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A thermal boundary control method for a flexible thin disk rotating over critical and supercritical speeds

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Abstract In practice a rotating flexible thin annular disk has to be operated at low speed, because three types of dynamic instabilities inevitably occur around critical and supercritical speeds, namely: aeroelastic, parametric and thermoelastic. The rotating disk is clamped and driven by a drive shaft attached to the disk inner edge. The external action of the flowing surrounding air causes the aeroelastic instability; a slider mass-damper-spring-friction moving load causes parametric instability; and disk/slider interface friction heat can cause thermoelastic instability. A thermal boundary control method is used to induce disk thermal membrane stresses utilizing drive shaft temperature increments to stabilize these dynamic instabilities. Fundamental investigations are made of disk temperature distribution, thermal stress, natural frequency, dynamic stability and steady state amplitude to validate and demonstrate the viability of the new control method. The thermal boundary control method offers valuable opportunities for rotating disk applications operating over critical and supercritical high speeds with high efficiency.

Keywords *Rotating disk • Boundary control • Thermal stress control • Dynamic stabilization • Parametric instability • Aeroelastic instability • Thermoelastic instability*

1 Introduction

A rotating disk is a widely used mechanical component in engineering, such as: computer optical/hard disk drives, automobile disk brake, machine tool sawing blade and grinding wheels. A flexible thin disk is often designed to minimize: product cost, resource utilization and energy wastage. However, in rotating disk applications, three types of dynamic instability can occur, which may result in an unacceptably large transverse- vibrational deflection; when a flexible thin disk rotates at high speed it will suffer from: parametric instability, aeroelastic instability and thermoelastic instability.

Firstly, the parametric instability takes place as the rotating flexible disk is subjected to a spatially-fixed slider (head and contact pad) loading system, which acts as a periodic parametric excitation. As indicated by Iwan and Stahl [1], Shen [2], Chen and Bogy [3], Young and Lin [4], the mass, damping and elasticity of the slider loading system and even the disk/slider interface friction [5-9] can all result in a parametric instability around critical and supercritical rotational speeds. In Young and Lin [4], the parametric instability of a rotating flexible disk was presented with a stationary slider oscillating unit with two spring-damper elements attached above and below a mass. Mottershead et al. [5] and Ouyang and Mottershead [6] investigated the parametric instability (parametric resonance) of an annular flexible disk subjected to a rotating slider loading system of distributed / lumped mass, elasticity, damping and friction as a moving load problem, and then some systematic numerical methods were presented for solving the dynamic instability of a car disk brake by Ouyang et al. [7] and Cao et al.

[8]. Ouyang and Mottershead [9] analyzed the dynamic instability of an elastic rotating disk subjected to a friction couple from a slider mass-spring-damper loading system. Pei et al. [10] investigated a rotating flexible thin disk perturbed by the reciprocating angular movement of a slider loading system, and observed not only the traditional parametric instability but also an additional parametric instability excited by some angular movement frequencies. Secondly, the rotating disk operates in viscous air, the surrounding air flows rotationally and results in an air pressure difference between the disk top and bottom surfaces due to disk rotation, the air pressure difference produces a lift force and causes an aeroelastic instability (flutter) of a rotating flexible disk around critical and supercritical rotating speeds. Theoretically and experimentally, Yasuda et al. [11] investigated the aeroelastic instability of a circular disk rotating in air as a dynamic problem of self-excited oscillations, and modeled the effects of flowing air on the rotating disk as a lift force with experimental confirmation. The aeroelastic instability (aerodynamic flutter instability) of various optical disks was experimentally observed at supercritical speeds in Lee et al. [12]. Using the wave equation for acoustic oscillations, Jana and Raman [13] and Kang and Raman [14, 15] studied the aeroelastic instability of a flexible disk rotating in a gas-filled enclosure, the aeroelastic instability was observed around critical and supercritical speeds theoretically and experimentally. The aeroelastic instability of a flexible disk rotating close to a rigid wall was simulated numerically by modeling the flowing air using the Navier-Stokes equations in Naganathan et al. [16]. With commercial CFD (Computational Fluid Dynamics) software, Kirpekar and Bogy

[17] and Cheng et al. [18] analyzed the aeroelastic rotating disk vibrations in hard/optical disk drives and provided some methods for vibration reduction. Thirdly, due to interface friction heat between the rotating disk and the stationary slider (head, pad) loading system, the so-called thermoelastic instability of a rotating flexible thin disk can be induced by the thin disk in-plane thermal membrane stresses at high speed. In addition, another type of thermoelastic instability takes place in a rotating thick disk due to a positive growth rate of disk temperature induced by friction heat as investigated in Afferrante et al. [19], Voldrich [20] and Honner et al. [21], but it is not the case considered in this paper. Krempaszky and Lippmann [22] and Davis et al. [23] indicated that the thermal membrane pre-stress/load can also cause the so-called thermoelastic instability in a clutch. Using a finite element method, dynamic thermoelasticity in rotating disks with stationary heat sources was discussed in Wauer and Schweizer [24] to demonstrate the interaction of temperature and displacement/stress fields.

Although these dynamic instabilities have been observed and studied experimentally and theoretically in the cited literature for a long time, there is no solution available to completely suppress them until now, hence the rotating disks have to be operated at low rotational speed with low efficiency.

In practice, the rotating disk is often clamped to a drive shaft attached to the disk inner edge. A drive shaft temperature increment can change the temperature distribution and induce thermal membrane stress in the rotating disk. Thus a drive shaft temperature increment can be used to induce disk thermal membrane stress that can be used to suppress instabilities. This paper presents a thermal boundary control method to stabilize dynamic instabilities in a rotating flexible annular thin disk.

An elementary feasibility investigation of the current method can be assessed by exploring the natural frequency and instability mechanisms of the rotating flexible disk. As indicated theoretically and experimentally by references [25-28], for a free rotating disk with a fixed slider loading system and friction heat, without any influence from the surrounding air, the shaft temperature increment only decreases the natural frequencies of low frequency disk modes, such as (0, 0) and (0, 1), and can cause thermoelastic instability for high shaft temperature increments, as presented by Pei et al. [27]. Nevertheless, the shaft temperature increment increases the natural frequencies of higher disk modes, such as (0, 2), (0, 3), (0, 4) and higher [27]. On the other hand, as reported in the above references these dynamic instabilities of a practical rotating disk coincidentally occur in those high order disk modes around the disk critical and supercritical speeds. This coincidence provides an opportunity to effectively suppress these dynamic instabilities by utilizing disk thermal membrane stress induced by using a drive shaft temperature increment. In this paper, some theoretical investigations are reported to verify and quantify this new method for the efficient control of rotating disk instabilities that develop at critical and supercritical speeds.

2 System of governing equations

A flexible annular thin disk is illustrated in Fig.1 (a). The disk rotates at an angular speed Ω under the action of a pair (top and bottom) of stationary distributed sliders (pads) loading the system [5]. Where h is the disk thickness that is much smaller than the disk radial length a-b. Fig. 1 (b) illustrates a possible drive shaft temperature increment feedback control schema. A non-contact infrared temperature sensor is used to measure the shaft (disk hub) temperature increment, and the shaft temperature increment is achieved via the heat supplied from a controlled induction coil. A feedback control system calibrates the power of the induction coil to obtain and maintain an assigned value of the shaft temperature increment.

The polar coordinate (r_0, θ_0) is defined as the center location of the pair of sliders on both sides of the disk in Fig.1 (a). The sectorial area corresponding to the disk/slider interface contact area $r\in[r_0-\varepsilon_r, r_0+\varepsilon_r]$ and $\theta\in[\theta_0-\varepsilon_\theta, \theta_0+\varepsilon_\theta]$ can be written as

$$\Delta_S = \frac{(\theta_0+\varepsilon_\theta)-(\theta_0-\varepsilon_\theta)}{2\pi}[\pi(r_0+\varepsilon_r)^2-\pi(r_0-\varepsilon_r)^2] = 4r_0\varepsilon_r\varepsilon_\theta$$

A distributed mass-damper-spring loading system [5, 10, 29] with distributed densities m_s/Δ_S, c_s/Δ_S and k_s/Δ_S is used to model each slider and its suspension. With an initial stress-free transversal runout $\tilde{w}$ [29], the transverse vibrational deflection of the rotating flexible disk is defined as w relative to $\tilde{w}$, and then the total deflection can be written as $W=w+\tilde{w}$.

