Choosing Monetary Sequences: Theory and Experimental Evidence - Technical and Data appendix

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A Appendix

B General properties of the $(\sigma - \delta)$ model

The linearly additive form in which the vagueness term enters the formula is equivalent to assuming that the following property on the primary criterion holds. For $a \in A$, let $L(a)$ be the lower contour set of a, that is the set of all sequences to which a is preferred according to the primary criterion:

$$L(a) = \{b \in A | aP_1 b\}$$

Discrimination: For $a, b \in A$, either $L(a) \subset L(b)$ or $L(b) \subset L(a)$.

The proof of equivalence and the additivity of P_1 follows from a simple adaptation of Manzini and Mariotti ([2], Proposition 1) or even from the classical results of Fishburn [1]. The assumption of Discrimination is tantamount to assuming that there is a hidden 'directionality' in the preferences expressed by P_1: if P_1 is a complete relation, Discrimination follows automatically from transitivity; otherwise it means that if a sequence a is not related by P_1 to another sequence b, either one 'improves' in moving from a to b in the sense of increasing the number of dominated alternatives, or viceversa. So either all alternatives dominated by a are also dominated by b, or viceversa. Note that this assumption makes P_1 more rational rather than less rational, so it does not particularly help *per se* in explaining irrational choices. The additional assumption of time-consistency embodied in the choice of the exponential discounting functional form makes the criterion P_1 even more standard.

Next, we discuss the applicability of our theoretical framework by showing how it can accommodate some apparent observed violations of the standard model.

Cycles

Even if *both* the primary and secondary criteria are transitive, it is not difficult to verify that the preference $\succ^*$ obtained from their sequential application is not necessarily transitive. For example, suppose that for a decision maker the following holds:

$$\begin{aligned}
\Sigma_t \delta^t u(b_t) &> \Sigma_t \delta^t u(a_t) > \Sigma_t \delta^t u(c_t) \\
\Sigma_t \delta^t u(b_t) &> \Sigma_t \delta^t u(c_t) + \sigma(c_t) \\
\Sigma_t \delta^t u(a_t) + \sigma(a_t) &\geqslant \Sigma_t \delta^t u(b_t) \\
\Sigma_t \delta^t u(c_t) + \sigma(c_t) &\geqslant \Sigma_t \delta^t u(a_t) \\
& \quad cP_2 aP_2 b
\end{aligned}$$

then the resulting complete preference relation $\succ^*$ is cyclical and given by

$$a \succ^* b \succ^* c \succ^* a$$

Tversky [5] first showed experimentally evidence of cycles in intertemporal choice. More recent and striking evidence of cyclical choices in an intertemporal context is due to Roelofsma and Read [3].

Rubinstein's experiment

Rubinstein [4] reports that subjects exhibited the following type of behavior: they chose x to be received at date t^* over $x+z$ to be received at date t^*+1 (they were impatient and preferred smaller reward earlier rather than larger reward later) but chose the sequence

$$a = (x+z, t'+1; x+z, t'+2; x+z, t'+3; x+z, t'+4)$$

over

$$b = (x, t'; x, t'+1; x, t'+2; x, t'+3)$$

where $t'+4 \leqslant t^*+1$. This contradicts not only exponential discounting but also hyperbolic discounting and in fact any model of discounting based on diminishing impatience (declining discount rates): if a subject were impatient at the late date t^* and not willing to trade off one unit of delay for an additional reward z, he should have been unwilling to perform all four trade-offs of this type involved in the comparisons between sequences.

Our simple model can explain these preferences. Suppose for example that the secondary criterion is the natural one proposed by Rubinstein himself, namely 'Pareto dominance' between the outcome sequences, and that

$$\begin{aligned} \delta^{t^*} u(x) &> \delta^{t^*+1} u(x+z) + \sigma(x+z, t^*+1) \\ \Sigma_{i=0}^{3} \delta^{t'+i} u(x) &\leqslant \Sigma_{i=1}^{4} \delta^{t'+i} u(x+z) + \sigma(x+z, t'+i) \end{aligned}$$

In this case the preference between date-outcome pairs can be explained by present discounted utility (primary criterion) and the preference between sequences can be explained by the secondary criterion.

B.1 Preference profiles in the $(\sigma - \delta)$ model

B.1.1 Three period sequences

We provide a full derivation of the results presented in sections 4 and 5 in the paper. Recall that we are making the following assumptions: u is monotonic increasing in outcome, concave and with positive third and the discounting function is monotonically non increasing.

Moreover, we are fixing the times at which outcomes are received as 0, 1 and 2, and for simplicity we denote $u_1 = u(8)$, $u_2 = u(16)$, $u_3 = u(24)$ and $u_4 = u(32)$; and for the discounting function let $\delta(0) = \delta_0$, $\delta(1) = \delta_1$, and $\delta(2) = \delta_2$.

The three period sequences of payments considered by experimental subjects were the following:

$I = (8, 16, 24)$

$K = (16, 16, 16)$

$D = (24, 16, 8)$

$J = (8, 8, 32)$

As we have already pointed out, any pure discounting criterion for choice $\succ_d$ should order them as either $D \succ_d K \succ_d I \succ_d J$ or $K \succ_d D \succ_d I \succ_d J$.

Denoting by $\succ_d$ the preference relation of a decision maker who discounts utility available at time t by some monotonically non decreasing discount function δ_t, then

$$D3 \succ_d K3 \Leftrightarrow \delta_2 < \frac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L \text{ and } D2 \succ_d K2 \Leftrightarrow \delta_2 < \frac{u_4 - u_3}{u_3 - u_2} \equiv \delta_S \tag{1}$$

holds since:

$$\begin{aligned} D \succ_d K \Leftrightarrow u_3 + \delta_1 u_2 + \delta_2 u_1 > u_2 + \delta_1 u_2 + \delta_2 u_2 \\ \Leftrightarrow \delta_2 < \frac{u_3 - u_2}{u_2 - u_1} \end{aligned}$$

Secondly,

$$D \succ_d I \text{ always} \tag{2}$$

holds since:

$$\begin{aligned} D \succ_d I \Leftrightarrow u_3 + \delta_1 u_2 + \delta_2 u_1 > u_1 + \delta_1 u_2 + \delta_2 u_3 \\ \Leftrightarrow u_3 - u_1 > \delta_2 (u_3 - u_1) \end{aligned}$$

while

$$K \succ_d I \text{ always} \tag{3}$$

holds since:

$$K \succ_d I \Leftrightarrow u_2 + \delta_1 u_2 + \delta_2 u_2 > u_1 + \delta_1 u_2 + \delta_2 u_3 \Leftrightarrow u_2 - u_1 > \delta_2 (u_3 - u_2)$$

Finally,

$$I \succ_d J \text{ always} \tag{4}$$

holds (with a nondecreasing discount function) since:

$$I \succ_d J \Leftrightarrow u_1 + \delta_1 u_2 + \delta_2 u_3 > u_1 + \delta_1 u_1 + \delta_2 u_4 \Leftrightarrow \delta_1 (u_2 - u_1) > \delta_2 (u_4 - u_3)$$

Then, the only two patterns of choice consistent with pure discounting theories are $D \succ_d K \succ_d I \succ_d J$ and $K \succ_d D \succ_d I \succ_d J$.

We can now turn to check what preference profiles are compatible with the $(\sigma - \delta)$ model. We begin by enumerating them, then we show how the various restrictions on the parameters have been derived.

As we explain in the paper, in the $(\sigma - \delta)$ model the secondary criterion alone implies $I \succ_2 K \succ_2 D$, where $\succ_2$ is transitive.

The profiles of choice based on all pairwise comparisons involving the three series I, K and D which are compatible with the model of vague time preferences are as follows:

1. $D \succ^* K \succ^* I$: this can be if $\sigma < (u_3 - u_2) - \delta^2 (u_2 - u_1)$ and $\delta_2 < \frac{u_3 - u_2}{u_2 - u_1}$. For this profile of preferences to obtain it is necessary that $D \succ_1 K \succ_1 I$ and $D \succ_1 I$. This corresponds to the standard preferences of a rational exponential discounting decision maker.

2. $K \succ^* D \succ^* I$: this can be if
   (a) $\sigma < \left(1 - \delta^2\right)(u_3 - u_1)$ and $\delta^2 \in \left(\frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)}, 1\right)$
   (b) $\sigma < \delta^2 (u_2 - u_1) - (u_3 - u_2)$ and $\delta^2 \in \left(\frac{u_3-u_2}{u_2-u_1}, \frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)}\right)$
   (c) $\sigma \in \left[(u_3 - u_2) - \delta^2 (u_2 - u_1), (u_3 - u_2) - \delta^2 (u_3 - u_2)\right)$ and $\delta^2 \in \left(0, \frac{u_3-u_2}{u_2-u_1}\right)$

   For this profile of preferences to obtain it is necessary that $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$.

3. $D \succ^* I \succ^* K \succ^* D$: this can be if $\sigma \in \left[(u_2 - u_1) - \delta^2 (u_3 - u_2), (1 - \delta^2)(u_3 - u_1)\right)$ and $\delta^2 < \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $I \succ_2 K \succ_{1,2} D \succ_1 I$.

4. $I \succ^* K \succ^* D$: this can be if $\sigma \geqslant \left(1 - \delta^2\right)(u_3 - u_1)$ and $\delta^2 < \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $I \succ_2 K \succ_{1,2} D$ and $I \succ_2 D$.

