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The simplest curvaton model

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We analyze the simplest possible realization of the curvaton scenario, where a nearly scale-invariant spectrum of adiabatic perturbations is generated by conversion of an isocurvature perturbation generated during inflation, rather than the usual inflationary mechanism. We explicitly evaluate all the constraints on the model, under both the assumptions of prompt and delayed reheating, and outline the viable parameter space.

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I. INTRODUCTION

There has recently been renewed interest in an alternative inflationary mechanism for generating an approximately scale-invariant adiabatic density perturbation spectrum to the usual one. Rather than immediately generating a curvature perturbation via perturbations in the inflaton field [1], instead the mechanism relies on isocurvature perturbations in another scalar field whose energy density is subdominant during inflation. After inflation ends this second scalar field comes to contribute significantly to the energy density, at which point the isocurvature perturbation converts to adiabatic even on superhorizon scales. Subsequent complete decay of this second field guarantees purely adiabatic perturbations, though variants on this scenario can leave a residual isocurvature component too. This mechanism of conversion from isocurvature to adiabatic perturbations was first discussed long ago by Mollerach [2], was briefly mentioned by Linde and Mukhanov [3] in a paper primarily focussing on scenarios for nongaussian isocurvature perturbations, and more recently received renewed attention in Refs. [4, 5, 6]. Amongst these, Lyth and Wands [5] considered the scenario in the broadest context, and named the second scalar field the *curvaton*.

While Lyth and Wands described the development of the perturbations in considerable detail, they sought to keep their discussion as model-independent as possible and did not discuss a specific inflationary scenario. In this paper we give a specific realization of the curvaton scenario and to evaluate all the constraints on model building that need to be satisfied for a successful scenario.

II. THE SIMPLEST CURVATON MODEL

While the general curvaton scenario allows perturbation generation featuring possible isocurvature components and possible nongaussianity, our aim here is to construct a simple curvaton model which creates a nearly scale-invariant spectrum of gaussian and purely adiabatic perturbations. The simplest possible curvaton model fea-

tures two massive non-interacting scalar fields, giving a potential

$$V(\phi, \sigma) = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2, \quad (1)$$

where we indicate the inflaton by ϕ and the curvaton by σ . The scenario requires that the curvaton energy density contributes negligibly during inflation (in particular the final stages). Since generically the curvaton will end up being the lighter of the two fields, it turns out that the curvaton must be close to its minimum, in order to prevent it driving a second period of inflation after the ϕ -driven inflation is complete. Under these constraints, σ will remain constant to a good approximation during the later stages of inflation, and we indicate this constant value by σ_* . This initial condition is not fixed by the theory, but rather represents an additional free parameter to be fixed by observations.¹ The curvaton's subdominance requires $m^2\sigma_*^2 \ll M^2\phi^2$ during inflation (where we will have $\phi \gtrsim m_{\text{Pl}}$ where m_{Pl} is the Planck mass).

The early stages of inflation set the global mean of σ_* in our observable Universe and arrange its classical homogeneity, and it then receives perturbations via the usual quantum mechanism, with the typical perturbation accrued in a Hubble time being $\delta\sigma \simeq H/2\pi$. In order for the eventual curvature perturbation to be gaussian, this perturbation must be small compared to the mean value of the field, and so we require $\sigma_*^2 \gg H^2/4\pi^2$. For these quantum perturbations to be the dominant influence on the curvaton, it must be effectively massless during inflation which requires $m^2 \ll H^2 \simeq 4\pi M^2\phi^2/3m_{\text{Pl}}^2$.

Once inflation is over and the inflaton energy density converted to radiation, the σ field will continue to remain constant while its mass is negligible compared to the Hubble parameter. Once $m^2 \simeq H^2$, it will begin to oscillate about the minimum of its potential, its energy

¹ Under this condition the oscillation mechanism of Ref. [7] is highly suppressed, and no mixture of correlated adiabatic and isocurvature perturbations will be relevant at the end of inflation.

density decaying at an average rate of $\rho_\sigma \propto 1/a^3$ (the Universe will have to still be radiation dominated by this stage, as otherwise the domination of σ will initiate a new period of inflation).