About here to insert **Fig. 1**

The forces acting [9, 10, 29] on the disk top and bottom surfaces can be expressed as:

$$p_t = \frac{P_S}{\Delta_S} + [\frac{m_s}{\Delta_S}\frac{\partial^2(w+\tilde{w})}{\partial t^2} + \frac{c_s}{\Delta_S}\frac{\partial(w+\tilde{w})}{\partial t} + \frac{k_s}{\Delta_S}(w+\tilde{w})],$$

$$p_b = \frac{P_S}{\Delta_S} - [\frac{m_s}{\Delta_S}\frac{\partial^2 (w+\tilde{w})}{\partial t^2} + \frac{c_s}{\Delta_S}\frac{\partial (w+\tilde{w})}{\partial t} + \frac{k_s}{\Delta_S}(w+\tilde{w})] \tag{1}$$

where P_S is an initial normal force acting on the slider with a uniform density P_S/Δ_S.

From Ouyang and Mottershead [6, 9] and Pei et al. [10, 27-29, 37], the governing equation of transversal vibration [30, 31] for the rotating thermoelastic disk subjected to the stationary slider loading system with interface friction can be written as

$$\begin{aligned}
&\rho h[\frac{\partial^2 (w+\tilde{w})}{\partial t^2} + 2\Omega\frac{\partial^2 (w+\tilde{w})}{\partial t\partial\theta} + \Omega^2\frac{\partial^2 (w+\tilde{w})}{\partial\theta^2}] + \frac{Eh^3}{12(1-v^2)}\nabla^4 w + \\
&c[\frac{\partial (w+\tilde{w})}{\partial t} + \Omega\frac{\partial (w+\tilde{w})}{\partial\theta}] - c_L\Omega\frac{\partial (w+\tilde{w})}{\partial\theta} + \frac{E\alpha_T}{1-v}\nabla^2\left(\int_{-h/2}^{h/2}\Theta\, z\, \mathrm{d}\, z\right) \\
&-\rho h a^2\Omega^2\{\frac{1}{r}\frac{\partial}{\partial r}[r\sigma_r\frac{\partial (w+\tilde{w})}{\partial r}] + \frac{1}{r^2}\frac{\partial}{\partial\theta}[\sigma_\theta\frac{\partial (w+\tilde{w})}{\partial\theta}]\} + \\
&\frac{\mathrm{n}_{r,r_0}}{\Delta_S}\left\{2[m_s\frac{\partial^2 (w+\tilde{w})}{\partial t^2} + c_s\frac{\partial (w+\tilde{w})}{\partial t} + k_s(w+\tilde{w}) + \mu P_S\frac{\partial (w+\tilde{w})}{r\partial\theta}]\mathrm{n}_{\theta,\theta_0}\right. \\
&\qquad\left. + \mu h\frac{\partial}{r\partial\theta}\{[m_s\frac{\partial^2 (w+\tilde{w})}{\partial t^2} + c_s\frac{\partial (w+\tilde{w})}{\partial t} + k_s(w+\tilde{w})]\mathrm{n}_{\theta,\theta_0}\}\right\} \\
&= \frac{\partial^2 (w+\tilde{w})}{\partial r^2}(\frac{\partial\Phi}{r\partial r} + \frac{\partial^2\Phi}{r^2\partial\theta^2}) + [\frac{\partial (w+\tilde{w})}{r\partial r} + \frac{\partial^2 (w+\tilde{w})}{r^2\partial\theta^2}]\frac{\partial^2\Phi}{\partial r^2} \\
&\quad - 2[\frac{\partial^2 (w+\tilde{w})}{r\partial r\partial\theta} - \frac{\partial (w+\tilde{w})}{r^2\partial\theta}](\frac{\partial^2\Phi}{r\partial r\partial\theta} - \frac{\partial\Phi}{r^2\partial\theta})
\end{aligned} \tag{2}$$

where σ_r and σ_θ are the disk membrane stress resultants [4, 10, 32]; c_L denotes the lift force induced by the rotating surround air as indicated by Yasuda et al. [11] and Lee et al. [12]; Φ is a partial stress function that excludes the stress induced by disk rotation [28, 37], it is a pure thermal stress resultant, only related to the disk temperature distribution Θ, whereas the membrane stress resultants σ_r and σ_θ are induced by disk rotation; ∇^4 is a plane bi-harmonic differential operator; $\mathrm{n}_{x,x0}$ is a step function of $\mathrm{n}_{x,x0}=1$ just in the interval $x\in[x_0-\varepsilon_x,\ x_0+\varepsilon_x]$, otherwise $\mathrm{n}_{x,x0}=0$. The boundary conditions [29, 33] of transverse deflection w are: at the clamped edge $r=b$ of the disk is $w|_{r=b}=0$

and $\partial w/\partial r|_{r=b}=0$, and the bending moment and shear force in the disk vanish at the free edge $r=a$.

From Cho and Ahn [34], a friction heat flux can be induced at the disk/slider contact area interface,

$$q_t=\mu p_t V=\mu p_t \Omega r, \quad q_b=\mu p_b V=\mu p_b \Omega r \tag{3}$$

where $V=\Omega r$ is the local relative velocity of friction.

From practice and Krempaszky and Lippmann [22] and Pei et al. [29], it is concluded that the initial normal force acting P_S, is much larger than the slider total inertial force $m_s \dfrac{\partial^2 (w+\tilde{w})}{\partial t^2} + c_s \dfrac{\partial (w+\tilde{w})}{\partial t} + k_s (w+\tilde{w})$, the temperature increment Θ can be assumed to be transversally symmetric, i.e. $q_t=q_b$, $\Theta(z)=\Theta(-z)$ and $\int_{-h/2}^{h/2} \Theta\, z\, \mathrm{d}z \equiv 0$. The total friction heat power shared by the rotating disk can be simplified and written as

$$Q_F=k_q(q_t+q_b)\Delta_S=2k_q\mu P_S \Omega r \tag{4}$$

where k_q is the ratio of friction heat shared by the rotating disk, the remaining part $(1-k_q)$ is shared by the slider.

The friction heat is induced at the disk/slider contact interface, and this provides the heat flux boundary condition at the interface surface. However, for the current flexible very thin metal (steel) disk in its initial starting stage, the friction heat and temperature increment at/between the interface areas ($r\in[r_0-\varepsilon_r,\ r_0+\varepsilon_r]$ and $\theta\in[\theta_0-\varepsilon_\theta,\ \theta_0+\varepsilon_\theta]$) can achieve a steady state temperature distribution instantaneously throughout the thin disk thickness for the case of a large metal thermal conductivity with a small characteristic dimension [35]. It is noted that the disk temperature distribution varies very slightly and can maintain its distribution for a long time after this short initial starting stage. To simplify the calculations of further analytical and numerical solutions of several coupled partial differential equations for a large disk speed range for several investigation cases (parametric, aeroelastic and thermoelastic instabilities), the total friction heat Q_F is considered as a heat source with power density Q_F/Δ_V, where $\Delta_V=\Delta_S h=4r_0\varepsilon_r\varepsilon_\theta h$ is a small sectorial volume corresponding to the friction area $r\in[r_0-\varepsilon_r, r_0+\varepsilon_r]$, $\theta\in[\theta_0-\varepsilon_\theta, \theta_0+\varepsilon_\theta]$ and the disk thickness $z\in[-h/2, +h/2]$. At the disk/slider contact area interface, the convection boundaries are assigned to reflect the convective heat loss through the sliders. However, as presented by Cho and Ahn [34] and Arafat et al. [36], since the convective heat loss is much smaller than the friction heat generation, this assignment will only have a slight effect on the final solutions, it is therefore quite helpful to construct a continuous disk surface boundary and yield valid analytical solutions of temperature increment and vibrational deflection fields in the following development.