5. $K \succ^* I \succ^* D$: this can be if $\sigma \in \left[\left(1 - \delta^2\right)(u_3 - u_1), \delta^2 (u_2 - u_1) - (u_3 - u_2)\right)$ and $\delta^2 > \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $K \succ_1 I \succ_2 D$ and $K \succ_{1,2} D$.

6. $D \succ^* K \succ^* I \succ^* D$: this is incompatible with the model $(\sigma - \delta)$ model, since it would require $K \succ_1 I \succ_2 D \succ_1 K$ which is impossible.

7. $I \succ^* D \succ^* K$: this is incompatible with the $(\sigma - \delta)$ model, since it would require $I \succ_2 D \succ_1 K$ and $I \succ_2 K$ which is impossible.

8. $D \succ^* I \succ^* K$: this is incompatible with the $(\sigma - \delta)$ model, since it would require $D \succ_1 I \succ_2 K$ and $D \succ_1 K$, which is impossible.

We now show how the conditions for the preference profiles above to be compatible or otherwise with the $(\sigma - \delta)$ model have been derived.

1. **Comparison between series I and K:** consider the following inequality (which, if true, would imply that I is preferred to K by the primary criterion):

$$u_1 + \delta u_2 + \delta^2 u_3 > u_2 + \delta u_2 + \delta^2 u_2 + \sigma$$

This can be rearranged as

$$u_1 + \delta^2 u_3 > u_2 + \delta^2 u_2 + \sigma \Leftrightarrow \sigma < \delta^2 (u_3 - u_2) - (u_2 - u_1) < 0.$$

The last inequality follows from the fact that $\delta^2 < 1$ and from our assumption of decreasing marginal utility, so that

$$(u_2 - u_1) > (u_3 - u_2) > \delta^2 (u_3 - u_2)$$

Consequently, given that σ must be positive, it can only be that

$$u_1 + \delta u_2 + \delta^2 u_3 < u_2 + \delta u_2 + \delta^2 u_2 + \sigma$$

in other words, it can never be that I 'beats outright' K. Therefore the comparison between the two sequences hinges on whether or not the following holds:

$$u_2 + \delta u_2 + \delta^2 u_2 \leqslant u_1 + \delta u_2 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma \geqslant (u_2 - u_1) - \delta^2 (u_3 - u_2) > 0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 K \Leftrightarrow \sigma \geqslant (u_2 - u_1) - \delta^2 (u_3 - u_2) \equiv A$$
$$K \succ_1 I \Leftrightarrow \sigma < (u_2 - u_1) - \delta^2 (u_3 - u_2) \equiv A$$

2. **Comparison between series I and D:** consider the following inequality (which, if true, would imply that a decision maker chooses I over D by the primary criterion)

$$u_1 + \delta u_2 + \delta^2 u_3 > u_3 + \delta u_2 + \delta^2 u_1 + \sigma$$

which is equivalent to

$$u_1 + \delta^2 u_3 > u_3 + \delta^2 u_1 + \sigma \Leftrightarrow \sigma < -(1-\delta^2)(u_3 - u_1) < 0$$

As above since we require $\sigma \geq 0$, it can only be that $u_1 + \delta u_2 + \delta^2 u_3 \leqslant u_3 + \delta u_2 + \delta^2 u_1 + \sigma$, i.e. it can never be that I beats D outright. Therefore the comparison hinges upon:

$$u_3 + \delta u_2 + \delta^2 u_1 \leqslant u_1 + \delta u_2 + \delta^2 u_3 + \sigma \Leftrightarrow u_3 + \delta^2 u_1 \leqslant u_1 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma \geqslant (1-\delta^2)(u_3 - u_1) > 0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 D \Leftrightarrow \sigma \geqslant (1 - \delta^2)(u_3 - u_1) \equiv B$$
$$D \succ_1 I \Leftrightarrow \sigma < (1 - \delta^2)(u_3 - u_1) \equiv B$$

3. **Comparison between series K and D:** consider the following inequality (which, if true, would imply that K is chosen over D by the primary criterion):

$$u_2 + \delta u_2 + \delta^2 u_2 > u_3 + \delta u_2 + \delta^2 u_1 + \sigma$$

equivalent to

$$u_2 + \delta^2 u_2 > u_3 + \delta^2 u_1 + \sigma \Leftrightarrow \sigma < \delta^2 (u_2 - u_1) - (u_3 - u_2)$$

The value on the left hand side is positive provided that $\delta^2 > \frac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L$. So if the discount function in period 2 is sufficiently large, then K can beat D outright provided the vagueness term is sufficiently *small*. If instead $\delta^2 \leqslant \delta_L$, then it can never be that K beats D outright, and as in the other cases the comparison hinges on whether:

$$u_3 + \delta u_2 + \delta^2 u_1 \leqslant u_2 + \delta u_2 + \delta^2 u_2 + \sigma \Leftrightarrow u_3 + \delta^2 u_1 \leqslant u_2 + \delta^2 u_2 + \sigma$$
$$\Leftrightarrow \sigma \geqslant (u_3 - u_2) - \delta^2 (u_2 - u_1) \geqslant 0$$

(where the last inequality follows from our assumption that $\delta^2 \leqslant \delta_L$). In short we can summarize as follows:

$$\text{if } \delta^2 > \tfrac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L, \text{ then: } \begin{cases} K \succ_1 D \text{ if } \sigma < \delta^2 (u_2 - u_1) - (u_3 - u_2) \equiv -C, \text{ and} \\ K \succ_2 D \text{ if } \sigma \geqslant \delta^2 (u_2 - u_1) - (u_3 - u_2) \equiv -C \end{cases}$$

$$\text{if } \delta^2 < \tfrac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L, \text{ then: } \begin{cases} D \succ_1 K \text{ if } \sigma < (u_3 - u_2) - \delta^2 (u_2 - u_1) \equiv C, \text{ and} \\ K \succ_2 D \text{ if } \sigma \geqslant (u_3 - u_2) - \delta^2 (u_2 - u_1) \equiv C \end{cases}$$

Next we compare the relative magnitudes of A, B, C and $-C$ to see which configurations of preferences of the eight listed above are compatible with the model. Recall:

$$A \equiv (u_2 - u_1) - \delta^2 (u_3 - u_2) > 0$$
$$B \equiv (1 - \delta^2)(u_3 - u_1) > 0$$
$$C \equiv (u_3 - u_2) - \delta^2 (u_2 - u_1)$$
$$-C \equiv < \delta^2 (u_2 - u_1) - (u_3 - u_2)$$

Observe that

$$A > B \Leftrightarrow \delta^2 > \tfrac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L$$

since

$$A > B \Leftrightarrow (u_2 - u_1) - \delta^2 (u_3 - u_2) > (1 - \delta^2)(u_3 - u_1) \Leftrightarrow \delta^2 (u_2 - u_1) > (u_3 - u_2)$$

Next observe that $A > C$ always, since

$$A > C \Leftrightarrow (u_2 - u_1) - \delta^2 (u_3 - u_2) > (u_3 - u_2) - \delta^2 (u_2 - u_1) \Leftrightarrow (1+\delta^2)(u_2 - u_1) > (1+\delta^2)(u_3 - u_2)$$

which is always true given our assumptions on the shape of the utility function). Finally, observe that whenever $\delta < \delta_L$, the threshold C is negative. In this case comparing A and $-C$ yields $A > -C$ always, since:

$$\begin{aligned} A &> -C \Leftrightarrow (u_2 - u_1) - \delta^2 (u_3 - u_2) > \delta^2 (u_2 - u_1) - (u_3 - u_2) \\ &\Leftrightarrow (1 - \delta^2)(u_2 - u_1) > -(1 - \delta^2)(u_3 - u_2) \end{aligned}$$

In the comparison between B and C we have $B > C$ always, since:

$$B > C \Leftrightarrow (1 - \delta^2)(u_3 - u_1) > (u_3 - u_2) - \delta^2 (u_2 - u_1) \Leftrightarrow (u_2 - u_1) > \delta^2 (u_3 - u_2)$$

and the last inequality holds by our assumption on the shape of the utility function. Finally,

$$B > -C \Leftrightarrow \delta^2 < \frac{(u_3 - u_1) + (u_3 - u_2)}{(u_2 - u_1) + (u_3 - u_1)} \equiv \overline{\delta} < 1$$

since

$$\begin{aligned} B &> -C \Leftrightarrow (1 - \delta^2)(u_3 - u_1) > \delta^2 (u_2 - u_1) - (u_3 - u_2) \\ &\Leftrightarrow (u_3 - u_1) + (u_3 - u_2) > \delta^2 [(u_2 - u_1) + (u_3 - u_1)] \end{aligned}$$

from which the condition on the discount factor follows.

Observe that given our assumption on diminishing increases in utility it is always the case that

$$\overline{\delta} > \delta_L$$

since

$$\overline{\delta} > \delta_L \Leftrightarrow \frac{(u_3 - u_1) + (u_3 - u_2)}{(u_2 - u_1) + (u_3 - u_1)} > \frac{u_3 - u_2}{u_2 - u_1} \Leftrightarrow u_2 - u_1 > u_3 - u_2$$

where the last inequality holds because of our assumption on the shape of the outcome utility function.

