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The Relationship between Gross Domestic Product (GDP), Inflation, Import and Export from a Statistical Point of View

Stephen Ayodele Oshungade

Thesis submitted for the degree of Doctor of Philosophy



June 2014

Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another University for the award of any other degree

Signature:

Dedication

This Thesis is dedicated to:

The Glory of God, The Head

(Matthew 28:19; 2 Corinthians 13:14)

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Abstract

The term relationship in a general statistical concept connotes a wide range of meanings and applications. However, the resultant meaning of the term usually focus on the principle of connectivity, association, causation, inter-relationship, or linkages between variables. In view of this, the thesis reports on the statistical relationships between GDP, Inflation, Export and import. The study utilized 65 countries with data ranging from 1970 to 2011.

The research, which is an applied empirical, involves two phases. The first phase dealt with the exploration of nature and pattern of Granger causality concept by using GDP and inflation. In this phase, we first ensured the stationarity and stability of our time series variables are maintained. The stationary and non-stationary instruments utilized include ADF, PP, KPSS, Chow and Quandt tests. After these, we carried out extensive computations using the Granger causality. It should be noted that the concept of Granger causality is concerned with how a variable X can enhance or better the prediction of other variable Y by using the principle of cause and effect.

In the second phase of the study, we explored the possible linkages of exports and imports to the Granger causality of GDP and Inflation that were established in Phase 1. To achieve this, we first carried out pairwise Granger causality tests on the four variables (GDP, Inflation, Export and Import) and then considered further computations and testing on the said variables by utilizing the principles of Bayes theorem, assignment problem models, coefficient of variation and other relevant statistical concepts. In fact, the results at this phase are the major contributions to knowledge.

The general description of the study embraced the conceptual steps, where we considered relevant literatures on Granger causality and theory of some statistical principles and practices as earlier mentioned above. Next, we have the empirical studies description in which the methodology, results/findings and interpretations on the study were considered.

Based on our findings, we conclude that Inflation “Granger causes” GDP most often occurred than the other combinations of Granger causality between Inflation and GDP. Also, it was established that countries with developed economies supported the Granger causality concept better than the developing economies. This result can be attributed to the stability of most of

the developed economy variables, while it is unstable with most of the developing economy countries. With countries supporting Granger causality, we have uniformly distributed pattern for the three types in the developed economies whilst skewed toward Inflation “Granger causes” GDP for the developing economies.

For other important conclusions, we could establish that less volatility of export over import supports the bidirectional Granger causality whilst higher volatility of exports over import is relationally linked to the unidirectional Granger causality. We inferred also that when there is unidirectional Granger causality between inflation and import (or export), there is also unidirectional causality between GDP and inflation by the Bayes’ Rule; and when there is bidirectional Granger causality between GDP and import only, there is bidirectional causality between GDP and inflation.

Contents

	Page
Declaration.....	i
Dedication.....	ii
Acknowledgements.....	iii
Abstract.....	iv
Table of contents	vi
List of figures	x
List of tables	xvi
List of Abbreviations	xx
 Chapter 1: Introduction.....	 1
1.1 Brief Description of GDP, Inflation, Unemployment, and Balance of Payment.	1
1.1.1 Gross domestic product (GDP).....	2
1.1.2 Inflation.....	6
1.1.3 Unemployment.....	8
1.1.4 Balance of payment (BOP).....	11
1.2 Business cycles and fluctuations.....	13
1.3 Aims and objectives of the study.....	15
1.4 Structure of the thesis.....	16
Chapter 2: Literature review.....	20
2.0 Introduction.....	20
2.1 Concept of Causality and its historical background	20
2.2 Definition of Granger Causality.....	24
2.3 Granger Causality review.....	26
Chapter 3: Stationary and non-stationary.....	38

3.0 Introduction.....	38
3.1 Some useful terms in regression analysis	38
3.1.1. Distributed lag and autoregressive models	40
3.1.2 Covariance, autocorrelation function, and partial autocorrelation function.	44
3.2 Stationary time series.....	44
3.2.1 Strict stationary.....	44
3.2.2 Weak stationary.....	45
3.3 Non-stationary.....	45
3.3.1 Unit root and trend.....	46
3.3.2 Outliers and structural break(s).....	47
3.4 Non-stationary tests.....	50
3.4.1 Tests on Unit roots and trend.....	50
3.4.1.1 Dickey-Fuller test.....	50
3.4.1.2 Phillips-Perron test.....	52
3.4.1.3 KPSS test.....	53
3.4.2 Outliers and structural break tests.....	54
3.4.2.1 Perron method.....	59
3.4.2.2 Chow's method.....	60
3.4.2.3 Cusum method.....	62
3.4.2.4 Quart method.....	63
3.5 Transformation of non-stationary to stationary.....	66
3.6 Type and determination of lag length.....	69
Chapter 4: Granger Causality methods and other tests.....	73
4.0 Introduction.	73
4.1 Granger Causality methods.....	73

4.1.1 Simple linear Granger Causality method.....	74
4.1.2 Multiple linear Granger Causality method.....	75
4.1.3 Vector auto-regression (VAR) method.....	77
4.2 Other statistical analysis on Granger Causality results.....	80
4.2.1 Proportionality test (With Normality and Binomial Assumptions)	80
4.2.2 Chi-square test.....	82
4.2.3 Bayes' inference statistics	83
4.2.4 Assignment problem optimization	84
4.2.5 Coefficient of variation.....	88
Chapter 5: Research Methodology.....	90
5.1 Introduction.....	90
5.2 Data description.....	90
5.3 Statistical instruments/tools.....	91
5.4 Further statistical analyses and tests on Granger Causality results.....	93
Chapter 6: Results, Findings, and Interpretations	96
6.0 Introduction	96
6.1 Phase 1 of the study.....	96
6.1.1 Results and findings (of Phase 1).....	96
6.1.2 Interpretations (on Phase 1 Results).....	143
6.2 Phase 2 of the study.....	146
6.2.1 Results and findings (in Phase 2).....	146
6.2.2 Interpretations (on Phase 2 Results).....	178
Chapter 7: Summary, Conclusions, and Suggestions.....	186
7.0 Introduction.....	186
7.1 Summary of the study.....	186
7.2 Conclusions	189

7.3 Suggestions.....	191
Bibliography.....	193
Appendices.....	207
A. Source codes of Granger Causality test.....	207
B. Time plots of countries using percentage change (Phase 1)...	210
C. List of countries with structural breaks and outliers...	235
D. Time plots of countries using original (level) data (Phase 2)	237

List of Figures

Figure 1.01:	Sketch of Business Cycles and Fluctuations
Figure 1.02:	The Structural Chart of Thesis
Figure 3.01(a):	Sketch of Additive Outlier (AO) Model
Figure 3.01(b):	Sketch of Innovative Outlier (IO) Model
Figure 3.01(c):	Sketch of Transient Changes (TC) Model
Figure 3.01(d):	Sketch of Level Changes (LC) Model
Figure 3.01(e):	Sketch of Variance Changes (VC) Model
Figure 5.01:	Methodological/Empirical Chart of the Study
Figure 6.01:	Plots of Algeria Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.02:	Plots of Angola Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.03:	Plots of Argentina Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.04:	Plots of Australia Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.05:	Plots of Austria Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.06:	Plots of Bangladesh Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.07:	Plots of Brazil Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.08:	Plots of Canada Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.09:	Plots of China Inflation Rate and GDP growth Percentage, 1970-2011
Figure 6.10:	Plots of France Inflation Rate and GDP growth Percentage, 1970-2011

- Figure 6.11: Plots of Germany Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.12: Plots of Hong Kong Sar Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.13: Plots of India Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.14: Plots of Iraq Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.15: Plots of Japan Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.16: Plots of New Zealand Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.17: Plots of Nigeria Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.18: Plots of Singapore Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.19: Plots of United Kingdom Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.20: Plots of United States (USA) Inflation Rate and GDP growth Percentage, 1970-2011
- Figure 6.21: Distribution into Granger Causality and Non-G.Causality Pie Chart
- Figure 6.22: Distribution of Granger Causality into Types Bar Chart
- Figure 6.23: Classification of Granger Causality into Developed & Developing Economies Chart
- Figure 6.24: Distribution of Developed and Developing Economies into Type of Granger Causality Chart

- Figure 6.25: Time Plots of Austria GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.26: Time Plots of Bangladesh GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.27: Time Plots of Canada GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.28: Time Plots of China GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.29: Time Plots of Finland GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.30: Time Plots of Japan GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.31: Time Plots of Luxembourg GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.32: Time Plots of Sweden GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.33: Time Plots of Switzerland GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.34: Time Plots of USA GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011
- Figure 6.35(a),(b)&(c): Prior Probabilities` Relationships and Linkages to Posterior Probabilities` Relationships on GDP, Inflation, Export, and Import

Figure 6.36: The Granger Causalities using Prior and Posterior Conditional Probabilities` Results and its Relational Connections to Export and Import via The Assignment Model

Figure 6.37: The Relational Linkages of Import and Export Co-efficient of Variation to the GDP and Inflation`s Granger Causality.

Figure 6.38: Economic meaning and Policy making of the study.

Appendix B.01 [Figure 1(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage 1970-2011 for countries: (a) Algeria; (b) Angola, (c) Argentina & (d) Australia

Appendix B.02 [Figure 2(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage, 1970-2011 for countries: (a) Austria, (b) Azerbaijan, (c) Bangladesh & (d) Barbados

Appendix B.03 [Figure 3(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage, 1970-2011 for countries: (a) Belgium, (b) Botswana, (c) Brazil, & (d) Bulgaria

Appendix B.04 [Figure 4(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage, 1970-2011 for countries: (a) Burkina Faso, (b) Cambodia, (c) Cameroon, & (d) Canada

Appendix B.05 [Figure 5(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage, 1970-2011 for countries: (a) Chile, (b) China, (c) Colombia, & (d) Costa Rica

Appendix B.06 [Figure 6(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth Percentage, 1970-2011 for countries: (a) Cyprus, (b) Czech Republic, (c) Denmark, & (d) Egypt

Appendix B.07 [Figure 7(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Estonia, (b) Ethiopia, (c) Fiji, & (d) Finland

Appendix B.08 [Figure 8(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) France, (b) Germany, (c) Ghana, & (d) Greece

Appendix B.09 [Figure 9(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Guyana, (b) Hong Kong Sar, (c) Hungary, & (d) Iceland

Appendix B.10 [Figure 10(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) India, (b) Indonesia, (c) Iran, (d) Iraq

Appendix B.11 [Figure 11(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Ireland, (b) Israel, (c) Italy, & (d) Jamaica

Appendix B.12 [Figure 12(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Japan, (b) Jordan, (c) Kazakhstan, & (d) Kenya

Appendix B.13 [Figure 13(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Korea, (b) Kuwait, (c) Latvia, & (d) Lebanon

Appendix B.14 [Figure 14(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Liberia, (b) Libya, (c) Lithuania, & (d) Luxembourg

Appendix B.15 [Figure 15(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Malawi, (b) Malaysia, (c) Malta & (d) Mexico

Appendix B.16 [Figure 16(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Morocco, (b) Nepal, (c) Netherlands, & (d) New Zealand

Appendix B.17 [Figure 17(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Nicaragua, (b) Nigeria, (c) Norway, & (d) Pakistan

Appendix B.18 [Figure 18(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Paraguay, (b) Peru, (c) Philippines, & (d) Poland

Appendix B.19 [Figure 19(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Portugal, (b) Romania, (c) Russia, & (d) Saudi Arabia

Appendix B.20 [Figure 20(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Senegal, (b) Sierra Leone, (c) Singapore, & (d) Slovak Republic

Appendix B.21 [Figure 21(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Slovenia, (b) South Africa, (c) Spain, & (d) Sri Lanka

Appendix B.22 [Figure 22(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Sudan, (b) Sweden, (c) Switzerland, & (d) Thailand

Appendix B.23 [Figure 23(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Taiwan, (b) Tonga, (c) Trinidad & Tobago, and (d) Tunisia

Appendix B.24 [Figure 24(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) Turkey, (b) United Kingdom, (c) Ukraine, & (d) United Arab

Appendix B.25 [Figure 25(a),(b),(c)&(d)]: Plots of Inflation Rate and GDP Growth

Percentage, 1970-2011 for countries: (a) United States (USA), (b) Venezuela, (c) Vietnam, & (d) Zimbabwe

Appendix D.01 [Figure 26(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Australia, (b) Austria, (c) Bangladesh, (d) Belgium

Appendix D.02 [Figure 27(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Botswana, (b) Canada, (c) Chile, & (d) China

Appendix D.03 [Figure 28(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Denmark, (b) Ethiopia, (c) Fiji, & (d) Finland

Appendix D.04 [Figure 29(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) France, (b) Germany, (c) Greece, & (d) Hungary

Appendix D.05 [Figure 30(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Iceland,

(b)India, (c) Iran, & (d) Italy

Appendix D.06 [Figure 31(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Japan, (b) Luxembourg, (c) Malaysia, & (d) Nepal

Appendix D.07 [Figure 32(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Netherlands, (b) New Zealand, (c) Portugal, & (d) Spain

Appendix D.08 [Figure 33(a),(b),(c)&(d)]: Time Plots of GDP Export, Import Values (in log)

and Inflation Indices (in log), 1970-2011 for countries: (a) Sweden, (b) Switzerland, (c) Tunisia, & (d) USA.

List of Tables

- Table 4.01: Assignment Model with n tasks and m assignment
- Table 6.01: Granger Causality Results on the Percentage Change of GDP and Inflation classified by countries, Economic Groups and Integration orders from 1970 to 2011
- Table 6.01 (a), (b), (c) and (d): Break down of Table 6:01 into Type of Granger Causality
- Table 6.02: Distribution of Granger Causality and Non-Granger Causality According to Economic Groupings
- Table 6.03: Classification into Granger Causality and Non-Granger Causality
- Table 6.04: Distribution of Granger Causality into Types
- Table 6.05: Classification of Granger Causality into Developed and Developing Economies
- Table 6.06: Distribution of Developed and Developing Economies into Type of Granger Causality
- Table 6.07: Granger Causality Results on Log of Actual Values of The Paired Variables of GDP, Inflation, Exports, and Imports from 1970 to 2011
- Table 6.08 (a), (b), (c) & (d): Classification Summary of Granger Causality Results at stages 1 and 2 for GDP/Inflation with other combinations of GDP/Exports, GDP/Imports, Inflation/Exports, and Inflation/Imports
- Table 6.09 (a), (b), (c) & (d): Tree Diagrams and associated computations of conditional Probabilities derived from Table 6:08

- Table 6.10: Summary of The Bayesian Results Supporting Granger Causality of Stages 1 (Prior) and 2 (Posterior)
- Table 6.11: List of countries classified by The Granger Causality of Inflation/GDP and Their Related coefficient of Variation on Exports and Imports
- Table 6.12: Statistical Tests on Results of co-efficient of Variation for the Group of Granger Causality
- Table 6.13: Summary Table of Pair Combinations of Export, Import, Inflation, and GDP in Terms of Granger Causality and Non-Granger Causality

List of Abbreviations

ACF	Autocorrelation Function
ADF	Augmented Dickey Fuller
AIC	Akaike Information Criterion
AO	Additive Outlier
AP	Assignment Problem
BIC/SIC	Bayesian Criterion of G. Schwarz
BOP	Balance of payment
CPI	Consumer Price Index
Cusum	Cumulative Sum Chart
CV	Coefficient of Variation
DF	Dickey Fuller
EADE	Emerging and Developing Economy
ECM	Error Correction Model
EL	Employed Labour
EU	European Union
FPE	Final Prediction-Error
Gcausality	Granger causality
GDP	Gross Domestic Product
HQ	Hannan and Quinn
ILO	International labour office
IO	Innovative Outlier

LC	Level Changes
LP	Linear Programming
MAE	Major Advanced Economy
OAE	Other Advanced Economy
OC	Other Countries
OLS	Ordinary least squares
PACF	Partial Autocorrelation Function
PP	Phillips- Perron
RSS	Residual Sum of Squares
TC	Transient Changes
TL	Total Labour Force
UEMOA	Union Economique et Monetaire Quest Africain
UL	Unemployed Labour
VAR	Vector auto regression
VC	Variance Changes
VEC	Vector Error Correction
VECM	Vector Error Correction Model

Chapter 1: Introduction

Macroeconomic and financial data have played important roles in the economic policy making for a country. According to Miles and Scott (2005), it is claimed that the study of macroeconomics is the study to know and understand the wealth of nations; and this induces a better understanding of issues on economic policy. By this, the economy performance, wealth, or financial position of a nation depends on a number of variables; most notably, the balance of payments, unemployment rate, gross domestic product (GDP), exchange rate and inflation rate. These metrics are important because of their usage in determining the welfare of a country in terms of standard of living, financial strength, per capita income and the buoyancy or otherwise of the economy.

As the government takes an active and very important parts in the management of circular flow of income (households, firms, public sector/government, financial sector and external foreign sector) in terms of national income and expenditure, the major concern of countries or governments are centred on economic growth, stability and good financial position. Hence, these main macro-metrics are utilized to monitor, as watchdog, in order to attain:

- (i) adequate and satisfactory economic growth rate (through GDP);
- (ii) steady and low inflation rate (through consumer price indices);
- (iii) steady and high employment (through unemployment statistics); and
- (iv) a reasonable and equilibrium level of balance of payment(BOP) (through the open economy and international trade by utilising the difference between the total values of exports and imports in terms of goods and services of the nations concerned. Exchange rate equally affects BOP).

1.1 Brief Description of GDP, Inflation, Unemployment and BOP

The following sub-sections discussed the meaning, and the effect or importance of the above variables on the economy of a nation as earlier mentioned.

1.1.1 Gross Domestic Product (GDP)

According to Hall and Papell (2005, Chapter 2), GDP is a useful econometric variable of knowing the economic growth of a country. Its value or determination can be achieved through the product of two components; namely, the volume and price of goods and services. Hence, either of the two (volume and price) can cause an increase (decrease) in GDP. When it is caused by increase in price with volume remaining constant, it is known as **nominal** GDP growth; while an increase in volume with constant price is known as **real** GDP growth.

The main difference between the two GDP is that nominal value is affected by inflation while the real value is not affected by inflation. Our research is utilising the real GDP. An increase in real GDP indicates real economic growth.

As GDP can be obtained through three main methods by employing the country's output, income, or expenditure, it can be equally stated that GDP has mathematical relationship with a country's output, income or expenditure. In light of this, we present the main components usually utilised to determine each of the said methods.

The **output approach** of GDP, which is most popular measure of national productivity, can be determined by summing up the added values of all final goods/productions of various industries (including self-employed producer) in a country. That is, to calculate the total values of production of goods and services in different industries within a country. The setback or challenge to this method is the issue of double counting. Double counting is an act

of counting item(s) more than once in the final production, most especially, the intermediate goods being used by other industries.

To prevent the said problem of double counting, value added concept was developed to take care of this method.

By using value added concept, the actual value of output can be obtained by subtracting the value of the intermediate inputs used in the final production from the value of final production. This is stated in the following mathematical form:

$$\text{GDP}_{\text{output}} = Y = \text{fp} - \text{ip} \quad (1.01)$$

where

fp = value of output in terms of final production in an economy; and

ip = value of intermediate production values used in the final production.

Another method for consideration is the **income approach**. In this approach, the GDP can be obtained or determined by adding up the total income earned by all domestic households and firms in a country at a particular period (yearly). Note that factors of production referred to all the households and firms in a country.

The four main components of income approach can be expressed in the algebraic form:

$$\text{GDP}_{\text{income}} = y = \text{ws} + \pi + r + i \quad (1.02)$$

Where y = value of GDP income;

ws = wages and salaries;

π = profits (including corporate profits);

r = other income from rental; and

i = interest/ dividend income from savings and investments.

It is generally acclaimed that ws is the largest component.

Lastly, the **expenditure approach** is computed through the addition of all expenditure incurred on domestic goods and services within a country in a particular period. Its components can be defined and expressed in the mathematical form:

$$Y = C + G + I + X - M \quad (1.03)$$

Where $Y = \text{GDP}_{\text{expenditure}}$;

C = consumption, (which form the largest component);

G = government spending;

I = investment;

X = export; and

M = import.

From equation (1.03), GDP is a linear function of other variables (which are on the right side of equality). That is,

$$Y = f(C, G, I, X, M)$$

Also, $X - M$ is the subtraction of imports from exports which results to the term “net-export” (X_{net}) or balance of trade (either deficit or surplus).

i.e $X_{\text{net}} = X - M$

Here, government can adjust its policies on international trade if balance of trade is being negatively affected, provided it does not change the nature of economy from open economy to closed economy.

It should be noted that the research will utilize export and import in the second phase of the study not to consider the ordinary linearity of the two (export and import) to GDP but to look at their enhance prediction power to GDP and inflation through Granger causality.

In all the methods, any changes in the components (f_p , i_p , w_s , π , r , i , C , G , I , X and M) will affect the GDP. By this, the changes can be seen as factors causing GDP to increase or decrease.

Other factors that influence or affect movement of GDP can be considered from the aggregate demand and supply pull forces. A synopsis of these factors includes:

- the degree of consumer confidence, either high or low, goes a long way to influence the GDP;
- a future anticipation of increase or decrease in asset price;
- lowering or raising the rate of interest as an encouragement or discouragement to consumer or investor to spend more or not; and
- the up and down movements of exchange rate will affect both the exports and imports into deficit or surplus in terms of trade balance.

As the above factors are influenced by the aggregate demand pull, the following are the factors affecting the aggregate supply pull:

- technological developments boost production and encourage more supply; while otherwise, it encourages less supply;

- improvement on the level of infrastructure can reduce the cost of industries and boost production whilst lack of infrastructure developments reduce production;
- improved skill in terms of human capital equally improve productivity, otherwise no improvement in productivity; and
- weather situations either bad or good in some countries can affect agricultural productions.

With various adjustments of fiscal policies or other measures, a government can establish and achieve an ideal and satisfactory economic growth. In fact, as claimed by the economists, an ideal real GDP growth rate has an adequate and sustainable (benchmark) level of growth which can stay within the expanding business cycle for as long as possible to maintain growth rate of two to four percentage range. If economic growth rises above four percent persistently, the inflation will increase.

1.1.2 Inflation

Inflation is another important macro-variable that must be taken into account because of its effects and constraints on the economic policy of a nation's economy.

Inflation can be simply defined as a general rise in prices to a higher level. It can be comprehensive or sporadic when there is rise in prices affecting all goods or some goods in the economy respectively. Economy wide inflation is another name for comprehensive inflation; while sporadic can occur; for example, due to bad weather which affects crops production and then induce increase in food prices. Inflation rate is the price level increase overtime. This is usually represented and determined by the consumer price index (CPI).

As CPI determines the cost of a range of goods relative to its cost at the base year, it has been used as a proxy of inflation rate. It can be calculated and expressed as:

$$\text{Inflation rate at time } t = CPI_{(t)} = \frac{P_t - P_{(t-1)}}{P_{(t-1)}} \quad (1.04)$$

Where P = the general average level of price;

t = reference year or the time; and

t – 1 = the preceding year to the reference year (base year).

The main source of inflation can be attributed to either an increase in the money supply without equivalent or commensurable output (production) increase, or general increase in the level of prices due to much demand of goods and services over stagnant or falling supplies.

Other economic forces that can trigger higher inflation rate include:

- increase in unit labour costs;
- pressure from high import prices;
- fluctuation of exchange rate;
- excess demand through consumption of goods and services;
- effect of indirect taxes such as tariff, excise duties, value added tax (VAT);
- high interest rates; and
- non-moderate increase in GDP.

Basically, in economic theory, there is a number of ways of categorising inflation. Some economists based their classifications on particular grounds or criteria. But we are discussing four types of inflation being categorised by their speed: the creeping inflation, walking inflation, galloping inflation and hyperinflation.

Creeping inflation is a mild type with the rising price change of less than three per cent, and that of walking one ranges between three and nine per cent. Both are of single digit change, which is being termed moderate inflation (Kumar, 2009). In fact, this range is seen as stable rate of inflation.

Galloping inflation ranges between two or three digit change, while the hyperinflation is of four and above digit change. These last two are harmful to the economy with more intense impact from the hyperinflation. At hyperinflation level, the value of currency reduces to almost zero making paper money worthless.

According to Samuelson and Nordhaus (1998), the rate or level of severity led to three types of inflation. These are the low, galloping and hyperinflations. The authors claimed and explained the three by their severities in terms of annual percentage change of a single digit, two/three digit range, and four and above digit range respectively. One can see that Samuelson and Nordhaus combined the creeping and walking inflations to form the low type in the same way as Kumar's moderate inflation type.

For instance, by a close observation on inflation rates of some countries in our study (See Appendix 2), one can see that Japan has one-digit or low inflation throughout the period of our study (1970 – 2011) except for 1973 to 1975 with galloping inflation. Also there was hyperinflation in Angola from 1993 to 1997 and in Argentina from 1988 to 1990.

Further, inflation with devalue effect on money/goods has impacts on unemployment and other macro-variables. These impacts are causing dynamic and stochastic effects resulting to business cycle movement. These effects will be later discussed.

1.1.3 Unemployment

Unemployment rate is crucial in determining the welfare of a country. It is generally believed to be closely tied to GDP growth and inflation.

Unemployment is a situation in which certain number of able people in a country that are qualified to engage in labour force are not engaged. This kind of situation causes both economic and social problems in a country. It is an unhealthy economic situation that a

country needs to handle with care. To overcome the problems, a stable and high employment rate must be maintained.

Unemployment rate is the ratio of unemployed number in a country to its total number of labour force. It can be generally expressed as -

$$\text{Unemployment} = \frac{UL \times 100}{TL} \quad (1.05)$$

Where UL = number of unemployed;

EL = number of employed labour; and

TL = total labour force ($UL + EL$).

Measurement of unemployment has been changed many times. However, the existing practice is of two types, namely, the claimant count approach and the survey of labour force approach (designed by the United Nations International Labour Office tagged the ILO unemployment).

In the claimant count approach, the unemployed comprises the sum of figures from those actively looking for jobs and those who registered for the benefits. UK utilised this approach for some time but now switched over to the other standard method of the United Nations International Labour office. The shortcomings of the claimant count approach include:

- not useful or applicable in a country where there is no accurate data and no benefit scheme;
- not taking care of people who intentionally do not register for benefits or come forward for employment; and
- the constant changes in benefits affect this approach.

In the ILO unemployment approach, people of age 16 (or 18 depending on definition of workforce in a country) and over are considered. They are categorised into ILO

“unemployed”, “inactive” and “inemployment”. The ILO unemployed include those out of jobs but are active job seekers. The ILO unemployment is determined monthly and it is defined by ILO as:

$$ILO_{unemp.} = \frac{N_{unemp}}{N_{totactive}} \times 100 \quad (1.06)$$

where equations (1.05) and (1.06) are respectively the general and ILO definitions, which makes the numerators of the two identical and likewise the denominators;

N_{unemp} = ILO unemployed figure; and

$N_{totactive}$ = total number of active labour force.

The major difference in the two methods is the constituent or definition of unemployment. Therefore, the issue of unemployment can be further explained by classification such as frictional, structural and cyclical unemployment.

The frictional unemployment exists in a situation where there is an unceasing or continuous change in movement of labour force (people) between locations and jobs. At the time of not gainfully employed due to movement, they are categorised and counted as unemployed. Also, there can be continuous change in jobs setup.

The structural unemployment exhibits the phenomenon of having a bad or unsuitable match between the demand and supply of labour force. This is a situation which require a particular skill labour but none commensurable supply of that skill or oversupply of a skill with less demand for it.

In cyclical unemployment, which is also called the demand-deficient unemployment, there is generally low demand for labour due to economic problems. When this happened, the total expenditure and production fall. It is usually common during recession times.

The health situation of the labour market is usually determined by these three types of unemployment. The cyclical is associated with recession in economy; while the other two (frictional and structural) formed the equilibrium unemployment. Equilibrium unemployment exists when the GDP is at its potential level, i.e. economy is at the level of possible improvement.

The consequences of unemployment on a country are of great significances. Among these, in the facet of economy are:

- i. less or high unemployment induced an increase or decrease in GDP . The Okun's law in macroeconomics support this. Here, the law asserts that the relationship is inversely proportional between GDP and unemployment [Blanchard (2006), p 186-188; Hall and Papell (2005), p 76,77); and
- ii. inflation rate is equally affected by unemployment. Based on the original Phillip's curve, there is trade-off between them leading to inversely proportional relationship (Chamberlin and Yueh, 2006).

Other non-economic issues emanated from unemployment include:

- a) poor welfare ensued, which lead to hard time for feeding and even getting essentials of life; and
- b) social problems increased, which include theft, robbery, prostituting, and other social vices.

From the above consequences, one can see that suitable rates of GDP and inflation can lead to adequate rate of unemployment; and otherwise of the suitable rates, an increase in the unemployment rate ensued.

1.1.4 Balance of payment

Within an open economy, the concept of balance of payments (BOP) is of great importance; and it entails an aggregate set of accounts of a country that summarised her economic transactions with other countries of the world. These transactions consist of trade and financial flow which are broadly categorised and summarised into current and capital accounts.

Among the constituents of current account, we have exports and imports. The two variables are impetuous components of balance of trade (BOT). Hence, exports and imports have impact or role to play in determination of economic wellbeing. Our equation (1.03), which is general economists' assumption and definition, equally supports this from the expenditure approach of GDP theory. It is worth mentioning that the balance of trade (through exports and imports) have profound influences on the level of consumptions and development of a country.

There is an inter-relationship between BOT and a country's exchange rate. The exchange rate has impact on BOT in the sense that high exchange rate induces high import values which can result in trade deficit and vice versa. Through volatile exchange rate, inflation and interest rate changes can creep into the economy and in return affect the consumption.

Another salient point on an open economy is the openness in the financial markets. The principle is to encourage financial investors to invest and hold both foreign and domestic assets as a way of diversifying their investment risks. Here, financial institutions act on behalf of people (investors) on issues of new financial decisions after necessary consultations with them.

Lastly, to support the claim on these variables as watchdog, the investors are expected to watch the performance of GDP and inflation of a country as the most prominent instrumental

indicators for decisions. They watch and study these metrics regularly in order to know the line of action to be taken on their investments in financial markets. In fact, these variables have high impacts on the movement of stock market.

In view of the effects of these variables, most especially the GDP and inflation, on stock exchange movement and other related variables we observe that:

- (i) there should be a relationship between GDP and inflation;
- (ii) unemployment should be closely tied to GDP growth. Okun's law supported this ;
- (iii) unemployment and inflation are twin-sisters type of macro-variables. According to original Phillip's curve (1958), which was later modified by the criticism of Milton Friedman (1968) and group in 1970s, there is still a short run trade-off between the variables. By this, unemployment directly or indirectly gets related to inflation in the short run; and
- (iv) exports and imports, seen as injector and linkages to economy respectively, are both inputs, factors ,or as parts of the function of GDP and inflation by direct or indirect effects.

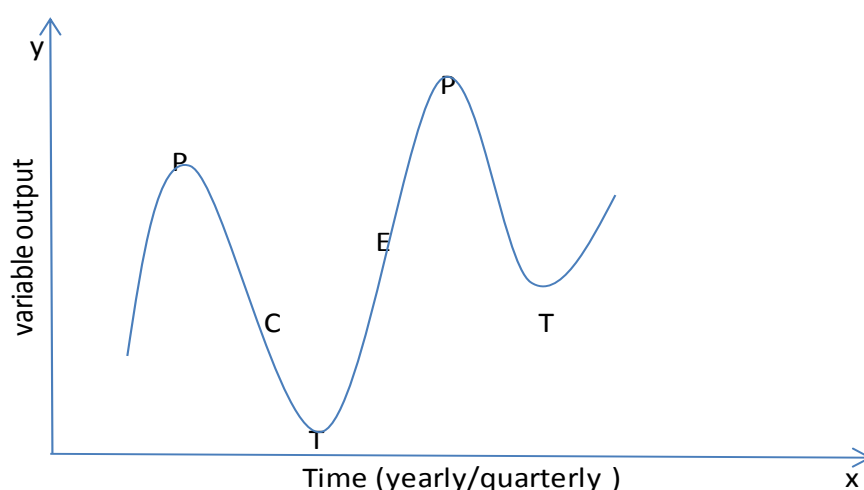
By these observations, we endeavour to examine possible Granger causality relationship between GDP and inflation at the first stage of our study. We consider in stage two the pairwise relationship for possible further links among exports, imports, GDP and inflation. At this stage (stage two), we use data from 33 countries that satisfied the Granger causality in the first stage. Furthermore on stage two, we explore for possible establishment of a sort of relationship using Bayesian's results and comparing the coefficient of variations for exports and imports as relating to inflation and GDP. On all our four focused variables, the need for isolating each component of change is assumed and maintained by relying on the 'ceteris paribus' assumption of holding everything else equal or remain constant.

1.2 Business Cycles and Fluctuations

Business cycles theory is concerned with the economic growth of a country in term of its GDP. As we have discussed earlier, many factors or other macro-variables are having impacts on the irregular movements of the GDP. By this, it is essential to take care of these irregularities in our analyses. In fact, the Keynesians presented it as a study of macroeconomic fluctuations (Blinder, 1986, 1987). They further claimed that business cycle is a pattern of movement with rapid growth and slowdown or decline in output fluctuations along a trend. These are marked and characterized by widespread of expansion, contraction, recession and recovery in the economy.

Furthermore, the macroeconomists classified business cycle into two main phases, namely, the recession and expansion. Peaks and troughs indicate the turning points for contraction and recovery respectively. See Figure 1.01 below:

Figure 1.01: Sketch of Business Cycles and Fluctuations



Note in Figure 1.01 that the paths P through C to T, and T through E to P respectively represent the recession and expansion periods.

It is worth mentioning that, in the business cycle theories, the sources of these movements are attributed to external (endogenous) and internal (exogenous) forces. The external forces include fluctuations of factors outside the economic system that may cause the movement. Examples of these factors include un-predictable bad weather, wars, revolutions, oil price (control through cartel) just to mention a few.

The internal forces include various policies and mechanisms introduced into the internal system of economy contribute to the movement. Government policies such as fiscal and monetary, political, and shifts in aggregate demand or supply of essential commodities or items within the economic system are good examples. Other mechanisms include various technology advancements in the economic system.

The resultant effects of these fluctuations can be seen in the change of inflation rate, in output (GDP) and employment variations.

Finally with the business cycle, one can see how fluctuations emerged in the economic growth/GDP. The main resultant effect of fluctuations on GDP and other related macro-variables is non-stability. By this, stochastic and dynamic effects ensued on these variables. Hence, the economic downturn and upturn on the variables lead to issue of trend and cycle. Trend indicates the long-run trend whilst cycle for short-run effects. From this explanation, it is expected to ensure the said macro-metrics are stable or stationary before any statistical analyses can be applied on them.

1.3 Aims and objectives of the study

This empirical-applied research examines the Granger causality in order to establish relationship of cause-effect principle on these variables, in a way to know which variable enhanced the prediction of the other. We further consider Bayes' rule on the results obtained from Granger causality tests. Also, we venture to carry out classical statistical tests such as

normal test, proportionality test and coefficient of variation; and also apply the optimization principle (assignment problem) on the results of stages one and two in order to establish linkage and interrelationship between our concerned variables in the study. In summary, we are utilising various forms of relationship, be it parametric, non-parametric, normality or non-normality approach but with much emphasis on Granger causality.

In the light of the above, our research considered “the relationship between GDP, inflation, import and export from a statistical point of view”. To achieve this, the research aimed at the following objectives in order to:

- (i) determine the pattern and nature of Granger causality between GDP and Inflation by utilising auto-regression on the annual percentage changes for the period 1970 to 2011;
- (ii) obtain the proportions of Granger causality patterns by classifications to developed and developing economies with appropriate statistical tests;
- (iii) know the supporting pattern of the Granger causality directions on each of these economies (developed and developing) by using some statistical tools;
- (iv) establish the pattern and nature of Granger causality between the pair combinations of GDP, Inflation, exports and imports at second stage utilising the actual values (in millions of US dollars) from 1970 to 2011. At this stage, exports and imports are introduced in order to know how they relate to GDP and inflation on the Granger causality concept;
- (v) explore the conditional probabilities in terms of cause and effect using Bayes’ rule;
- (vi) find the optimal combinations of results emanated from the Bayes’ rule in (v); and
- (vii) carry out comparisons of coefficient of variations of exports and imports of various countries in order to see their effects on the pattern established in (i); that

is, as a way of presenting impact or effect of some components (exports and imports) of balance of trade on the results of (i).

1.4 Structure of the Thesis

The arrangement of materials in this thesis comprises of seven chapters. See Figure 1.02 on the structural chart of the thesis. After Chapter 1, we have the conceptual description consisting of literature review and theory of some statistical principles/practices used in the study. This consists of Chapters 2 to 4. Also, there exists the empirical studies block which gives the account of methodology, findings/results and interpretations on our various computations and results in Chapters 5 and 6.

In summary, we have:

Chapter 1 presents the introduction with discussion on some macroeconomic variables as the watchdogs on economy performance in terms of wellbeing, financial strength and economy buoyancy of a nation. This led to our brief descriptions of GDP, inflation, unemployment and balance of payment in Section 1.1 whilst the concept of business cycle and fluctuations considered in Section 1.2. Also, the aims and objectives, and structure of thesis are highlighted in Sections 1.3 and 1.4 respectively.

Chapter 2 gives the literature review, comprising of historical background of causation in Section 2.1, definition of Granger causality in Section 2.2, and Granger causality review in Section 2.3. Some philosophical views or thoughts on causation were discussed.

Chapter 3 provides some useful terms in regression analysis in Section 3.1 and then investigates the stationary and non-stationary situations of the variables. Types of stationarity are discussed in Section 3.2 and that of non-stationarity in Section 3.3. Appropriate tests for unit roots, trend, structural breaks and outliers are considered in Section 3.4. Among the tests

we have Augmented Dick-Fuller, Phillips-Perron, KPSS, Chow's and Quart methods. Section 3.5 presents the transformation of non-stationary whilst Section 3.6 discusses the type and determination of lag length.

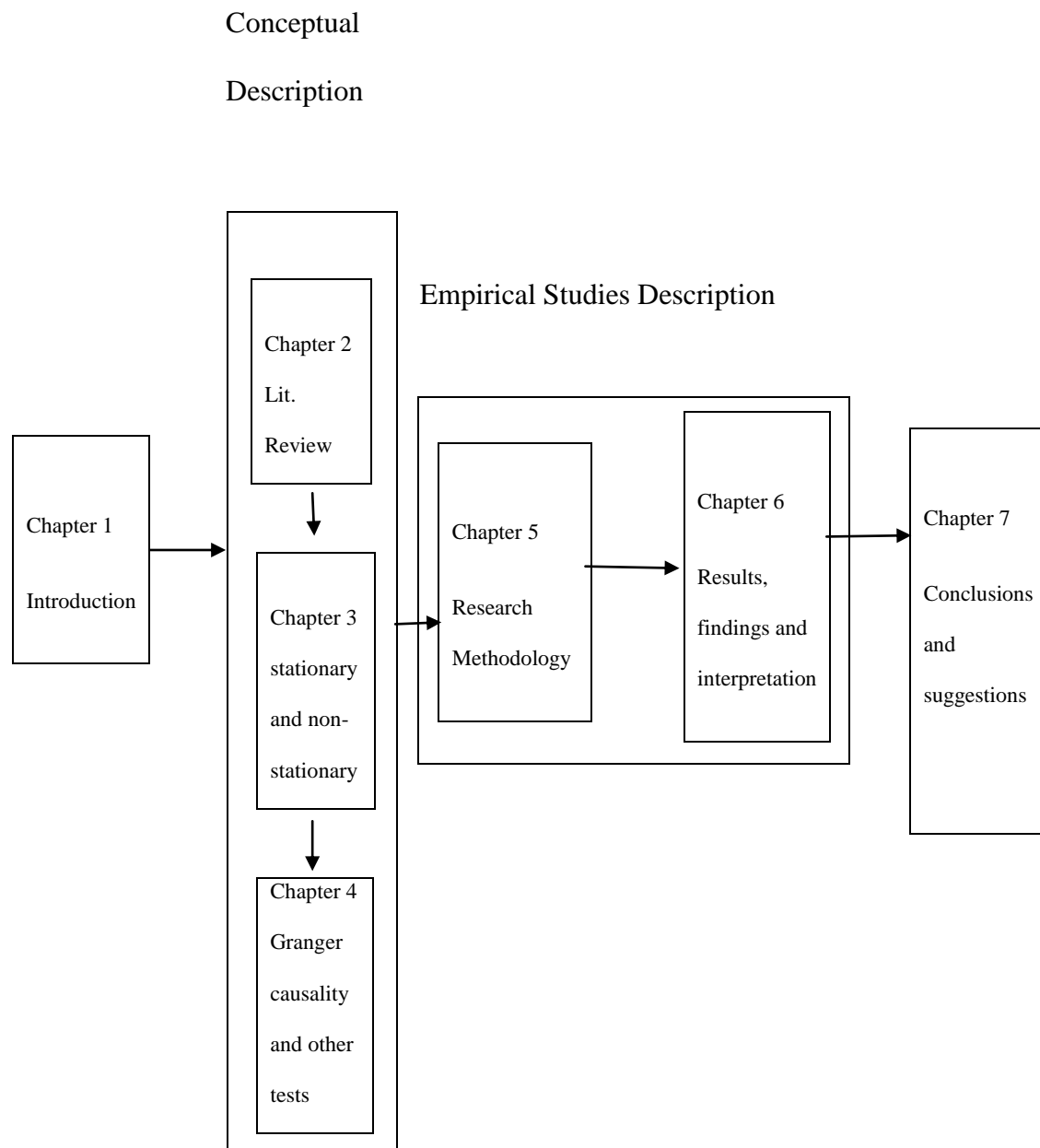
Chapter 4 discusses the various statistical tools utilised in computations. Section 4.1 witnesses some of Granger causality methods; while other statistical analyses and tests are presented in Section 4.2 include Bayes' computations, Normal and Binomial tests and so on.

Chapter 5 outlines the data description and methodology, where the statistical instruments/tools for the research are included. Other statistical analyses on the Granger causality results are given. In summary, the chapter described all the necessary empirical steps.

Chapter 6 covers respectively the results and findings, and the interpretations in Sections 6.1 and 6.2. It is the chapter that gives the outcomes of the empirical computations and its full interpretations.

Chapter 7 finally makes available the summary, conclusion and suggestions respectively for the study in Sections 7.1, 7.2 and 7.3. Here, the conclusion which reports the final research findings and possible further studies on the research is discussed. See the structural chart (Figure 1.02) on the next page.

Figure 1.02 – The Structural Chart of the Thesis



Chapter 2: Literature Review

2.0 Introduction

This chapter presents the historical background of causality and the Granger Causality in terms of meaning and its general review.

2.1 Concept of Causality and its Historical Background

According to Hulswit (2002), the ideas of cause, causation and causality have a long history; likely emanated from the time of Plato. However, Aristotle (384BC – 322BC) was the first person being recognised to state and explain the idea of causation in an elaborate way. Aristotle was able to identify and distinguish four causes, and be able to adduce their interpretations on these ideas. The identified causes are the material, efficient, final and formal. He was concerned with the issue of a single existing thing by asking ‘‘what is this?’’. As a way of response to the question, he gave the causes in order to address it from four perspectives. Specifically, the identified causes respectively answered the questions- ‘‘what is this made of’’, ‘‘what is this made by’’, ‘‘what is this made for’’, and ‘‘what is it that makes this what it is and not something else’’.

By these causes, Aristotle was able to use them to establish an explanation for how a thing came about. By these causes, he was able to attribute:

- i. material cause to the source at which something arises;
- ii. efficient cause to a thing or activity causing a change. The result of this change is the effect;
- iii. final cause to end or purpose of a thing; and

- iv. formal cause to the importance and necessity of a thing being organized or created. In fact, the formal cause precedes the efficient cause.

Further to epitomize these ideas, the material cause can be seen as raw items needed for creating or making things. For instance, a chair made from plank or iron. The plank or iron is the material cause. The efficient cause is the carpenter or welder that made the chair, changing it from plank/iron to chair. The formal cause is the necessary steps or organized way the carpenter/welder handles the building of the chair. Lastly, final cause is the concern of making chair not table. The purpose is to use the chair to sit.

In view of the above explanations, it is worthy to note that the strengths of Aristotle causations can be seen in terms of:

- reflecting and being compatible to the science thought involving material, organising and empirical phases;
- having effective influence on various disciplines. Note that majority of research areas discussed relationship in term of the various causes nowadays; and
- supporting different reasons, in terms of the four causations, to back up the existence of a thing.

After Aristotle philosophy on causality concept, there were a large number of philosophers that discussed the concept. Among them, we have Newton (1643 – 1727) who gave the mathematical laws of physics and the system of the world. In summary, the material and efficient causes were equally identified by him.

In another contributor's view on type of relationships, Runes (1962) gave nine definitions of causality using Aristotle ideas. These definitions of causality were reproduced and summarised by Hinkelmann and Kempthorne (2008), and Awe (2012) as:

“(i) a set of entities, events or course of actions that related in the same time series which is being affected by many different conditions;

(ii) a set of entities, events or course of actions that related in a time series in a way that when one happens, the other always followed;

(iii) a related association of variables such that one has an effective ability to produce or change another variable;

(iv) a related variable such that non occurrence of one allows the other to happen;

(v) a related association of prior knowledge of entities, events or course of actions to that of experimental entities, events or course of actions;

(vi) a relationship within a variable itself (auto-regression with one variable);

(vii) a related association that explained or give reason for the existence of entity, event or course of action;

(viii) a set of ideas and experiences being related; and

(ix) an idea or group of knowledge being related to the past ones."

From Runes' (1962) definitions, one can understand his views and give interpretations of causality concept as sort of relationships or associations which show or demonstrate some connections or linkages between variables.

According to Hoover (2006), in his paper on Causality in Economics and Econometrics, he recognised David Hume (1711-1776) as the greatest economist/historian/philosopher that laid the foundation for the later causality developments in economics. He also gave a long history about the study of causality and discussed it under two main types; namely the structural and

process approaches. Under each of these approaches a common sub-group of a priori and inferential were identified.

In the theory of Structural- a Priori, he mentioned the Cowles Commission with the causality contributions of Koopmans(1953), Hood and Koopmans(1953); while that of Structural-Inferential consists of contributions from Angrist and Krueger(1999, 2001), Favero and Hendry(1992), Hoover(1990, 2001), and Simon(1953).

In the case of process approach, he attributed the process- a priori to Zellner (1979) whilst that of Process-Inferential to causality idea through Granger (1969) and Vector auto-regression of Sims (1980). Sims' approach is an extension of standard Granger causality to vector form.

Hoover (2006) also claimed that Granger causality is the most influential and celebrated method of all the causality because of its wide applications. He further described it as modern probabilistic and inferential approach which is commonly used in autoregressive or dynamic models. It depends on empirical behaviour of the data without much or direct referencing to the basic theory of economy.

Edward Leamer, an econometrician, preferred the term precedence over causality according to Zellner (1979); while Diebold (2001) preferred the term predictive causality. In his paper, he made the statement- " y_i causes y_j is just shorthand for more precise, but long-winded statement y_i contains useful histories for predicting y_j in the linear least squares over and above the past histories of the other variables in the system".

Other sources such as Bressier and Seth (2010), Gujarati-Porter (2008) claimed that the originator of cause-effect relationship for prediction was that famous mathematician, Norbert

Wiener through his paper of 1956 titled “The theory of prediction, in Modern Mathematics for Engineers”. The assertion of the authors was based on the fact that Granger causality approach is based on cause-effect relationship. Hence, the authors called it Wiener-Granger causality instead of Granger causality alone.

Granger (1969) made a possible exploration on cause-effect relationship, tagged “Granger causality”, with the econometric models by utilising cross-spectral methods. He aimed at establishing possible relationship between some econometric models involving functions and feedback. The purpose of the paper is to establish a relationship that can enhance prediction through the principle of cause and effect. By this, he applied two stationary variables in the study and came up with four established testable definitions on -

1. *Causality definition*, which defined unidirectional causality;
2. *Feedback (causality) definition*, which established bidirectional causality;
3. *Instantaneous causality definition*, which described the causality that allowed for the inclusion of present or current value of the explanatory/independent variable; and
4. *Causality lag definition*, which discussed optimal lag length for causality.

In our study, we applied all the definitions except the third one.

2.2 Definitions of Granger Causality.

Definition 1

Granger causality, in its simple form, can be defined as:

A variable X_t “Granger causes” another variable Y_t when the past values of X_t [that is X_{t-i} , $i = 1, 2, 3, \dots$] contain information that enhance prediction of Y_t better and higher than the information contained in the past values of Y_t [i.e. Y_{t-i} , $i = 1, 2, \dots$] alone. This definition (of Granger causality) assumes stationarity for the variables X_t , Y_t .

As an illustration with the stationary bivariate X_t and Y_t , the linear autoregressive model with one lag AR(1) can be expressed as:

$$y_t = a_1 + B_{11}y_{t-1} + e_{1t} \quad (\text{for restricted model})$$

$$y_t = a_2 + B_{21}y_{t-1} + B_{22}x_{t-1} + e_{2t} \quad (\text{for unrestricted model})$$

where a_i ($i=1,2$) are intercepts, B_{ij} ($i=j=1,2$) the coefficients and e_{it} ($i=1,2$) are the errors with time sequence t . If the residual sum of square of e_{2t} is not “equal” and it is “better” than that of e_{1t} by statistical test at an appropriate significant level, then X_t Granger causes Y_t . This is a test of means equality on the two models. It implies:

$$E(Y_t | \bar{Y}_{t-1}, \bar{X}_{t-1}) \neq E(Y_t | \bar{Y}_{t-1})$$

Definition 2 :

Assume having information set $\{S_t\}$ with stationary variables $(X_t, X_{t-j}; Y_t, Y_{t-i}), X_t$

Granger causes Y_t if the variance of the optimal linear predictor of Y_{t+h} based on X_{t-j} and Y_{t-i} is smaller than that of the optimal linear predictor of Y_{t+h} based on Y_{t-i} alone.

This is another way to define Granger causality and it is based on the volatility principle.

Forest (2007) equally restated it in his paper. The said principle can be expressed as, using the same models as in definition 1:

$$\sigma^2(Y_t | Y_{t-1}, X_{t-1}) < \sigma^2(Y_t | Y_{t-1})$$

From these definitions, one can see that the main focus and benefit of Granger causality is the enhanced prediction power which depends on cause and effect relationships. This is precedence relationship which led to the view of cause happening before effect and a cause contains unique information about an effect not available elsewhere. Hence, an inequality

conditional statement of $f(Y_t|X_{t-1}, Y_{t-1}) > f(Y_t|Y_{t-1})$ for Granger causality ensued. Here, the usage of *conditional statement* (for Granger causality) does not imply *conditional probability*. The two is not the same but similar for an acknowledgement of situation in which both depends on issue of precedence. For instance in Granger causality, effect depends on cause whilst in the conditional probability, the posterior depends on new sample space (from the occurrence of prior).

2.3 Granger Causality Review

As a result of a wide application of Granger causality concept in many fields involving time series data, there exist a plethora of studies on the concept. However, most of the relevant literature to our focused area of research will be summarised in this section.

Friedman (1973) explored possible relationship between economic growth and inflation in an extensive manner. He came up with the following summarised statement: “historically, all possible combinations have occurred: inflation with and without development, no inflation with and without development”. By this study, the relationships between the two series are not fixed and consistent; that is, it may exist or it may not.

Jung and Marshall (1986) considered the effects, either positively or negatively, of inflation on economic growth. The study arose from the debate of structural, distortionist and natural economists of different opinions on these effects. The authors carried out Granger causality tests on these variables in order to verify their claims. It is the belief of the structuralists that inflation has positive effect, and this discussion was based on Georgescu-Roegen (1970) view. More precisely, they asserted that inflation is a force which induces savings. An alternative view by the distortionists is that inflation has a negative effect on economic growth (Mundell 1971, Taylor 1979). The naturalists neither support the other two’s views (Lucas 1972, 1973). The outcome of their findings, using Granger causality test on annual

data ranging from 15 to 34 years, substantially supported distortionist view most especially in developed countries; whereas too little and few for structuralist among the developing countries.

Bruno and Easterly (1996) carried out an empirical study on the determinants of economic growth using annual CPI (Inflation) of 26 countries which experienced inflation crises from 1961 to 1992. They were able to conclude that the threshold level for an inflation crisis begins at an Inflation rate of 40 per cent and above. In addition, the authors claimed that it was inconsistent or somehow inconclusive on the relationship between inflation and economic growth below this threshold level; whereas there was a temporal negative relationship between inflation and economic growth beyond this threshold level. Lastly, they found that reduction of high inflation helped countries to recover their pre-crisis economic growth rates.

Paul, Kearney and Chowdhury (1997) used 70 countries of which 48 were developing economies in their research for the relationship between economic growth and inflation. With the data from 1960 to 1989, they found no causality between the two variables in 40 percent of the countries considered; 20 per cent were bidirectional while the rest (40 per cent) were unidirectional (either ways). More interestingly, the relationship between economic growth and inflation was found to be positive in some cases, while it is negative for other cases.

Malik and Chowdhury(2001) explored the relationship between inflation and economic growth in the framework of four South Asian economies (Pakistan, Sri Lanka, Bangladesh and India) to study whether there exists a relationship and if so, its nature. They applied co-integration and error models on the yearly data sourced from International Financial Statistics (IFS) unit of International Monetary Fund (IMF) and came up with two meaningful outcomes. The first one is at economic growth and inflation are positively related; the second

finding is “that the sensitivity of inflation to changes in growth rates is larger than that of growth to changes in inflation rates”.

Fatai, Oxley and Soriwzgeour (2004) utilised Standard Granger method, and Toda and Yamamoto`s (1995) approach to empirically investigate the relationship between GDP and energy consumption in Australia, New Zealand and four Asian countries (Thailand, The Philippines, India, and Indonesia). The data used is an annual data set from 1960 to 1999. Granger results support unidirectional causality from real GDP to energy consumption in Australia and New Zealand, whereas in Asian countries, it is unidirectional line from energy to GDP in India and Indonesia whilst a bidirectional link in Thailand and Philippines. A simple inference from this study is that energy conservation policies did not have significant impact on the industrialized countries like Australia and New Zealand when compared to the Asian countries.

Konya (2004) explored the possibility of causality in terms of export-led GDP growth and GDP growth-driven export in twenty-five countries of OECD. The author utilised two strategies to carry out its analysis on vector auto-regression (VAR) of Granger Causality tests. The first involved variables in levels or first difference with Wald procedure and then considered the modified Wald procedure with the augmented level of VAR. The findings indicated exports cause growth (ecg) in Iceland, but less certain with ecg in Australia, Austria and Ireland; while in Canada, Japan and Korea growth causes export (gce) with less gce certainty in Finland, Portugal and USA. It was both directional causality between the two in Luxembourg and the Netherlands. The results of other countries such as Belgium, Italy, Mexico, New Zealand, Spain and Switzerland are of controversy to decide whether there is causality or not.

Ahmed and Mortaza (2005) examined the relationship between economic growth and inflation in Bangladesh using co-integration and error correction models for the study. With the yearly data on CPI (Inflation) and real GDP from 1980 to 2005, the empirical result supports the existence of long-run negative relationship between economic growth and inflation for the country.

Liu (2006) investigated the Granger causality on real GDP and four types of air emissions [Sulphur dioxide (SO_2), Oxides of Nitrogen (NO_x), Carbon monoxide (CO), and Carbon dioxide (CO_2)] by utilising the Norwegian annual data from 1973-2003. Co-integration (including ECM) and VAR approaches are applied, and resulting to only unidirectional causal relationships between GDP and air emissions. CO_2 and CO are of long run causality from GDP to emissions, while SO_2 and NO_x are of short run causality from emissions to GDP. By comparing the results above with the conventional standard of EKC (Environmental Kuznets Curve, which can be defined as a hypothesized relationship between various indicators of environmental degradation and income per capita) analyses, the paper concluded in accepting CO_2 and CO and in rejecting SO_2 and NO_x of the said convention.

Singh and Konya (2006) examined the Granger causality tests in terms of export/import-led growth and growth-driven export/import utilising India data from 1950/51 to 2003/04. The Granger causality methodology used is mainly based on the paper of Konya (2004) and other contributions from the literature. VAR and VEC, with Wald and modified Wald tests respectively are applied. The outcomes support exports/imports caused GDP in terms of Granger causality either individually or jointly. Also, there was an indication of joint GDP/exports Granger caused imports whilst exports are being “Granger caused” by imports/GDP. Lastly, the growth-driven export/import could not be established.

Wen (2007) applied Granger causality concept to investigate whether there is equilibrium between the demand shocks and business cycle models. He made use of the imbedded information such as: (i) employment and output, and (ii) investment demands. The author carried out standard Granger causality tests on these variables utilising U.S. quarterly data from 1947 Q₁ to 2006 Q₁. The results shown GDP is being “Granger caused” by consumption growth but not vice versa. However, business investment growth “Granger-caused” by GDP; and also it is not reversible. In conclusion of the study, it was established that unidirectional Granger causalities support: (i) consumption has enhanced information for shocks, (ii) output has better information for shocks than the investment. Thus by the approach, Granger causality test has an advantage with better explanation which the standard real business cycle models cannot render.

Saaed (2007) examined the economic growth-inflation relationship using the annual data on CPI (Inflation) and real GDP (economic growth) from 1985 to 2005. He made use of co-integration and error correction models. In his findings, he was able to establish a long-run and strong inverse relationship in Kuwait.

Erbaykal and Okuyan (2008) investigated the relationship between economic growth and inflation. The authors used the quarterly data of Turkey from first quarter of 1987 to the second quarter of 2006. In the research, they utilised the framework of Bound Test developed by Pesaran et al (2001) and the causality test developed by Toda Yamamoto (1995). Their findings showed that no long term relationship exists between the two variables despite the variables were co-integrated. Also, bidirectional causality was established with the two variables.

Agu and Chukwu (2008) carried out empirical research on the relationships between economic growth and bank-based financial deepening. Toda and Yamamoto (1995) causality

test, an augmented Granger causality approach, was utilised with Nigerian annual data between 1970 -2005. In addition, the authors used Multivariate Johansen and Juselius (1988, 1992) and Juselius (1990) methods to determine any long-run equilibrium relationships. The outcomes with Toda-Yamamoto tests supported the supply-leading hypothesis for “bank-based” financial variables like bank deposit liabilities and loan deposit ratio; while the demand- following hypothesis for “bank-based” financial deepening variables like broad money and private sector credit. Hence, the authors concluded that the choice of bank-based financial deepening variable influences the causality results. Also, with co-integration tests, the study supported financial deepening and economic growth as positively co-integrated.

Shahbaz, Awan and Ali (2008) investigated the directions of Granger causality using Toda and Yamamoto (1995), and impulse response function and variance decomposition approaches. The study utilised the annual data of Pakistan from 1972 to 2007. The inference from the two approaches supported bi-direction causality between foreign direct investment (FDI) and domestic savings (DS) but with DS to FDI being stronger.

Shombe (2008) examined the causal relationships among exports, agricultural and manufacturing products of Tanzania. VAR and VEC were utilised with the annual data from 1970 to 2005. The results from VAR support (i) agriculture causes both exports and manufacturing; (ii) exports cause both agriculture and manufacturing; while (iii) any two variables out of the three jointly cause the third one. On VEC, the following pairs are co-integrated: (i) agriculture and export, (ii) export and manufacture; and (iii) agriculture and manufacture (with lag sensitiveness). Also, the three variables are co-integrated with long run equilibrium.

Ullah, Zaman, Farooq and Javed (2009) examined the possibility of export-led growth relationship using cointegration (including Vector Error Correction Model) and Granger

causality methods. The Pakistan annual data from 1970 to 2008 was utilised. The results supported export expansion “Granger-causes” the economic growth. Also, carrying out the standard Granger causality tests on real export, real gross fixed capital, real import, real per capita income and economic growth, the relationships revealed one-directional Granger causality between exports, imports and economic growth.

Uddin (2009) explored the behavioural pattern and relationships in the Bangladesh’s imports and exports by utilising co-integration analysis and error correction model. An annual data spanning through 1972/73 and 2007/08 was used. The variables, totals of exports and imports, revealed random walk pattern through the unit root tests. Johansen co-integration tests supported both ways causality of the long-run equilibrium type whilst short term for the unidirectional causality for the variables.

Oladipo (2009) unveiled the Granger causality relationship between savings and economic growth in a small open economy. Nigerian data, as one of the developing countries, was utilised spanning from 1970 to 2006. He employed Toda-Yamamoto (1995) and Dolado-Lutkepohi (1996)-(TYDL) methodology for the said relational tests. Savings and economic growth revealed positively co-integration with a stable long-run equilibrium result. In addition, unidirectional causality was established between savings and economic growth with the complementary role of FDI (foreign direct investment).

Keho (2009) uncovered the relationship, in terms of long-run and causality, between inflation and financial development in the countries of the UEMQA (which is now renamed West Africa Economic and Monetary Union in 1994). The UEMQA countries considered for this study were Senegal, Niger, Togo, Burkina Faso, Cote d’Ivoire, Mali, and Benin Republic. The empirical research utilised co-integration proposed by Pesaran et al (2001) and Toda-Yamamoto (1995) Granger causality tests. Results of the tests supported no long-run

relationship between two variables in six countries and no causality for two countries. Also financial development causes inflation in four countries and bidirectional causality in two countries were established. Hence, the patterns of causality in UEMQA varied across the countries.

Chimobi (2010) empirically examined the relationship between Inflation and economic growth in Nigeria using annual data from 1970 to 2005. Consumer price index (CPI) and GDP were respectively utilised for inflation and economic growth as proxies. By applying co-integration methods and VAR-Granger causality to these data, he was able to come up with the superiority of Johansen-Juselius co-integration technique than Engle and Granger co-integration method. Also, unidirectional causality from inflation to economic growth was established.

Chimobi and Uche (2010) explored the possibility of any Granger causality relationship between economic growth, domestic demand and export. The authors employed the co-integration and the pair Granger causality tests on the Nigerian annual data from 1970 to 2005. The outcomes supported economic growth “Granger caused” export and domestic demand whilst bilateral causality with export and domestic demand.

Ismail et al (2010) empirically examined the possible linkages between economic growth, inflation, exports and investment in Pakistan. The research utilised VAR and co-integration (including Johansen`s and ECM) tests on the data for over the period 1980-2009. The results’ evidence supported exports and investment of having positive impacts on GDP; whereas inflation had negative impact on the Pakistan economy. Also, there was no long-run effect on exports led growth.

Jayachandran and Seilan (2010) studied the relationship between foreign direct investment (FDI), economic growth and trade. The Indian annual data from 1970 to 2007 was utilised to

carry out the said study by applying co-integration analysis. The empirical results supported existence of non-reciprocal causality relationship among the variables, exports Granger-causes GDP, there was no causality relationship from FDI to exports and also no causality from GDP to exports. Co-integration analysis uncovered long-run equilibrium relationship with the variables.

Kogid et al (2010) carried out an empirical study on factors affecting economic growth by utilising a case study on Malaysia. The annual data from 1970 to 2007 was collected on consumption, foreign direct investment, government expenditure, exchange rate, and export. Co-integration causality approach, by Johansen and ECM tests, were conducted on the variables using the annual data. The outcomes unveiled the existence of long-run co-integration and multiple short-run causal relationships between economic growth and determinant factors. Also, the combinations of different factors cause economic growth in short-run. Hence, the authors concluded that consumption/ expenditure/ export played active and important part on economic growth. That is, these variables or factors have effective impact on the economic growth.

Kogid, Mulok, Cling and Lily (2011) investigated whether import affect economic growth of Malaysia. Bivariate co-integration and causality analysis based on Engle-Granger and Johansen were applied on the annual data from 1970 to 2007. The results supported no co-integration between the two variables; while economic growth Granger-causes the imports.

Hussain and Malik (2011) examined the possibility of any Granger causality relationship between economic growth and inflation of Pakistan. The study employed co-integrated methods to investigate the causality between the said variables by utilising yearly data over the period 1960- 2006. The outcomes established that inflation is positively related to

economic growth and vice versa. On causality, uni-directionality was exhibited but for the error correction model (ECM), inflation is not in equilibrium.

Afaha and Oluwatobi (2012) empirically investigated possible relationships between economic growth and foreign trade through the export led growth in Nigeria. Linear multiple regression analysis was utilised with the annual data from 1980 to 2010. The outcome of the analysis revealed that exchange rate, per capita income and export were positively related; whereas imports and economic openness were related negatively to economic growth. The paper then concluded by recommending to the government of Nigeria for the need to fine-tune various macro-economic variables in a way to create or promote more avenue for foreign trade.

Saad (2012) investigated possible Granger causality between the variables: export, external debt servicing and economic growth in Lebanon. The empirical study utilised annual data ranging from 1970 to 2010 with an addition of exchange rate to the other variables making the fourth. By employing Granger causality and vector error correction model (VECM) analyses on the data, a long run and short run relationships were established among these variables. Also, there were bi-directional Granger causality between external debt servicing and economic growth; uni-directional from external debt to exports; exports to economic growth; and likewise exchange rate to economic growth.

Awe (2012) explored the pairwise Granger causality model to examine relationship between the seven economic indicators in Nigeria. These indicators were GDP, government investment, exchange rate, government expenditure, interest rate, money supply and inflation rate. Co-integration and Granger causality tests were carried out on an annual data from 1970 to 2004. The findings alternated between all the possible outcomes of Granger causality, i.e. uni-directional (the two types), bi-directional and non causality. In conclusion, the paper

revealed that government expenditure, real money supply and government investment “Granger causes” output growth in Nigeria.

Muktadir-Al-Mukit, Shafiulla and Ahmed (2013) investigated the hypothesis that inflation led import for Bangladesh. The authors utilised methods of co-integration, VECM and Granger causality on the monthly data from 2000 to 2011. The results indicated stable and positive relationship between the variables, and the Granger causality supported existence of a uni-directional causality from inflation to import.

Agboluaje (2013) employed the structural tools analysis on the inflation and GDP of Nigerian data. Among the structural tests utilised were the Granger causality, impulse response and forecast error variance decomposition. By using these tests, the findings shown no causality between GDP and inflation rate and by forecast error variance, innovations in per capita GDP contributed a little to explain the variation of nominal GDP and inflation rate. Hence, low rate of the economic growth was established.

In summary of the above literature review, there exists Granger causality with diverged outcomes on the inflation-economic growth relationship with some supporting causality theory while some at variance. With those supporting, there is no uniform pattern of Granger causality between GDP and inflation.

Other observations include:

- the results of Granger causality on the structural and distortionist economists views did not totally conform to their claims. However, the outcomes supported distortionist, most especially in developed economy, whereas it was scanty for structuralist among the developing countries;

- some authors included more variables in addition to GDP and inflation. Among the variables, we have export, import, interest rate, and exchange rate;
- various methods of Granger causality tests were used. Among these, we have (a) Standard Granger causality (for a pair of variable only), (b) Vector auto-regression methods (an extension of Granger to multiple variables) in various forms e.g. VAR, SVAR, VECM, co-integration [with Engle-Granger, Johansen and Juilus], etc; and
- lastly, some wrong combinations of variables were noted, such as nominal GDP and inflation rate. Also, some of the methods mentioned and utilised in the above literature are with various setbacks.

To conclude the above summary, it can be seen that various methods of Granger causality had been utilised on GDP and inflation with different outcomes; hence we observed that there is wide spread use of Granger causality principle. Hence, in this research, we equally carry out Granger causality but with a step further to utilise the components that are commonly affecting GDP and inflation as part of their constituents (export and import) to study possible relational linkages on Granger causality results of the first stage. Then we continue to the second stage focusing on an extension with export and import superimposed on stage one results. Here, the principles of Bayes' theory and coefficient of variation are explored to establish the possible linkages between our results.

Lastly, this extension study of export and import on Granger causality results of GDP and Inflation is a unique work and a better motivation for novelty on the study of Granger causality concept.

Chapter 3: Stationary and Non-Stationary Models

3.0 Introduction

In time series analysis, especially in macro-econometric and financial time series data, the series depends much on assumption that the future is like the past. By this, studies in time series usually necessitate that the assumption is maintained. Hence, it is necessary and essential to ensure the involved variables are stationary or stable before further analyses. Without stationary conditions, some issues leading to problems such as spurious regression (with the exception of co-integration case of Engle-Granger (1987)), wrong estimates, incorrect decision and invalid forecasts will arise. To guide against the non-stationary, which is caused by stochastic effect, there is the need to first study the movement of the individual variable in order to understand its properties. This can be achieved through the time-plot, further verification and tests.

In light of these problems, some statistical terms which are related and useful in time series analysis, will be first presented before discussing the issues of stationary and non-stationary concepts.