From Pei et al. [27], Pei [37] and Cho and Ahn [34], the steady state ($\partial\Theta/\partial t=0$) governing equation of disk heat conduction can be stated as:

$$\rho c_v \Omega \frac{\partial \Theta}{\partial \theta} = k\left(\nabla^2\Theta + \frac{\partial^2 \Theta}{\partial z^2}\right) + \mathrm{n}_{r,r_0}\mathrm{n}_{\theta,\theta_0}\frac{Q_F}{\Delta_V} \tag{5}$$

For a theoretical comparison, a constant temperature boundary (for shaft temperature increment control schema, see Fig. 1(b)) and a free convective boundary (without temperature control) are specified respectively at the clamped edge $r=b$, hence the two boundary conditions [24, 27, 37, 38] can be written as:

$$\text{With temperature control:} \quad \Theta\big|_{r=b} = \Theta_D \tag{6.1}$$

Without temperature control: $$\left.\left(h_T\Theta - k\frac{\partial\Theta}{\partial r}\right)\right|_{r=b} = 0 \tag{6.2}$$

At the free edge $r = a$ and at the top $z = h/2$ and at the bottom $z = -h/2$ surfaces of the disk, the convection boundary conditions are assumed to be [27],

$$\left.\left(h_T\Theta + k\frac{\partial\Theta}{\partial r}\right)\right|_{r=a} = 0 \tag{7}$$

$$\left.\left(h_T\Theta + k\frac{\partial\Theta}{\partial z}\right)\right|_{z=h/2} = 0,\ \left.\left(h_T\Theta - k\frac{\partial\Theta}{\partial z}\right)\right|_{z=-h/2} = 0 \tag{8}$$

From Pei [37] and Arafat et al. [36], the equation of disk deformation continuity can be modeled as

$$\nabla^4\Phi = -E\alpha_T\nabla^2\left(\int_{-h/2}^{h/2}\Theta\,\mathrm{d}\,z\right) \tag{9}$$

At the clamped edge r=b and free edge r=a, the boundary conditions of the partial stress function Φ can be written as

$$\left.\left[\frac{\partial^2\Phi}{\partial r^2} - v\left(\frac{\partial\Phi}{r\partial r} + \frac{\partial^2\Phi}{r^2\partial\theta^2}\right) + E\alpha_T\left(\int_{-h/2}^{h/2}\Theta\,\mathrm{d}\,z\right)\right]\right|_{r=b} = 0,$$

$$\left.\left[\frac{\partial}{\partial r}(\nabla^2\Phi) + \frac{1+v}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial\Phi}{\partial r} - \frac{\Phi}{r}\right) + E\alpha_T\frac{\partial}{\partial r}\left(\int_{-h/2}^{h/2}\Theta\,\mathrm{d}\,z\right)\right]\right|_{r=b} = 0 \tag{10}$$

$$\left.\left(\frac{\partial\Phi}{\partial r} + \frac{1}{r}\frac{\partial^2\Phi}{\partial\theta^2}\right)\right|_{r=a} = 0,\ \left.\left(\frac{\partial^2\Phi}{\partial r\partial\theta} - \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right)\right|_{r=a} = 0 \tag{11}$$

3 Mathematical solution procedures

From references [27, 28] for the transversally symmetric temperature increment $\Theta(z)=\Theta(-z)$, the solution of Eq. (5) can be assumed to be

$$\Theta(r,\theta,z) = \sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty}\mathrm{e}^{\mathrm{i}n\theta}\cos(\omega_s z)y_{s,n}(r) \tag{12}$$

where ω_s can be determined from $\omega_s h+2\tan^{-1}(k\omega_s/h_T)=(2s-1)\pi$ [27], which is derived from the surface convection boundary conditions Eq. (8); $y_{s,n}(r)$ in the finite difference method can be found in the Appendix.

From the solution of Eq. (12), the integration of the temperature increment $\Theta(r,\theta,z)$ in the disk deformation continuity Eq. (9) can be collected as

$$\int_{-h/2}^{h/2}\Theta\,\mathrm{d}z=\sum_{n=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}n\theta}hY_n(r) \tag{13}$$

where $Y_n(r)=\sum_{s=1}^{\infty}2h^{-1}\omega_s^{-1}\sin(\omega_s h/2)y_{s,n}(r)$.

Then substituting Eq. (13) into Eqs. (9-11) yields

$$\nabla^4\Phi=-E\alpha_T h\sum_{n=-\infty}^{\infty}\nabla^2[\mathrm{e}^{\mathrm{i}n\theta}Y_n(r)] \tag{14}$$

$$\left.\left[\frac{\partial^2\Phi}{\partial r^2}-v\left(\frac{\partial\Phi}{r\partial r}+\frac{\partial^2\Phi}{r^2\partial\theta^2}\right)+E\alpha_T h\sum_{n=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}n\theta}Y_n(r)\right]\right|_{r=b}=0,$$

$$\left.\left[\frac{\partial}{\partial r}(\nabla^2\Phi)+\frac{1+v}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial\Phi}{\partial r}-\frac{\Phi}{r}\right)+E\alpha_T h\sum_{n=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}n\theta}Y_n'(r)\right]\right|_{r=b}=0 \tag{15}$$

$$\left.\left(\frac{\partial\Phi}{\partial r}+\frac{1}{r}\frac{\partial^2\Phi}{\partial\theta^2}\right)\right|_{r=a}=0,\quad \left.\left(\frac{\partial^2\Phi}{\partial r\partial\theta}-\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right)\right|_{r=a}=0 \tag{16}$$

Mathematically, the solution of the partial stress function $\Phi(r,\theta)$ can be separated into its homogenous and non-homogenous parts [27, 28],

$$\Phi=\Psi_H+\Psi_N \tag{17}$$

where the homogenous part Ψ_H and non-homogenous part Ψ_N of the partial stress function can be solved by the method in Pei et al. [27].

Finally, the partial stress function $\Phi(r,\theta)$ can be represented as:

$$\Phi = E\alpha_T h \sum_{n=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}n\theta} \Phi_n(r) \tag{18}$$

$$\Phi_n(r) = \boldsymbol{R}_n(r)\boldsymbol{c}_n^H + \sum_{m=0}^{\infty} \boldsymbol{S}_{m,n}(\gamma_{m,n} r)\boldsymbol{d}_{m,n} z_{m,n}^N \tag{19}$$

From Pei [37], the solution for vibrational deflection w is governed by Eq. (2) and the initial stress-free transversal runout $\tilde{w}$ can be assumed to be

$$w(r,\theta,t) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty} \mathrm{e}^{\mathrm{i}n\theta} \varphi_{m,n}(r) x_{m,n}(t) \tag{20}$$

$$\tilde{w}(r,\theta,t) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty} \mathrm{e}^{\mathrm{i}n(\theta-\Omega t)} \varphi_{m,n}(r) \tilde{a}_{m,n} \tag{21}$$

where $x_{m,-n} = \mathrm{conj}(x_{m,n})$, $\tilde{a}_{m,-n} = \mathrm{conj}(\tilde{a}_{m,n})$, the disk shape function $\varphi_{m,n}(r)$ can be found in Pei et al. [29].