Summing up:

1. If $\delta^2 \in (0, \delta_L)$, then $B > A > C > 0 > -C$.

2. If $\delta^2 \in \left(\delta_L, \overline{\delta}\right)$, then $A > B > -C > 0 > C$.

3. If $\delta^2 \in \left(\overline{\delta}, 1\right)$, then $A > -C > B > 0 > C$.

We can now derive the conditions such that the various preference profiles listed at the beginning are compatible with the $(\sigma - \delta)$ model.

1. Profile $D \succ_1 K \succ_1 I$ and $D \succ_1 I$: it requires $\sigma < C$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$), $\sigma < A$ (to have $K \succ_1 I$) and $\sigma < B$ (to have $D \succ_1 I$). Since we need $\delta^2 < \delta_L$, then $B > A > C > 0 > -C$, so that $\sigma < \min\{A, B, C\} = C$.

2. Profile $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$: it requires either $\sigma < -C$ and $\delta^2 > \delta_L$, or $\sigma \geqslant C$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), $\sigma < B$ (to have $D \succ_1 I$) and $\sigma < A$ (to have $K \succ_1 I$). If $\delta^2 > \delta_L$, then we require $\sigma < \min\{A, B, -C\}$, which is either B or $-C$ depending on how large δ^2 is (i.e. whether or not it is greater than $\overline{\delta}$). If instead $\delta^2 < \delta_L$, we know that $B > A > C > 0 > -C$, in which case the requirement is $\sigma \in [C, A)$.

3. Profile $D \succ_1 I \succ_2 K \succ_{1,2} D$: this requires $\sigma \geqslant A$ (to have $I \succ_2 K$), either $\sigma < -C$ and $\delta^2 > \delta_L$, or $\sigma \geqslant C$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), $\sigma < B$ (to have $D \succ_1 I$). Since at the very least we need $A \leqslant \sigma < B$, we must be in the situation where $\delta^2 < \delta_L$, in which case we also need $\sigma \geqslant C$, which is however not binding. Consequently this preference profile is compatible with $\sigma \in [A, B)$ and $\delta^2 < \delta_L$.

4. Profile $I \succ_2 K \succ_{1,2} D$ and $I \succ_2 D$: this would require $\sigma \geqslant A$ (to have $I \succ_2 K$), either $\sigma < -C$ and $\delta^2 > \delta_L$, or $\sigma \geqslant C$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), and $\sigma \geqslant B$ (to have $I \succ_2 D$). So, if $\delta^2 < \delta_L$ as we saw above we have $B > A > C > 0 > -C$, so

that in this case all conditions are satisfied provided that $\sigma \geqslant \max\{A, B, C\} = B$, whereas if $\delta^2 > \delta_L$ there is no possibility that the profile can be justified, since the interval (A, C) is in this case empty.

5. Profile $K \succ_1 I \succ_2 D$ and $K \succ_{1,2} D$: this requires $\sigma < A$ (to have $K \succ_1 I$), either $\sigma < -C$ and $\delta^2 > \delta_L$, or $\sigma \geqslant C$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), and $\sigma \geqslant B$ (to have $I \succ_2 D$). Since at the very least we must have $\sigma \in (B, A)$, it must be that $\delta^2 > \delta_L$, so that $A > B$. Then the comparison between K and D is resolved on the basis of the primary criterion, and we need $\sigma < \min\{A, -C\} = -C$. In short, for this profile of preferences we need $\sigma \in [B, C)$ and $\delta^2 > \delta_L$.

6. Profile $D \succ_1 K \succ_1 I \succ_2 D$: this requires $\sigma < A$ (to have $K \succ_1 I$), $\sigma \geqslant B$ (to have $I \succ_2 D$), and $\sigma < C$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$), so that $B > A > C > 0 > -C$. Since σ cannot be at the same time smaller than A and greater than B, this case is not compatible with the $(\sigma - \delta)$ model.

7. Profile $I \succ_2 D \succ_1 K$ and $I \succ_2 K$: would require $\sigma \geqslant A$ (to have $I \succ_2 K$), $\sigma \geqslant B$ (to have $I \succ_2 D$), and $\sigma < C$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$), so that the interval (A, C) is empty, incompatible with the $(\sigma - \delta)$ model.

8. Profile $D \succ_1 I \succ_2 K$ and $D \succ_1 K$: requires $\sigma < B$ (to have $D \succ_1 I$), $\sigma \geqslant A$ (to have $I \succ_2 K$), and $\sigma < C$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$), implying $B > A > C > 0 > -C$, incompatible with the $(\sigma - \delta)$ model.

Figure 1 depicts these choice profiles for given combinations of the preference parameters. Inspection of this figure shows that when C is negative the relative position of $-C$ with respect to A and B is irrelevant. This is why in the paper we present the more compact version, ignoring $-C$.

B.1.2 Two period sequences

The analysis of two period sequence is parallel to that carried out so far. Now the periods involved are only 0 and 2, where recall that $u_2 = u(16)$, $u_3 = u(24)$ and $u_4 = u(32)$.

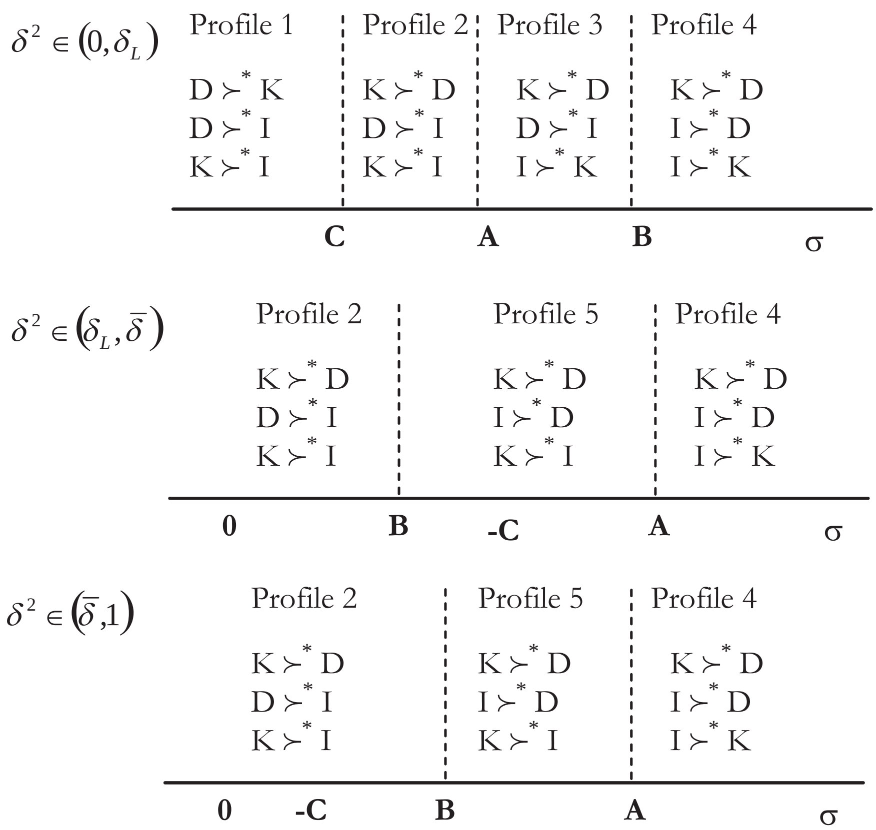

Figure 1: Admissible choice profiles in the $(\sigma - \delta)$ model for three period sequences

Moreover we denote $u_5 = u(40)$. The two period sequences of payments considered by the experimental subjects were the following:

$I = (16, 32)$

$K = (24, 24)$

$D = (32, 16)$

$J = (8, 40)$

Here too any pure discounting criterion for choice $\succ_d$ should order them as either $D \succ_d K \succ_d I \succ_d J$ or $K \succ_d D \succ_d I \succ_d J$.

Denoting by $\succ_d$ the preference relation of a decision maker who discounts utility available at time t by some monotonically non decreasing discount function δ_t, then condition (1) now holds since:

$$D \succ_d K \Leftrightarrow u_4 + \delta_2 u_2 > u_3 + \delta_2 u_3 \Leftrightarrow \delta_2 < \frac{u_4 - u_3}{u_3 - u_2}$$

Checking back the conditions for the choice between D and K for long sequences, recall that D is chosen over K if and only if

$$\delta_2 < \frac{u_3 - u_2}{u_2 - u_1} < \frac{u_4 - u_3}{u_3 - u_2}$$

where the last inequality follows from our assumption of positive third derivative. Consequently, a choice of D over K for the long sequences implies a choice of D over K for the short sequences, too. Equivalently, a choice of K over D for the short sequences means that the subject is patient enough that he should also choose K over D in the long sequences. In short, then, a cross-tabulation of $DK2$ and $DK3$ should have at least a zero off-diagonal element (corresponding to the $D3 \succ K3/K2 \succ D2$ cell).

Secondly, condition (2) holds given that:

$$D \succ_d I \Leftrightarrow u_4 + \delta_2 u_2 > u_2 + \delta_2 u_4 \Leftrightarrow u_4 - u_2 > \delta_2(u_4 - u_2)$$

while condition (3) holds since:

$$K \succ_d I \Leftrightarrow u_3 + \delta_2 u_3 > u_2 + \delta_2 u_4 \Leftrightarrow u_3 - u_2 > \delta_2 \left(u_4 - u_3\right)$$

Finally, condition (4) holds since:

$$I \succ_d J \Leftrightarrow u_2 + \delta_2 u_4 > u_1 + \delta_2 u_5 \Leftrightarrow u_2 - u_1 > \delta_2 \left(u_5 - u_4\right)$$

Once more then the only two patterns of choice consistent with discounting theories are as for the three period sequences.

