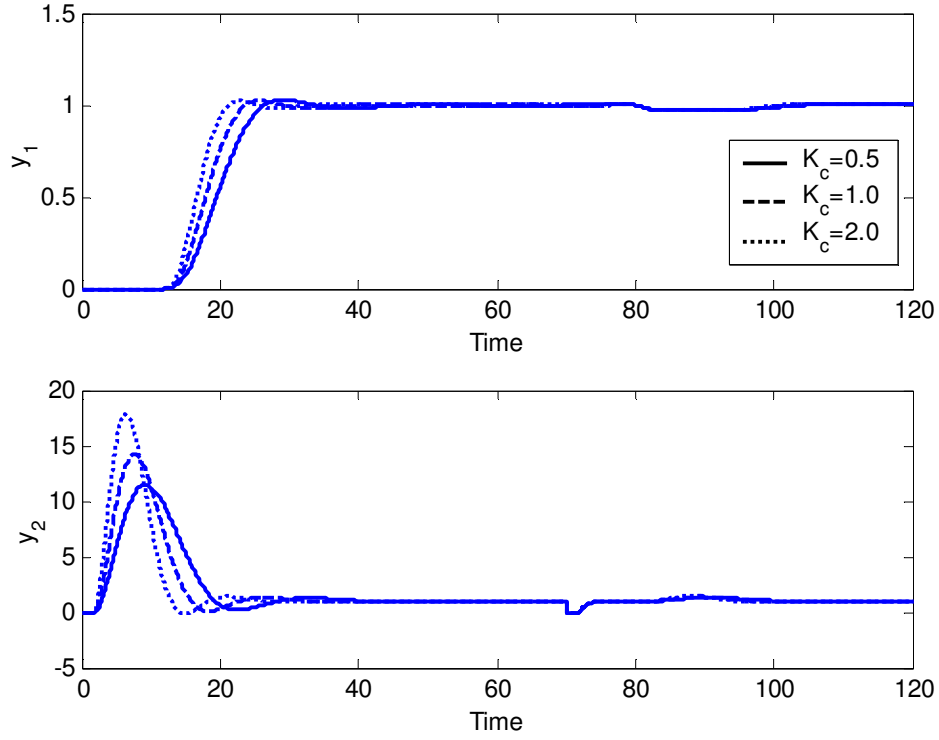


Fig. 10: Responses for the proposed design method to a unity step set point change and disturbance

$D_2$  for example 1

Fig. 11 compares the performance of the closed loop system for the proposed design method with the design method of Lee *et al.* [6], where the IMC principles are used to design both the inner and outer loop controllers, and the conventional cascade control. For the proposed method controller parameters corresponding to  $K_c = 0.5$  were used. Lee *et al.* [6] suggested controller parameters of  $K_{pi} = 3.44$ ,  $T_{ii} = 20.66$  and  $T_{di} = 0.64$  for the inner loop and  $K_{po} = 5.83$ ,  $T_{io} = 105.00$  and  $T_{do} = 4.80$  for the outer loop. For the traditional cascade control, a P only controller for the inner loop and a PI controller for the outer loop were used. The well known Ziegler- Nichols tuning method [21] was used to find the controller parameters, which were found to be  $K_{pi} = 3.35$  for the inner loop and  $K_{po} = 5.57$ ,  $T_{io} = 40.38$  for the outer loop. The traditional cascade control gives the worst result in the sense of

maximum overshoot and settling time. The proposed structure when compared to design method of Lee *et al.* [6] gives a faster response. Although, both design methods gives similar overshoots, the design method of Lee *et al.* [6] is very sluggish and do not settle down for a very long time period.