The final stage is the decay of the curvaton, which in this paper we will assume is a complete decay into conventional matter which thermalizes with the existing radiation. The decay occurs on a timescale Γ_σ , which is a further free parameter of the scenario. The most conservative constraint on the decay rate is that conventional radiation domination had better be in place by nucleosynthesis, though baryogenesis scenarios are likely to require a much earlier decay. Decay will happen when $\Gamma_\sigma \simeq H$, so this requires $\Gamma_\sigma > H_{\text{nuc}} \simeq 10^{-40} m_{\text{Pl}}$. The time until decay sets the magnitude of the curvature perturbation generated, because it determines what fraction of the mean energy density comes from the curvaton when it decays. We require to match the COBE normalization; this sets a minimum requirement on the size of the curvaton fluctuations because they must be large enough to generate the required perturbations in the limit where the curvaton field is completely dominant when it decays. The inflaton will also generate a curvature perturbation at some level, and while a mixed perturbation scenario is permitted we will only consider here the case where the inflaton-generated perturbation is negligible, which requires $M \ll 10^{-6} m_{\text{Pl}}$.

III. MODEL CONSTRAINTS

A. The case of prompt reheating

In this subsection, we shall assume that after inflation ends reheating occurs promptly, with the inflaton decaying into radiation. In that case, no further parameters are necessary to specify the scenario; we have four parameters which are the two masses m and M , the curvaton value during inflation σ_* , and the curvaton decay constant Γ_σ .

In a quadratic potential, inflation ends by violation of slow-roll at $\phi_{\text{end}} \simeq m_{\text{Pl}}/\sqrt{4\pi}$, corresponding to a Hubble parameter $H_{\text{end}}^2 = M^2/3$. During the subsequent radiation-dominated era $H^2 = H_{\text{end}}^2 a_{\text{end}}^4/a^4$ where a is the scale factor.

The next event to take place is for the curvaton to become effectively massive, $m^2 = H^2$. This happens when

$$\left(\frac{a_{\text{mass}}}{a_{\text{end}}}\right)^4 = \frac{M^2}{3m^2}. \quad (2)$$

In order to prevent a period of curvaton-driven inflation, the Universe must still be radiation dominated at that point, which implies a significant constraint

$$\sigma_*^2 \ll \frac{3}{4\pi} m_{\text{Pl}}^2. \quad (3)$$

This is a substantial restriction amongst all the possible values that σ might have taken (most of which would

result in a long epoch of σ -driven inflation after the ϕ field has reached its minimum).

The most important constraint on the parameters is the requirement of reproducing the observed perturbation amplitude. Denoting the ratio of the curvaton energy density to that of radiation by $r \equiv \rho_\sigma/\rho_{\text{rad}}$, in the limit where $r < 1$ (i.e. the curvaton decays during radiation domination), Lyth and Wands [5] demonstrated that the spectrum of the Bardeen parameter \mathcal{P}_ζ , whose observed value is about 2×10^{-9} , is given by

$$\mathcal{P}_\zeta \simeq \frac{r_{\text{decay}}^2}{16} \frac{H_*^2}{\pi^2 \sigma_*^2}, \quad (4)$$

where

$$H_*^2 \simeq \frac{100}{3} M^2 \quad (5)$$

is the Hubble parameter when observable perturbations were generated, around 50 e -foldings before the end of inflation, and

$$r = \frac{\rho_\sigma^{\text{end}}}{\rho_{\text{rad}}^{\text{end}}} \left(\frac{a_{\text{mass}}}{a_{\text{end}}}\right)^4 \frac{a}{a_{\text{mass}}} = \frac{4\pi}{3} \frac{\sigma_*^2}{m_{\text{Pl}}^2} \frac{a}{a_{\text{mass}}}. \quad (6)$$

Continuing to presume that the Universe is still radiation dominated at decay, this will be at $\Gamma_\sigma^2 = H^2 = H_{\text{end}}^2 a_{\text{end}}^4/a^4$, giving

$$r_{\text{decay}} = \frac{4\pi}{3} \frac{\sigma_*^2}{m_{\text{Pl}}^2} \sqrt{\frac{m}{\Gamma_\sigma}}, \quad (7)$$

and hence

$$\mathcal{P}_\zeta \simeq 4 \frac{m M^2 \sigma_*^2}{\Gamma_\sigma m_{\text{Pl}}^4} \quad (\text{for } r_{\text{decay}} < 1). \quad (8)$$

We find that achieving the correct perturbation amplitude, in combination with other constraints, excludes all the regions of parameter space where the curvaton decays while still effectively massless.

