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Publication date
01-01-2008

Published in
Physical Review D

Link to external publisher version
https://doi.org/10.1103/PhysRevD.77.121301

PHYSICAL REVIEW D 77, 121301(R) (2008)

Triple unification of inflation, dark matter, and dark energy using a single field

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(Received 8 April 2008; published 19 June 2008)

We construct an explicit scenario whereby the same material driving inflation in the early universe can comprise dark matter in the present universe, using a simple quadratic potential. Following inflation and preheating, the density of inflaton/dark matter particles is reduced to the observed level by a period of thermal inflation, of a duration already invoked in the literature for other reasons. Within the context of the string landscape, one can further argue for a nonzero vacuum energy of this field, thus unifying inflation, dark matter, and dark energy into a single fundamental field.

DOI: 10.1103/PhysRevD.77.121301 PACS numbers: 98.80.Cq, 95.35.+d, 95.36.+x

I. INTRODUCTION

In a recent paper [1], two of us proposed a general scenario for unification of dark matter and inflation into a single field. The key ingredient is the survival of a residual amount of the inflaton field's energy density, which undergoes coherent oscillations and can serve as a cold dark matter candidate. In the context of the string landscape, one can further argue for a nonzero vacuum energy of this field on anthropic grounds, thus providing a single description of the three key unknowns of modern cosmology, namely, dark energy, dark matter, and the material responsible for early universe inflation.

In practice, however, realizing this scenario is nontrivial, due to the need for a long radiation-dominated era of the Universe encompassing the nucleosynthesis period. This requires that the amplitude of scalar field oscillations be extremely small after the energy trapped in the inflaton is released into normal particles. Preheating scenarios can provide part of the required reduction of the oscillation amplitude, but still leave it too high and in conflict with the observed dark matter to radiation density ratio.[1]

In this paper, we explore a modification to the original scenario of Ref. [1]. As we shall discuss below, after preheating the Universe undergoes a short period of radiation quickly followed by a period of matter domination driven by the relic energy density of the inflaton field itself. This early matter domination period is interrupted by a short second period of inflation, known as thermal inflation, driven by a separate field and perhaps associated with the supersymmetry breaking transition. We find that thermal inflation can reduce the oscillation amplitude of the scalar field to the desired level, and then provide a proper reheating of the Universe.

II. COSMOLOGICAL EVOLUTION

For definiteness, we consider throughout the model of Ref. [1] where the inflaton ϕ has potential $V_0 + \frac{1}{2}m^2\phi^2$. Here V_0 has the small value needed to explain the observed dark energy density, and otherwise does not play a significant role. For sufficiently large $|\phi| \gtrsim m_{\rm Pl}$, this potential drives inflation and produces density perturbations in agreement with observations provided $m \simeq 10^{-6} m_{\rm Pl}$. Subsequently $H \ll m$ at all times, where H is the Hubble parameter, and the ϕ field oscillates rapidly on the Hubble time scale. Such an oscillating field behaves as cold dark matter, both in the redshifting of the mean density $\rho_\phi \propto a^{-3}$ and in the evolution of perturbations.

Unless some mechanism exists to reduce the energy density of the oscillating field, and indeed to transform some of it into conventional material, it is not possible to recover a satisfactory big bang cosmology. The original resolution was reheating—the complete transfer of energy from the inflaton via single-particle decays. Later coherent decays, known as preheating [2–4,6], were invoked as well. Such decays may be extremely efficient when the inflaton oscillations are large, but if the only interactions present are annihilations, the process will necessarily shut off once the density reduces. This led Kofman, Linde, and Starobinsky [2,4] to propose that the residual field could act as dark matter, but in fact detailed calculations [1] show the relic abundance is far too high under standard assumptions. It has usually thus been considered that preheating is followed by a period of reheating leading to complete decay of the inflaton field.

Having recognized that an inefficient reheating is a main concern in our unification scenario, the authors in Refs. [7,8][2] suggest that plasma mass effects [10] could

[1]This holds for the original four-legs interaction studied in the preheating literature [2–4], though a complete decay of the inflaton can be obtained from the introduction of other couplings [5].