3.1 Some useful terms in regression analysis

It is generally assumed in linear regression that the error terms (e_i), $i = 1, 2, \dots, n$, are of:

- (i) zero mean, i.e. $E(e_i) = 0$;
- (ii) constant variance, $\text{var}(e_i) = \sigma^2$; and
- (iii) un-correlated errors, $E(e_i e_j) = 0$, ($i \neq j$).

But in the case of time series data, these assumptions, especially the uncorrelated errors, are not applicable in the sense that the errors in time series usually result to have serial

correlation. That is, $E(e_i e_j) \neq 0$. This serial correlation phenomenon affects the mean and variance of error as well.

The term serial correlation is sometimes called autocorrelation in time series. However, some authors further made distinction between the two terms. Tintner (1965), defined serial correlation as the “Lag correlation between two different series” whilst the autocorrelation as “Lag correlation of a given series with itself”. The said autocorrelation or serial correlation has adverse effects on the time series analyses in general.

Let's illustrate how the adverse effects can creep in:

For a set of linear regressions, there is a set of error terms (e_i). If we consider autoregressive model of order one AR (1) for the error terms, it leads to:

$$e_t = \alpha + \rho e_{t-1} + \varepsilon_t \quad (3.1.1)$$

where e_t is the error term as a variable at time t , ε_t is error term which is normally and independently distributed [NID $(0, \sigma_\varepsilon^2)$], and ρ is the auto-correlation within the range $(-1 \leq \rho \leq 1)$.

By the AR(1) model of equation (3.1.1), it suffices to note that the model is conditioned on the past value of e_{t-1} such that

$$E(e_t | e_{t-1}) = \alpha + \rho e_{t-1} ,$$

$$\text{Var}(e_t | e_{t-1}) = \text{Var}(\varepsilon_t) = \sigma_\varepsilon^2 .$$

Assuming the series {errors (e_t)} is weakly stationary and taking the expectation of the model in equation (3.1.1), we obtain

$$E(e_t) = \alpha + \rho E(e_{t-1}) + E(\varepsilon_t)$$

$$= \alpha + \rho E(e_{t-1}) + 0, \text{ because } E(e_t) = 0$$

Under the stationary condition, $E(e_t) = E(e_{t-1}) = \mu$, hence

$$E(e_t) = \mu = \alpha + \rho\mu$$

$$\mu = \frac{\alpha}{1-\rho} \quad (3.1.2)$$

From equation (3.1.2), we can see if $\rho = 0$, it is good and free from serial correlation/ autocorrelation; but it is not usually possible in time series. As $\rho > 0$ and getting closer to one, the serial correlation/autocorrelation effects become more pronounced on the model. The worst case is when ρ is almost 1.

As a further discussion on some useful terms, we have other time series related terms such as distributed lag, autoregression (AR), autocovariance function, and partial autocorrelation. All these terms are useful instruments in the time series analysis. Hence, the need to briefly discuss them will be useful in our further discussions in some sections.

3.1.1. Distributed Lag and Autoregressive Models

Distributed lag plays important role in time series analysis by allowing a period of time between two or more events. It is usually accompanied with past values of events. By this, we can have a number of periods of events within the series and each period representing the time sequence t .

The distributed lag can be finite or infinite when the number of periods of time (t) is known or unknown respectively. Let's consider the following finite distributed lag model:

$$Z_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + b_3X_{t-3} + \dots + b_pX_{t-p} + e_t \quad (3.1.1.1)$$

Where Z_t is the dependent variable (regressand), X_i ($i=1,2,\dots,p$) is the independent variable (regressor) with finite integer p lags, $a, b_0, b_1, b_2, b_3, \dots, b_p$ are coefficients and e_t is the error term.

If p is unknown in equation (3.1.1.1), the equation becomes an infinite distributed lag model.

Another concept related to distributed lag is the autoregression. It is a type of regression that depends on the previous values of the regressand and the regressor. A typical example of this model is:

$$Z_t = a + b_0X_t + b_1X_{t-1} + c_1Z_{t-1} + e_t, \text{ (for two variables)} \quad (3.1.1.2); \text{ or}$$

$$Z_t = a + b_1Z_{t-1} + b_2Z_{t-2} + b_3Z_{t-3} + \dots + b_pZ_{t-p} + e_t, \text{ (for single variable)} \quad (3.1.1.3)$$

Where a, b, e_t and p as defined in equation (3.1.1.1), and c_1 is the coefficient for Z_{t-1} . This model (the autoregressive) is also known as dynamic models. We will intensively utilise this model in this study. It is worth noting the similarity and difference between the two models. We observed that the main similarity is the dependency of the two on past successive points of the variable(s). For the difference, distributed lag is having only lags in the explanatory variables, while autoregression is using lags in both the independent and dependent variables as regressors.

3.1.2 Covariance, Autocorrelation Function (ACF), and Partial Autocorrelation Function (PACF)

The term covariance is a measure of linear dependency between two variables. It is an important concept because it forms the basis of discussion for all the concepts discussed above, and it will also form the foundation for our subsequent discussion. In fact, covariance and mean are the central issues in time series analysis because of the Gaussian (Normality) assumption. This assumption is based on the Central Limit Theorem (CLT) which supports

the normality of a sample mean of the distribution of observations [which have the same probability density with the defined population mean (μ) and variance (σ^2)] upon which the same mean is calculated. In the light of this, we shall pay some attention to it and briefly define it as follows:

Grimmett and Stirzaker (2001) defined “the covariance of variables X and Y as

$$\text{Cov}(X,Y) = E[(X - EX)(Y - EY)],$$

while the correlation (coefficient) between X and Y is defined as

$$\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X).\text{Var}(Y)}}$$

as long as the variances are non-zero”.

Further, covariance can be generalized to variance as

$$\text{Cov}(X,X) = \text{Var}(X).$$

The covariance of variables X and Y can also be simplified and expressed as:

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y);$$

and if $E(XY) = E(X)E(Y)$,

$\text{Cov}(X,Y) = 0$, which implies X and Y are uncorrelated.

Hence, independent variables are uncorrelated, meaning $\text{Cov}(X,Y)=0$; but the converse is not true. That is, zero covariance does not imply independence of the variables.

Autocorrelation Function (ACF)

When the linear dependence between X_t and its past values of X_{t-k} , (k is an integer), is of interest, the concept of correlation is generalised to autocorrelation. According to Tsay (2010), the correlation between X_t and X_{t-k} is called the lag- k autocorrelation and denoted by ρ_k . By the assumption of weak stationarity, ρ_k is a function of k and is defined as :

$$\rho_k = \frac{\text{cov}(x_t, x_{t-k})}{\sqrt{[\text{var}(x_t)\text{var}(x_{t-k})]}} ,$$

where k is an integer and $k \neq 0$.

Partial Autocorrelation Function (PACF)

The PACF, an extension to ACF, is a measure that examine the autocovariance of a variable x_t and its lagged version (x_{t+k}) after their intervening lags ($x_{t+1}, x_{t+2}, \dots, x_{t+k-1}$), which are mutually linear independent, are removed. It implies of being conditional correlation such that:

$$\text{Corr}(x_t, x_{t+k} \mid x_{t+1}, x_{t+2}, \dots, x_{t+k-1})$$

That is, PACF is the partial correlation coefficients between a variable and a lag of itself over time which is not explained by correlations at all lower order lags. It implies that correlation at lag 1 “propagates” to lag 2 and presumably to higher order lags. By this, the partial autocorrelation at lag 2 is difference between the actual correlation at lag 2 and the expected correlation due to propagation of correlation at lag 1.

These two concepts (ACF and PACF) are useful in time series analysis in order to determine the stationarity and adequate/ optimal lag length of the series.

According to Box and Jenkins (1976), the ACF and PACF are utilized in ARMA (Autoregression moving average) to determine the lag length and the stationarity of the

series. Here, we present how they used the (ACF/PACF) to determine the lag and stationarity/invertible of the series respectively for AR, MA and ARMA. The first two steps are as follows:

Step 1: determine the autocorrelation coefficients for a fair number of lags, and similarly for partial auto-correlation coefficients for ACF and PACF respectively;

Step 2: plot separate graph of ACF (correlogram) and PACF (partial correlogram) against the lags;

The next steps are meant for AR, MA and ARMA respectively.

Step 3: watch their movements (the correlogram and partial correlogram) within 5 percent confidence limits, for the points to fall within the limits, and to see how they are conforming to the following rules:

- (a) for the $AR(p)$ when (i) ACF movement tails off as exponential decay or damped sine wave, and (ii) the PACF cuts off after lag p ; then there is stationary at lag p and the adequate/optimal lag is p ;
- (b) in the case of $MA(q)$, when the (i) ACF cuts off after lag q , and (ii) PACF tails off as exponential decay or damped sine wave; then the series is invertible at lag q and the lag length is q ; next the
- (c) $ARMA(p,q)$ needs the conformity of (i) ACF tailing off after lag $(q-p)$, and (ii) PACF also tailing off after lag $(p-q)$; then the ARMA is stationary/invertible at p and q respectively.

3.2 Stationary Time Series

In this thesis, the focus is on time series. They can be modelled as realisations of discrete-time stochastic processes. A discrete-time stochastic process is a sequence of random

variables defined on the same probability space. To learn more on stochastic processes, the reader can consult Grimmett and Stirzaker (2001) and Stirzaker (2005).

There are two types of stationary time series; namely, the strictly (strong) stationary and the weakly (covariance/second-order) stationary. By stationary, we meant a time series x_t with a constant probability distribution over a given time period and its joint distribution is invariant to any displacement in time. To be more explicit, let's consider the following definitions.

3.2.1 A time series ($x_t : x_1, x_2, \dots, x_t$; and $t \geq 1$) is said to be **STRICTLY STATIONARY** if:

- (i) the distributions of ($x_t : x_1, x_2, \dots, x_t$) and ($x_{t+k} : x_{1+k}, x_{2+k}, \dots, x_{t+k}$) are equal for all k ; x_t (as defined above) for every value of k (k an integer) and t is the time index; and
- (ii) the joint distribution of the two is invariant.

By implication, it means there is invariance under time shift, no systematic change in mean or trend and no systematic change in variance and the mean of stochastic error is zero. In practice, these conditions are hard to check and that is the reason why the weak stationary is the preferred option for empirical study.

3.2.2 A time series x_t (x_t as defined in section 3.2.1) is **WEAKLY STATIONARY** if it satisfies the following conditions.

- (i) $E(x_t) = \mu$ (the mean)
- (ii) $\text{var}(x_t) = E(x_t - \mu)^2 = \delta^2$
- (iii) Covariance $\gamma_k = E[(x_t - \mu)(x_{t+k} - \mu)]$

Where k is an integer, γ_k stands for covariance (or auto-covariance) at lag k and is a finite function of k but not of t , i.e $k < t$ (because t is not the lag length and it should be greater

than k). Here, the first two moments (i.e. mean and variance) are invariant overtime but in the case of strict stationary, all its moments must be invariant. This is the main difference between the two (strict and weak stationaries). A violation to any of the above conditions leads to non-stationarity. This can be attributed to non-constant mean and/or invariance (in terms of covariance or heteroskedasticity) in the series. For instance, if the means and variances of a number of the series' subset groups are not approximately the same as that of the whole series, there exists non-stationarity in the series.

3.3 Non-stationary Time Series

When any of the above conditions for a stationary time series is violated, non-stationarity ensues with its accompanied problems. As earlier said, this phenomenon (non-stationarity) is very common with the macro-econometrics and financial time series data. Hence, there is the need to handle the concept with care.

The major causes of non-stationarity in time series data can be attributed to unit root, trend, outliers and structural break(s). Each of these attributes is discussed in the following sub-sections.

3.3.1 Unit Root and Trend

The non-stationary variables with unit roots or trend general take two forms of models. These are with (i) random walk (with/without drift) process; and (ii) the deterministic trend process.

Using an auto-regression model of order one AR(1) for variable X_t , the random walk (with or without drift) can respectively take:

$$x_t = \alpha + \beta x_{t-1} + e_t \text{ -----(3.3.1.1)} \quad \text{according to (i) with drift ;}$$

$$x_t = \beta x_{t-1} + e_t \text{ -----(3.3.1.2)} \quad \text{according to (i) without drift}$$

with constant α (the drift) and β (the regression coefficient) , no intercept (the pure random) in (3.3.1.2) and e_t is stochastic error, independently and identically distributed (i.i.d) with mean zero and variance δ^2 . These two cases have their problems emanating from the stochastic effects resulting to unit roots.

The second one, deterministic trend process, can also be represented with our AR(1) model as either of:

$$x_t = \alpha + \beta t + e_t \text{ -----(3.3.1.3)}$$

$$x_t = \alpha + \beta_1 x_{t-1} + \beta_2 t + e_t \text{ -----(3.3.1.4)}$$

In (3.3.1.3), the deterministic trend regressed on a time trend βt and its mean grows around a fixed trend; while in (3.3.1.4) there is random walk combined with drift component α and deterministic trend $\beta_2 t$.

In all, (3.3.1.1) and (3.3.1.2) led to unit roots problems called stochastic trend while (3.3.1.3) and (3.3.1.4) led to trend problems called deterministic trend. These two will require different treatments to induce stationarity. The first (unit roots) can be handled by differencing while the second (trend) by de-trending.

A **unit root** can be explained as an AR(p) which has a root equals 1 (one), in which, the series has a unit autoregressive root. By this, an existence of a unit root in the series implies the presence of stochastic trend. For example on the unit root concept, let us recall our auto-regression equation (3.3.1.1),

$$x_t = \alpha + \beta x_{t-1} + e_t$$

In this equation, if β (the regression coefficient) is equal to 1 (i.e. $\beta = \pm 1$), it gives unit root resulting to non-stationarity; while if $-1 < \beta < 1$, it is stationarity. When β is greater than 1

($\beta > 1$), it becomes an explosive process and this has unappealing properties such as increasingly large shocks which cannot be used to describe data series in economics and finance. Hence, it is not being relevant for consideration.

3.3.2 Outliers and Structural Break.

There are shocks in the macro-economic/financial time series data that lead to non-stationarity apart from unit root. These shocks are patches that are generally termed outliers and structural breaks. In unit root testing for univariate autoregressive time series, they affect the series mean and thereby resulting to non-linearity of the series, spurious model or incorrect decision.

The two concepts can be distinguished by the pattern of their movements and behaviours. An outlier movement is characterized by jumping upward (downward) and then moving downward (upward) immediately or gradually to its normal pattern. But in the case of structural break, it can jump upward (downward) and maintain a new level of movement for a while.

According to Maddala and Kim (1988, P 425), it was claimed that structural break is one type of outliers. In other words there exists a number of outliers types as claimed by Balke and Fomby (1991, 1994), Fox (1972) and Tsay (1988). Let's give a brief description of structural breaks.

Structural break occurs when there is an unexpected or sudden change or shift in the time series data. It can lead to a misleading inference and forecast due to the unreliability of the model caused by this break. Therefore, necessary and adequate attention must be given to it in order to have a reliable model.

The said breaks and outliers are usually caused by sudden change in government policies such as open market, tax rate, minimum wage rate, monetary, exchange rate. Others are economic recession, change in weather conditions as one of agricultural factors, growth in terms of skills and services-intensity, oil embargoes cartel (e.g. 19 73 and 1979) and so on. For more details, see the list of affected countries (with the causes) in Appendix 3.

According to Perron(1989, 1994), a structural break in the deterministic trend data leads to wrong or misleading inference or conclusion when unit root test is conducted on them. This claim was based on the view that both the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests on unit root are insensitive to break and even their inability and weakness to detect and handle coefficient of unit root closer to one.

It should be noted further that as random walk model is associated with unit autoregressive root, there is a number of times the model may be differentiated in order to make it stationary. This number of times is known as integration order, $I(d)$, where I and d stand for integration and order number respectively. For example, if X_t is integration of order one, i.e $I(1)$, then X_t has a (one) unit autoregressive root and its first difference, $\Delta X_t = X_t - X_{t-1}$, is stationary. For X_t which is stationary at level, $I(0)$, it is integration of order zero with no unit autoregressive root. Here, there is no need of differentiation for $I(0)$ because it is already stationary.

By the identified weaknesses of ADF and PP, a series found to be integration of order one $\{I(1)\}$ may be in fact stationary of zero integration order $\{I(0)\}$ around the structural break(s) but erroneously classified as $I(1)$. Hence, spurious non-rejection of $I(1)$ occurred which leads to incorrect model.

Another problem identified by Hansen (2001) is that the size and location of structural break(s) affect the regression function which is usually differed from the actual and true

regression function. The reason is that the Ordinary Least Square (OLS) regression estimator of the sample depends on mean (average) of the function in order to have the parameter estimates for the relationship. Hence, with structural break(s), the regular pattern of the variable is distorted and lead to wrong estimates and inference.

Let's look at the following illustration as a vehicle of explanation:

For a stationary variable y_t , the simple linear autoregression model of AR(1) is

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t \quad \text{----- (3.3.2.1)}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

and $E(\varepsilon_t^2) = \sigma_\varepsilon^2$;

where ε_t is a time series of serially uncorrelated shocks, α , ρ and σ^2 are parameters.

By assumption of stationarity, these parameters are constant over time and we can derive from equation (3.3.2.1):

$$E(y_t) = \mu = \alpha / (1-\rho) , \text{ for } |\rho| < 1 \quad \text{----- (3.3.2.2)}$$

Where μ stands for mean

$$V(y_t) = (\sigma^2) / (1- \rho^2) , \text{ for } |\rho| < 1 \quad \text{----- (3.3.2.3)}$$

But when there is structural break in y_t , we have non-stationarity. Its presence (structural break) may affect any or all of the model parameters with different resulting effects or implications. For instance, changes in parameter ρ will result in changes in the serial correlation of y_t ; i.e. if $|\rho| = 1$, equations (3.3.2.2) and (3.3.2.3) are not defined. Also, the intercept controls the mean (μ) of y_t ; i.e as α changes, the value of equation (3.3.2.2) changes. The changes in σ^2 indicate volatility changes. In all, we see how structural break

may affect a variable. Hence, the existence of structural break leads to estimation problems affecting the parameters α (intercept), μ (mean), σ^2 (variance) and even e_t (correlation shocks).

The methods of determining the structural breaks and outliers shall be presented in Section 3.4.2.

3.4 Non-Stationary Tests

We present the non-stationary tests in line with its major causes as stated in Section 3.2 .

3.4.1 Tests on Unit Roots and Trend.

The common tests for unit root (as described on page 48) and trend which are generally termed stochastic and deterministic trends respectively are: (i) Augmented Dickey-Fuller (1979), (ii) Phillips-Perron(1988), and (iii) Kwiatkowski-Phillips-Schmidt-Shin(KPSS)(1992). We utilized these tests in order to know and ascertain whether a series is stationary or not before its further usage in the analysis.

3.4.1.1 Dickey-Fuller (DF) Test

The authors of this test are Dickey and Fuller (DF, 1979). In their paper, they were able to come up with the test on a unit root. In fact, they are the pioneers of test on a unit root.

In their paper, the main objective was to test the null hypothesis (applying equations (3.3.1.1) to (3.3.1.4)) that $\beta = 1$ in an AR (1) model:

$$x_t = \beta x_{t-1} + \varepsilon_t \text{ -----(3.4.1.1.1)}$$

So that

Null Hypothesis = $H_{\text{null}} : \beta = 1$ (there is unit root in the series)

Alternative Hypothesis = $H_{alt} : \beta < 1$ (stationary series)

Using the AR(1) model of (3.4.1.1.1) and by subtracting x_{t-1} from both sides, we can equally have:

$$\Delta x_t = (\beta - 1)x_{t-1} + \varepsilon_t$$

The following types of Dickey-Fuller test are identified and known as τ tests: τ , τ_μ , τ_τ .

Using our AR (1), we have the following:

- (i) testing for a random walk against a stationary auto-regressive process with the hypotheses:

$$\text{Null Hypothesis} = H_{null} : x_t = x_{t-1} + \varepsilon_t$$

$$\text{Alternative Hypothesis} = H_{alt} : x_t = \beta x_{t-1} + \varepsilon_p, \quad \beta < 1$$

- (ii) a random walk against a stationary AR(1) with drift (α).

$$\text{Null Hypothesis} = H_{null} : x_t = x_{t-1} + \varepsilon_t$$

$$\text{Alternative Hypothesis} = H_{alt} : x_t = \beta x_{t-1} + \alpha + \varepsilon_p, \quad \beta < 1$$

- (iii) a random walk against a stationary AR(1) with drift and a trend ($\beta_2 t$).

$$\text{Null Hypothesis} = H_{null} : x_t = x_{t-1} + \varepsilon_t$$

$$\text{Alternative Hypothesis} = H_{alt} : x_t = \beta x_{t-1} + \alpha + \beta_2 t + \varepsilon_p, \quad \beta < 1$$

In general for the above null hypotheses, it can be stated that:

$\Delta x_t = \varepsilon_t$ (an autoregression of one variable X using the null hypothesis; also see equation (3.1.1.3))

where $\Delta x_t = x_t - x_{t-1}$ and the alternatives may be expressed as:

$$\Delta x_t = \rho x_{t-1} + \alpha + \beta_2 t + \varepsilon_t$$

with $\alpha = \beta_2 = 0$ in case(i); and $\beta_2 = 0$ in case (ii) and $\rho = \beta - 1$.

For all these cases, the Dickey-Fuller (DF) test is based on the t-test statistic in terms of the estimation of Δx_t being regressing on x_{t-1} so that:

$$DF = \frac{\hat{\rho}}{se(\hat{\rho})}$$

where se is the standard error.

As DF statistic is not of usual t-distribution, in shape with the null hypothesis, a table of critical values was derived from Monte Carlo experiments. See Dickey-Fuller (1981).

Augmented Dickey- Fuller (ADF) Test.

This is another type of Dickey-Fuller test where it accommodates more than one lag of AR(p). The type arose as a need for more lag instead of one lag in the model as in the case of DF.

Another reason is that the white noise assumption of ε_t above will be affected when more lags is used in DF. This usually leads to auto-correlation effect on the dependent variable when Δy_t is being regressed on them. Thus, an alternative model is now written. For example in case(i), we have:

$$\Delta y_t = \rho y_{t-1} + \sum_{i=1}^p \alpha_i \Delta y_{t-i} + \varepsilon_t, \quad i=1, 2, 3, \dots, p.$$

By introducing or adding $\alpha_i \Delta y_{t-i}$ to each of other cases (ii), (iii) in DF tests will make these cases augmented. Here, the same test statistics and critical values (from the DF table) are used and determination of lag length is essential.

3.4.1.2 Phillips-Perron Test

Another useful unit root test is the Phillips-Perron [PP, (1988)].

For the reason that ADF test became more complicated as the lag term increasing, in terms of auto-correction effects on error, the Phillips-Perron method was suggested. In this approach, instead of adding extra lag term to the series, a non-parametric correction to the t-test statistic was incorporated in order to take care of autocorrelation effects. Although the test is also utilizing the same null hypothesis like ADF test, the test is robust even in the presence of non homogeneous errors. The null hypothesis can be expressed as:

H_0 : the series is integration of order 1 , i.e existence of unit root.

H_1 : the series is stationary

To explain further, let's consider AR(1) with DF test. By the OLS regression fitting we have:

$$\Delta y_t = \rho y_{t-1} + (\text{constant, trend}) + \varepsilon_t \text{ -----(3.4.1.2.1)}$$

But the PP test in the same pattern of (3.4.1.2.1) while using AR (1), we set or use

$$y_t = \Pi y_{t-1} + (\text{constant, trend}) + \varepsilon_t \text{ -----(3.4.1.2.2)}$$

where ε_t is $I(0)$ and may be heteroskedastic as lag increases in (3.4.1.2.1); but in case of (3.4.1.2.2) PP test corrects any serial correlation and heteroskedastic in the error ε_t non-parametrically by modifying the Dickey-Fuller test statistic. Hence, under the null hypothesis that $\rho = 0$, the PP test Z_t and Z_π statistics have the same asymptotic distribution as the ADF t-statistic and normalized bias statistics.

Here, PP test can be seen as DF statistics being made robust to serial correlation by using Newey-West (1987) heteroskedastic and auto-correlation-consistent covariance matrix estimator.

The advantages of PP test over ADF test include:

- (i) Robustness result to general forms of heteroskedasticity in the error term ε_t ;

- (ii) The user does not have to specify a lag length for the test regression.

3.4.1.3 KPSS Test.

KPSS is an alternative procedure for testing stationary (deterministic trend) properties of time series originated and developed by D.Kwiatkowski, P.C.B, Phillips, P.Schmidt and Y.Shin [KPSS, (1992)]. Bhargava was the first person to propose this test (in his Ph.D thesis) on model which is stationary around a deterministic trend.

It is useful to know that the KPSS has been developed to complement unit root tests as ADF has lower power with respect to near unit root and long-run trend processes. The test considered 3 components of the series as a test on the deterministic trend, a random walk and a stationary residual.

Later, the KPSS test was developed in which the null hypothesis was stationary around deterministic trend, but not of unit root (non-stationary) as in ADF and PP tests as thus:

H_0 : the time series is stationary around a deterministic trend.

H_1 : not stationary around a deterministic trend.

The authors derived their test by using

$$y_t = \beta d_t + \mu_t + e_t$$

$$\mu_t = \mu_{t-1} + \varepsilon_t; \quad [\varepsilon_t \sim N(0, \delta_\varepsilon)]$$

where d_t is the deterministic components (constant or constant plus trend), e_t is $I(0)$ and may be heteroskedastic; and μ_t is a pure random walk with variance δ_ε^2 . By this, the null hypothesis that y_t is $I(0)$ is formulated as:

$$H_0 : \delta_\varepsilon^2 = 0$$

$$H_1 : \delta_e^2 > 0$$

with the test statistic

$$KPSS = (T^{-2} \sum_{t=1}^T \hat{s}_t^2) / \hat{\lambda}^2$$

where $\hat{s}_t^2 = \sum_{j=1}^t \hat{e}_j$, e_t is the residual of a regression y_t on d_t and $\hat{\lambda}^2$ is the consistent estimate of a long-run variance of e_t using \hat{e}_t .

3.4.2 Outliers and Structural Breaks Tests

In furtherance to our discussion on structural breaks and outliers as discussed in Section 3.3 .2, some of the methods on how to determined or detect outliers and structural breaks shall be presented. However, we are to note that there are a lot of statistical tests that can be utilised to determine the existence of outliers and structural breaks. Among them, which we are to use, we have the Chow test, the Cusum chart approach, Perron method, and Quandt method (1958).

Maddala and Kim (1998) discussed the issues of unit roots, outliers and structural change to a reasonable length. According to these authors, we take note of the following suggestions in relation to resolve or detect the issue of outliers in time series:

- Legendre (1805) in his least squares paper, he suggested throwing out the outliers; which was equally supported by Edgeworth(1887).
- Later; Donald and Maddala (1993) were able to come up with the following suggestions:

- (i) Throwing out outliers is not a good course of action or practice in time series.

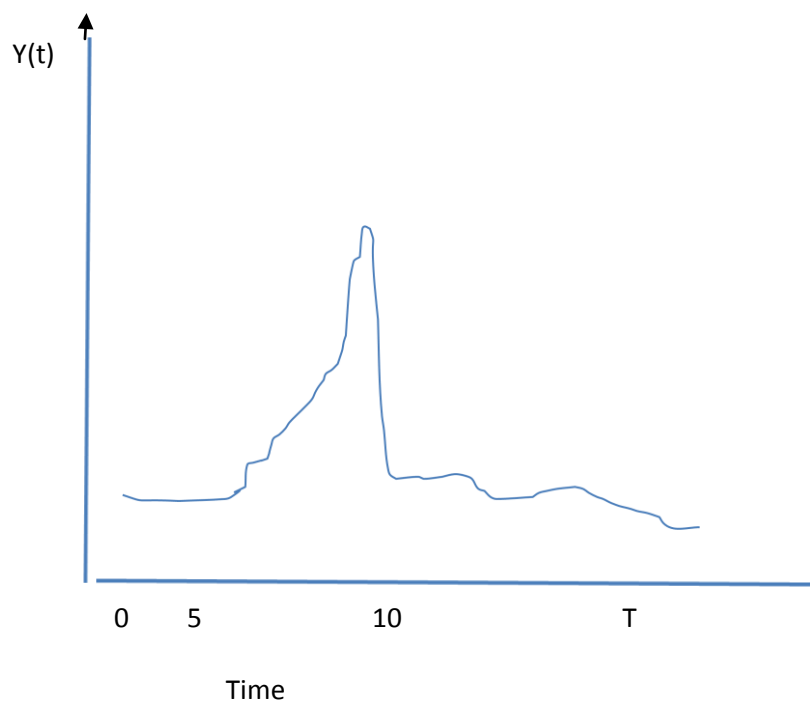
The reason is that time series has high adherence and restriction to time order or sequence of its data. By any alteration, omission or deletion of a point, it has effect on the statistical decision of the series.

- (ii) Leave the outliers in the series, but handled by the robust method.
- (iii) Change the model that generated the series in case of an initial error or mis-specification.

Further on Maddala and Kim claims, Fox (1972) was the first person to classify outliers into additive outlier (AO) and innovative outlier (IO). Later Tsay (1988) extended the classification, in order to accommodate structural changes, to transient changes (TC), level changes (LC), and variance changes (VC).

Additive outlier occurs when the series suddenly jumps up or down and returns immediately to its normal course or pattern. See Fig 3.01 (a). Here, a factor usually caused the jumping which may be due to error or other strong forces such as drought (on agricultural products), economic recession, government policy change, but to mention a few

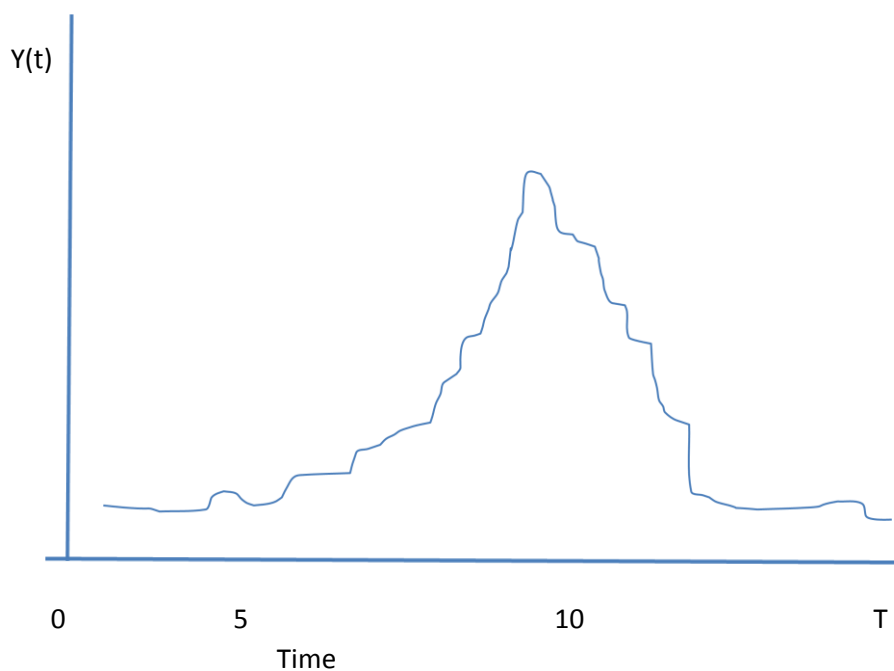
Fig 3.01(a):Sketch of AO model



The innovative outliers (IO) occurs when a point jumps up or down and gradually adjusting to its normal course or pattern. See Fig 3.01(b). Also, the effects of a large IO can cause the dynamic effects on the model.

Further to our discussions in the previous page, the main difference between **additive outlier (AO)** and **innovative outlier (IO)** is that when the two jump up AO will quickly return to the normal course of movement, while the (IO) is gradually coming to normal course or pattern..

Fig. 3.01(b): Sketch of IO Model



Tsay classifications of TC, LC and VC respectively considered changes in short period, at level and in slope variance. See Figures 3.01(c), 3.01(d), and 3.01(e). Further, he claimed the nature and type of outliers (for all the five) have different type of effects.

Fig. 3.01(c): Sketch of TC Model

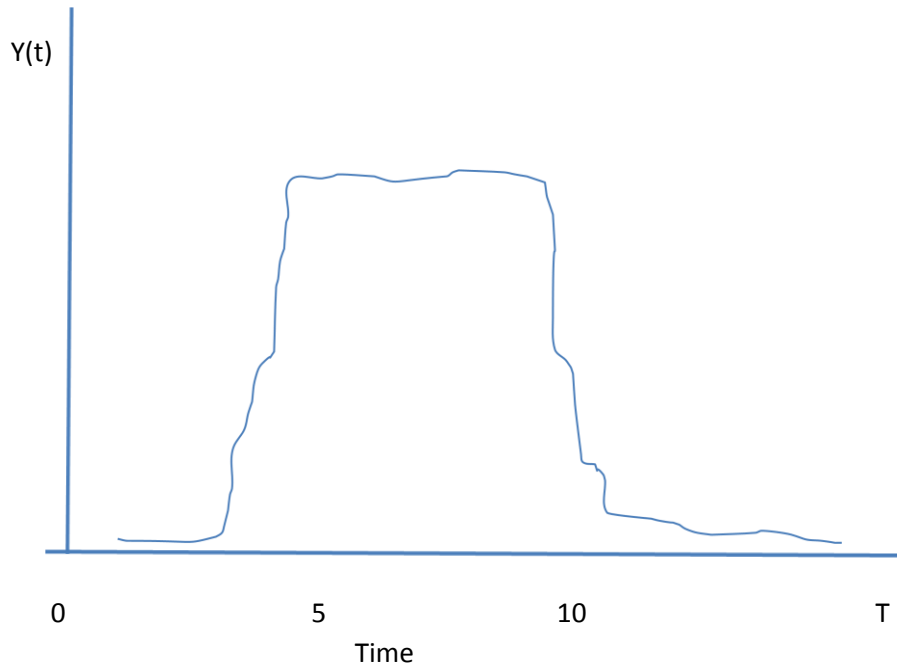


Fig. 3.01(d): Sketch of LC Model

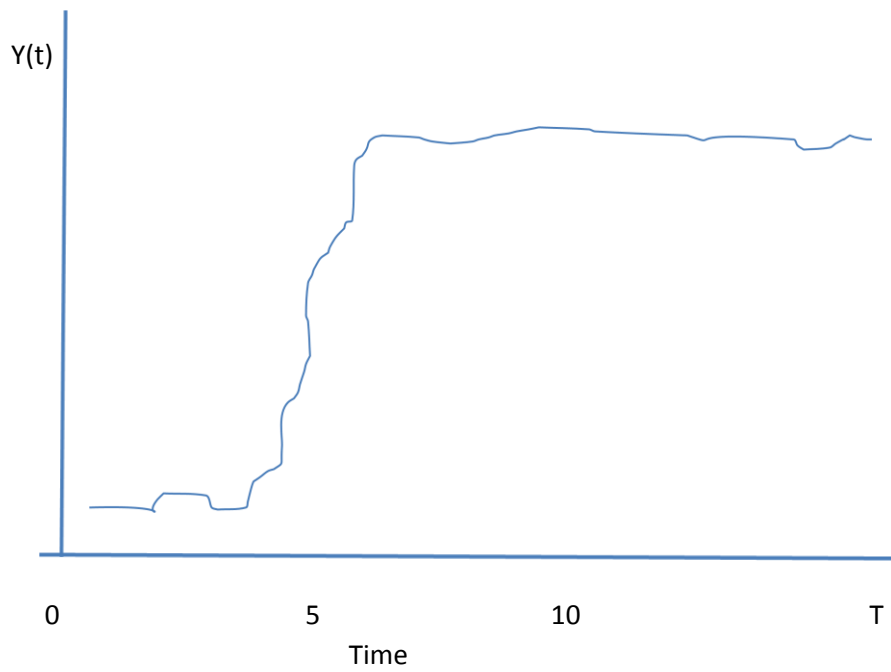
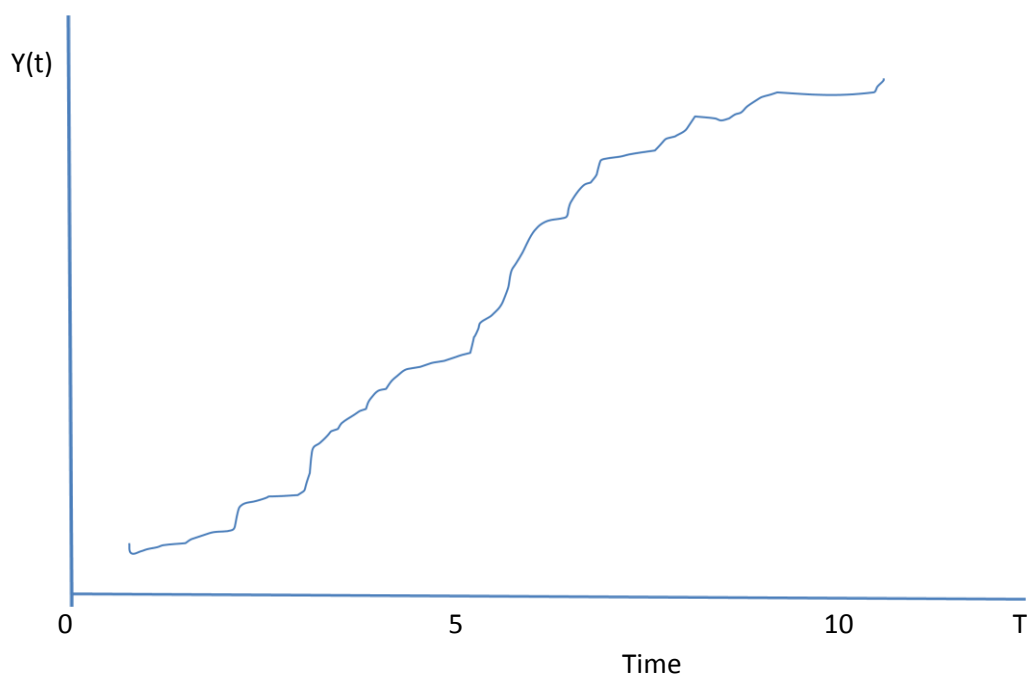


Fig. 3.01(e): Sketch of VC Model



3.4.2.1 Perron Method

Perron (1994) considered breaks in intercept, slope or both by utilising the AO and IO methods. Other researchers such as Zivot Andrew (1992) and Banerjee et al (1992) carried out the same study. They were able to come out with similar results.

By adherence to Perron approach, the following cases are identified:

Case 1 : The AO models allowing a change in the intercept in the variable trend of y_t such that:

$$Y_t = \alpha_1 + \beta t + (\alpha_2 - \alpha_1)D_t + e_t \quad \dots\dots\dots (3.4.2.1.1)$$

Where $D_t = 1$, if $t > T_b$; otherwise 0; T_b is the time of breaking.

Case 2 : The AO models allowing change in intercept and slope of y_t resulting to:

$$\alpha_1^* + \beta t^* + (\alpha_2^* - \alpha_1^*)D_t + (\beta_2^* - \beta_1^*)D_{1t} + e_t^* \dots\dots\dots(3.4.2.1.2)$$

Where $D_t = t - T_b$, if $t > T_b$; otherwise 0; i.e D_t and D_{1t} respectively.

Case 3: The IO model allowing both cases 1 and 2 above with gradual change in trend function of Y_t . Here, by application of moving average (MA) expansion in terms of polynomial lag operator

$\Delta(L) = \Phi_0 + \Phi_1 L + \Phi_2 L^2 + \dots \Phi_q L^q$ on the terms $(\alpha_2 - \alpha_1)D_t + e_t$ (in case 1) and

$(\alpha_2^* - \alpha_1^*)D_t + (\beta_2^* - \beta_1^*)D_{1t} + e_t^*$ (in case 2), we respectively have

$$\Delta_1(\alpha_t)D_t + e_t \quad (\text{in case 1}) \dots\dots\dots (3.4.2.1.3); \text{ and}$$

$$\Delta_1(\alpha_1^*)D_t + \Delta_1(\beta_t^*)D_{1t} + e_t^* \quad (\text{in case 2}) \dots\dots\dots(3.4.2.1.4).$$

Next to test if there is structural break in AO and IO models, the above equations are to be utilized. For AO models, equations (3.4.2.1.1) and (3.4.2.1.2) are being used to estimate the error terms e_t and e_t^* . Then the error is used in the autoregression.

$$\Delta(e_t) = \theta e_{t-1} + \sum_{i=1}^{p-1} \theta_i \Delta e_{t-i} + \sum_{j=0}^{p-1} d_j D(T_b)_{t-j} + \varepsilon_t$$

Also for IO models, results in equations (3.4.2.1.3) and (3.4.2.1.4) are utilised in order to obtain the error for its further usage in auto-regression of this error.

3.4.2.2 Chow's Method

Chow (1960) discussed a possible way of testing the equality between sets of coefficients in two linear regressions, as an approach for testing and confirming a structural break with a known date, in a given series where two regression lines can be identified.

The method entails finding or identifying a date (as a suspected date) of structural break first. Suppose our time series data resulted to a model:

$$Y_t = \alpha + \beta x_t + \varepsilon_t, \quad t = 1, 2, \dots, n \quad (3.4.2.2.1)$$

On the assumption that break date is known, two segmented regression lines can be identified in (3.3.2.2.1) due to the said structural break, then we have the split regressions:

$$Y_t = \alpha_1 + \beta_1 x_t + \varepsilon_{1t} \quad (3.4.2.2.2)$$

$$Y_t = \alpha_2 + \beta_2 x_t + \varepsilon_{2t} \quad (3.4.2.2.3)$$

The Chow test on the split regressions utilizes the null hypothesis $\alpha_1 = \alpha_2, \beta_1 = \beta_2$ with the assumption that errors ε_{1t} and ε_{2t} are independently and identically normal distribution, with unknown variance.

According to Gujarati (2004), the Chow test is presented in the following stages utilizing the above information:

Stage 1 : Let the number of $t = 1, 2, \dots, T$, be n_1 , and that of $t = T+1, T+2, \dots, n$, i.e. $[n - T]$ be n_2 . Then combine n_1 and n_2 ; and determine its residual sum of squares (RSS) as S_1 , with degree of freedom (df) = $n_1 + n_2 - k$, where k is the number of parameter estimated.

Stage 2 Determine the individual residual sum of squares (RSS)

as S_2 and S_3 respectively for series with n_1 and n_2 data using $(n_1 - k)$ and $(n_2 - k)$ as df in that order.

Stage 3: Add the two RSS obtained in Stage 2 and label it as S_4 , ($S_4 = S_2 + S_3$).

Stage 4: Determine the difference between S_1 and S_4 , and label it as S_5 ($S_5 = S_1 - S_4$).

Stage 5 : Carry out the test using the stated hypothesis with

$$F = \frac{S_5/K}{S_4/(p)}$$

Where $p = n_1 + n_2 - 2k$.

As Chow test follows the F distribution with k and $n_1 + n_2 - 2k$ degrees of freedom, decision can be made at a desired significant level.

3.4.2.3 CUSUM Chart Approach.

Page (1954) developed the CUSUM chart which was announced in *Biometrika*. CUSUM chart has the full name of cumulative sum chart. It is a charting utilising the principle of interaction between the two disciplines of statistical process control and the econometric time series. It is useful in monitoring the process mean of the series as a statistical tool to detect outlier or structural break.

Few years later after Page's CUSUM, Barnard (1959) came up with an idea of a visualization method tagged V-mask chart which is related to Page's work. In his paper, he described the V-mask as an instrument which can be superimposed on the CUSUM plot for a better decision to detect a small change in the CUSUM plot.

Mesnil and Petitgas (2008) described the usage of CUSUM chart to detect changes in time series data as an indicator of change in the process mean. They used the state of marine ecosystems data in their empirical study and came up with a decision. In the decision, it was established that the performance measure of CUSUM depends on adherence to some key assumptions such as independent and normality; and its violation effects lead to wrong decisions.

Durbin, Brown and Evans (1975) discussed two types of CUSUM charts in their paper on techniques of testing the constancy of regression relationships over time. The tests are the CUSUM (ordinary) and CUSUM of Square (SQ). They found them useful when the date of structural break is known or not.

The identified features of CUSUM chart include: (i) very good at identifying small shifts or changes in the process average; (ii) applications to both variables and attributes are possible; (iii) utilising all the historical data in the given series and (iv) interpreting the chart by analyzing its shape.

The CUSUM chart (ordinary) utilises all the past and present data by plotting the cumulative sums of the deviations of the sample values of the said data from a target value [usually the mean (\bar{x})] against time t . That is, the deviation can be stated as $S_t = x_i - \bar{x}$ for variable x_i . Therefore for variable x_i , which is independently and normally distributed, the cumulative sum (CS_t) can be expressed as:

$$CS_t = \sum_i^t (S_t)$$

Where $S_t = x_i - \bar{x}$ for variable x_i , $i = 1, 2, \dots, t$ and \bar{x} is the mean.

But for the case of CUSUM of Square chart, the square of $S_t (S_t^2)$ is used. Hence, we have:

$$CS_t (SQ) = \sum_i^t (S_t^2)$$

Next set the maximum and minimum limits of tolerance or the control limits for the CS_t using the decision interval (H) with a significant level. Then plot the CS_t and see the point(s) falling out of the limits as outlier or the structural break.

It is important to state that the decision and interpretation depends on the type of interpretation weapon utilizing; is it of V-mask or non V-mask? In practice, V-mask is not easy to apply but the non V-mask is usually utilised. Hence, we used the non V-mask tagged the “Tabular/ Algorithm approach”.

In the tabular approach of CUSUM chart, we utilized two types of cumulative sums (the upper and the lower). These cumulative sum statistics are respectively called the upper cumulative sum (C_i^+) and the lower cumulative sum (C_i^-), where

$$C_i^+ = \max \{0, [X_i - (\mu_0 + K) + C_{i-1}^+]\} ; \text{ and}$$

$$C_i^- = \max \{0, [(\mu_0 - K) - X_i + C_{i-1}^-]\} ;$$

and also, $i = 1, 2, \dots, n$; μ_0 is the grand mean and K is the slack value (reference value) which is often chosen about halfway between the target μ_0 and the out-of-control value of the mean μ_1 that we are interested in detecting quickly (Montgomery, 2001).

So, if the shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$ (or $\delta = |\mu_1 - \mu_0| / \sigma$), then K is one-half the magnitude of the shift or $K = (\delta\sigma) / 2 = |\mu_1 - \mu_0| / 2$.

Note that it is essential to select the right and appropriate value for K . The reason is that a large value of K will allow for large shifts in the mean without detection, whereas a small value of K will increase the frequency of false alarms. Usually, K is often selected to be equal to 0.5σ .

Therefore, in the Tabular CUSUM, it is necessary to choose the right values for the reference value K and the decision interval H so that $K = k\sigma$ and $H = h\sigma$. By using $h=4$ or $h=5$ and $k=1/2$, it usually results to a CUSUM that has good ARL (average run length) properties against a shift of about 1σ in the process mean (Montgomery, 2001).

3.4.2.4 Quandt Test

Quandt (1958) paper discussed how to determine the unknown date of structural change. The author first identified or recognised the existence of a shift in the regression line within the

time period of $t = 1, 2, \dots, n$. Then, the regression line is split into two leading to two new distinct linear regression lines.

These new lines span over a subset periods $(1, \dots, T)$ and its complementary period $(T+1, \dots, n)$ respectively within the time frame $(t=1, 2, \dots, n)$. Here, the unknown time is T .

In order to estimate T , he applied the likelihood ratio estimation principle with the assumptions of error (e_t) normally and independently distributed.

Formally, Quandt paper is presented in the following order:

By utilising a bivariate time series Y_t and X_t with time period t ($t=1, 2, \dots, n$), he established a linear regression model:

$$Y_t = \alpha + \beta X_t + e_t, \quad \text{for } (t=1, 2, \dots, n) \quad (3.4.2.4.1)$$

The likely identified unknown time t in equation (3.4.2.4.1) is labelled T ; and this lead to formation of two linear regression lines spanning over the periods $(t = 1, 2, \dots, T)$ and $(t=T+1, T+2, \dots, n)$ respectively as thus:

$$Y_{1t} = \alpha_1 + \beta_1 X_t + e_{1t}, \quad \text{for } t (1 \leq t \leq T) \quad (3.4.2.4.2)$$

$$Y_{2t} = \alpha_2 + \beta_2 X_t + e_{2t}, \quad \text{for } t (T+1 \leq t \leq n) \quad (3.4.2.4.3)$$

Where α , α_1 , α_2 , β , β_1 , and β_2 are constants/coefficients, and e_t , e_{1t} and e_{2t} are error terms in equations (3.4.2.4.1) to (3.4.2.4.3). Also with the assumptions that: (i) the data of the bivariate series are observational errors free; (ii) e_t , e_{1t} , e_{2t} are normally and independently distributed with mean zero and standard deviation σ_t , σ_{1t} and σ_{2t} [i.e $N(0, \sigma_t, \sigma_{1t}, \sigma_{2t})$].

To estimate T , the densities of e_t , e_{1t} and e_{2t} are to be obtained. By this, an application of normal assumption of error term is made. Hence, $e_t \sim N(0, \sigma^2)$. Therefore we have:

$$\left(\frac{1}{\sqrt{2\pi\sigma_t}}\right)\exp\left[-\left(\frac{1}{2\sigma_t^2}\right)(y_t - \alpha - \beta x_t)^2\right],$$

where $e_t = y_t - \alpha - \beta x$ (from equation (3.4.2.4.1) and is equally applied to e_{1t} and e_{2t}

respectively as:

$$\left(\frac{1}{\sqrt{2\pi\sigma_{1t}}}\right)\exp\left[-\left(\frac{1}{2\sigma_{1t}^2}\right)(y_{1t} - \alpha_1 x_t - \beta_1)^2\right], \quad \text{for } t (1 \leq t \leq T) \quad (3.4.2.4.4)$$

$$\left(\frac{1}{\sqrt{2\pi\sigma_{2t}}}\right)\exp\left[-\left(\frac{1}{2\sigma_{2t}^2}\right)(y_{2t} - \alpha_2 x_t - \beta_2)^2\right], \quad \text{for } t (T+1 \leq t \leq n) \quad (3.4.2.4.5)$$

The likelihood functions of sample period $t=1, \dots, T$ and $t= T+1, T+2, \dots, n$ are respectively

$$\left(\frac{1}{\sqrt{2\pi\sigma_{1t}}}\right)^T \exp\left[-\left(\frac{1}{2\sigma_{1t}^2}\right) \sum_{t=1}^T (y_{1t} - \alpha_1 x_t - \beta_1)^2\right], \quad \text{for } t (1 \leq t \leq T) \quad (3.4.2.4.6)$$

$$\left(\frac{1}{\sqrt{2\pi\sigma_{2t}}}\right)^{n-T} \exp\left[-\left(\frac{1}{2\sigma_{2t}^2}\right) \sum_{t=T+1}^n (y_{2t} - \alpha_2 x_t - \beta_2)^2\right], \quad \text{for } t (T+1 \leq t \leq n) \quad (3.4.2.4.7)$$

But the likelihood function of the entire period $t=1, \dots, n$ is the same as sum of the periods

$t=1, \dots, T$ and $t= T+1, T+2, \dots, n$. Hence, the entire likelihood function is

$$\left(\frac{1}{\sqrt{2\pi\sigma_{1t}}}\right)^T \left(\frac{1}{\sqrt{2\pi\sigma_{2t}}}\right)^{n-T} \exp\left[-\left(\frac{1}{2\sigma_{1t}^2}\right) \sum_{t=1}^T (y_{1t} - \alpha_1 x_t - \beta_1)^2 - \left(\frac{1}{2\sigma_{2t}^2}\right) \sum_{t=T+1}^n (y_{2t} - \alpha_2 x_t - \beta_2)^2\right] \dots \dots \dots (3.4.2.4.8)$$

Next, take the logarithm of the entire sample

$$L = -T \log \sqrt{2\pi} - t \log \sigma_{1t} - (T-t) \log \sigma_{2t} - \left(\frac{1}{2\sigma_{1t}^2}\right) \sum_{t=1}^T (y_{1t} - \alpha_1 x_t - \beta_1)^2 - \left(\frac{1}{2\sigma_{2t}^2}\right) \sum_{t=T+1}^n (y_{2t} - \alpha_2 x_t - \beta_2)^2 \dots \dots \dots (3.4.2.4.9)$$

Obtain the partial derivatives of $\alpha_1, \alpha_2, \beta_1$ and β_2 from equation (3.3.2.4.9) and equate to zero. The resulting equations give the estimates of these parameters ($\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\beta}_1$ and $\tilde{\beta}_2$) by using the least squares.

Substitute the estimates back into the equation (3.3.2.4.9), then obtain the partial derivatives of σ_{1t}^2 , σ_{2t}^2 and equate to zero in order to get their estimates utilizing the least squares.

Finally after substituting all the estimated values back into the original equation (3.3.2.4.9), a new logarithm of maximum likelihood function comes up as thus:

$$L(t) = -T \log \sqrt{2\pi} - t \log \sigma_{1t} - (T-t) \log \sigma_{2t} - \frac{T}{2} \dots\dots\dots (3.4.2.4.10)$$

In order to determine value of T from equation (3.3.2.4.10), calculate the likelihood function of (3.3.2.4.10) for all possible values of t and pick the maximum estimate which will be the required T.

Quandt Likelihood Ratio (QLR) Statistic (Quandt,1960).

Sequel to the above paper (Quandt, 1958), the author came up with an extension paper in which an approach for testing a break at unknown date was established. The said paper is tagged “Quandt likelihood ratio (QLR) test (Quandt,1960), or sometimes called “Sup-Wald Statistic”. It has a better practical applications than the previous one (Quandt,1958).

This paper (Quandt, 1960) was seen as a modified Chow test in which the possible break date of unknown or known only within a range can be determined.

In essence, the method entails identifying all possible date t in-between t_0 and t_1 , and then using the largest of the resulting F-statistic to test for a break at unknown date. The method is summarized as follows:

- (i) Let denote F(t) as the F-statistic testing the hypothesis of a break in the regression coefficients at date t. Suppose the regression coefficients are $\beta_0, \beta_1, \beta_2, \dots$, then the F-statistic testing the null hypothesis is

$$H_0 = \beta_0 = \beta_1 = \beta_2 = 0.$$

The QLR (or Sup-Wald) test is the largest of statistics in the range $t_0 \leq t \leq t_1$ such that:

$$QLR = \max[F(t_0), F(t_0+1), \dots, F(t_1)];$$

- (ii) Using the critical values of the QLR statistic table at a desire significant level, then the QLR statistic can be utilized to test for a break in all or just a few of the regression coefficients;
- (iii) It should be noted that in large samples, the distribution of the QLR statistic under the null hypothesis depends on the number of restrictions (K) being tested and on the end points t_0 and t_1 as a fraction T (total number of observations). These extremes t_0 and t_1 are being 15 % trimmed so that $t_0 = 0.15T$ and $t_1 = 0.85T$, rounding up to the nearest integer; and
- (iv) A single discrete break, multiple discrete breaks and/or slow evolution of the regression function can be detected by using the QLR test.

3.5 Transformation of Non-Stationary to Stationary.

The non-stationary trend can be transformed to stationary by differencing and de-trending respectively for the stochastic and deterministic trends.

By the act of differencing a non-stationary stochastic or random walk variable X_t to the stationary form of the variable can be induced as follows:

Consider an annual time series with AR(1) model with unit root such that:

$$X_t = \alpha + X_{t-1} + e_t \quad (3.5.1)$$

The first difference is obtained by subtracting X_{t-1} from both sides of (3.5.1)

$$X_t - X_{t-1} = \alpha + e_t$$

$$\Delta X_t = X_t - X_{t-1} = \alpha + e_t$$

$$E(\Delta X_t) = \alpha$$

$\delta_{\Delta X}^2 = \delta_e^2$, which implies covariance $X_t = X_{t+s} = 0$ and the series X_t is integrated of order one (i.e $s=1$).

There is the possibility of having more than one order of integration; this depends on the number of times differencing being carried out to have a stationary model. Hence, in general, $I(d)$ represents the integration order where d is the number of times difference took place.

To relate the Lag operator to difference, which is commonly called the difference operator in time series analysis, consider the following:

Let L (or B) represents the Lag (or Backshift) operator. By backshift in time series, there is a back movement of time in the series lag. This can be stated in lag operator using time series X_t as follows:

$$L(X_t) = X_{t-1}$$

Now relating the lag to difference operator (Δ), it yields:

$$\begin{aligned} \Delta X_t &= X_t - X_{t-1} \\ &= X_t - L(X_t) = (1 - L) X_t \end{aligned}$$

Further difference operations resulting to the general term :

$$\Delta^k (X_t) = (1 - L)^k X_t, \text{ k is an integer and number of differencing} \quad (3.5.2)$$

However, the operation performed on equation (3.5.1) with the $AR(1)$ is called regular (non-seasonal) differencing while there is another one called seasonal differencing.

Formally, we distinguish the two as follows:

Given an annual data with variable X_t , the regular difference can be expressed as:

Using the general term of equation (3.5.2), that is:

$$\Delta^k(X_t) = (1 - L)^k X_t$$

If $k=1$ here for $I(1)$, we have

$$g_t = \Delta(X_t) = X_t - X_{t-1} \quad (\text{for } I(1)); \text{ and}$$

the Second differencing by

$$g^* = g_t - g_{t-1} \quad (\text{for } I(2)); \text{ and}$$

the third differencing by

$$g^{**} = g_t^* - g_{t-1}^* \quad (\text{for } I(3)), \text{ and so on for higher differencing.}$$

In the case of seasonal differencing, let have a variable x_t which is seasonal, i.e. weekly, monthly, quarterly, or half-yearly, we have:

$$g_t = x_t - x_{t-12} \quad \text{----- (for monthly)}$$

$$g_t = x_t - x_{t-4} \quad \text{----- (for quarterly)}$$

$$g_t = x_t - x_{t-2} \quad \text{----- (for half-yearly)}$$

so that in general $g_t = x_t - x_{t-s}$,

where s is the season. By this, seasonal difference is denoting by (d, s) where d is the degree or order number of seasonal difference and s stands for seasonal period.

A caution is to avoid over-differencing as this often results in a pronounced negative first order autocorrelation effects leading to an increase in the estimated variance. Also, there are

some variable models that will demand both regular and seasonal differencing and it should be applied accordingly.

Let consider the following hypothetical illustration as a vehicle for further explanation:

Given a time series X_t with the values: 20, 28, 25, 20, 16, 18, 22, 25, 30, 32, 31, 29, 25, 23, 26, 27, 24, 20, 18, 15.

For the first difference, assuming the values are yearly data, we have:

8, -3, -5, -4, 2, 4, 3, 5, 2, -1, -2, -4, -2, 3, 1, -3, -4, -2, -3

Second difference gives:

$\Delta^2 X_t = \Delta(\Delta X_t) = -11, -2, 1, 6, 2, -1, 2, -3, -3, -1, -2, 2, 5, -2, -4, -1, 2, -1$

Also, by assuming the data is quarterly, the quarterly differencing results to:

$\Delta X_t = X_t - X_{t-4} = -4, -10, -3, 5, 14, 14, 9, 4, -5, -9, -5, -2, -1, -3, -8, -12$

$\Delta^2 X_t = \Delta(\Delta X_t) = 18, 24, 12, -1, -19, -23, -14, -6, 4, 6, -3, -10$

In de-trending, let us consider a variable y_t with deterministic trend

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \varepsilon_t \quad (3.5.3)$$

where $\alpha_1 < 1$

Here, we detrend y_t by subtracting $\alpha_2 t$ from y_t (where $\alpha_2 t$ is obtained by regression of y_t on time t). i.e $y_t = \alpha_2 t$

$y_t - \alpha_2 t$ gives residuals of y_t

Another caveat on these two transformations is that one must have a clear distinction of which to apply. By wrong transformation, overestimation or underestimation ensued and it affects the results.

It is important to note another form of transformation, namely the logarithm transformation. In this transformation, the linearization property can be achieved. The goal is to convert or reduce variable to additive from multiplicative relationships, and exponential (compound growth) trend to linear trend.

As positivity property and choice of base are required in logarithm, it is also helpful to know the following transformation:

- (i) First difference of log is equal to percentage change; -----(3.5.4)
- (ii) Trend in logged unit is equal to percentage growth; and
- (iii) Error in logged unit is equal to percentage error.

3.6 Type and Determination of Lag Length.

As lag length is important in model specification, it is also of no exception to Granger causality. In fact, Granger causality is very sensitive to lag length and by this, it affects the outcome or result of the Granger causality. This is a cogent reason to ensure that correct lag length is being used.

Stock and Watson (2012) cautioned on wrong or non-optimal lag length usage. They maintained that under-lagging leads to loss or omission of potential valuable information; while in over-lagging, estimating more coefficients than necessary leads to additional error for the model. In the light of this, it is essential and necessary to guide against the said caution.

In time series analysis, some methods or criteria in literature are utilised to determine the lag length. However, we are considering two popular ways of determining the lag length. These are through the:

- (a) use of graphs of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) as in Box-Jenkins (1976) method ;
- (b) use of information criteria.

We shall discuss each of these two ways.

Box-Jenkins' Method:

In furtherance to our discussions on ACF/PACF in Section 3.1.2, we are employing these concepts to determine the lag length p of AR(p) model. That is, to find an adequate p .

Box and Jenkins (1976) used the principle of plotting the graphs of ACF and PACF, and then observed the pattern of the movements to determine the lag length. The plots of ACF and PACF are respectively called correlogram and partial correlogram.

These authors utilized their approach to determine lag length for AR, MA models and even the mixed models of AR and MA (ARMA, ARIMA). See page 47 in Section 3.1.2 for more details.

As we are much concerned with AR(P) lag determination, let restate the Box-Jenkins' steps and rules guiding the usage of ACF/PACF as in Steps 1, 2 and 3(a) for our desired model:

Step1: determine the autocorrelation coefficients for a fair number of lags, and similarly for partial auto-correlation coefficients for ACF and PACF respectively;

Step 2: plot separate graph of ACF and PACF against the lags;

Step 3: watch their movements within 5 percent confidence limits, for the points to fall within the limits, and to see how they are conforming to the following rules:

- (a) if the PACF displays a sharp cut-off (or spike off) from the series at lag P, then all zeros;
- (b) also ACF decays very slowly or tailing off like damping sine wave at lag P;
- (c) then AR(P) at lag P is the required lag length.

Here, the lag is determined at p and also stationary at that point.

It is widely noted that the Box-Jenkins' approach will give an idea of adequate lag length but not optimal like information criteria.

Use of Information Criteria:

Tsay (2010) and Lutkepohl (1991) discussed some information criteria and then summarised by Kirchgassner and Wolters (2007) as thus:

- (i) “ the Final Prediction-Error (FPE, 1964) as

$$FPE(p) = \left(\frac{N+m}{N-m}\right) \frac{1}{N} \sum_{t=1}^N (\hat{e}_t^{(p)})^2$$

- (ii) the Akaike Information Criterion (AIC,1974) as

$$AIC(p) = \ln \frac{1}{N} \sum_{t=1}^N (\hat{e}_t^{(p)})^2 + m \frac{2}{N}$$

- (iii) the Bayesian criterion of G.Schwarz (BIC/SC, 1978) as

$$BIC/SC(p) = \ln \frac{1}{N} \sum_{t=1}^N (\hat{e}_t^{(p)})^2 + m \left(\frac{\ln N}{N}\right)$$

- (iv) the Hannan and Quinn (HQ, 1979) as

$$HQ(p) = \ln \frac{1}{N} \sum_{t=1}^N (\hat{e}_t^{(p)})^2 + m \left(\frac{2 \ln(\ln N)}{N}\right) "$$

Where p is lag length, $\hat{\epsilon}_t^{(p)}$ is the estimated residuals of AR(p) process, while m is the number of estimated parameters and N is the series length. If m contains constant term, then

$$m = p + 1 \text{ for the AR}(p).$$

The purpose of the criteria method is to have a better optimal lag over the Box-Jenkin's method. In all criteria, there is penalty on the auto-regression estimates as the lag increases; and they are based on the same principle. The p lag that minimises the estimates of each of the above criteria is the optimal lag.

The authors further claimed that the first two criteria overestimate the true order asymptotically, while the last two criteria estimate the true order of the process consistently.

To estimate the lag in information criteria, for all the types stated above, one needs the use of autoregressive processes, through one of the following means:

1. Use of maximum likelihood (ML) method.

In this method, one needs to know the distribution of the white noise that generates the AR(p) process, then the parameter can be estimated by the maximum likelihood (ML) method.

In the ML method, it is suggested to adhere to the following stages-

- (i) For a given density function $f(x; \theta)$ of a random variable X , where θ is the parameter to be estimated, and variable X is of n observations (i.e. x_1, x_2, \dots, x_n), then the likelihood function (L) is established and given as:

$$L(x_1, x_2, \dots, x_n; \theta) = \prod_{i=1}^n f(x_i; \theta);$$

- (ii) For easy handling of L, take the log of the likelihood function; and then differentiate with respect to the desired parameter θ and equate to zero in order to obtain the estimate ($\hat{\theta}$);
 - (iii) Next, vary the estimated $\hat{\theta}$ and obtain a list of corresponding values from them. Pick the smallest of these corresponding values as the optimal parameter θ .
2. Use of ordinary least squares (OLS) through autoregressive modelling with the following steps:
- (i) Let utilize the autoregression model AR(p) of variable x such that –

$$x_t = \alpha + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \dots + \beta_p x_{t-p} + \varepsilon_t$$
 where α is the intercept and β_i (i=1,2, ..., p) are the regression coefficients.
 - (ii) By the assumption of variable stability (i. e. stationary), an application of OLS provides consistent estimates. Even, $\sqrt{n}(\hat{\alpha} - \alpha)$ and $\sqrt{n}(\hat{\beta}_i - \beta_i)$ are asymptotically normal distributed.
 - (iii) For the order of the AR process, it can be estimated by successively applying the information criteria, with an increasing p, and then pick the smallest of the obtained values.

Chapter 4: Granger Causality Methods and other Statistical Analyses and Tests.

4.0. Introduction.

This chapter presents various methods of Granger causality and other statistical analyses that are carried-out on Granger causality results.

For the Granger causality concept with stationary variables x_t and y_t , there exists four possible outcomes; and these are:

(i) The unidirectional causality which are of two types; namely

(a) x “Granger causes” y ($x \longrightarrow y$) or

(b) y “Granger causes” x ($y \longrightarrow x$).

(ii) The bi-directional indicates Granger causality in both directions, ($x \longleftrightarrow y$); and

(iii) The non-directional or non-Granger causality, ($x \text{ — } y$ or $x \nleftrightarrow y$). Here, the variables are termed as independent and exogenous in causality sense.

4.1 Granger Causality Methods

As there are a number of methods for determining Granger causality, our discussion will focus on the three major approaches as highlighted by Foresti (2006). These are:

(i) Simple /Standard Granger causality method

(ii) Multiple linear Granger causality method

(iii) Vector autoregression (VAR) method

Kirchgassner and Wolters (2007) described the simple Granger causality by Granger (1968) as traditional and statistical bottom - up approach causality; while that of VAR by Sim (1999) as an alternative to top-down approach causality in econometric philosophy. VAR was seen as an approach contrary to that of Granger. The Sim's method is also an alternative to the old traditional simultaneous equations system approach of the econometric theory. It utilises the reduced form of the system.

4.1.1 Simple/Standard Linear Granger Causality Method

Simple Granger causality method is a test approach which depends on two variables and their lags. This approach uses the auto-regressive specification on the said variables and using the ordinary least squares (OLS) for the parameters' estimates.

The computational steps involved in this method are stated below:

Utilising stationary variables x_t and y_t with optimal lag length (as discussed in Section 3.6):

1. Autoregress current y_t on the past values of y_t , (y_{t-i}), not including lags x_t , to form the nexus :

$$y_t = a_1 + \sum B_{1i}y_{t-i} + e_{1i} \dots\dots\dots (4.1.1.1)$$

where a_1 is the intercept, B_{1i} ($i = 1, \dots, p$) the autoregression coefficients and e_{1i} the error which is uncorrelated. Then obtain the restricted residual sum of square (RSS_r) from equation (4.1.1.1).

2. Autoregress current y_t on the past values of y_t , (y_{t-i}) and x_t , (x_{t-i}) to have the expression:

$$y_t = a_2 + \sum B_{2i}y_{t-i} + \sum C_i x_{t-i} + e_{2i} \dots\dots\dots (4.1.1.2)$$

where C_i is also autoregression coefficient and others (a_2 , B_{2i} and e_{2i}) as defined before in equation (4.1.1.1). The unrestricted Residual sum of squares (RSS_{ur}) is also obtained from equation (4.1.1.2).