In the opposite regime, where r_{decay} exceeds one and so decay occurs after curvaton domination, this formula no longer holds and instead the perturbation produced becomes independent of Γ_σ , being [5]

$$\mathcal{P}_\zeta \simeq \frac{1}{9} \frac{H_*^2}{\pi^2 \sigma_*^2} \simeq \frac{M^2}{4\sigma_*^2} \quad (\text{for } r_{\text{decay}} > 1). \quad (9)$$

The two expressions agree at the transition $r_{\text{decay}} \sim 1$. Note that if this last expression is normalized as quoted above, the gaussianity condition $\sigma_*^2 \gg H_*^2/4\pi^2 \simeq M^2$ is automatically satisfied.

With four parameters to vary and only one equality, Eq. (8) or (9), imposed upon them, we expect quite a bit of freedom in choosing suitable parameters. However the set of inequalities the parameters must satisfy is a large one, and it turns out that viable parameter space is quite restricted. For definiteness, we set thresholds that the inflaton perturbation must be no more than ten percent of

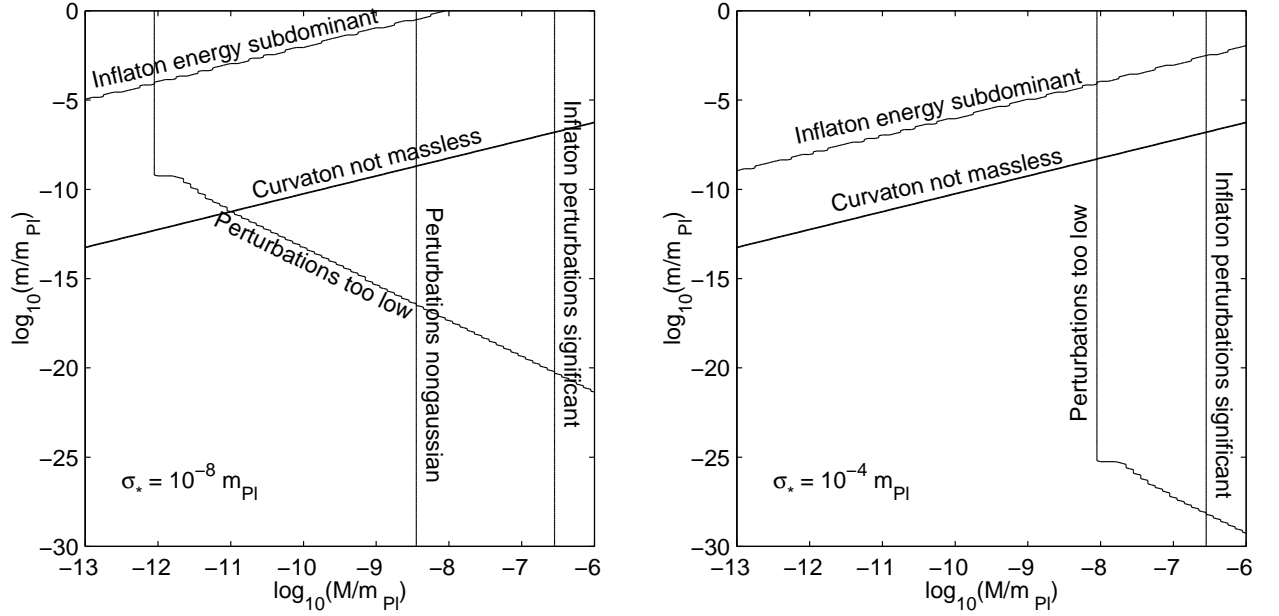


FIG. 1: Allowed regions of parameter space for $\sigma_* = 10^{-8} m_{\text{Pl}}$ (left panel) and $\sigma_* = 10^{-4} m_{\text{Pl}}$ (right panel). In all cases, the constraint lines are identified with a label on the outside of the allowed region.

the curvaton perturbation (i.e. $M < 3 \times 10^{-7} m_{\text{Pl}}$) and that the gaussianity condition $\sigma_*^2 \gg H_*^2/4\pi^2$ on observable scales be satisfied by an order of magnitude.