With Galerkin's method, substituting Eqs. (20) and (21) into Eq. (2), and multiplying Eq. (2) by $\mathrm{e}^{-\mathrm{i}n\theta} \varphi_{m,n}(r)$ then integrating over the disk solution region $r \in [b, a]$ and $\theta \in [0, 2\pi]$ yields

$$\begin{aligned}
&\ddot{\boldsymbol{x}}_n + (c/\rho/h + \mathrm{i}\,2n\Omega)\dot{\boldsymbol{x}}_n + \\
&\{\mathbf{S}_n + \Omega^2\mathbf{L}_n + [\mathrm{i}n\Omega(c-c_L)/\rho/h - n^2\Omega^2]\mathbf{I}_1\}\boldsymbol{x}_n \\
&-E\rho^{-1}\alpha_T \sum_{j=-N_n}^{N_n} \mathbf{T}_{n,j}(\boldsymbol{x}_j + \mathrm{e}^{-\mathrm{i}j\Omega t}\tilde{\boldsymbol{a}}_j) + \sum_{j=-N_n}^{N_n} \mathbf{H}_{n,j}(r_0)\cdot \\
&[m_s(1+\mathrm{i}nr_0^{-1}\mu h/2)(\ddot{\boldsymbol{x}}_j - j^2\Omega^2\mathrm{e}^{-\mathrm{i}j\Omega t}\tilde{\boldsymbol{a}}_j) + \\
&\quad c_s(1+\mathrm{i}nr_0^{-1}\mu h/2)(\dot{\boldsymbol{x}}_j - \mathrm{i}\,j\Omega\,\mathrm{e}^{-\mathrm{i}j\Omega t}\tilde{\boldsymbol{a}}_j) + \\
&\quad k_s(1+\mathrm{i}nr_0^{-1}\mu h/2 + \mathrm{i}\,jr_0^{-1}\mu w_P)(\boldsymbol{x}_j + \mathrm{e}^{-\mathrm{i}j\Omega t}\tilde{\boldsymbol{a}}_j)] \\
&+(\Omega^2\mathbf{L}_n - \mathrm{i}n\Omega c_L\mathbf{I}_1/\rho/h)\mathrm{e}^{-\mathrm{i}n\Omega t}\tilde{\boldsymbol{a}}_n = \boldsymbol{0}
\end{aligned} \tag{22}$$

where the initial normal force P_S is assigned as P_S=$k_S w_P$ resulting in a slider spring initial deformation w_P equivalently; $\boldsymbol{x}_n$=[$x_{0,n}$, $x_{1,n}$, $x_{2,n}$, …, $x_{m,n}$, …], $\tilde{\boldsymbol{a}}_n$ =[$\tilde{a}_{0,n}$, $\tilde{a}_{1,n}$, $\tilde{a}_{2,n}$, …, $\tilde{a}_{m,n}$, …], $\mathbf{I}_1$ denotes the identity matrix,

$$\mathbf{T}_{n,j}=\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & \int_b^a \varphi_{m_1,n} f_{m_1,n,m_2,j}(r)\,r\,\mathrm{d}r & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix}$$ with

$$f_{m_1,n,m_2,j}(r)=\varphi''_{m_2,j}[r^{-1}\Phi'_{n-j}-(n-j)^2 r^{-2}\Phi_{n-j}]+(r^{-1}\varphi'_{m_2,j}-j^2r^{-2}\varphi_{m_2,j})\Phi''_{n-j} \\ +2j(n-j)(r^{-1}\varphi'_{m_2,j}-r^{-2}\varphi_{m_2,j})(r^{-1}\Phi'_{n-j}-r^{-2}\Phi_{n-j}),$$

$$\mathbf{H}_{n,j}=\frac{\mathrm{i}[\mathrm{e}^{\mathrm{i}(-n+j)(\theta_0-\varepsilon_\theta)}-\mathrm{e}^{\mathrm{i}(-n+j)(\theta_0+\varepsilon_\theta)}]}{\pi(-n+j)\rho h\Delta_S}\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & \int_{r_0-\varepsilon_r}^{r_0+\varepsilon_r}\varphi_{m_1,n}\varphi_{m_2,j}\,r\,\mathrm{d}r & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix},$$ and the matrices $\mathbf{S}_n$ and $\mathbf{L}_n$ can be found in Pei et al. [10].

Let $\boldsymbol{X}=[\cdots\ \ \boldsymbol{x}_n^{\mathrm{T}}\ \ \cdots]^{\mathrm{T}}$ and $\boldsymbol{a}=[\cdots\ \ \boldsymbol{a}_n^{\mathrm{T}}\ \ \cdots]^{\mathrm{T}}$ for $n=-N_n,\cdots,-1,0,1,\cdots,N_n$, and rearranging Eq. (7) yields

$$\begin{aligned}&(\mathbf{I}+m_s\mathbf{R})\ddot{\boldsymbol{X}}+(c\mathbf{I}/\rho/h+\mathrm{i}\,2\Omega\mathbf{I}_n+c_s\mathbf{R})\dot{\boldsymbol{X}}+\\ &[(\mathbf{S}-\mathbf{T})+\Omega^2(\mathbf{L}-\mathbf{I}_{n^2})+\mathrm{i}(c-c_L)\Omega\mathbf{I}_n/\rho/h+k_s(\mathbf{R}+\mathrm{i}\,\mu w_P\mathbf{Q})]\boldsymbol{X}=\\ &-\sum_{n=-N_n}^{N_n}\mathrm{e}^{-\mathrm{i}n\Omega t}[\Omega^2\mathbf{L}-\mathrm{i}\,\Omega c_L\mathbf{I}_n/\rho/h+(k_s-\mathrm{i}\,n\Omega c_s-n^2\Omega^2m_s)\mathbf{R}+\mathrm{i}\,\mu k_s w_P\mathbf{Q}]\mathbf{E}_n\tilde{\boldsymbol{a}}\end{aligned} \tag{23}$$

where $\mathbf{S}$=diag([… , $\mathbf{S}_n$, …]), $\mathbf{L}$=diag([… , $\mathbf{L}_n$, …]), $\mathbf{R}=\mathbf{H}+\mathrm{i}\mu h\mathbf{P}/2$,

$$\mathbf{T}=\frac{E\alpha_T}{\rho}\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & \mathbf{T}_{n,j} & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix},\quad \mathbf{H}=\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & \mathbf{H}_{n,j}(r_0,\theta_0) & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix},\quad \mathbf{P}=\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & nr_0^{-1}\mathbf{H}_{n,j}(r_0,\theta_0) & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix}$$

and $\mathbf{Q}=\begin{bmatrix}\ddots & \vdots & \iddots\\ \cdots & jr_0^{-1}\mathbf{H}_{n,j}(r_0,\theta_0) & \cdots\\ \iddots & \vdots & \ddots\end{bmatrix}$.

Some dimensionless parameters are introduced as:

$$\tau=\Omega_{0,0}t,\quad \zeta=\Omega/\Omega_{0,0},\quad \eta=c/(2\rho h\Omega_{0,0}),\quad \chi=c_L/(2\rho h\Omega_{0,0}),$$

$$\eta_s=c_s/(2m_s\xi_s\Omega_{0,0}),\quad \xi_s^{\,2}=k_s/(m_s\Omega_{0,0}^{\,2}),\quad \kappa_F=w_P/h \tag{24}$$

where $\Omega_{0,0}$ is the fundamental frequency of the non-rotating free disk mode (0,0); With respect to $\Omega_{0,0}$, τ is the dimensionless time t, ζ is the dimensionless disk rotating speed Ω,

η and χ is the dimensionless disk viscous damping c and air lift force c_L; η_s and ξ_s are the dimensionless damping ratio and natural frequency of the slider mass-damper-spring loading system $m_s\ddot{x}+c_s\dot{x}+k_s x=m_s{\Omega_{0,0}}^2\left(\frac{\mathrm{d}^2 x}{\mathrm{d}\tau^2}+\frac{c_s}{m_s\Omega_{0,0}}\frac{\mathrm{d}x}{\mathrm{d}\tau}+\frac{k_s}{m_s{\Omega_{0,0}}^2}x\right)$; with respect to the disk thickness h, κ_F is the dimensionless slider spring initial deformation w_P, and it determines the total friction heat power Q_F via Eq. (4) at the disk/slider contact interfaces. Similar non-dimensionalization can also be found in references [2, 4-6, 9-11, 13-14, 19, 22, 27-32, 36, 38].

Using the dimensionless parameters in Eq. (24), Eq. (23) become:

$$\begin{aligned}&(\mathbf{I}+m_s\mathbf{R})\frac{\mathrm{d}^2\boldsymbol{X}}{\mathrm{d}\tau^2}+2(\eta\mathbf{I}+\mathrm{i}\zeta\mathbf{I}_n+m_s\eta_s\xi_s\mathbf{R})\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}\tau}+\\&[(\mathbf{K}-\mathbf{W})+\zeta^2(\mathbf{L}-\mathbf{I}_{n^2})+2\mathrm{i}(\eta-\chi)\zeta\mathbf{I}_n+m_s{\xi_s}^2(\mathbf{R}+\mathrm{i}\mu\kappa_F h\mathbf{Q})]\boldsymbol{X}=\\&-\sum_{n=-N_n}^{N_n}\mathrm{e}^{-\mathrm{i}n\zeta\tau}[\zeta^2\mathbf{L}-2\mathrm{i}\chi\zeta\mathbf{I}_n+m_s({\xi_s}^2-2\mathrm{i}n\zeta\eta_s\xi_s-n^2\zeta^2)\mathbf{R}+\mathrm{i}\kappa_F\mu m_s h{\xi_s}^2\mathbf{Q}]\mathbf{E}_n\tilde{\boldsymbol{a}}\end{aligned}\tag{25}$$

where $\mathbf{K}=\mathbf{S}/{\Omega_{0,0}}^2$ and $\mathbf{W}=\mathbf{T}/{\Omega_{0,0}}^2$.