[2]There exists an sneutrino (which is a scalar field) unification model for inflation and dark matter [8,9], with similar properties to our phenomenological model; under certain conditions, our approach also applies to it.

provide the mechanism for an incomplete reheating after inflation. The idea is that the decay of the inflaton field is kinematically forbidden in part of the reheating phase. However, the inflaton field is free to decay once it becomes subdominant with respect to the radiation fluid (see the paragraph after Eq. (9) in Ref. [10]), so a thermal mass cannot be thought of by itself as a mechanism for incomplete reheating.

We can consider three main possibilities for reducing this excess abundance, while leaving a relic level of oscillations capable of acting as cold dark matter. The first is to modify the shape of the inflaton potential. However, it is easy to show that the required level of post-inflationary oscillations is too small for such a modification to work; inflation must end long before the field is near enough the minimum to give the right abundance. This approach is therefore fruitless. The second possibility is to modify the reheating process so that it leaves a relic abundance level; this was the approach of Ref. [1], who chose a phenomenological form for the decay rate intended to correspond to particles which only had annihilation routes rather than decay routes, thus permitting incomplete reduction of the inflaton oscillations. However, fine-tuning of the decay rate, unmotivated by fundamental theory, is required to make this scenario work.

In this paper we consider a third possibility, which appears more attractive and natural, which is to consider a brief period of inflation at lower energy densities. Such a period, often called *thermal inflation* [11–13], was introduced in order to remove possible relic abundance problems left over by the original high-energy inflation period. This second period would be too short to imprint any new large-scale perturbations, but would reduce the abundance of any relic particles compared to the ultimate radiation background. An oscillating scalar field would have its density reduced by this mechanism.

For future reference, Fig. 1 shows a schematic of the Universe's evolution for our proposed scenario. As we will show, the required reduction, assuming a period of preheating after inflation but no reheating, is achieved provided thermal inflation lasts for around 12 e-foldings. This is in agreement with the duration already suggested in the literature [11,12].

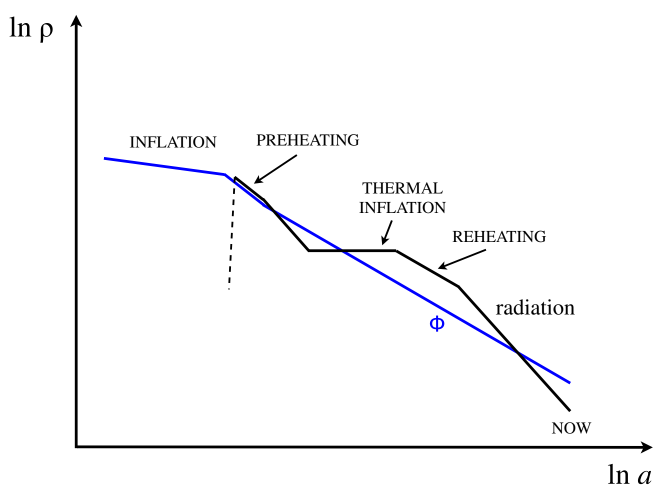

FIG. 1 (color online). A schematic of the evolution of the densities throughout the Universe's evolution. One (blue) line shows the density of the ϕ field, and the other (black) line the combined density of all other materials. The latter changes shape depending whether this combined density is dominated by radiation or by the material driving thermal inflation. We also include a reheating period after thermal inflation. (To avoid confusing the diagram, we do not show the emergence of nonrelativistic baryons from the relativistic fluid at late times.)

III. A DETAILED SCENARIO

We first revisit the calculation of the dark matter mass per photon for our scalar field, which ultimately gives the strongest constraint on the parameters of our model.

Let us denote by t_* the time after which the required hot big bang (HBB) cosmology is recovered[3]; hence, the averaged scalar field energy density will be given by $\rho_\phi = m^2\phi_*^2 a_*^3/a^3$ for $t > t_*$. Hereafter, all quantities with an asterisk denote values at time t_*.

As in Ref. [1], we define the scalar field dark matter mass per photon as $\xi_{\rm dm} \equiv \rho_\phi/n_\gamma$, and we assume expansion at constant entropy implying that $\xi_{\rm dm}/g_{\rm S}$ remains constant where $g_{\rm S}$ is the entropic degrees of freedom, usually very similar to the relativistic degrees of freedom that we denote here by $g_{\rm E}$ [14]. It is straightforward to show that, for any time $t > t_*$,

$$\frac{\xi_{\rm dm}}{m_{\rm Pl}} = \frac{\pi^2}{2\zeta(3)} \frac{g_{\rm S}(T)}{g_{\rm S}(T_*)} \frac{m^2}{m_{\rm Pl}^2} \frac{\phi_*^2}{m_{\rm Pl}^2} \frac{m_{\rm Pl}^3}{T_*^3}, \tag{1}$$

where T is the temperature of the Universe, measured from the relativistic particles in thermal equilibrium at time t.