We proceed by considering triplets of values (m, M, σ_*) . For such a triplet, we fix Γ_σ using the power spectrum normalization. In some areas of parameter space this cannot be achieved, either because the curvaton perturbations are too small even if the curvaton energy density becomes dominant, or because one would violate $\Gamma_\sigma \gtrsim 10^{-40} m_{\text{Pl}}$ as required by nucleosynthesis. Otherwise, we then test whether the many other requirements to build a successful model are achieved, namely we must guarantee that the inflaton dominates during inflation, that the curvature perturbation from the inflaton is negligible, that the curvaton is effectively massless during inflation and that the curvature perturbations resulting from the curvaton are gaussian, and that the curvaton energy density is still subdominant when it begins oscillating. This set of constraints slices off regions of the parameter space, leaving the region in which viable models can be constructed.

Fig. 1 shows the allowed regions for two choices of σ_* , with the main constraints plotted. The perturbation amplitude constraint has two branches; the vertical part indicates that the models cannot reach the required perturbation amplitude even once the curvaton becomes fully dominant,² while the lower part of the curve indicates

that the perturbations have not grown sufficiently by nucleosynthesis. For low σ_* , the nongaussianity constraint sweeps leftwards across the allowed region and there are no viable models once $\sigma_* \lesssim 10^{-10} m_{\text{Pl}}$. For high σ_* it becomes impossible to generate sufficient perturbations, cutting off parameter space above $\sigma_* \simeq 3 \times 10^{-3} m_{\text{Pl}}$. There is however a significant parameter space of viable models satisfying all our requirements.

For the special case of the curvaton decaying when its energy density is dominating over radiation, an analytical analysis of the parameter space is possible. In this case Γ_σ no longer enters into the relevant expression for \mathcal{P}_ζ , Eq. (9). Thus, using the power spectrum normalization to fix the value of σ_* in terms of M as $\sigma_*^2 = 1.2 \times 10^8 M^2$, the relevant constraints reduce to

$$M < 3 \times 10^{-7} m_{\text{Pl}}, \quad m^2 \ll \frac{M^2}{3} \quad (10)$$

and

$$H_{\text{nucl}} \simeq 10^{-40} m_{\text{Pl}} < \Gamma_\sigma < \left(\frac{4\pi}{3}\right)^2 m \left(\frac{\sigma_*}{m_{\text{Pl}}}\right)^4. \quad (11)$$

The first two conditions require that the inflaton-generated perturbations be subdominant and that the curvaton be massless during inflation respectively, and are the same as before. If they are satisfied, the remaining constraints automatically follow. The condition on Γ_σ derives from requiring $r_{\text{decay}} > 1$, where in this case

$$r_{\text{decay}} = \left(\frac{4\pi}{3}\right)^{4/3} \left(\frac{\sigma_*}{m_{\text{Pl}}}\right)^{8/3} \left(\frac{m}{\Gamma_\sigma}\right)^{2/3}, \quad (12)$$

as a consequence of the σ decay after curvaton domination.

² On the vertical line itself lie models where the curvaton can dominate at decay; these are best analyzed separately which we do in the following paragraphs.

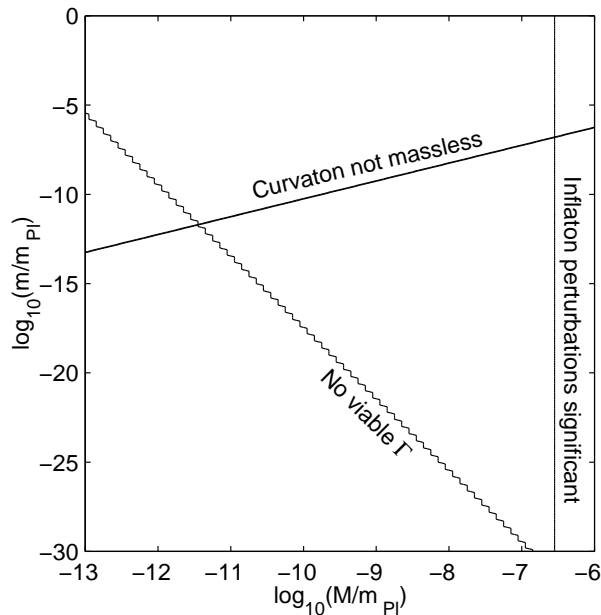


FIG. 2: The parameter space in the case of the curvaton dominating before decay. Models within the triangular region are viable; they have $\sigma_*^2 = 1.2 \times 10^8 M^2$, and Γ_σ may take on any value in the range given by Eq. (11).