4 Dynamic stability and steady state amplitude

A solution of the homogenous form of Eq. (25) is assumed to be $\boldsymbol{X}=\mathrm{e}^{\lambda\tau}\boldsymbol{A}$, and then a quadratic eigenvalue problem can be obtained as

$$\begin{aligned}&\{\lambda^2(\mathbf{I}+m_s\mathbf{R})+2\lambda(\eta\mathbf{I}+\mathrm{i}\zeta\mathbf{I}_n+m_s\eta_s\xi_s\mathbf{R})+[(\mathbf{K}-\mathbf{W})+\\&\zeta^2(\mathbf{L}-\mathbf{I}_{n^2})+2\mathrm{i}(\eta-\chi)\zeta\mathbf{I}_n+m_s{\xi_s}^2(\mathbf{R}+\mathrm{i}\mu\kappa_F h\mathbf{Q})]\}\boldsymbol{A}=\boldsymbol{0}\end{aligned}\tag{26}$$

where λ is the system eigenvalue.

As a result, system natural frequency is calculated by $\omega_k=\mathrm{Im}(\lambda_k)$, where λ_k is k-th eigenvalue, and system dynamic stability [10] can be determined by the system stability factor $s = \max_k \mathrm{Re}(\lambda_k)$, the dynamic system is unstable when s>0.

The steady state solution of Eq. (25) can be solved to be

$$\boldsymbol{X} = \sum_{n=-N_n}^{N_n} \mathrm{e}^{-\mathrm{i}n\zeta\tau} \boldsymbol{Y}_n \tag{27}$$

$$\begin{aligned}\boldsymbol{Y}_n = &-\{(\mathbf{K}-\mathbf{W})+\zeta^2(2n\mathbf{I}_n-\mathbf{I}_{n^2}-n^2\mathbf{I})+2\mathrm{i}\zeta[(\eta-\chi)\mathbf{I}_n-\eta n\mathbf{I}]+\\ &\zeta^2\mathbf{L}-2\mathrm{i}\chi\zeta\mathbf{I}_n+m_s[(\xi_s{}^2-2\mathrm{i}n\zeta\eta_s\xi_s-n^2\zeta^2)\mathbf{R}+\mathrm{i}\kappa_F\mu h\xi_s{}^2\mathbf{Q}]\}^{-1}\cdot\\ &\{\zeta^2\mathbf{L}-2\mathrm{i}\chi\zeta\mathbf{I}_n+m_s[(\xi_s{}^2-2\mathrm{i}n\zeta\eta_s\xi_s-n^2\zeta^2)\mathbf{R}+\mathrm{i}\kappa_F\mu h\xi_s{}^2\mathbf{Q}]\}\mathbf{E}_n\tilde{\boldsymbol{a}}\end{aligned} \tag{28}$$

Finally, the total deflection of the disk is governed by:

$$W(r,\theta,\tau) = w(r,\theta,\tau) + \tilde{w}(r,\theta,\tau) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty} \mathrm{e}^{\mathrm{i}n(\theta-\zeta\tau)}\varphi_{m,n}(r) z_{m,n} \tag{29}$$

where $\mathbf{z}_n$=[$z_{0,n}$, $z_{1,n}$, $z_{2,n}$, …, $z_{m,n}$, …], and

$$\boldsymbol{Z} = \boldsymbol{X} + \sum_{n=-N_n}^{N_n} \mathrm{e}^{-\mathrm{i}n\zeta\tau}\, \mathbf{E}_n\tilde{\boldsymbol{a}} = \sum_{n=-N_n}^{N_n} \boldsymbol{z}_n\, \mathrm{e}^{-\mathrm{i}n\zeta\tau} \tag{30}$$

$$\begin{aligned}\boldsymbol{z}_n = &\{(\mathbf{K}-\mathbf{W})+\zeta^2(2n\mathbf{I}_n-\mathbf{I}_{n^2}-n^2\mathbf{I})+2\mathrm{i}\zeta[(\eta-\chi)\mathbf{I}_n-\eta n\mathbf{I}]+\\ &\zeta^2\mathbf{L}-2\mathrm{i}\chi\zeta\mathbf{I}_n+m_s[(\xi_s{}^2-2\mathrm{i}n\zeta\eta_s\xi_s-n^2\zeta^2)\mathbf{R}+\mathrm{i}\xi_s{}^2\mu\kappa_F h\mathbf{Q}]\}^{-1}\cdot\\ &\{(\mathbf{K}-\mathbf{W})+\zeta^2(2n\mathbf{I}_n-\mathbf{I}_{n^2}-n^2\mathbf{I})+2\mathrm{i}\zeta[(\eta-\chi)\mathbf{I}_n-\eta n\mathbf{I}]\}\mathbf{E}_n\tilde{\boldsymbol{a}}\end{aligned} \tag{31}$$

As indicated by Eq. (30), with the initial stress-free transversal runout, with a disk mode (m, n), the total deflection $W_{(m,n)}(r_P, \theta_P, \tau)$ for a given position (r_P, θ_P) vibrates with period $2\pi/\zeta$ and amplitude $A_{(m,n)}(r_P, \theta_P)= \max_{0\le\tau\le2\pi/\zeta} W_{(m,n)}(r_P,\theta_P,\tau)$, as indicated by Pei et al. [29]. Moreover, let the ratio $a_{(m,n)}(r_P, \theta_P) = A/A_0$ be the dimensionless amplitude, where A_0 is the amplitude of the initial transverse runout at the disk outer edge $r=a$, for the disk without any slider and load, i.e. the case for disk rotational speed ζ=0, slider mass m_s=0, disk air lift force χ=0 and slider spring initial deformation κ_F=0 simultaneously.

5 Numerical results and discussions

In the following numerical results, some fundamental and dimensionless parameters are selected, specifically: disk mass density ρ=7840 kg/m^3, Young's modulus E=200×10^9 Pa, thermal expansion coefficient a_T=12×10^{-6} K^{-1}, thermal conductivity k=60 W/(m·K), specific heat c_v=450 J/(kg·K), convective heat transfer coefficient h_T=60 W/(m^2·K), outer radius a=95 mm, inner radius b=16 mm, thickness h=0.5 mm, Poisson's ratio v=0.3, viscous damping ratio η=0.03, friction coefficient μ=0.20, allocation ratio of friction heat shared by the disk k_q=0.6, dimensionless slider damping ratio η_s=0.01 and natural frequency ξ_s=10, and the fundamental frequency $\Omega_{0,0}$=409.4044 rad/s in current case. There are several potential references to practical application of the current thermal boundary control method: optical/hard disk drives, circular blade saws, disk brakes, and turbine disk rotors. Without loss of generality, the above values of these parameters are assigned taking into account the widely different applications for a rotating flexible thin disk and a convenient results comparison for academic investigation simultaneously, it is noted that these values are quite useful to demonstrate how the current new method stabilizes the dynamic instabilities in a rotating flexible annular thin disk.

Using a natural frequency comparison with the commercial software ANSYS Workbench for a free rotating disk in the rotating coordinate system - without the external action of the slider and the surrounding air flow - Table 1 presents a fundamental numerical validation of the current analytical model and solution procedure, the results demonstrate excellent agreement in evaluating the disk natural frequencies, with a maximum disagreement of 0.7431%.