3. Set the hypotheses for Granger causality test using equations (4.1.1.1) and (4.1.1.2) as thus:

$H_0: \sum_{i=1}^p C_i = 0$, for x_t does not Granger cause y_t ;

$H_1: \sum_{i=1}^p C_i \neq 0$, for x_t Granger causes y_t

4. Calculate the F- statistic (using the normal Wald test)

$$F = \left(\frac{RSS_r - RSS_{ur}}{p} \right) / \left(\frac{RSS_{ur}}{N-K} \right) \sim F_{\alpha} (p, N-K)$$

where p = lag length, N =total number of observations and K = number of parameters.

Note that the above calculated F-statistic is the decomposition of variability of the data in terms of sums of squares. This is in line with the original work of

Fisher-Snedecor seen as the distribution of ratios of two independent estimators of population variances

5. Decision

If one arrives at F-value to be less than the $F_{\alpha} (p, N-K)$ -value or p-value is greater than the alpha (α), where α is the significant level, accept H_0 . Otherwise, accept H_1 and conclude x_t “Granger causes” y_t .

6. To test whether y_t Granger cause x_t by making x_t the regressand (dependent variable), make a repetition of steps 1 to 5.

4.1.2 Multiple Linear Granger Causality Method

In this method, there is not much difference to that of simple Granger Causality except for an increased number of variables or the regressors and non-existence of co-linearity. Here, more than two variables are involved. Assuming that the variables are stationary with the optimal lag p , the following steps are adhered to:

1. Set up the unrestricted model for three variables x , y , z and then auto-regress such that:

$$z_t = \Theta_1 + \sum A_{1i}x_{t-1} + \sum B_{1i}y_{t-1} + \sum C_{1i}z_{t-1} + e_{1t} \dots \dots \dots (4.1.2.1)$$

where Θ_1 (intercepts), A_{1i} , B_{1i} , C_{1i} are autoregression coefficients, $i=1,\dots, p$, and e_{1t} is the error (as defined in equation 4.1.1.1). Then obtain the unrestricted Residual Sum of squares (RSS_{ur}).

2. Set up the Restricted Model for z_t Granger causes x_t and y_t as in 1; carry out:

$$z_t = \Theta_2 + \sum A_{2i}x_{t-1} + \sum B_{2i}y_{t-1} + e_{2t} \dots \dots \dots (4.1.2.2)$$

where Θ_2 , A_{2i} , B_{2i} and e_{2t} are defined as in step 1.

Then obtain the Restricted Residual Sum of Squares (RSS_{2r}) as case 1.

3. Set up the restricted model for Z_t Granger causes X_t or Y_t as Case 2, and then carry out:

$$z_t = \Theta_3 + \sum B_{3i}y_{t-1} + \sum C_{3i}z_{t-1} + e_{3t} \dots \dots \dots (4.1.2.3)$$

Or

$$z_t = \Theta_4 + \sum A_{4i}x_{t-1} + \sum C_{4i}z_{t-1} + e_{4t} \dots \dots \dots (4.1.2.4)$$

where Θ_3 , Θ_4 , A_{4i} , B_{3i} , C_{3i} , C_{4i} , e_{3t} and e_{4t} are also defined as in step 1.

Next determine the restricted residual sum of squares from equations (4.1.2.3) or (4.1.2.4) respectively as RSS_{3r} or RSS_{4r} .

4. Set up the hypotheses for Granger causality tests on the two cases as thus:

Case1: $H_0: \Sigma C_i = 0$, for x_t and y_t (combined) do not Granger cause z_t

$H_1: \Sigma C_i \neq 0$, for both x_t and y_t Granger causes z_t

Case 2(a) $H_0: \Sigma A_i = 0$, for x_t does not Granger cause z_t

$H_1: \Sigma A_i \neq 0$, for x_t does Granger causes z_t

(b) $H_0: \Sigma B_i = 0$, for y_t does not Granger cause z_t

$H_1: \Sigma B_i \neq 0$, for y_t does Granger causes Z_t

5. Compute the F-statistic (using the normal Wald test)

$$F = \frac{(RSS_r - RSS_{sur})/P}{RSS_{sur}/(N-K)} \sim F_{\alpha}(p, N-K)$$

where RSS_{2r} is utilised for Case 1, RSS_{3r} or RSS_{4r} for Case 2(a) or (b) respectively; while P, N and K are as defined in Step 4 of Section 4.1.1

6. Decision

If one arrives at F-value to be less than the $F_{\alpha}(p, N-K)$ -value or p-value is greater than the alpha (α), where α is the significant level, accept H_0 . Otherwise, accept H_1 .

7. Steps 1 to 6 can also be repeated to test whether other variables x_t or y_t Granger causes Z_t .

We are to note that RSS_{2r} and RSS_{3r} are to be used in place of RSS_r for the case hypotheses 2(a) and 2(b) respectively in Step 5.

4.1.3 Vector Auto-regression (VAR) Method.

Granger causality test can also be carried out through the framework of VAR. VAR, developed by Sim (1980), is an extension of univariate autoregression (AR) in which multiple variables are being treated as endogenous (dependent variables).

In VAR method, all the variables are linear functions of past values being regressed simultaneously. As VAR(P) is in standard reduced form, no contemporaneous variables included as explanatory variables on the right-hand side of its equations.

For a set of m time series $Z_t = (Z_{1t}, Z_{2t}, Z_{3t}, \dots, Z_{mt})$ the VAR(P) can be expressed or denoted by

$$Z_t = c_i + D_1 Z_{t-1} + D_2 Z_{t-2} + \dots + D_p Z_{t-p} + e_t \dots \dots \dots$$

(4.1.3.1)

where Z_t is an (mx1) vector containing each of the 'm' variables

c_i is an (mx1) vector of intercept terms.

D_i is the coefficients of (mxm) matrix, (i=1,2,...,p).

m= the number of variables to be considered in the system.

p = number of lags to be considered in the model.

e_t is the error terms in (mx1) vector.

For instance, let's set up a two-variable VAR(1) with variables x_t and y_t such that:

$$y_t = c_1 + D_{11}y_{t-1} + D_{12}x_{t-1} + e_{1t}$$

$$x_t = c_2 + D_{21} y_{t-1} + D_{22} x_{t-1} + e_{2t}$$

In matrix notation, we have

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

$$\text{where } Z_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}, Z_{t-1} = \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix}, c = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \text{ and } e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

Then determine the parameters c_i , D_{ii} and the error (e_t).

Next, perform the Granger causality test utilising the parameters such that:

If x_t does not Granger cause y_t , $D_{12} = D_{21} = 0$. In other words, this corresponding to the restrictions that all cross-lags' coefficients are all zeros which can be tested by Wald statistics.

The following are the possible Granger causality results:

1. If x_t "Granger causes" y_t only, we have:

$$(x_t \longrightarrow y_t) = \begin{pmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{pmatrix}$$

2. If y_t "Granger causes" x_t only, we have:

$$(x_t \longleftarrow y_t) = \begin{pmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{pmatrix}$$

3. Both ways causality, also called feedback, we have:

$$(x_t \longleftrightarrow y_t) = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

4. No Granger Causality both ways, seen as independence of the variables, we have:

$$x_t \text{ — } y_t \quad \text{or} \quad (x_t \longleftrightarrow y_t) = \begin{pmatrix} D_{11} & 0 \\ 0 & D_{22} \end{pmatrix}$$

4.2 Other Statistical Analyses and Tests on Granger Causality Results

As a further step to establish new ideas, statistical analyses are carried out on the obtained Granger causality results at Phases 1 and 2 . This is to enable us to have statistical inference on these results. By this, proportionality and chi-square tests are utilised in order to ascertain: (1) the equality between Granger causality and non-Granger causality proportions; (2) the uniformity of the Granger causality results within its various types and to; (3) have the distribution pattern of Granger causality being classified into developed and developing economies. Also, we have inference on Bayes' Theorem and coefficient of variation computations using assignment problem model and Binomial probabilities respectively.

4.2.1 Proportionality Test

The proportionality test is usually conducted on categorical/nominal data. If the categories are two, binomial distribution is assumed as a related distribution pattern. Hence, binomial probability distribution function is used for carrying out the proportionality test. That is, the test employs the binomial distribution to determine the likelihood that x or more (or, x or less) of n observations that comprise a sample will fall in one of two categories designated by Category 1 {success} with proportion P_1 , while the other designated as Category 2 {failure} with proportion P_2 ($P_2 = q = 1 - P_1$).

Therefore, the probability of exactly x from n observations will fall into one of the two categories which make way for computation of probability through the Binomial distribution.

$$P(x) = {}^nC_x p_1^x p_2^{(n-x)};$$

where nC_x stands for n combination x ; that is, the number of combinations of n things taken at a time selecting x, and p_i (i=1,2) is probability for the two categories respectively (with $p_1+p_2=1$).

However, when the sample size is large ($n>30$) a Normal approximation to Binomial can be used to carry out the test. Hence, issues of large and small samples are to be recognised and handled accordingly.

Suppose we are to test whether a random sample of size n (n is large), with proportion of success p_1 , could have been drawn from a population with proportion of success p.

By an application of the sampling distribution of proportions, with normal approximation to binomial, we have:

$$p \sim N(p_1, p_1 p_2 / n)$$

where N is the normality assumption and other letters as early defined.

Then one can carry out the proportionality test by utilizing the test statistic:

$$Z = \frac{P - p_1}{\sqrt{\frac{P(1-P)}{n}}}$$

Which is distributed as $N(0, 1)$ under the null hypothesis H_0 that the proportion of success in the population is P. Then decide according to the desired significant level.

On a small sample, normal approximation could not be applied. However, the actual Binomial distribution as stated above will be applied.

Suppose we want to test on the basis of a small sample observation of x at α percentage level of significance, with proportion P_1 from population proportion p , then the statistical tests with necessary decisions are given below:

For a variable X which is Binomial, i.e $X \sim \text{Binomial}(n, p)$, the test statistics can be handled with the following cases.

(a) One tailed test

$$H_0 : P = p_1 \quad \text{vs} \quad H_1 : P > p_1$$

$$\text{Reject } H_0 \quad \text{if } \text{Prob}(X \geq r) < \frac{\alpha}{100}, \quad \text{where } X = r;$$

OR

$$H_0 : P = p_1 \quad \text{vs} \quad H_1 : P < p_1$$

$$\text{Reject } H_0 \quad \text{if } \text{Prob}(X \leq r) < \frac{\alpha}{100}; \quad \text{and}$$

(b) Two tailed test

$$H_0 : P = p_1 \quad \text{vs} \quad H_1 : P \neq p_1$$

$$\text{Reject } H_0 \quad \text{if } \text{Prob}(X \leq r) < \frac{0.5\alpha}{100} \quad \text{OR} \quad \text{if } \text{Prob}(X \geq r) < \frac{0.5\alpha}{100}$$

4.2.2 Chi – Square Test

The Chi – square test, with the symbol χ^2 , depends on the properties of Chi- square distribution and it is widely used to:

- (i) test whether it is valid to use a particular distribution in order to know or ascertain the correctness of the assumed model for the data under study. This is also called the ‘goodness of fit test’;
- (ii) decide whether two variables are independent of each other; and generally to
- (iii) handle various other categorical data tests.

The following procedural stages are to be adhered to for this test:

Step 1: Obtain the observed frequency (O) from the data and state the Null hypothesis (H_0) concerning the distribution of this data.

Step 2: Determine the expected frequency (E) according to the stated hypothesis.

Step 3: Determine the significant level (α) and the degree of freedom.

Step 4: Compute the test statistic:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Step 5: Compare the computed statistic with the χ^2 – table value and decide either to accept H_0 or not.

4.2.3 Bayesian Inference

For non-empty events F and G, the probability of F after the occurrence of G is known as conditional probability. It can be expressed as:

$$P(F|G) = \frac{P(F \cap G)}{P(G)} \quad , \quad \text{for } p(G) > 0$$

Where P represents probability, $P(F \cap G)$ is the joint probability of F and G; and it stands for the probability of F intersection G.

By an application of conditional probability, one can state that:

(i) $P(F|G) = \frac{P(F \cap G)}{P(G)}$, if $P(G) > 0$; and also

(ii) $P(G|F) = \frac{P(G \cap F)}{P(F)}$, if $P(F) > 0$

{(i) and (ii) are known as Kolmogorov definition.}

Since $P(F \cap G) = P(G \cap F)$, we have the Bayesian expression:

$$P(F \cap G) = P(F|G) P(G) = P(G|F) P(F)$$

Using (i) ,

$$P(F|G) = P(G|F) \left(\frac{P(F)}{P(G)} \right) \quad , \text{ if } P(G) > 0$$

Where $P(F)$ and $P(F|G)$ are representing prior and posterior probabilities respectively.

By an extension, suppose F_1, F_2, \dots, F_n are n mutually exclusive and exhaustive events so that $F_1 \cup F_2 \cup \dots \cup F_n = \Psi$, the possibility space, and F_j is an arbitrary event of Ψ . Then for $i = 1, 2, \dots, p$ and $j = 1, 2, \dots, q$, and the event G is also in Ψ , then

$$Q = P(F_i | G) = \frac{P(G|F_i)P(F_i)}{\sum_{j=1}^q P(G|F_j)P(F_j)} \quad (4.2.3.01)$$

is called the Bayes Theorem.

This theorem tells us how to compute or determine $P(F|G)$ if $P(G|F)$ and a few other things are known.

It also gives an additional knowledge about a prior event in terms of posterior. This is seen as a degree of probability measure of belief. Hence, $P(F|G) > P(F)$ is more likely to occur, that is posterior supporting the prior claim ; while $P(F|G) < P(F)$ is less likely and not supporting prior.

4.2.4 Assignment Problem Optimization

Assignment problem (A.P) arises in the course of assigning a task to a job in an optimal way of allocation. This is to enable one to assign p objects/tasks/ machines to q other objects /jobs/ assignee in an injective manner to achieve an optimal allocation. It is a combinatorial optimization that entails one – to – one mapping of task to assignee. Hence, it is essential that p equals q (balance) in order to achieve the said mapping.

The general form of an assignment model with p tasks and q assignee is given in table 4.01 below:

Table 4.01: Assignment Model with p tasks and q assignee

		Tasks					
Assignee	1	C_{11}	C_{12}	C_{13}	-	-	C_{1p}
	2	C_{21}	C_{22}	C_{23}	-	-	C_{2p}
	3	C_{31}	C_{32}	C_{33}	-	-	C_{3p}
	-	-	-	-			-
	-	-	-	-			-
	-	-	-	-			-
	M	C_{q1}	C_{q2}	C_{q3}	-	-	C_{qp}

Where C_{ij} [$i = 1, 2, \dots, p$ and $j = 1, 2, 3, \dots, q$] represents the cost/time that associated with assignee i performing task j .

As a linear programming problem, A.P is a special class of transportation problem where its sources (supply) and destinations (demand) capacities are respectively one. Its goal is to minimize the total cost subject to the condition that each task goes to exactly one assignee and each assignee to exactly one task.

With a given set of costs (C_{ij}) (as in Table 4.01 above) and cells allocations (X_{ij}), an A.P can be mathematically expressed as:

$$\text{Min } Z = \sum_{i=1}^q \sum_{j=1}^p C_{ij} X_{ij} \quad (\text{the objective function}) \quad (4.2.4.1)$$

$$\text{Subject to } \left. \begin{array}{l} \sum_{i=1}^p X_{ij} = 1, \text{ for } j = 1, 2, \dots, p \\ \sum_{j=1}^q X_{ij} = 1, \text{ for } i = 1, 2, \dots, q \end{array} \right\} \quad (\text{the constraints})$$

where $X_{ij} \geq 0$, (and binary $\{0 \text{ or } 1\}$ for all i and j);

C_{ij} is the cost associated with assignee i performing task j .

Z is the objective function to be minimized (which is concerned with the overall allocation of cells that leads to smallest total cost); and p equals q is required for balanced assignment.

The A.P. can be solved by various methods of linear programming; but however, these methods take longer time to accomplish the required solution. Hence, Harold Kuhn in 1955 came up with the Hungarian method. It is an optimization algorithm with the following steps:

Step 1: For a given matrix table, called the cost matrix and p equals q , locate the minimum value and subtract it from every element (cell) in that row. Then subtract the column minimum from each column from the reduced matrix. A collection of these operations is termed the opportunity cost matrix. [Note that if p is not equal to q , balance the table by creating dummy tasks or jobs].

Step 2: Determine whether the reduced matrix is optimal. For optimal assignment, the reduced matrix should have p zero entries of which no two of them are in the same row or column. To achieve this, draw the minimum number of straight lines (vertical or/and horizontal) on the opportunity cost matrix to cover all the zeros as many as possible at a time. This is the same as having total number of straight lines equal to p in the reduced matrix. If the optimal condition is not met, go to the next step.

Step 3: Once the reduced matrix is not optimal, one needs to revisit the last reduced matrix of step 2 and locate the smallest (minimum) number uncovered by the lines drawn. Then subtract this number from every element (cell) uncovered by a line and add it to every cell covered by the intersection of two lines.

Step 4: Repeat Steps 2 and 3 until the allocation is optimal.

Remark: A collection of Steps 1 to 4 is utilised for minimization of the assignment problem. However, when there is maximization problem (for profit or effectiveness), one of the following additional steps needs to be taken:

- (i) Converting the problem from maximization to minimization by multiplying the assignment problem by -1 ; that is :

$$\mathbf{Min\ Z' = Max\ (-Z)}$$

Then apply Steps 1 to 4.

- (ii) Subtracting every number or element in the profit matrix from the largest number in that matrix. The resulting entries produced the cost matrix and next to apply Steps 1 to 4.

In order to support and justify the Hungarian idea of adding (subtracting) a constant to (from) cost matrix (C_{ij}) , the LP model of AP stated above will be used as follows:

Let a_i and b_j be constants added (subtracted) to (from) row i and column j of cost matrix C_{ij} in equation (4.2.4.1), then the new cost matrix is

$$\dot{C}_{ij} = C_{ij} - a_i - b_j$$

Now replacing the new cost matrix into equation (4.2.4.1), with the new cost matrix \dot{C}_{ij} , we have

$$\begin{aligned}\sum_i \sum_j \dot{C}_{ij} X_{ij} &= \sum_i \sum_j (C_{ij} - a_i - b_j) X_{ij} \\ &= \sum_i \sum_j C_{ij} X_{ij} - \sum_i a_i (\sum_j X_{ij}) - \sum_j b_j (\sum_i X_{ij}) \\ &= \sum_i \sum_j C_{ij} X_{ij} - \sum_i a_i (1) - \sum_j b_j (1)\end{aligned}$$

$$\sum_i \sum_j \dot{C}_{ij} X_{ij} = \sum_i \sum_j C_{ij} X_{ij} - \text{constant} \quad (4.2.4.2)$$

Where constant = $\sum_i a_i + \sum_j b_j$, $\sum_i a_i$ or $\sum_j b_j$.

By comparing the objection functions in equations 4.2.4.1 (old) and 4.2.4.2 (new), one can see that the difference is the constant. It implies that optimum values of X_{ij} for the allocations remain the same in both cases. This is to show that the iterations carried out on the matrix cost, whatever constants involved in the iterations, will still give optimum allocations of X_{ij} .

4.2.5 Coefficient of Variation.

As mean and standard deviation/variance are important parameters or characteristics in any distribution, we utilise these parameters for the consideration of coefficient of variation concept. This concept is very useful in statistical analysis, inference and prediction.

A coefficient of variation (CV) is a standardised or normalised measure of dispersion of a probability distribution or frequency distribution. It is a ratio scale where the variable value of CV is non-dimensional and independent of unit. Hence, it is conducive and useful for comparing two or more distributions.

We present two possible ways in which CV can be defined and utilized. These are:

- i. With a single variable, a model can be formed and interpreted. It is calculated by:

$$CV = \sigma/\mu \quad \text{or} \quad S/\bar{X} \dots\dots\dots (4.2.5.1)$$

where the pairs (σ, μ) and (S, \bar{X}) are standard deviation and mean respectively for population and sample of variable X.

In another way, using the sample, we can have:

$$CV = \frac{\frac{\sum(x-\bar{x})^2}{n-1}}{\frac{\sum x}{n}} = \frac{n \sum(x-\bar{x})^2}{(n-1)(\sum x)}$$

- ii. With the linear regression model of more than one variables, for example

$$y_t = ax_1 + bx_2 + e_t$$

CV is obtained as the ratio of the root mean square error (rmse) to the mean of the dependent variable y, (\bar{y}) . That is

$$CV = \text{rmse}/\bar{y} \dots\dots\dots (4.2.5.2)$$

In both cases, the CV can be expressed in percentage.

The advantages of CV include -

- i. Bring about the dimensional units of different variables into the same non-dimensional unit;
- ii. By this non-dimensional unit, comparison can be carried out.
- iii. A variable or model with smaller CV has smaller volatility and this amounts to predicted values closer to the actual values. This demonstrates and supports a better prediction power of one variable over the other.

However, CV has the following demerits:

- i. non-negative values for S and \bar{X} are not permitted.
- ii. \bar{X} cannot be zero. If it is zero, the CV diverges.

It is of note that the reciprocal or inverse of CV is μ/σ and referred to as the signal-to-noise-ratio (in signal processing).

Chapter 5: Research Methodology

5.1 Introduction

The first part of this chapter considered the data, in terms of source, type and period covered; while the second part dealt with the research methods being utilised in this study.

5.2 Data Description and Sources

All data utilised in this study were secondary and sourced from International Financial Statistics (IFS) of the International Monetary Fund (IMF). IMF also got the data through:

- World Bank Development
- IHS Global Insight
- Oxford Economic Forecasting
- ERS Baseline Regional Aggregations

The internet connections to these data are from website:

(i) [www.internationalmonetaryfund.com/HistoricalRealGDPvalues]

(ii) [www.internationalmonetaryfund.com/HistoricalCPIvalues]

(iii) [http://esdc80.mcc.ac.uk/wds_ifs/TableViewer/tableView.aspx?ReportId=54780]

for import.

(iv) [http://esdc80.mcc.ac.uk/wds_ifs/TableViewer/tableView.aspx?ReportId=54721]

for export.

The data selection involved two stages. In the first stage, a stratified sampling of 100 countries was made, using the IMF (World Economic Outlook) grouping, into ratio 2:3 for developed and developing countries respectively. The data set utilised in this stage comprises

of the real gross domestic product (real GDP at 2005 prices) percentage change and the inflation rate (proximally represented by consumer price indices (CPI 2005 = 100) percentage change from 1970 to 2011.

By application of structural breaks and outliers tests, the non-stationary test to the said data, sixty-five countries were found to be non-structural break. It is this set of countries that we considered for Granger causality test. The countries are: Canada, France, Germany, Italy, Japan, U.K, U.S.A, Australia, Czech Republic, Denmark, Hong Kong SAR, Iceland, Israel and Korea (south). Others are New Zealand, Norway, Singapore, Sweden, Switzerland, Taiwan Province of China, Austria, Belgium, Cyprus, Finland, Greece, Hungary, Ireland, Luxemburg, Malta, Netherlands, Spain, Portugal, India, China, South Africa, Chile, Egypt, Iran, Kenya, Kuwait, Malaysia, Nigeria, Pakistan, Saudi Arabia, Thailand, Tunisia, United Arab, Vietnam, Sri Lanka, Botswana, Libya, Trinidad & T, Bangladesh, Cambodia, Fiji, Nepal, Tonga, Barbados, Colombia, Paraguay, Algeria, Jordan, Morocco, Burkina Faso and Ethiopia.

At the second stage, a quota sampling (a non-random sampling) was utilised to select all countries that supported the existence of Granger causality in stage one. The actual data values (in millions of USA dollar) or realizations of Real GDP, Inflation Rate, Export and Import of these countries (33 in number), covering the period 1970 to 2011, were considered. The countries are: Canada, France, Germany, Italy, Japan, USA, Australia, Denmark, Iceland, New Zealand, Sweden, Switzerland, Austria, Belgium and Finland. Others include Greece, Hungary, Luxembourg, Netherlands, Spain, Portugal, India, China, Chile, Iran, Malaysia, Tunisia, Botswana, Bangladesh, Fiji, Nepal, Algeria and Ethiopia.

5.3 Statistical Instruments/Tools

The first tool used in this study is the pictorial/graphical representation. Time plots for the countries considered in stages one and two, by their data type were studied. This is to enable us to see the pattern and movement of the variables. See Figure 5.01-Methodological / Empirical Chart of the study. It gives the systematic and sequential steps of the study.

The next statistical tools were mainly on stationary and non-stationary issues of the variables. All the statistical steps in Chapter 3 were carried out on these variables in order to make them stationary, where necessary, for further analysis (the Granger causality). Here, data that are non-stationary were transformed to stationary.

The set of statistical tools discussed in Chapter 4 were utilised to establish the Granger causality. To be specific, the pair-wise Granger causality test using Standard Granger causality method was applied.

Due to large data and large number of countries considered in our study, a number of programmes as functions in Matlab were developed. We utilised both developed functions and those functions that already exist within Matlab. The developed functions are (a) Stationary test (for unit roots and trend); (b) Chow test, Cusumchart test and Quandt test (for outliers and structural break(s); and (c) Granger Causality test (to carry out computations and tests after establishing and making the variables to be stationary).

In developing (c), we adhered to the stated steps of Standard Granger causality in Section 4.1.1. By this, equations (4.1.1.1) and (4.1.1.2) became our models.

The involved stages in the Matlab function of (c) include:

- To ensure variables y and x are of the same length column vectors.

- To determine the optimal number of lags using Bayesian information criterion (BIC).
- Determination of Data lag for variables y and x respectively.
- Computation of autoregression for unrestricted model of y on x and determination of its RSS ($RSS_{UR_{yx}}$).
- Computation of autoregression for restricted model of y on x and determination of its RSS represented by ($RSS_{R_{yx}}$).
- Determination of F-Statistics (for both F_{cal} and F_{table}).
- Computation of autoregression for un-restricted model of x on y and determination of its RSS indicated as ($RSS_{UR_{xy}}$).
- Computation of autoregression for restricted model of x on y and determination of its RSS stated as ($RSS_{R_{xy}}$).
- Determination of F-Statistics (for both F_{cal} and F_{table}).
- Decision on Granger causality.
- Print the outputs of the Granger causality results.

Source codes are given in Appendix A.

5.4 Further Statistical Analyses and Tests on Granger Causality Results.

Further statistical analyses were carried out on the results of the phases involved in this study. We are able to apply the paradigms of frequency and Bayesian procedures in our tests.

In phase one, distribution/classification tables were formed and some statistical tests were carried out.

The tables formed with their accomplished statistical analyses are given below:

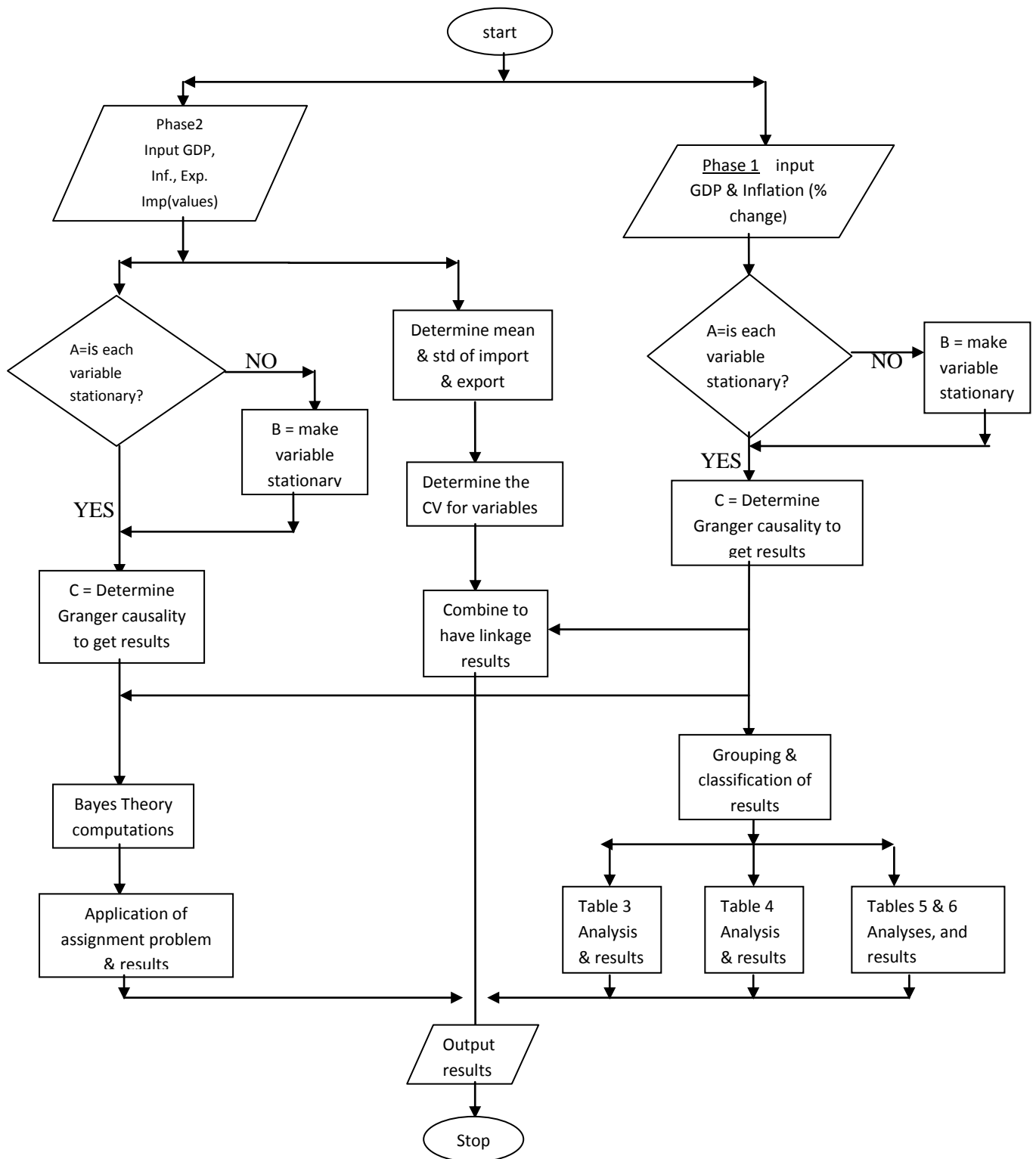
- (i) Classification of results into Granger causality and non-Granger causality presented. The test performed here was the proportionality test.
- (ii) Classification into type of Granger causality with the accomplished test of Chi-square by considering the biased and dependency of the results.
- (iii) Classification of Granger causality into developed and developing economies. Test on proportion was carried out.
- (iv) Classification of non-Granger causality results into developed and developing economies. Proportionality test utilised.
- (v) Distribution of developed and developing economies into type of Granger causality. Chi-Square and Proportionality (Using Binomial Distribution) tests are used.

For Phase 2, three main statistical tools were used. These are:

- (i) Bayesian Theory utilised in order to confirm our results on Granger causality in Phase 1.
- (ii) Further analysis on Bayesian results is carried out by using assignment optimization. This is to enable us have one to one mapping from multiple Bayesian results in terms of GDP/inflation Granger causality's combinations as linkages to export and import (from the Bayesian supportive approach).
- (iii) Coefficient of variation (CV) is utilised on export and import for comparison purpose and to relate or link the results to Granger causality outcomes in Phase1.

It is worth mentioning that exports and imports came into the picture of this study as a way of having new ideas, as how the said variables are relating to the Granger causality between GDP and inflation but not exports (imports) to imports (exports). From the macroeconomic theory, it is a known fact that exports and imports are component parts of GDP and inflation. By this, our research venture to examine how these variables (exports and imports) are related or linked to the Granger causality results of GDP and inflation statistically. The Methodological/Empirical Chart follows:

Figure 5.1: Methodological/Empirical chart of the study



KEY: A= is each variable stationary? (The stationary tests including ADF, KPSS, Chow, Quandt are carried out). B= Make variable stationary (for the non-stationary variable using differencing, de-trending). C= Determine Granger causality (Here, the optimal lag length determined by using BIC criteria before application of auto-regression). Inf = Inflation, Exp = Export, Imp = Import, var=variable.

Chapter 6: Results, Findings and Interpretation

6.0 Introduction

The chapter is concerned with the presentation of results, findings and interpretations along each phase of the study. See Figure 5.01 on the methodological/empirical description of the study. These outcomes emanated from the plotting, computations and analyses carried out on the collated data by adherence to the methodology.

6.1 Phase 1 of the Study

In Phase 1, we first utilised the percentage change of data on GDP and inflation. Our results, findings and interpretation are presented under the following sub-sections

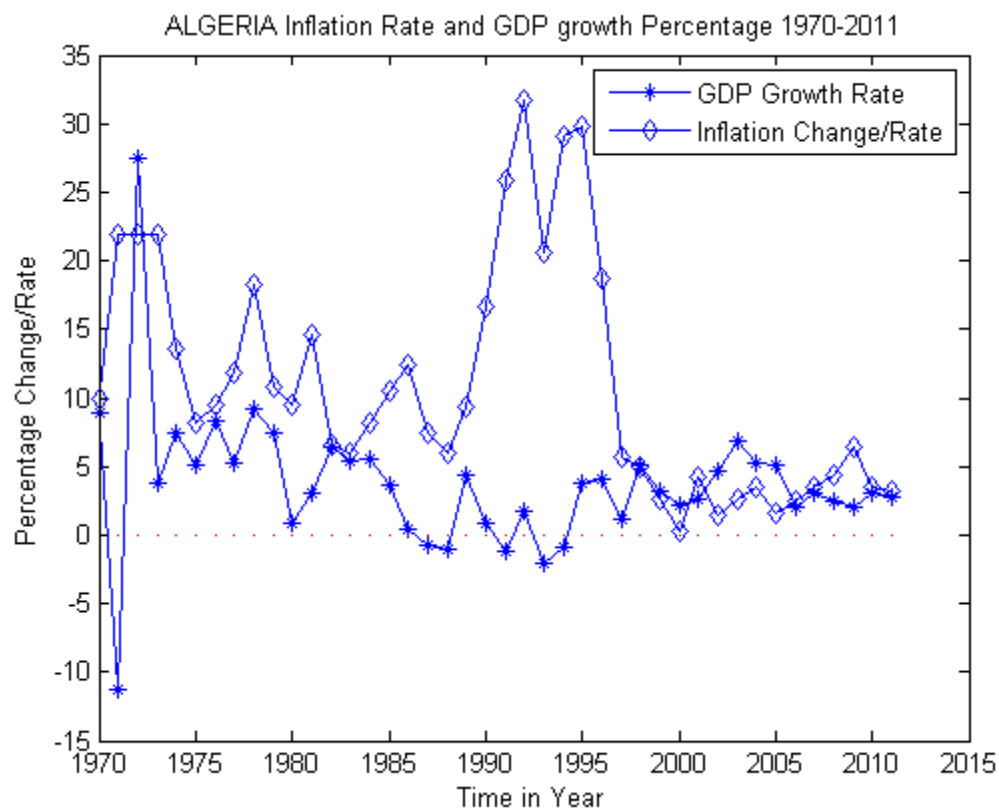
6.1.1 Results and Findings

The time-plots drawn showcased the pictorial presentation of the variables' movement. For the Phase 1 time-plots, see Figures 6.01 to 6.20 as part of the countries considered in the study. Other countries' time-plots can be found in Appendix B consisting of Figures 1 to 25

Remark 1 on time series plots:

This remark affects all time series plots in Phase 1. It is necessary to re-state that the data utilized in Phase 1 are in percentage change which involved both positive and negative values. By applying logarithm to these percentages, especially the negative, it resulted to complex numbers. When these complex numbers are plotted, the imaginary parts are ignored. This will not augur well for plotting. Hence, we maintain the actual percentage change without applying logarithm.

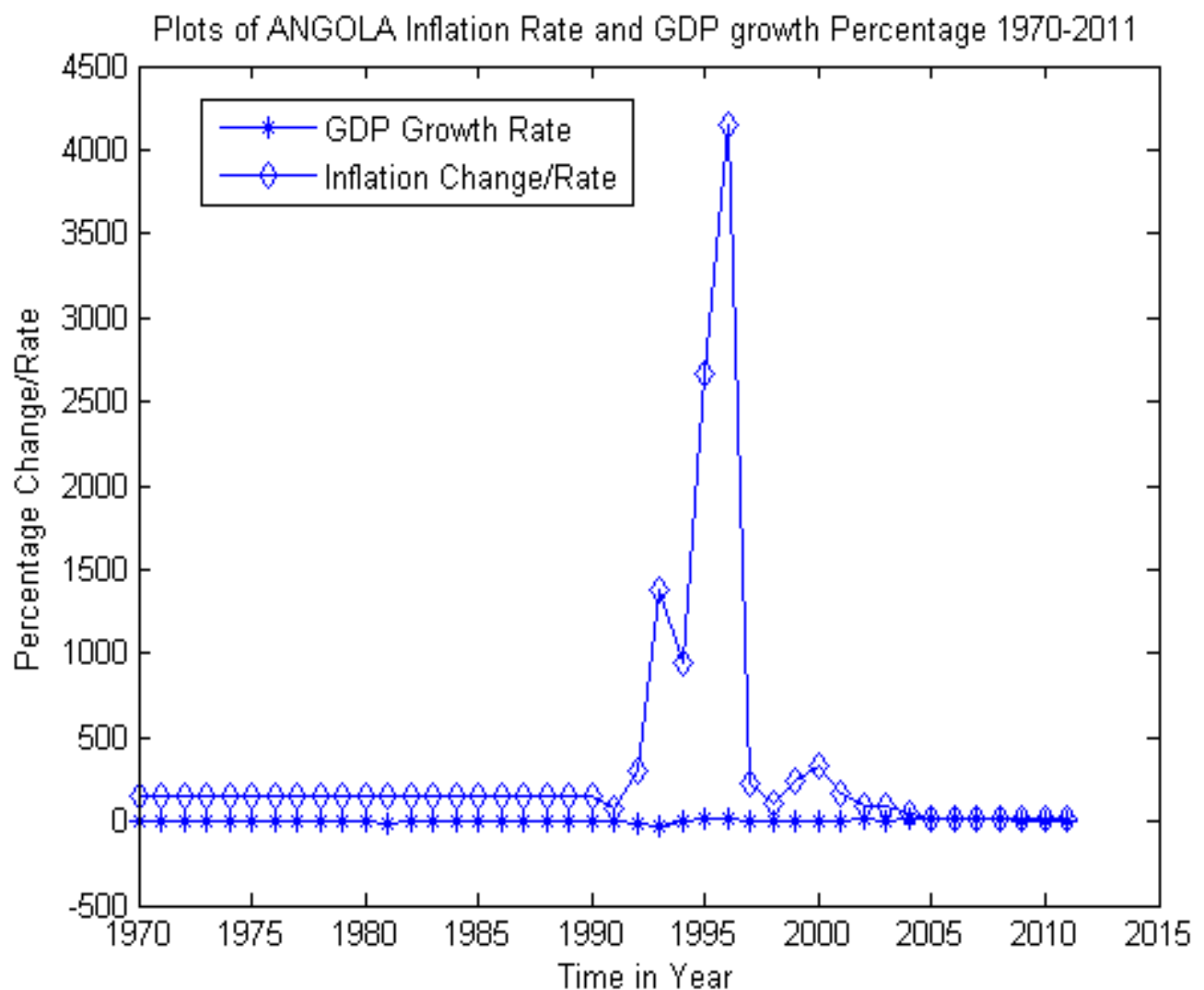
Figure 6.01



Comments: The percentage changes of inflation rate are higher than that of GDP. For instance, most periods ranging from 1970 to 1996 experienced galloping inflation because the percentages are of two digits. Whereas, it is of one digit in GDP except for 1971 and 1972.

It is also observed the existence of irregular movements in the two variables, but more pronounced in Inflation. These irregularities can be termed as fluctuations which can lead to non-stationarity

Figure 6.02

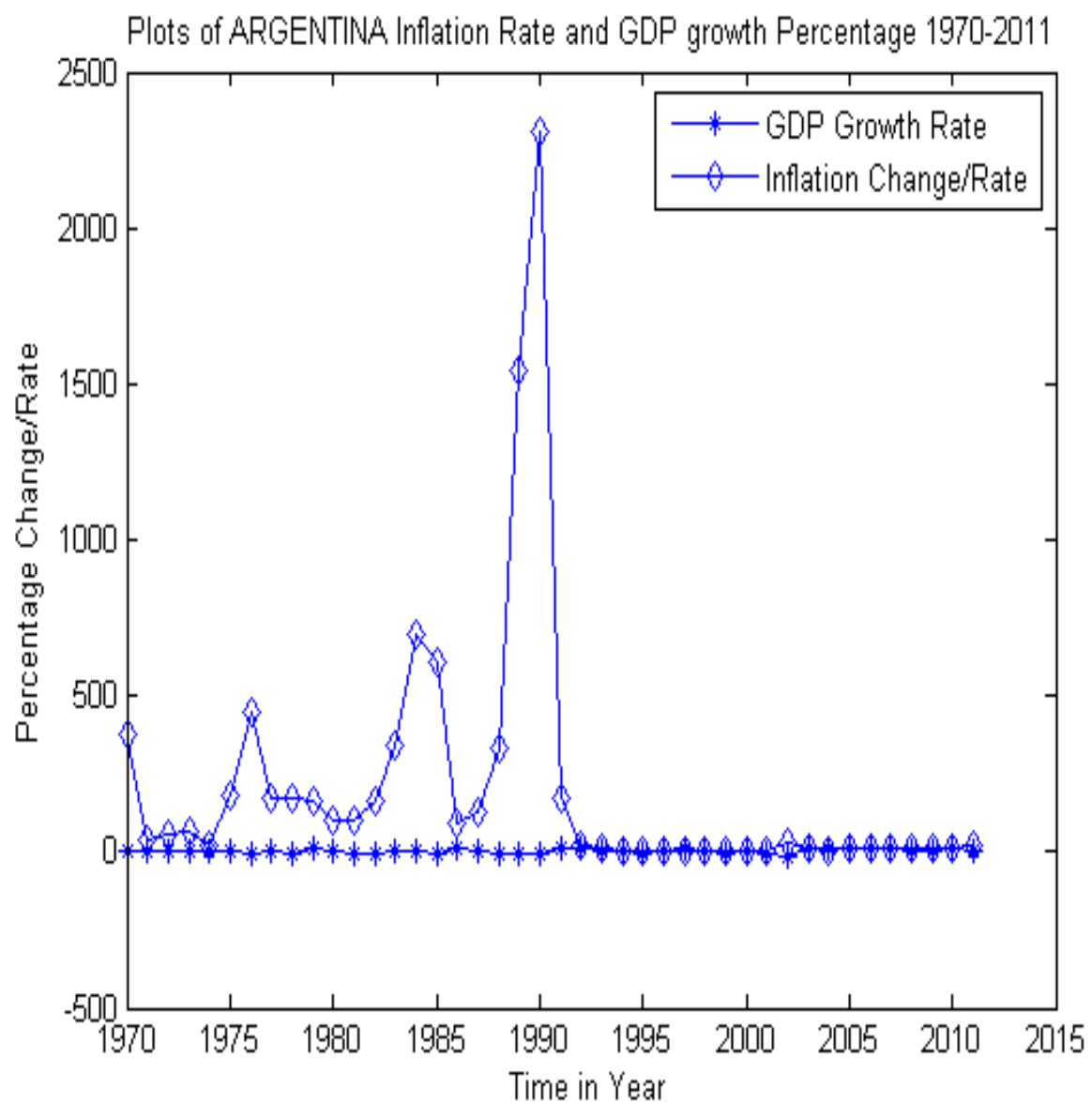


NB: Here, we have some negative percentages. Hence, we adhere to “Remark 1 on time series plots” (See Page 103).

Comments: The inflation rates ranging between 2 and 4 digits, with the hyperinflation from 1991 to 1995, are observed. But the GDP rates of 1/ 2 digit range are equally noted.

It is suspected that the inflation rates have some features like outliers and structural breaks which may not be amenable to further analyses. Appropriate tests will be utilized to check the said features.

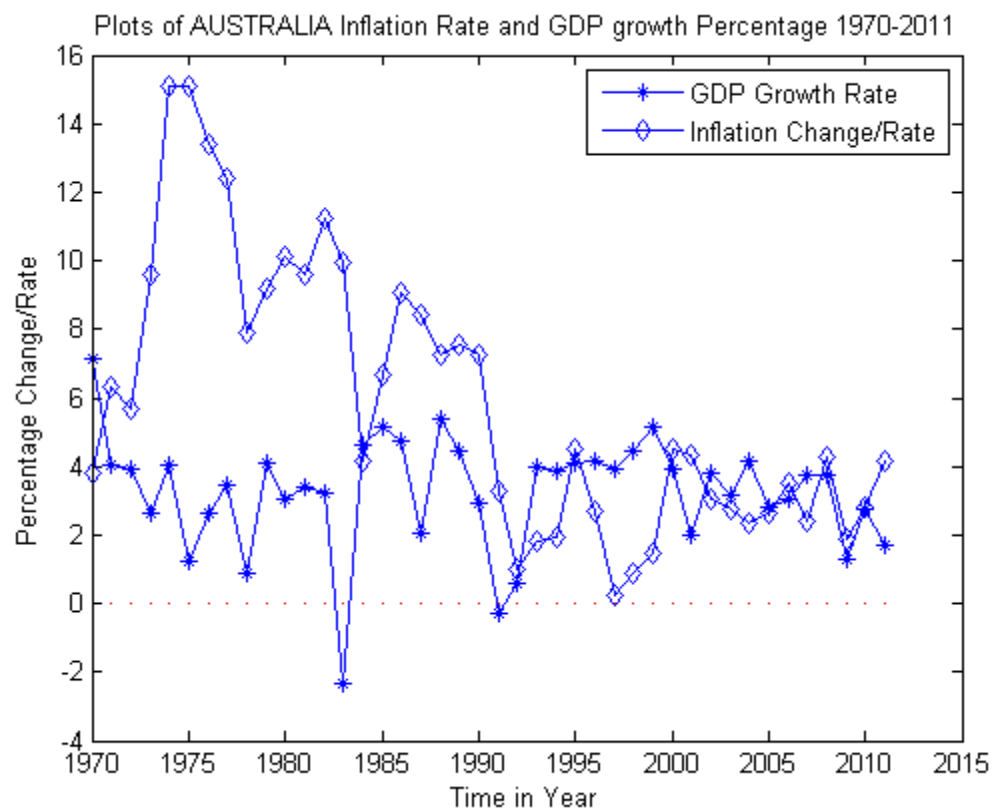
Figure 6.03



NB: Here, we have some negative percentages. Hence, we adhere to “Remark 1 on time series plots” (See Page 103).

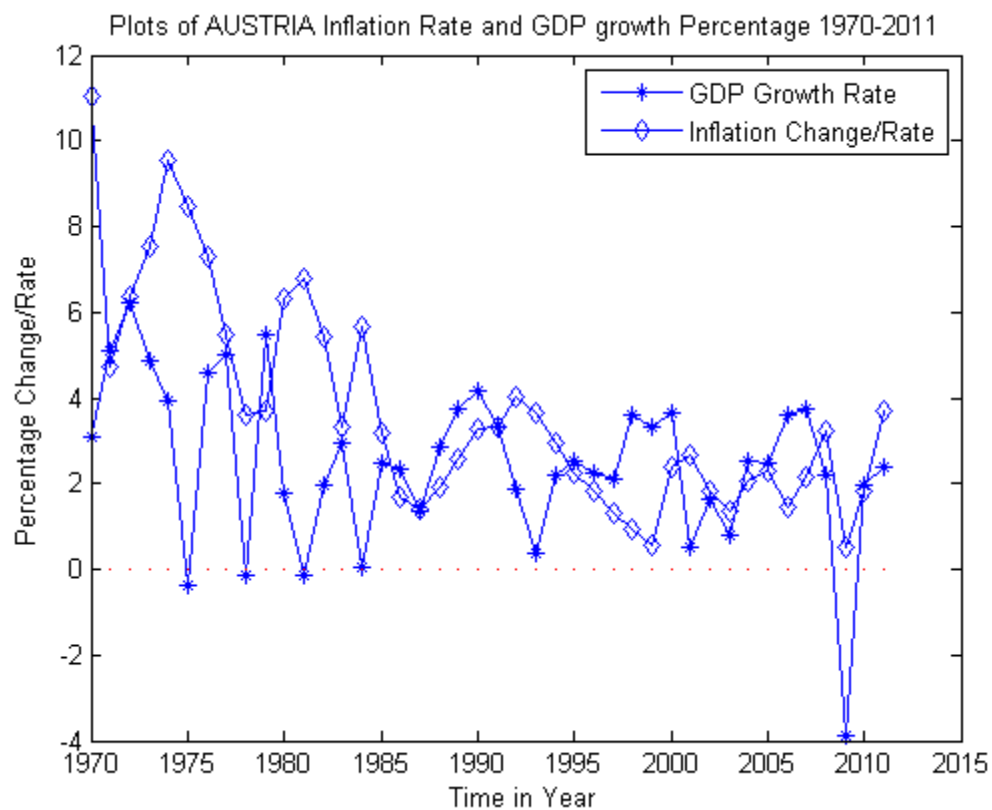
Comments: Majority of the GDP rates are of 1 digit, while a few are of 2 digits. Inflation has 2 to 4 digits of percentage change (i.e galloping and hyperinflation). This may equally induces some problems in further analyses because of the high irregular fluctuations. Infact, outliers and structural breaks are suspected.

Figure 6.04



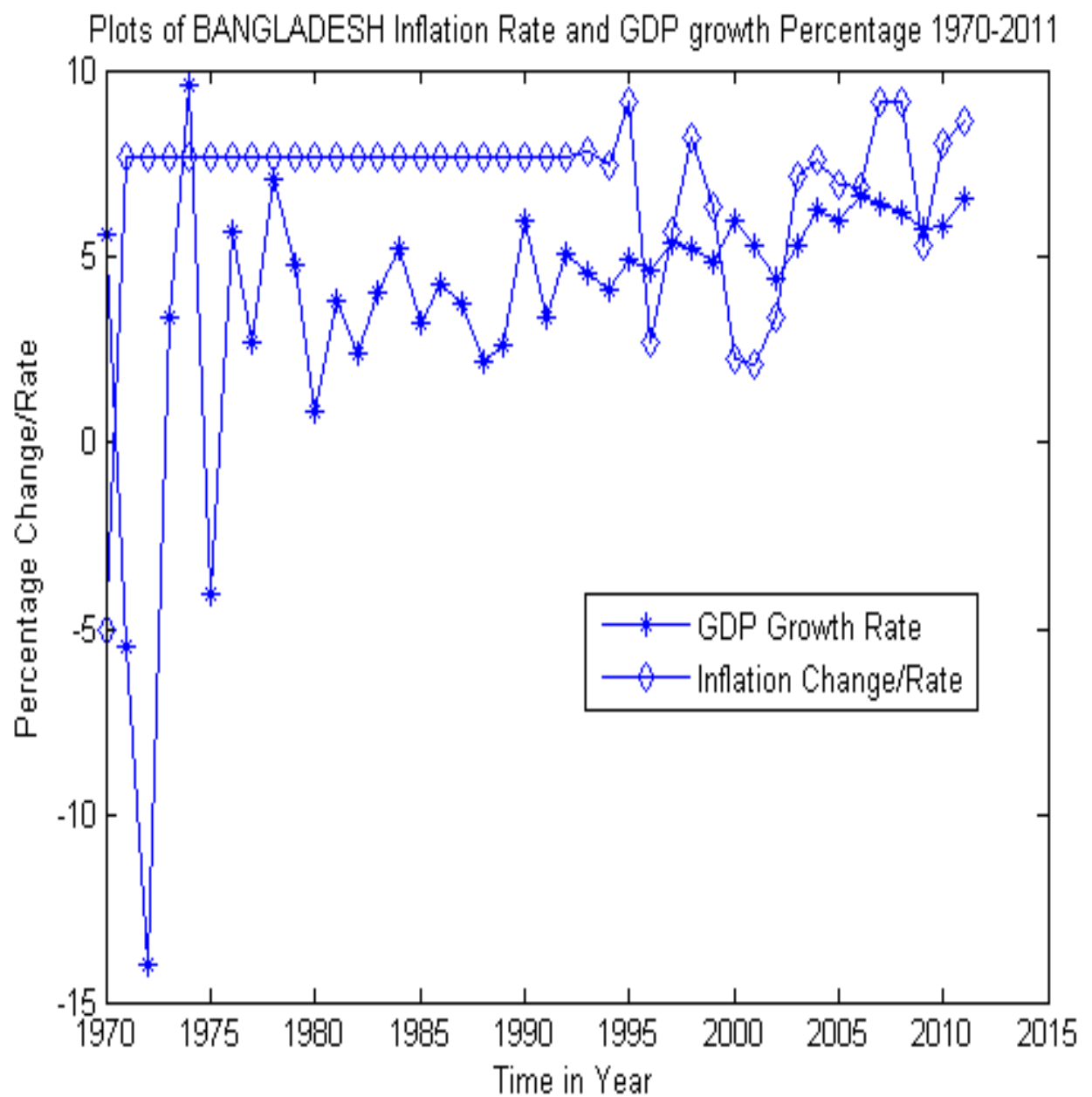
Comments: The fluctuations in GDP rate movement is of 1 digit (i.e from 1 to 9). It means the chance of it being stationarity is high. In the case of Inflation, the highest rate is below 16 which can be amenable if it is of non-stationary.

Figure 6.05



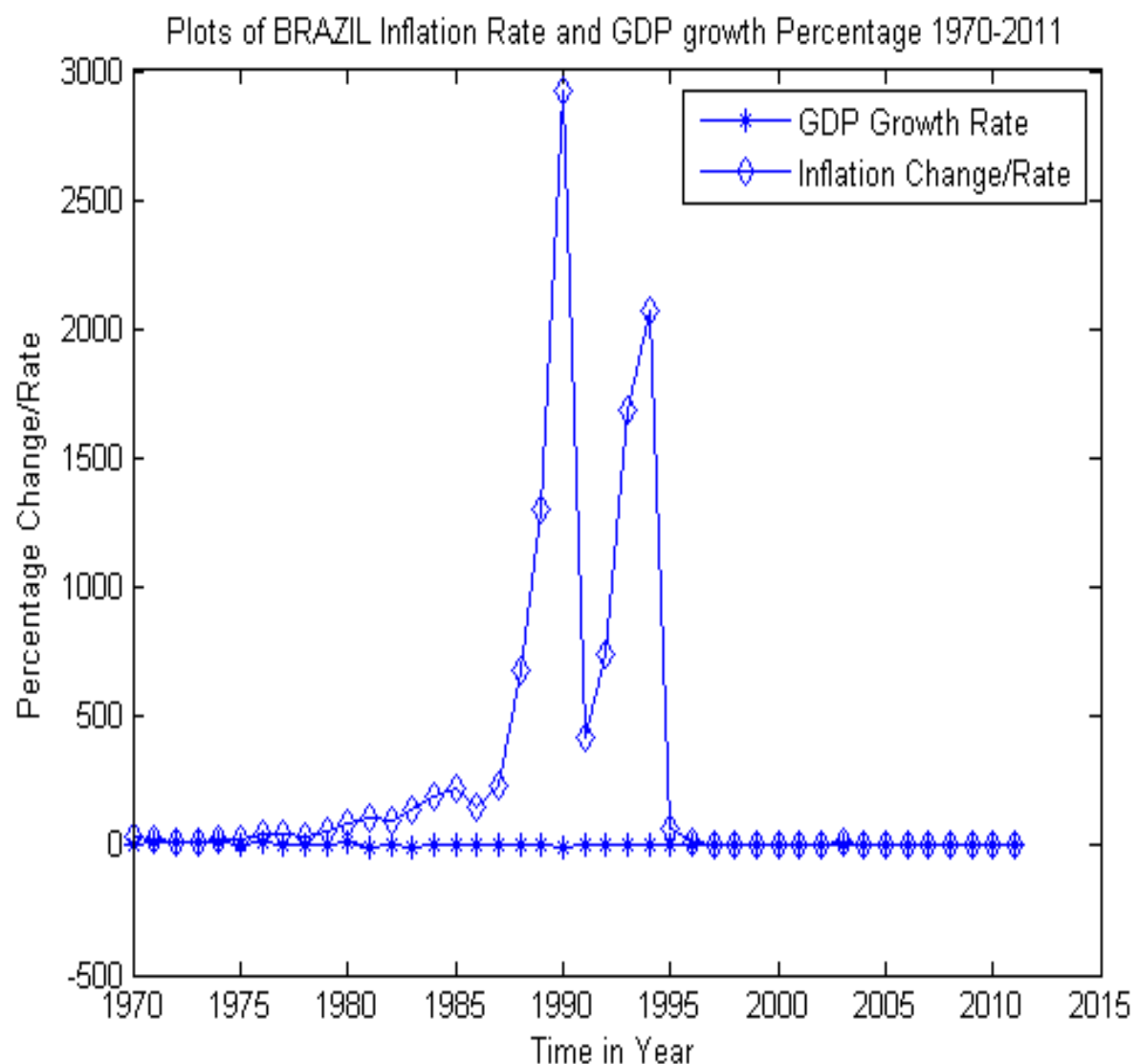
Comments: GDP exhibits 1 digit of percentage change with less than 7 as highest. The movement here looks like a little bit of regular fluctuations. GDP stationarity is suspected. Also, inflation equally has 1 digit of rate with less than 10 as the highest. The inflation movement is not regular as that of GDP.

Figure 6.06



Comments: The Inflation rate is of 1 digit with its constancy from 1971 to 1993. It further exhibits fluctuations from 1994 to 2011. GDP has about 14 and 10 percentage changes in 1972 and 1974 respectively whilst the others are of 1 digit. The GDP fluctuations look a little bit regular from 1976 onwards.

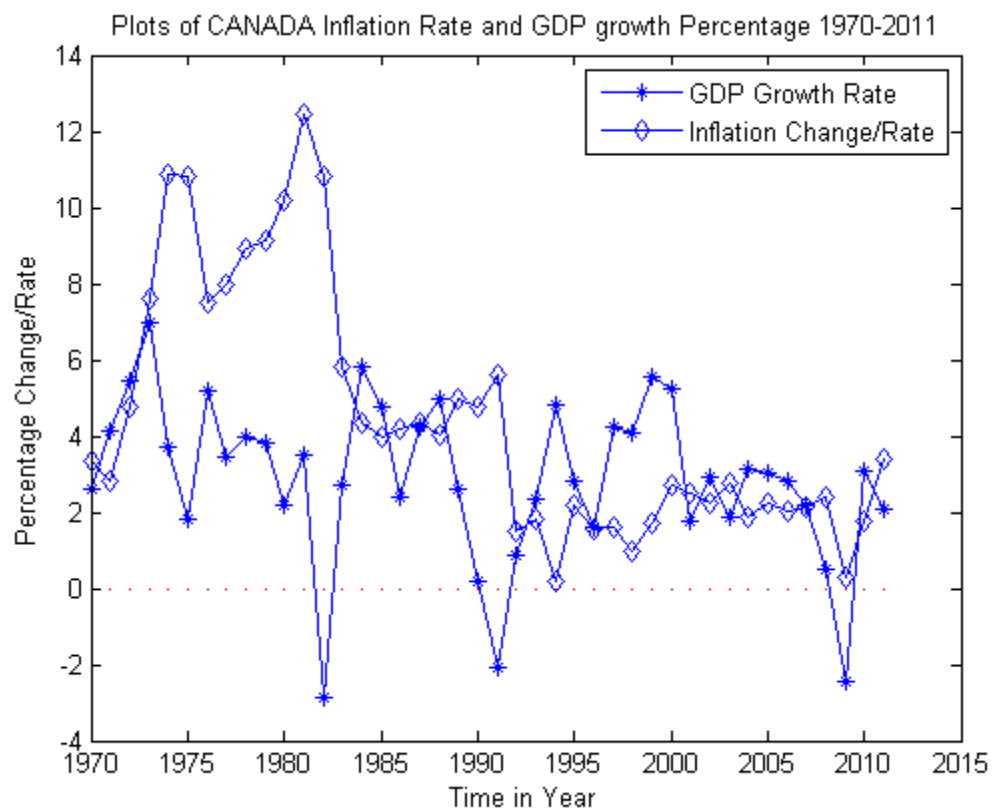
Figure 6.07



NB: Here, we have some negative percentages. Hence, we adhere to “Remark 1 on time series plots” (See Page 103).

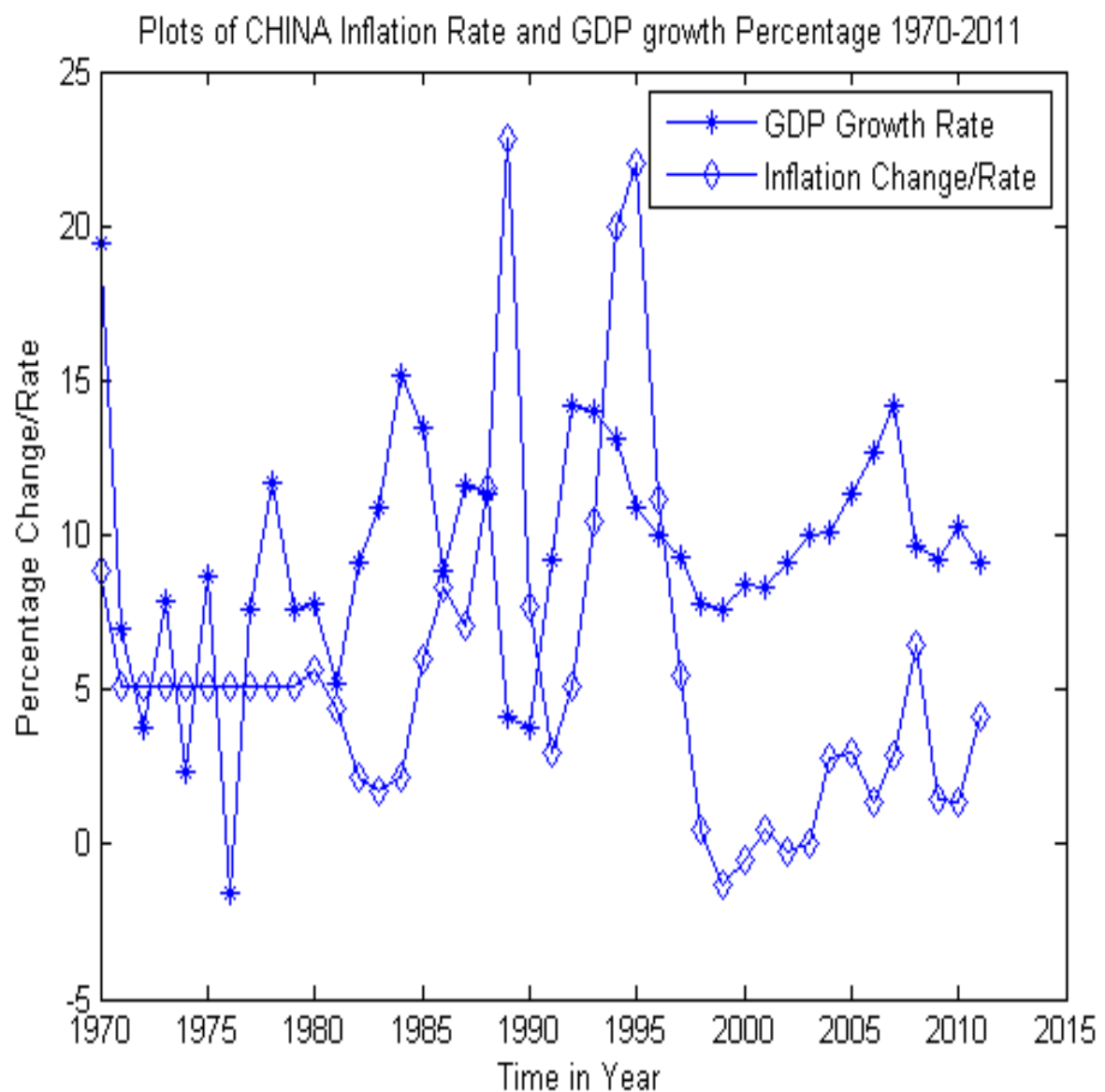
Comments: The inflation rates of this country experienced both galloping and hyperinflation (i.e of 2 to 4 digits) from 1987 to 1994. By 1995, the inflation rates come down to 1 digit. The outliers and structural breaks are suspected for this variable. On the other hand, the GDP shown the percentage changes of 1 unit with the exception from 1972 to 1974.

Figure 6.08



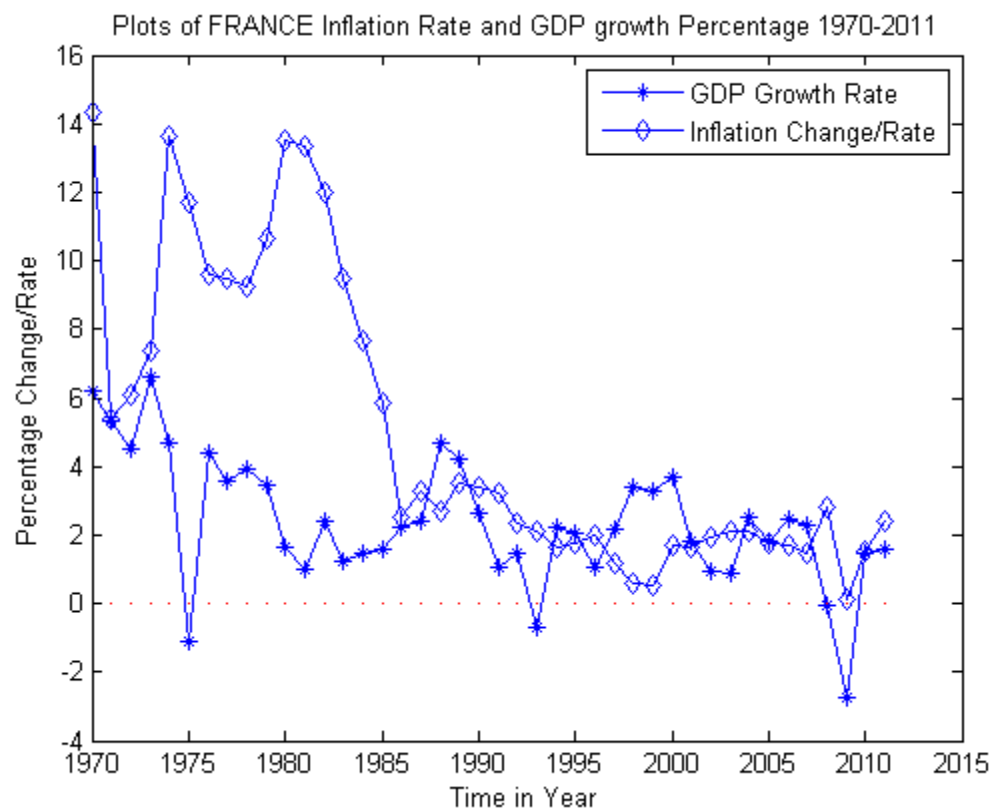
Comments: The GDP growth rates exhibited unit digit, i.e. ranging between 1 and 9 percentage changes. The fluctuation movements of GDP look a little bit more regular. Also, for the inflation it is unit with the exception of about 5 points having 2 digits.

Figure 6.09



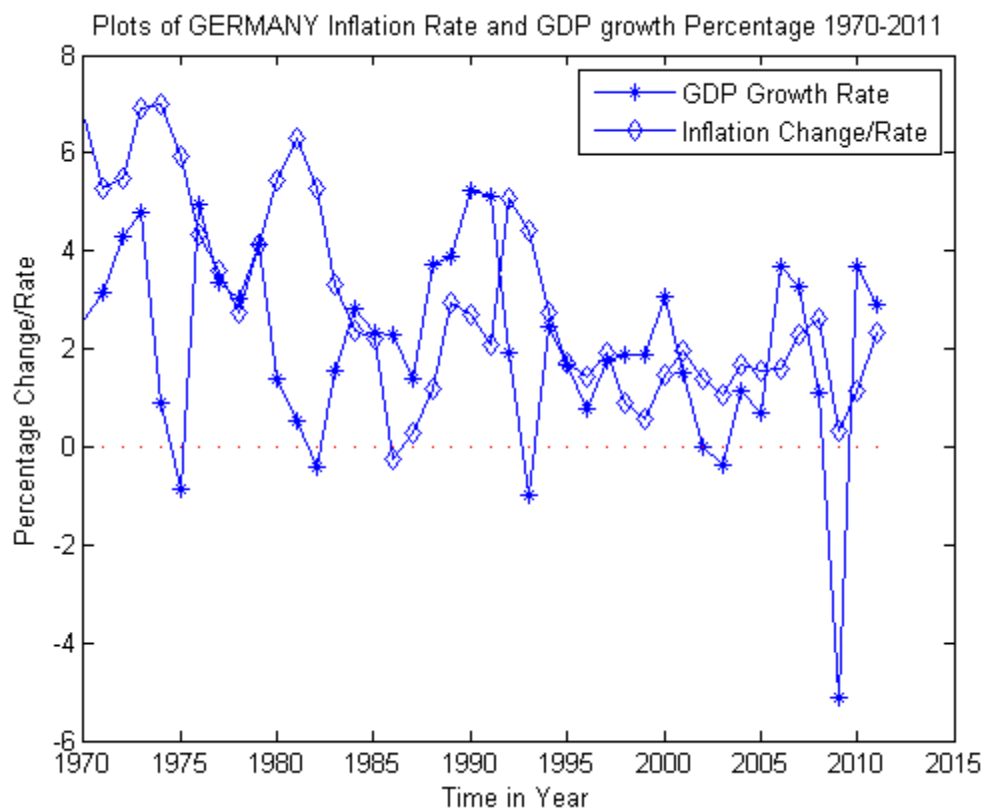
Comments: Both variables' rates ranged between 1 and 2 digits. However, it is observed that the GDP rates are higher than that of Inflation in a large number of years. For instance, we have 1970 to 1986 (except 1976) and 1997 to 1980 periods for the said higher rates. In the case of inflation, the rates were constant for the period 1971-1980. Also, there are about 5 points where inflation is higher than GDP.