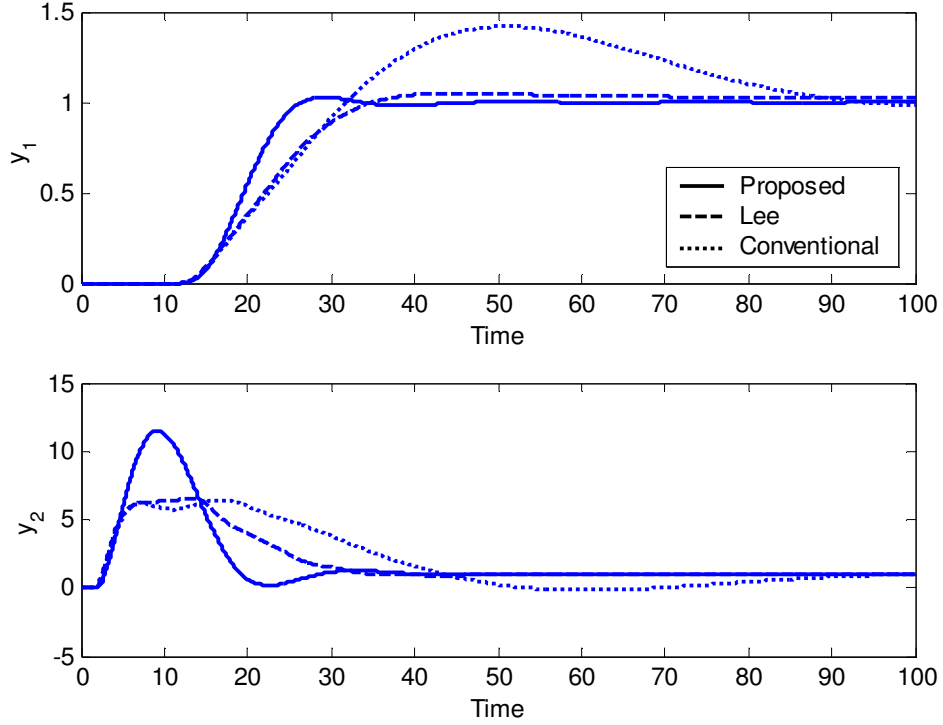


Fig. 11: Responses to a unity step set point change for example 1

To illustrate the robustness to parameter variations, a  $\pm 10\%$  change in the outer loop time delay is assumed, as this is the most deteriorative on the system performance, and results for this case are given in Fig. 12. Disturbance rejection capabilities of three structures and controller design methods are given in Fig. 13 for  $D_1$  and  $D_2$ . In both cases, the disturbance magnitudes were assumed to be unity. Control signals for all methods are compared in Fig. 14.

In practice, signals may contain measurement noises. Responses for band-limited white noise with maximum power of 0.02 are given in Fig. 15. Among all methods, the conventional method is the most insensitive to noises, which is understandable as it does not involve the derivative term in the

controllers. However, if a conventional cascade control scheme with a PI or PID controller in the inner loop and a PID controller in the outer loop is used, this insensitivity to noise may not be the case. It must be noted that for the method of Lee *et al.* [6], both the inner and outer loop PID controllers must be used with approximate derivative terms. Otherwise, completely unacceptable results are obtained. On the other hand, the proposed method gives quite moderate responses.