About here to insert **Table. 1**

Without any external action, i.e. in the idealized case of slider mass m_s=0, disk air lift force χ=0 and slider spring initial deformation κ_F=0 simultaneously, the dynamic characteristics of a rotating flexible free disk as a function of the shaft temperature increment Θ_D are illustrated in Fig. 2 in order to provide a reference for the following discussions and comparisons. The flexible disk becomes unstable when the system stability factor s>0 for high or low shaft temperature increments Θ_D, however, there is a stable range of Θ_D as shown in Fig. 2(a). With the increase of disk rotational speed ζ, the stable range of Θ_D widens and spreads to give a stable region of s<0 on the ζ-Θ_D parameter plane in Fig. 2(b). In the stable region, the dimensionless steady state amplitudes $a_{(0,3)}$ of the flexible disk with initial transversal runout of the disk mode (0,3) are calculated and shown in Fig. 2(c) and (d) at the radial middle position $r_P = r_0$ and at the edge a. If the thermal boundary control method was not used, as given by Eq. (6.2), the imaginary part (natural frequency) and real part (dynamic stability) of eigenvalue λ, vary with ζ as plotted in Fig. 2(e). As a result, it can be observed that this idealized system with a free convective inner edge boundary Eq. (6.2) is always stable when s<0 due to the disk inherent viscous damping, and the corresponding steady state amplitude $a_{(0,3)}$, which consistently decreases with ζ.

About here to insert **Fig. 2**

Including the external action of the surrounding air flow, i.e. in the case when the slider mass m_s=0, disk air lift force χ=0.03 and slider spring initial deformation κ_F=0, Fig. 3 shows the effects of dynamic stabilization by shaft temperature increment Θ_D on the rotating flexible thin disk. Without temperature control of the shaft temperature increments Θ_D in Eq. (6.2), the rotating disk is unstable when the system stability factor s>0 as the dimensionless disk rotational speed ζ>0.97 in Fig. 3(e), that is to say the aeroelastic instability occurs inevitably due to the lift force induced by the rotating surround air [11, 12]. The dimensionless speed value ζ_c=0.97 is named as the common critical speed, which is a general speed limit for the disk rotation. Compared with the idealized case in Fig. 2, the stable region on the ζ-Θ_D parameter plane becomes small in Fig. 3(a) and (b) due to the effects of aeroelastic instability. However, there is a stabilized region when the disk rotational speed ζ is greater than the general speed limit 0.97 and even above 2 in Fig. 3(b), this is due to the contribution of dynamic stabilization of the disk thermal membrane stress induced by the shaft temperature increment Θ_D. Therefore, the general speed limit in this case is broken with the help of the dynamic stabilization created by Θ_D, which permits the flexible disk to rotate and work stably above critical and even supercritical speeds as illustrated in Fig. 3(c), (d) and (f).

About here to insert **Fig. 3**

In the following discussion, two configurations of a rotating flexible thin disk are subjected to the stationary slider loading system for different slider pad radial locations, as shown in Fig.4. These two disk/slider configurations are investigated to prove the universal validity of disk dynamic stabilization using a shaft temperature increment Θ_D for a range of different applications. The first configuration Fig. 4(a) is analyzed in Figs. 5, 6 and 7, and the second, Fig. 4(b), is analyzed in Fig. 8 for a middle slider pad radial location r_0, respectively.

About here to insert **Fig. 4**

The effects of dynamic stabilization by shaft temperature increment Θ_D on the rotating flexible thin disk with slider mass m_s=0.01, disk air lift force χ=0 and slider spring initial deformation κ_F=0 are illustrated in Fig. 5. This case is used to investigate a rotating flexible disk subjected to the external action of a slider mass-damper-spring-friction moving load but without the friction heat Q_F. Without temperature control of the shaft temperature increments Θ_D, as shown by the appearance of some Re(λ_k)>0 in Fig. 5(g) middle subplot, the moving load action can result in parametric instability, the system stability factor s>0, of the rotating flexible disk at some disk speeds ζ>1.52, which correspond to the discontinuous lines in Fig. 5(g) lowest subplot. The stable region on the ζ-Θ_D parameter plane in Fig. 5(a) and (b) also becomes small when compared with Fig. 2 due to the effects of parametric instability. Nevertheless, a stabilized region appears for ζ greater than 1.52 and even over 2 in Fig. 5(b) due to the dynamic stabilization by the shaft temperature increment Θ_D, thus the flexible disk under the moving load action of the slider mass-damper-spring-friction system can rotate and work stably over critical and even supercritical speeds. In Fig. 5(c-f) and (h), the steady state amplitudes $a_{(0,3)}$ at different disk radial locations are normal and do not include any extreme large peak value throughout the stabilized region, that is to say the dynamic stabilization does not cause any significant negative influence on the steady state amplitude.

About here to insert **Fig. 5**

The double external action of the slider mass-damper-spring-friction moving load and friction heat Q_F, and the effects of dynamic stabilization using shaft temperature increments Θ_D are presented in Fig. 6 when slider mass m_s=0.01, disk air lift force χ=0 and slider spring initial deformation κ_F=0.02. Besides the parametric instability induced by the moving load, thermoelastic instability can be caused by the friction heat for this case. By comparing Fig. 6(g) with Fig. 5(g) without control of the shaft temperature increments Θ_D, the additional thermoelastic instability results in s>0 of the rotating disk at some much lower disk speeds ζ>0.99. The stable region on the ζ-Θ_D parameter plane in Fig. 6(a) and (b) becomes small, and there also is a stabilized region when ζ is greater than 0.99 and even larger than 2 with normal steady state amplitudes $a_{(0,3)}$. Therefore, the flexible disk subjected to thermoelastic instability induced by the slider friction heat can also be stabilized using the drive shaft temperature increment.

About here to insert **Fig. 6**

Fig. 7 illustrates the effectiveness of dynamic stabilization using the shaft temperature increment Θ_D, when all the external actions are active, that is: the surrounding air flow, slider mass-damper-spring-friction moving load and disk/slider interface friction heat when slider mass m_s=0.01, disk air lift force χ=0.03 and slider spring initial deformation κ_F=0.02. In Fig. 7 the aeroelastic, parametric and thermoelastic instabilities are fully coupled. It can be observed that with fully-coupled instabilities occurring, the dimensionless speed is as low as ζ>0.58 without temperature control, and using dynamic stabilization via the shaft temperature increment Θ_D, disk rotation is stabilized up to a much higher disk speed of ζ=1.85.

About here to insert **Fig. 7**

The fully-coupled instabilities for the other disk/slider configuration of Fig. 4 (b) are illustrated in Fig. 8. For a middle slider radial location, the fully-coupled instabilities occur at a disk speed ζ>1.39 without temperature control, whereas the stabilized region is increased to dimensionless disk rotational speed ζ=2, by the dynamic stabilization via the shaft temperature increment Θ_D. As with the cases in Figs. 3, 5-7, the steady state amplitudes $a_{(0,3)}$ are normal throughout the stabilized regions in Fig. 8.

About here to insert **Fig. 8**

Therefore, the current flexible disk can rotate stably over almost double its critical speed and those instabilities can all be stabilized by using a shaft (disk hub) temperature increment Θ_D of less than about 150 K, as indicated in Figs. (3, 5-8). Thus the thermal boundary control method is practical and achievable for the rotating flexible disk.

6 Conclusions

(1) The parametric, aeroelastic and thermoelastic instabilities in a rotating flexible thin

disk all can be stabilized using the disk thermal membrane stress induced via the shaft temperature increment.

(2) The dynamic stabilization by shaft temperature increments does not cause any significant negative influence on the steady state amplitude of the rotating flexible disk throughout the stabilized disk speed region.

(3) With the current thermal boundary control method using a shaft temperature increment, a flexible disk can exceed its general speed limit, being able to rotate and operate stably over critical and even much higher supercritical speeds.