Equation (1) contains two free parameters, which are the scalar field ϕ_* and the temperature T_* at the beginning of the HBB; equivalently, we shall call this the time at the end of reheating. It is then necessary to predict the aforementioned values and determine whether they can match the observed value of $\xi_{\rm dm}$.

In the early Universe, there is first a stage of slow-roll inflation, after which the inflaton field value is $\phi_{\rm end} \simeq 0.28 m_{\rm Pl}$. Then a preheating stage starts in which part of the inflaton energy density is converted into relativistic degrees of freedom. We assume the simplest model of preheating [6], in which the inflaton field is coupled to a massless scalar field χ through the four-legs interaction term

[3]Notice that the meaning of t_* is changed with respect to Ref. [1], where it was intended to denote the time at which the equality $H = m$ was achieved.

$$V_{\rm int} = \frac{g^2}{2}\phi^2\chi^2, \quad (2)$$

where $10^{-10} < g^2 < 10^{-5}$ is the typically considered range for the coupling constant.

The preheating process ends once the inflaton amplitude is of the order $\phi_{\rm pr} \simeq m/g$, at which point the ratio between relativistic (χ and ϕ quanta) and nonrelativistic degrees of freedom (coherent oscillations of ϕ) ρ_r/ρ_ϕ is of order of a few [15]. In such a case, we cannot expect a prolonged radiation-dominated era after preheating; rather, we expect the appearance of a matter-dominated era just a few e-folds after the end of preheating when the coherent ϕ field comes back into domination.

Alternative coupling terms in the potential, such as three-leg decay interactions, can lead to a complete decay of the inflaton field [5]; that would also happen in cases where the inflaton field is coupled to fermionic fields [4]. However, we do not allow such couplings for the inflaton field in our model, for instance by presuming that the Z_2 symmetry $\phi \leftrightarrow -\phi$ is (almost) exact. The ϕ field therefore survives right through to the present; however, if the radiation simply redshifts away as normal its density will be far too low relative to that of ϕ by the present.

Instead, our proposal is that the subsequent evolution of the Universe raises the radiation energy density back above that of the ϕ field, so as to reestablish a standard HBB evolution. After the preheating process, the energy density of the Universe is composed of relativistic particles and nonrelativistic matter represented by the coherent oscillations of the inflaton field. We now assume that there is a second scalar field, initially part of the relativistic thermal bath, that will drive thermal inflation in a later stage. This second field, known as the *flaton* field, has thermal corrections to its potential which trap it in a false vacuum with energy density denoted by $\hat{V}$. A hat will be used henceforth to denote quantities related to the flaton field.

According to the standard picture of thermal inflation [11–13], an inflationary stage starts once the false vacuum energy dominates over the radiation fluid; this happens once the temperature of the latter is $T < \hat{V}^{1/4}$. Thermal inflation then ends once the thermal corrections to the potential are insufficient to oppose the underlying symmetry-breaking (SB) potential, so that the thermal inflaton can escape from its false vacuum and undergoes a SB transition. This happens once the temperature of the Universe is below the flaton mass scale, $T < \hat{m}$.

In our scenario the sequence is a little different, as seen in Fig. 1, because the flaton density is initially subdominant to the oscillating ϕ field. However after some interval the ϕ density falls below it and thermal inflation starts; sometime afterwards the SB transition then takes place.

After the preheating process the inflaton field redshifts as cold dark matter, $\phi \propto a^{-3/2}$, and we can calculate the total dilution of the inflaton field from the end of preheating up to the SB process. The square of the inflaton field at the end of thermal inflation is given by

$$\phi^2_{\rm SB} = \phi^2_{\rm pr}\left(\frac{a_{\rm pr}}{a_{\rm SB}}\right)^3 = \phi^2_{\rm pr}\frac{g_{\rm S}(T_{\rm SB})}{g_{\rm S}(T_{\rm pr})}\left(\frac{\hat{m}}{T_{\rm pr}}\right)^3 \quad (3)$$

To obtain the above equation we are assuming both entropy conservation and that the radiation fluid is in thermal equilibrium. $T_{\rm pr}$ and $T_{\rm SB} = \hat{m}$ are the values of the temperature at the end of the preheating stage and at the SB process, respectively; likewise, $a_{\rm pr}$ and $a_{\rm SB}$ are the corresponding values of the scale factor.