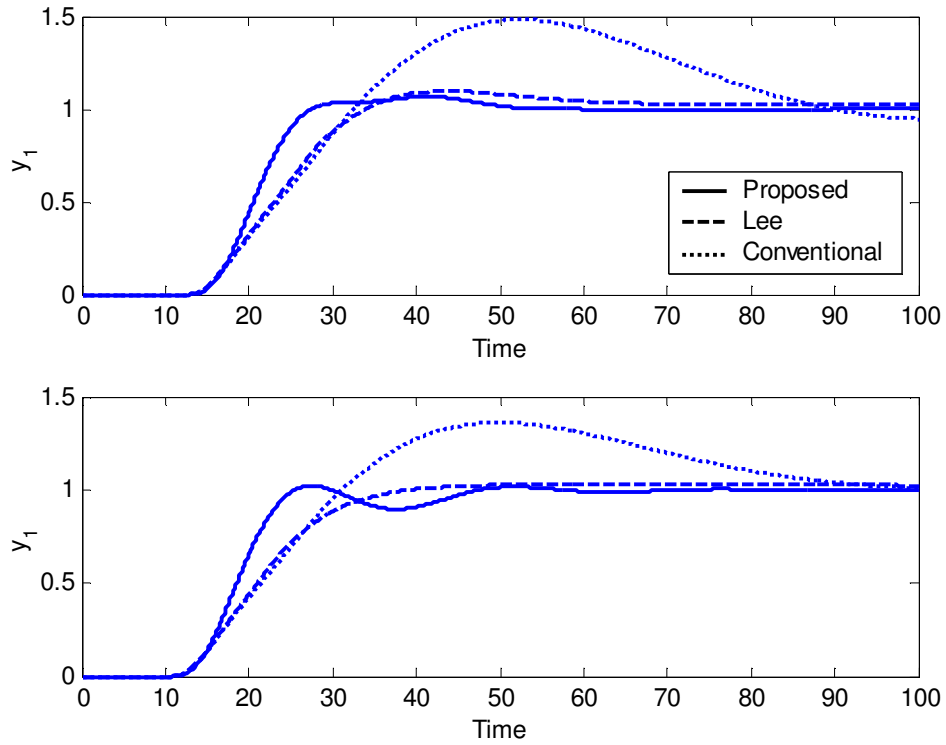


Fig. 12: Responses for example 1 with assumed +10% change (upper) and -10% change (lower) in the outer loop time delay

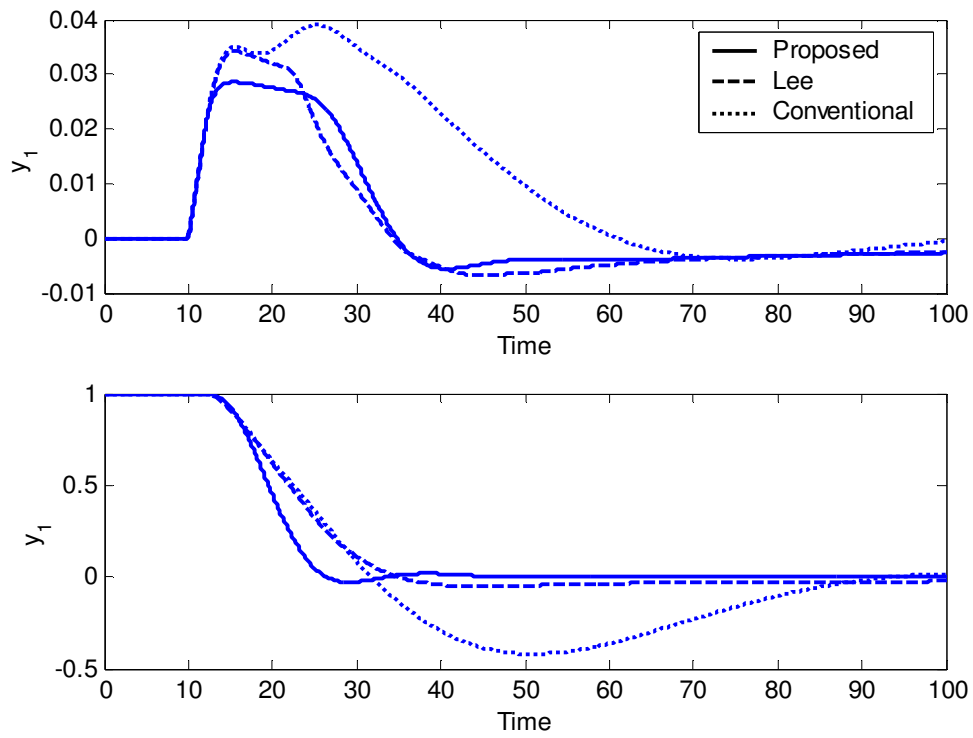


Fig. 13: Responses to disturbance  $D_2$  (upper) and  $D_1$  (lower) with unity magnitude for example 1