The formulae Eqs. (10) and (11) give an allowed region in the m - M plane shown in Fig. 2. At each point within the allowed region there is a range of permitted Γ_σ indicated by Eq. (11) which gives the correct density perturbation normalization. These regions correspond to the vertical part of the perturbation amplitude constraint in Fig. 1 for the corresponding σ_* .

B. The case of prolonged reheating

Reheating is not expected to be instantaneous, and in this subsection we generalize the previous results to allow for a delay before reheating is complete. However there are no qualitative differences to the scenario and so we will keep the discussion fairly brief.

In the case of prolonged reheating, after inflation there is a significant period during which the inflaton oscillates coherently at the bottom of its potential. Its ultimate decay products will be considered much lighter than ϕ itself thus constituting the radiation, and we assume that ϕ decays into radiation with a rate Γ_ϕ . We make the assumption that there are no significant decays of the inflaton into the curvaton field.³ The new parameter Γ_ϕ

enters to modify the previous constraints.

To prevent a new period of inflation driven by the curvaton, its energy density must still be subdominant during reheating. Following some standard approximations [8], we can say that the inflaton ϕ starts oscillating at the end of inflation (when $H_{\text{end}}^2 = M^2/3$) and it behaves as $\rho_\phi \propto a^{-3}$ until $H \approx \Gamma_\phi$, when it decreases exponentially ($\rho_\phi \propto e^{-\Gamma_\phi t}$) producing most of the radiation. When $H \approx \Gamma_\phi$ reheating is completed. Before that point some radiation will be produced, but it is subdominant and ρ_{rad} scales as $a^{-3/2}$. After reheating the universe will be radiation dominated.

Concerning the curvaton we can consider two situations, one where σ starts oscillating *after* reheating, and the other where it begins oscillating *during* reheating. Just for illustrative purposes we assume that σ finally decays with a rate Γ_σ after reheating when it is still subdominant with respect to the produced radiation. Then P_C is given by Eq. (4) with $r_{\text{decay}} < 1$. The constraints defining the curvaton model during inflation, as described in Sec. II, still hold as does the requirement $\Gamma_\sigma > H_{\text{nucl}} \simeq 10^{-40} m_{\text{Pl}}$. When σ starts oscillating it should be a subdominant component in order to prevent a period of curvaton-driven inflation, and it can be checked that this constraint remains the same as Eq. (3).

In the case where the oscillations of the curvaton start after reheating $\Gamma_\phi > m$ and

$$\left(\frac{a_{\text{reh}}}{a_{\text{mass}}}\right)^4 = \frac{m^2}{\Gamma_\phi^2}, \quad (13)$$

where a_{reh} is the scale factor at the end of reheating.

On the other hand the produced radiation has an energy density $\rho_{\text{rad}}^{\text{reh}} = 3m_{\text{Pl}}^2 \Gamma_\phi^2 / 8\pi$. As a consequence the expression for r_{decay} is exactly the same of Eq. (7), and it is independent of Γ_ϕ

$$r_{\text{decay}} = \frac{4\pi}{3} \sigma_*^2 \frac{m^2}{m_{\text{Pl}}^2} \sqrt{\frac{1}{m^3 \Gamma_\sigma}}. \quad (14)$$

Note, however, that if $\Gamma_\phi > m$ and the constraint in Eq. (3) is satisfied, then

$$\frac{4\pi}{3} \sigma_*^2 \frac{m^2}{m_{\text{Pl}}^2} \ll \Gamma_\phi^2. \quad (15)$$

This amounts to saying that throughout reheating the energy density of the curvaton has to be much smaller than the radiation energy density at the end of reheating. The strong constraint in Eq. (15) implies that the ratio r at the moment of the curvaton decay can indeed be much smaller than in the case of prompt reheating. In fact this case is recovered when $\Gamma_\phi^2 = H_{\text{end}}^2 = M^2/3$, but for a prolonged period of reheating $\Gamma_\phi^2 < H_{\text{end}}^2$. Thus a similar analysis of the parameter space could be done as in the previous subsection, taking into account now that $m^2 < \Gamma_\phi^2 < M^2/3$.