Figure 6.10



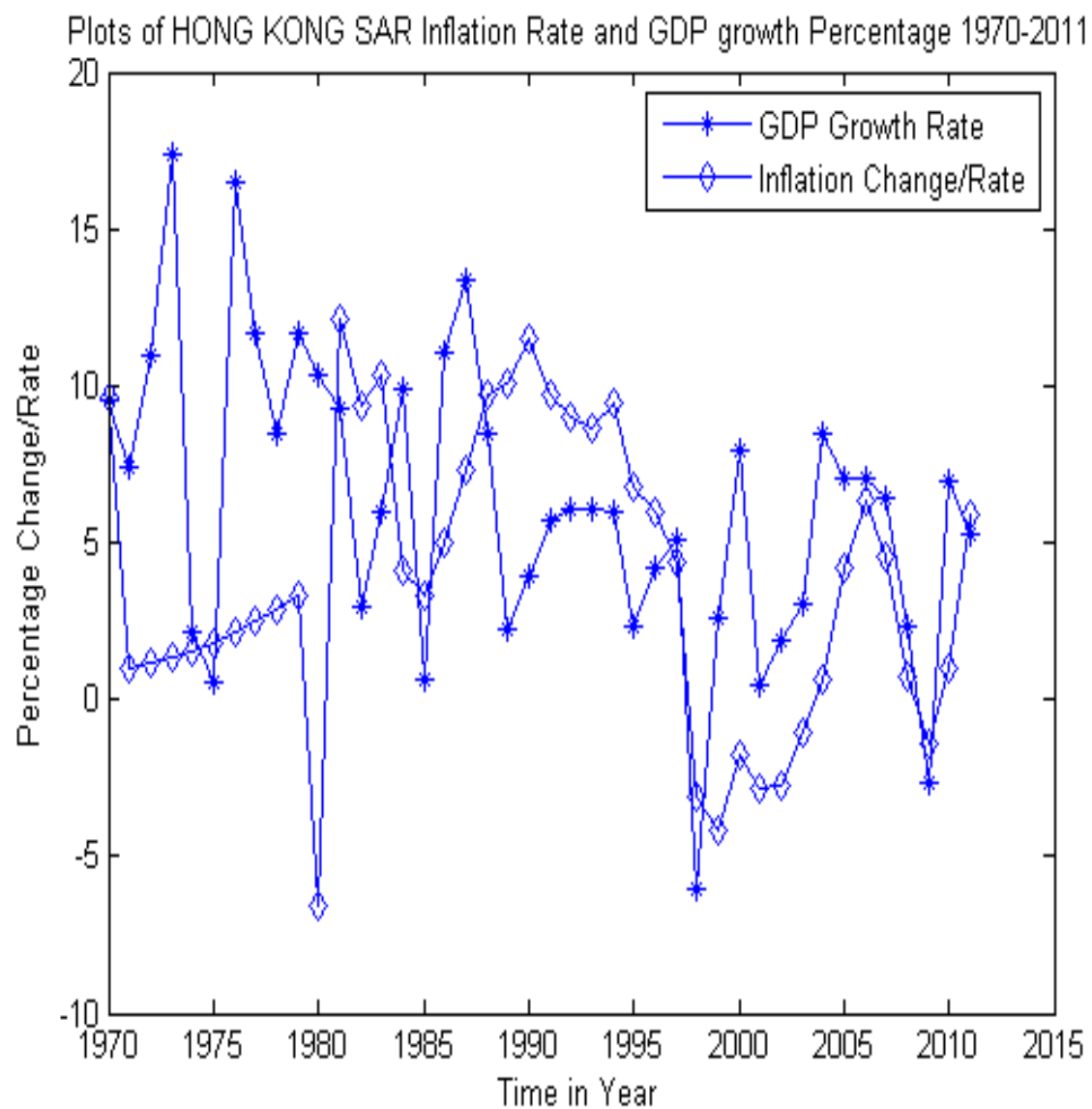
Comments: GDP rates are of unit digit with highest rate at about 7 percent. But Inflation rates ranged between 1 / 2 digit. We have a 2 digit mostly at the earlier period of the study whilst the other years stayed at unit digit level.

Figure 6.11



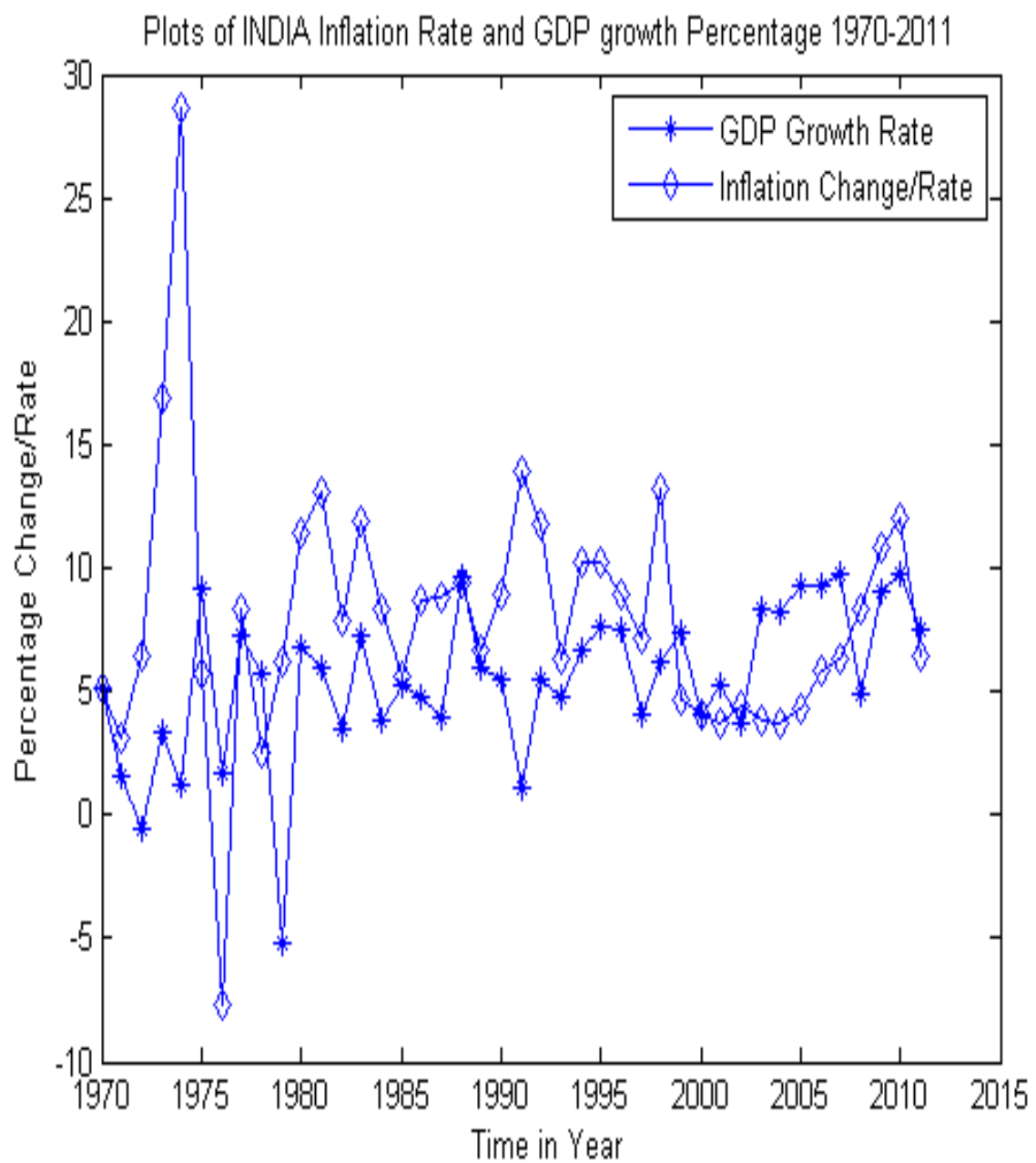
Comments: The percentages or rates of changes demonstrated fluctuations with a moderate value for the two variables in the sense that they are both of unit digit.

Figure 6.12



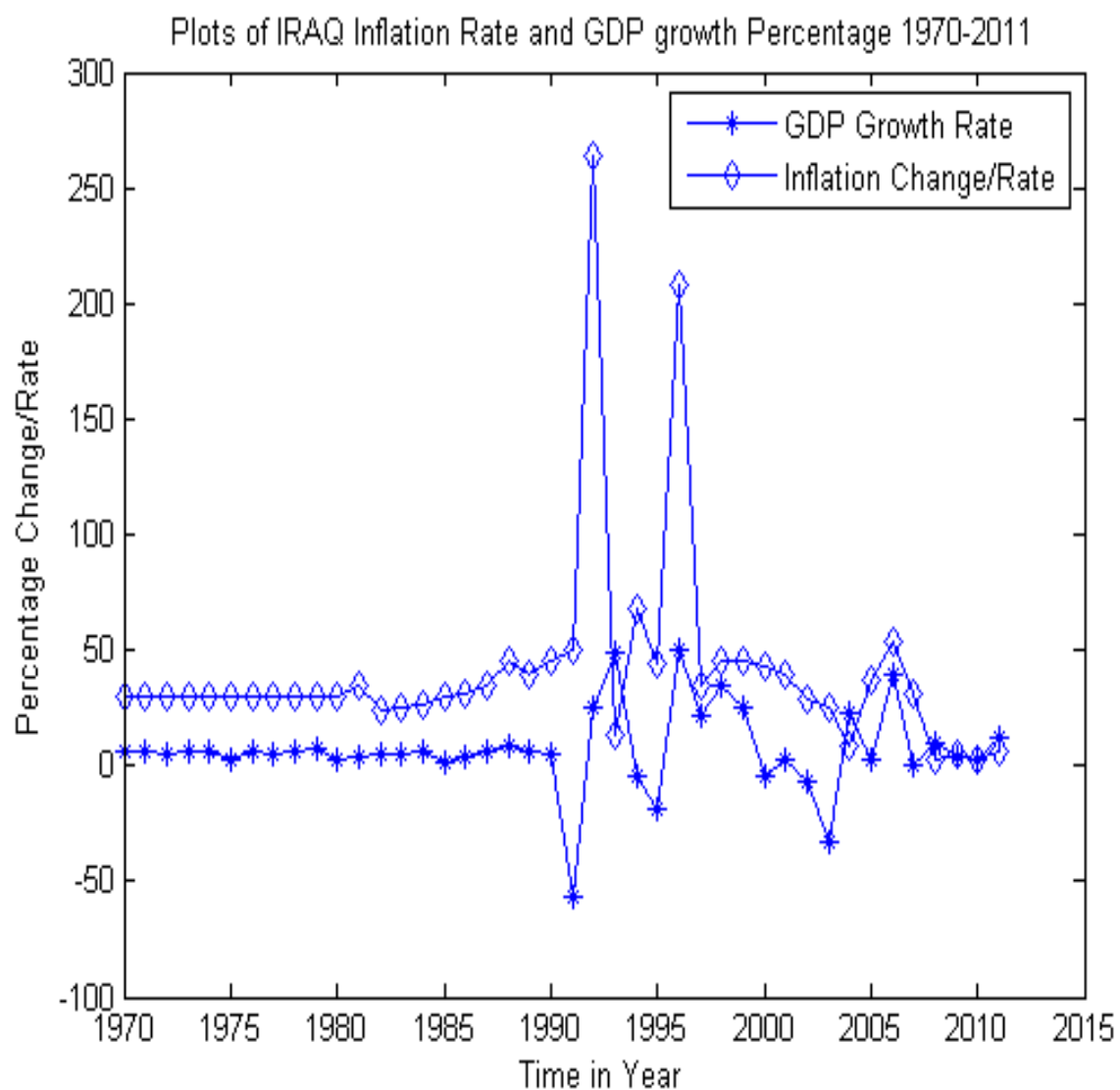
Comments: The GDP and Inflation rates shown 2 digits rate. Initially, from 1971 to 1980, the GDP rates were higher than the Inflation rates, but later the two variables were interchanging in terms of relationship movement.

Figure 6.13



Comments: In the country, the Inflation rates exhibit the range 1/ 2 digit' fluctuations; but for the GDP it is of unit digit with fairly regular fluctuations. By comparing the two, it is obvious that Inflation rates are higher than that of GDP.

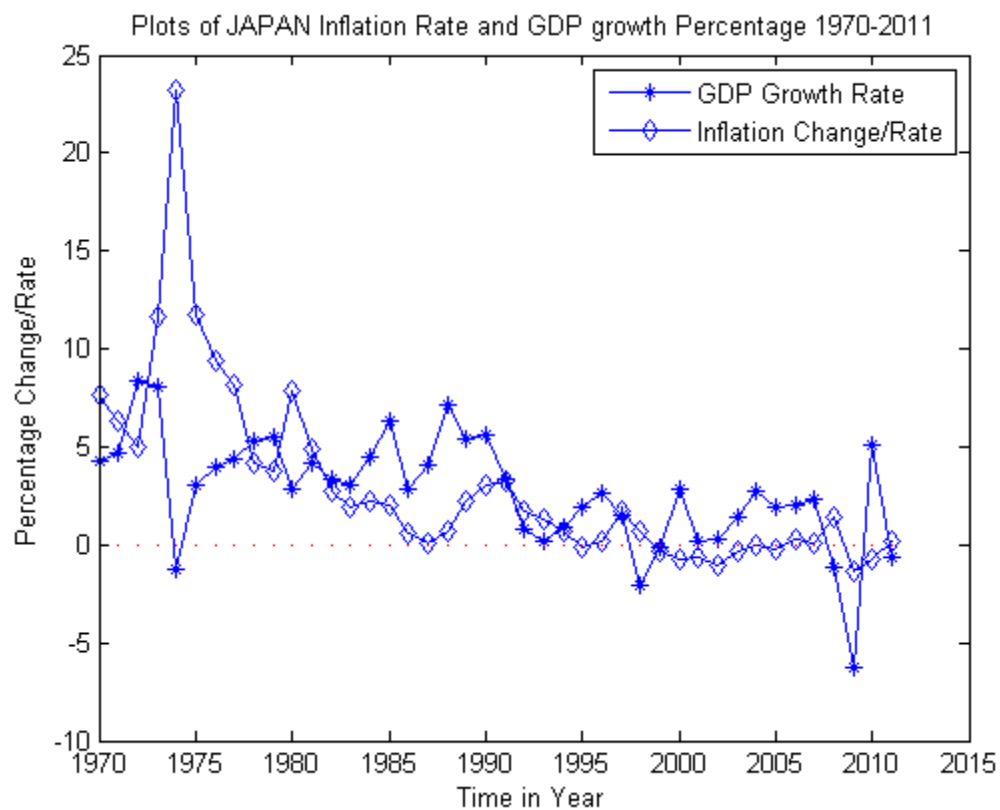
Figure 6.14



NB: Here, we have some negative percentages. Hence, we adhere to “Remark 1 on time series plot remark 1” (See Page 103).

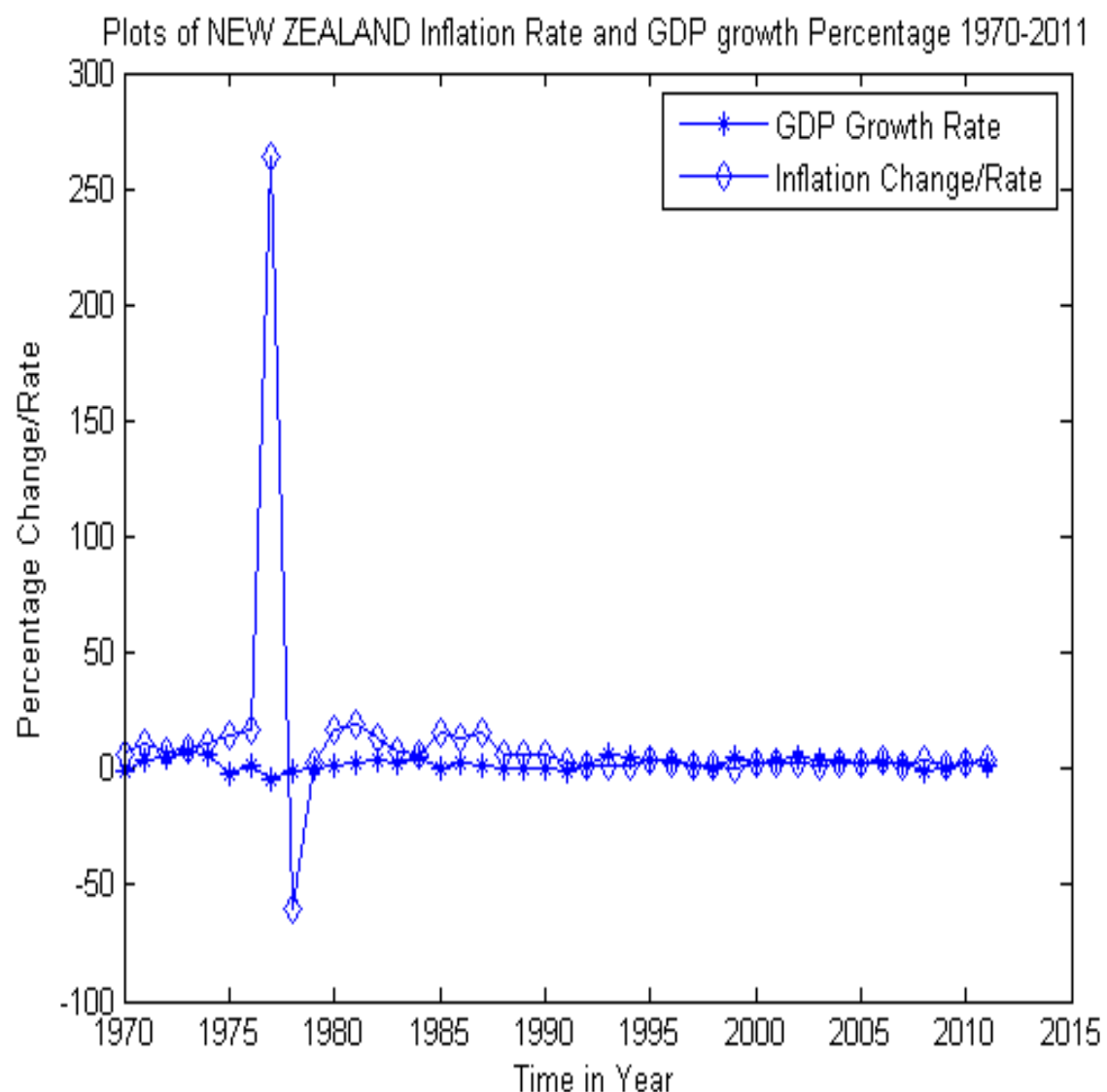
Comments: Inflation rates mostly fell in between 2 and 3 unit digits, while at about 5 points were of 1 unit digit. In addition, the rates in years 1992 and 1996 suggest the likelihood of outliers/structural breaks in Inflation. On GDP, the rates are between 1 and 2 unit digits. Even, we have about 6 negative rates for GDP. This shows how terrible the economic situation was for that country.

Figure 6.15



Comments: The GDP rates are of unit digit whilst that of Inflation rates are in between 1 and 2 digits range. By a closer look, one could see that Inflation rates are higher than that of GDP at earlier period but later GDP rates became higher. This is an indication of gradual economic improvement.

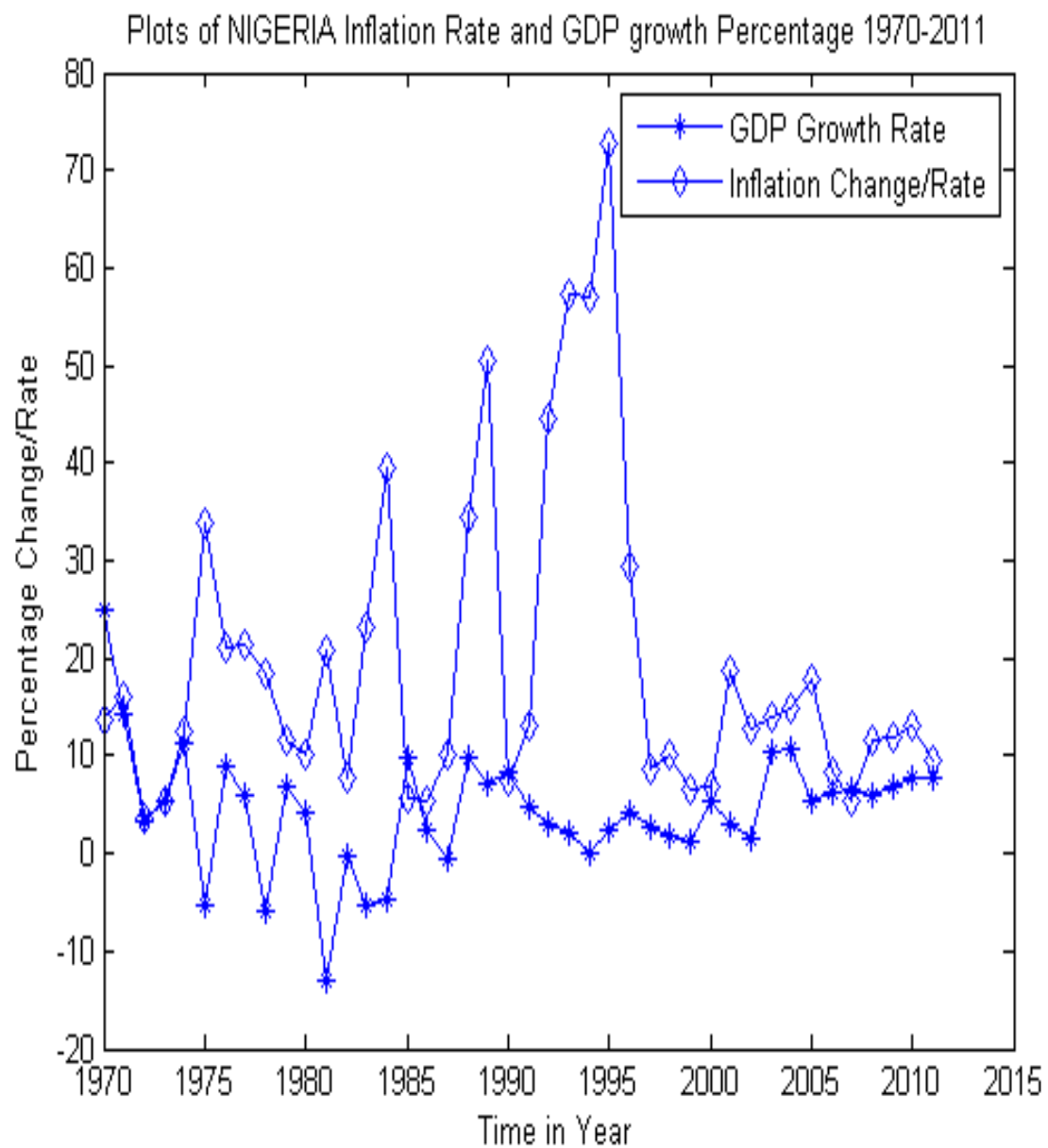
Figure 6.16



NB: Here, we have some negative percentages. Hence, we adhere to “Remark 1 on time series plots” (See Page 103).

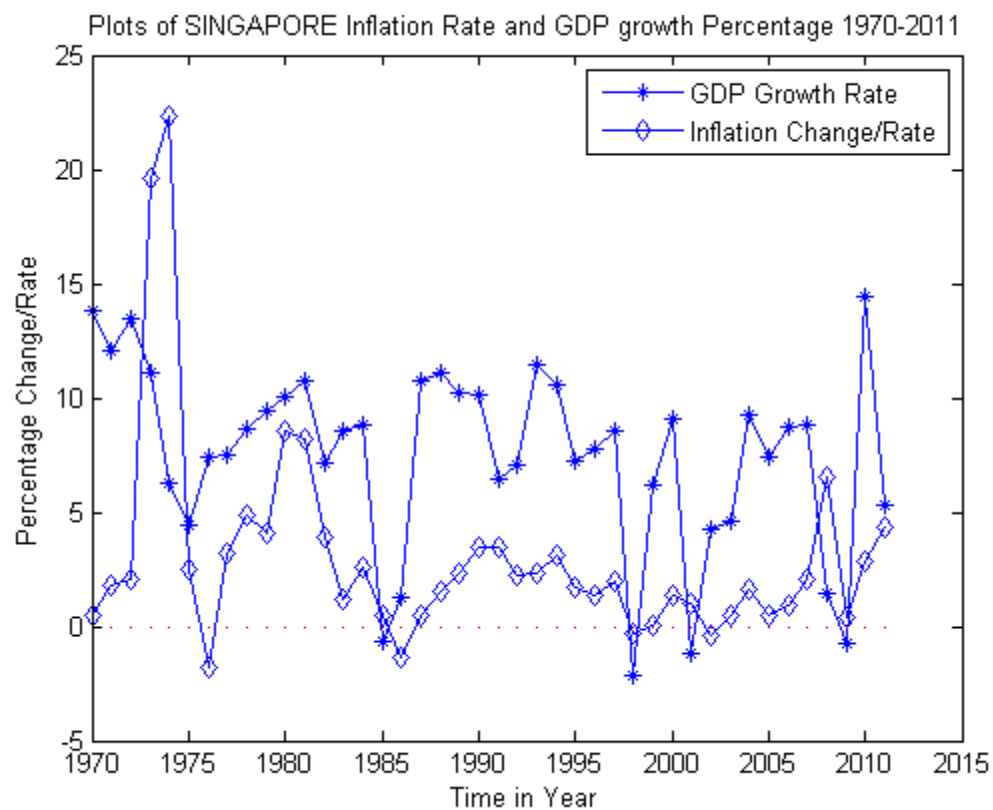
Comments: The Inflation rates of this country are in the range of 1 to 3 digits, while that of GDP rates are unit digit. The movement of the GDP is likely to be stationarity. However, Inflation rates in years 1977 and 1978 are outrageous which can lead to outliers/structural breaks in earlier part of the said series.

Figure 6.17



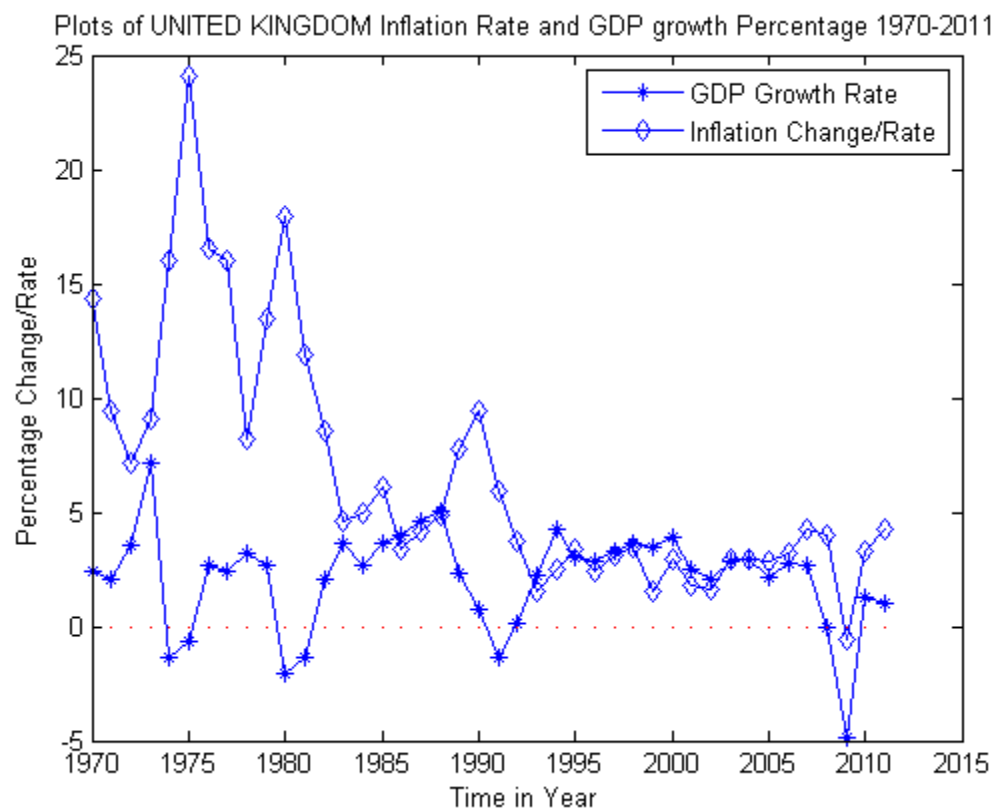
Comments: Both variables' rates are in 1 /2 digit range, but most of the GDP rates fall within the unit digit. It is also observed that the Inflation rates are on high side. It means inflation has dominant effects on that country.

Figure 6.18



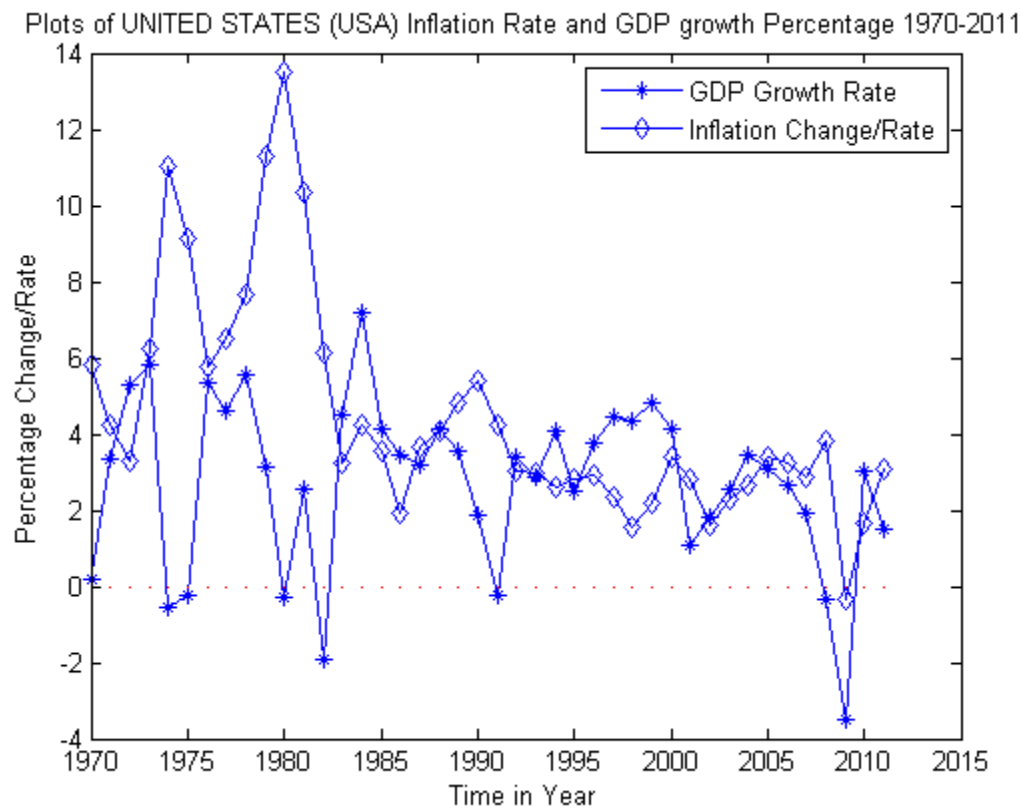
Comments: This is a country where the inflation rates are of unit digit with the exception of 1973 and 1974. The GDP rates are of 1 / 2 digit range. It is also noted that GDP rates are on high side when compare to Inflation.

Figure 6.19



Comments: GDP rates exhibit unit digit throughout the period of our study whilst 1 /2 unit range for the Inflation rates. The rates shown Inflation has dominant effects on the country with an exception of a few years. Irregular movements are much pronounced with Inflation.

Figure 6.20



Comments: The Inflation rates in this country are on the high side, most the time, when compare to the GDP. The highest rate is around 14 percent. The GDP maintained a unit digit range with fairly regular fluctuations.

In summary on all the time-plots' movements, we observed that variables were mostly characterized with irregular movements and displayed strong non-stationarity.

In order to authenticate the non-stationarity of the variables, the structural break(s)/outliers are investigated first. The results had shown that 65 countries are qualified for the study. The remaining 35 countries are not qualified and are affected by structural breaks and outliers.

See Appendix 3 for these countries with the factors causing their problems. We then carried out stationary tests, we conclude that the variables for the qualified countries are of integration order $I(0)$ and $I(1)$.

The ones with $I(1)$ are made stationary by differencing and de-trending before applying the Granger causality tests on the stationary variables.

Table 6.01 gives the results in terms of economic groupings, integration orders and type of Granger causality.

Table 6.01: Granger causality results on the percentage change of GDP and inflation classified by countries, economic groups and integration orders from 1970 to 2011

S/N (Col.1)	COUNTRY (Col.2)	ECONOMIC GROUP (Col.3)	INTEGRATION ORDER I(d) (Col.4)		S.S (Col.5)	TYPE OF GRANGER CAUSALITY (Col.6)
			GDP(y)	INF.(x)		
1	GERMANY	MAE	I(0)	I(1)	41	GDP \leftarrow INF
2	DENMARK	OAE	I(0)	I(1)	41	GDP \leftarrow INF
3	SWEDEN	OAE	I(0)	I(1)	41	GDP \leftarrow INF
4	AUSTRIA	EU	I(0)	I(1)	41	GDP \leftarrow INF
5	HUNGARY	EU	I(1)	I(1)	41	GDP \leftarrow INF
6	LUXEMBURG	EU	I(1)	I(1)	41	GDP \leftarrow INF
7	SPAIN	EU	I(1)	I(1)	41	GDP \leftarrow INF
8	CHILE	EADE	I(0)	I(1)	32	GDP \leftarrow INF
9	TUNISIA	EADE	I(0)	I(1)	41	GDP \leftarrow INF
10	BOTSWANA	EADE	I(1)	I(1)	41	GDP \leftarrow INF
11	BANGLADESH	OC	I(0)	I(1)	41	GDP \leftarrow INF
12	FIJI	OC	I(0)	I(1)	41	GDP \leftarrow INF
13	NEPAL	OC	I(0)	I(0)	42	GDP \leftarrow INF
14	ALGERIA	OC	I(0)	I(1)	41	GDP \leftarrow INF
15	JAPAN	MAE	I(0)	I(1)	41	GDP \rightarrow INF
16	AUSTRALIA	OAE	I(0)	I(1)	41	GDP \rightarrow INF
17	ICELAND	OAE	I(1)	I(1)	41	GDP \rightarrow INF

18	NEW ZEALAND	OAE	I(0)	I(1)	32	GDP \rightarrow INF
19	SWITZERLAND	OAE	I(0)	I(1)	41	GDP \rightarrow INF
20	NETHERLANDS	EU	I(0)	I(1)	41	GDP \rightarrow INF
21	INDIA	EADE	I(0)	I(1)	36	GDP \rightarrow INF
22	CHINA	EADE	I(1)	I(1)	41	GDP \rightarrow INF
23	IRAN	EADE	I(1)	I(1)	41	GDP \rightarrow INF
24	MALAYSIA	EADE	I(0)	I(0)	42	GDP \rightarrow INF
25	CANADA	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
26	FRANCE	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
27	ITALY	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
28	U.S.A	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
29	BELGIUM	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
30	FINLAND	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
31	GREECE	EU	I(1)	I(1)	41	GDP \leftrightarrow INF
32	PORTUGAL	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
33	ETHIOPIA	OC	I(0)	I(0)	42	GDP \leftrightarrow INF
34	U.K	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
35	CZECH REPUBLIC	OAE	I(0)	I(1)	41	GDP \leftrightarrow INF
36	HONG KONG SAR	OAE	I(0)	I(1)	41	GDP \leftrightarrow INF
37	* ISRAEL	OAE	I(1)	I(1)	24	GDP \leftrightarrow INF
38	KOREA (SOUTH)	OAE	I(0)	I(1)	41	GDP \leftrightarrow INF
39	NORWAY	OAE	I(1)	I(1)	41	GDP \leftrightarrow INF

40	SINGAPORE	OAE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
41	TAIWAN PROVINCE	OAE	I(1)	I(0)	41	GDP $\leftarrow//\rightarrow$ INF
42	CYPRUS	EU	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
43	IRELAND	EU	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
44	MALTA	EU	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
45	SOUTH AFRICA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
46	EGYPT	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
47	KENYA	EADE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
48	KUWAIT	EADE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
49	NIGERIA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
50	PAKISTAN	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
51	SAUDI ARABIA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
52	THAILAND	EADE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
53	UNITED ARAB	EADE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
54	VIETNAM	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
55	SRI LANKA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
56	LIBYA	EADE	I(1)	I(1)	40	GDP $\leftarrow//\rightarrow$ INF
57	TRINIDAD & TOB.	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
58	CAMBODIA	OC	I(1)	I(0)	41	GDP $\leftarrow//\rightarrow$ INF
59	TONGA	OC	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
60	BARBADOS	OC	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
61	COLOMBIA	OC	I(I)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF

62	PARAGUAY	OC	I(1)	I(1)	41	GDP \nleftrightarrow INF
63	JORDAN	OC	I(1)	I(1)	41	GDP \nleftrightarrow INF
64	MOROCCO	OC	I(0)	I(1)	41	GDP \nleftrightarrow INF
65	BURKINA FASO	OC	I(0)	I(0)	42	GDP \nleftrightarrow INF

*Indicates that earlier part of Israel data is outliers/structural break; but the remaining part (about half) is not.

KEY: MAE = Major Advanced Economies; OAE = Other Advanced Economies; EU = European Union (excluding advanced economies); EADE = Emerging & Developing Economies; OC = Other Countries; S.S = Sample Size; GDP \leftarrow INF = Inflation “Granger causes” GDP; GDP \rightarrow INF = GDP Granger causes Inflation; GDP \leftrightarrow INF = Both ways Granger cause; GDP \nleftrightarrow INF = Neither Granger cause.

Table 6.01 is further broken into Tables 6.01(a), 6.01(b), 6.01(c) and 6.01(d) respectively for the types of Granger causality with the associated countries.

Table 6.01(a): Table of inflation “Granger causes” GDP growth on the percentage change of GDP and inflation, classified by countries, economic groups and integration orders from 1970 to 2011

S/N (Col.1)	COUNTRY (Col.2)	ECONOMIC GROUP (Col.3)	INTEGRATION ORDER I(d) (Col.4)		S.S (Col.5)	TYPE OF GRANGER CAUSALITY (Col.6)
			GDP(y)	INF.(x)		
1	GERMANY	MAE	I(0)	I(1)	41	GDP \leftarrow INF
2	DENMARK	OAE	I(0)	I(1)	41	GDP \leftarrow INF
3	SWEDEN	OAE	I(0)	I(1)	41	GDP \leftarrow INF
4	AUSTRIA	EU	I(0)	I(1)	41	GDP \leftarrow INF
5	HUNGARY	EU	I(1)	I(1)	41	GDP \leftarrow INF
6	LUXEMBURG	EU	I(1)	I(1)	41	GDP \leftarrow INF
7	SPAIN	EU	I(1)	I(1)	41	GDP \leftarrow INF
8	CHILE	EADE	I(0)	I(1)	32	GDP \leftarrow INF
9	TUNISIA	EADE	I(0)	I(1)	41	GDP \leftarrow INF
10	BOTSWANA	EADE	I(1)	I(1)	41	GDP \leftarrow INF
11	BANGLADESH	OC	I(0)	I(1)	41	GDP \leftarrow INF
12	FIJI	OC	I(0)	I(1)	41	GDP \leftarrow INF
13	NEPAL	OC	I(0)	I(0)	42	GDP \leftarrow INF
14	ALGERIA	OC	I(0)	I(1)	41	GDP \leftarrow INF

KEY: MAE = Major Advanced Economies; OAE = Other Advanced Economies; EU = European Union (excluding advanced economies); EADE = Emerging & Developing Economies; OC = Other Countries; S.S = Sample Size; $GDP \leftarrow INF$ = Inflation Granger causes GDP.

Comments: From the table, we have the following economy grouping summary:

MAE=1, OAE=2, EU=4, EADE=3, and OC=4.

The MAE and OAE of developed economies had smaller figures than the other groupings. This implied that the concept of “Inflation lead GDP” did not have much impact on the developed economies.

Table 6.01(b): Table of GDP “Granger causes” inflation results on the percentage change of GDP and inflation, classified by countries, economic groups and integration orders from 1970 to 2011

S/N (Col.1)	COUNTRY (Col.2)	ECONOMIC GROUP (Col.3)	INTEGRATION ORDER I(d) (Col.4)		S.S (Col.5)	TYPE OF GRANGER CAUSALITY (Col.6)
1	JAPAN	MAE	I(0)	I(1)	41	GDP → INF
2	AUSTRALIA	OAE	I(0)	I(1)	41	GDP → INF
3	ICELAND	OAE	I(1)	I(1)	41	GDP → INF
4	NEW ZEALAND	OAE	I(0)	I(1)	32	GDP → INF
5	SWITZERLAND	OAE	I(0)	I(1)	41	GDP → INF
6	NETHERLANDS	EU	I(0)	I(1)	41	GDP → INF
7	INDIA	EADE	I(0)	I(1)	36	GDP → INF
8	CHINA	EADE	I(1)	I(1)	41	GDP → INF
9	IRAN	EADE	I(1)	I(1)	41	GDP → INF
10	MALAYSIA	EADE	I(0)	I(0)	42	GDP → INF

KEY: MAE = Major Advanced Economies; OAE = Other Advanced Economies; EU = European Union (excluding advanced economies); EADE = Emerging & Developing Economies; OC = Other Countries; S.S = Sample Size; GDP → INF = GDP Granger causes Inflation.

Comments: The following summary statistics of the groupings are: MAE= 1, OAE= 4, EU= 1, and EADE= 4.

From these figures, one can see that the chance of “GDP lead Inflation “ is higher in developed economies than the developing economies. It implies that GDP can be used to enhance better prediction of Inflation in the affected countries of developed economies.

Also, the affected countries in EADE are countries with strong GDP.

Table 6.01(c): Table of bi-directional Granger causality results on the percentage change of GDP and inflation, classified by countries, economic groups and integration orders from 1970 to 2011

S/N (Col.1)	COUNTRY (Col.2)	ECONOMIC GROUP (Col.3)	INTEGRATION ORDER I(d) (Col.4)		S.S (Col.5)	TYPE OF GRANGER CAUSALITY (Col.6)
1	CANADA	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
2	FRANCE	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
3	ITALY	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
4	U.S.A	MAE	I(0)	I(1)	41	GDP \leftrightarrow INF
5	BELGIUM	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
6	FINLAND	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
7	GREECE	EU	I(1)	I(1)	41	GDP \leftrightarrow INF
8	PORTUGAL	EU	I(0)	I(1)	41	GDP \leftrightarrow INF
9	*+ ETHIOPIA	OC	I(0)	I(0)	42	GDP \leftrightarrow INF

*+ indicates our result in Ethiopia is compared with the result of Girma (2012) paper. Go to next page for the outcome of this comparison.

KEY: MAE = Major Advanced Economies; OAE = Other Advanced Economies; EU = European Union (excluding advanced economies); EADE = Emerging & Developing Economies; OC = Other Countries; S.S = Sample Size; GDP \leftrightarrow INF = Both ways Granger cause.

Comments: In the groupings, we have: MAE= 4, EU= 4, and OC= 1.

From these figures, it is evident that bi- directional Granger causality had upper hand in the developed economies (MAE and EU) than developing economies. By implication, either of the two variables can be used to predicate better in the developed economies.

Outcome of the comparison on *+

From Grima's paper, we observed that the paper covered 1980- 2011, while our own covered 1970 to 2011. Even on the data utilized by both of us, there is a difference in the figures. For instance, Grima used Ethiopia currency (Birr in billion) for GDP whilst we used USA dollar (in millions); and for inflation, there is a significant difference with ours on the high side.

For the analysis, Engle-Granger cointegration (an ordinary least square based) and the standard Granger causality were utilized by us, while Grima used Johansen method of cointegration (vector autoregression based). Our result supported bidirectional causality between GDP and inflation whilst Grima result supported unidirectional from GDP to inflation. It is our view that the difference can be due to difference in data and the period covered. Hence, the issue of robustness on our result cannot be discussed because of the stated differences in data and period.

Table 6.01(d): Table of Non- Granger causality results on the percentage change of GDP and inflation classified by countries, economic groups and integration orders from 1970 to 2011

S/N (Col.1)	COUNTRY (Col.2)	ECONOMIC GROUP (Col.3)	INTEGRATION ORDER I(d) (Col.4)		S.S (Col.5)	TYPE OF GRANGER CAUSALITY (Col.6)
1	U.K	MAE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
2	CZECH REPUBLIC	OAE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
3	HONG KONG SAR	OAE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
4	* ISRAEL	OAE	I(1)	I(1)	24	GDP $\leftarrow//\rightarrow$ INF
5	KOREA (SOUTH)	OAE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
6	NORWAY	OAE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
7	SINGAPORE	OAE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
8	TAIWAN PROVINCE	OAE	I(1)	I(0)	41	GDP $\leftarrow//\rightarrow$ INF
9	CYPRUS	EU	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
10	IRELAND	EU	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
11	MALTA	EU	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
12	SOUTH AFRICA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
13	EGYPT	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
14	KENYA	EADE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF

15	KUWAIT	EADE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
16	*++ NIGERIA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
17	**+ PAKISTAN	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
18	SAUDI ARABIA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
19	THAILAND	EADE	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
20	UNITED ARAB	EADE	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
21	VIETNAM	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
22	SRI LANKA	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
23	LIBYA	EADE	I(1)	I(1)	40	GDP $\leftarrow//\rightarrow$ INF
24	TRINIDAD & TOB.	EADE	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
25	CAMBODIA	OC	I(1)	I(0)	41	GDP $\leftarrow//\rightarrow$ INF
26	TONGA	OC	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF
27	BARBADOS	OC	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
28	COLOMBIA	OC	I(I)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
29	PARAGUAY	OC	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
30	JORDAN	OC	I(1)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
31	MOROCCO	OC	I(0)	I(1)	41	GDP $\leftarrow//\rightarrow$ INF
32	BURKINA FASO	OC	I(0)	I(0)	42	GDP $\leftarrow//\rightarrow$ INF

*Indicates that earlier part of Israel data is outliers/structural break; but the remaining part (about half) is not. It is the second part that being utilized.

*++ and **+ indicating the comparison of results from Nigeria and Pakistan respectively using Inyiama (2013), and Ahmad and Joyia (2012) papers. See the result on the next page.

KEY: MAE = Major Advanced Economies; OAE = Other Advanced Economies; EU = European Union (excluding advanced economies); EADE = Emerging & Developing Economies; OC = Other Countries; S.S = Sample Size; GDP \leftrightarrow INF = Neither Granger cause.

Comments: The summary break down of the table gives: MAE=1, OAE= 7, EU= 3, EADE=13, and OC= 8.

The issue of non- Granger causality is highly pronounced with the EADE and followed by OC. The two groups belong to developing economy. It means the non- Granger causality had higher impact on the developing economies.

Comments on the Papers:

- (i) Comment on Inyama paper: The paper covered 1979 to 2011 whilst we covered 1970 -2011. The data source is the same. On the method of analysis, the author utilized Johansen, while we used the Engle-Granger cointegration method and the standard Granger causality methods. The results are the same for Nigeria with no Granger causality between GDP and inflation. Hence, the robustness of our method and results are upheld when compared to Inyama paper.
- (ii) Ahmad and Joyia paper: The authors covered 1971 to 2011 whilst we covered 1970 to 2011. The data source of the authors is from the World Development Indicator (WDI), International Financial Statistics (IFS) and Economic Survey of Pakistan. But our data source does not include the Economic Survey of Pakistan. We google the said Economic survey of Pakistan, no tangible data can be found. All the data are given in various economic breakdowns. By this non availability of data from Economy survey of Pakistan, we cannot carry out any comparisons.

Further groupings and classifications are made on the Granger causality results leading to the creation of tables 6.02 to 6.06.

The titles of the tables are:

Table 6.02 – Distribution of Granger causality and Non-Granger causality according to economic groupings. See the table below:

**TABLE 6.02: DISTRIBUTION OF G.CASUALITY AND NON-G.CASUALITY
ACCORDING TO ECONOMIC GROUPINGS (EXTRACTED FROM TABLE 6.01).**

G.Casuality/ Non - G.Casuality	Economic Groupings				TOTAL
	Adv. Econ	EU	Developing	Others	
Case1 (Inf \longrightarrow GDP)	3	4	3	4	14
Case2 (GDP \longrightarrow Inf)	5	2	4	-	11
Case3 (GDP \longleftrightarrow Inf)	4	3	-	1	8
Case 4 (GDP \longleftrightarrow Inf)	8	3	13	8	32
TOTAL	20	12	20	13	65
G.Casuality	12	9	7	5	33
Non- G.Casuality	8	3	13	8	32

The following tables (6.03 to 6.06) are extracted from Table 6.02 for the purpose of some statistical analyses.

Table 6.03: –Classification into Granger causality and Non-Granger causality.

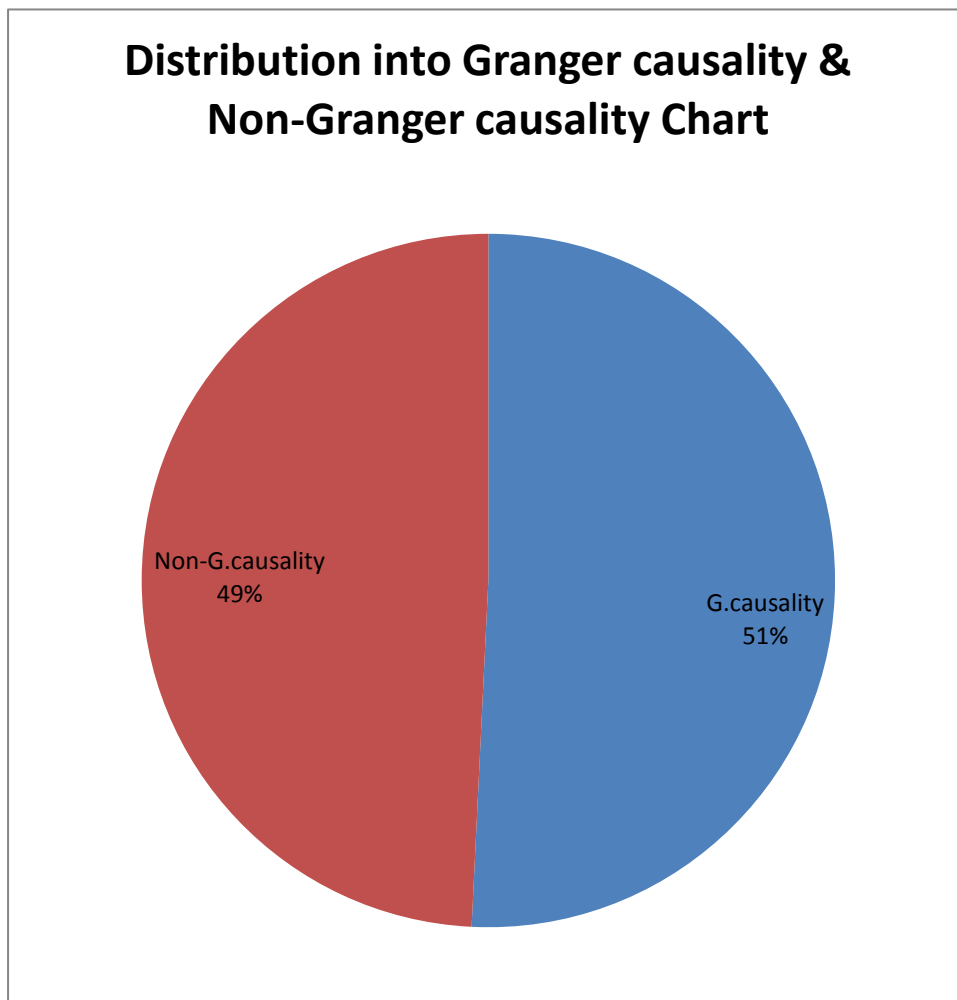
See the following table:

TABLE 6.03: CLASSIFICATION INTO GRANGER CASUALITY AND NON-GRANGER CASUALITY.

Granger causality	Non-Granger causality	TOTAL
33	32	65

Table 6.03 can be shown in the following chart:

Figure 6.21



For the purpose of having a statistical decision on this table, let test the equality of these proportions.

Let P_1 = proportion of Granger causality = $33/65 = 0.5077$ and

$q = (1 - p_1)$ = Porportion of non-Granger causality = $1 - 0.5077 = 0.4923$.

By Normal approximation to Binomial, we have

$\hat{\sigma}(p) = \pi = 0.5$ (i.e half-half equality);

$$\hat{\sigma}(p) = \sqrt{\frac{0.5(0.5)}{65}} = 0.0620$$

$H_0 : p_1 = \pi$ vs $H_1 : p_1 \neq \pi$

$$Z = \frac{P_1 - \pi}{\hat{\sigma}(p)} = \frac{0.5077 - 0.5000}{0.0620} = 0.1242$$

Normal table at 5% sign level = 1.96(two-tailed).

Decision: Since $0.1242 < 1.96$, H_0 is accepted and there is equality between the two. Hence, it is a 50-50 probability of Granger causality and Non-Granger causality.

Table 6.04 – Distribution of Granger causality into types.

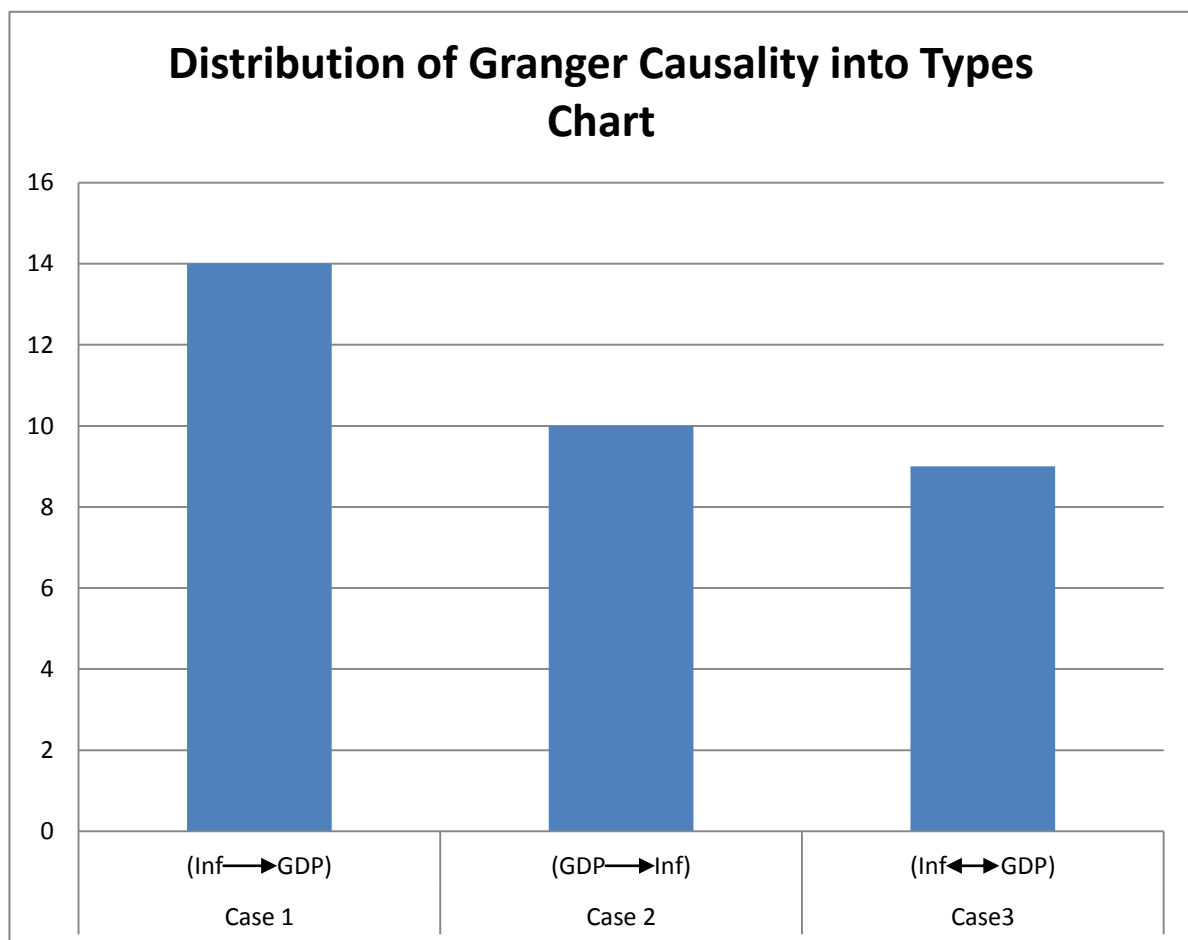
The following figure depicts the table:

TABLE 6.04: DISTRIBUTION OF GRANGER CAUSALITY INTO TYPES

	Case 1 (Inf \rightarrow GDP)	Case 2 (GDP \rightarrow Inf)	Case3 (Inf \leftrightarrow GDP)	TOTAL
Granger causality	14	10	9	33

The following Bar Chart shows the figures in the table:

Figure 6.22



Here, we want to test the independence of these figures. Categorical data test will be carried out using Chi-square statistics.

Test hypothesis:

H_0 : Number of Granger causality results is not biased and independently distributed.

H_1 : Number of Granger causality results is biased and dependently distributed.

The test statistics is

$$\chi^2 = \sum \frac{(o-e)^2}{e}$$

Where o, (the observed) is the number of Granger causality in each cell; while e (the expected) is obtained from the assumption of uniform distribution.

Cases	O	e.	$o - e$	$(o - e)^2$	$\frac{(o - e)^2}{e}$
1	14	11	3	9	$\frac{9}{11}$
2	10	11	-1	1	$\frac{1}{11}$
3	9	11	-2	4	$\frac{4}{11}$
TOTAL					$\frac{14}{11} = 1.2727$

χ^2 table at $\alpha = 0.05$ is 7.815

Since $1.2727 < 7.815$, we accept H_0 and conclude that the result in the table are not biased and being independently distributed.

Hence, we can say the chance of Case 1 (with probability $\frac{14}{33}$) is the highest and then followed by Case 2 and Case 3 respectively.

Table 5 – Classification of Granger causality into developed and developing economies.

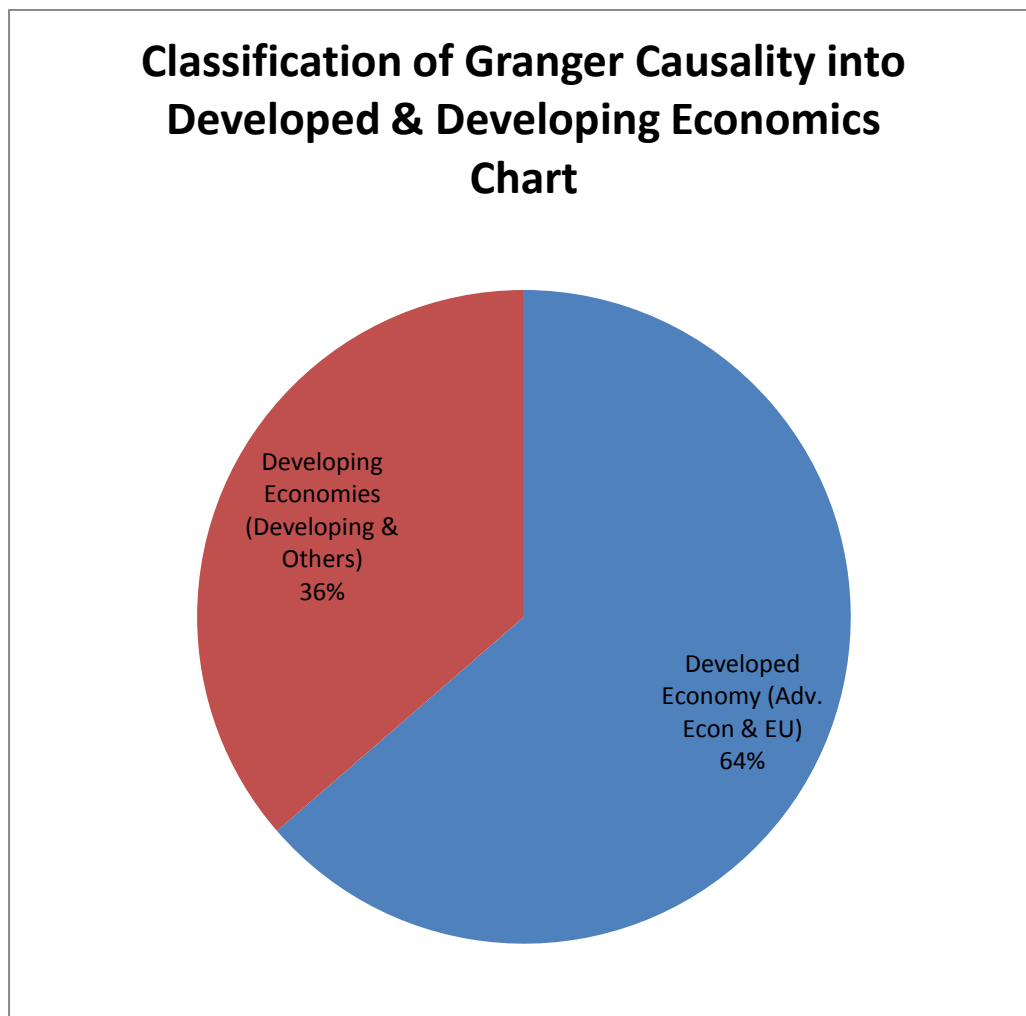
The table follows:

TABLE 6.05: CLASSIFICATION OF GRANGER CAUSALITY INTO DEVELOPED AND DEVELOPING ECONOMIES.

	Developed Economies (Adv. Econ & EU)	Developing Economies (Developing & Others)	TOTAL
Granger causality	21	12	33

See the pie chart for the table below:

Figure 6.23



The test on equalities of Developed and Developing economies is considered here. By this, ratio 50 to 50 should be maintained. Hence, proportion $\pi = 0.5$.

$$H_0 : \hat{p} = \pi (=0.5) \text{ vs } H_1 : \hat{p} > \pi; \quad \left(\hat{p} = \frac{x}{n} \right)$$

With Normal approximation to Binomial,

$$\text{Mean } \mu = n\pi = 33 \times 0.5 = 16.5$$

$$\text{Standard deviation} = \sqrt{n\pi(1-\pi)} = 33 \times 0.5 \times 0.5 = \sqrt{8.25} = 2.8723$$

By continuity principle in Normal Approximation to Binomial, we have

$$Z = \frac{x - n\pi}{\sqrt{n\pi(1-\pi)}}$$

$$\text{with } x = 21, Z = \frac{21.5 - 16.5}{2.8723} = 1.7408$$

Normal table at 5% sign level = 1.65 (one-tailed).

Decision: since $1.7408 > 1.65$, H_0 is rejected. Hence, there is difference between developed and developing economies.

Table 6.06 – Distribution of developed and developing economies into types of the Granger causality.

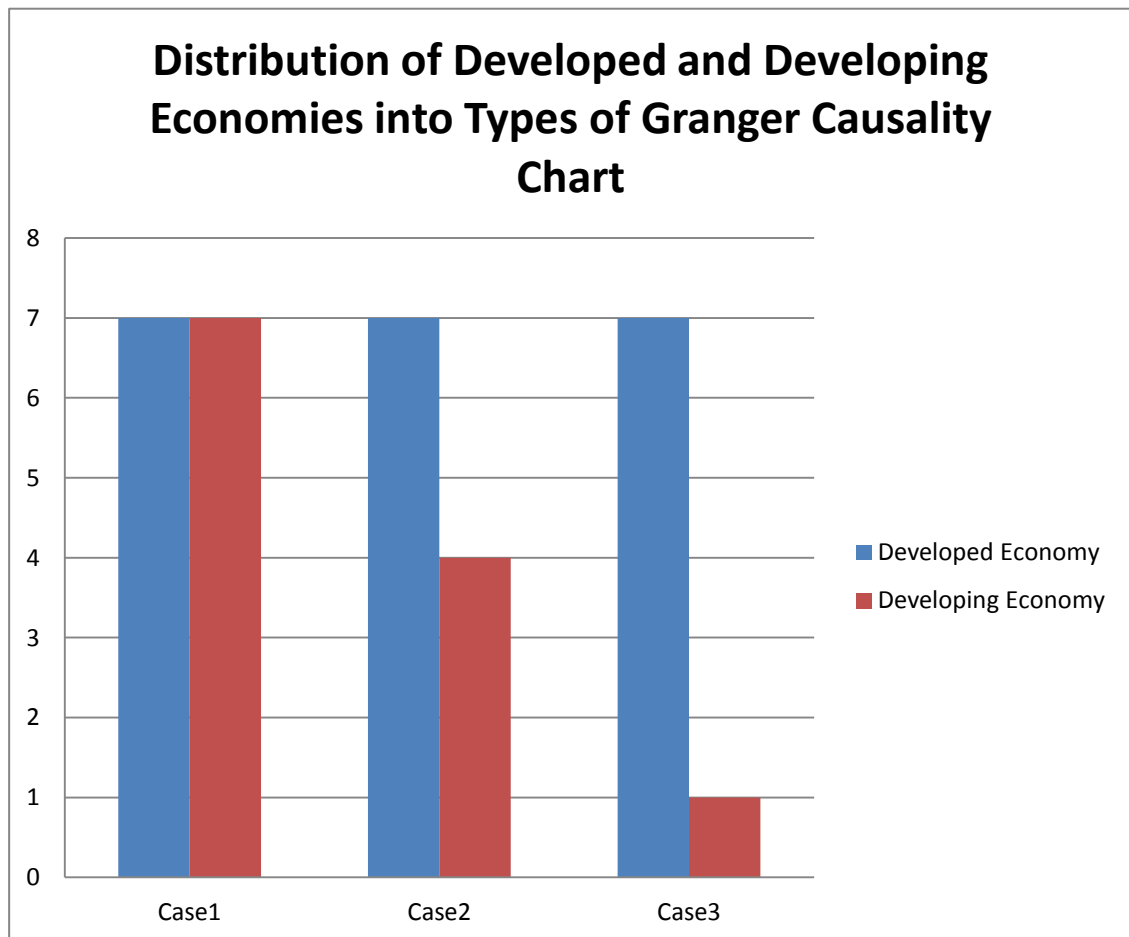
The following table gives the distribution:

TABLE 6.06 : DISTRIBUTION OF DEVELOPED AND DEVELOPING ECONOMIES INTO TYPES OF GRANGER CAUSALITY

Case	Type of Granger causality	Developed Economies	Developing Economies	Total
Case 1	GDP \longleftarrow Inf	7	7	14
Case 2	GDP \longrightarrow Inf	7	4	11
Case 3	GDP \longleftrightarrow Inf	7	1	8
	Total	21	12	33

The following chart shows the distribution:

Figure 6.24



NB: Cases 1 to 3 are similarly defined as in Table 6.06. The first bar in each case represents the Developed Economies while the second bar is for the Developing Economies.

Observations:

- (1) In developed economies, the distribution of the Granger causality is evenly or uniformly distributed. To check this, a Chi-Square test of uniformly distributed will be carried out. See Test 6.1 below.

(2) The developing economy tends to support inflation “Granger causes” GDP most frequently. In order to confirm this, the test of equality is to be carried out. Test 6.2 below is utilised.