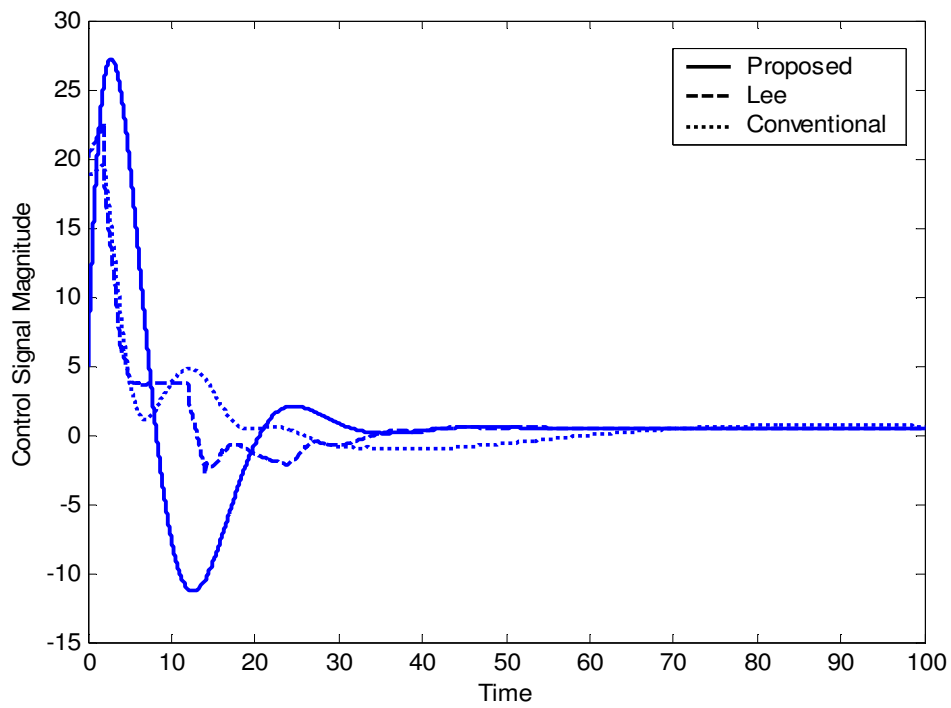


Fig. 14: Control signal magnitudes for example 1

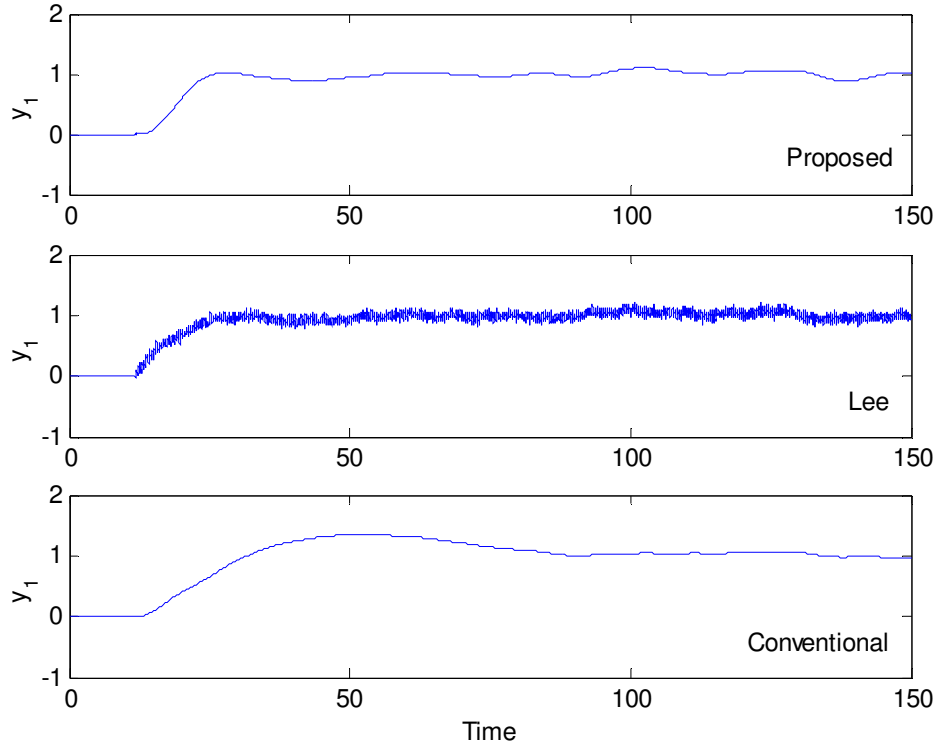


Fig. 15: Responses under measurement noises for example 1

**Example 2:** The following plant transfer functions  $G_{p1}(s) = \frac{e^{-4s}}{s^2 + 0.2s + 1}$ ,  $G_{p2}(s) = \frac{e^{-2s}}{s + 0.1}$  are considered. Note that the process transfer function in the outer loop has complex poles. Taking  $\lambda$  equal to the half inner loop time delay, i.e.  $\lambda=1$ , gives  $G_{c2}(s) = (s + 0.1)/(s + 1)$ . Following the procedure given in section 3.2,  $k = 1.0$ ,  $a = 1.2$ ,  $b = 1.2$  and  $c = 1.0$ . In order to obtain a suitable value of  $d_3$  to give a standard form for  $J_2$ , as shown in Fig. 5, requires  $\alpha < 0.6$ . Selecting  $\alpha = 0.4$ , gives  $d_3 = 3.00$  from eqn. (27b). The standard form from Fig. 5 requires  $c_1 = 3.391$ ,  $d_2 = 5.67$  and  $d_1 = 4.81$ . These values can be obtained with  $T_i = 8.48$  from eqn. (27a),  $K_c = 0.22$  from eqn. (25),  $T_f = -0.29$  from eqn. (27c) and  $K_f = -0.91$  from eqn. (27d). With these calculated controller parameters the performance of the proposed controller design method, together with design method of Lee et al. [6] and the conventional cascade control, are shown in Fig. 16 for a unit magnitude of set-



point change. The method proposed by Lee et al. [6] have controller parameters of  $K_{pi} = 0.36$ ,  $T_{ii} = 10.67$  and  $T_{di} = 0.63$  for the inner loop and  $K_{po} = 0.36$ ,  $T_{io} = 3.20$  and  $T_{do} = 1.13$  for the outer loop. The conventional cascade control have controller parameters of  $K_{pi} = 0.35$  for the inner loop and  $K_{po} = 0.34$ ,  $T_{io} = 12.50$  for the outer loop, which were found from Ziegler-Nichols tuning rules. The proposed design method clearly gives better performance than the other two methods in the sense of overshoot and settling time. Responses for all structures to disturbance  $D_1$  and  $D_2$ , with magnitude of 1, are shown in Fig. 17. Fig. 18 illustrates the control effort for all design methods. Performances of the closed loop system for band-limited noise with maximum amplitude of 0.02 are illustrated in Fig. 19. Again, for the method of Lee *et al.* [6], PID controllers with approximate derivative term were used in the inner and outer loop of the cascade control system. In this case, the proposed cascade control scheme is the most insensitive to noise among the three control structures.