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Nomenclature

a	disk outer radius
$a_{(m,n)}$	dimensionless steady state amplitude
$\tilde{a}_{m,n}$	initial runout amplitude of disk mode (m,n)
A_0	disk initial runout amplitude
b	disk inner radius
c	disk viscous damping
c_L	disk surrounding air lift force coefficient
c_s	slider damping
c_v	disk specific heat at constant volume
conj	complex conjugate
diag	diagonal matrix
E	disk Young's modulus
h	disk thickness
h_T	disk convective heat transfer coefficient
Im	imaginary part of complex variable
k	disk thermal conductivity
k_q	allocation ratio of friction heat
k_s	slider stiffness
m	number of nodal circles
m_s	slider mass

n	number of nodal diameters
$n_{x,x0}$	finite length step function
P_S	slider initial normal force
Q_F	total friction heat power
r	disk radial polar coordinate
r_0	slider radial center
Re	real part of complex variable
s	system stability factor
t	time
w	disk vibrational deflection
$\tilde{w}$	disk initial runout
w_P	slider spring initial deformation
W	disk total deflection
z	disk transversal coordinate

α_T	disk thermal expansion coefficient
Δ_r	radial constant gap in finite difference
Δ_S	disk/slider interface sectorial area
Δ_V	disk/slider interface sectorial volume
ε_r	disk/slider interface radial length
ε_θ	disk/slider interface circumferential length

ζ dimensionless disk speed

ζ_c dimensionless common critical speed

η dimensionless disk damping

η_s dimensionless slider damping

θ disk circumferential polar coordinate

θ_0 slider circumferential center

Θ disk temperature increment

Θ_D shaft temperature increment

κ_F dimensionless slider spring initial deformation

λ system eigenvalue

μ disk/slider friction coefficient

ν disk Poisson's ratio

ξ_s dimensionless slider frequency

ρ disk mass density

σ_r disk radial membrane stress resultant

σ_θ disk circumferential membrane stress resultant

τ dimensionless time

$\varphi_{m,n}$ disk shape function of disk mode (m,n)

Φ disk partial stress function

χ dimensionless disk air lift force

ω system natural frequency

Ω disk rotating speed

$\Omega_{0,0}$ fundamental frequency of free disk

Constant	normal upright ‘A’
Matrix	bold upright ‘**A**’
Variable	normal italic ‘*A*’
Vector	bold italic ‘***A***’

Appendix. Temperature increment field

Substituting Eq. (12) into Eq. (7) and collecting terms yields

$$y'_{s,n}(a) + h_T k^{-1} y_{s,n}(a) = 0 \tag{A.1}$$

where y' denotes the derivative dy/dr.

And then substituting Eq. (12) into Eqs. (6.1) and (6.2) yields

$$\sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty} \mathrm{e}^{\mathrm{i}n\theta} \cos(\omega_s z) y_{s,n}(b) = \Theta_D \tag{A.2}$$

$$y'_{s,n}(b) - h_T k^{-1} y_{s,n}(b) = 0 \tag{A.3}$$

To solve Eq. (A.2) in its solution region $\theta \in [0,\ 2\pi]$ and $z \in [-h/2,\ h/2]$, it can be assigned as

$$\sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty} \beta_{s,n}\, \mathrm{e}^{\mathrm{i}n\theta} \cos(\omega_s z) = 1 \tag{A.4}$$

where the coefficient $\beta_{s,n}$ in Eq. (A.4) can be solved by the method of least squares fitting over the solution region.

And then $y_{s,n}(b)$ is determined from Eq. (A.2) and Eq. (A.4),

$$y_{s,n}(b) = \beta_{s,n}\Theta_D \tag{A.5}$$

Substituting Eq. (12) into Eq. (5) yields

$$\begin{aligned}&\sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty}\mathrm{e}^{\mathrm{i}n\theta}\cos(\omega_s z)\{y''_{s,n}+r^{-1}y'_{s,n}-[\omega_s{}^2+n^2r^{-2}+\mathrm{i}n\Omega(\rho c_v k^{-1})]y_{s,n}\}\\&+(r/r_0)\,\mathrm{n}_{r,r_0}\mathrm{n}_{\theta,\theta_0}\frac{\mu P_S k_q k^{-1}\Omega}{2\varepsilon_r\varepsilon_\theta h}=0\end{aligned} \tag{A.6}$$

In the solution region $r\in[b, a]$, $\theta\in[0, 2\pi]$ and $z\in[-h/2, h/2]$ of Eq. (A.6), it also can be assigned as

$$\sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty}\gamma_{s,n}(r)\mathrm{e}^{\mathrm{i}n\theta}\cos(\omega_s z)+\mathrm{n}_{r,r_0}\mathrm{n}_{\theta,\theta_0}=0 \tag{A.7}$$

where a solution of $\gamma_{s,n}(r)$ in Eq. (A.7) is assumed as $\gamma_{s,n}(r)=\alpha_{s,n}\,\mathrm{n}_{r,r_0}$, and then Eq. (A.7) is reduced as

$$\sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty}\alpha_{s,n}\,\mathrm{e}^{\mathrm{i}n\theta}\cos(\omega_s z)=-\mathrm{n}_{\theta,\theta_0} \tag{A.8}$$

where the coefficient $\alpha_{s,n}$ in Eq. (A.8) also can be solved by the method of least squares fitting over the solution region $\theta\in[0, 2\pi]$ and $z\in[-h/2, h/2]$.

As a result, Eq. (A.6) can be rewritten as

$$\begin{aligned}&y''_{s,n}+r^{-1}y'_{s,n}-[\omega_s{}^2+n^2r^{-2}+\mathrm{i}n\Omega(\rho c_v k^{-1})]y_{s,n}\\&=\alpha_{s,n}(r/r_0)\,\mathrm{n}_{r,r_0}(\mu P_S k_q k^{-1}\Omega)/(2\varepsilon_r\varepsilon_\theta h)\end{aligned} \tag{A.9}$$

The finite difference method can be used to solve the ordinary differential equation Eq. (A.9). Along the disk radial direction, the solution region $r\in[b, a]$ is discretized into some points $[b, r_{(1)}, r_{(2)}, \ldots, r_{(i)}, \ldots, r_{(N)}, a]$ with a constant gap Δ_r, then Eqs. (A.5, A.3, A.9, A.1) are collected into a finite difference system as

$$y_{s,n}(b) = \beta_{s,n}\Theta_D \tag{A.10}$$

$$-(1+\varDelta_r h_T k^{-1}) y_{s,n}(b) + y_{s,n}(r_{(1)}) = 0 \tag{A.11}$$

$$\begin{aligned}&(2-\varDelta_r r_{(i)}{}^{-1}) y_{s,n}(r_{(i-1)}) - \{4 + 2\varDelta_r{}^2[\omega_s{}^2 + n^2 r_{(i)}{}^{-2} + \mathrm{i}\, n\Omega(\rho c_v k^{-1})]\} y_{s,n}(r_{(i)}) \\ &+ (2+\varDelta_r r_{(i)}{}^{-1}) y_{s,n}(r_{(i+1)}) = 2\varDelta_r{}^2 \alpha_{s,n}(r_{(i)}/r_0)\,\mathrm{n}_{r,r_0}(\mu P_S k_q k^{-1}\Omega)/(2\varepsilon_r\varepsilon_\theta h)\end{aligned} \tag{A.12}$$

$$-y_{s,n}(r_{(N)}) + (1+\varDelta_r h_T k^{-1}) y_{s,n}(a) = 0 \tag{A.13}$$

where $d_{s,n,(i)} = 4 + 2\varDelta_r{}^2[\omega_s{}^2 + n^2 r_{(i)}{}^{-2} + \mathrm{i}\, n\Omega(\rho c_v k^{-1})]$.