Test 6.1: For Developed Economies

Here, we want to test the uniformity of frequencies on the type / pattern of the developed economies in terms of Granger causality using Chi-Square test.

Test hypothesis:

H_0 : The frequencies of Granger causality results in the 3 cases are evenly or uniformly distributed for developed economies.

H_1 : The frequencies of Granger causality results in the 3 cases are not uniformly distributed for developed economies.

The test statistics is

$$\chi^2 = \sum \frac{(o-e)^2}{e}$$

where o (the observed) is the number of Granger causality in each cell; while e (the expected) is obtained from the assumption of uniform distribution.

See the computational table below:

Cases	O	e.	O – e	(o – e) ²	$\frac{(o - e)^2}{e}$
1	7	7	0	0	$\frac{0}{7}$
2	7	7	0	0	$\frac{0}{7}$
3	7	7	0	0	$\frac{0}{7}$
TOTAL	21	21			$\frac{0}{7} = 0.0000$

χ^2 table at $\alpha = 0.05 = 5.991$

Decision: Since $0.0000 < 5.991$, we accept H_0 and conclude that the results on the pattern of Granger causality for developed economies are uniformly distributed.

Test 6.2: For Developing Economies

We are to carry out this test by utilising the Binomial Distribution in order to obtain the probabilities (because the sample size is not large). Case 1 (GDP \leftarrow Inf) is suspected to be most frequently occurring among the 3 cases. Test of equality is to be applied.

Case 1	Case 2	Case 3	TOTAL
GDP \leftarrow Inf	GDP \longrightarrow Inf	GDP \longleftrightarrow Inf	
No (P_1)	No (P_2)	No (P_3)	No ($P_1+P_2+P_3$)
7 (0.5833)	4 (0.3333)	1 (0.0834)	12 (1.0000)

Where P_i ($i=1,2,3$) are the probabilities of the 3 cases respectively.

Since we are interested in P_1 alone, the equality test between P_1 and the other cases (combined) will be carried out. By this, equality test between P_1 and $P^* = 0.5$ is conducted as follows:

Hypothesis	P Value from Cumulative probabilities of Binomial Dist.	Decision at 5% significant level
$H_0 = P^* \leq P_1$ vs $H_1 = P^* > P_1$	P= 0.3872	H_0 is accepted. It implied P_1 is greater than P^* (which is 0.5).

Decision: Case 1 is greater than the other cases (combined). Hence, Case 1 has highest frequency.

Remark: By using Tests 6.1 and 6.2, we conclude that the Granger causality distribution is uniformly distributed for the developed economies whilst it skewed to case1 (GDP \leftarrow Inf) for developing economies.

Appropriate tests were carried out on Tables 6.03 to 6.06 into order to have clear and well defined statistical inferences on relationships.

6.1.2 Interpretations on Phase 1

This sub-section gives the interpretations on our findings in sub-section 6.1.1.

Findings on the time-plots indicated irregular movements of the variables. This is an indication of fluctuations on the variables. It supports the Keynesians' belief of existence of the business circle and fluctuations in macro-variables. We find out factors causing the problem of structural breaks and others in those countries to include wars, reforms in economic policies and currency, excessive liquidations, change of government, economic recessions, default in external debt, recession, but to mention a few.

The stationary tests involving structural breaks/outliers, unit roots and trends test, were carried out. The ones which have structural breaks cannot be amended were dropped. See Appendix 2 for the set of countries.

We also carried out stationary tests on the countries which are freed from structural breaks/outliers and the outcomes gave the integration order $I(0)$ (stationary) and $I(1)$ (non-stationary). It means some variables are stable $I(0)$ while others are not. See Table 6.01 column 2 for integration orders.

Next, we applied the Matlab programme for the Granger causality test (written to our own specification) to the said stable variables.

The outcomes of the Granger causality test are summarized with their respective interpretations thus:

- (i) 14 countries supported inflation “Granger causes” GDP; i.e. $GDP \longleftarrow Inflation$. See Table 6.01(a). This can be interpreted as inflation led GDP, which means inflation can enhance the prediction of GDP in those countries;
- (ii) 10 countries supported GDP led inflation; i.e. $GDP \longrightarrow Inflation$. See Table 6.01(b). It can be also be interpreted as GDP can cause better or enhance predication of Inflation for those 10 countries;
- (iii) another group of 9 countries supported bi-directional Granger causality; i.e $GDP \longleftrightarrow Inflation$.

See Table 6.01(c). This is interpreted as either of the variables can be used to enhance the prediction of the other;

(iv) lastly, 32 countries supported non-Granger causality with the variables;

i.e. GDP \longleftrightarrow Inflation. See Table 6.01(d). Neither of the two variables can enhance the prediction of the other. It implied the variables are exogenous.

Tables 6.02 to 6.06 presented the results in Section 6.1.1 are being utilised to accomplish the following tests and make the decision on:

- equality chance of Granger causality and non-Granger causality. Table 6.03 is used. Here, the test of normal approximation to binomial is utilized. Our test outcome or decision is equality chance of the two at 5% significance level. That is, the chance is 50 – 50 for Granger causality and non-Granger causality;
- the Case 1 (GDP \leftarrow Inf) of Table 4, has the highest chance in the Granger causality.

A categorical test using chi-square statistic is applied and the test supported Case 1 with the highest probability. It implies inflation “Granger causes” GDP occurred most often than the others;

- there exists difference between developed and developing economies in terms of the Granger causality. Table 6.05 is used with an application of Normal approximation to Binomial. The test supported existence of differences between developed and developing economies in terms of Granger causality. The developed economy has better support for Granger causality concept than the developing economy on the basis of the ratio being 21:12; and
- also in Table 6.06, the pattern of Granger causality is uniformly distributed for developed economies (i.e equal chance of having the 3 types of G.Causality) whilst skewed towards inflation “Granger causes” GDP for developing economies. One can attribute the test result to an indication of more stability of the developed economy variables, and the pattern of Granger causality for developing economies indicates inflation inducing GDP for most of its countries. Or in another way, one can say inflation induces GDP growth most in developing economies.

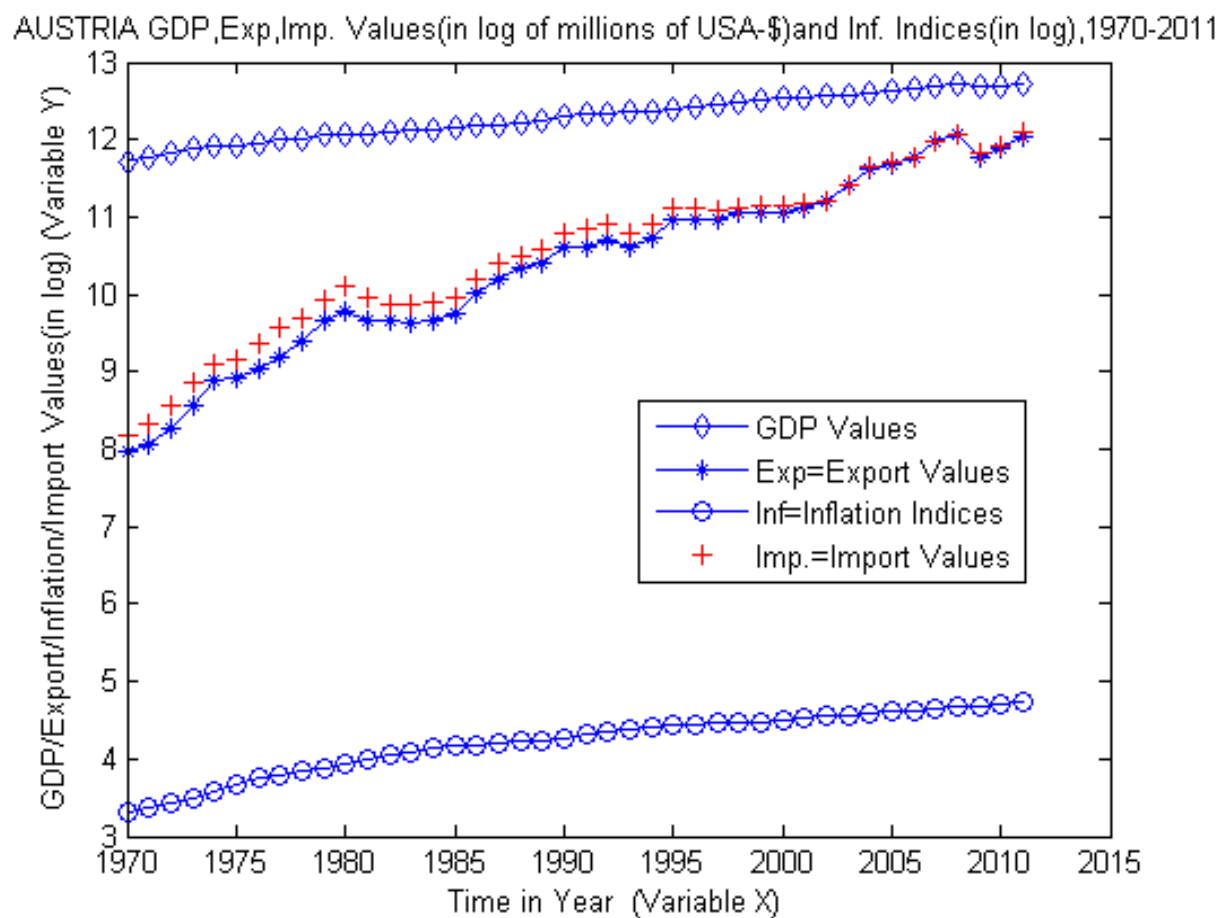
6.2 Phase 2 of the Study

This phase utilised the logarithm of the actual values of the data on GDP, inflation, exports and imports. The following sub-sections are presenting the results, findings and interpretation on Phase 2

6.2.1 Results and Findings in Phase 2

Here, we also draw the time-plots for the four variables (GDP, inflation, export and import) using the 33 countries that supported the existence of Granger causality in Phase 1. The actual values of the variables utilized in this phase are millions of USA dollar. See Figures 6.25 to 6.34 for a handful number of the time-plots of some the countries whilst the others are in Appendix D.

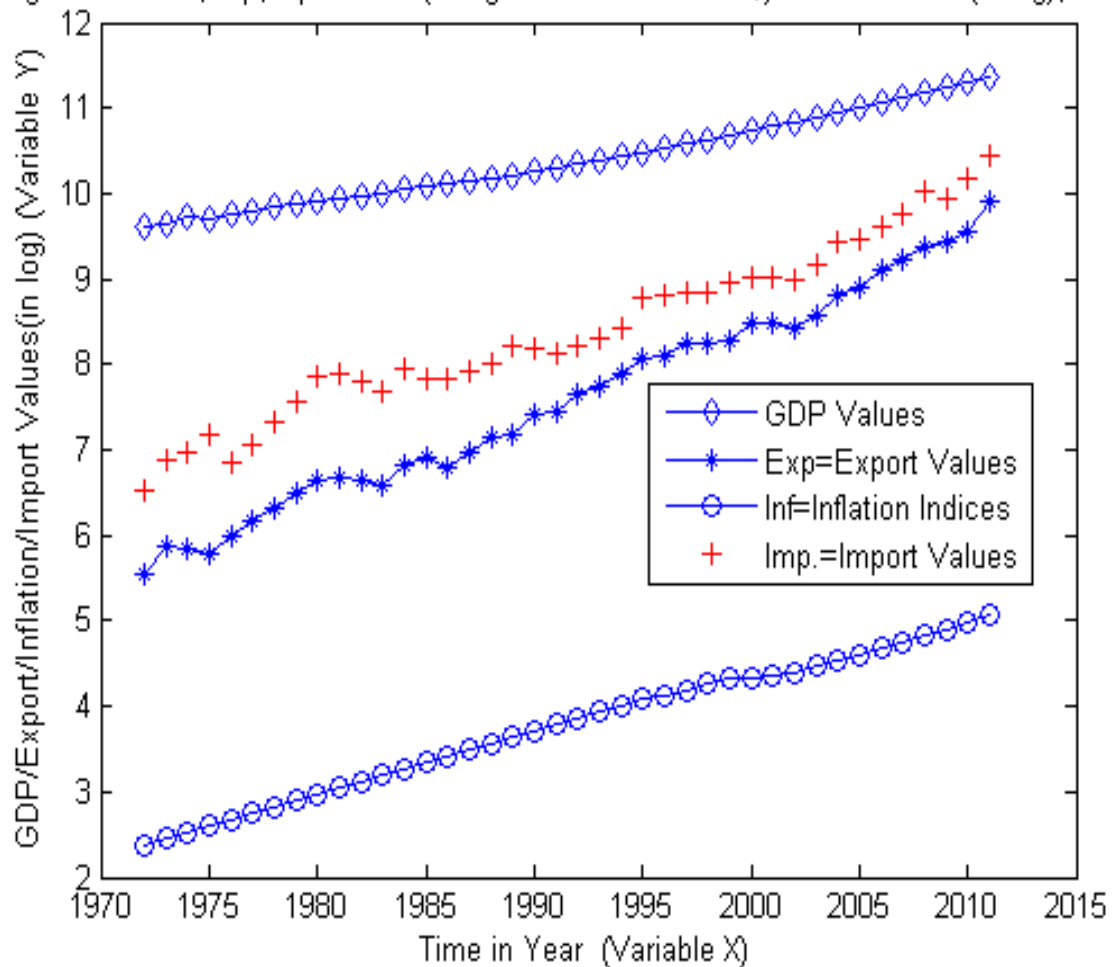
Figure 6.25



Comments: The values of GDP, export and import show rising trends, but the rate of increments in terms of slope steepness, are higher in export and import than that of GDP. The time-plot of import is above that of export. This implies import values are higher than that of export. On inflation, its incremental step is equally of steeper slop than that of GDP.

Figure 6.26

Bangladesh GDP,Exp,Imp. Values(in log of millions of USA-\$)and Inf. Indices(in log),1972-2011



Comments: The values of all the variables are increasing, and showing rising trends.

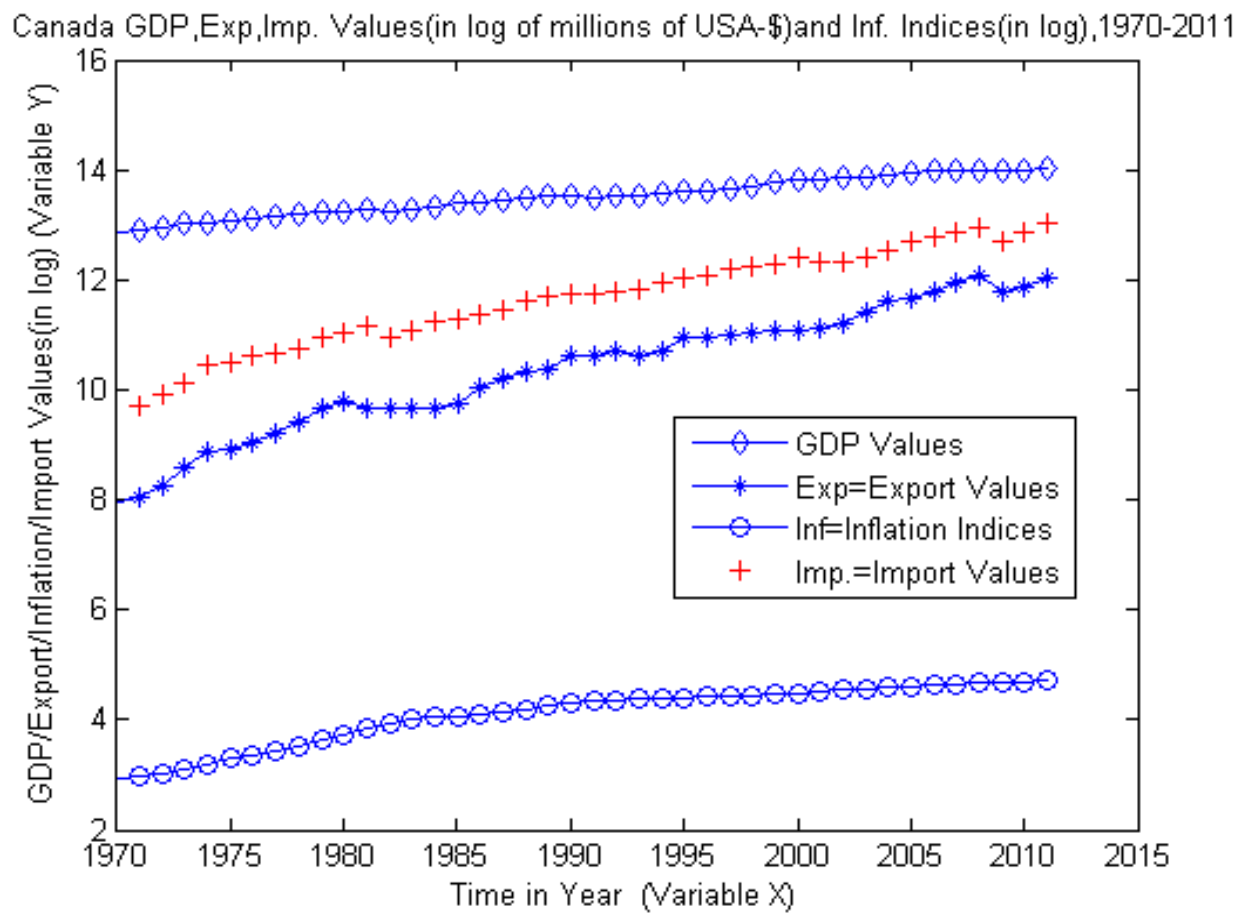
However, the steepness in export, import and inflation are higher than that of GDP. The

movements of the export and import values are not smooth when compare to other two

variables (GDP and inflation). Also from the time-plot, the import values are higher than that

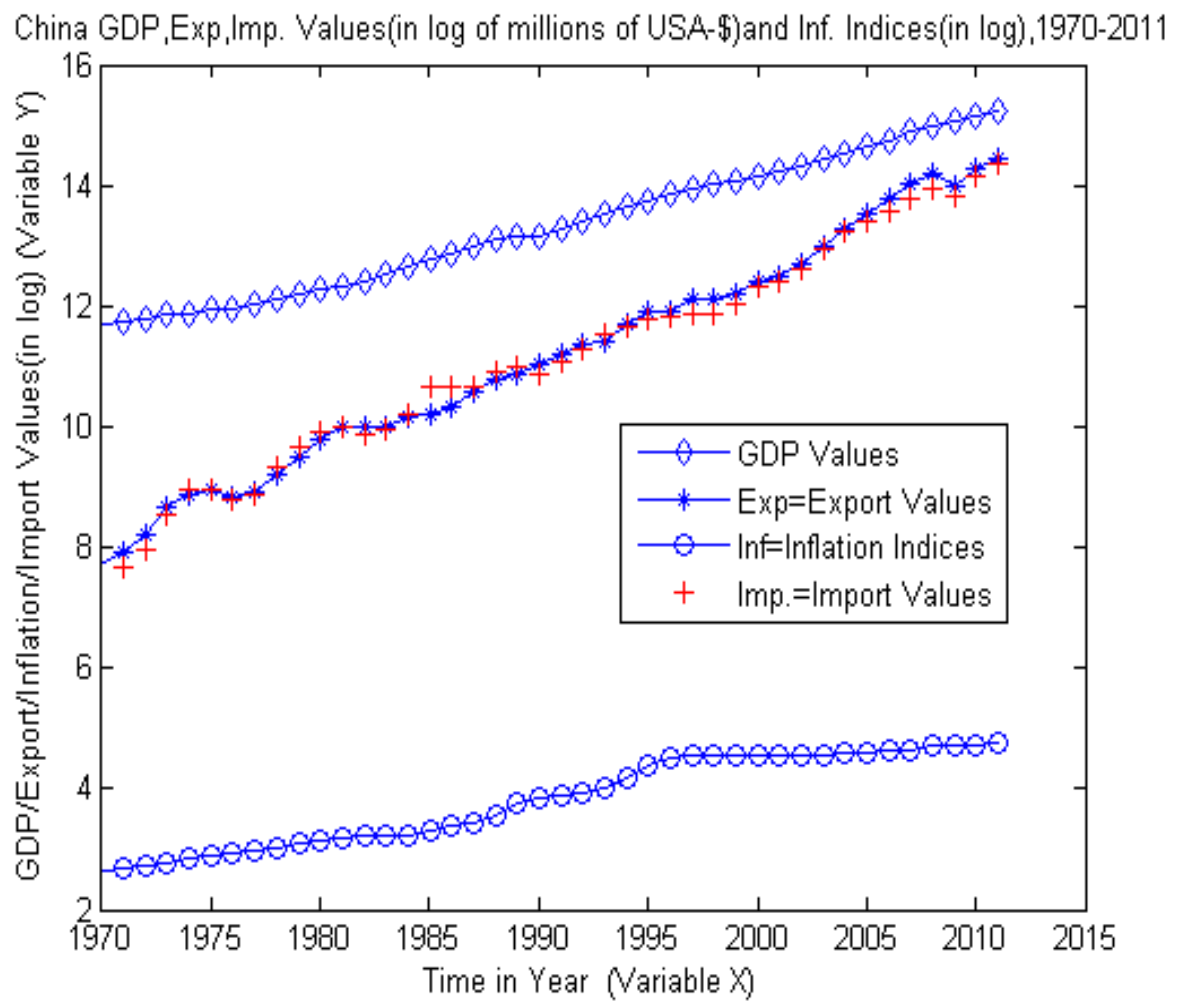
of export.

Figure 6.27



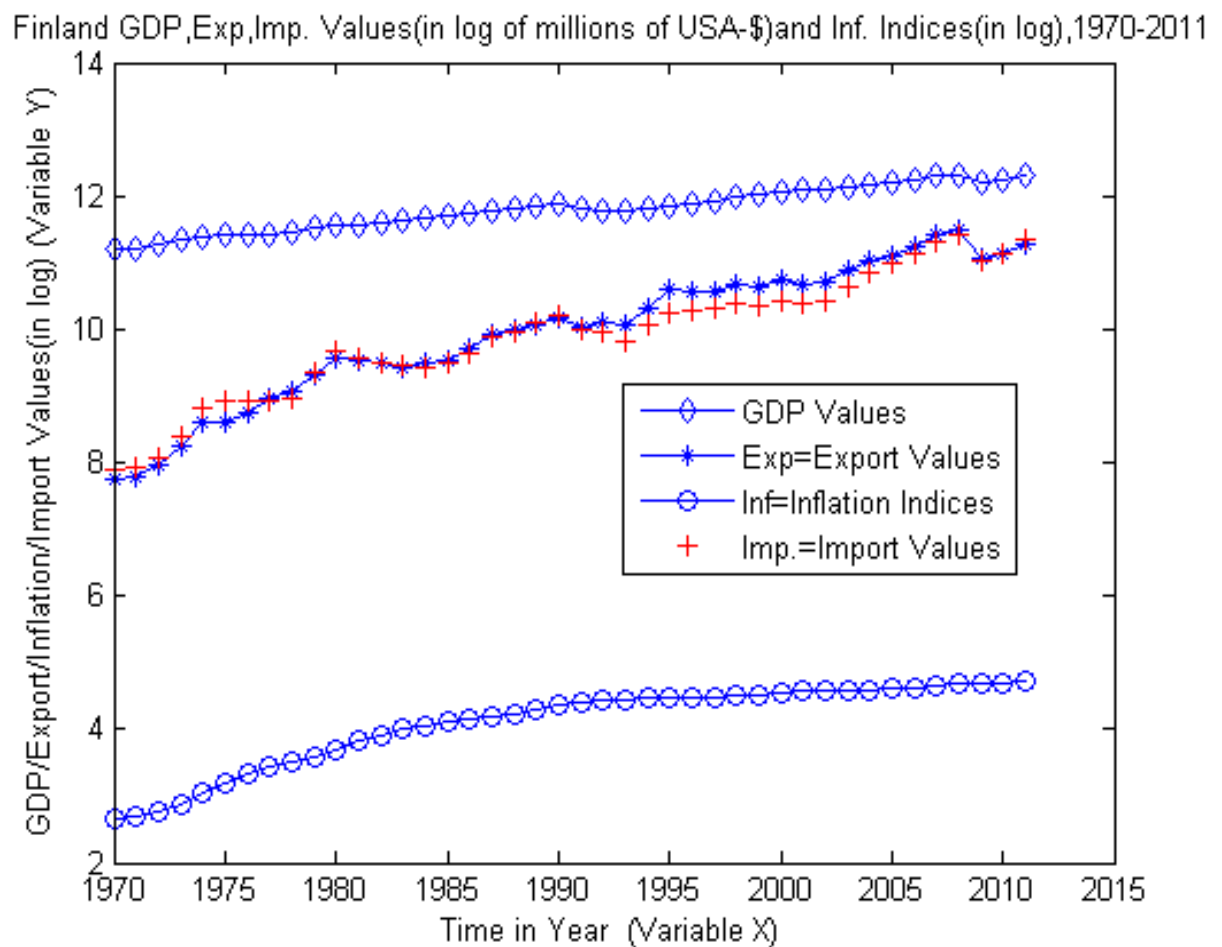
Comments: All the variables' values are increasing with rising trends. It is observed that the values of import are on high side when compare with export, giving room for wide difference between the two.

Figure 6.28



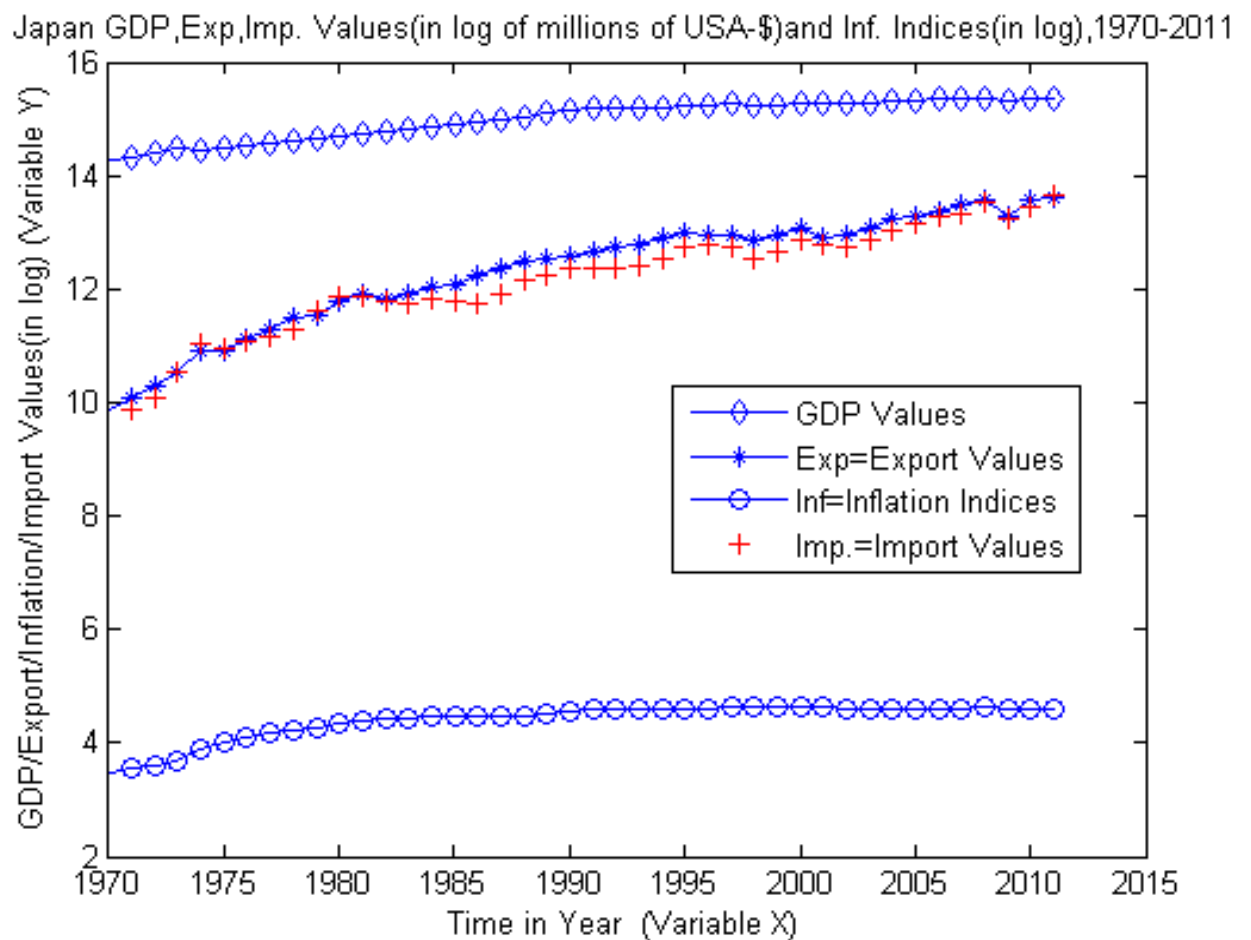
Comments: Rising trends characterized the values of all the variables. However, it is observed that the values of export and import are closely tied.

Figure 6.29



Comments: There are rising trends in all the variables. Export and import are closely tied, but at the tail end of the period the export values are higher than the import. It means export exhibits better values at the tail end of the trend.

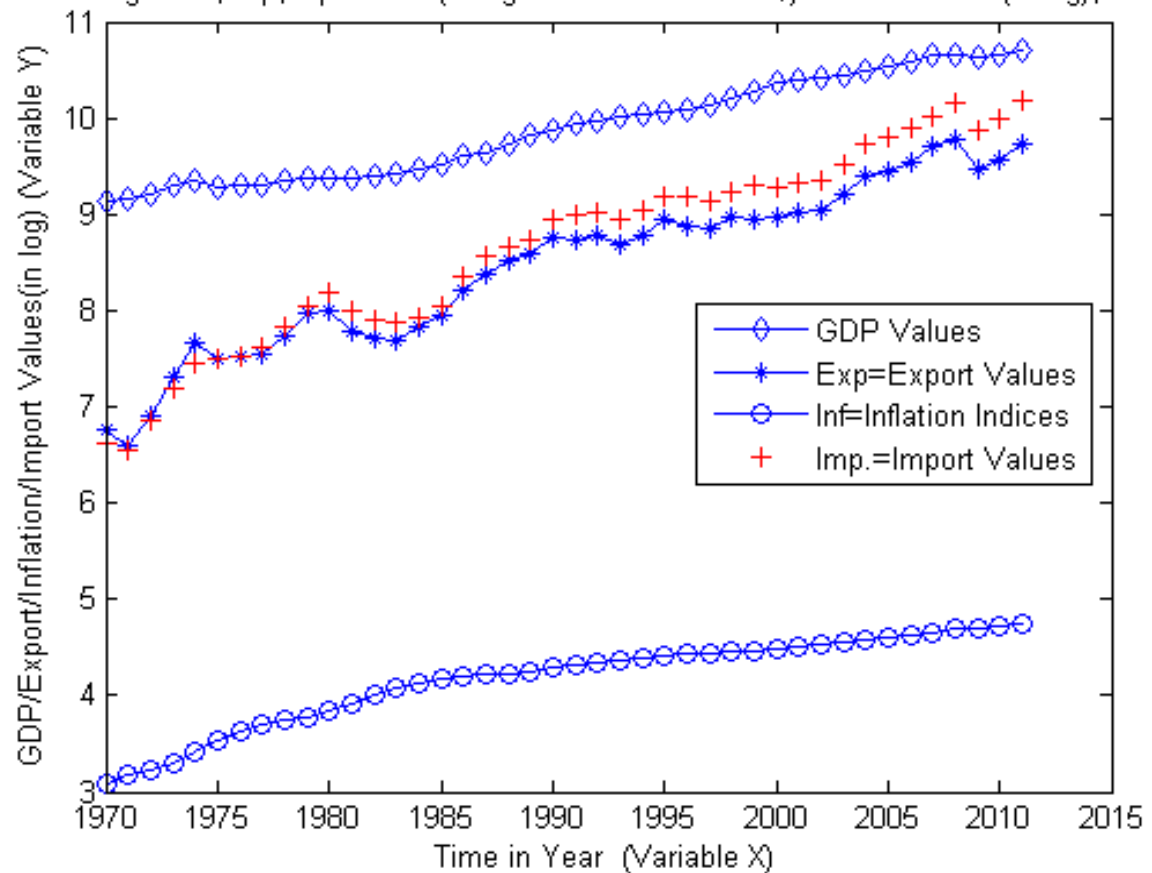
Figure 6.30



Comments: All the variables experienced rising trends. Export and import values are closely tied at earlier period but later export values became greater. It means an improved export over the import leading to surplus balance of trade during that period.

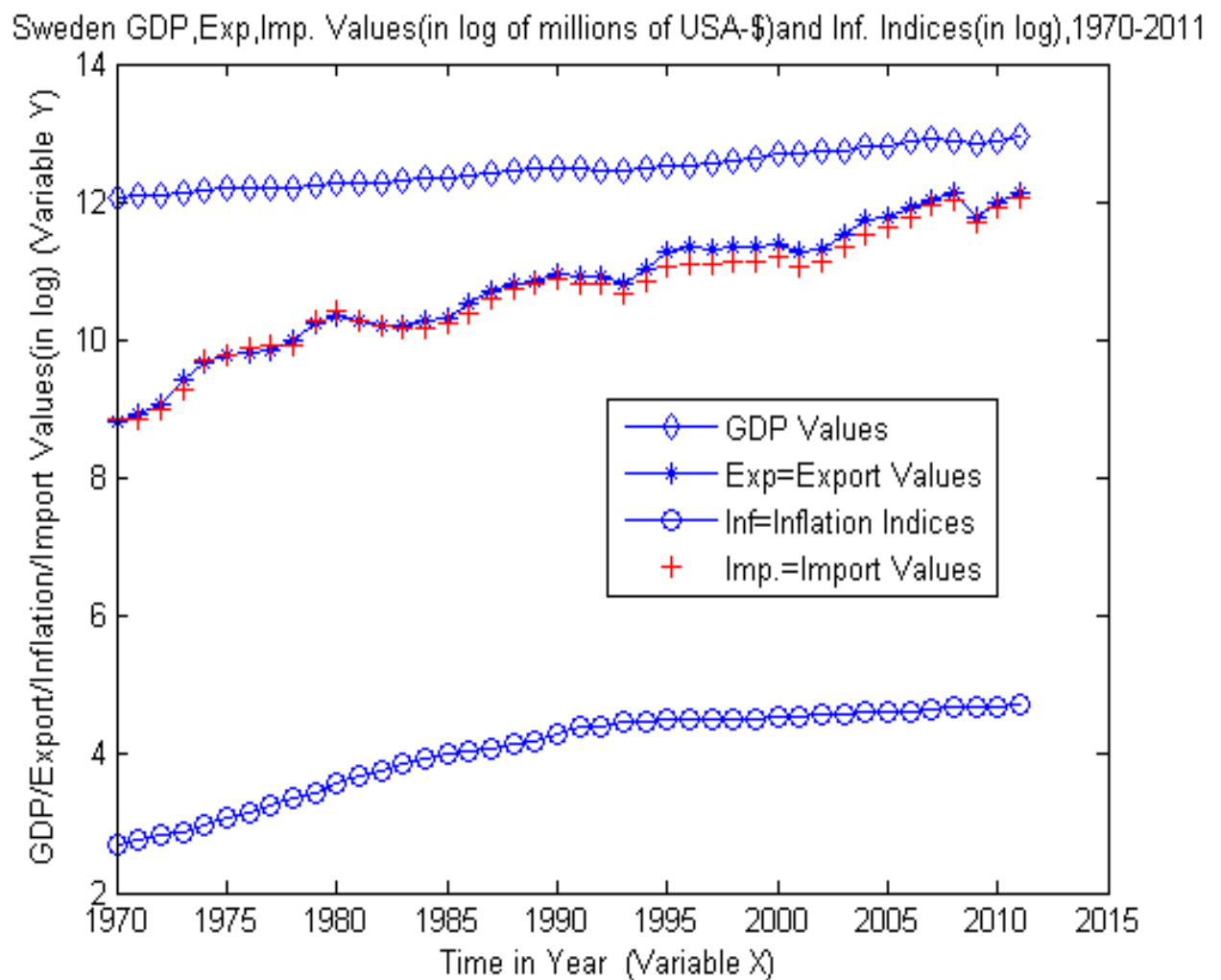
Figure 6.31

Luxembourg GDP,Exp,Imp.Values(in log of millions of USA-\$)and Inf. Indices(in log),1970-2011



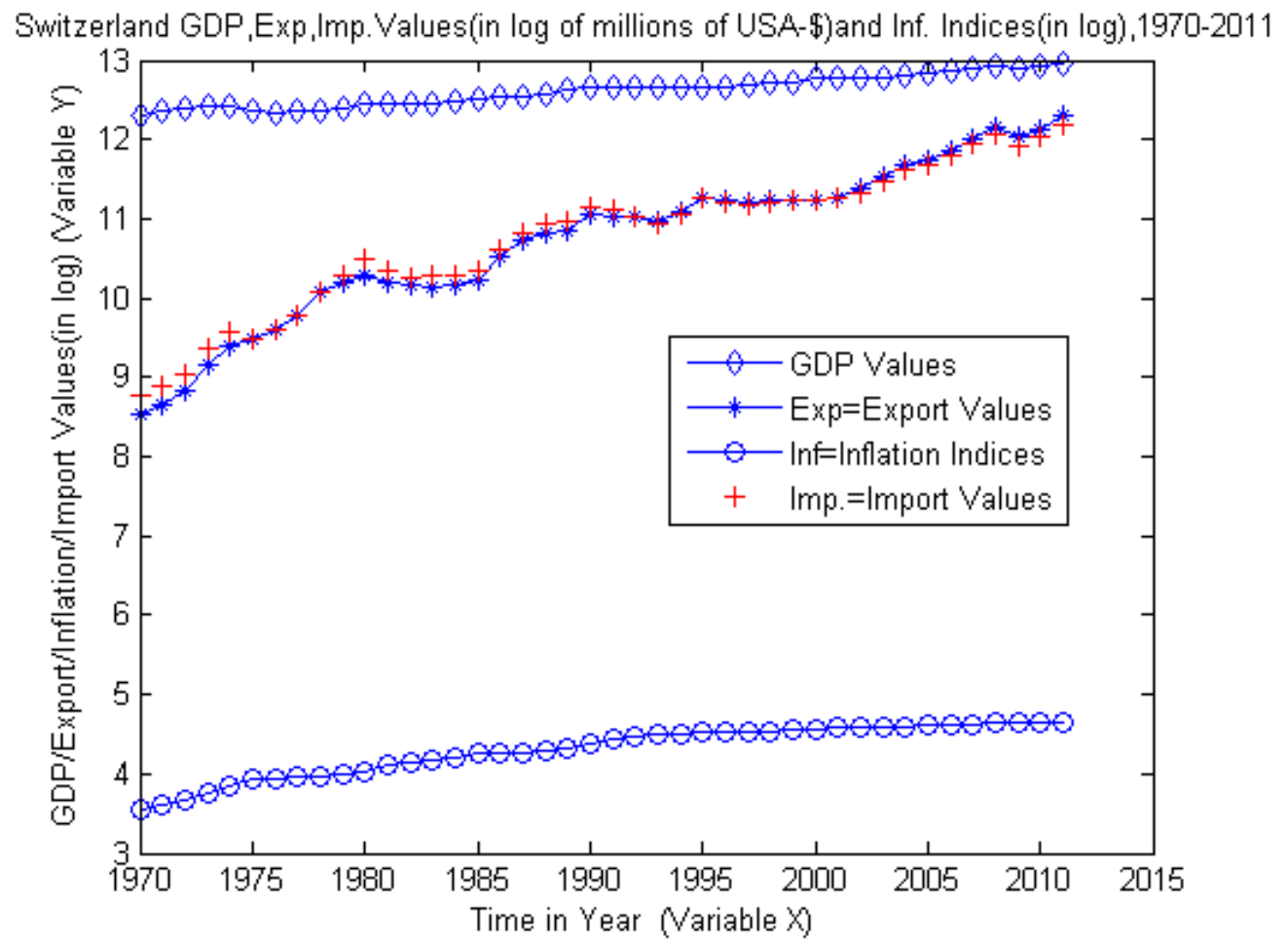
Comments: There are rising trends in the four variables. However, we observed in the early period export values are a little bit higher than that of import, but at the later part of the period import values are having upper hand. It implies there is deficit balance of trade at the later part of the period.

Figure 6.32



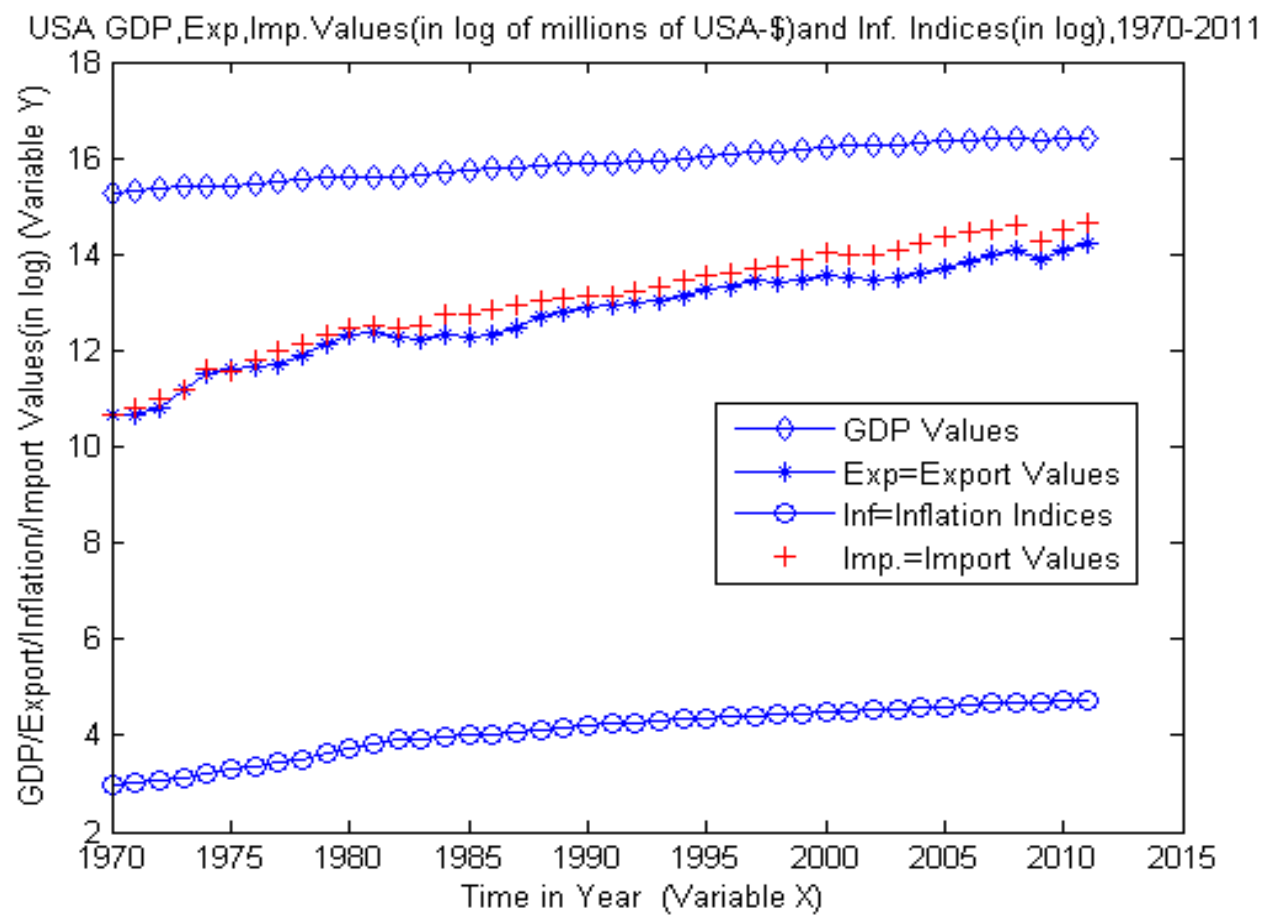
Comments: The set of all involved variables exhibits rising trend. That is, they are of increasing values. It is also noted that export and import are in close tie, but export had values greater than that of import towards the end tail of the period.

Figure 6.33



Comments: There is an increasing value on all the variables. Import is a little bit higher than export in the earlier part but later tied up at the tail end.

Figure 6.34



Comments: Rising trends are observed on all the variables, meaning increasing values ensured. The country experienced higher import over export throughout the study period.

All the plots equally witnessed irregular movements leading to the suspicion of possible non-stationarity in the variables.

To ascertain this, we carried out stationary tests on these variables utilising Cusum chart, Chow's test for structural breaks and outliers, while ADF, PP, KPSS for the unit roots and trend tests. The tests confirmed the existence of non-stationary with integration orders $I(1)$ and $I(2)$.

The acts of transforming the non-stationary variables to stationary were carried out. The transformation involved differencing or/and de-trending. By this, the variables were made ready for Granger causality test; we then carried out Granger causality tests and the pair-wise results are shown in Table 6.07.

Table 6.07: Granger causality results on log of actual values of the paired variables (of GDP, inflation, exports and imports) from 1970 to 2011.

S/N	COUNTRY	GDP (y) and Inflation (x)	GDP (y) and Exports (x)	GDP (y) and Imports (x)	Inflation (y) and Exports (x)	Inflation (y) and Imports (x)	REM ARK
1.	Germany	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
2.	Denmark	GDP \leftarrow Inf.	GDP \leftarrow Exp.	GDP \leftarrow Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
3.	Sweden	GDP \leftarrow Inf.	GDP \leftarrow Exp.	GDP \leftarrow Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
4.	Austria	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. \rightarrow Exp.	Inf. \rightarrow Imp.	
5.	Hungary	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. \rightarrow Exp.	Inf. \leftarrow Imp.	
6.	Luxembourg	GDP \leftarrow Inf.	GDP \leftarrow Exp.	GDP \leftarrow Imp.	Inf. \rightarrow Exp.	Inf. \rightarrow Imp.	
7.	Spain	GDP \leftarrow Inf.	GDP \leftarrow Exp.	GDP \leftarrow Imp.	Inf. — Exp.	Inf. — Imp.	
8.	Chile	GDP \leftarrow Inf.	GDP — Exp.	GDP \leftarrow Imp.	Inf. — Exp.	Inf. — Imp.	
9.	Tunisia	GDP \leftarrow Inf.	GDP \leftarrow Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
10.	Botswana	GDP \leftarrow Inf.	GDP — Exp.	GDP \rightarrow Imp.	Inf. \leftrightarrow Exp.	Inf. \rightarrow Imp.	
11.	Bangladesh	GDP \leftarrow Inf.	GDP — Exp.	GDP \rightarrow Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
12.	Fiji	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	

13.	Nepal	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
14.	Algeria	GDP \leftarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
15.	Japan	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. \rightarrow Imp.	
16.	Australia	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
17.	Iceland	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
18.	New Zealand	GDP \rightarrow Inf.	GDP \leftarrow Exp.	GDP \leftarrow Imp.	Inf. \leftarrow Exp.	Inf. \leftarrow Imp.	
19.	Switzerland	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. \leftarrow Imp.	
20.	Netherlands	GDP \rightarrow Inf.	GDP \rightarrow Exp.	GDP — Imp.	Inf. \rightarrow Exp.	Inf. \rightarrow Imp.	
21.	India	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
22.	China	GDP \rightarrow Inf.	GDP — Exp.	GDP \leftrightarrow Imp.	Inf. — Exp.	Inf. — Imp.	
23.	Iran	GDP \rightarrow Inf.	GDP — Exp.	GDP \leftrightarrow Imp.	Inf. — Exp.	Inf. — Imp.	
24.	Malaysia	GDP \rightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. \leftarrow Exp.	Inf. — Imp.	

25.	Canada	GDP \leftrightarrow Inf.	GDP \rightarrow Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
26.	France	GDP \leftrightarrow Inf.	GDP — Exp.	GDP \leftarrow Imp.	Inf. \leftarrow Exp.	Inf. \leftarrow Imp.	
27.	Italy	GDP \leftrightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
28.	U.S.A	GDP \leftrightarrow Inf.	GDP \leftarrow Exp.	GDP \leftrightarrow Imp.	Inf. — Exp.	Inf. — Imp.	
29.	Belgium	GDP \leftrightarrow Inf.	GDP — Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
30.	Finland	GDP \leftrightarrow Inf.	GDP \leftarrow Exp.	GDP — Imp.	Inf. — Exp.	Inf. — Imp.	
31.	Greece	GDP \leftrightarrow Inf.	GDP — Exp.	GDP \rightarrow Imp.	Inf. — Exp.	Inf. — Imp.	
32.	Portugal	GDP \leftrightarrow Inf.	GDP — Exp.	GDP \rightarrow Imp.	Inf. — Exp.	Inf. \rightarrow Imp.	
33.	Ethopia	GDP \leftrightarrow Inf.	GDP — Exp.	GDP \leftrightarrow Imp.	Inf. \leftarrow Exp.	Inf. \leftrightarrow Imp.	

KEY:

Unidirectional :- (1) = $x \rightarrow y$; (2) = $x \leftarrow y$; Bidirectional :- (3) = $x \leftrightarrow y$; and No
 Granger Causality :- (4) = $x — y$; GDP= GDP Growth; Inf. = Inflation; Exp.=
 Export; Imp. = Import

Next, we combined the Granger causality results in Phases 1 and 2 to form a classification
 summary table for the two phases. See Table 6.08 .Then we used the table to compute the
 Bayes’ formula rule in terms prior and posterior probabilities respectively for Phases 1 and 2.
 See Table 6.09 for Bayes’ computations.

The two tables are in ascending order below:

Table 6.08: Classification summary of Granger Causality results at Stages 1 and 2 for GDP/inflation with other combinations of GDP/exports, GDP/ imports, inflation/exports and inflation/imports. (Extracted from Table 6.07)

(a) – GDP/Inflation and GDP/Exports

STAGE 1- F_i	STAGE 2 – G_j				
Type of Granger Causality on GDP & Inflation (Inf)	Type of Granger Causality on GDP & Export (Exp)				TOTAL
	(1) (GDP \leftarrow Exp.)	(2) (GDP \rightarrow Exp.)	(3) (GDP \leftrightarrow Exp.)	(4) (GDP – Exp.)	
(1) (GDP \leftarrow Inf.) 14 Countries	5	-	-	9	14
(2) (GDP \rightarrow Inf.) 10 Countries	1	1	-	8	10
(3) (GDP \leftrightarrow Inf.) 9 Countries	2	1	-	6	9
TOTAL	8	2	-	23	33

(b) GDP/Inflation and GDP/Imports

STAGE 1- F_i	STAGE 2 – U_j				
Type of Granger Causality on GDP & Inflation (Inf).	Type of Granger Causality on GDP & Imports (Imp).				TOTAL
	(1) (GDP \leftarrow Imp.)	(2) (GDP \rightarrow Imp.)	(3) (GDP \leftrightarrow Imp.)	(4) (GDP – Imp.)	
(1) (GDP \leftarrow Inf.) 14 Countries	5	2	-	7	14
(2) (GDP \rightarrow Inf.)	1	-	2	7	10
(3) (GDP \leftrightarrow Inf.)	1	2	2	4	9
TOTAL	7	4	4	18	33

(c) GDP/Inflation and Inflation/Exports

STAGE 1- F_i	STAGE 2 – V_j				
Type of Granger Causality on GDP & Inflation (Inf)	Type of Granger Causality on Inflation (Inf) & Export (Exp).				TOTAL
	(1) Inf. \leftarrow Exp.	(2) Inf. \rightarrow Exp.	(3) Inf. \leftrightarrow Exp.	(4) Inf. – Exp.	
(1) (GDP \leftarrow Inf.) 14 Countries	-	3	1	10	14
(2) (GDP \rightarrow Inf.) 10 Countries	2	1	-	7	10
(3) (GDP \leftrightarrow Inf.) 9 Countries	2	-	-	7	9
TOTAL	4	4	1	24	33

(d) GDP/Inflation and Inflation/Imports

STAGE 1- F_i	STAGE 2 – W_j				
Type of Granger Causality on GDP & Inflation(Inf)	Type of Granger Causality on Inflation (Inf) & Import (Imp).				TOTAL
	(1) (Inf. \leftarrow Imp.)	(2) Inf. \rightarrow Imp.	(3) Inf. \leftrightarrow Imp.	(4) Inf. – Imp.	
(1) (GDP \leftarrow Inf.) 14 Countries	4	3	-	7	14
(2) (GDP \rightarrow Inf.) 10 Countries	4	2	-	4	10
(3) (GDP \leftrightarrow Inf.) 9 Countries	1	1	1	6	9
TOTAL	9	6	1	17	33

Table 6.09: Tree Diagram and associated computations of conditional probabilities derived from Table 6.08.

(a) Inf/ GDP &/ GDP/EXP (from Table 6.08(a))

s/n	STAGE 1 - F_i	STAGE 2 - G_j	$P(G_j F_i)P(F_i)$	Q (from equation 4.2.3.01)	Remark
1	<p> $P(F_1)=14/33=0.4243$ $P(F_2)=10/33=0.303$ $P(F_3)=9/33=0.2727$ </p>	$P(G_1 F_1)=5/14=0.357$	0.1515	$(0.1515)/(0.2424)=0.625$	$P(F_1 G_1)>P(F_1)$
2		$P(G_2 F_1)=0/14=0$	0.0000	0.0000	0
3		$P(G_3 F_1)=0/14=0$	0.0000	0.0000	0
4		$P(G_4 F_1)=9/14=0.642$	0.2728	$(0.2728)/(0.697)=0.3914$	$P(F_1 G_4)<P(F_1)$
5		$P(G_1 F_2)=1/10=0.100$	0.0303	$(0.0303)/(0.2424)=0.1250$	$P(F_2 G_1)<P(F_2)$
6		$P(G_2 F_2)=1/10=0.100$	0.0303	$(0.0303)/(0.0606)=0.5000$	$P(F_2 G_2)>P(F_2)$
7		$P(G_3 F_2)=0/10=0$	0.0000	0.0000	0
8		$P(G_4 F_2)=8/10=0.8$	0.2424	$(0.2424)/(0.697)=0.3472$	$P(F_2 G_4)>P(F_2)$
9		$P(G_1 F_3)=2/9=0.2222$	0.0606	$(0.0606)/(0.2424)=0.2500$	$P(F_3 G_1)<P(F_3)$
10		$P(G_2 F_3)=1/9=0.1$	0.0303	$(0.0303)/(0.0606)=0.5000$	$P(F_3 G_2)>P(F_3)$
11		$P(G_3 F_3)=0/9=0$	0.0000	0.0000	0
12		$P(G_4 F_3)=6/9=0.6667$	0.1818	$(0.1818)/(0.697)=0.2608$	$P(F_1 G_4)<P(F_3)$

Summary of supporting points – $P(F_1|G_1) > P(F_1)$; $P(F_2|G_2) > P(F_2)$; $P(F_3|G_2) > P(F_3)$ and $P(F_2|G_4) > P(F_2)$.

(b) Inf/GDP & GDP/Imp (from Table 6.08(b))

s/n	STAGE 1 - F_i	STAGE 2 - U_j	$P(U_i F_i)P(F_i)$	Q (from equation 4.2.3.01)	Remark
1	$P(F_1)=14/33=0.4243$ $P(F_2)=10/33=0.303$ $P(F_3)=9/33=0.2727$	$P(U_1 F_1)=5/14=0.357$	0.1515	$(0.1515)/(0.2121)=0.7143$	$P(F_1 U_1) > P(F_1)$
2		$P(U_2 F_1)=2/14=0.1429$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_1 U_2) > P(F_1)$
3		$P(U_3 F_1)=0/14=0$	0.0000	0.0000	0
4		$P(U_4 F_1)=7/14=0.500$	0.2122	$(0.2122)/(0.5455)=0.3890$	$P(F_1 U_4) < P(F_1)$
5		$P(U_1 F_2)=1/10=0.100$	0.0303	$(0.0303)/(0.2121)=0.1429$	$P(F_2 U_1) < P(F_2)$
6		$P(U_2 F_2)=0/10=0$	0.0000	0.0000	0
7		$P(U_3 F_2)=2/10=0.20$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_2 U_3) > P(F_2)$
8		$P(U_4 F_2)=7/10=0.70$	0.2121	$(0.2121)/(0.5455)=0.3890$	$P(F_2 U_4) > P(F_2)$
9		$P(U_1 F_3)=1/9=0.111$	0.0303	$(0.0303)/(0.2121)=0.1429$	$P(F_3 U_1) < P(F_3)$
10		$P(U_2 F_3)=2/9=0.2222$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_3 U_2) > P(F_3)$
11		$P(U_3 F_3)=2/9=0.2222$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_3 U_3) > P(F_3)$
12		$P(U_4 F_3)=4/9=0.4445$	0.1212	$(0.1212)/(0.5455)=0.2222$	$P(F_3 U_4) < P(F_3)$

Summary of supporting points – $P(F_1|U_1) > P(F_1)$; $P(F_1|U_2) > P(F_1)$; $P(F_2|U_3) > P(F_2)$;

$P(F_2|U_4) > P(F_2)$; $P(F_3|U_2) > P(F_3)$; $P(F_3|U_3) > P(F_3)$

(c) Inf/GDP & Inf/Exp.(from Table 6.08(c))

s/n	STAGE 1 - F_i	STAGE 2 - V_j	$P(V_i F_i)P(F_i)$	Q (from equation 4.2.3.01)	Remark
1	$P(F_1)=14/33=0.4243$ $P(F_2)=10/33=0.303$ $P(F_3)=9/33=0.2727$	$P(V_1 F_1)=0/14=0$	0.0000	0	0
2		$P(V_2 F_1)=3/14=0.2143$	0.0909	$(0.0909)/(0.1212)=0.7500$	$P(F_1 V_2)>P(F_1)$
3		$P(V_3 F_1)=1/14=0.0714$	0.0303	$(0.0303)/(0.0303)=1.0000$	$P(F_1 V_3)>P(F_1)$
4		$P(V_4 F_1)=10/14=0.7143$	0.3031	$(0.3031)/(0.7273)=0.4166$	$P(F_1 V_4)<P(F_1)$
5		$P(V_1 F_2)=2/10=0.2000$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_2 V_1)>P(F_2)$
6		$P(V_2 F_2)=1/10=0.1000$	0.0303	$(0.0303)/(0.0303)=1.0000$	$P(F_2 V_2)>P(F_2)$
7		$P(V_3 F_2)=0/10=0$	0.0000	0.0000	0
8		$P(V_4 F_2)=7/10=0.70$	0.2121	$(0.2121)/(0.7273)=0.2916$	$P(F_2 V_4)<P(F_2)$
9		$P(V_1 F_3)=2/9=0.2222$	0.0606	$(0.0606)/(0.1212)=0.5000$	$P(F_3 V_1)>P(F_3)$
10		$P(V_2 F_3)=0/9=0$	0.0000	0.0000	0
11		$P(V_3 F_3)=0/9=0$	0.0000	0.0000	0
12		$P(V_4 F_3)=7/9=0.778$	0.2121	$(0.2121)/(0.7273)=0.2916$	$P(F_3 V_4)>P(F_3)$

Summary of supporting points – $P(F_1|V_2) > P(F_1)$; $P(F_1|V_3) > P(F_1)$; $P(F_2|V_1) > P(F_2)$; $P(F_2|V_2) > P(F_2)$; $P(F_3|V_1) > P(F_3)$; $P(F_3|V_4) > P(F_3)$

(d) Inf/GDP & Inf/Imp.(from Table 6.08(d))

s/n	STAGE 1 - F_i	STAGE 2 - W_j	$P(W_i F_i)P(F_i)$	Q (from equation 4.2.3.01)	Remark
1	$P(F_1)=14/33=0.4243$ $P(F_2)=10/33=0.303$ $P(F_3)=9/33=0.2727$	$P(W_1 F_1)=4/14=0.285$	0.1212	$(0.1212)/(0.2727)=0.4444$	$P(F_1 W_1) > P(F_1)$
2		$P(W_2 F_1)=3/14=0.2143$	0.0909	$(0.0909)/(0.1818)=0.5000$	$P(F_1 W_2) > P(F_1)$
3		$P(W_3 F_1)=0/14=0$	0.0000	0.0000	0
4		$P(W_4 F_1)=7/14=0.500$	0.2122	$(0.2122)/(0.5152)=0.4119$	$P(F_1 W_4) < P(F_1)$
5		$P(W_1 F_2)=4/10=0.400$	0.1212	$(0.1212)/(0.2727)=0.4444$	$P(F_2 W_1) > P(F_2)$
6		$P(W_2 F_2)=2/10=0.200$	0.0606	$(0.0606)/(0.1818)=0.3333$	$P(F_2 W_2) > P(F_2)$
7		$P(W_3 F_2)=0/10=0$	0.0000	0.0000	0
8		$P(W_4 F_2)=4/10=0.4$	0.1212	$(0.1212)/(0.5152)=0.2352$	$P(F_2 W_4) < P(F_2)$
9		$P(W_1 F_3)=1/9=0.1111$	0.0303	$(0.0303)/(0.2727)=0.1111$	$P(F_3 W_1) < P(F_3)$
10		$P(W_2 F_3)=1/9=0.1111$	0.0303	$(0.0303)/(0.1818)=0.1667$	$P(F_3 W_2) < P(F_3)$
11		$P(W_3 F_3)=1/9=0.1111$	0.0303	$(0.0303)/(0.0303)=1.0000$	$P(F_3 W_3) > P(F_3)$
12		$P(W_4 F_3)=6/9=0.6667$	0.1818	$(0.1818)/(0.5152)=0.3529$	$P(F_3 W_4) > P(F_3)$

Summary of supporting points – $P(F_1|W_1) > P(F_1)$; $P(F_1|W_2) > P(F_1)$; $P(F_2|W_1) > P(F_2)$; $P(F_2|W_2) > P(F_2)$; $P(F_3|W_3) > P(F_3)$; $P(F_3|W_4) > P(F_3)$

For further statistical inference, we applied the assignment problem model on the obtained results of Bayes' computations. See Table 6.10.

Table 6.10: Summary of The Bayesian Results Supporting Granger Causality at Stages one (Prior) and two (Posterior), (Extracted from Table 6.09)

	Second Stage (Posterior Probability)			
First Stage(Prior Probability) F_i, ($i = 1,2,3$)	G_j , ($j= 1,2,3$)	U_j , ($j= 1,2,3$)	V_j , ($j= 1,2,3$)	W_j , ($j= 1,2,3$)
F_1 (0.423)	$G_1=0.625$	$U_1=0.7143$, $U_2=0.500$	$V_2=0.7500$, $V_3= 1.000$	$W_1= 0.500$, $W_2=0.500$
F_2 (0.303)	$G_2= 0.500$	$U_3= 0.500$	$V_1= 0.5000$, $V_2= 1.000$	$W_1= 0.500$, $W_2=0.333$
F_3 (0.274)	$G_3= 0.500$	$U_2=0.500$, $U_3=0.500$	$V_1= 0.2857$	$W_3= 1.000$

In this table, we are to note that the combinations of conditional probabilities involving G_4 , U_4 , V_4 and W_4 are not included because they are of non-G.Causality.

Note:

-- $(GDP/ Inf) = F_i$, $i = 1,2,3$; that is, $F_1 = GDP \leftarrow Inf$, $F_2 = GDP \rightarrow Inf$, $F_3 = GDP \leftrightarrow Inf$.

-- $(GDP/Exp) = G_i$, $i = 1,2,3$; that is, $G_1 = GDP \leftarrow Exp$, $G_2 = GDP \rightarrow Exp$, $G_3 = GDP \leftrightarrow Exp$.

-- $(GDP/ Imp) = U_i$, $i = 1,2,3$; that is, $U_1 = GDP \leftarrow Imp$, $U_2 = GDP \rightarrow Imp$, $U_3 = GDP \leftrightarrow Imp$

-- $(Inf/ Exp) = V_i$, $i = 1,2,3$; that is, $V_1 = Inf \leftarrow Exp$, $V_2 = Inf \rightarrow Exp$, $V_3 = Inf \leftrightarrow Exp$.

-- (Inf/ Imp)= W_i , $i = 1,2,3$; that is, $W_1 = \text{Inf} \leftarrow \text{Imp}$, $W_2 = \text{Inf} \rightarrow \text{Imp}$, $W_3 = \text{Inf} \leftrightarrow \text{Imp}$.

GDP = Gross domestic product; Inf = Inflation; Exp = Export; Imp = Import.

The following Table 6.10-1 is extracted from table 6.10 by counting the number of points in each cell.

Table 6.10-1

	G_j	U_j	V_j	W_j
F_1	1	2	2	2
F_2	1	1	2	2
F_3	1	2	1	1

The above supporting points in the table can equally be seen as profit or effectiveness for the posterior outcomes. Hence, the table is taken as profit matrix in the assignment problem. By this, we solve the table by Hungarian method in order to have one-to-one assignment of the two stages as a way of combinatorial optimization.

Table 6.10-2

	G_j	U_j	V_j	W_j
F_1	1	2	2	2
F_2	1	1	2	2
F_3	1	2	1	1
S_{dummy}	0	0	0	0

S_{dummy} is the dummy variable to make the table balance.

Table 6.10-3

	G_j	U_j	V_j	W_j
F_1	1	0	0	0
F_2	1	1	0	0
F_3	1	0	1	1
S_{dummy}	2	2	2	2

Subtract each element of table 6.10-2 from the highest in that tableau (to make it cost table)

Note that by crossing zeros in Table 6.10-3, the tableau is not yet optimal because the number crossed lines are supposed to be 4 instead of 3. By this, we need another iteration of row or column operation to make it optimal. Here, we prefer the row operation in the next table.

Table 6.10-4

	G_j	U_j	V_j	W_j
F_1	1	0	0	0
F_2	1	1	0	0
F_3	1	0	1	1
S_{dummy}	0	0	0	0

Table 6.10-4 is optimal now; hence, we have the following route- combinations of assignments:

--- (a) $F_1 - W_1$, $F_2 - V_2$, $F_3 - U_3$, $S_{dummy} - G$.

With total profit = $2+2+2+0 = 6$

--- (b) $F_1 - V_1$, $F_2 - W_2$, $F_3 - U_3$, $S_{dummy} - G$.