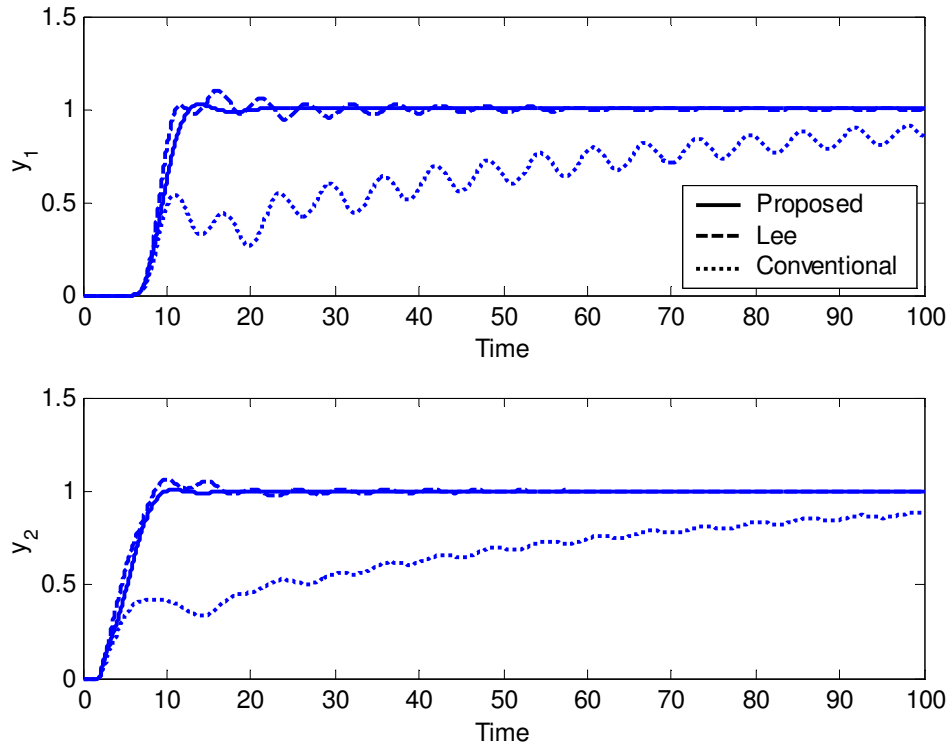


Fig. 16: Responses to a unity step set point change for example 2

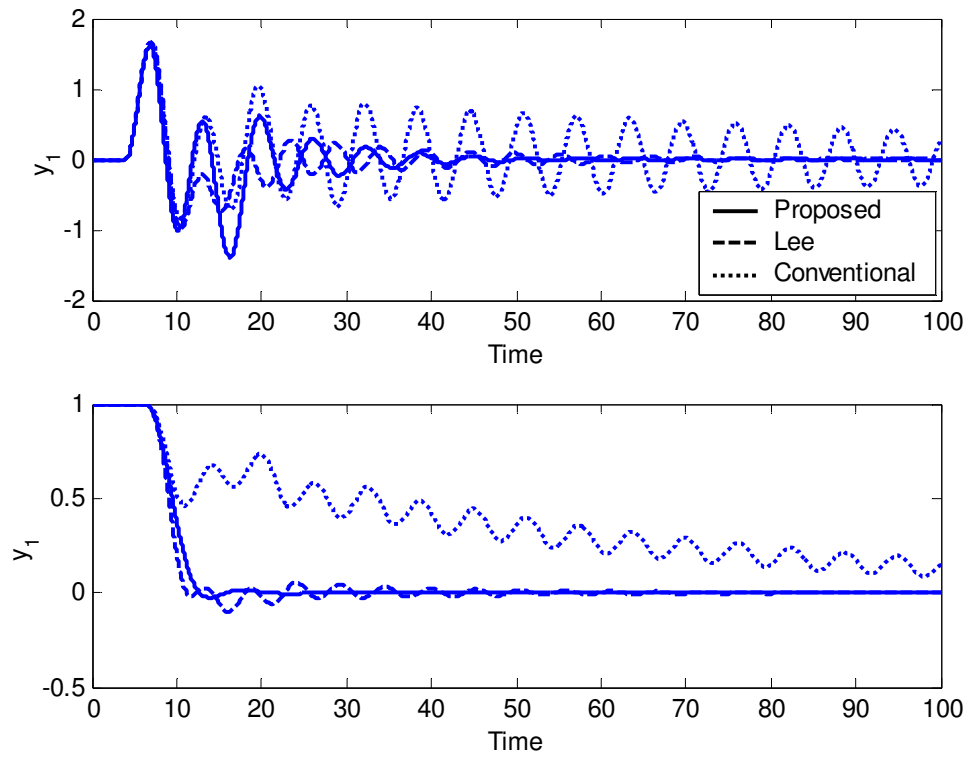


Fig.17: Responses to disturbance  $D_2$  (upper) and  $D_1$  (lower) with unity magnitude for example 2.