Turning now to the $(\sigma - \delta)$ model, the profiles of choices based on all pairwise comparisons involving the three series are derived in an analogous way as for the three period sequences. Define:

$$a \equiv (u_3 - u_2) - \delta^2 (u_4 - u_3) > 0$$
$$b \equiv \left(1 - \delta^2\right)(u_4 - u_2) > 0$$
$$c \equiv (u_4 - u_3) - \delta^2 (u_3 - u_2)$$
$$-c \equiv \delta^2 (u_3 - u_2) - (u_4 - u_3)$$
$$\delta_S \equiv \frac{u_4 - u_3}{u_3 - u_2} \text{ and} \overline{\gamma} \equiv \frac{(u_4 - u_3) + (u_4 - u_2)}{(u_3 - u_2) + (u_4 - u_2)}$$

and observe that $\overline{\gamma} > \delta_S$, and that[1]:

- If $\delta^2 \in (0, \delta_S)$, then $b > a > c > 0 > -c$.
- If $\delta^2 \in (\delta_S, \overline{\gamma})$, then $a > b > -c > 0 > c$.
- If $\delta^2 \in (\overline{\gamma}, 1)$, then $a > -c > b > 0 > c$.

Then the choice profiles compatible with the $(\sigma - \delta)$ model are as follows:

1. $D \succ^* K \succ^* I$: this can be if $\sigma < c$ and $\delta^2 \in (0, \delta_S)$;
2. $K \succ^* D \succ^* I$: this can be if
   (a) $\sigma < b$ and $\delta^2 \in (\overline{\gamma}, 1)$
   (b) $\sigma < -c$ and $\delta^2 \in (\delta_S, \overline{\gamma})$;
   (c) $\sigma \in [c, a)$ and $\delta^2 \in (0, \delta_S)$;
3. $D \succ^* I \succ^* K \succ^* D$: this can be if $\sigma \in [a, b)$ and $\delta^2 \in (0, \delta_S)$;
4. $I \succ^* K \succ^* D$: this can be if $\sigma \geqslant b$ and $\delta^2 \in (0, \delta_S)$;
5. $K \succ^* I \succ^* D$: this can be if $\sigma \in [b, a]$ and $\delta^2 > \delta_S$;
6. $D \succ^* K \succ^* I \succ^* D$;
7. $I \succ^* D \succ^* K$;
8. $D \succ^* I \succ^* K$.

[1] A full derivation of these values follows later on.

where the last three profiles are not compatible with the $(\sigma - \delta)$ model. We derive these results below. Before doing so, though, observe the relationship that our model postulates between choice profiles in the two and three period cases. As we show below, it is the case that $\delta_L < \delta_S$, $\overline{\delta} < \overline{\gamma}$, $A > a$, $B > b$ and $C > c$.

Figure 3 in the paper illustrates this point, showing that a switch either from $D \succ^* K \succ^* I$ to $K \succ^* D \succ^* I$ or the opposite switch from $K \succ^* D \succ^* I$ to $D \succ^* K \succ^* I$ with sequence length is possible.

Next we show how the various thresholds have been derived for the case of choice between two period sequences.

1. **Comparison between series I and K:** sequence I is chosen over K by the primary criterion whenever

$$u_2 + \delta^2 u_4 > u_3 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma < \delta^2 (u_4 - u_3) - (u_3 - u_2) < 0$$

where the last inequality follows from the fact that $\delta^2 < 1$ and from our assumption of decreasing marginal utility, so that

$$(u_3 - u_2) > (u_4 - u_3) > \delta^2 (u_4 - u_3)$$

Consequently, given that σ must be non negative, it can only be that

$$u_2 + \delta^2 u_4 < u_3 + \delta^2 u_3 + \sigma$$

in other words, I can never beat K outright. Therefore the comparison between the two sequences hinges on whether or not the following holds:

$$u_3 + \delta^2 u_3 \leqslant u_2 + \delta^2 u_4 + \sigma \Leftrightarrow \sigma \geqslant (u_3 - u_2) - \delta^2 (u_4 - u_3) > 0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 K \Leftrightarrow \sigma \geqslant (u_3 - u_2) - \delta^2 (u_4 - u_3) \equiv a < A \equiv (u_2 - u_1) - \delta^2 (u_3 - u_2)$$
$$K \succ_1 I \Leftrightarrow \sigma < (u_3 - u_2) - \delta^2 (u_4 - u_3) \equiv a < A \equiv (u_2 - u_1) - \delta^2 (u_3 - u_2)$$

where the comparison between a and A follows from our assumptions on the concavity of the utility function. This means that if an agent chooses K over I (by the primary criterion) when confronted with short sequences, he must do so when confronted with longer sequences too.

2. **Comparison between series I and D:** Sequence I is chosen over series D by the primary criterion if

$$u_2 + \delta^2 u_4 > u_4 + \delta^2 u_2 + \sigma \Leftrightarrow \sigma < -(1-\delta^2)(u_4 - u_2) < 0$$

Again, because of the restriction on σ, it can only be that

$$u_2 + \delta^2 u_4 \leqslant u_4 + \delta^2 u_2 + \sigma$$

i.e. it can never be that I beats D outright. Therefore the comparison hinges on:

$$u_4 + \delta^2 u_2 \leqslant u_2 + \delta^2 u_4 + \sigma \Leftrightarrow \sigma \geqslant (1-\delta^2)(u_4 - u_2) > 0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$\begin{aligned} I \succ_2 D &\Leftrightarrow \sigma \geqslant (1-\delta^2)(u_4 - u_2) \equiv b < B \equiv (1-\delta^2)(u_3 - u_1) \\ D \succ_1 I &\Leftrightarrow \sigma < (1-\delta^2)(u_4 - u_2) \equiv b < B \equiv (1-\delta^2)(u_3 - u_1) \end{aligned}$$

where again the comparison between b and B follows from the assumptions on the third derivative of the utility function. As a consequence, if an agent chooses D over I in the short sequences, he must do so in the long sequences, too.

3. **Comparison between series K and D:** Sequence K is chosen over sequence D by the primary criterion if

$$u_3 + \delta^2 u_3 > u_4 + \delta^2 u_2 + \sigma \Leftrightarrow \sigma < \delta^2 (u_3 - u_2) - (u_4 - u_3)$$

The left hand side is positive provided that $\delta^2 > \frac{u_4 - u_3}{u_3 - u_2} \equiv \delta_S$. So if the discount function in period 2 is sufficiently large, then K can beat D outright provided the vagueness term is sufficiently *small*. If instead $\delta^2 \leqslant \delta_S$, then it can never be that K beats D outright, and as in the other cases the comparison hinges on whether or not

$$u_4 + \delta^2 u_2 \leqslant u_3 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma \geqslant (u_4 - u_3) - \delta^2 (u_3 - u_2) \geqslant 0$$

where the last inequality follows from our assumption that $\delta^2 \leqslant \delta_S$. Incidentally, observe that $\delta_S > \delta_L \equiv \frac{u_3 - u_2}{u_2 - u_1}$.

In short we can summarize as follows:

$$\text{if } \delta^2 > \tfrac{u_4 - u_3}{u_3 - u_2} \equiv \delta_S \text{, then: } \begin{cases} K \succ_1 D \text{ if } \sigma < \delta^2 (u_3 - u_2) - (u_4 - u_3) \equiv -c\text{, and} \\ K \succ_2 D \text{ if } \sigma \geqslant \delta^2 (u_3 - u_2) - (u_4 - u_3) \equiv -c \end{cases}$$

$$\text{if } \delta^2 < \tfrac{u_4 - u_3}{u_3 - u_2} \equiv \delta_S \text{, then: } \begin{cases} D \succ_1 K \text{ if } \sigma < (u_4 - u_3) - \delta^2 (u_3 - u_2) \equiv c\text{, and} \\ K \succ_2 D \text{ if } \sigma \geqslant (u_4 - u_3) - \delta^2 (u_3 - u_2) \equiv c \end{cases}$$

Now let us compare the relative positions of a, b, c and $-c$ to see which configuration of preferences of the eight listed above are compatible with the model. Recall:

$$\begin{aligned} a &\equiv (u_3 - u_2) - \delta^2 (u_4 - u_3) > 0 \\ b &\equiv \left(1 - \delta^2\right) (u_4 - u_2) > 0 \\ c &\equiv (u_4 - u_3) - \delta^2 (u_3 - u_2) \\ -c &\equiv \delta^2 (u_3 - u_2) - (u_4 - u_3) \end{aligned}$$

Observe that

$$a > b \Leftrightarrow \delta^2 > \delta_S \equiv \tfrac{u_4 - u_3}{u_3 - u_2} > \tfrac{u_3 - u_2}{u_2 - u_1} \equiv \delta_L$$

That is, $a > b$ implies that $A > B$, too. The derivation is analogous to that for three period sequences, and is thus omitted. Similarly, one can show that

$$a > \max\{-c, c\}$$

while

$$b > c \text{ and } b > -c \Leftrightarrow \delta^2 < \frac{(u_4 - u_2) + (u_4 - u_3)}{(u_3 - u_2) + (u_4 - u_2)} \equiv \overline{\gamma} < 1$$

In this case too it is straightforward to verify that

$$\overline{\gamma} > \delta_S$$

Similarly to what we saw for the three period sequences, then:

1. If $\delta^2 \in (0, \delta_S)$, then $b > a > c > 0 > -c$.
2. If $\delta^2 \in (\delta_S, \overline{\gamma})$, then $a > b > -c > 0 > c$.
3. If $\delta^2 \in (\overline{\gamma}, 1)$, then $a > -c > b > 0 > c$.