With total profit = $2+2+2+0 = 6$

For route (a), we have the following:

Stage 1 (Prior Probabilities' Relationships)	Stage 2 (Posterior Probabilities' Relationships)
GDP \leftarrow Inflation (F_1)	Export \rightarrow Inflation (V_1)
GDP \rightarrow Inflation (F_2)	Inflation \rightarrow Import (W_2)
GDP \leftrightarrow Inflation (F_3)	GDP \leftrightarrow Import (U_3)

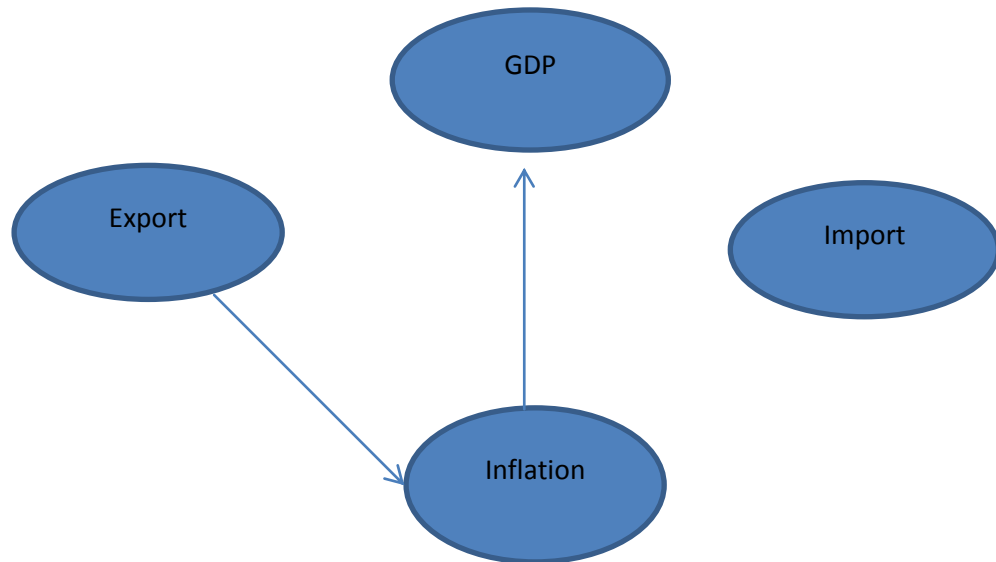
For route (b), we have the following:

Stage 1 (Prior Probabilities' Relationships)	Stage 2 (Posterior Probabilities' Relationships)
GDP \leftarrow Inflation (F_1)	Import \rightarrow Inflation (W_1)
GDP \rightarrow Inflation (F_2)	Inflation \rightarrow Export (V_2)
GDP \leftrightarrow Inflation (F_3)	GDP \leftrightarrow Import (U_3)

Figure 35

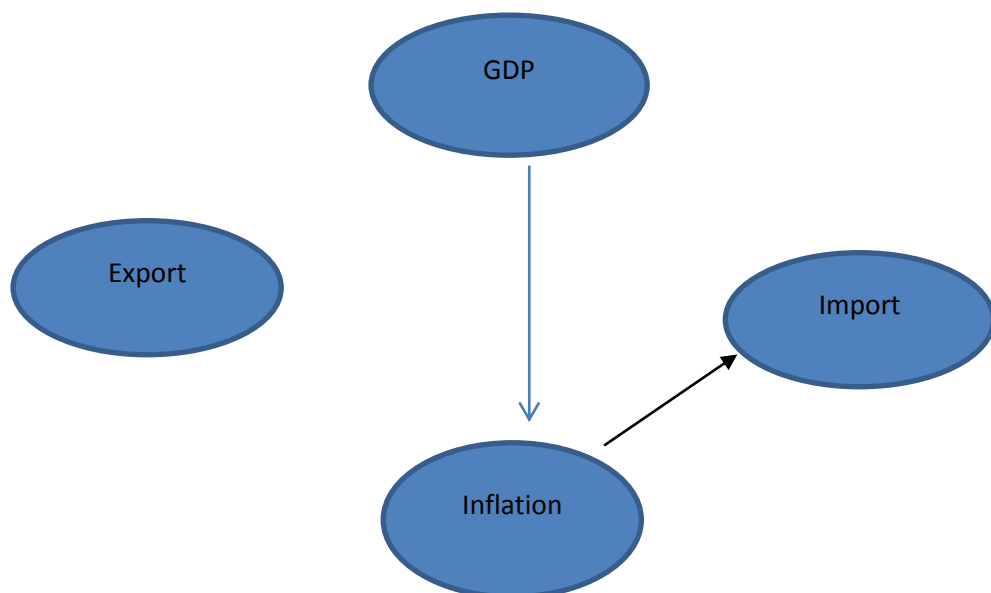
Route (a)_(i)

F₁ and V₁



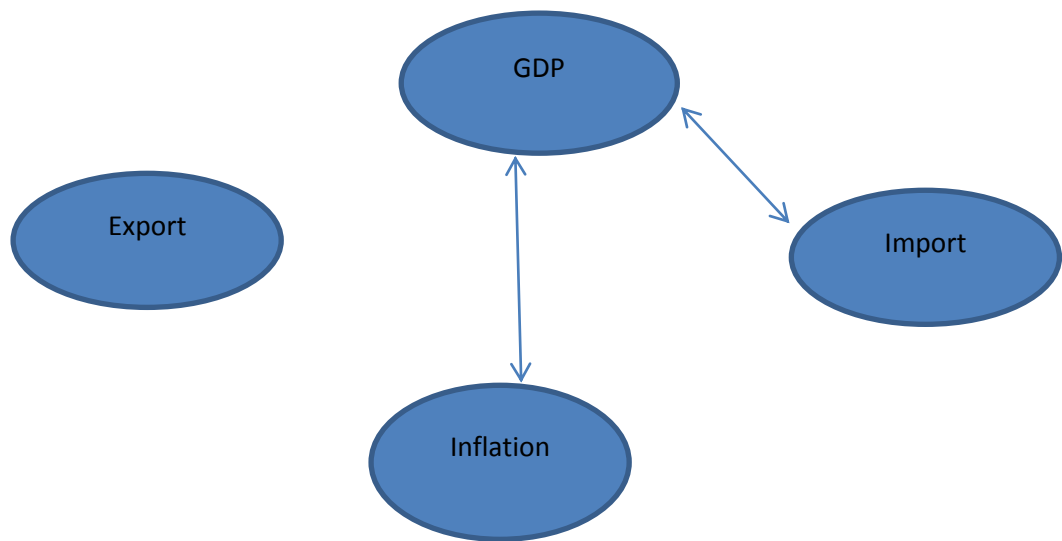
(a)_(ii)

F₂ and W₂



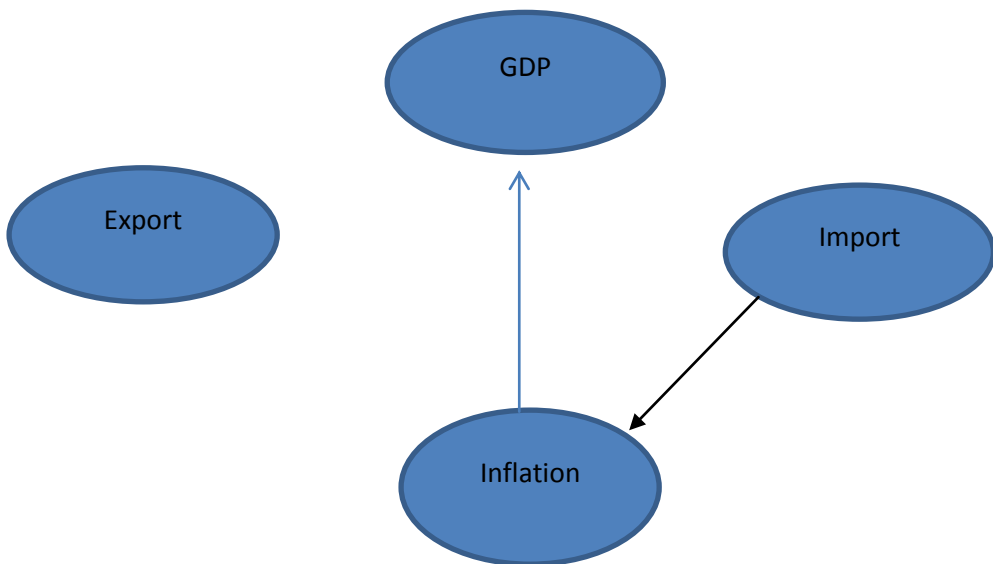
(a)_(iii)

F_3 and U_3



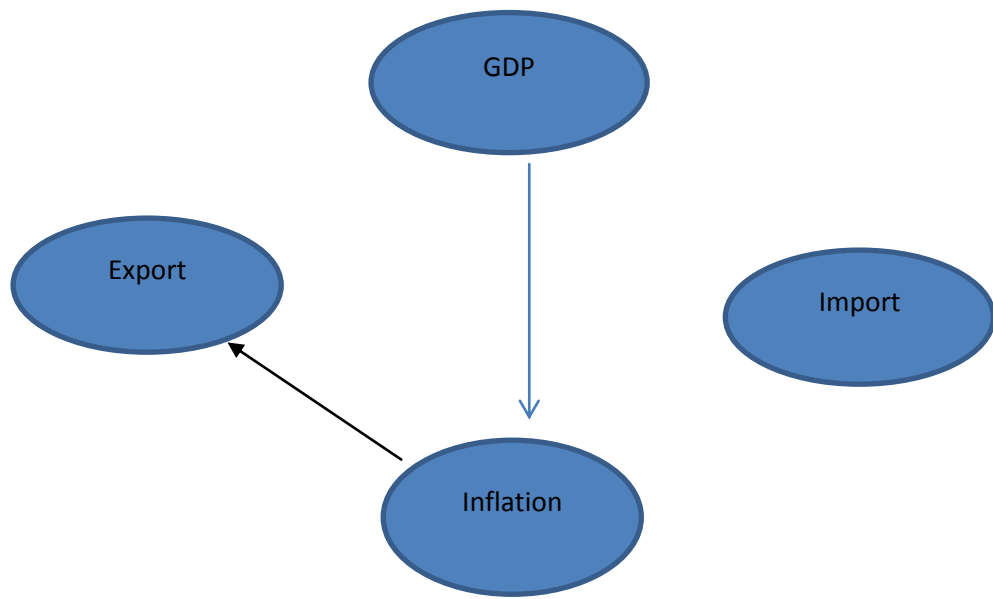
Route (b)_(i)

F_1 and W_1



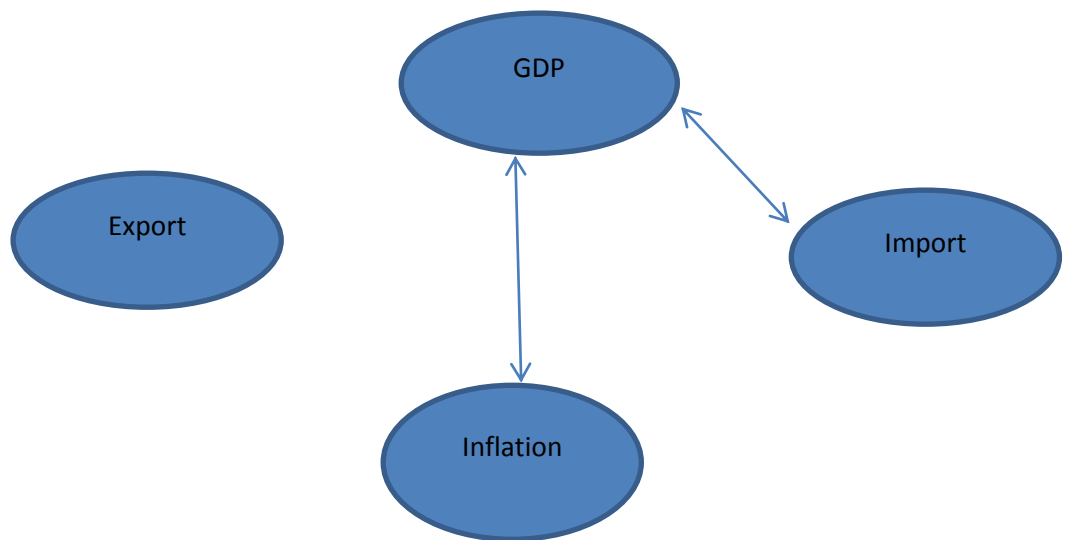
(b)_(ii)

F₂ and V₂



(b)_(iii)

F₃ and U₃



Decisions: The following decisions are made that:

---- when there is unidirectional causality between inflation and import (or export), there is also unidirectional causality between GDP and inflation by the Bayes' Rule; and

--- when there is bidirectional causality between GDP and import only, there is bidirectional causality between GDP and inflation.

In another approach, with Phase 2, we determined the means and standard deviations of exports and imports. These central measure parameters are utilized to determine the coefficient of variation (CV) for the two variables. The results here are then combined with Phase 1 Granger causality results leading us to have the following tables:

Table 6.11 – List of countries classified by the Granger causality of Inflation/GDP and their CV on exports and imports.

Table 6.12 – Statistical tests on results of coefficient of variation for the groups of Granger causality.

The two tables are presented in sequential order below:

Table 6.11: List of countries classified by the Granger Causality of inflation/GDP and their related coefficient of variation (CV) on exports and imports

S/N	COUNTRY	TYPE OF G. CAUSALITY ON GDP & INFLATION	Export (in Millions of US-\$)			Import (in millions of US-\$)			Comparing cv_e and cv_i	Remarks
			Mean (\bar{x}_e)	Std. dev (σ_e)	$cv_e = (\sigma_e / \bar{x}_e)$	Mean (\bar{x}_i)	Std. dev (σ_i)	$cv_i = (\sigma_i / \bar{x}_i)$		
1	Germany	Inflation "Granger causes" GDP (GDP \leftarrow Inflation)	476200	409580	0.8601	401090	335260	0.8358	$cv_e > cv_i$	✓
2	Denmark		40930	32097	0.7842	37819	27889	0.7375	$cv_e > cv_i$	✓
3	Sweden		66483	50927	0.7660	58918	45090	0.7653	$cv_e > cv_i$	✓
4	Austria		53045	49718	0.9373	57675	49837	0.8640	$cv_e > cv_i$	✓
5	Hungary		26126	31513	1.2061	26781	30440	1.1366	$cv_e > cv_i$	✓
6	Luxembourg		6458.3	4763.5	0.7376	8660.6	7241.6	0.8362	$cv_e < cv_i$	X
7	Spain		84995	85174	1.0021	117790	11797	1.0015	$cv_e > cv_i$	✓
8	Chile		-	-	-	-	-	-	-	M
9	Tunisia		52173	5129.7	0.9832	47387	61701	1.3021	$cv_e < cv_i$	X
10	Botswana		-	-	-	-	-	-	-	M
11	Bangladesh		3803	4488.7	1.1804	7146	7624.30	1.0669	$cv_e > cv_i$	✓
12	Fiji		-	-	-	-	-	-	-	M
13	Nepal		360.76	312.36	0.8658	1233.2	1407.3	1.1410	$cv_e < cv_i$	X
14	Algeria		-	-	-	-	-	-	-	M
15	Japan	GDP "Granger causes" Inflation (GDP \rightarrow Inflation)	321340	229080	0.7129	270150	211650	0.7835	$cv_e < cv_i$	X
16	Australia		57313	59399	1.0364	61672	59703	0.9681	$cv_e > cv_i$	✓
17	Iceland		1716.2	1372.1	0.7995	1941.7	1542.5	0.7943	$cv_e > cv_i$	✓
18	New Zealand		-	-	-	-	-	-	-	M
19	Switzerland		6737	5587	0.829	6571	4922	0.7491	$cv_e > cv_i$	✓

			0	0	5	5	5			
20	Netherlands		1764 00	1513 90	0.858 2	1621 10	1337 00	0.8247	$cv_e > cv_i$	√
21	India		5233 0	7044 7	1.346 3	7548 7	1104 00	1.4624	$cv_e < sm_i$	x
22	China		2999 60	4886 80	1.629 2	2614 10	4197 10	1.6057	$cv_e > cv_i$	√
23	Iran		2768 3	3186 3	1.151 0	2047 6	1638 9	0.8004	$cv_e > cv_i$	√
24	Malaysia		6122 0	6432 9	1.050 8	5214 6	5261 0	1.0089	$cv_e > cv_i$	√
25	Canada	Bi-directional causality (GDP ↔ Inflation)	1699 00	1302 40	0.766 6	1592 90	1240 70	0.7789	$cv_e < cv_i$	√
26	France		2306 80	1691 10	0.733 1	2492 60	1946 50	0.7809	$cv_e < sm_i$	√
27	Italy		1897 70	1512 80	0.797 2	1930 40	1547 20	0.8015	$cv_e < cv_i$	√
28	USA		4993 70	3878 30	0.776 6	7660 00	6643 50	0.8673	$cv_e < cv_i$	√
29	Belgium		2721 20	1180 90	0.434 0	2575 40	1182 70	0.4592	$cv_e < cv_i$	√
30	Finland		3177 6	2581 3	0.812 3	2842 4	2365 9	0.8323	$cv_e < cv_i$	√
31	Greece		9257 .8	7321 .3	0.790 8	2541 6	2252 9	0.8864	$cv_e < cv_i$	√
32	Portugal		4606 5	4516 3	0.980 4	3074 5	2339 6	0.7610	$cv_e > cv_i$	X
33	Ethiopia		-	-	-	-	-	-	-	M

KEY: Std. dev. = Standard deviation, cv_e = Coefficient of Variation for export, cv_i = Coefficient of Variation for import, G. Causality = Granger Causality, BOT= Balance of Trade, √= supporting the same course in each group, x=opposing course in each group, m= missing data in either export or import.

Table 12: Statistical tests on coefficient of variation (CV) results for the Granger causality. (Extracted from table 10).

s/n	G.Causality	$cv_e > cv_i$		$cv_e < cv_i$		Total	Hypothesis ($P^* = 0.5$)	P-value & Decision at 5% significance level.	Remark/Interpretation on better option
		no	P_1	no	P_2				
1	GDP \leftrightarrow Inf	1	0.125	7	0.875	8	$H_0: P^* \leq p_1$ vs $H_1: P^* > p_1$	P=0.0351 H ₁ accepted, which implied p ₂ is supported.	$cv_e < cv_i$
2	GDP \rightarrow Inf	7	0.778	2	0.222	9	$H_0: P^* \geq p_1$ vs $H_1: P^* < p_1$	P=0.0899 H ₀ accepted, which implied p ₁ is supported.	$cv_e > cv_i$
3	GDP \leftarrow Inf	7	0.700	3	0.300	10	$H_0: P^* \geq p_1$ vs $H_1: P^* < p_1$	P=0.1719 H ₀ accepted, which implied p ₁ is supported.	$cv_e > cv_i$

Note : (i) That $p^* = 0.5$ because we are testing the equality of p_1 and p_2 , hence the theoretical chance of 50-50 being represented by p^* .

(ii) p is obtained by using Binomial Distribution (the Cumulative Probabilities).

Decision:

(i) Bi-directional G. Causality resulting to $cv_e < cv_i$, meaning smaller volatility of export over import supports Bi-directional $[GDP \leftrightarrow Inf]$ Granger causality.

(ii) Single directional Granger causality implied $cv_e > cv_i$, indicating larger volatility of export over import supports uni-directional $[GDP \rightarrow Inf]$ or $[GDP \leftarrow Inf]$ Granger causality.

6.2.2 Interpretations

The interpretations to our results in the previous sub-section are presented as follows:

The time-plots and stationary tests results supported existence of non-stationarity with integration orders I(1) and I(2). It implies that all the variables are of irregular movements, which is typical with presence of fluctuations. It is important to note that there is higher order of integration in Phase 2 than Phase1. This is due to data type; that is Phase 1 data is in percentage change and the Phase 2 in the actual values. The one with percentage change has utilized the property of transformation stated in Equation 3.5.3 of Section 3.5 to have the reduced order.

After making the variables stationary, the Granger causality computations and tests were carried out. The results are given in Table 6.07. To interpret this Table 6.07, another table was generated as Table 6.13 entitled “Summary table of pair combinations of exports, imports, inflation and GDP in terms of Granger causality”. The table is presented with the accomplished discussions below:

Table 6.13: Summary Table of pair combinations of export, import, inflation and GDP in terms of Granger causality and non-Granger causality.

Variable combinations	Granger causality		Non-Granger causality		Total
	No	%	No	%	
GDP & Exp.	10	30.3	23	69.7	33
GDP & Imp.	15	45.5	18	54.5	33
Inf. & Exp.	09	27.3	24	72.7	33
Inf. & Imp.	16	48.5	17	51.5	33

From Table 6.13, a simple interpretation one can make is that there is fair percentages of these variables` combinations. The Inf./ Imp. and GDP/Imp. are of 48.5 and 45.5 respectively, and it can be seen having better chances than the other combinations. By a further close look , we have imports being combined with GDP and Inflation to have better percentages. Hence, one can say imports have higher impacts on these variables for the concerned countries which in turn can be used to have enhanced predictions of GDP and inflation.

Table 6.08 was formed by combining the Granger causality results of Phases 1 and 2. This table form the bases of our computations on Bayes' theory in Table 6.09, where results in Phases 1 and 2 are respectively the prior and posterior probabilities. Tree diagrams probabilities were utilised to generate the conditional probabilities. The results in Phases 1 and 2 emanated from Tables 1 and 7 respectively.

Bayes` Theorem dealt with degree of belief which we use to be more ascertain of the results in phase one. See Table 6.09(a, b, c, d) for the necessary computations. The following discussions present the interpretations of the computations in terms of better/higher posterior than prior probabilities using the results from Table 6.07.

With combinations of inflation/GDP and GDP/export, we have:

- $P(F_1|G_1) > P(F_1)$ which implies that there is higher belief that inflation causes GDP.
- $P(F_2|G_2) > P(F_2)$ and $P(F_2|G_4) > P(F_2)$ have better support of GDP causing Inflation.
- $P(F_3|G_2) > P(F_3)$ indicates greater impression of belief on both directional causality of GDP and inflation.

In the case of inflation/GDP and GDP/import combination, it resulted to:

- $P(F_1|U_1) > P(F_1)$ and $P(F_1|U_2) > P(F_1)$ implying high-top notion in support of Inflation lead GDP;
- $P(F_2|U_3) > P(F_2)$ and $P(F_2|U_4) > P(F_2)$ indicating GDP causing inflation at higher belief.
- $P(F_3|U_2) > P(F_3)$ and $P(F_3|U_3) > P(F_3)$ support both directional Granger Causality between inflation and GDP at greater level.

For the combinations inflation/GDP and inflation/export, we have:

- $P(F_1|V_2) > P(F_1)$ and $P(F_1|V_3) > P(F_1)$ supporting at better belief of inflation causing GDP;
- $P(F_2|V_1) > P(F_2)$ and $P(F_2|V_2) > P(F_2)$ supporting GDP lead inflation with higher belief;

- $P(F_3|V_1) > P(F_3)$ and $P(F_3|V_4) > P(F_3)$ support bi-directional causality of the two variables at higher superiority.

Lastly, the inflation/GDP and inflation/imports combinations resulted to:

- $P(F_1|W_1) > P(F_1)$ and $P(F_1|W_2) > P(F_1)$ indicate improved notion of inflation causing GDP;

- $P(F_2|W_1) > P(F_2)$ and $P(F_2|W_2) > P(F_2)$ resulting in superior belief of GDP causing inflation.

- $P(F_3|W_3) > P(F_3)$ and $P(F_3|W_4) > P(F_3)$ show higher supporting level of bi-directional causality for the two variables.

In summary, the combinations of inflation/GDP and GDP/export, and inflation/GDP and GDP/import are respectively having 4 and 6 supporting points. In the cases of inflation/GDP and inflation/export, and inflation/GDP and inflation/import, we have 6 and 6 supporting points respectively.

However, as a further study and a way of having more concrete views on the results in Table 9 and interpretations, an optimal assignment problem model was explored leading to generation of Table 6.10. See Table 6.10 and its accomplished computations, and Figure 6.35 gave the pictorial presentation of the results.

A simple inference on Table 6.10 is that the existence of unidirectional Granger causality between inflation and import (or_export), there is also higher conditional probability supporting the unidirectional Granger causality relationship between GDP and inflation. On the other hand, the existence of bidirectional causality between GDP and import only, there is better conditional probability supporting both directional Granger causality between GDP and inflation.

For the coefficient of variation (CV) relationship, which is another approach of study in Phase 2, Table 6.11 was formed to generate the results of the means, standard deviation and CV for exports and imports. Still on Table 6.11, we have combined results of Granger causality of GDP/inflation and the CV for the purpose of comparison (as a way of linkage).

Table 6.12 is utilised to make a concrete decision by using binomial distribution test on the results in column 10 of Table 6.11. Our probability tests supported smaller export volatility over import for both directional Granger causality whilst larger volatility of export over imports supported unidirectional Granger causality.

Figures 6.36 and 6.37 respectively present the linkages' charts on:

- The Granger causality results with an application to the prior and posterior conditional probabilities, and its relational connections to export and import via the assignment model.
- The relational linkages of import and export CV to the GDP and inflation's Granger causality.

Figure 6.36: The granger causality using a prior and posterior con probabilities results and its relational connection to export and import via the assignment models.

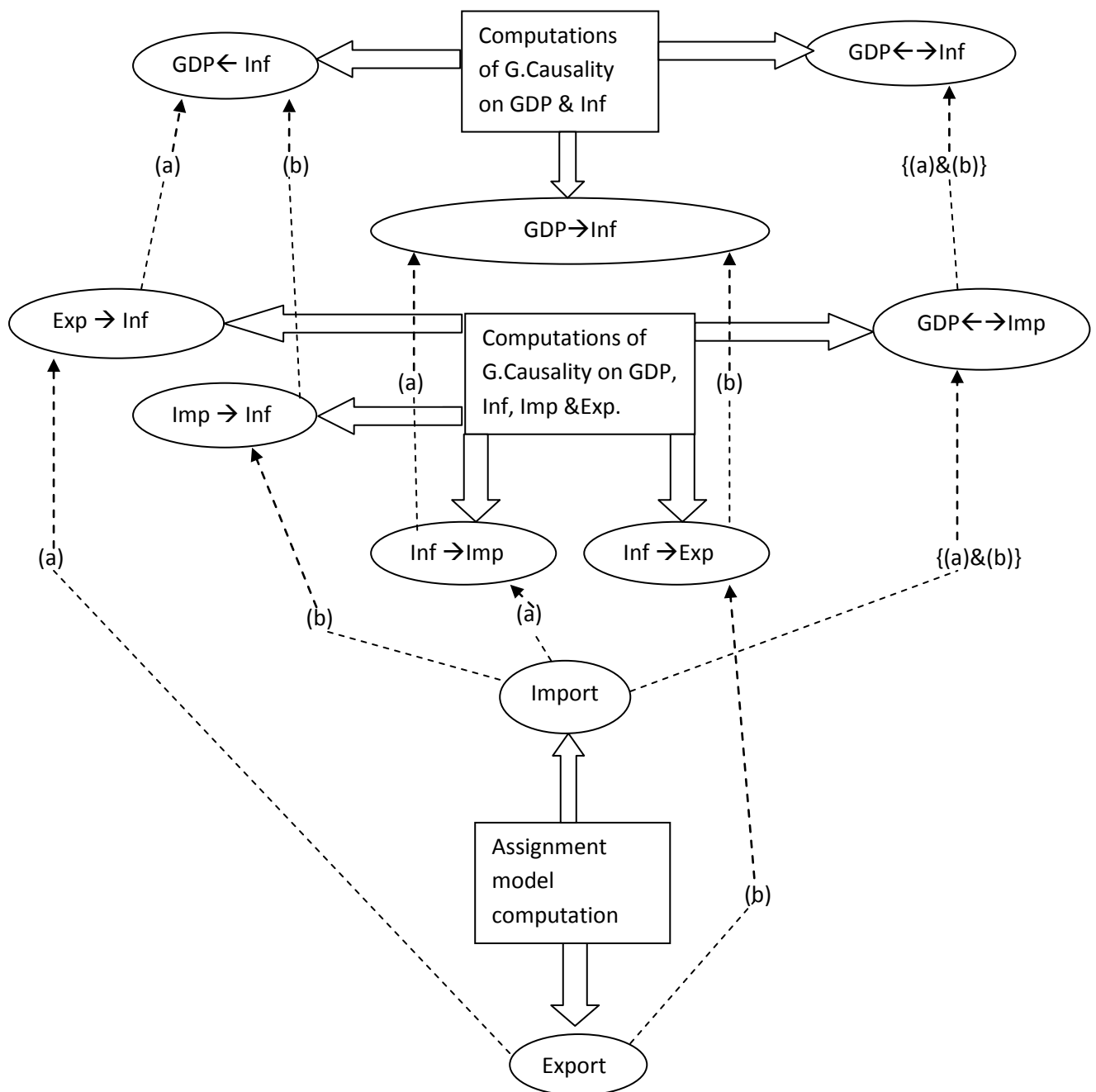
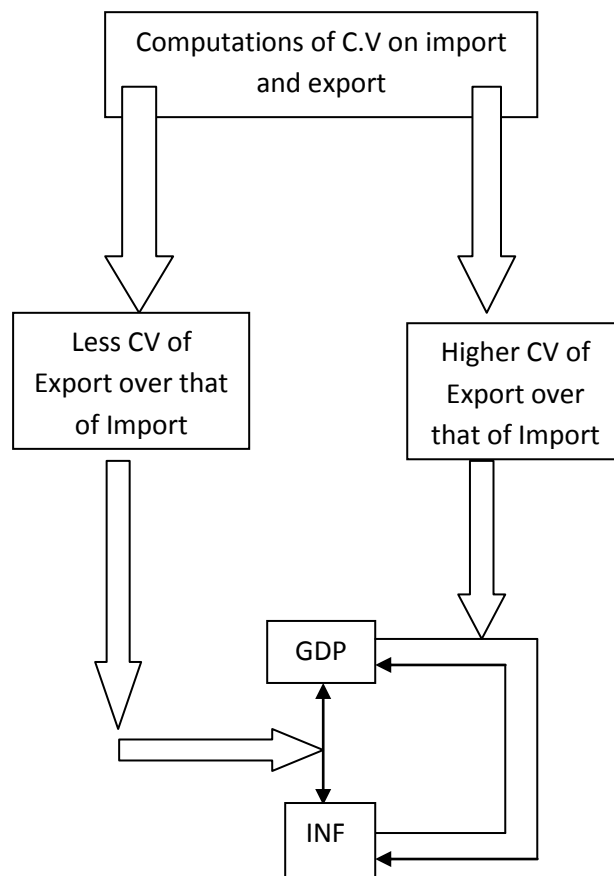


Figure 6.37

The rational linkages of import and export CV to the GDP and inflation's Granger causality



Key: \Rightarrow = relational linkage

\rightarrow Or \leftarrow = unidirectional G.causality

\longleftrightarrow = Bi-directional G.causality.

C.V = Coefficient of Variation

N.B : As export and import are inputs or contributing factors to GDP and Inflation, the resultant effects of their impacts on GDP and Inflation are established using the CV and Granger causality statistical relationships. This shows less volatility of export CV over that of import induces better and

enhancing prediction in bi-directional Granger causality whilst higher volatility export over that of import enhancing better prediction in unidirectional Granger causality.

Chapter 7: Summary, Conclusions and Suggestions

7.0 Introduction

The main focus of the research centered on the study of relationship between GDP, inflation, export and import in terms of their interrelationships, linkages and causations. A number of statistical tools were utilized to examine and explore the said relationships.

The following subsequent sections discussed the summary and conclusion of the study; while the suggestions for further study are equally given in the chapter.

7.1 Summary of the study.

The study passed through various stages and this enabled us to present the summary in the following sequential order.

Chapter 1 opened up the study with the purpose and its relevance to some macro-economic variables of a nation's economy. The variables considered are GDP, inflation, export and import.

In the study, we venture to examine how these macro-metrics are related. The said variables are important and useful to the management policy of national income and expenditure of a country. A number of other related macro-variables were identified and how our research variables are affected by these other variables were studied. To be specific, the related variables are unemployment, balance of payment and exchange rate. A brief detail of these quantities were discussed. It was observed that these variables have rather irregular movements, which can be termed fluctuations with its accompanying characteristics of expansion, contraction, recession and recovery in the economy. These phenomena lead to the stochastic problems with their dynamic effects on our macro-variables.

Succinctly, the chapter ends with aims and objectives of the study, and the structure of the thesis.

In Chapter 2, we discussed the concept of the causality and its historical background. Here, we were able to present a number of schools of thoughts by philosophers on causation. Among them, we have Aristotle, who identified four causes; Newton, Romes (1962), Hoover, Wiener-Granger on cause-effect and so on.

Also, formal definition of Granger causality was presented; and necessary and relevant survey on historical research was made. From our survey, we could see that various methods of Granger causality were utilized and there were no uniform outcomes on the Granger causality of the relationship. In addition, we stated our own new idea of exploring the statistical linkages of exports and imports to the Granger causality of GDP/inflation.

Chapter 3 considered the stationary and non-stationary concepts with other related issues. The resultant effects of non-stationarity on time series analysis were identified. The effects include spurious regression, wrong estimates, incorrect decision and even invalid forecasts.

Formal definitions of stationarity in terms of strict and weak stationarities were given. Here, we were able to learn that weak stationarity is applicable and amenable to empirical study; while the strict definition is difficult to apply in practice because of being defined in terms of the distribution function.

Further, the instruments for testing the non-stationarity were stated. Among them we have ADF, PP, KPSS, Chow and Quarndt tests with the appropriate methods of transforming non-stationary to stationary. Also, the determination of the lag length was presented.

Granger causality methods and other utilized statistical analyses and tests in the study were described in Chapter 4. Here, we identified four possible outcomes of Granger causality. Then the discussion on the procedural steps of simple standard, multiple linear and vector auto regression methods of the Granger causality were presented. To have a set of inferences on the study, a number of statistical analyses and tests were discussed and later carried out in Chapter 5. They are the proportionality, normality, binomial

and Chi -square test. Others are the Bayes' role and coefficient of variation computations, and the application of assignment problem model.

Chapters 5 and 6 embraced methodological/empirical description of the study which consists of data description, research methodology, results/findings and interpretation of the study. We utilized secondary time series data from 1970 to 2011 for our variables. The data are sourced through internet, from IFS, IMF, World Bank development, HIS global insight and etc.

On the methodological part, the time-plots of the variables on the selected countries, in the two phases, were drawn in order to see the movements. We observed irregular movement which can be termed fluctuations.

To verify and take necessary action on the said fluctuations, a number of stationary tests were carried out. Among them, we have the ADF, PP, KPSS, Chow's, Cusum and Quart tests. The outcomes indicated integration order of $I(0)$ and $I(1)$ in phase1 whilst $I(1)$ and $I(2)$ in Phase2. Differencing and/or de-trending were executed on them to make these non-stationary variables stable and stationary.

Next, we computed the Granger causality for both phases with results in Tables 6.01 and 6.07 respectively for Phases 1 and 2. With Table 1, we further classified the outcomes into Tables 6.02 to 6.06. Necessary tests were carried out on the classified tables to have statistical inferences.

The inferences arrived at are for:

Table 6.03 - equality change of both Granger causality and non-Granger causality.

Table 6.04 - Inflation "Granger causes" GDP has highest chance for the total Granger causality.

Table 6.05 - difference exists between the developed and developing economics in terms of the Granger causality.

Table 6.06 – any possible difference in the pattern of Granger causality for the two types of economies.

While with Phase 2, we combined the Granger causality results of both phases to form Table 7. Then, from this table we generated Table 6.09 with an application of Bayes theory. The outcomes gave various combinations of Phase 2 (posterior) supporting Phase 1 (a prior) outcomes of the Granger causality. By a step further, Table 6.10 was formed to make a further inference. On this table, we utilized assignment problem model and we were able to establish that when there is unidirectional causality between inflation and import (or export), there is also unidirectional causality between GDP and inflation by the Bayes' Rule; and when there is bidirectional causality between GDP and import only, there is bidirectional causality between GDP and inflation.

Lastly on Phase 2, we utilized coefficient of variation computations with exports and imports and then related them to Phase 1.

See Tables 6.11 and 6.12 for the said computations. The resultant inference is that smaller exports volatility over imports supported bi-directional Granger causality whilst larger volatility of export over imports supported unidirectional Granger causality.

We go further to compare our results in Phase 1 with the papers of Girma (2012), Inyama (2013) and Ahmad-Joyia (2012). Our findings agreed with Inyama (2013) that there is no Granger causality between GDP and inflation in Nigeria. This confirms the robustness of our method and results (see page 134 for more details). But for Girma (2012) on Ethiopia, we could not discuss the issue of robustness because of differences in the data and period covered by the paper (go to page 132 for more details). However, we have bidirectional Granger causality between GDP and inflation whilst Girma (2012) comes up with unidirectional from GDP to inflation. Lastly, there is non-availability of data from Economic Survey of Pakistan for Ahmad and Joyia (2012). Hence, we could not compare our results

with the paper.

As the above discussed papers are concerned with some countries, we endeavor further to compare the overall results of Phase 1 with the results of Paul, Kearney and Chowdhury (1997). Paul, Kearney and Chowdhury (1997) utilized 70 countries and came up with 60 per cent of the countries supported all types of Granger causality; while we used 65 countries and established 51 percent in support of Granger causality of all types. On equality of the two outcomes, a statistical test established no difference in the two results at 5% significant level.

7.2 Conclusions

The purpose of this study is centered and focused on “the relationship between GDP, inflation, export and import from a statistical point of view”. For this reason, we present the final conclusions in reference to our earlier stated objectives in Section 1.3.

In order to achieve the said aims, we utilised various statistical computations within the frame work of the conceptual and empirical studies descriptions. The descriptions consist of literature review, some statistical theories and concepts, methodology and empirical computations.

To be specific, we used the statistical relationship in terms of Granger causality concept and other statistical linkages to establish our purpose along the stated aims.

From objective (i), the pattern and nature of Granger causality between GDP and inflation, we used Tables 6.03 and 6.04 respectively for (a) Classification into Granger causality and non- Granger causality; (b) Distribution of Granger causality into types.

For (a), we used Table 6.03 which results to 33 and 32 respectively for Granger causality and non-Granger causality. The test on the said table supported equality of their proportions; that is, 50-50 chance of occurrence; and in (b), we have the following summary:

- 14 countries supported inflation “Granger causes” GDP (Case1);

- 10 countries supported GDP lead inflation (Case2); and
- 9 countries supported bi-directional Granger causality (Case3).

With the statistical test on Table 6.04, the test supported Case 1 with probability (14/33 or 0.472) as the highest. It implies that Case 1 has the chance of occurring most often than the others.

For Objective (ii), we utilized Table 6.05 with the appropriate statistical test. The test supported existence of difference between developed and developing economies in terms of Granger causality; because developed and developing economies has the ratio of 21:12 (or 7:4) in their distributed numbers.

In the case of Objective (iii), some tests were applied on Table 6.06. The developed economy pattern of Granger causality is uniformly distributed whilst skewed towards inflation “Granger causes” GDP for the developing economy with highest frequency.

The Objective (iv) arose from Table 6.07 to get the Granger causality on pair wise combinations of GDP, inflation, export and import. It is to be noted that we have this in Phase 2.

In summary, the outcomes from Table 6.08 supported:

- GDP/Exp with 10 Granger causality and 23 non-Granger causality;
- GDP/Imp with 15 Granger causality and 18 non-Granger causality;
- Inf/Exp. with 9 Granger causality and 24 non-Granger causality; and
- Inf/Imp with 16 Granger causality and 17 non-Granger causality.

Objective (v) used the concept of Bayes’ computations and tree diagrams to make its inference (see Table 6.09). The Granger causality computations made in Phases 1 and 2 respectively represented the prior and posterior probabilities. The outcomes gave a long possible list as stated in Chapter 6.

On Objective (vi), we make use of Table 10 for further analysis on the results in Table 9 by

applying the assignment problem model.

The outcomes are:

- an existence of unidirectional Granger causality between inflation and import (or export), there is also unidirectional Granger causality between GDP and inflation by the Bayes' rule; and

- the existence of bidirectional Granger causality between GDP and import only, there is also bidirectional Granger causality between GDP and inflation.

Lastly on Objective (vii), we obtained the coefficient of variation for exports and imports and related the results to the Granger causality in Phase 1. See Tables 6.11 and 6.12 for the computations and tests. The test supported:

- smaller export volatility over import for bidirectional Granger causality; and
- larger export volatility over import for unidirectional Granger causality.

However, it should be noted that Objectives (vi) and (vii) are the major contributions to the existing works on Granger causality.

Therefore, the possible applications of these objectives (vi) and (vii) on economic policy making for a country, can be summarized as thus:

- (1) . The relationship between G.causlity and Bayes' supportive principle –for [Objective (vi)] :

- The supportive principle of Bayes' rule is utilized to ascertain the results of Phase1 through Phase 2; and then apply an assignment model to have:

- (i) “uni-directional G.causality between inflation and import (or export)” has statistical evidence that “GDP and inflation has uni-directional G.causality”; and
- (ii) that “bi-directional G.causality between GDP and import only” statistically supports “bi-directional G.causality between GDP and inflation. The two [1(i)

and (ii)] can be used to have an enhanced prediction for economy policy on GDP and inflation;

(2)The relationship between coefficient of variation (CV) and Granger causality

(G.causality)- for [Objective (vii):

- Here, we discovered that the CV and G.causality can be used to predict the inter-dependence and inter-implication between GDP, inflation, import and export as follows:

(i) If $\left(\frac{CV_{export}}{CV_{import}}\right) < 1$, we have statistical evidence that GDP and inflation has bi-directional G.causality.

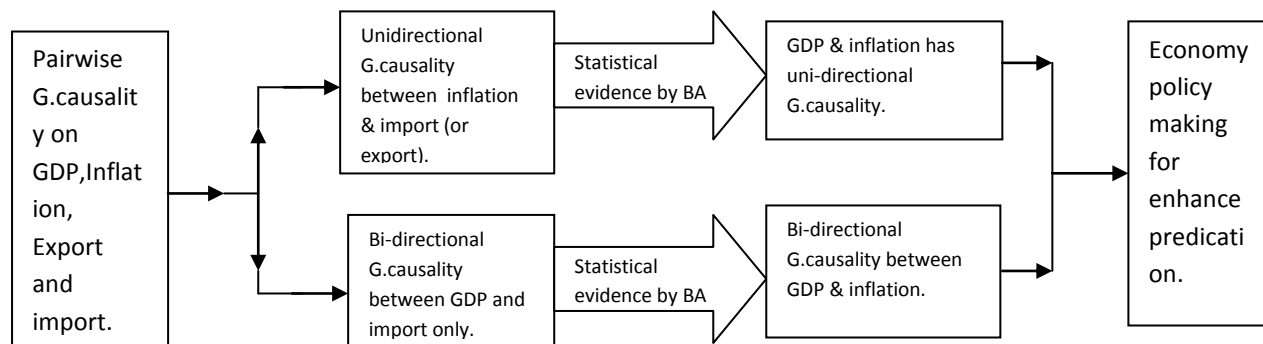
(ii) If $\left(\frac{CV_{export}}{CV_{import}}\right) > 1$, we have statistical evidence that there is uni-directional G.causality between GDP and inflation.

Both 2(i) and (ii) can be useful weapons for enhanced prediction policy on economy.

See Figure 38 (a) and (b) for Objectives (vi) and (vii) respectively.

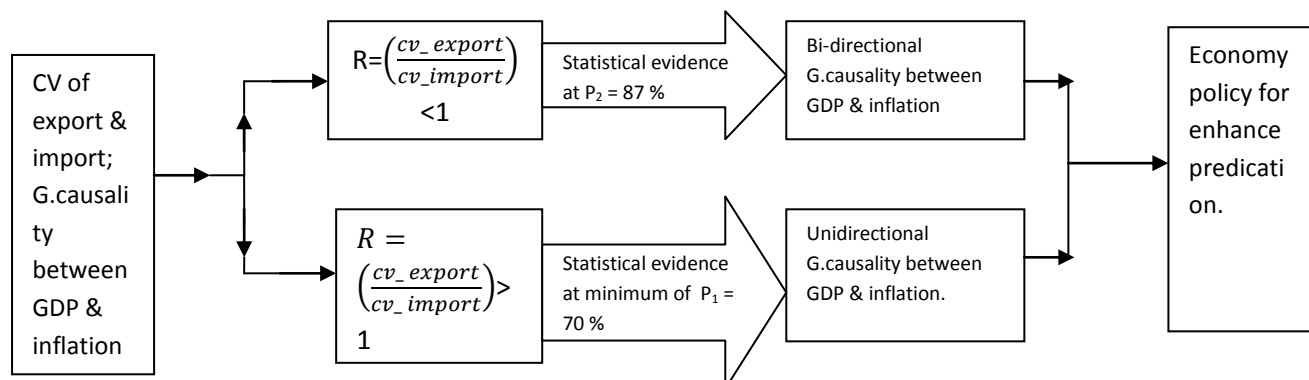
Figure 6.38; Economic meaning and Policy making of the Study

(a) The relationship between Granger causality and Bayes' supportive principle.



where BA stands for Bayes and assignment model optimization. [See Tables 6.09 (a), (b), (c), (d), and 6.10]

(b) The relationship between coefficient of variation (CV) and Granger causality (G.causality).



Where P_1 represents the probability of unidirectional with $R > 1$, and P_2 representing the probability of bidirectional with $R < 1$. Note that P_1 and P_2 are tested at 5 % significant level. (See Table 12).

7.3 Suggestions

As we have stated that our study is an extension work on the Granger causality, it can be useful to explore further studies on other common components or constitutes that affecting these two variables, the GDP and Inflation.

Also, the disparity between the results of developed and developing economies in terms of Granger causality can be investigated further through more research studies.

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Data Sources:

(i) [[www.internationalmonetaryfund/HistoricalRealGDPvalues](http://www.internationalmonetaryfund.org/HistoricalRealGDPvalues)]

(ii) [[www.internationalmonetaryfund/HistoricalCPIvalues](http://www.internationalmonetaryfund.org/HistoricalCPIvalues)]

(iii) [http://esdc80.mcc.ac.uk/wds_ifs/TableViewer/tableView.aspx?ReportId=54780]
for import.

(iv) [http://esdc80.mcc.ac.uk/wds_ifs/TableViewer/tableView.aspx?ReportId=54721]
for export.

Appendix A

-(Source Codes of Granger Causality Test- in Matlab Function)

```
function [D1,D2,D3,D4]=GrangerCause(x,y,alpha,max_lag)
D1=0;
D2=0;
D3=0;
D4=0;
% function
[D1,D2,D3,D4]=GrangerCause(x,y,alpha,max_lag)determines the
% Granger Causality tests for autoregression of variable y on
variable x,and vice versa that of x on y respectively.
% Input Specifications:
% x --- is a column vector of time series data that must be
stationary.
% y --- is a column vector of time series data that must be
stationary.
% alpha is the significant level of the test.
% max_lag ---is the maximum number of lags to be considered.
% Outputs:
% These are Decisions on Granger Causality tests;where
D1,D2,D3 and D4 stand for
% Decision Statements with 0 as untrue statement and
otherwise, true.
% Points to Note:
% When all the outputs (D1,D2,D3 and D4) are zeros (0),it
means there is
% no Granger Causality in both ways of autoregression.
% The optimal lag length is determined by Bayesian Information
Criterion
% (BIC).
% Comparision is made between Computed value of F-Statistics
(F_cal) and F-Statistics table(F_table,at alpha;ie(c_v)) to
have the Decisions.

% The Programme:
% To ensure variables y and x are of the same length and also
of column
% vectors
if (length(y)~= length(x))
    error('y and x must be the same length');
end
% Now to column vector checking of y
[a,b]=size(y);
if (b>a)
    % y is a row vector -----fix to column vector
```

```

        y=y';
end
% To check for x
[a,b]=size(x);
if (b>a)
    % x is a row vector -----fix to column vector
    x=x';
end
% To determine the optimal number of lags using BIC
N=length(y);
Numobs=N;
nARmax=max_lag;
g=[y,x];
g0=g(1:nARmax,:);
g1=g(nARmax+1:end,:);
BICtest=zeros(nARmax,1);
for i=1:nARmax
    spec=vgxset('n',2,'constant',true,'nAR',i);
    [spec,specstd,LLF,W]=vgxvarx(spec,g1,[],g0);
    [NumParam,NumActive]=vgxcount(spec);
    BICtest(i)=aicbic(LLF,NumParam,Numobs);
end
[BICmin,nAR]=min(BICtest);
m=nAR;
% Determination of Data_lag for variables y and x
respectively.
A=DataLag(y,m);
B=DataLag(x,m);
T=length(A);
% Computation of autoregression for Unrestricted model of y on
x and
% determination of its RSS (RSS_UR)
CONS=ones(1,N-m)';
[b,BINT,R]=regress(A(:,1),[CONS,A(:,2:m+1),B(:,2:m+1)]);
RSS_UR=R'*R;
% Computation of autoregression for Restricted model of y on x
and
% determination of its RSS (RSS_R)
[b,bint,r]=regress(A(:,1),[CONS,A(:,2:m+1)]);
RSS_R=r'*r;
% Determination of F-Statistics (both for F_cal and F_table)
F_cal_1=[((RSS_R-RSS_UR)/m)/((RSS_UR)/(T-(2*m)))];
F_yx=F_cal_1;
c_v=finv(1-alpha,m,(T-(2*m)));
% Computation of autoregression for Unrestricted model of x on
y and
% determination of its RSS (RSS_UR)
CONS=ones(1,N-m)';
[b,BINT,R]=regress(B(:,1),[CONS,B(:,2:m+1),A(:,2:m+1)]);
RSS_UR=R'*R;

```

```

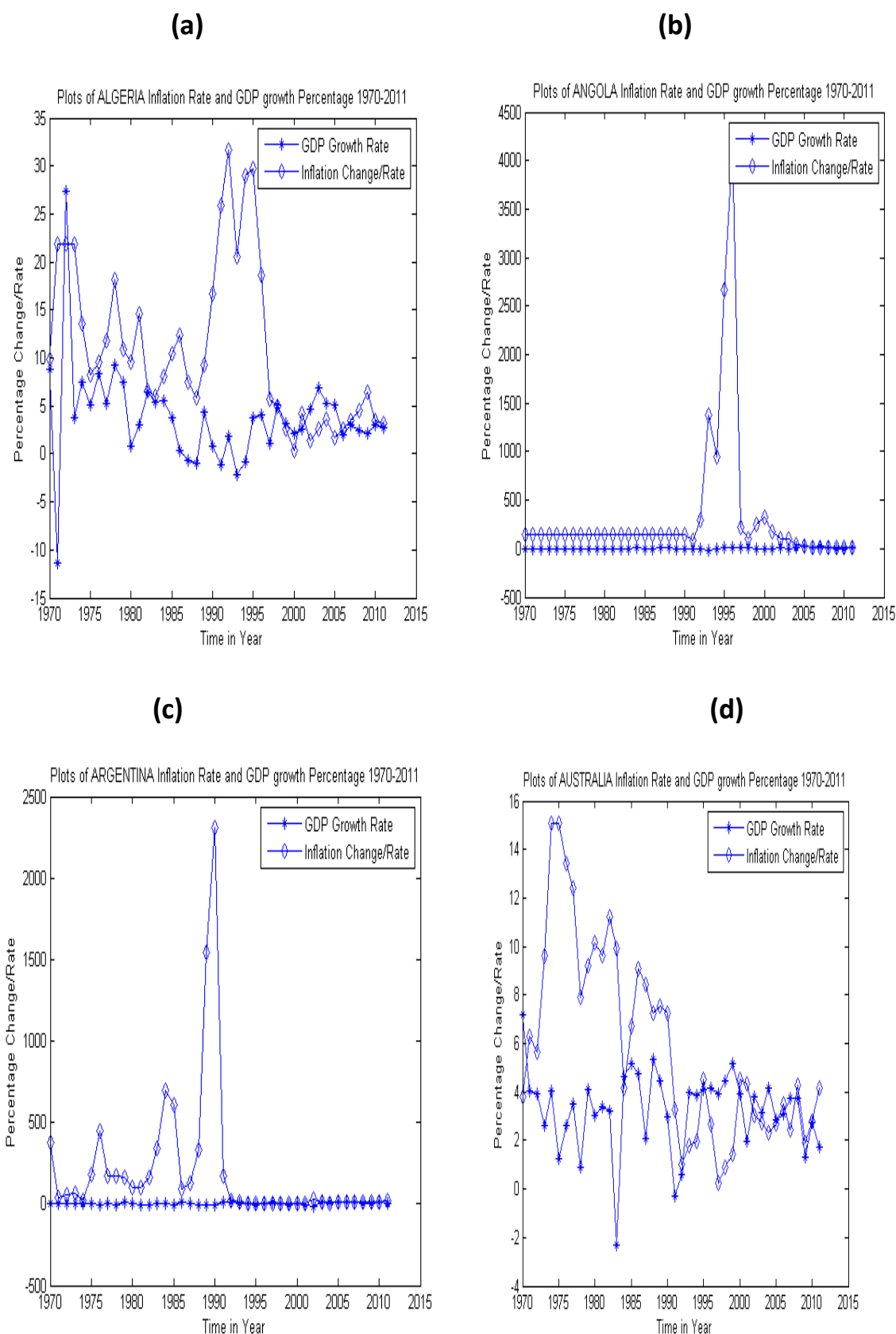
% Computation of autoregression for Restricted model of x on y
and
% determination of its RSS (RSS_R)
[b,bint,r]=regress(B(:,1),[CONS,B(:,2:m+1)]);
RSS_R=r'*r;
% Determination of F-Statistics (both for F_cal and F_table)
F_cal_2=[((RSS_R-RSS_UR)/m)/((RSS_UR)/(T-(2*m)))];
F_xy=F_cal_2;
% Decision on Granger cause
if abs(F_yx) < c_v;
    D1=('x does not Granger_cause y');
else;
    abs(F_yx) > c_v;
    D2=('x Granger_cause y');
end;
if abs(F_xy) < c_v;
    D3=('y does not Granger_cause x');
else;
    abs(F_xy) > c_v;
    D4=('y Granger_cause x');
end;
fprintf('Granger-causality results of y on x and x on y are
%d.\n',statement)
return

function Data=DataLag(y,n)
y=y(:)';
clear Data
for i=1:n+1
    Data(:,i)=y(n+2-i:end+1-i);
end
return

```

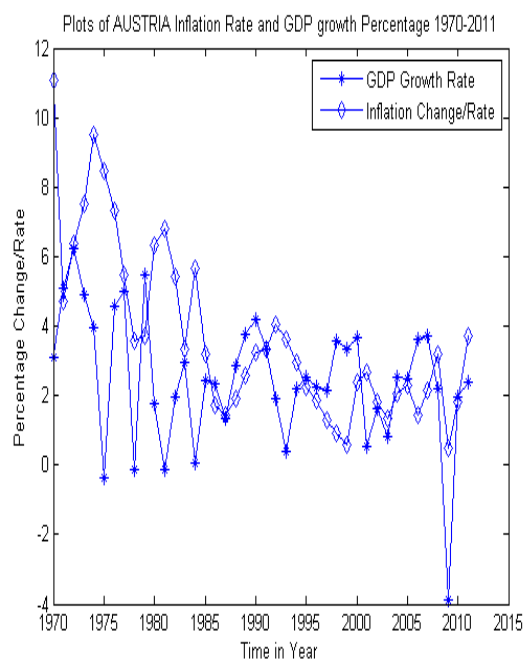
Appendix B (It covered Figures 1 to 25)

Appendix B.01: Figure 1

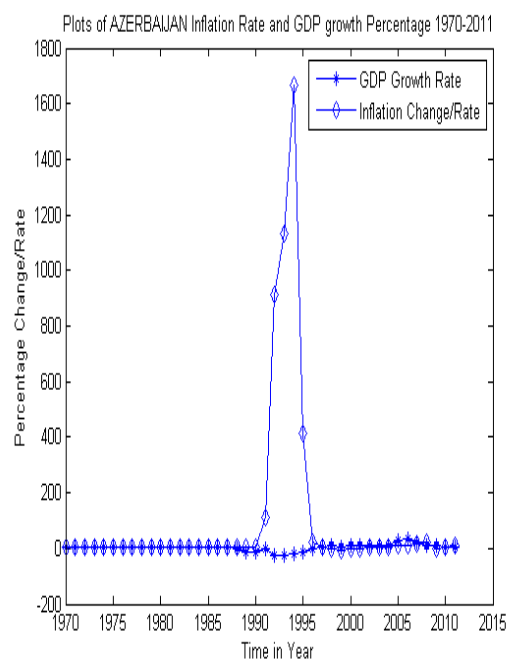


Appendix B.02 : Figure 2

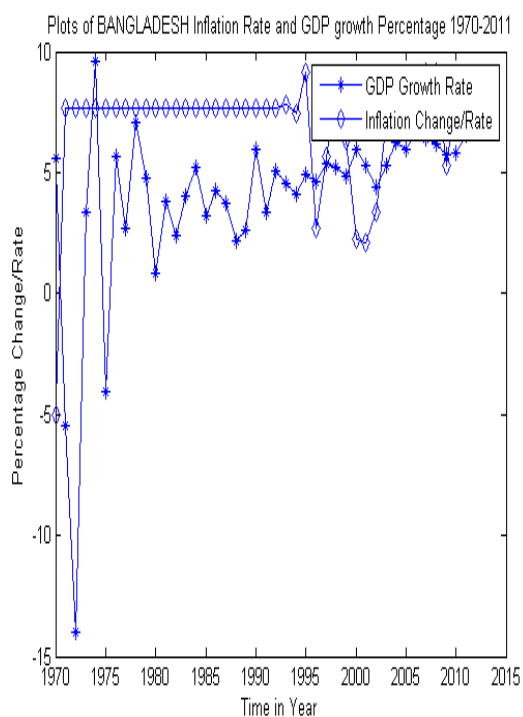
(a)



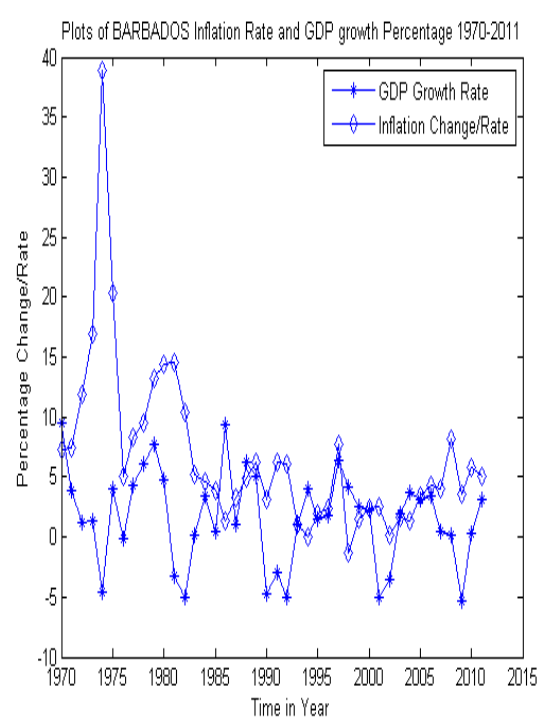
(b)



(c)

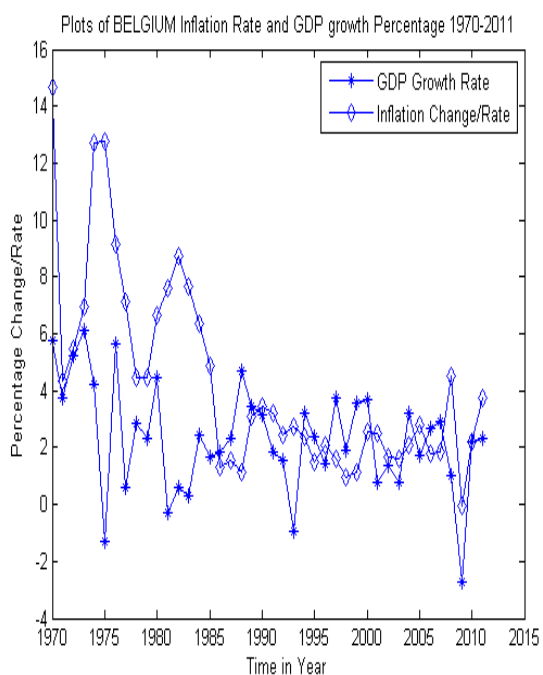


(d)

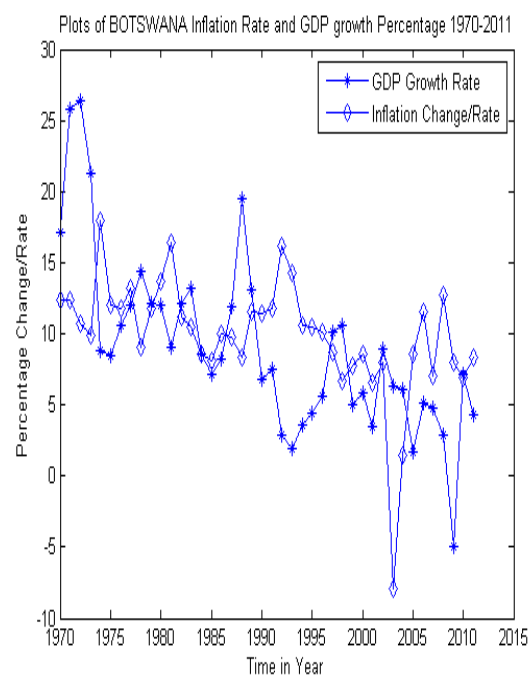


Appendix B.03: Figure 3

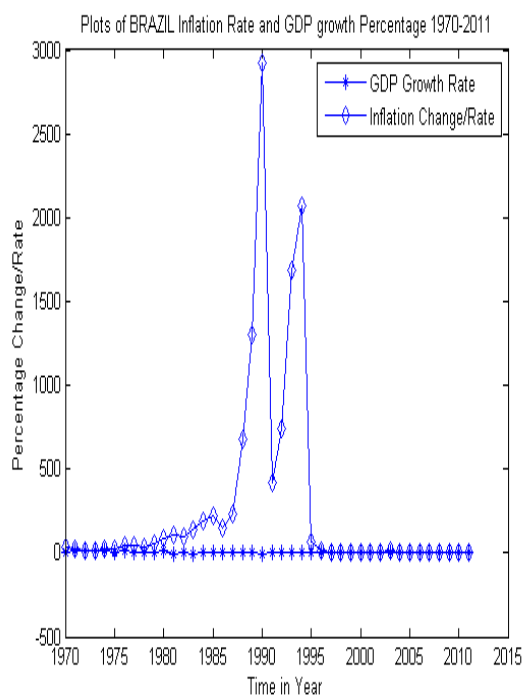
(a)



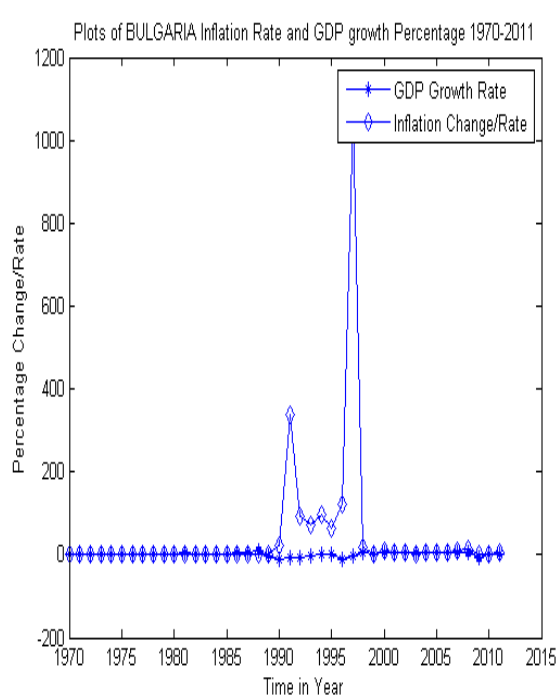
(b)



(c)

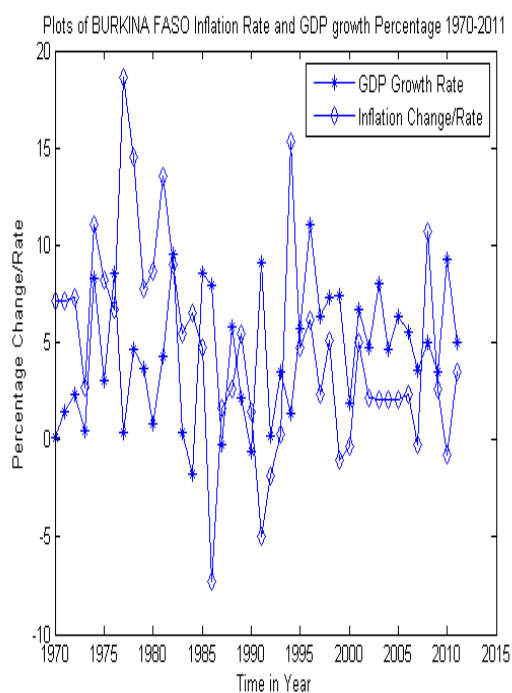


(d)

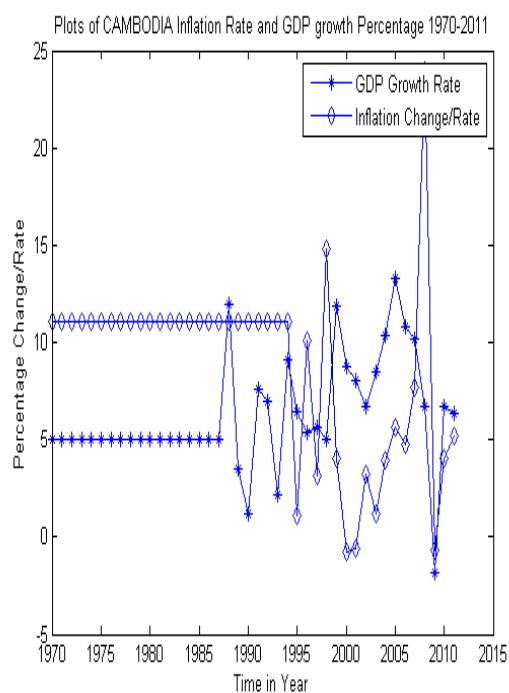


Appendix B.04: Figure 4

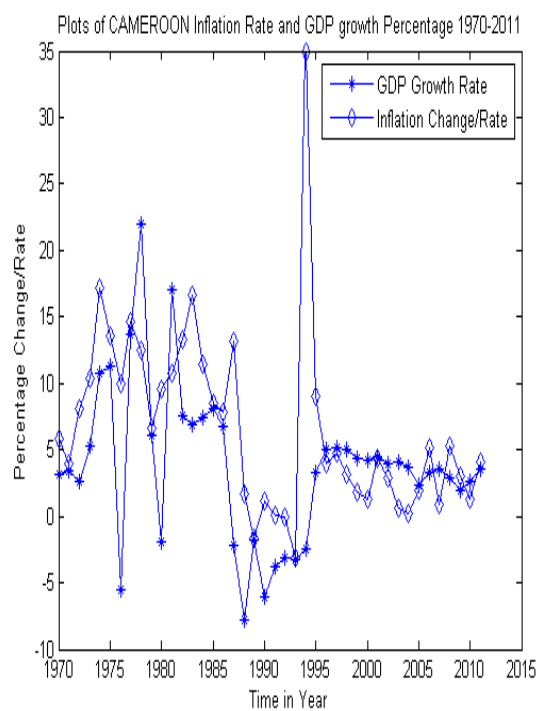
(a)



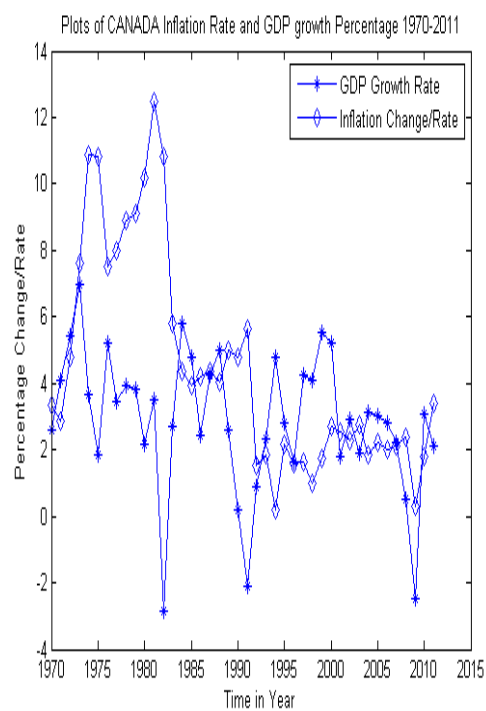
(b)



(c)

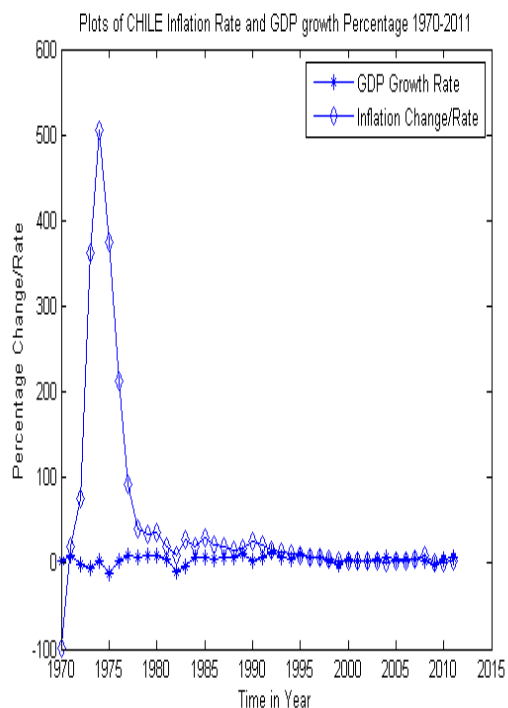


(d)

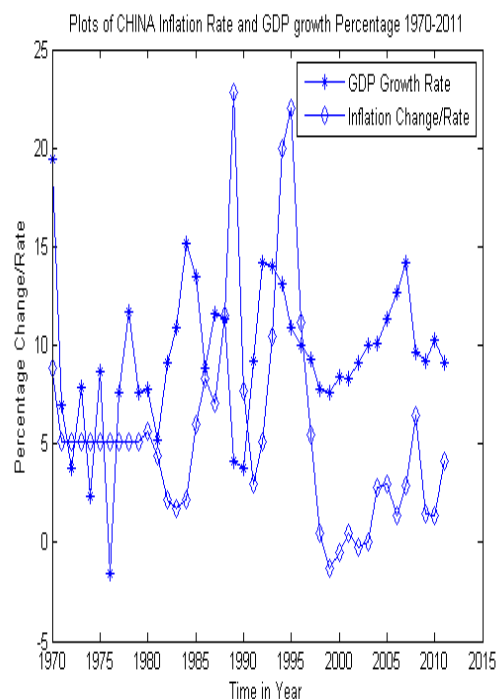


Appendix B.05: Figure 5

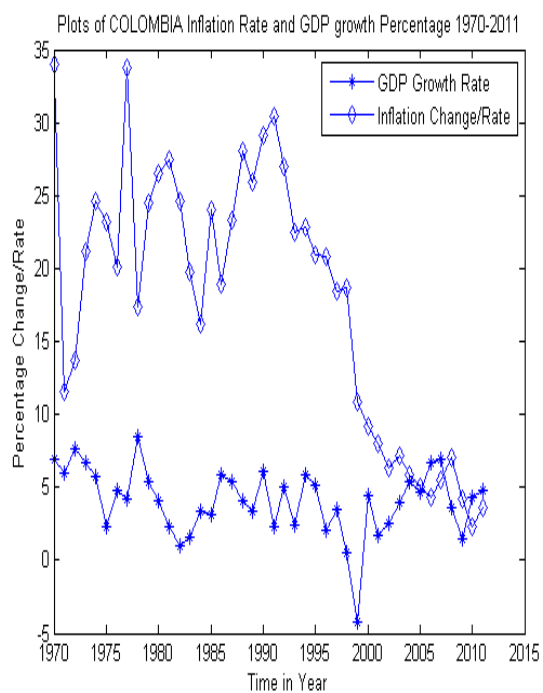
(a)