Once thermal equilibrium is attained at the end of preheating [6,15], the usual formula for the temperature of the radiation fluid applies, $\rho_{r,\rm pr} = (\pi^2/30)g_{\rm E}(T_{\rm pr})T^4_{\rm pr}$. Recalling that $\rho_{r,\rm pr} \simeq \rho_{\phi,\rm pr}$, then $T_{\rm pr} \simeq (30/\pi^2)^{1/4}\times g^{-1/2}g_{\rm E}^{-1/4}(T_{\rm pr})m$.[4] Thus, from Eq. (3), the total dilution of the inflaton field is largely determined by the mass scales of the inflationary fields,

$$\phi^2_{\rm SB} \simeq \frac{\pi^{3/2}}{30^{3/4}}\frac{g_{\rm S}(T_{\rm SB})}{g_{\rm S}(T_{\rm pr})}\frac{g_{\rm E}^{3/4}(T_{\rm pr})}{g^{1/2}}\frac{\hat{m}^3}{m^3}m^2. \quad (4)$$

The last process is the reheating of the Universe at the end of thermal inflation. We shall assume that each flaton particle decays at a single-particle decay rate Γ, which is a new free parameter in our phenomenological approach. In principle the value of Γ can be estimated in terms of $\hat{m}$ and $\hat{V}$ [12], as we discuss later.

The Universe is reheated when $\Gamma \simeq H_*$, where H_* is the Hubble rate at the beginning of the HBB. In between, the Universe is dominated by the energy density of the oscillating flaton field (which redshifts as a^{-3}), so that the change in the scale factor is given by

$$\frac{a^3_{\rm SB}}{a^3_*} \simeq \frac{H^2_*}{H^2_{\rm SB}} \simeq \frac{3m^2_{\rm Pl}\Gamma^2}{8\pi\hat{V}}. \quad (5)$$

The inflaton field is further affected by this expansion as well, so that we get

$$\frac{\phi^2_*}{m^2_{\rm Pl}} = \frac{\phi^2_{\rm SB}}{m^2_{\rm Pl}}\frac{a^3_{\rm SB}}{a^3_*} \simeq \phi^2_{\rm SB}\frac{3\Gamma^2}{8\pi\hat{V}}, \quad (6)$$

where $\phi^2_{\rm SB}$ is given in Eq. (4). Finally, the reheating temperature T_* is estimated to be [4]

$$T_* = (90/8\pi^3)^{1/4}g_{\rm E}^{-1/4}(T_*)\sqrt{m_{\rm Pl}\Gamma}. \quad (7)$$

We are now in a position to use the dark matter constraint from Eq. (1), which now takes the form

[4] Incidentally, thermal inflation can resolve the relic abundance troubles, e.g. the gravitino, that such a high temperature $T_{\rm pr} \sim m \simeq 10^{13}$ GeV may lead to [16].

$$\frac{\xi_{\rm dm}}{m_{\rm Pl}} \simeq \frac{3\pi}{16\zeta(3)} \frac{g_{\rm S}(T_{\rm SB})}{g_{\rm S}(T_{\rm pr})} \frac{g_{\rm S}(T)}{g_{\rm S}(T_*)} g_{\rm E}^{3/4}(T_*) g_{\rm E}^{3/4}(T_{\rm pr}) \times \left(\frac{3}{8\pi}\right)^{1/4} g^{-1/2} \frac{m}{\hat{m}} \left(\frac{\hat{m}}{\hat{V}^{1/4}}\right)^4 \sqrt{\frac{\Gamma}{m_{\rm Pl}}}. \qquad (8)$$

The measured value of the current dark matter mass per photon is $\xi_{\rm dm,0} = 2.4 \times 10^{-28} m_{\rm Pl}$ using values from the five-year Wilkinson Microwave Anisotropy Probe observations [17]. We shall take that $g_{\rm E}(T) \simeq g_{\rm S}(T) \simeq 100$ for temperatures $T \geq T_*$, and $g_{\rm S}(T_0) = 3.9$, where "0" indicates present values; Eq. (8) then becomes

$$g^{-1/2} \frac{m}{\hat{m}} \left(\frac{\hat{m}}{\hat{V}^{1/4}}\right)^4 \sqrt{\frac{\Gamma}{m_{\rm Pl}}} \simeq 1.4 \times 10^{-29}. \qquad (9)$$