We can now derive the conditions such that the various preference profiles listed at the beginning are compatible with the model.

1. Profile $D \succ_1 K \succ_1 I$ and $D \succ_1 I$: this requires $\sigma < c$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$), $\sigma < a$ (to have $K \succ_1 I$) and $\sigma < b$ (to have $D \succ_1 I$). Since we need $\delta^2 < \delta_L$, then $b > a > c > 0 > -c$, so that it must be $\sigma < \min\{a, b, c\} = c$.

2. Profile $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$: it requires either $\sigma < -c$ and $\delta^2 > \delta_L$, or $\sigma \geqslant c$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), $\sigma < b$ (to have $D \succ_1 I$) and $\sigma < a$ (to have $K \succ_1 I$). If $\delta^2 > \delta_L$, then we require $\sigma < \min\{a, b, -c\}$, which is either b or $-c$ depending on how large δ^2 is (i.e. whether or not it is greater than $\overline{\delta}$). If instead $\delta^2 < \delta_L$, we know that $b > a > c > 0 > -c$, in which case the requirement is $\sigma \in [c, a)$.

3. Profile $D \succ_1 I \succ_2 K \succ_{1,2} D$: this requires $\sigma \geqslant a$ (to have $I \succ_2 K$), either $\sigma < -c$ and $\delta^2 > \delta_L$, or $\sigma \geqslant c$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), $\sigma < b$ (to have $D \succ_1 I$). Since at the very least we need $a \leqslant \sigma < b$, we must be in the situation where $\delta^2 < \delta_L$, in which case we also need $\sigma \geqslant c$, which is however not binding. Consequently this preference profile is compatible with $\sigma \in [a, b)$ and $\delta^2 < \delta_L$.

4. Profile $I \succ_2 K \succ_{1,2} D$ and $I \succ_2 D$: this would require $\sigma \geqslant a$ (to have $I \succ_2 K$), either $\sigma < -c$ and $\delta^2 > \delta_L$, or $\sigma \geqslant c$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), and $\sigma \geqslant b$ (to have $I \succ_2 D$). So, if $\delta^2 < \delta_L$ as we saw above we have $b > a > c > 0 > -c$, so that

in this case all conditions are satisfied provided that $\sigma \geqslant \max\{a,b,c\} = b$, whereas if $\delta^2 > \delta_L$ there is no possibility that the profile can be justified, since the interval $[a,c)$ is in this case empty.

5. Profile $K \succ_1 I \succ_2 D$ and $K \succ_{1,2} D$: this requires $\sigma < a$ (to have $K \succ_1 I$), either$\sigma < -c$ and $\delta^2 > \delta_L$, or $\sigma \geqslant c$ and $\delta^2 < \delta_L$ (to have $K \succ_{1,2} D$), and $\sigma \geqslant b$ (to have $I \succ_2 D$). Since at the very least we must have $\sigma \in [b,a)$, it must be that $\delta^2 > \delta_L$, so that $a > b$. Then the comparison between K and D is resolved on the basis of the primary criterion, and we need $\sigma < \min\{a, -c\} = -c$. In short, for this profile of preferences we need $\sigma \in [b,c)$ and $\delta^2 > \delta_L$.

6. Profile $D \succ_1 K \succ_1 I \succ_2 D$: this requires $\sigma < a$ (to have $K \succ_1 I$), $\sigma \geqslant b$ (to have $I \succ_2 D$), and $\sigma < c$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$). Since $\delta^2 < \delta_L$ is necessary, we saw above that in this case $b > a > c > 0 > -c$, so that σ cannot be at the same time smaller than a and greater than b, so that this case is not compatible with the $(\sigma - \delta)$ model.

7. Profile $I \succ_2 D \succ_1 K$ and $I \succ_2 K$: this would require $\sigma \geqslant a$ (to have $I \succ_2 K$), $\sigma \geqslant b$ (to have $I \succ_2 D$), and $\sigma < c$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$). But if $\delta^2 < \delta_L$ the interval $[a,c)$ is empty, so that this profile is not compatible with the $(\sigma - \delta)$ model.

8. Profile $D \succ_1 I \succ_2 K$ and $D \succ_1 K$: this requires $\sigma < b$ (to have $D \succ_1 I$), $\sigma \geqslant a$ (to have $I \succ_2 K$), and $\sigma < c$ with $\delta^2 < \delta_L$ (to have $D \succ_1 K$). Again, $\delta^2 < \delta_L$ implies $b > a > c > 0 > -c$, so that this case is not compatible with the $(\sigma - \delta)$ model.

B.1.3 The role of plan J

We now show how the three implications for choices involving plan J discussed in section 6 in the paper have been derived. Recall that $J2 = (8, 40)$, $I2 = (16, 32)$, $SJ = (8, 32, 8)$ and $SI = (16, 24, 8)$. Then:

1. **the choice of J2 over I2 implies the choice of SJ over SI**. If a subject is vague between them and his preferences conform to our model, he will choose the

former based on the secondary criterion as it is steeper than I2. Since $u(16)-u(8) > \delta^2(u(40)-u(32))$, a choice of $J2$ over $I2$ can occur only by the secondary criterion, so that it must be that $u(16)+\delta^2 u(32) \leqslant u(8)+\delta^2 u(40)+\sigma$, that is the decision maker must have

$$\sigma \geqslant (u(16)-u(8)) - \delta^2(u(40)-u(32)) \tag{5}$$

Note that by our assumptions on the shape of the utility function, it is the case that

$$(u(16)-u(8)) - \delta^2(u(40)-u(32)) > (u(16)-u(8)) - \delta^2(u(32)-u(24))$$

so that condition (5) implies also

$$\sigma \geqslant (u(16)-u(8)) - \delta^2(u(32)-u(24)) \tag{6}$$

Rearranging, this yields

$$u(16)+\delta u(24)+\delta^2 u(8) \leqslant u(8)+\delta u(32)+\delta^2 u(8)+\sigma$$

that is the discounted utility of SI does not exceed the discounted utility of SJ by more than σ. In short, then, choosing $J2$ over $I2$ implies the choice of SJ over SI.

2. **the choice of J2 over I2 implies the choice of J3 over I3**. To show this just observe that condition (6) implies also

$$\sigma \geqslant \delta(u(16)-u(8)) - \delta^2(u(32)-u(24)) \tag{7}$$

which can be rearranged as

$$u(8)+\delta u(16)+\delta^2 u(24) \leqslant u(8)+\delta u(8)+\delta^2 u(32)+\sigma$$

so that the discounted utility of $I3$ does not exceed the discounted utility of $J3$ by more than σ. In short, then, choosing $J2$ over $I2$ implies the choice of $J3$ over $I3$.

3. **the choice of J2 over D2 implies the choice of J3 over D3**. Recall that $J2 = (8, 40)$ while $D2 = (32, 16)$. Since the choice of J2 over D2 can occur only because of the secondary criterion, it is necessary that

$$u(32) + \delta^2 u(16) \leqslant u(8) + \delta^2 u(40) + \sigma$$

which can be rearranged as

$$\sigma \geqslant u(32) - u(8) - \delta^2 (u(40) - u(16)) \equiv A \tag{8}$$

Similarly for the longer series, that is it must be that

$$u(24) + \delta u(16) + \delta^2 u(8) \leqslant u(8) + \delta u(8) + \delta^2 u(40) + \sigma$$

which can be rearranged as

$$\sigma \geqslant u(24) - u(8) + \delta(u(16) - u(8)) - \delta^2(u(32) - u(8)) \equiv B \tag{9}$$

We now show that $A > B$, so that condition (8) implies condition (9), and our claim is proved. Observe that $A > B$ if and only if:

$$\begin{aligned} & u(32) - u(8) - \delta^2(u(40) - u(16)) > u(24) - u(8) + \delta(u(16) - u(8)) - \delta^2(u(32) - u(8)) \\ & \Leftrightarrow u(32) - u(24) - \delta(1-\delta)(u(16) - u(8)) - \delta^2 (u(40) - u(32)) > 0 \end{aligned} \tag{10}$$

The last term is decreasing in the discount factor: the first derivative with respect to δ is given by

$$\begin{aligned} & 2\delta \left[(u(16) - u(8)) - (u(40) - u(32))\right] - (u(16) - u(8)) < 0 \\ & \Leftrightarrow \delta < \tfrac{u(16)-u(8)}{(u(16)-u(8))-(u(40)-u(32))} \end{aligned}$$

which is always verified since the right hand side is greater than unity. Consequently expression (10) is smallest in the limit as δ approaches unit. In this limit, however, expression (10) has value $u(32) - u(24) - (u(40) - u(32)) > 0$, which establishes our claim.