Let $\boldsymbol{y}_{s,n} = [y_{s,n}(b), y_{s,n}(r_{(1)}), y_{s,n}(r_{(2)}), \cdots, y_{s,n}(r_{(N)}), y_{s,n}(a)]^{\mathrm{T}}$, Eqs. (A.10), (A.12) and (A.13) can be rewritten in matrix form as

$$\boldsymbol{y}_{s,n} = \Theta_D(\mathbf{A}_{s,n}{}^{-1}\boldsymbol{b}_{s,n}^{\Theta_D}) + \left(\frac{\mu P_S k_q k^{-1}\Omega}{2\varepsilon_r\varepsilon_\theta h}\right)(\mathbf{A}_{s,n}{}^{-1}\boldsymbol{b}_{s,n}) \tag{A.14}$$

where $\boldsymbol{b}_{s,n} = 2\varDelta_r{}^2\alpha_{s,n}[0, \frac{r_{(1)}}{r_0}\mathrm{n}_{r_{(1)},r_0}, \frac{r_{(2)}}{r_0}\mathrm{n}_{r_{(2)},r_0}, \cdots, \frac{r_{(N)}}{r_0}\mathrm{n}_{r_{(N)},r_0}, 0]^{\mathrm{T}}$, $\boldsymbol{b}_{s,n}^{\Theta_D} = \beta_{s,n}[1, 0, \cdots, 0]^{\mathrm{T}}$

and

$$\mathbf{A}_{s,n} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 2-\varDelta_r r_{(1)}{}^{-1} & -d_{s,n,(1)} & 2+\varDelta_r r_{(1)}{}^{-1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2-\varDelta_r r_{(2)}{}^{-1} & -d_{s,n,(2)} & 2+\varDelta_r r_{(2)}{}^{-1} & \cdots & 0 & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \cdots & 2-\varDelta_r r_{(N)}{}^{-1} & -d_{s,n,(N)} & 2+\varDelta_r r_{(N)}{}^{-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1+\varDelta_r h_T k^{-1} \end{bmatrix}.$$

And Eqs. (A.11), (A.12) and (A.13) can be rewritten in matrix form as

$$\boldsymbol{y}_{s,n} = \left(\frac{\mu P_S k_q k^{-1}\Omega}{2\varepsilon_r\varepsilon_\theta h}\right)(\mathbf{A}_{s,n}{}^{-1}\boldsymbol{b}_{s,n}) \tag{A.15}$$

where

$$
\mathbf{A}_{s,n} = \begin{bmatrix} -(1+\Delta_r h_T k^{-1}) & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 2-\Delta_r r_{(1)}^{-1} & -d_{s,n,(1)} & 2+\Delta_r r_{(1)}^{-1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2-\Delta_r r_{(2)}^{-1} & -d_{s,n,(2)} & 2+\Delta_r r_{(2)}^{-1} & \cdots & 0 & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \cdots & 2-\Delta_r r_{(N)}^{-1} & -d_{s,n,(N)} & 2+\Delta_r r_{(N)}^{-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1+\Delta_r h_T k^{-1} \end{bmatrix}.
$$

Figure caption and key

Fig.1 (a) A schematic diagram of the rotating disk subjected to a stationary slider loading system; (b) Thermal boundary feedback control schema using an induction coil to heat the drive shaft.

Fig.2 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0, disk air lift force χ=0 and slider spring initial deformation κ_F=0. (a) Dynamic stability on the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stable region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at slider radial center r_0; (d) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: r_P at disk outer edge a; (e) Eigenvalue λ and $a_{(0,3)}$ without temperature control: r_P at disk outer edge a.

Fig.3 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0, disk air lift force χ=0.03 and slider spring initial deformation κ_F=0. (a) Dynamic stability on the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stabilized region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at slider radial center r_0; (d) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: r_P at disk outer edge a; (e) Eigenvalue λ and $a_{(0,3)}$ without temperature control; (f) Eigenvalue λ and $a_{(0,3)}$ under shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In (a-d), Red dotted line: the assigned shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In the lowest subplot of (e-f), black line $a_{(0,3)}$: r_P at disk outer edge a.

Fig.4 Two disk/slider configurations of a rotating disk subjected to a stationary slider loading system. Slider circumferential center θ_0=0, disk/slider interface radial length ε_r=15h and circumferential length ε_θ=2ε_r/r_0. (a) slider radial center r_0=a-ε_r; (b) slider radial center r_0=b+(a-b)/4.

Fig.5 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0.01, disk air lift force χ=0 and slider spring initial deformation κ_F=0. Disk/slider configuration: Fig. 4(a). (a) Dynamic stability of the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stabilized region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at disk outer edge a, circumferential position θ_P=0°; (d) $a_{(0,3)}$: r_P=a, θ_P=90°; (e) $a_{(0,3)}$: r_P=a, θ_P=180°; (f) $a_{(0,3)}$: r_P=a, θ_P=270°; (g) Eigenvalue λ and $a_{(0,3)}$ without temperature control; (h) Eigenvalue λ and $a_{(0,3)}$ with shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In (a-f), Red dotted line: the assigned shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In the nethermost subplot of (g-h), Blue line $a_{(0,3)}$: radial position r_P at disk outer edge a, circumferential position θ_P=0°; Green line $a_{(0,3)}$: r_P=a, θ_P=90°; Red line $a_{(0,3)}$: r_P=a, θ_P=180°; Cyan line $a_{(0,3)}$: r_P=a, θ_P=270°.

Fig.6 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0.01, disk air lift force χ=0 and slider spring initial deformation κ_F=0.02. Disk/slider configuration: Fig. 4(a). (a) Dynamic stability of the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stabilized region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at disk outer edge a, circumferential position θ_P=0°; (d) $a_{(0,3)}$: r_P=a, θ_P=90°; (e) $a_{(0,3)}$: r_P=a, θ_P=180°; (f) $a_{(0,3)}$: r_P=a, θ_P=270°; (g) Eigenvalue λ and $a_{(0,3)}$ without temperature control; (h) Eigenvalue λ and $a_{(0,3)}$ with shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In (a-f), Red dotted line: the assigned shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In the nethermost subplot of (g-h), Blue line $a_{(0,3)}$: radial position r_P at disk outer edge a, circumferential position θ_P=0°; Green line $a_{(0,3)}$: r_P=a, θ_P=90°; Red line $a_{(0,3)}$: r_P=a, θ_P=180°; Cyan line $a_{(0,3)}$: r_P=a, θ_P=270°.

Fig.7 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0.01, disk air lift force χ=0.03 and slider spring initial deformation κ_F=0.02. Disk/slider configuration: Fig. 4(a). (a) Dynamic stability of the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stabilized region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at disk outer edge a, circumferential position θ_P=0°; (d) $a_{(0,3)}$: r_P=a, θ_P=90°; (e) $a_{(0,3)}$: r_P=a, θ_P=180°; (f) $a_{(0,3)}$: r_P=a, θ_P=270°; (g) Eigenvalue λ and $a_{(0,3)}$ without temperature control; (h) Eigenvalue λ and $a_{(0,3)}$ with shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In (a-f), Red dotted line: the assigned shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In the nethermost subplot of (g-h), Blue line $a_{(0,3)}$: radial position r_P at disk outer edge a, circumferential position θ_P=0°; Green line $a_{(0,3)}$: r_P=a, θ_P=90°; Red line $a_{(0,3)}$: r_P=a, θ_P=180°; Cyan line $a_{(0,3)}$: r_P=a, θ_P=270°.

Fig.8 Dynamic characteristics of the rotating disk using shaft temperature increments for the case of slider mass m_s=0.01, disk air lift force χ=0.03 and slider spring initial deformation κ_F=0.02. Disk/slider configuration: Fig. 4(b). (a) Dynamic stability of the dimensionless disk rotational speed ζ - for shaft temperature increments Θ_D, ζ-Θ_D plane; (b) Stabilized region on ζ-Θ_D plane; (c) Steady state amplitude $a_{(0,3)}$ on ζ-Θ_D plane: radial position r_P at disk outer edge a, circumferential position θ_P=0°; (d) $a_{(0,3)}$: r_P=a, θ_P=90°; (e) $a_{(0,3)}$: r_P=a, θ_P=180°; (f) $a_{(0,3)}$: r_P=a, θ_P=270°; (g) Eigenvalue λ and $a_{(0,3)}$ without temperature control; (h) Eigenvalue λ and $a_{(0,3)}$ with shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In (a-f), Red dotted line: the assigned shaft temperature increment control $\underline{\Theta}_D(\zeta)$. In the nethermost subplot of (g-h), Blue line $a_{(0,3)}$: radial position r_P at disk outer edge a, circumferential position θ_P=0°; Green line $a_{(0,3)}$: r_P=a, θ_P=90°; Red line $a_{(0,3)}$: r_P=a, θ_P=180°; Cyan line $a_{(0,3)}$: r_P=a, θ_P=270°.

Figure caption

Table 1 Numerical model validation using the natural frequency for a free rotating disk