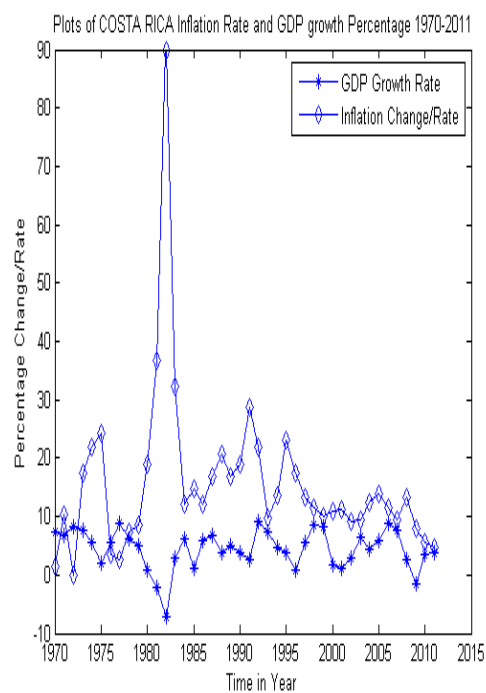
(b)



(c)

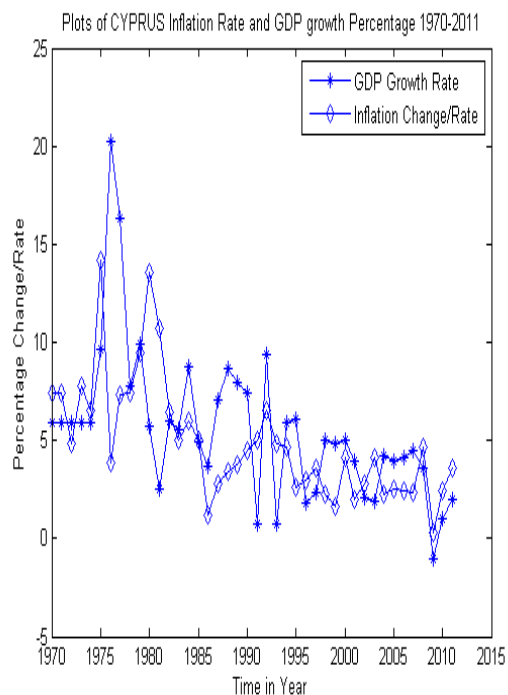


(d)

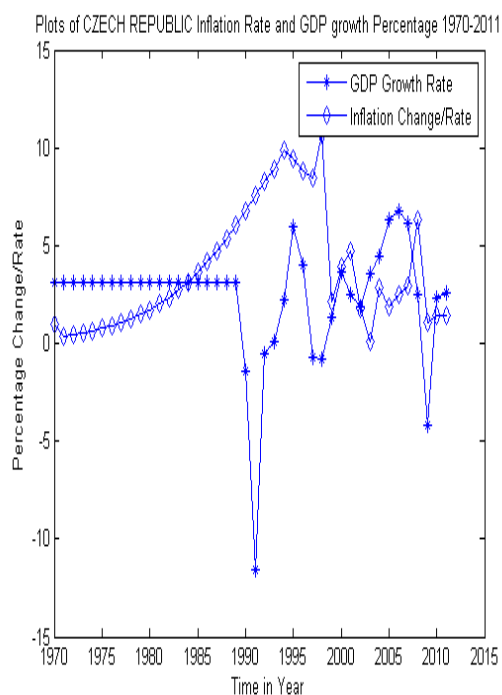


Appendix B.06: Figure 6

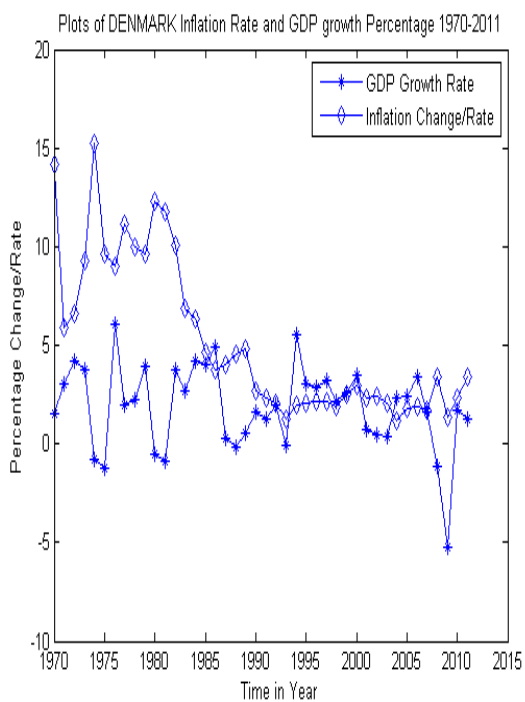
(a)



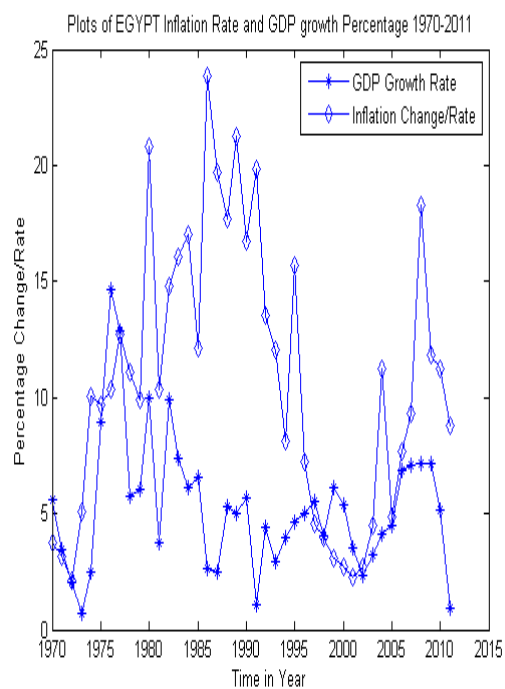
(b)



(c)

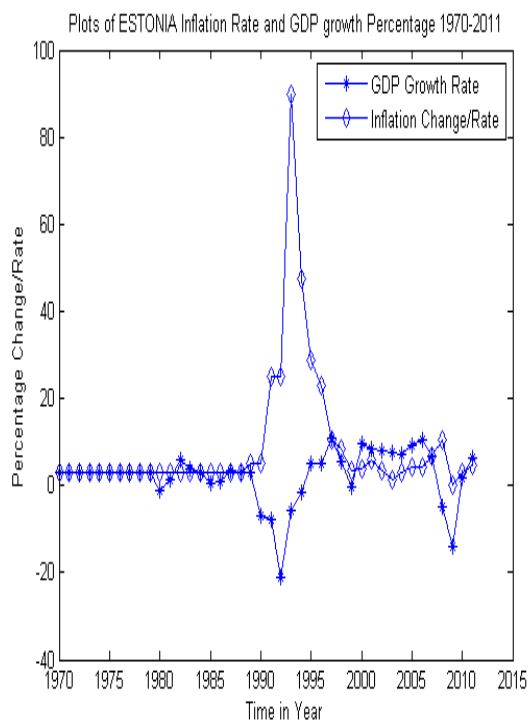


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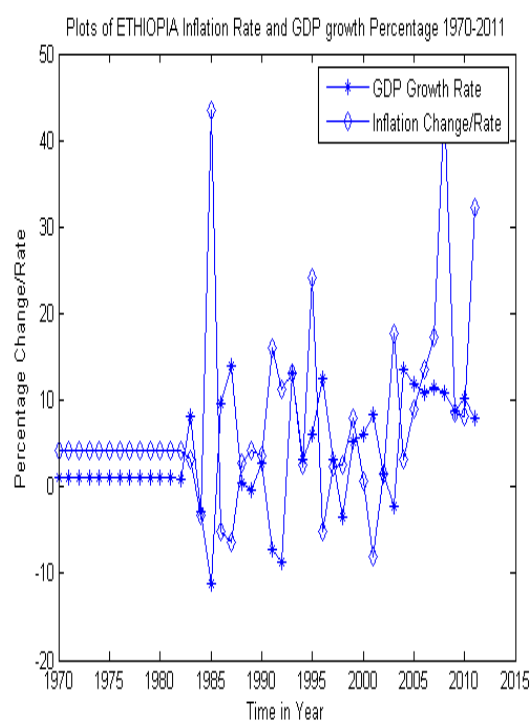


Appendix B.07: Figure 7

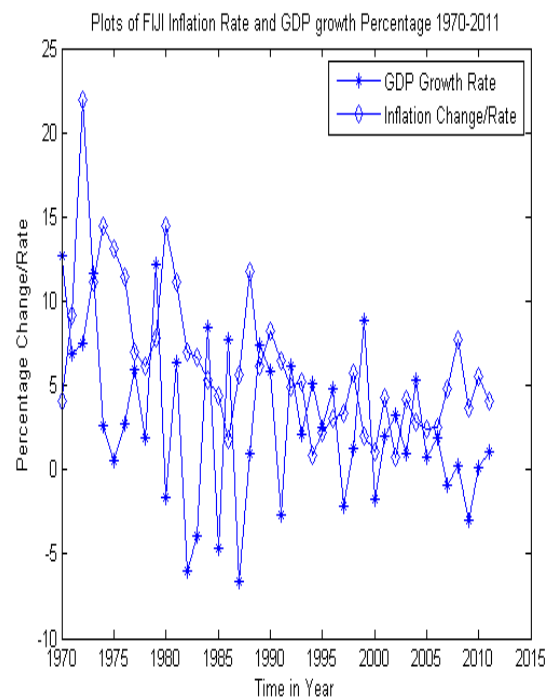
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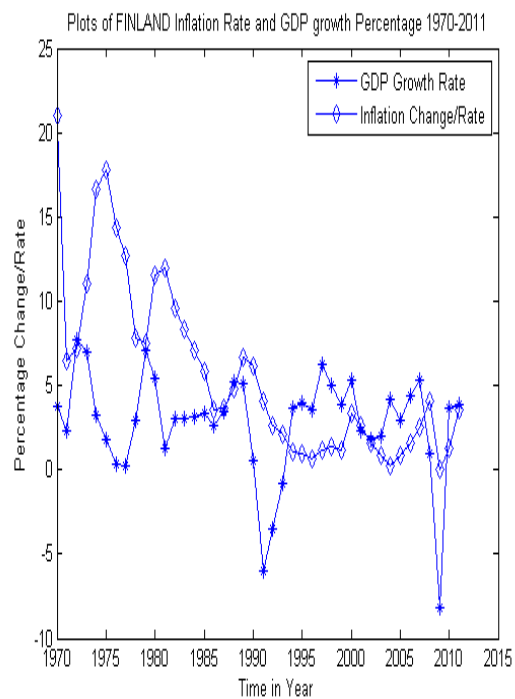
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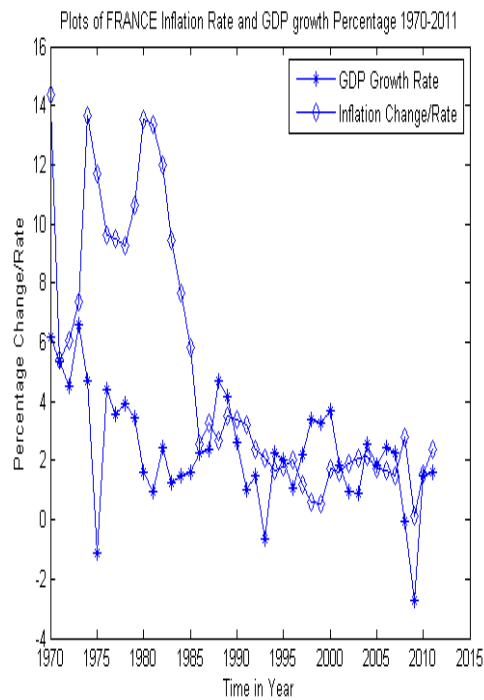


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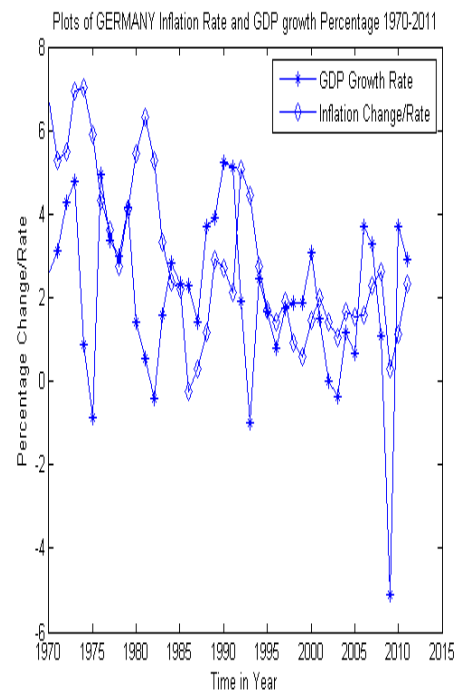


Appendix B.08: Figure 8

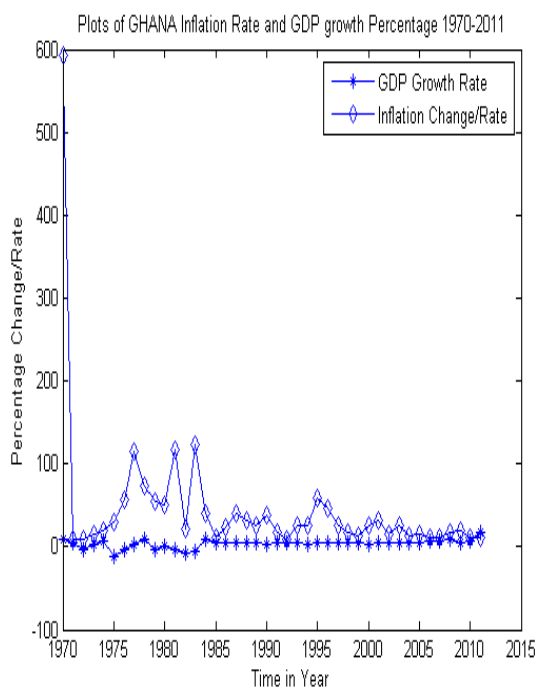
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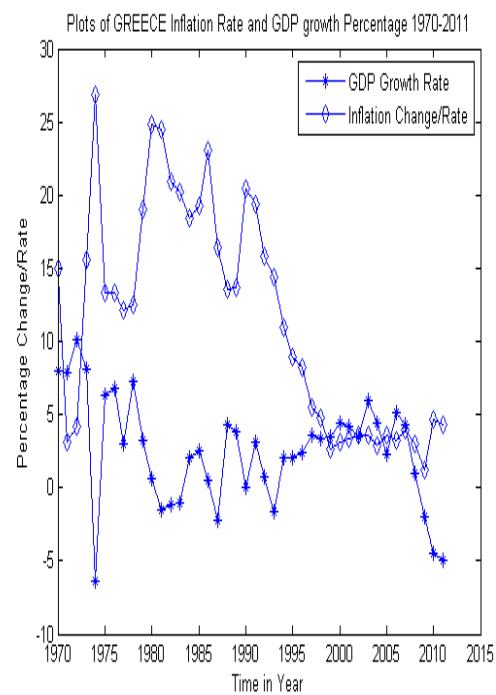
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(c)

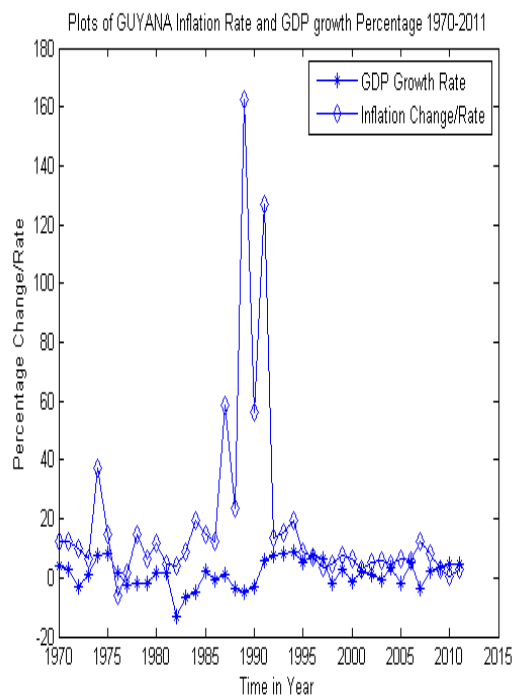


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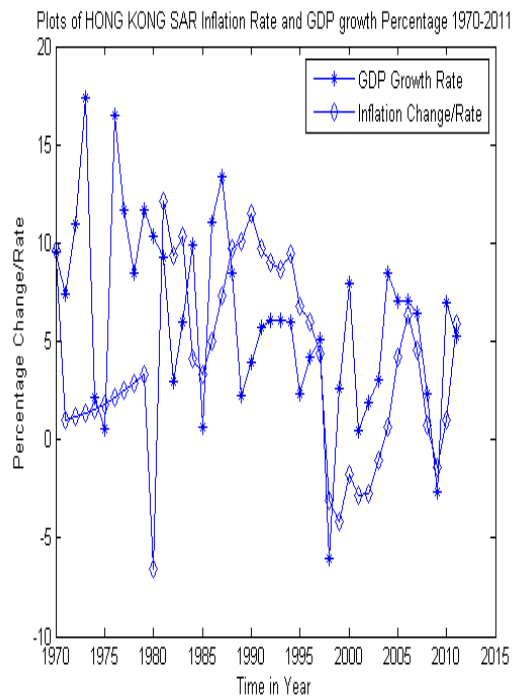


Appendix B.09 : Figure 9

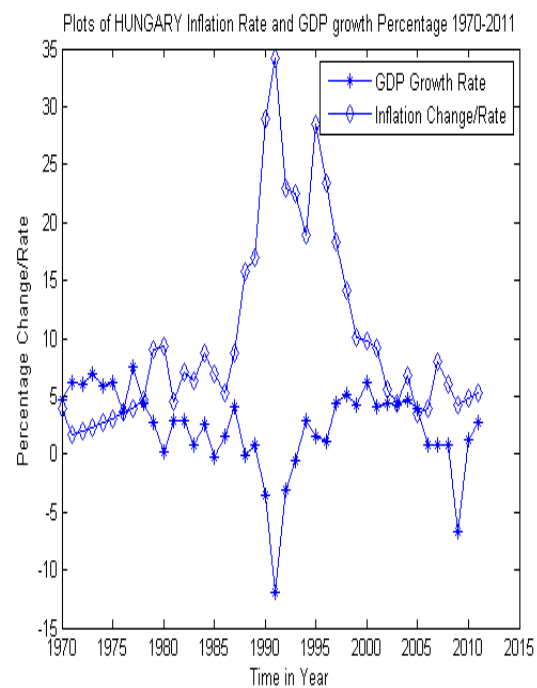
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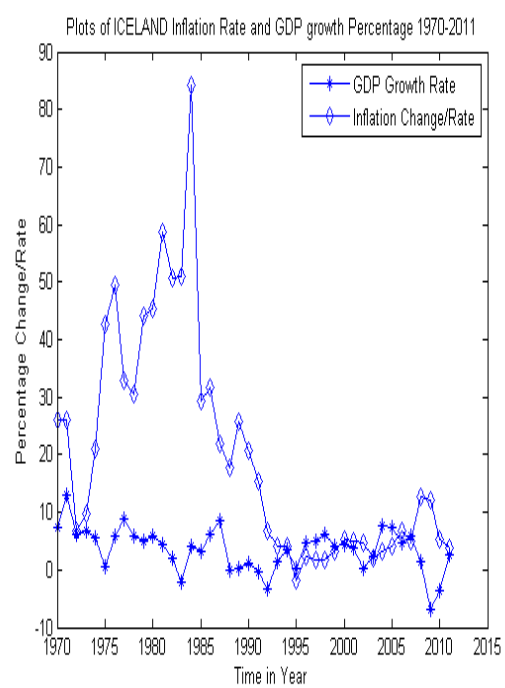
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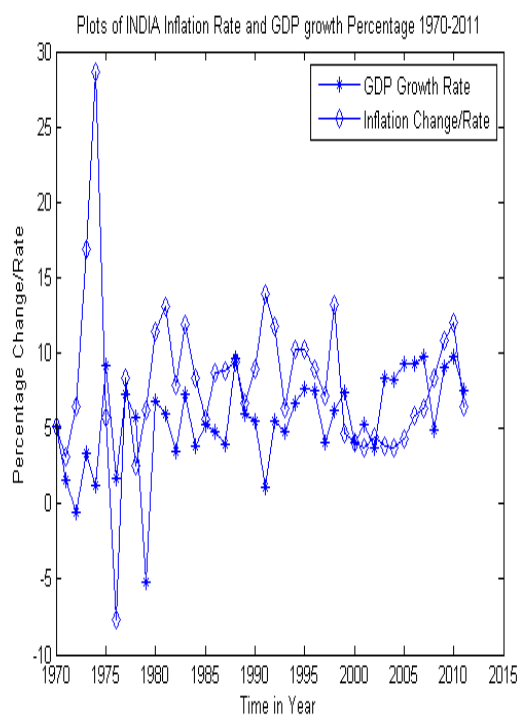


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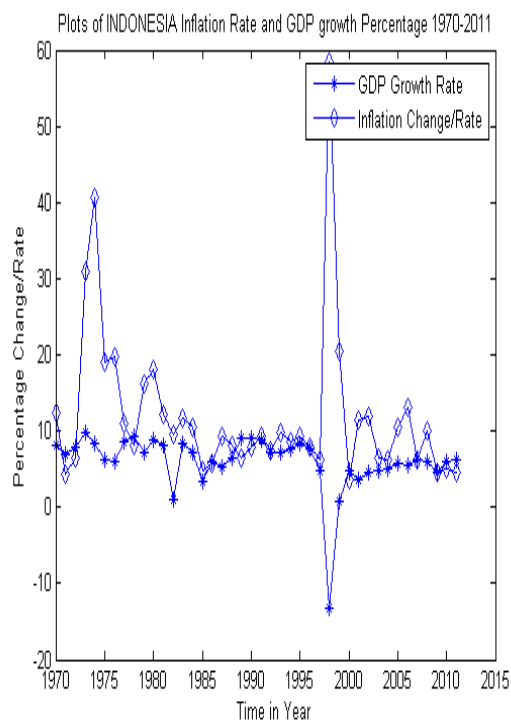


Appendix B.10: Figure 10

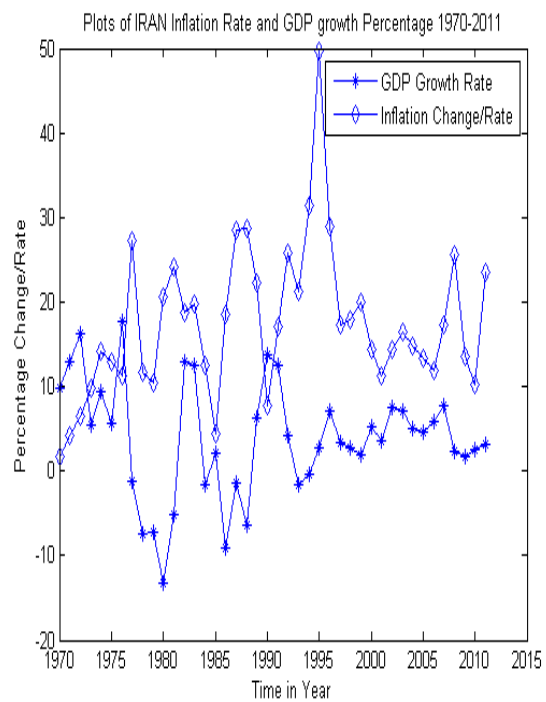
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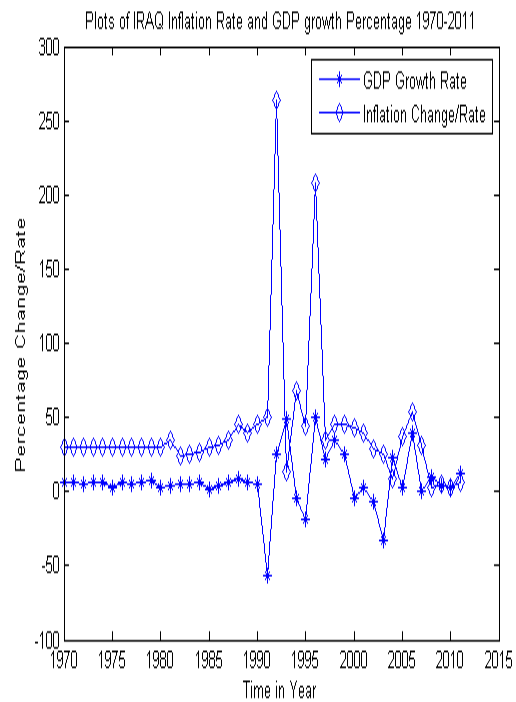
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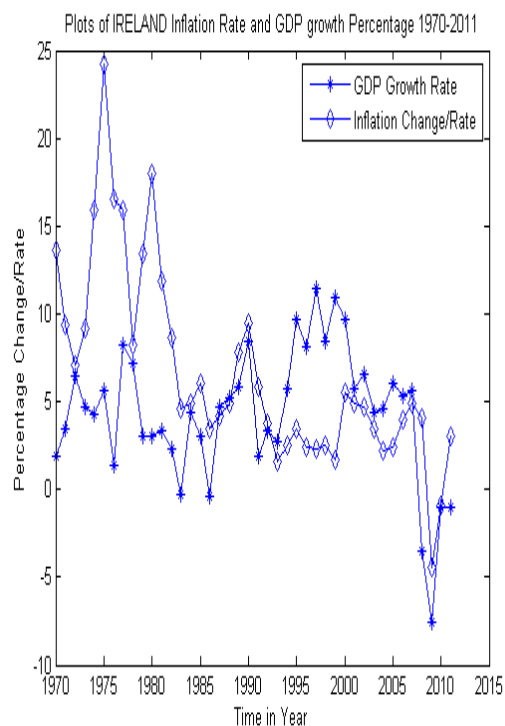


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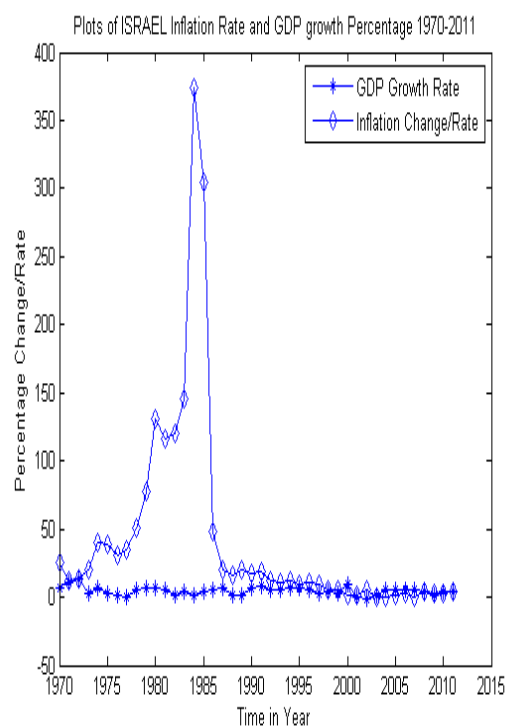


Appendix B.11: Figure 11

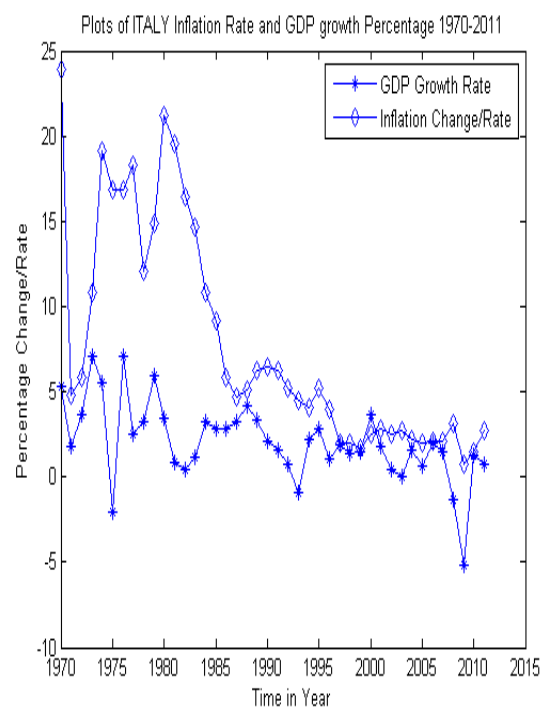
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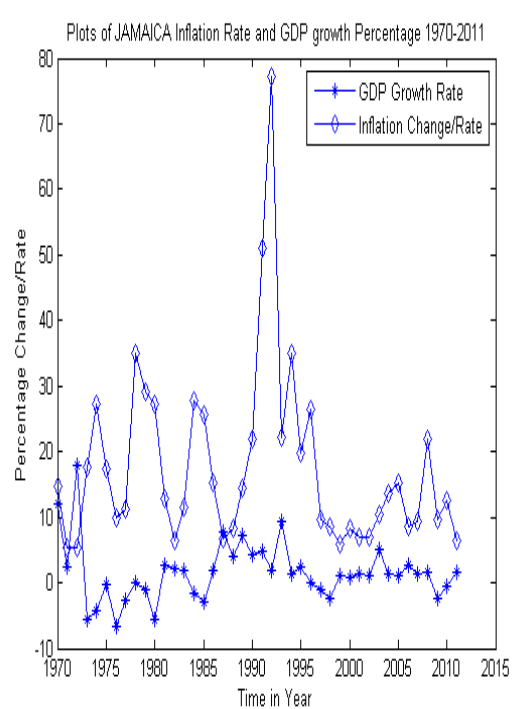
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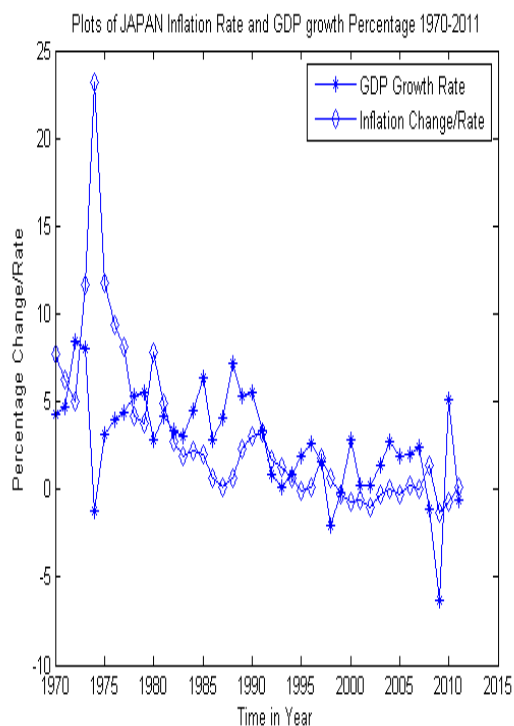


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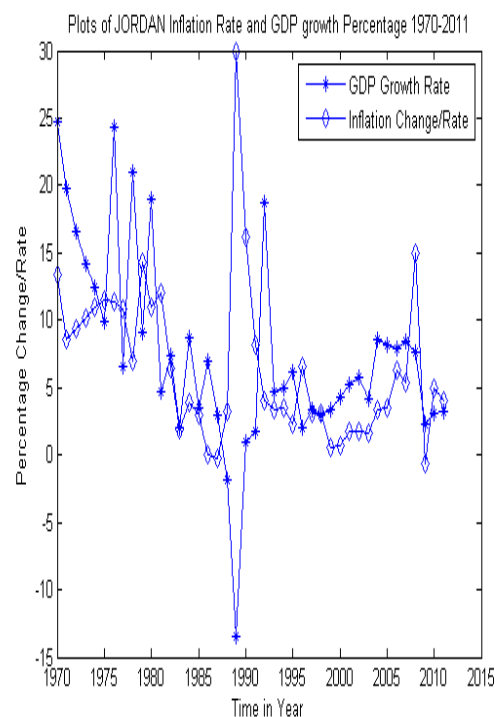


Appendix B.12: Figure 12

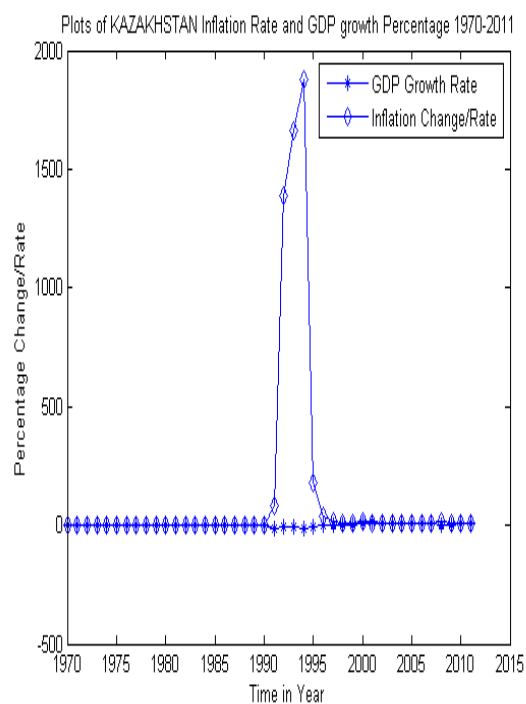
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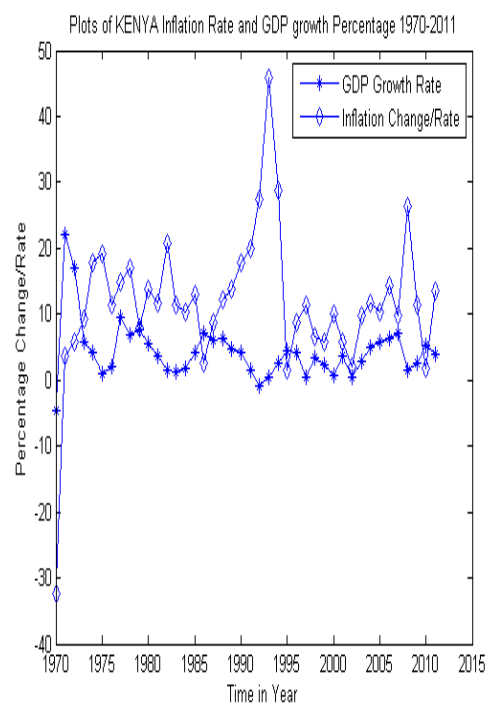
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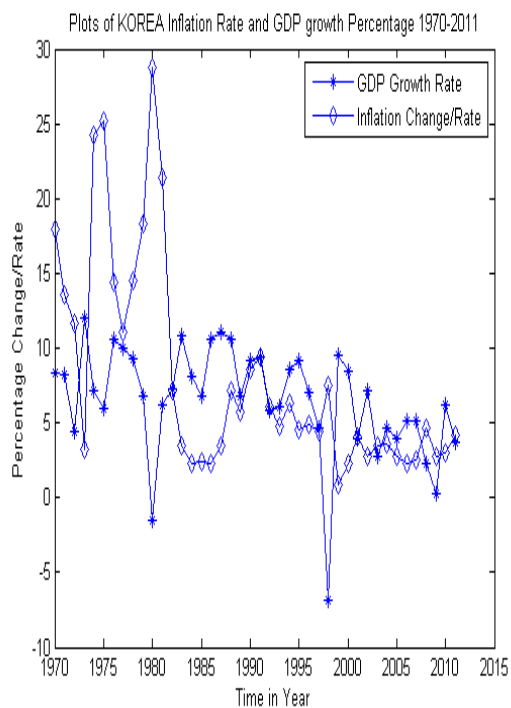


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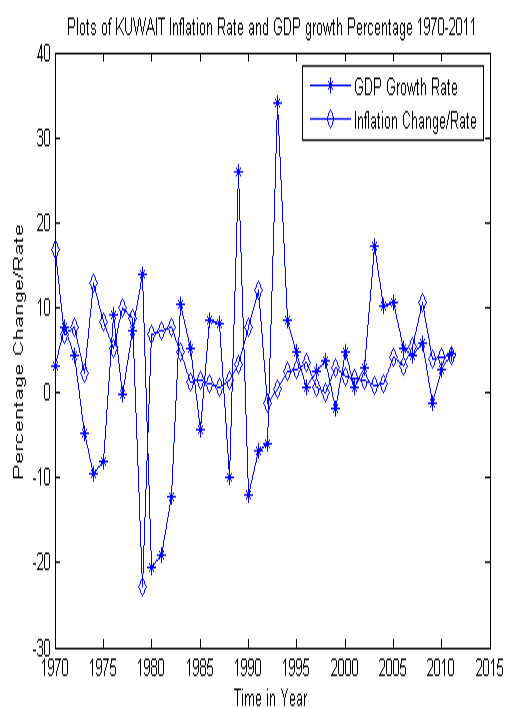


Appendix B.13: Figure 13

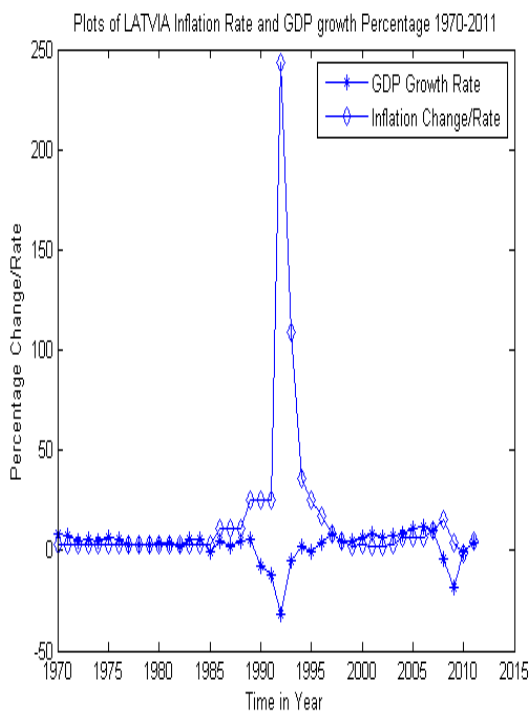
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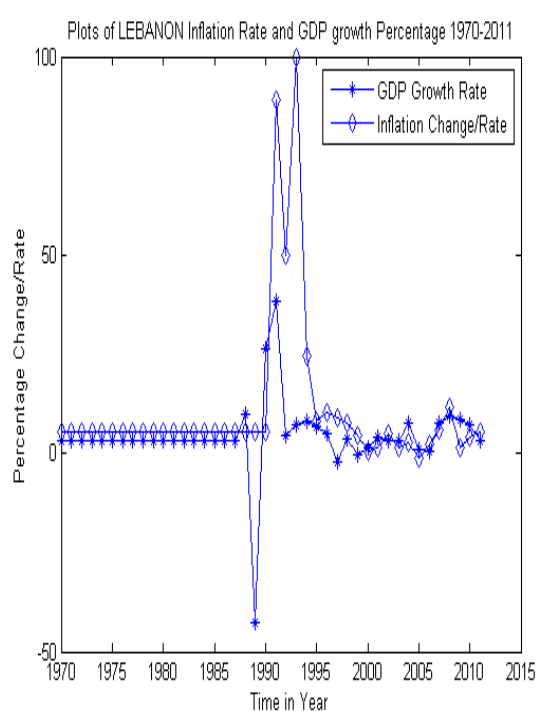
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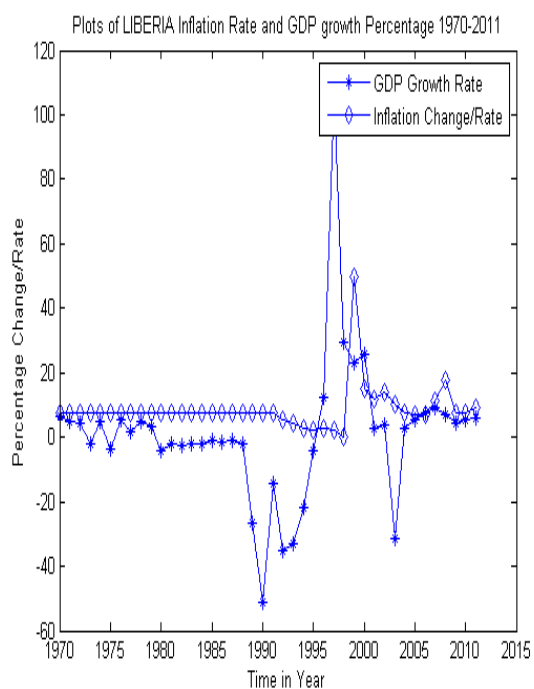


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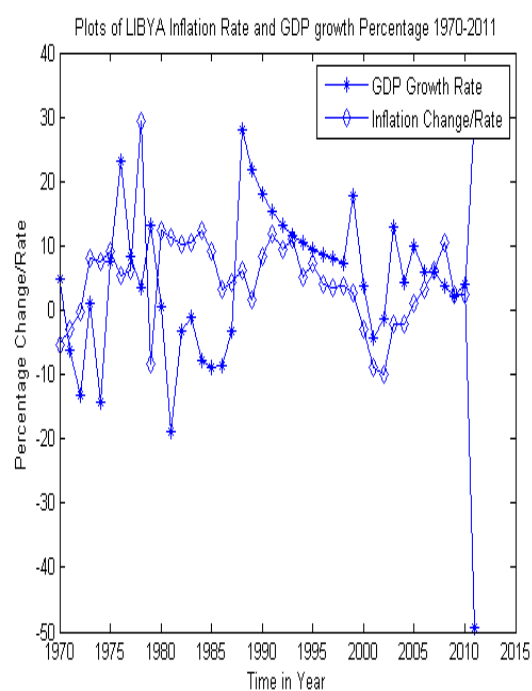


Appendix B.14: Figure 14

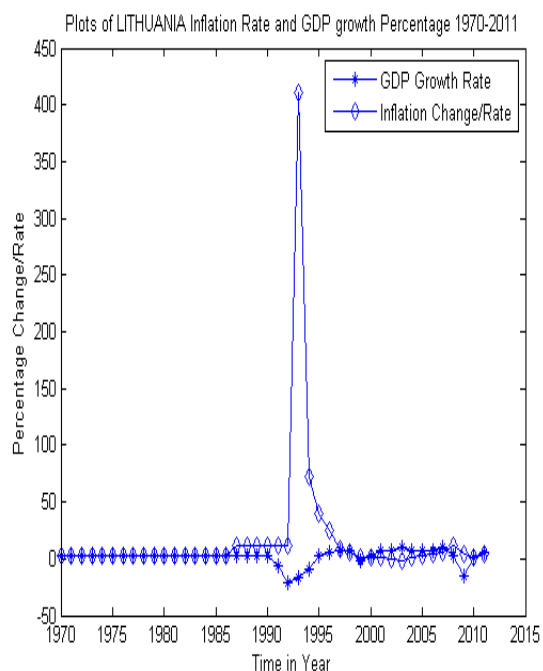
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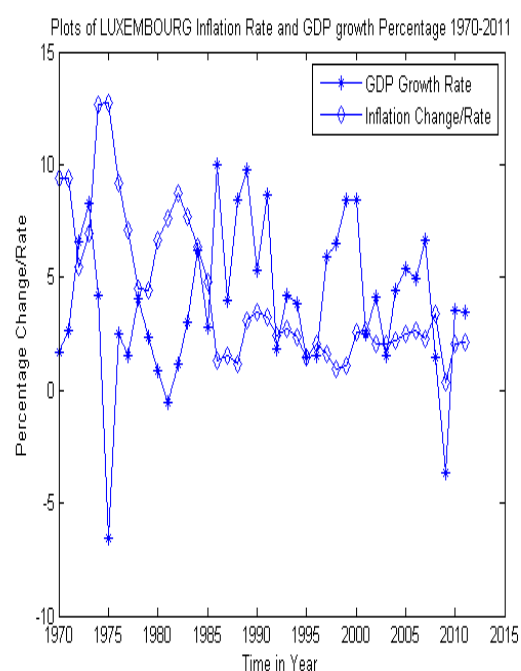
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(c)

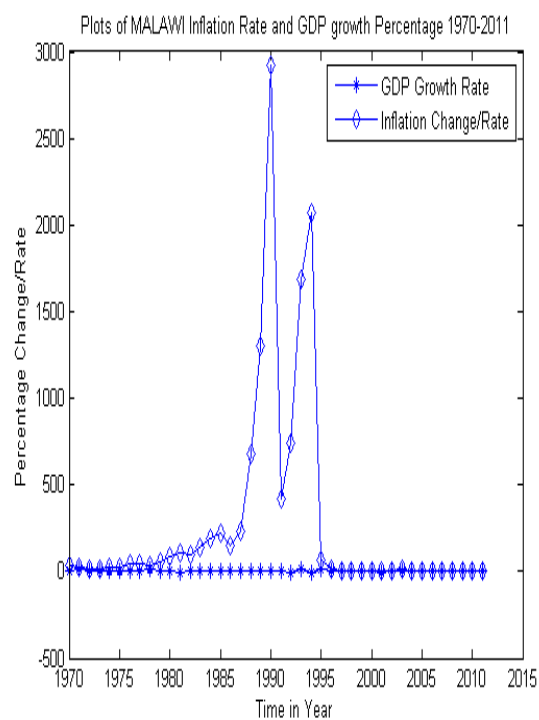


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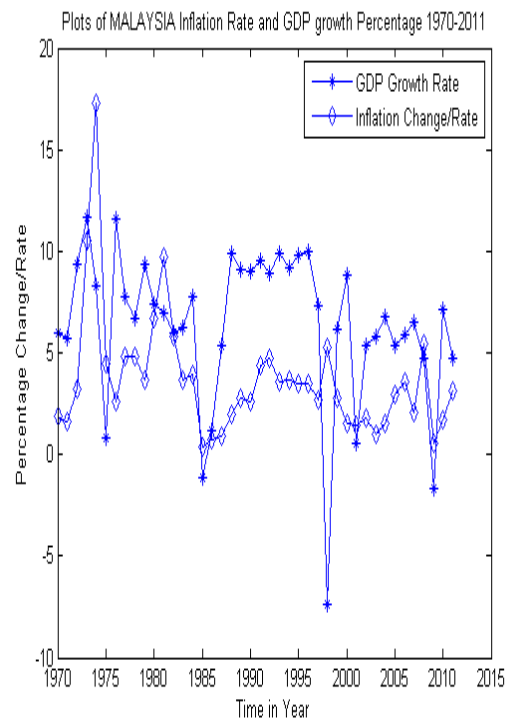


Appendix B.15: Figure 15

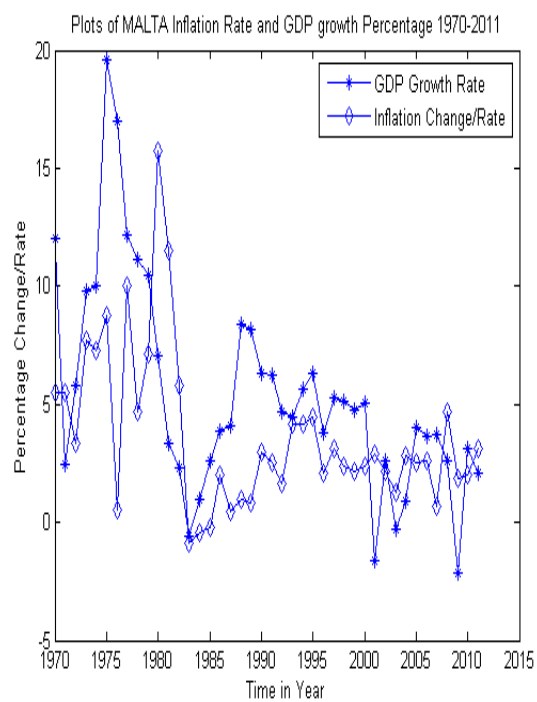
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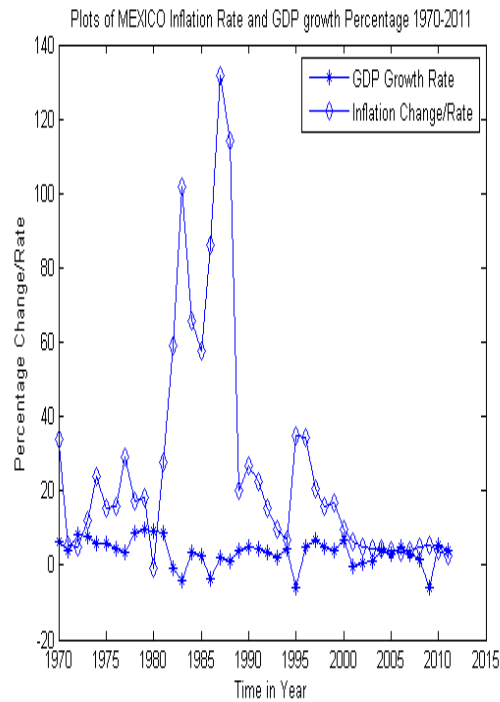
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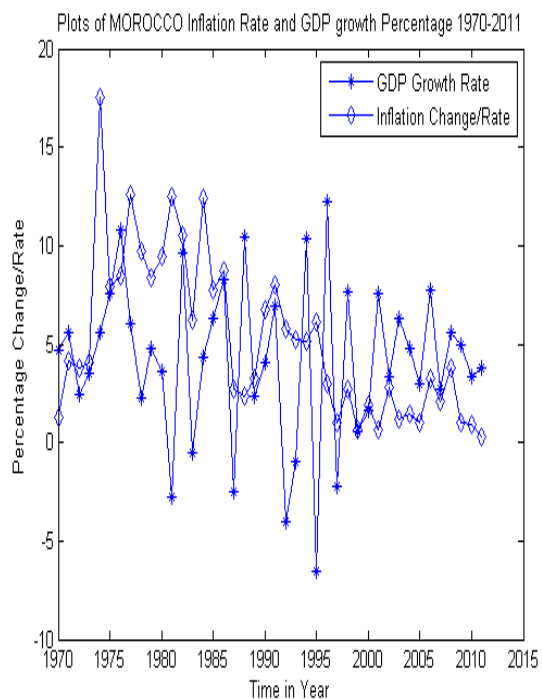


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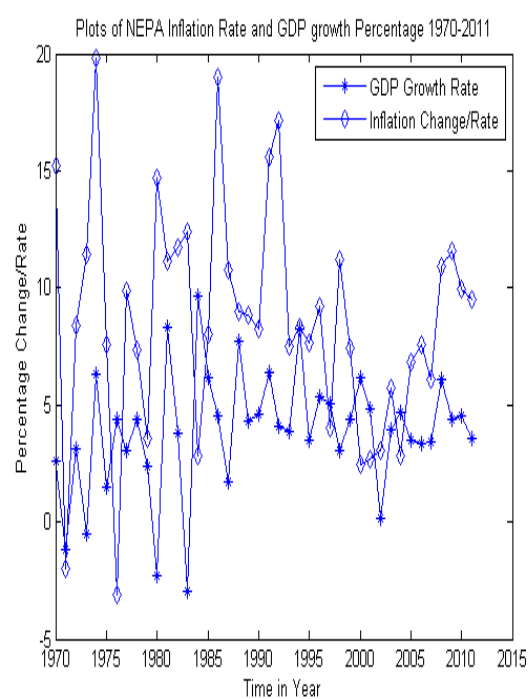


Appendix B.16: Figure 16

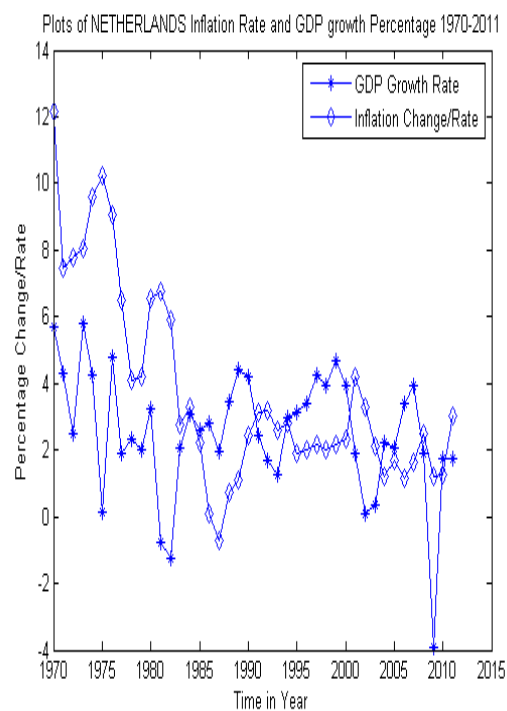
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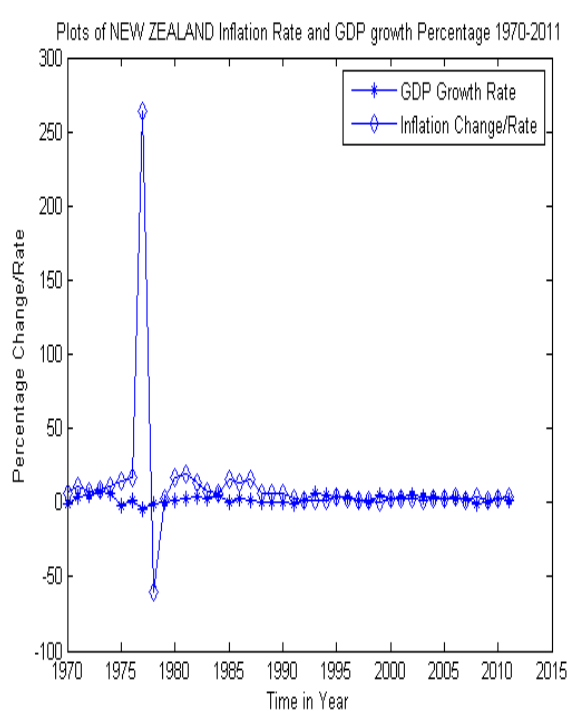
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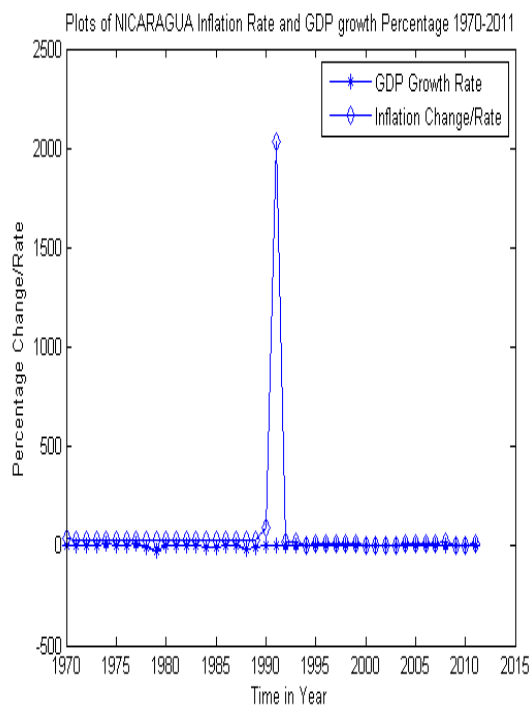


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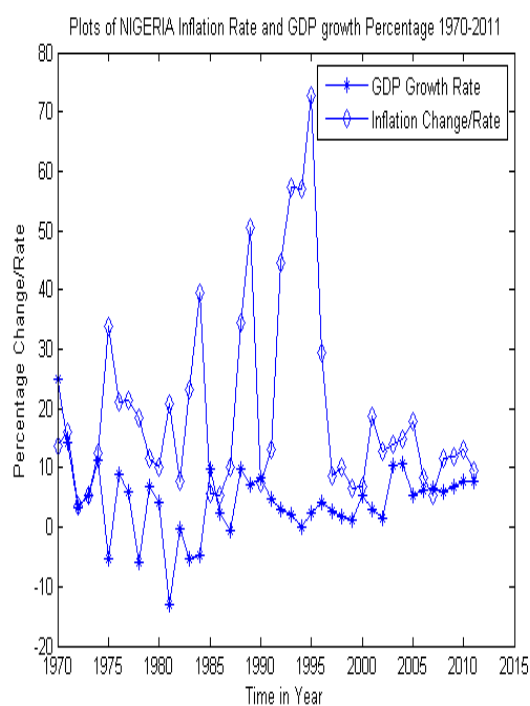


Appendix B.17: Figure 17

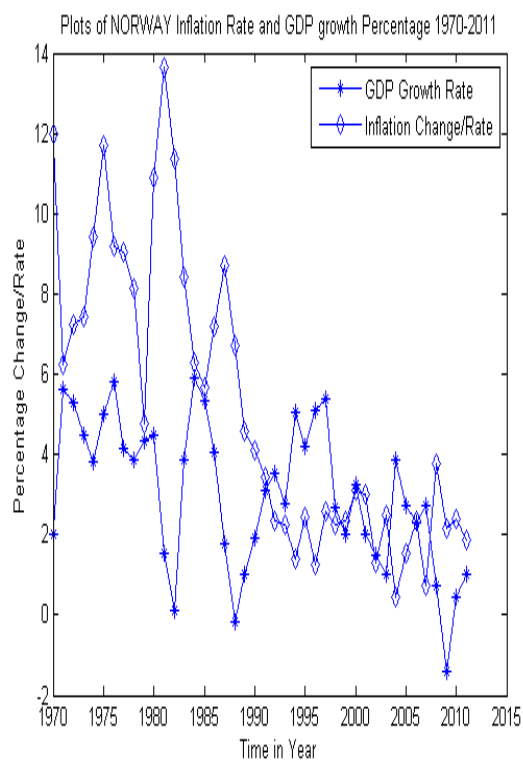
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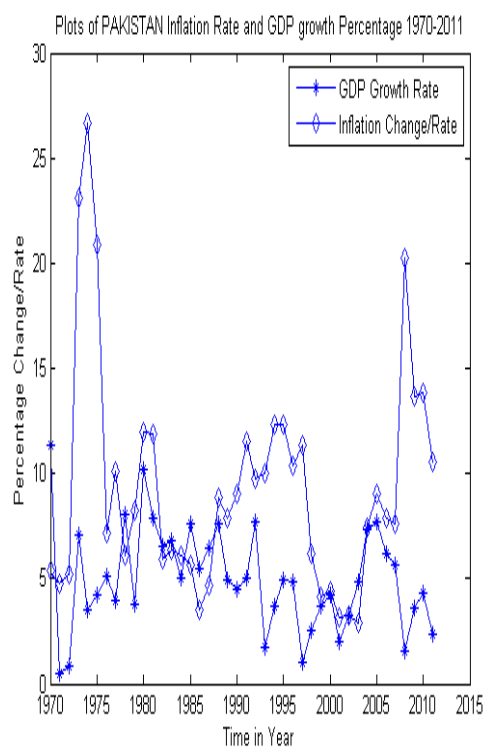
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(c)

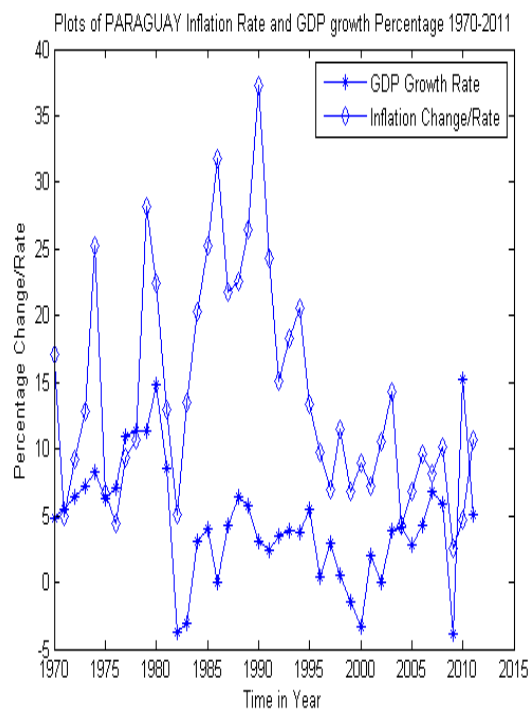


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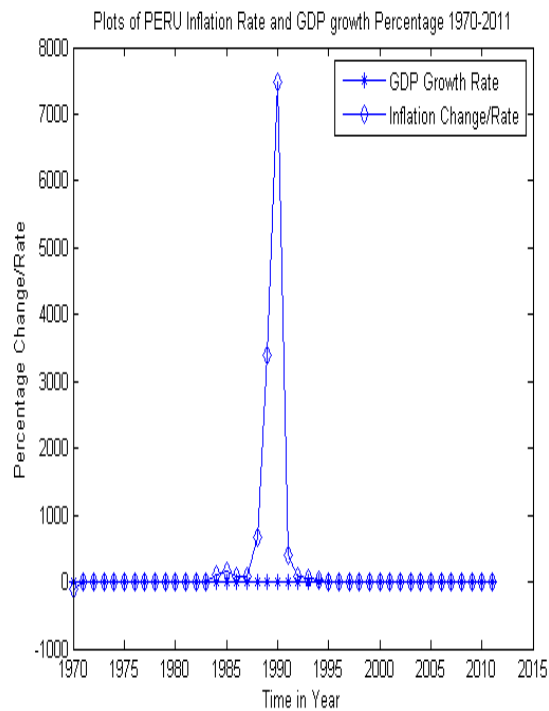


Appendix B.18: Figure 18

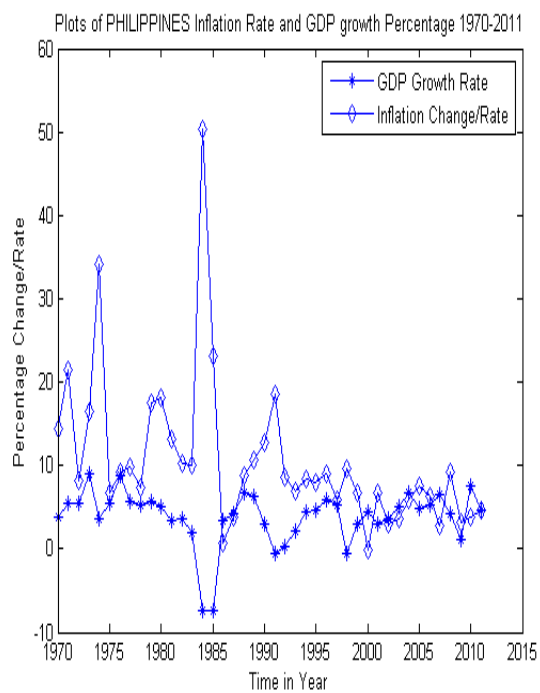
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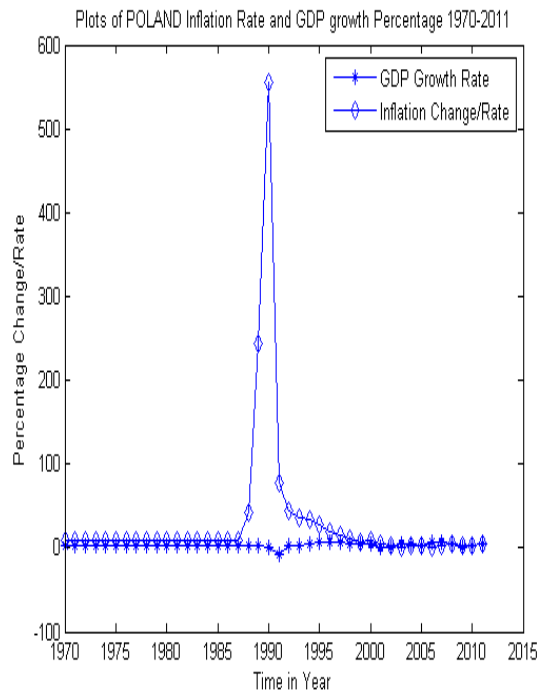
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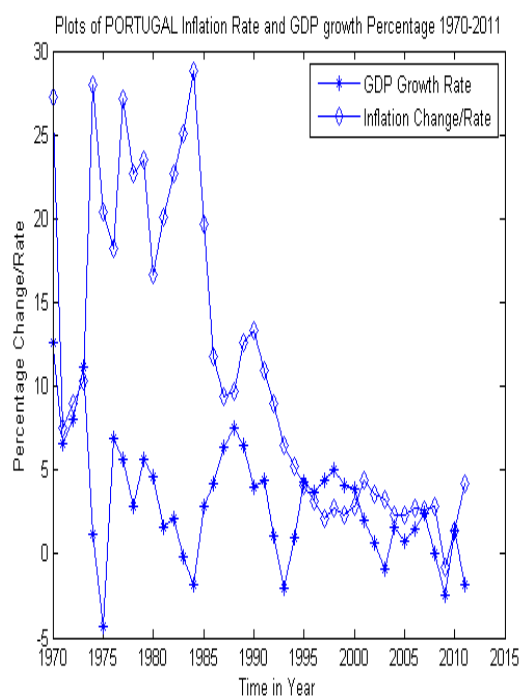


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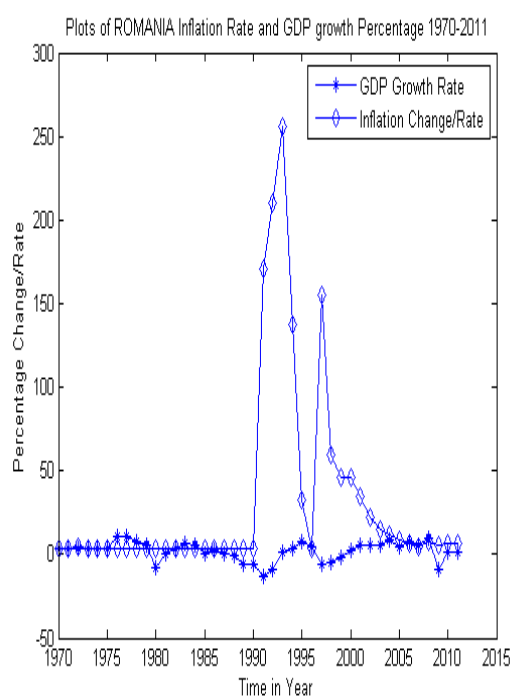


Appendix B.19: Figure 19

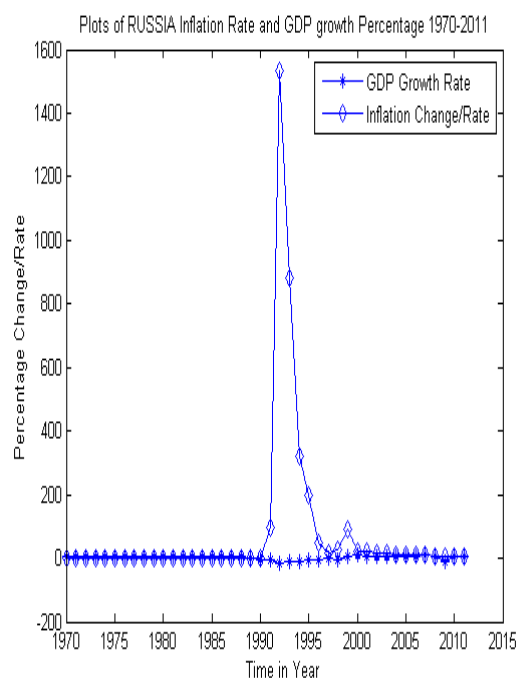
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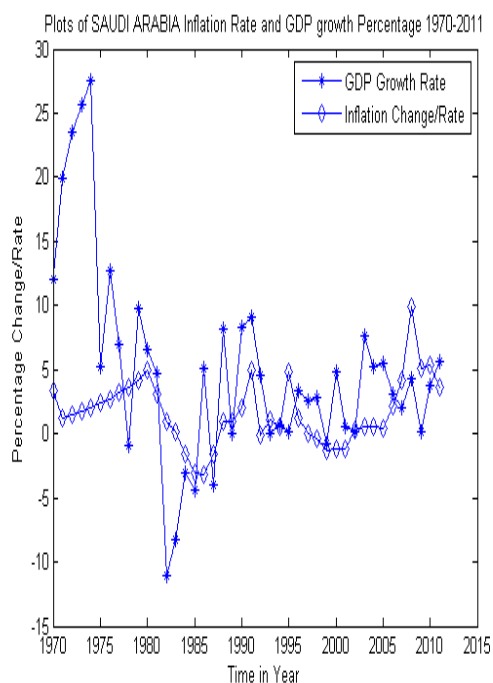
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(c)