In order to compare plans I and J, observe that:

$$u_1+\delta_1 u_2+\delta_2 u_3 \geqslant u_1+\delta_1 u_1+\delta_2 u_4+\sigma \Leftrightarrow \delta_1 u_2+\delta_2 u_3 \geqslant \delta_1 u_1+\delta_2 u_4+\sigma \Leftrightarrow \sigma \leqslant \delta_1 (u_2 - u_1) - \delta_2 (u_4 - u_3)$$

Note that the value on the left hand side is positive provided that $\delta_1 \geqslant \delta_2$. In this case I can beat J outright provided the vagueness term is sufficiently *small.* If instead σ is large enough, then the two sequences are vague, and based on the secondary criterion our decision maker will prefer J to I. In short we can summarize as follows:

$$I \succ J \Leftrightarrow \sigma < \delta (u_2 - u_1) - \delta^2 (u_4 - u_3) \quad \equiv d > 0$$

C Additional experimental results

C.1 Robustness check

As we mention in the main text, in order to verify that our results are independent of the size of the monetary amounts used, we ran two additional treatments, one where all payments were hypothetical (HYP treatment) and one with real payments which were just half of those in the main treatment (PAY_L treatment). The results are qualitatively similar to those for the main treatment, and are reported below.

C.1.1 HYP treatment

In the HYP treatment a total of 56 subjects (in roughly equal proportion across sexes) were paid just for taking part to the experiment, i.e. the choice they made were purely hypothetical. We ran this treatment to control whether any differences would arise in comparison with the incentive compatible choices of the PAY treatment. Frequencies are reported in Table 1. The figures in parentheses refer to the corresponding frequencies in the PAY treatment.

	2 periods (%)	3 periods (%)
D chosen over K	64.3 (66.7)	60.7 (64.7)
D chosen over I	80.4 (79.4)	71.4 (81.4)
D chosen over J	89.3 (90.2)	85.7 (84.3)
K chosen over I	87.7 (92.2)	87.5 (93.1)
K chosen over J	89.3 (89.2)	91.1 (91.2)
I chosen over J	98.2 (92.2)	92.9 (91.2)

Table 1: Frequency distribution of binary choice, aggregate data, HYP treatment (total of 56 subjects)

As for the PAY treatment, sequence length seems not to matter much: the only change in proportions which is statistically significant (at 10% confidence level) concerns the choice between I and D, where the proportion of irrational choices increases with sequence length (McNemar's p-value is 0.089).

In comparisons *across* treatments, the percentage of subjects choosing I over either K or J is higher in the HYP than in the PAY treatment, regardless of sequence length. The proportions of subjects choosing I over D are similar across treatments in the case of short sequences, while for long sequences the proportions of agents preferring I over D in the HYP treatment is much higher than in the PAY treatment. These differences, though, are only significant at 10% level for the differences in proportions across treatments for choice ID3 and for choice IJ2 (Fisher's mid-p values are 0.08 and 0.063, respectively). All in all, then, there is some evidence that incentive compatible choices do make a difference, and that when real money is involved 'irrational' choices are less frequent. However this evidence is weak, and the fact remains that the overwhelming majority of subjects does prefer a 'rational' (i.e. either constant of decreasing) over an 'irrational' sequence. In addition, as for the PAY treatment, in the HYP treatment too the proportions of subjects choosing I is greater when the choice is against D than against K (proportions of I over D and I over K are 19.6% and 14%, respectively, for the short sequences and 28.8 and 12.5%, respectively, for the long sequences[2]). Finally, the end point effect observed in the PAY treatment is considerably 'damped down' in the HYP treatment, in the sense that there is a modest increase with sequence length in the percentage of subjects preferring J over D. There is no difference in the choice between J and K.

[2]Statistically significant only for long sequences (McNemar's p-value is 0.01).

The distribution of preference profiles is similar to that observed in the PAY treatment:

Choice profiles	Two period sequences (%)	Three period sequences
1: D≻K≻I	35	31
2: K≻D≻I	8	7
3: (D≻I≻K)	1	1
4: I≻K≻D	6	5
5: K≻I≻D	5	9
6: (D≻K≻I)	*0*	*2*
7: I≻D≻K	*0*	*0*
8: D≻I≻K	*1*	*1*

Table 2: frequency distribution of choice profiles for two and three period sequences, HYP treatment (56 subjects)

In the HYP treatment, too, we find again that the distribution of choices between I and D and I and K is strongly associated to the choice between D and K (see table 3). As we saw earlier, this is in perfect agreement with the $(\sigma - \delta)$ model, unlike pure discounting theories.

Two period sequences

	K2 ≻ I2, I2 ≻ D2	I2 ≻ K2, I2 ≻ D2	K2 ≻ I2, D2 ≻ I2	I2 ≻ K2, D2 ≻ I2
D2 ≻ K2	0	0	35	1
K2 ≻ D2	5	6	8	1

Three period sequences

	K3 ≻ I3, I3 ≻ D3	I3 ≻ K3, I3 ≻ D3	K3 ≻ I3, D3 ≻ I3	I3 ≻ K3, D3 ≻ I3
D3 ≻ K3	2	0	31	1
K3 ≻ D3	9	5	7	1

Table 3: rational versus irrational sequences, HYP treatment

Table 4 reports the cross-tabulation of choice profiles by sequence length, where again we underline the combinations of choice profiles compatible with the $(\sigma - \delta)$ model, while Table 5 summarizes the explanatory power of alternative theories.[3]

[3] Please refer to the discussion of the PAY treatment for the conditions establishing which combinations of preference profiles are compatible with the $(\sigma - \delta)$ model

3\2	D ≻K≻I	K ≻D ≻I	(D≻I ≻K)	I ≻K≻D	K≻I≻D	(D≻K≻I)	I≻D≻K	D≻I≻K	Total
D≻K≻I	***30 (53.6)***	1	0	0	0	0	0	0	31 (55.6)
K≻D≻I	2	***4 (7.1)***	0	0	0	0	0	1	7 (12.5)
(D≻I≻K)	0	0	**0**	1	0	0	0	0	1 (1.8)
I≻K≻D	0	0	0	***5 (8.9)***	0	0	0	0	5 (8.9)
K≻I≻D	2	3(5.3)	0	0	***4 (7.1)***	0	0	0	9 (16.7)
(D≻K≻I)	1	0	1	0	0	**0**	0	0	2 (3.6)
I≻D≻K	0	0	0	0	0	0	**0**	0	0
D≻I≻K	0	0	0	0	1	0	0	**0**	1 (1.8)
Total	35 (62.5)	8 (14.3)	1 (1.8)	6 (10.7)	5 (8.9)	0	0	1	56 (100)

Table 4: choice profiles for two and three period sequences, HYP treatment

Our findings with respect to the explanatory power of the competing frameworks are summarised in table 5

	Explained	Unexplained	total
Any discounting	36 (64.3%)	20 (35.7%)	56 (100%)
$(\sigma - \delta)$ model	47 (83.9%)	9 (16.1%)	56 (100%)

Table 5: explanatory power of competing theories, HYP treatment

Although in this treatment both classes of explanations fare worse than in the PAY treatment, the ability of the $(\sigma - \delta)$ model to explain the data is far greater than that of any pure discounting theory (combinations of preference profiles incompatible with the $(\sigma - \delta)$ model are still less than half those incompatible with standard discounting theories). For this treatment evidence between the different 'explanatory' power of each theory is less straightforward than for the PAY treatment, in the sense that the 90% Blyth-Still-Casella exact confidence intervals for the proportions of choice profiles compatible with the two models are $[0.529, 0.747]$ for the pure discounting model and $[0.747, 0.907]$ for the $(\sigma - \delta)$ model, that is they intersect just at one extreme.[4] However, Selten's index confirms the primacy of the $(\sigma - \delta)$ model, as in this treatment $s_d = 0.643 - 0.047 = 0.596$ while $s_{(\sigma-\delta)} = 0.839 - 0.172 = 0.667$.

C.1.2 PAY$_L$ treatment

A total of 60 subjects (30 males and 30 females) took part in this treatment, with real monetary payments totalling €24 (plus the participation fee, as in the other two treat-

[4]The overlap is obviously larger with 95% confidence intervals, which are $[0.514, 0.766]$ and $[0.719, 0.916]$ for pure discounting and $(\sigma - \delta)$ model, respectively.

ments), with the reward sequences specified as in table 6

Two period sequences				
	I2	D2	K2	J2
in three months	8	16	12	4
in nine months	16	8	12	20

Three period sequences				
I3	D3	K3	J3	
4	12	8	4	in three months
8	8	8	4	in six months
12	4	8	16	in nine months

Table 6: the base remuneration plans in the PAY_L treatment

Binary choices are similar to the PAY treatment:

	2 periods (%)	3 periods (%)
D chosen over K	65	65
D chosen over I	83.3	78.3
D chosen over J	86.7	86.7
K chosen over I	91.7	88.3
K chosen over J	93.3	96.7
I chosen over J	88.3	90

Table 7: Frequency distribution of binary choice, aggregate data (60 subjects)

In particular, there is no statistically significant difference in binary choice behavior when moving from two to three period sequences; **a** majority of subjects prefer decreasing to increasing sequences. As for the other two treatments, a majority of subjects prefer rational to irrational sequences: the constant sequence is preferred to the increasing sequence more than 88% of the times and over the jump sequence more than 90% of the times; the decreasing sequence is preferred to both the increasing and the jump ones, though, as in the PAY treatment, somewhat less decisively (between 78% and 87% of the times). Indeed, regardless of length, the subjects who chose I over D are almost thrice as many as those choosing I over K[5] (the corresponding proportions are 16.7% against 8.3% for the short sequences and 21.3% against 11.3% for the long sequences). Finally, here too for the long sequences subjects choosing the jump series over the decreasing one are just over four times as many as those choosing the jump sequence over the constant one[6] (13.3% against 3.3%) - for the short sequences the frequency is approximately the same (recall that for short sequences, J and I are both increasing, with J steeper than I).