³ Were there such decays, they would typically generate an extra, nearly homogeneous, component of the curvaton energy density, which might affect the ability of the curvaton to generate sufficiently large perturbations.

The curvaton begins oscillating during reheating when

$$\left(\frac{a_{\text{mass}}}{a_{\text{end}}}\right)^3 = \frac{M^2}{3m^2}. \quad (16)$$

In the approximation $\rho_{\text{rad}}^{\text{reh}} \simeq \rho_{\phi}^{\text{end}}(a_{\text{end}}/a_{\text{reh}})^3$, one finds

$$r_{\text{decay}} = \frac{4\pi}{3} \frac{\sigma_*^2}{m_{\text{Pl}}^2} \sqrt{\frac{\Gamma_{\phi}}{\Gamma_{\sigma}}}. \quad (17)$$

Since $\Gamma_{\phi} < m$, a comparison with Eq. (14) shows that r_{decay} can be even smaller than in the case when σ starts oscillating after reheating: once σ_* , Γ_{σ} and Γ_{ϕ} are fixed, the curvaton energy density starts decreasing as a^{-3} at an earlier time.

IV. CONCLUSIONS

The curvaton model is an interesting new proposal for generating the approximately scale-invariant curvature perturbations which presently give the best match to observational data. While nongaussianity would not necessarily be an observational disaster, we have chosen to restrict our attentions to gaussian models. We have constructed the simplest possible realization of this idea, and demonstrated how various requirements close off regions of parameter space while leaving a substantial area of viable models. Even this simplest model, with prompt reheating, features four parameters, and so the predicted perturbations in different regions of parameter space are highly degenerate.

While three of these parameters (m , M and Γ_{ϕ}) are parameters of the underlying theory, the fourth, σ_* , refers to the initial conditions for our patch of the Universe. The required values of σ_* for a successful curvaton model have magnitude less than about $10^{-3} m_{\text{Pl}}$, which represents only a small region of the plausible values for σ_* that might exist during inflation, as its potential has too low a magnitude to influence the dynamics. Nevertheless, in a typical chaotic inflation scenario one expects regions which do satisfy this criterion by chance. If one wishes, there are also opportunities to introduce anthropic principle considerations; for fixed values of m and M , regions with large σ_* would typically lead to the inflaton evolving to the bottom of its potential followed by a period

of slow-roll inflation driven by the curvaton, which given the small value of m would generate a much lower level of curvature perturbations than in the curvaton domain and hence not give rise to structure in the Universe.

As far as the density perturbations are concerned, the predictions of these models are indistinguishable from slow-roll inflation models arranged to give the same spectral index. However, the curvaton model has the feature that the gravitational wave amplitude is predicted to be low, because the Hubble rate during inflation is less than in an equivalent slow-roll model. In particular, the scalar and tensor perturbations typically will not obey the usual consistency relation (see e.g. Ref. [1]). In fact, in a natural curvaton model where σ is extremely light ($m^2 \ll H^2$) and subdominant with respect to the inflaton, the scalar and tensor indices are predicted to be the same: $n_s - 1 = n_t + 2m^2/3H_*^2 \simeq n_t$ [5]. Equal spectral indices is a prediction of the power-law class of conventional inflation models, but they predict a high amplitude of gravitational waves, contrary to the curvaton model. Unfortunately however the low amplitude of tensors in the curvaton model makes it difficult to detect the tensors at all, and one cannot expect a useful measure of their spectral index.

While one can certainly design slow-roll inflation models with any spectral index for the density perturbations and negligible gravitational waves, so that the curvaton model predictions cannot be viewed as distinct, a detection of a significant amplitude of gravitational waves would be sufficient to rule out these curvaton models. It is however interesting to note that present observations of large-scale structure and cosmic microwave background anisotropies tend to prefer a slight red tilt and a low contribution of tensor modes [9], as predicted by models of the type we have discussed.

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