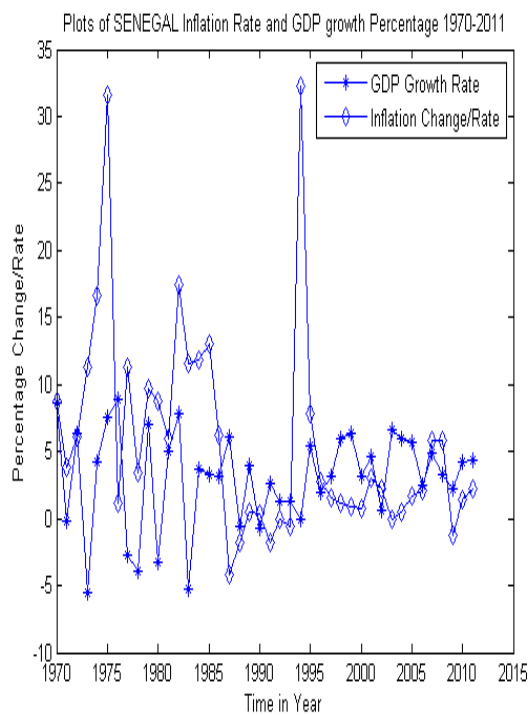


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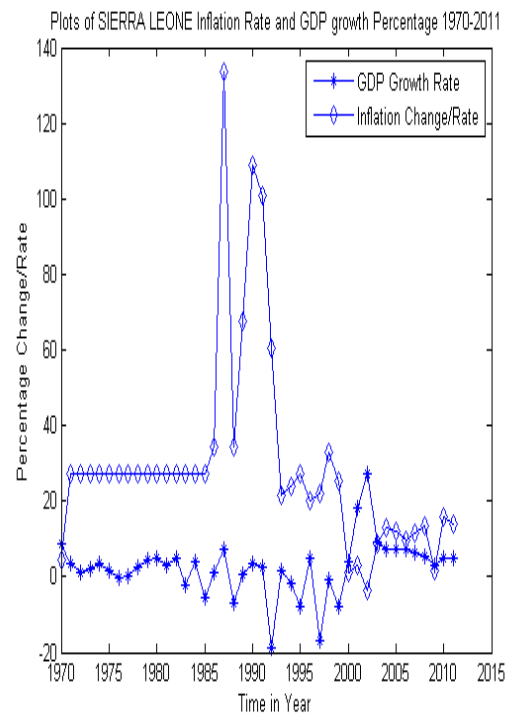


Appendix B.20: Figure 20

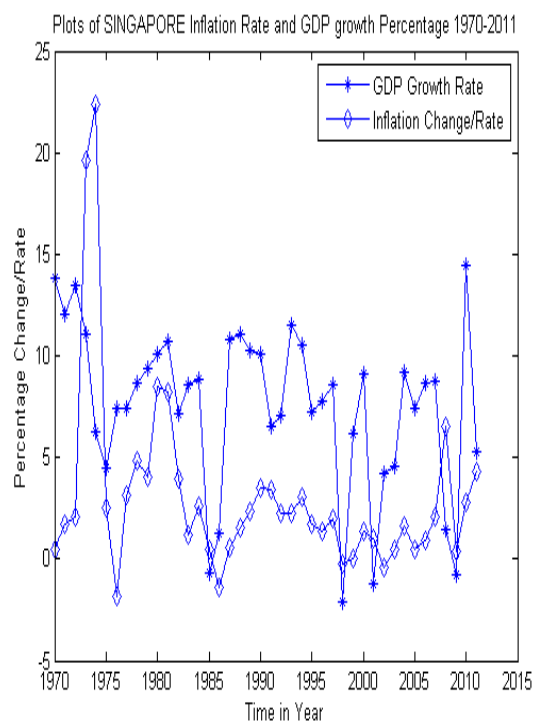
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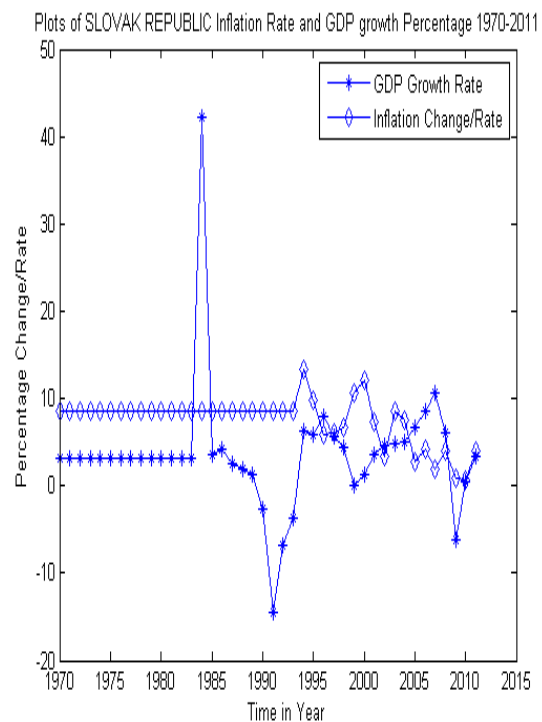
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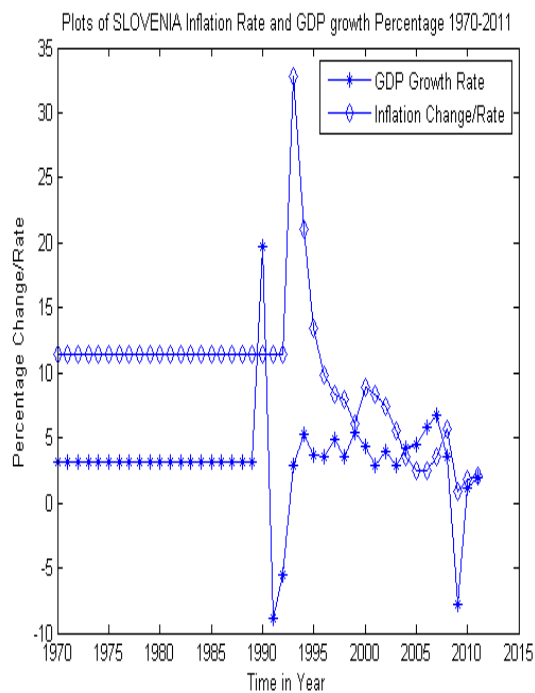


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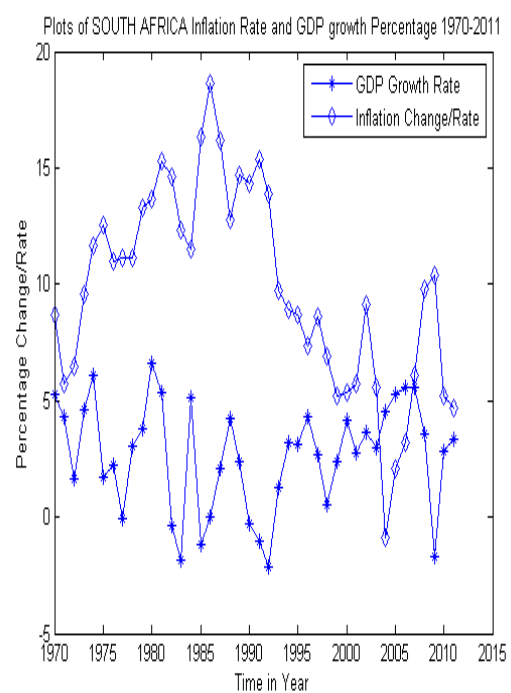


Appendix B.21: Figure 21

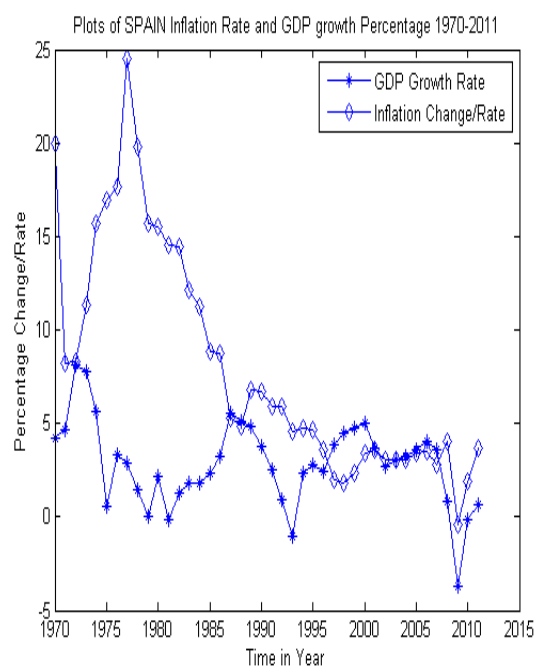
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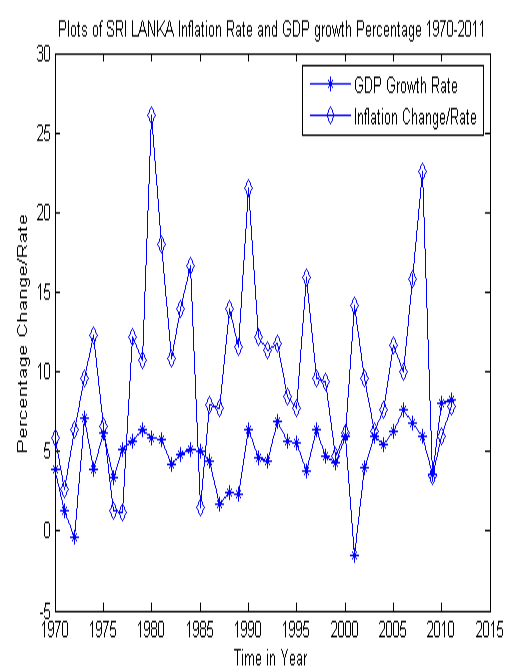
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(c)

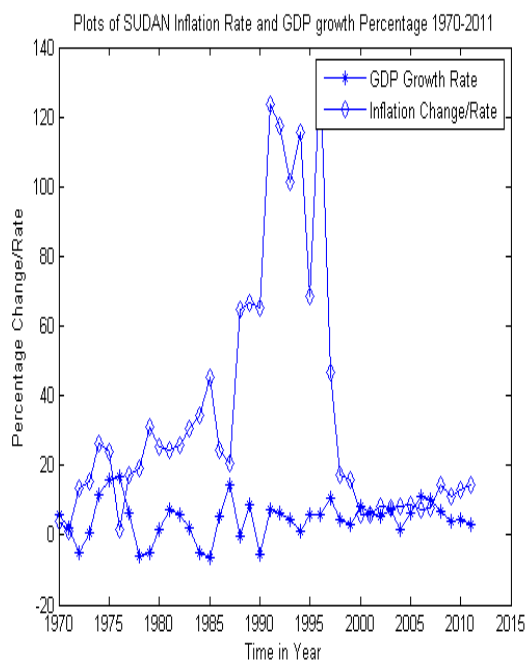


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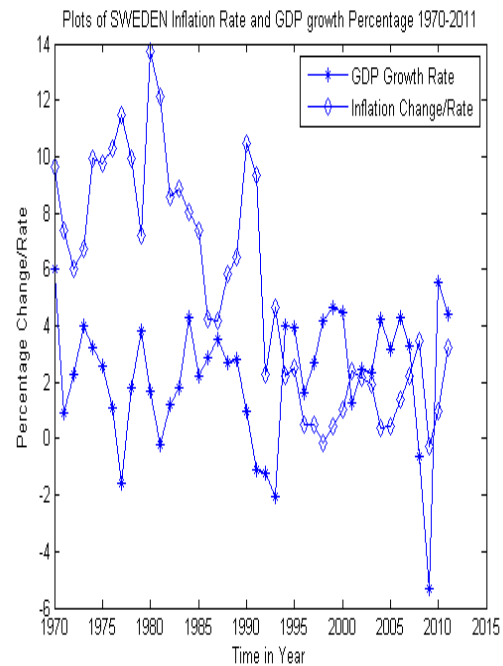


Appendix B.22: Figure 22

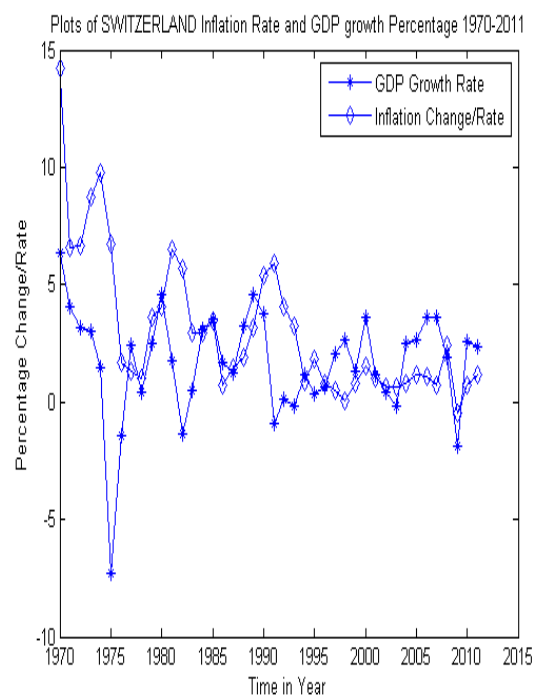
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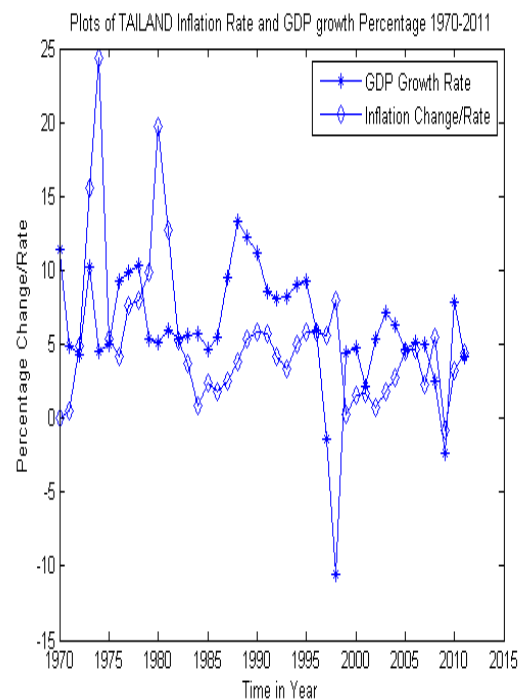
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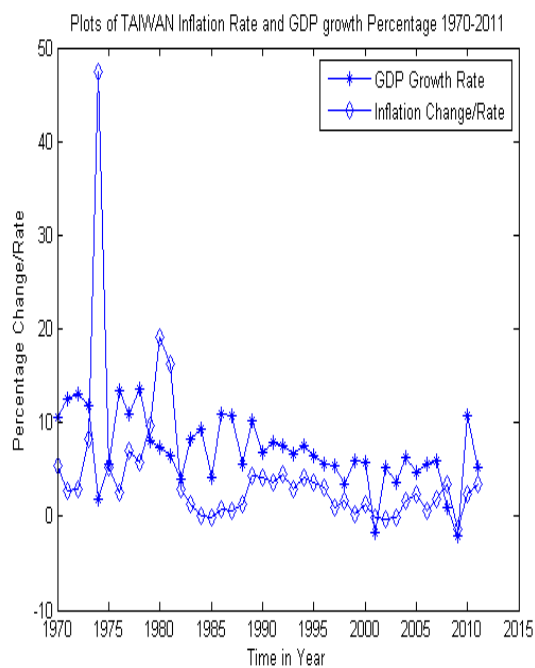


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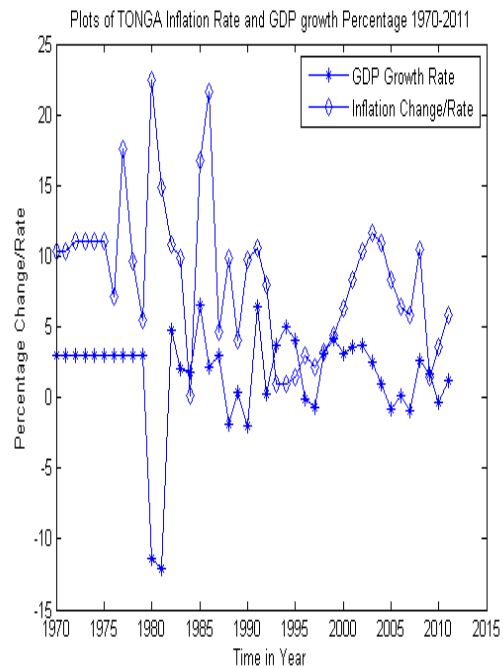


Appendix B.23: Figure 23

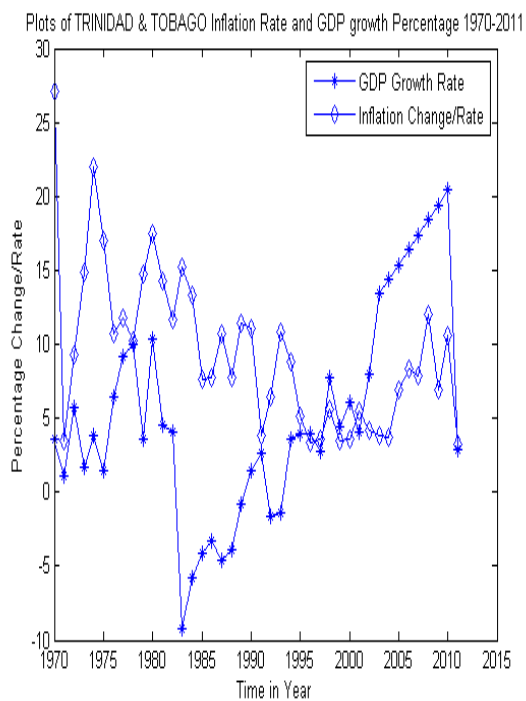
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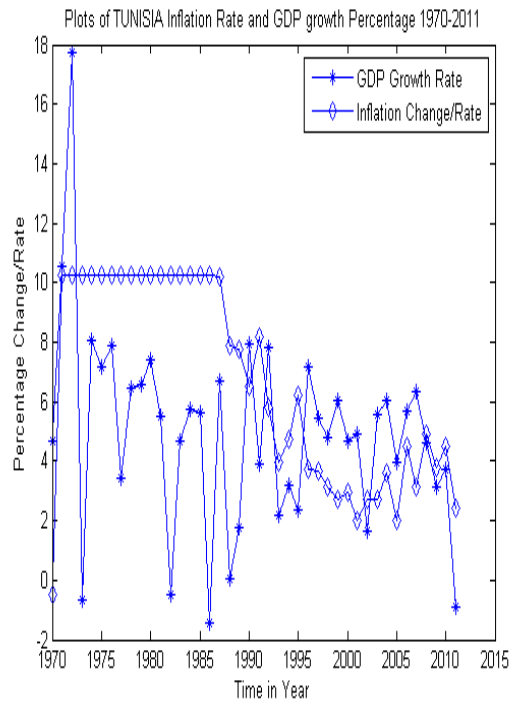
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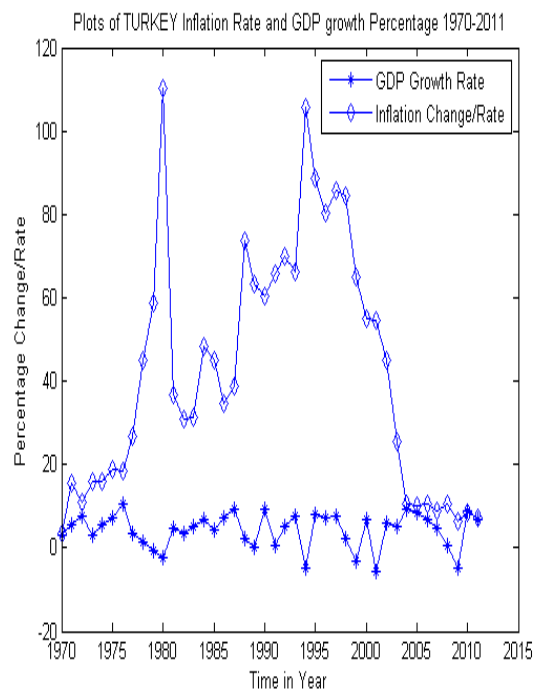


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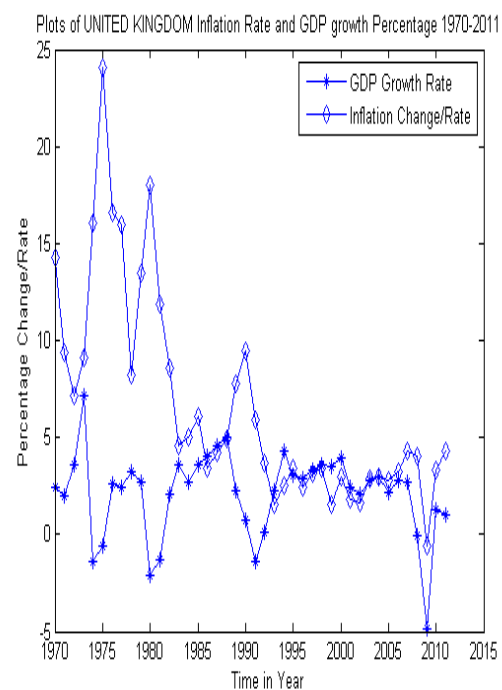


Appendix B.24: Figure 24

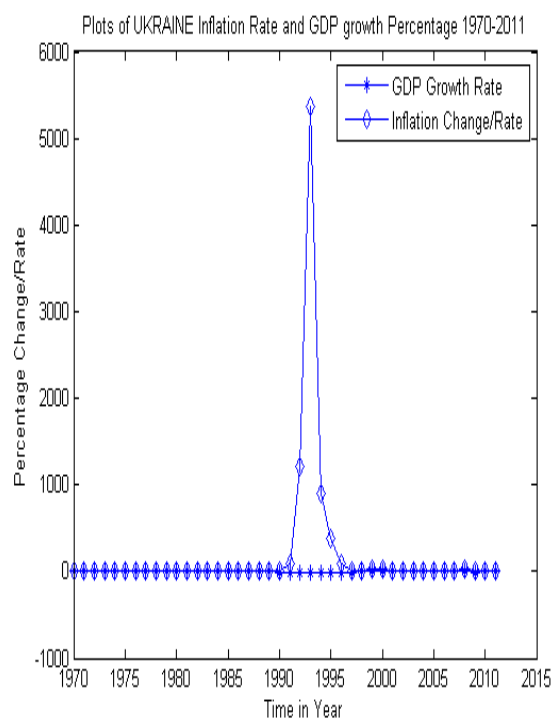
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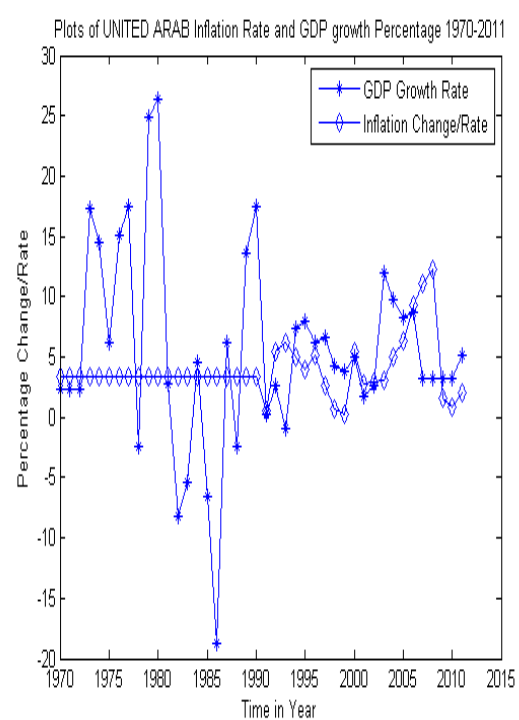
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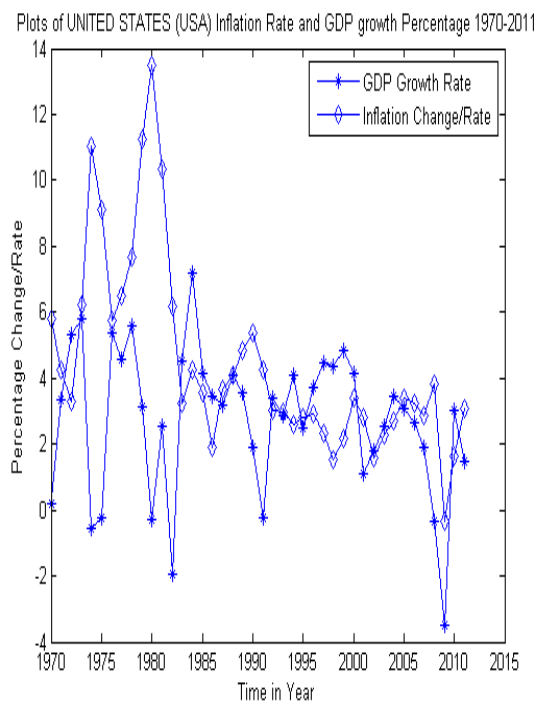


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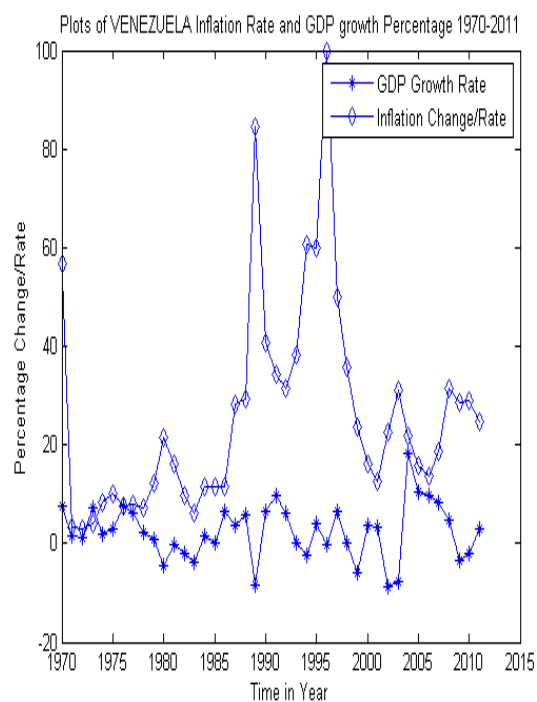


Appendix B.25: Figure 25

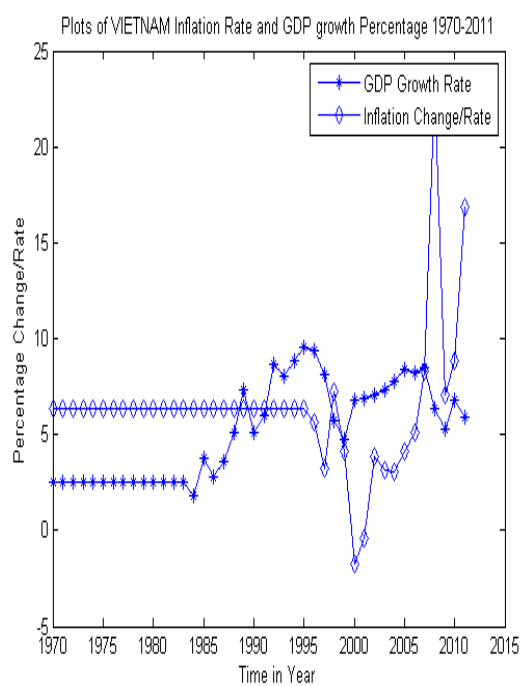
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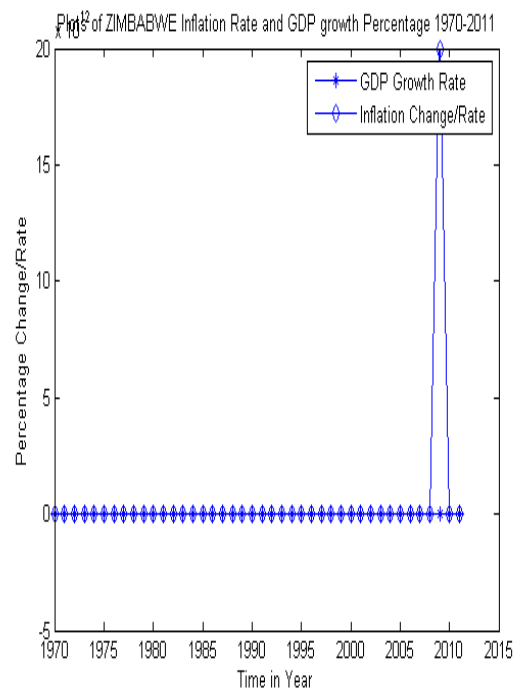
(b)



(c)



(d)



Appendix C

LIST OF COUNTRIES WITH STRUCTURAL BREAKS AND OUTLIERS

S/N	GROUP	COUNTRY	CAUSES OF STRUCTURAL BREAKS ON INFLATION
1	European Union (Excluding Advanced Economies) (EU)	Bulgaria	1997(Feb/March) worst hyperinflation occurred.
2		Estonia	The country experienced hyperinflation as a result of using the Russian ruble after the fall of the Soviet union.
3		Latvia	High inflation in 1990-1995 and also 2007.
4		Lithuania	High inflation in 1992.
5		Poland	Hyperinflation due to transition economies in 1989-1990.
6		Romania	Hyperinflation in the 1990s.
7		Slovak Republic	High GDP in 1984 when it was part of Czechoslovakia (communist) due to planned economy.
8		Slovenia	High inflation due to transition economy/ Euro change over.
9	Emerging and Developing Economies. (EADecon)	Brazil	From 1967-1994 hyperinflation with unit currency shifted 7 times.
10		Russia	Economic reform 1991/1992 caused Inflation
11		Indonesia	Monetary policy before 2000 & its effect on inflation.
12		Argentina	High Inflation from 1975-1991. Currency reform.
13		Azerbaijan	High Inflation from 1992-1994 due to Russian ruble when it introduced the Azerbaijani manat.
14		Ghana	Economic recession in 1970s
15		Kazakhstan	Hyperinflation 1992-1993
16		Philippines	Inflation caused by excessive liquidity (much money supplied with less goods and services).
17		Turkey	Severe Inflation rates throughout 1990s
18		Venezuela	High inflation in 1990s with excessive government spending and much money supply compared to output.
19		Ukraine	1992-1994 high inflation due to change from soviet ruble to Ukrainian Karbovenets
20		Lebanon	Excessive dependent on imported

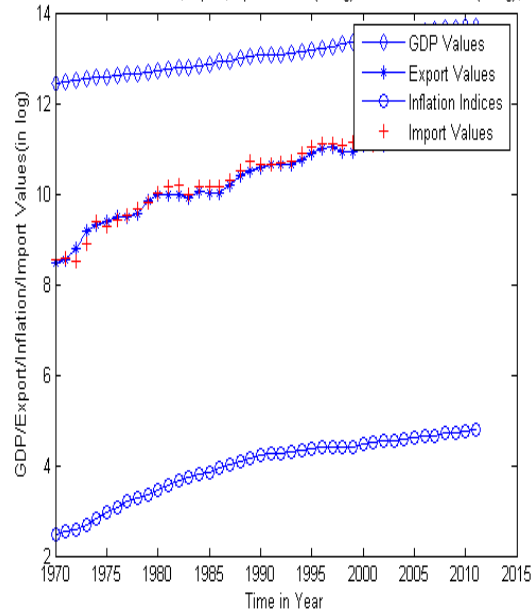
			goods (over 80% of the country's consumption) much pronounced in the 1990s.
21		Cameroon	Inflation mostly foreign compare to domestic cost.
22		Peru	Worst inflation 1988-1990 due to currency reform
23	Other Countries. (OC)	Costa Rica	High inflation due to much money supply compare to available goods.
24		Guyana	Inflation caused by fiscal deficits decision of government.
25		Jamaica	Inflation caused by government policies.
26		Mexico	Mexico defaulted on its external debt in 1982 and this led to severe high inflation. Devaluation of currency also contributed to the problem.
27		Nicaragua	Inflation caused by global recession.
28		Iraq	1987-1995 with continuous high inflation increasing each year
29		Sudan	Inflation caused by the southern sudan secession.
30		Angola	1991-1995 high inflation due to exchange restrictions as a result of currency reform.
31		Liberia	War in 1980s
32		Malawi	Inflation caused by food shortage and non-availability of other essentials like petrol, housing, etc.
33		Senegal	Inflation caused by poor weather, poor electricity supply.
34		Sierra Leone	Inflation caused by war for 11 years, imported goods and services.
35		Zimbabwe	After independence in 1980, the economy collapsed and led to devaluation of currency in 2000-2008

Appendix D (It consists of Figures 26 to 33)

Appendix D.01: Figure 26

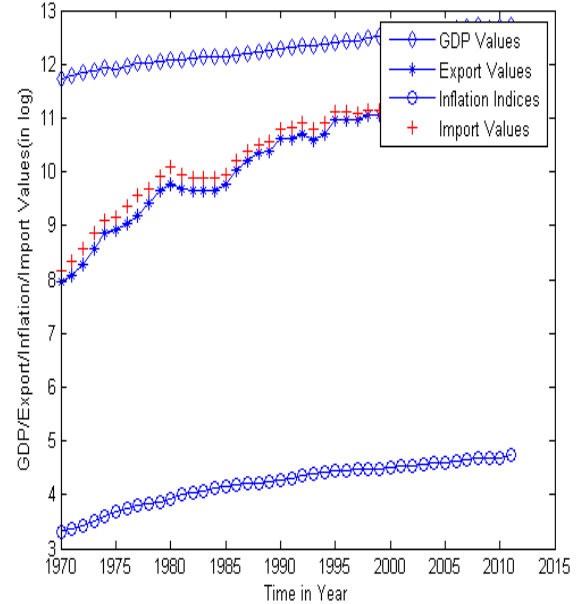
(a)

Time Plots of AUSTRALIA GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-201



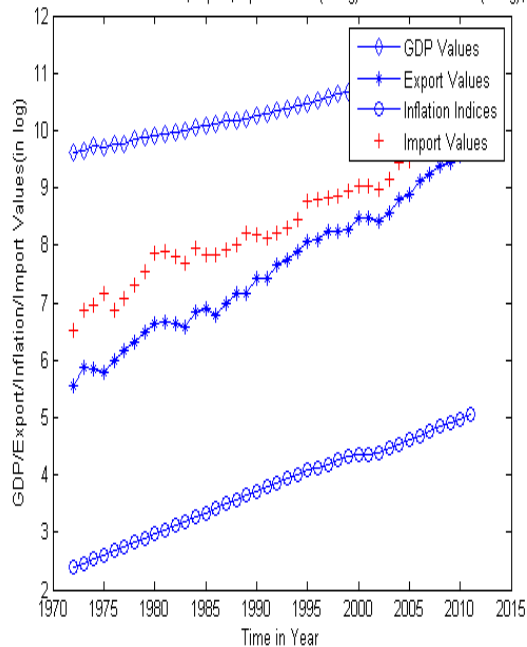
(b)

Time Plots of AUSTRIA GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



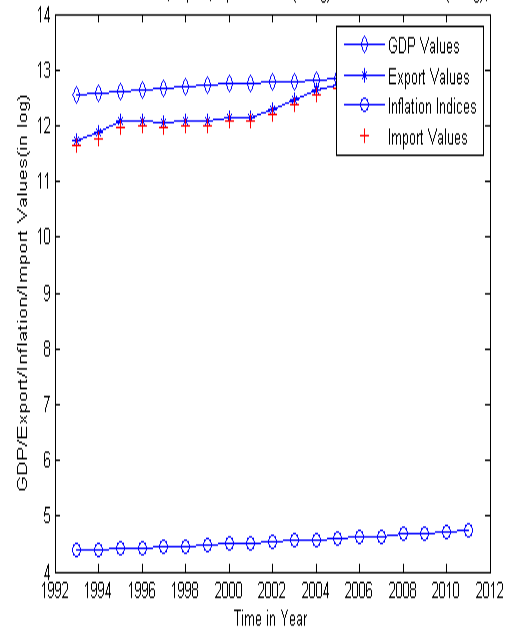
(c)

Time Plots of BANGLADESH GDP,Export,Import Values(in log)and Inflation Indices(in log),1972-20



(d)

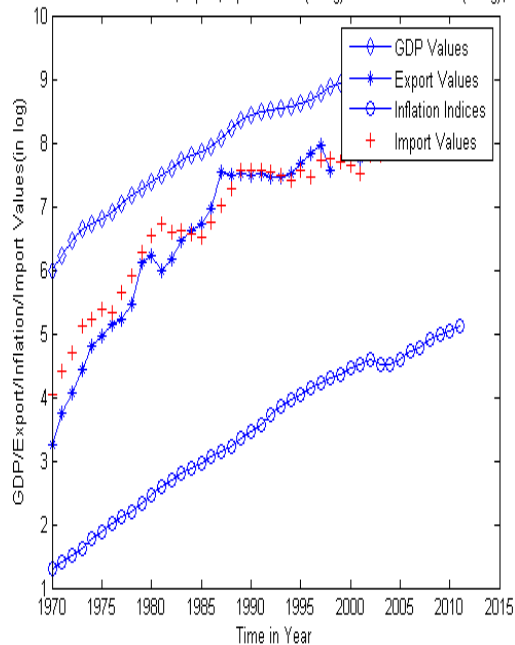
Time Plots of BELGIUM GDP,Export,Import Values(in log)and Inflation Indices(in log),1993-201



Appendix D.02: Figure 27

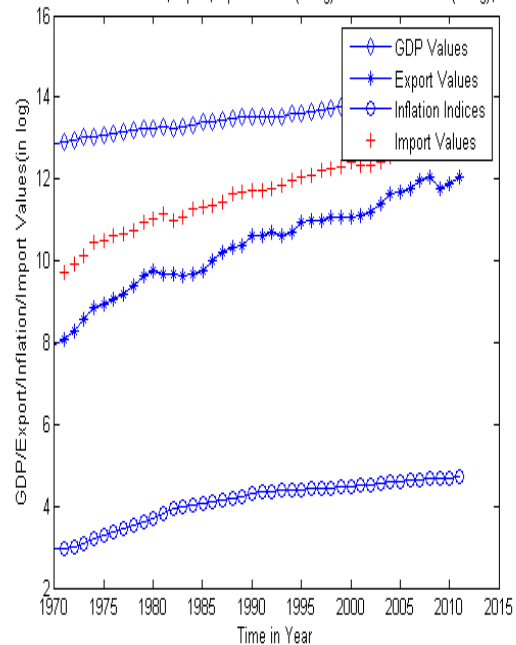
(a)

Time Plots of BOTSWANA GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-20



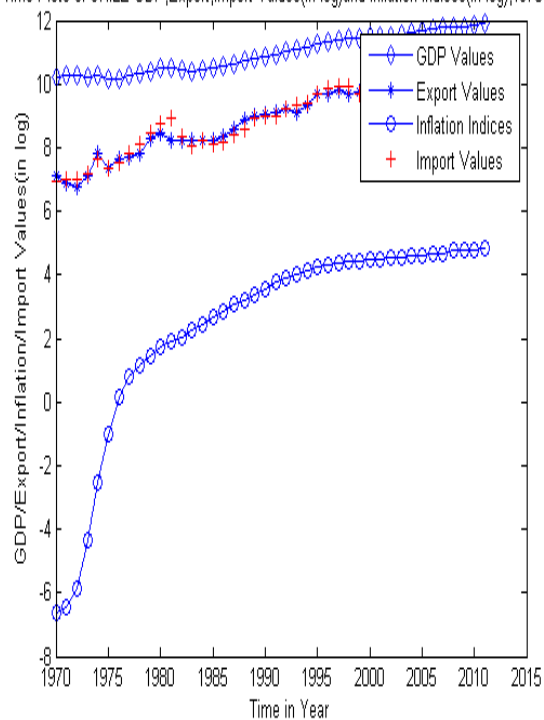
(b)

Time Plots of CANADA GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



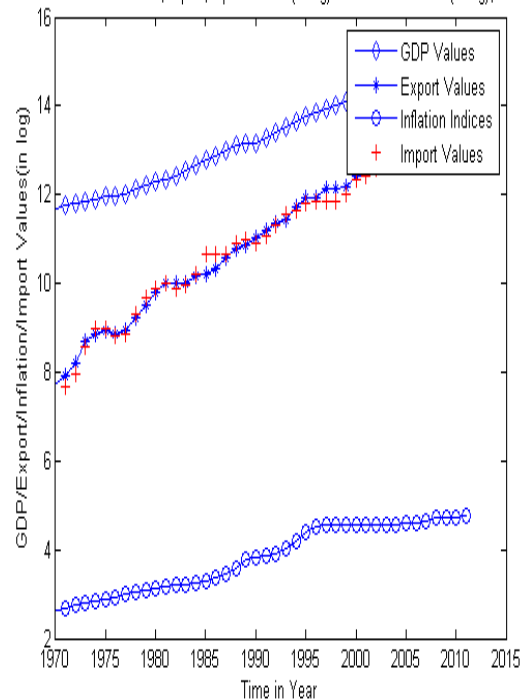
(c)

Time Plots of CHILE GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



(d)

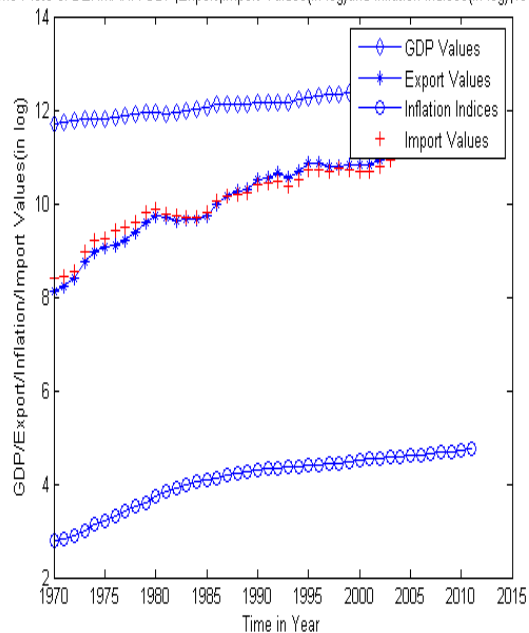
Time Plots of CHINA GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



Appendix D.03: Figure 28

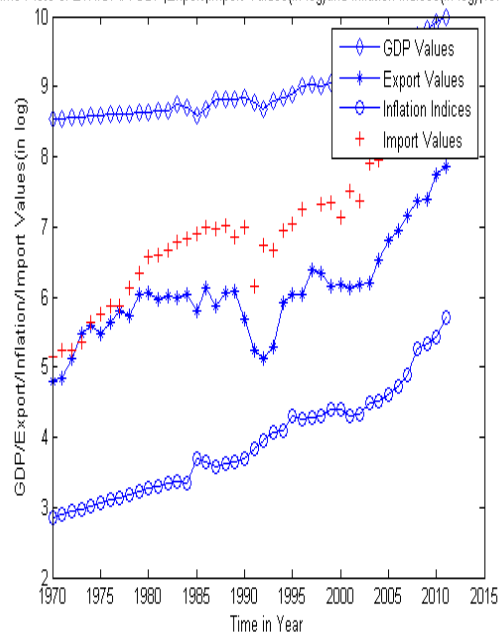
(a)

Time Plots of DENMARK GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-201



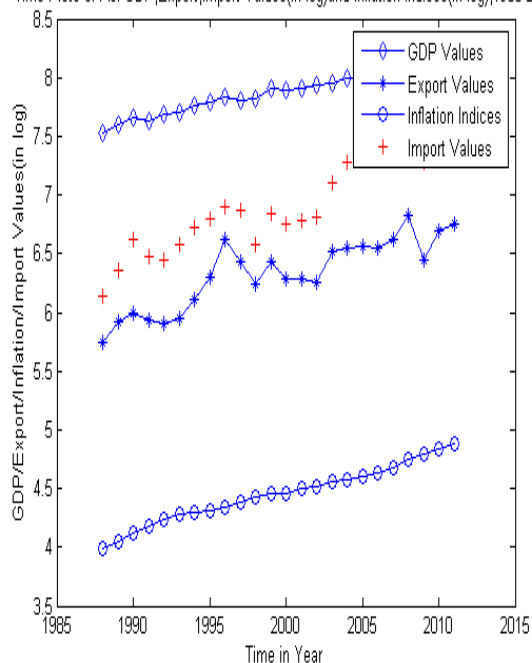
(b)

Time Plots of ETHIOPIA GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-201



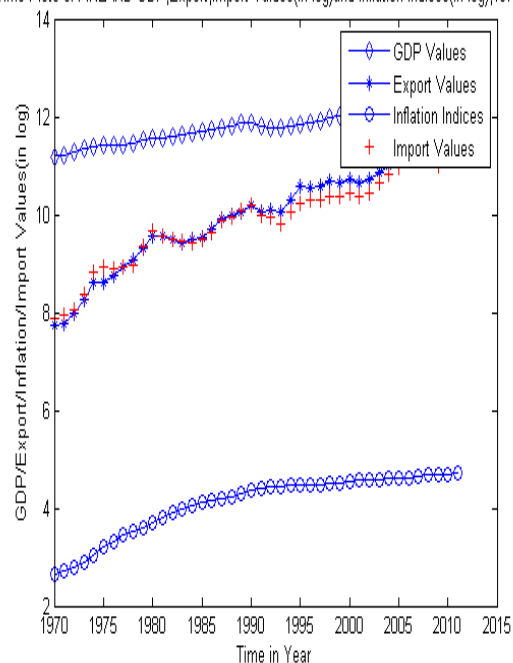
(c)

Time Plots of FIJI GDP, Export, Import Values (in log) and Inflation Indices (in log), 1988-2011



(d)

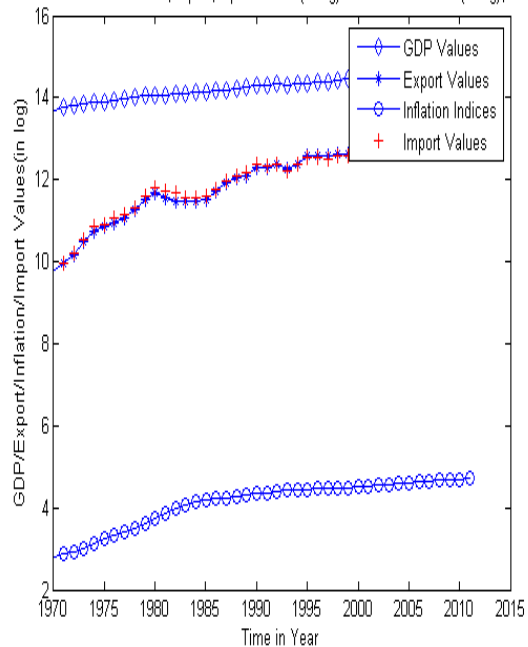
Time Plots of FINLAND GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011



Appendix D.04: Figure 29

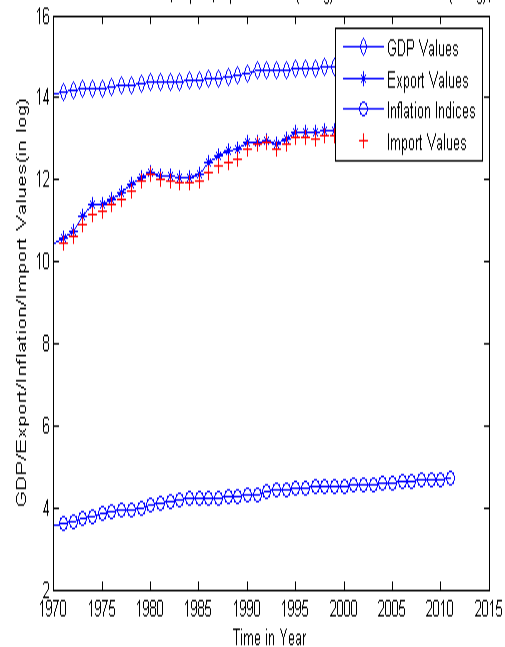
(a)

Time Plots of FRANCE GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



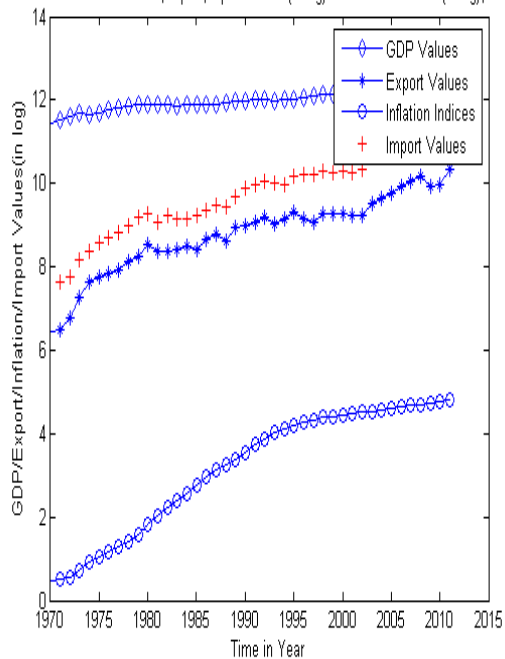
(b)

Time Plots of GERMANY GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



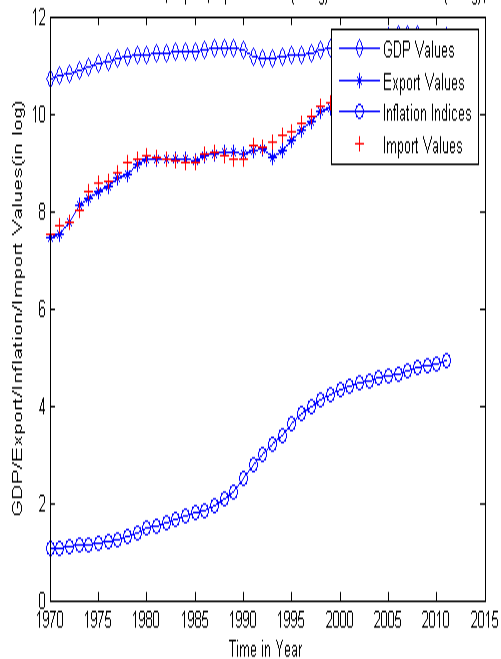
(c)

Time Plots of GREECE GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



(d)

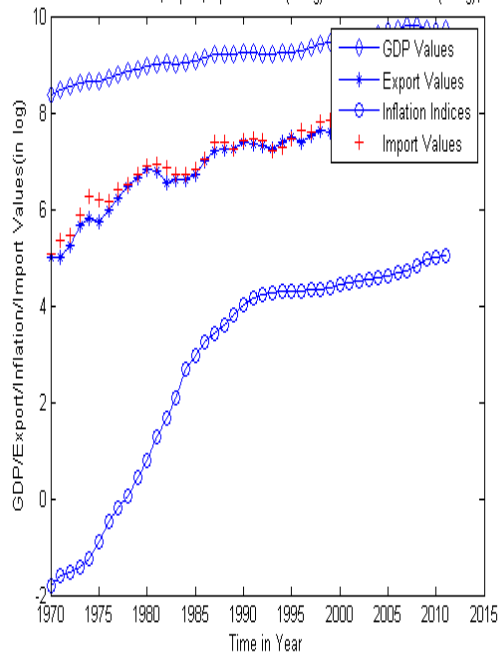
Time Plots of HUNGARY GDP,Export,Import Values(in log)and Inflation Indices(in log),1970-2011



Appendix D.05: Figure 30

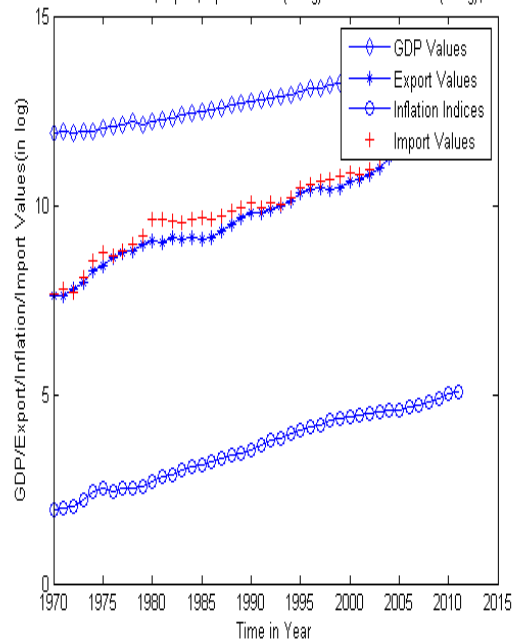
(a)

Time Plots of ICELAND GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011



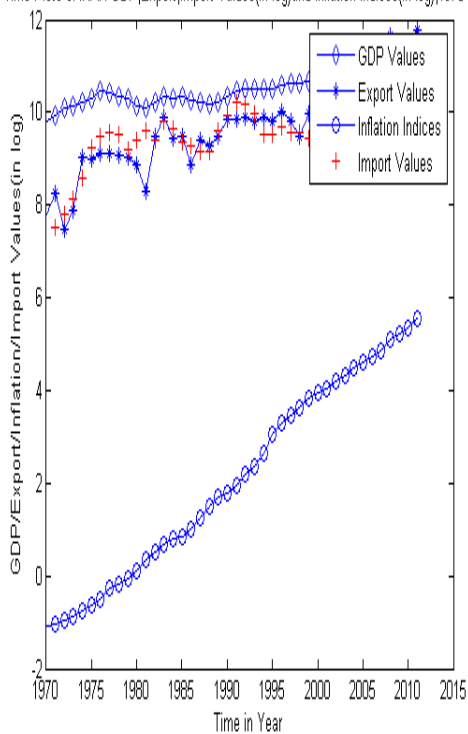
(b)

Time Plots of INDIA GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011



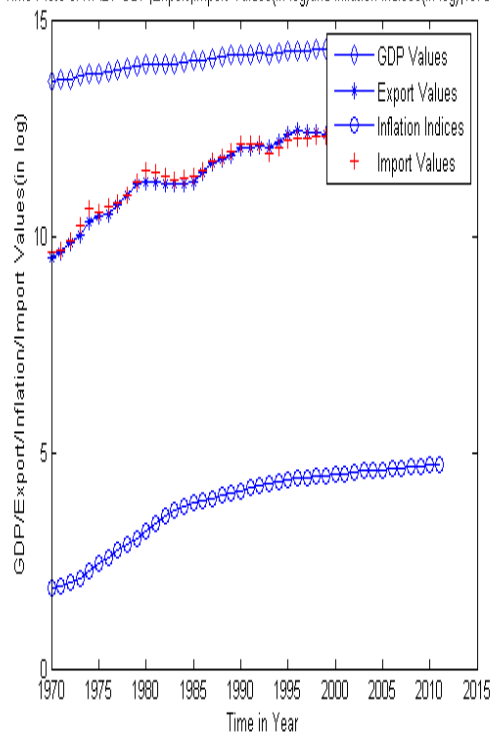
(c)

Time Plots of IRAN GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011



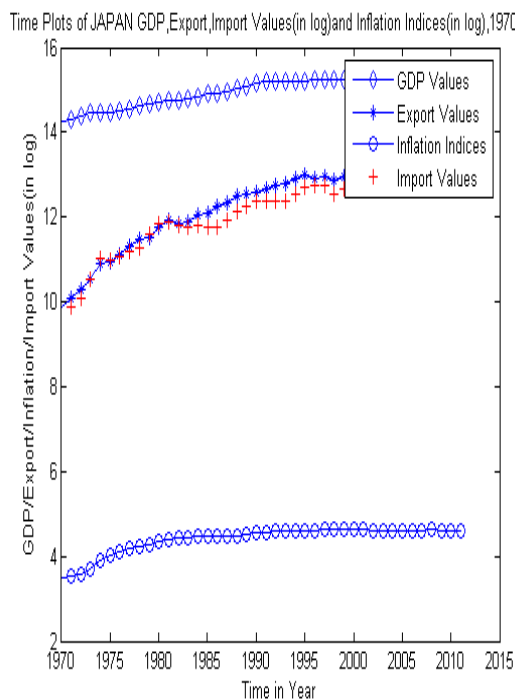
(d)

Time Plots of ITALY GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011

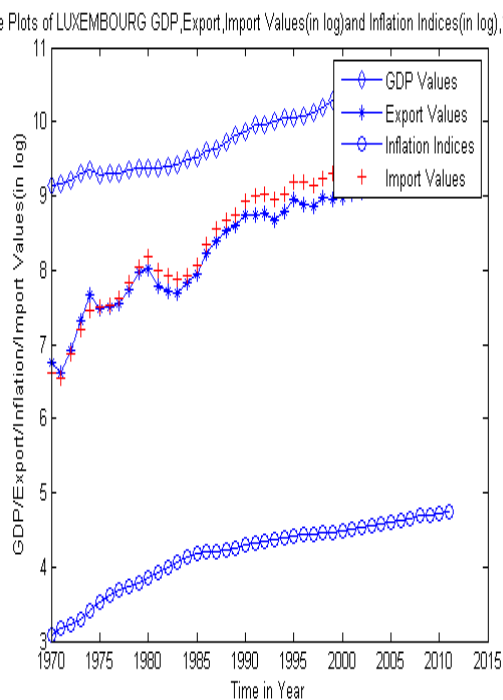


Appendix D.06: Figure 31

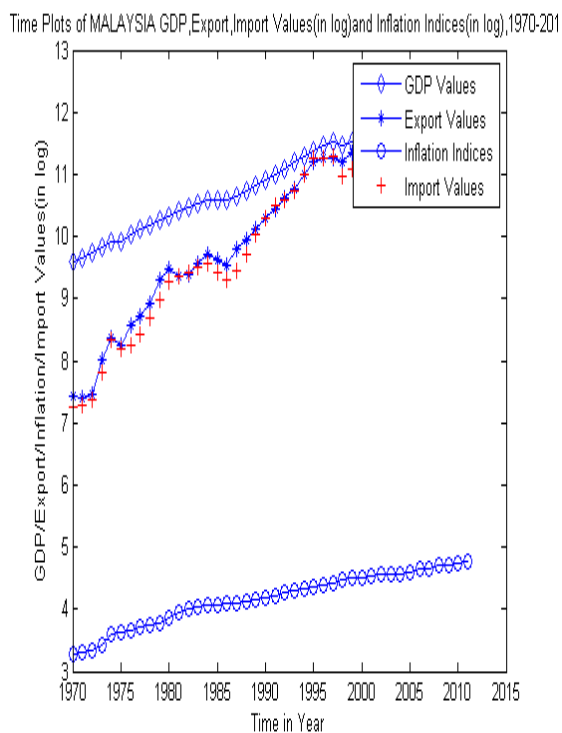
(a)



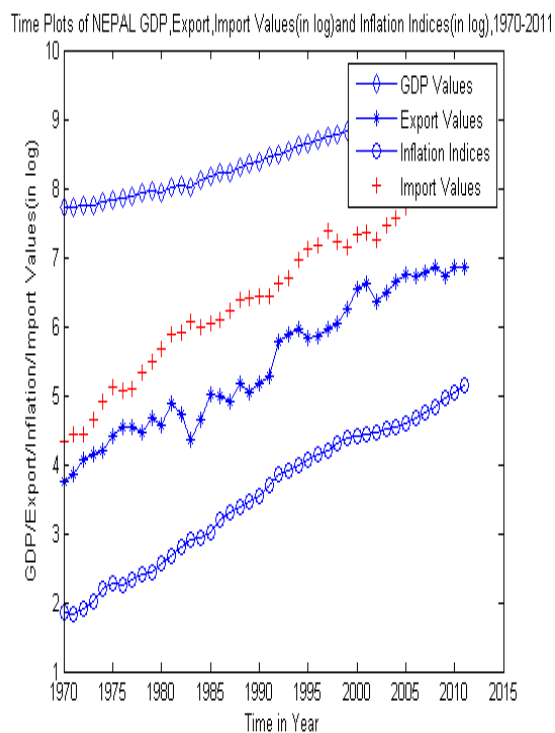
(b)



(c)



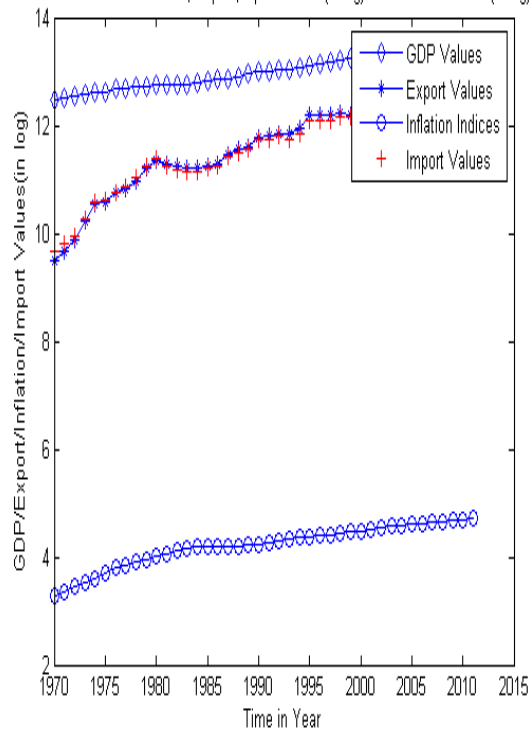
(d)



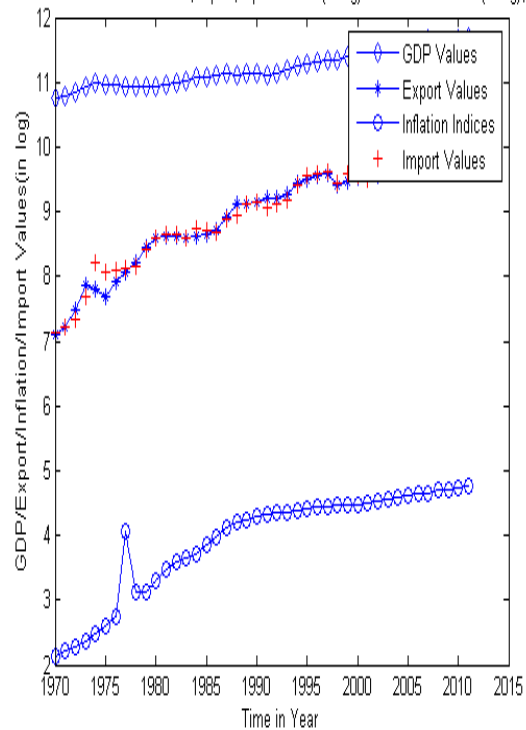
Appendix D.07: Figure 32

(a)

Time Plots of NETHERLANDS GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011

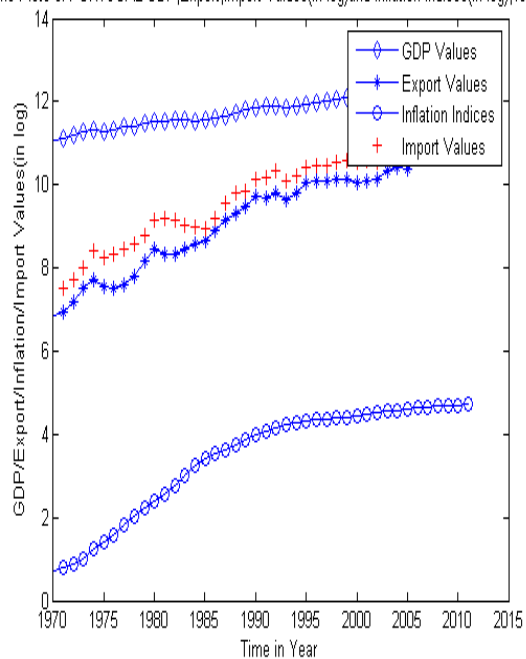


(b)



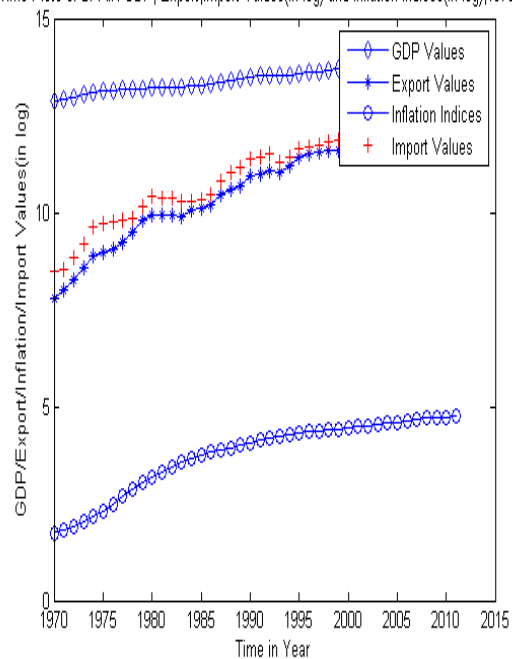
(c)

Time Plots of PORTUGAL GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011



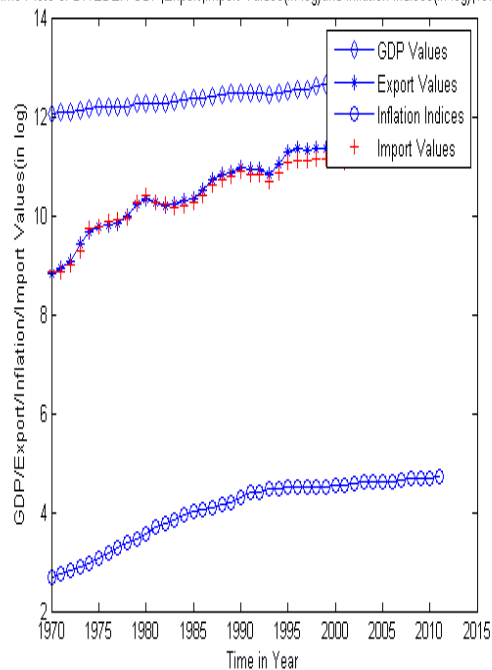
(d)

Time Plots of SPAIN GDP, Export, Import Values (in log) and Inflation Indices (in log), 1970-2011

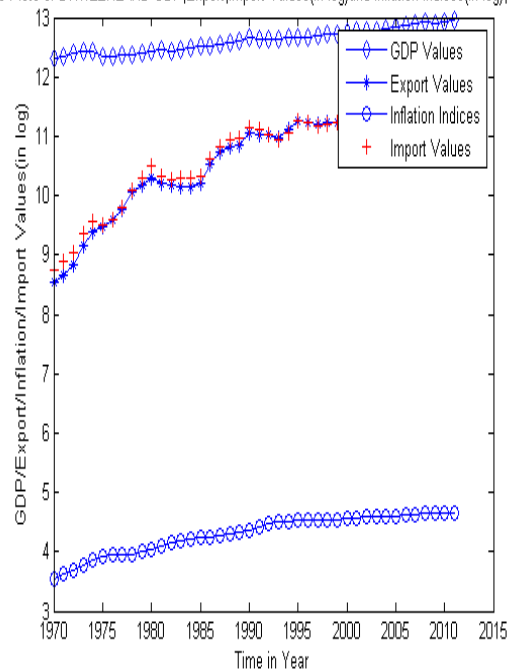


Appendix D.08: Figure 33

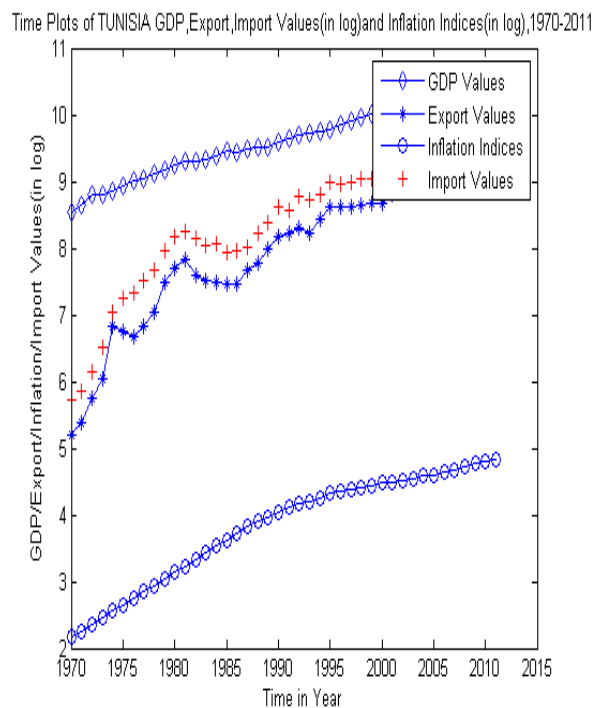
(a)



(b)



(c)



(d)