[5] This difference is statistically significant (though at the 10% confidence level for the short sequences): a McNemar test of the difference between the proportion of subjects choosing I over K and those choosing I over D returns a p-value of 0.089 for the short and 0.054 for the long sequences.

[6] This difference is statistically significant: a McNemar test of the difference between the proportion of subjects choosing K over J and those choosing D over J returns a p value of 0.035.

When it comes to testing theories, choice profiles show the same patters as the other two treatments:

Choice profiles	Two period sequences (%)	Three period sequences (%)
1: D≻K≻I	60	60
2: K≻D≻I	20	15
3: (D≻I≻K)	1.67	3.33
4: I≻K≻D	3.32	6.67
5: K≻I≻D	10	10
6: (D≻K≻I)	1.67	3.33
7: I≻D≻K	1.67	1.67
8: D≻I≻K	1.67	0

Table 8: frequency distribution of choice profiles for two and three period sequences, PAY treatment (60 subjects)

When considering the choice profiles for each subject across both sequence lengths, we obtain a similar distribution to those for the other two treatments.

3\2	D≻K≻I	K≻D≻I	(D≻I≻K)	I≻K≻D	K≻I≻D	(D≻K≻I)	I≻D≻K	D≻I≻K	Total
D≻K≻I	***33 (55)***	1 (1.67)	0	0	0	1 (1.67)	0	1 (1.67)	36 (60
K≻D≻I	2 (3.33)	***6 (10)***	1 (1.67)	0	0	0	0	0	9 (15)
(D≻I≻K)	1 (1.67)	1 (1.67)	**0**	0	0	0	0	0	2 (3.3
I≻K≻D	0	1 (1.67)	0	**1 (1.67)**	2 (3.33)	0	0	0	4 (6.6
K≻I≻D	0	2 (3.33)	0	1 (1.67)	**2 (3.33)**	0	1 (1.67)	0	6 (10)
(D≻K≻I)	0	1 (1.67)	0	0	1 (1.67)	**0**	0	0	2 (3.3
I≻D≻K	0	0	0	0	1 (1.67)	0	**0**	0	1 (1.6
D≻I≻K	0	0	0	0	0	0	0	**0**	0
Total	36 (60)	12 (20)	1 (1.67)	2 (3.33)	6 (10)	1 (1.67)	1 (1.67)	1 (1.67)	60 (10

Table 9: choice profiles for two and three period sequences, PAY treatmentwith low stakes

The explanatory power of the $(\sigma - \delta)$ model is compared to that of other discounting theories in table 10

	explained	Unexplained	Total
Any discounting	41 (68.3%)	19 (31.7%)	60 (100%)
$(\sigma - \delta)$ model	49 (81.7%)	11 (18.3)	60 (100%)

Table 10: explanatory power of competing theories, PAY treatment with low stakes

Now Selten's indices are $s_d = 0.683 - 0.047 = 0.636$ and $s_{(\sigma-\delta)} = 0.817 - 0.172 = 0.645$, so that the $(\sigma - \delta)$ model still performs better.

Again we can look at the pattern of association between rational and 'irrational' sequences, reported in table 11.

	D2≻K2	K2≻D2
I2≻K2	2	3
K2≻I2	37	18

	D2≻K2	K2≻D2
I2≻D2	2	8
D2≻I2	37	13

	D3≻K3	K3≻D3
I3≻K3	1	6
K3≻I3	38	15

	D3≻K3	K3≻D3
I3≻D3	3	10
D3≻I3	36	11

Table 11: choices with and without 'irrational' sequences

Independence between rows and columns is rejected in all but one case (Fisher's exact p-value in the above tables going from top to bottom and from lesft to right are 0.227, 0.005, 0.002 and less than 0.001). Rearranging the data to take into account the whole profile yields the following:

Two period sequences

	K2 ≻ I2, D2 ≻ I2	K2 ≻ I2, I2 ≻ D2	I2 ≻ K2, I2 ≻ D2	I2 ≻ K2, D2 ≻ I2
D2 ≻ K2	36	1	1	1
K2 ≻ D2	12	6	2	1

Three period sequences

	K3 ≻ I3, D3 ≻ I3	K3 ≻ I3, I3 ≻ D3	I3 ≻ K3, I3 ≻ D3	I3 ≻ K3, D3 ≻ I3
D3 ≻ K3	36	2	1	0
K3 ≻ D3	9	6	4	2

Table 12: rational versus irrational sequences

As in the PAY treatment, in the case of low stakes too the same association pattern between irrational and rational choices emerges, and independence is rejected regardless of sequence length (Fisher-Freeman-Halton test provides p values of 0.002 and less than 0.001 for the short and long sequences, respectively), though as in the PAY treatment the effect appears stronger for longer sequences.

Two period sequences

	I2 $\succ$ K2	K2 $\succ$ I2
I2 $\succ$ D2	3	7
D2 $\succ$ I2	2	48

Three period sequences

	I3 $\succ$ K3	K3 $\succ$ I3
I3 $\succ$ D3	5	8
D3 $\succ$ I3	2	45

Table 13: rational versus increasing sequences

Two period sequences

	J2$\succ$ K2	K2 $\succ$ J2
J2 $\succ$ D2	2	6
D2 $\succ$ J2	2	50

Three period sequences

	J3 $\succ$ K3	K3 $\succ$ J3
J3 $\succ$ D3	1	7
D3 $\succ$ J3	1	51

Table 14: rational versus jump sequences

Both Tables 13 and 14 display association[7] between choices between I and D and between I and K (Fisher test p-values of 0.02for the short sequences in table 3 hte p-value is 0.05.

C.2 Choices and sex

In this section we report for completeness the experimental results distinguishing by sex. To save space we only report distributions for our main treatment (PAY).[8]

We will show shortly that the two sexes exhibit distinct patterns of choice: women tend to prefer the constant sequence to the decreasing one significantly more often than men; in addition, women tend to choose steeper (i.e. more irrational) sequences significantly less often than men. Yet, in spite of this heterogeneity among the sexes, the $(\sigma - \delta)$ model describes well choice behavior for both sexes. The key seems to lie in the fact that for both sexes there is a non negligible proportion of subjects whose irrational choices (i.e. preference for increasing sequences) is associated with a preference for the constant to the decreasing sequence, a pattern which can be accommodated within the $(\sigma - \delta)$ model but not within any pure discounting theory of time preferences. The frequency distribution of pairwise choices in Table 15.

Out of the 102 experimental subjects in this treatment, 55 (i.e. around 54%) were males and 47 (i.e. around 46%) females. In addition:

[7] A Fisher test of independence returns p-values of 0.029, 0.003 in table 13 and 0.08 and 0.25 in table 14. so that we cannot reject independence in the last panel of table 14.

[8] The analysis for the HYP and PAY$_L$ treatment yields qualitatively similar results and is thus omitted (but available from the authors upon request).

	2 periods (%)		3 periods (%)	
	Females	Males	Females	Males
D chosen over K	61.7	70.9	55.3	72.7
D chosen over I	72.3	85.5	74.5	87.3
D chosen over J	93.6	87.3	85.1	83.6
K chosen over I	91.5	92.7	89.4	96.4
K chosen over J	93.6	85.5	89.4	92.7
I chosen over J	91.5	92.7	89.4	87.3

Table 15: frequency distribution of binary choice by sex, PAY treatment (47 female subjects and 55 male subjects)

1. *Women prefer flatter sequences:* For the two period sequences, the proportion of women preferring I to D is much higher than those preferring the steeper sequence J to D (these proportions are 27.7.3% and 6.4%, respectively). This difference is statistically significant (McNemar's p-value is smaller than 0.001 in both cases). For three period sequences, though, where the 'jump' dimension of sequence J kicks in, the difference in the proportion of women choosing I over D and choosing J over D (25.5% and 14.9%, respectively) is not significant;

2. *Women like K better than D more than men do*: the proportion of women preferring the K over the D sequence is much larger than for men (38.3% against 29.1% for the short sequences and 44.7% against 27.3% in the long sequences); this difference is significant for the longer sequences (Fisher's mid p-values are 0.169 and 0.037 and for short and longer sequences, respectively).

3. *Women like I better than D more than men do*: the proportion of women preferring I over D is roughly twice the corresponding proportion of men independently of sequence length (i.e. 27.7% for women and 14.5% for men in the case of short sequences, and 25.5% and 12.7% for women and men, respectively, in the case of long sequences), and this difference is statistically significant at 10% confidence level (Fisher's mid-p values are 0.057, resp. 0.055 for short and long sequences).

4. *Both women and men choose I over D more often than they choose I over K*: the proportion of women preferring I to K is much smaller than the proportion of women choosing I over D, and this difference is statistically significant (McNemar's p-values are 0.002 and 0.019 for short and long sequences, respectively). For men this dif-

ference is not statistically significant (McNemar's p-values are and 0.144 and 0.062 for the short and long sequences, respectively). The difference in the proportions of men and women choosing I over D is not statistically significant at 5% level.