We define the number of e-folds of thermal inflation as $N_{\rm TI} \equiv \ln(\hat{V}^{1/4}/\hat{m})$, whereas we denote the number of e-folds between the end of thermal inflation and the completion of reheating, from Eq. (5), as

$$N_{\rm reh} \equiv \frac{1}{3} \ln \frac{8\pi \hat{V}}{3 m_{\rm Pl}^2 \Gamma^2}. \qquad (10)$$

Thus, an equivalent expression for Eq. (9) is, in terms of the above-defined e-folding numbers,

$$N_{\rm TI} + \tfrac{1}{4} N_{\rm reh} \simeq 18 - \ln g^{1/6}, \qquad (11)$$

where we have used $m/m_{\rm Pl} \simeq 10^{-6}$. For the expected range $10^{-10} < g^2 < 10^{-5}$, the last term on the right-hand side contributes one to two extra e-folds.

Equation (11) is our main result, giving the duration of thermal inflation and subsequent reheating required to give a viable universal history. We now investigate how achievable it is. The only genuinely free parameter of our model is the decay width Γ, which determines $N_{\rm reh}$ (the dependence on g over its expected range is modest). The reason is that thermal inflation parameters are expected to lie in more or less definite ranges of energy [11]. The mass of the flaton field should be of the order of $\hat{m} \simeq 10^2$ to 10^3 GeV, and on general grounds we expect $\hat{V}^{1/4} \simeq 10^7$ to 10^8 GeV, so that $N_{\rm TI} \simeq 11$ with an uncertainty of one or two in either direction. This could be increased by having more than one period of thermal inflation, but we do not need this.

The decay width is sandwiched by two limits: that the decay should take place after thermal inflation is complete, $\Gamma < H_{\rm SB} \simeq 10^{-24} m_{\rm Pl}$, and that it should be complete before the run-up to nucleosynthesis begins at around 10 MeV, requiring $\Gamma > 10^{-42} m_{\rm Pl}$. Figure 2 shows the required value of Γ to satisfy the observational constraint (11), as a function of $\hat{V}$ and for some different values of $N_{\rm TI}$. We see that the nucleosynthesis constraint can readily be satisfied provided $N_{\rm TI}$ and $\hat{V}$ are large enough, and that suitable values lie well within the expected range.

Actually, one can arguably justify that the typical decay width of flaton particles is of the form $\Gamma \simeq 10^{-2} \hat{m}^5/\hat{V}$ [12]. If we plot this in combination with Eq. (11) in Fig. 2, we find that the favored flaton parameters are $\hat{m} \simeq 10^3$ GeV and $\hat{V}^{1/4} \simeq 10^8$ GeV.