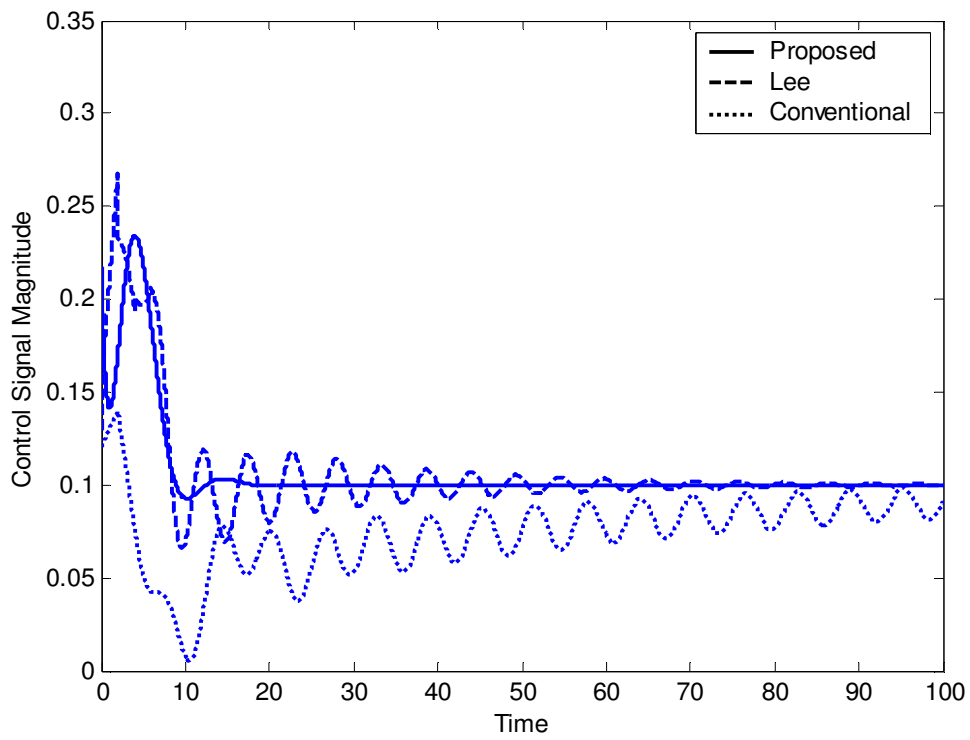


Fig.18: Control signal magnitudes for example 2.

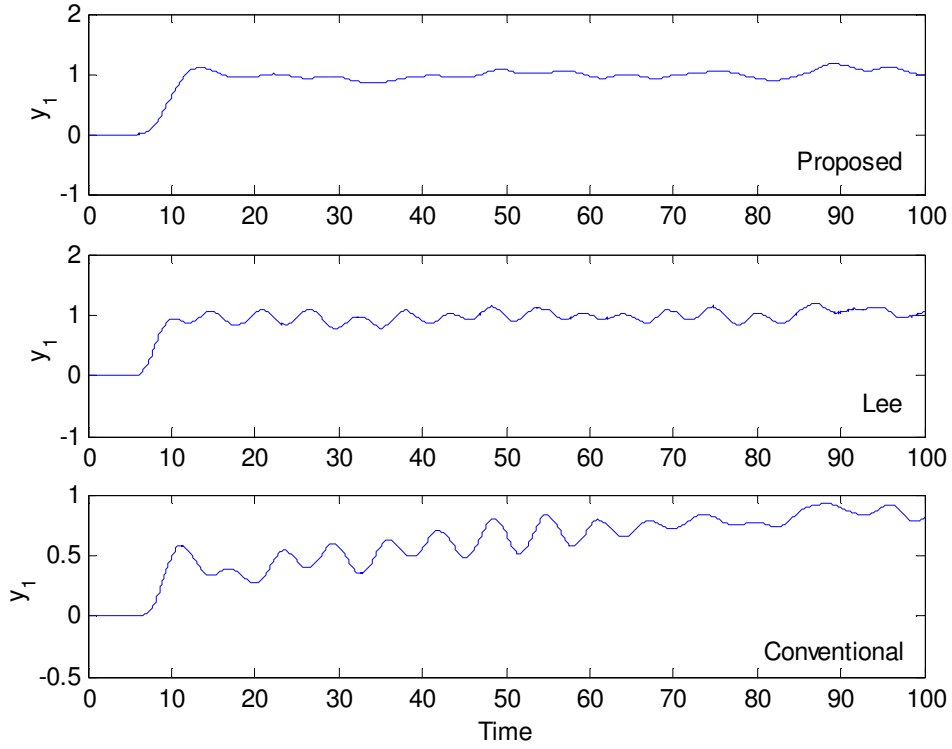


Fig. 19: Responses under measurement noises for example 2

## V. CONCLUSIONS

In this paper, an improved cascade control structure has been introduced. A PI-PD Smith predictor scheme is used in the outer loop, and internal model control in the inner loop of the cascade control. This has two advantages: First, the best merits of the cascade control and a Smith predictor scheme were combined in one structure. Second, the use of PI-PD controller accomplished to obtain improved performance when the process has large time constants or poorly located poles, i.e. lightly damped. Several procedures for obtaining the parameters of the PI-PD controllers are possible, but one of the simplest approaches is to employ standard forms as this enables the design to be completed using simple algebra.

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