5. *The two sexes respond to end effects in a different way.* When the comparison is between J and D, there is no significant difference in proportions of men preferring J over D in short and long sequences; for women this proportion increases from 6.4% to 14.9%, and this difference is statistically significant at 10% level (McNemar's p-value is 0.062). However, when considering the comparison between J and K, about twice as many men choose J over D than they choose J over K in the case of three period sequences. This difference is significant at 10% level (Mc Nemar's p-value is 0.062).

Putting together this evidence with the points on aggregate data from Table 2 in the main text, we conclude the following:

I. Points 3, c and d suggest that the higher frequency with which subjects choose I over D as compared to I over K in the aggregate data is driven by female subjects;

II. Points 1 and e suggest that the increase with sequence length in the proportion of subjects observed choosing J over D in the aggregate data is mainly driven by women, who seem to be more sensitive than men to end effects. In this same direction, note that – unlike men - the proportion of women choosing J over K increases with sequence length, when the end effect kicks in (though this change is not statistically significant).

III. Points 3 and e suggest that men are more sensitive to end effects (in a negative way) involving sequence J when the comparison is with D rather than K.

IV. Points 3 and c suggest that the large proportion of subjects who prefer I to D in aggregate data is mainly due to female subjects.

Preference profiles show similar patterns for both sexes, namely there are more preference profiles compatible with the $(\sigma - \delta)$ model than with pure discounting theories. The preference profiles are summarized in table 16.

		Two period sequences		Three period sequences	
		Females (%)	Males (%)	Females (%)	Males (%)
Code	Profile				
1	D≻K≻I	53.2	69.2	51.1	70.9
2	K≻D≻I	19.2	12.7	21.3	14.6
3	D≻I≻K≻D	0	1.8	0	1.8
4	I≻K≻D	8.5	3.6	8.5	1.8
5	K≻I≻D	10.6	10.9	14.9	9.1
6	D≻K≻I≻D	8.5	0	2.1	1.8
7	I≻D≻K	0	0	0	0
8	D≻I≻K	0	1.8	2.1	0
	Total	100.0	100.0	100.0	100.0

Table 16: choice profiles distinguishing by sex, PAY treatment (47 female subjects and 55 male subjects)

Finally, regarding the various forms of irrationality, the same kind of pattern that we saw for aggregate data is repeated within each sex: a preference for I over either K or D is far more frequent in subjects who also choose K over D (Tables 17 and 18):

Females

	K2≻I2	I2≻K2	D2≻I2	I2≻D2
D2≻K2	29	0	25	4
K2≻D2	14	4	9	9

Males

	K2≻I2	I2≻K2	D2≻I2	I2≻D2
D2≻ K2	38	1	39	0
K2≻ D2	13	3	8	8

Table 17: rational versus irrational choices for two period sequences, by sex

Females

	K3≻I3	I3≻K3	D3≻I3	I3≻D3
D3≻K3	25	1	25	1
K3≻D3	17	4	10	11

Males

	K3≻I3	I3≻K3	D3≻I3	I3≻D3
D3≻K3	40	0	39	1
K3≻D3	13	2	9	6

Table 18: rational versus irrational choices for three period sequences, by sex

D Instructions

Please note: you are not allowed to communicate with the other participants for the entire duration of the experiment.

The instructions are the same for all you. You are taking part in an experiment to study intertemporal preferences. The project is financed by the ESRC.

Shortly you will see on your screen a series of displays. Each display contains various remuneration plans worth the same total amount of 48 Euros each, staggered in three, six and nine months installments. For every display you will have to select the plan that you prefer, clicking on the button with the letter corresponding to the chosen plan. (HYP: These remuneration plans are purely hypothetical. At the end of the experiment you'll be given a participation fee of 5 €.) (PAY: At the end of the experiment one of the displays will be drawn at random and your remuneration will be made according to the plan you have chosen in that display).

In order to familiarise yourself with the way the plans will be presented on the screen, we shall now give you a completely hypothetical example, based on a total remuneration of 7 Euros.

Plan A

How much When

3 € in one year

1 € in two years

1 € in three years

2 € in four years

Plan B

How much When

1 € in one year

2 € in two years

3 € in three years

1 € in four years

In this example plan A yields 7 € in total in tranches of 3 €, 1 €, 1 € and 2 € in a year, two years, three years and four years from now, respectively, while plan B yields 7 Euros in total in tranches of 1 €, 2 €, 3 € and 1 € in a year, two years, three years and four years from now, respectively.

References

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[3] Roelofsma, P. H. and D. Read, 2000, "Intransitive Intertemporal Choice", *Journal of Behavioral Decision Making*, 13: 161-177.

[4] Rubinstein, A., 2003, "Is it 'Economics and Psychology'? The case of hyperbolic discounting", *International Economic Review*, 44: 1207-1216.

[5] Tversky, A., 1969, "Intransitivity of Preferences", *Psychological Review*, 76: 31-48.

E Raw data

We report our data in the table below. Variable names are as follows:

T: treatment (0 for PAY and 1 for HYP)

SS: session number

SB: subject number

SX: subject's sex (0 for Female and 1 for Male)

Choices between plans are coded as follows: ABn indicates the choice between plans A and B of length n periods. A value of 0 indicates that A was chosen over B, whereas a value of 1 indicates that B was chosen over A. The only exception to this coding strategy is for plan SJSI, which was only available for three period sequences and where 0 indicates the choice of the SJ sequence over the SI sequence, while 1 denotes the opposite choice.

T	SS	SB	SX	KI3	ID3	DK3	IJ3	JK3	JD3	SJSI	KI2	ID2	DK2	IJ2	JK2	JD2
0	1	0	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	1	0	0	0	1	0	0	0	0	0	1	1	1	1	1
0	1	2	0	1	0	1	0	1	1	1	0	0	0	0	1	1
0	1	3	0	0	0	1	0	1	0	1	0	0	1	0	1	1
0	1	4	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	5	0	0	1	0	0	1	1	1	0	1	1	0	1	1
0	1	6	1	0	1	0	0	1	1	1	0	1	0	0	0	1
0	1	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	9	0	1	0	1	1	0	0	1	1	0	1	0	0	0
0	2	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	1	0	1	1	0	0	0	1	1	0	0	1	0	1	1
0	2	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	4	1	0	1	1	0	1	1	1	0	1	1	0	1	1
0	2	5	0	0	0	1	0	1	1	1	0	0	1	0	1	1
0	2	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	7	0	0	1	0	0	1	1	1	0	1	0	0	1	1

0	2	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	9	1	1	0	1	0	1	0	1	0	0	1	0	1	0
0	2	10	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	11	0	0	1	0	0	1	1	1	0	0	0	0	1	1
0	2	12	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	13	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	14	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	15	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	3	0	1	1	1	1	0	1	1	1	1	0	1	0	1	1
0	3	1	0	0	1	0	0	1	1	0	0	1	0	0	1	1
0	3	2	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	3	3	0	0	1	1	0	1	1	1	0	1	1	0	1	1
0	3	4	1	0	1	0	0	1	1	1	0	1	1	0	1	1
0	3	5	1	0	1	1	0	1	1	1	0	1	0	0	1	1
0	3	6	0	0	1	1	0	0	1	1	0	1	0	0	1	1
0	3	7	0	0	1	1	0	1	1	1	0	1	0	0	1	1
0	3	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	0	0	0	1	1	0	1	1	1	0	1	0	0	1	1
0	4	1	1	0	1	0	1	1	1	1	0	1	0	0	1	1
0	4	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	3	1	0	1	1	0	1	1	1	0	1	0	0	1	1
0	4	4	1	0	1	0	0	1	1	1	0	1	0	0	0	1
0	4	5	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	9	1	0	0	1	0	1	0	1	0	0	1	0	0	0
0	4	10	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	11	0	0	0	1	0	1	1	1	0	0	0	0	1	1
0	5	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1
0	5	1	0	0	0	1	0	1	0	0	1	0	1	1	0	0

0	5	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	5	3	0	0	1	0	0	1	1	0	0	1	0	0	1	1
0	5	4	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	5	5	1	0	1	0	0	1	1	0	0	1	0	0	1	1
0	5	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	5	7	0	0	0	1	0	1	0	1	1	0	1	0	1	0
0	6	0	0	0	1	0	0	1	1	0	0	1	0	1	1	1
0	6	1	1	0	1	0	1	0	0	0	1	1	0	1	0	1
0	6	2	0	0	1	0	0	1	1	0	0	1	1	0	1	1
0	6	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	4	1	0	0	1	0	1	0	1	0	0	1	1	1	1
0	6	5	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	6	1	0	0	0	0	0	0	0	0	0	1	0	0	1
0	6	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	9	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	10	0	0	1	0	0	1	1	1	0	1	1	0	1	1
0	6	11	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	7	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	7	1	0	0	1	1	0	1	1	1	0	1	1	0	1	1
0	7	2	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	7	3	0	0	1	1	0	1	1	1	0	1	0	0	1	1
0	7	4	0	0	1	1	0	1	1	1	0	1	1	0	1	1
0	7	5	1	0	1	1	0	1	1	1	0	1	1	0	1	1
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