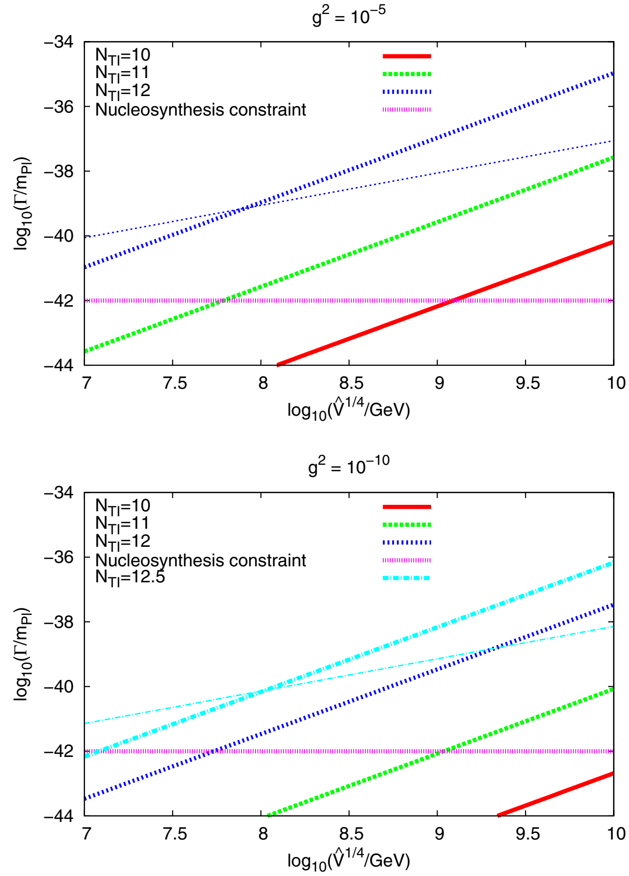

FIG. 2 (color online). The lines in both figures show the value of Γ required to satisfy the abundance constraint Eq. (11), as a function of $\hat{V}$, for some fixed values of $N_{\rm TI}$ (note that the value of $N_{\rm TI}$ itself depends on both $\hat{V}$ and $\hat{m}$) and the preheating coupling term g^2. Only models with large enough values of $N_{\rm TI}$ lie above the nucleosynthesis constraint line $\Gamma > 10^{-42} m_{\rm Pl}$. We also show the thermal inflation estimation of the decay width Γ (thin lines) for the cases $N_{\rm TI} = 12$ (top) and $N_{\rm TI} = 12.5$ (bottom); see text for details. As read from the crossing of the corresponding lines, models with flaton parameters of the order of $\hat{m} \simeq 10^3$ GeV and $\hat{V}^{1/4} \simeq 10^8$ GeV are able to satisfy all constraints.

We therefore conclude that thermal inflation, with properties already well established in the literature, can indeed dilute the inflaton density sufficiently that it can act as dark matter.

IV. CONCLUSIONS AND DISCUSSION

The task of arranging that a residual inflaton density survives to act as dark matter is a challenging one, but unification of two normally unconnected sectors of cosmological modeling would be a valuable reward (within the string landscape picture we can even argue that the same potential also gives rise to dark energy [1]). In this paper, we have shown that one option to achieve this is to exploit the uncertainty in cosmological dynamics during the long period from the end of inflation up to nucleosynthesis. In particular, we have found that a period of thermal inflation during this epoch has exactly the desired effect, reducing the residual inflaton density after preheating from a dominant level down to one where the desired late radiation to matter transition can be achieved. Moreover, the amount of thermal inflation needed to achieve this is pretty much the amount already taken as standard in the literature, for completely different reasons.

Further, since the thermal inflation scenario comes quite close to failing the nucleosynthesis constraint, it is clear that less drastic modifications to early universe dynamics, such as a protracted period of matter domination due to temporary domination by some long-lived massive particle species, would not be sufficient to achieve our goals. Extra periods of early universe inflation appear essential.

It is of course not so attractive that we have had to invoke a second period of inflation, in order to unify the first type of inflaton with dark matter. But at least the thermal inflaton is more grounded in conventional particle physics, specifically supersymmetry. Additionally, even conventional high-scale inflation models may too need thermal inflation in order to solve extra relic abundance problems such as the gravitino [16].

The scenario that we have described is based around the quadratic potential, but the construction is of course more general and can be applied to a wide range of inflation models. Indeed, at least within the context of the string landscape, the quadratic potential is actually quite unattractive as its form has to hold over field values many times greater than the (reduced) Planck mass, which is the scale on which we expect the potential to have features [18]. It may be much more plausible to consider inflation as occurring near a hilltop [19] between neighboring minima in the landscape; we anticipate the calculation going through more or less as in this paper, but perhaps different in the fine numerical details [for instance, Eq. (9) depends on the inflaton mass, whose value depends on the shape of the potential during inflation].

Another reason to consider different potentials is that thermal inflation significantly reduces the number of inflationary e-folds corresponding to the present horizon, perhaps to 40 rather than the usual 50 to 60 [20]. This forces the predictions for the observables n and r further from the slow-roll limit $n = 1$ and $r = 0$, and WMAP5 is starting to exert significant observational pressure against the quadratic potential for low e-folding numbers [21]. While this is not yet conclusive, it certainly motivates study of potentials which can produce smaller values of r.

ACKNOWLEDGMENTS

A. R. L. was supported by STFC, C. P. by the Swiss Sunburst Fund and by the Marie Curie programme at IAP, and L. A. U.-L. by CONACYT (46195, 47641, 56946), DINPO and PROMEP-UGTO-CA-3, and by STFC during a visit to Sussex. L. A. U.-L. is part of the Instituto Avanzado de Cosmología (IAC) Collaboration.

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