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Teaching mathematics for social justice: translating theories into practice

Pete Wright

Thesis

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“Education is the most powerful weapon which you can use to change the world” (Nelson Mandela)

Statement

I hereby declare that this thesis has not been, and will not be, submitted in whole or in part to another University for the award of any other degree.

Pete Wright

Signature: .....

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Summary

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This study reports on a project exploring how a commitment towards teaching mathematics for social justice amongst teachers can be translated into related classroom practice. It recounts how a group of teacher researchers set about achieving this through developing, trying out and evaluating a series of teaching ideas and activities. It contrasts the abundance of research literature on theories of mathematics education and social justice with the relative scarcity of studies on developing practice in this area.

Mathematics lessons are generally characterised by too much focus on factual recall and procedural understanding, resulting in unacceptable levels of disengagement and disaffection amongst learners. A critical methodological stance is adopted in arguing that this current situation should not be taken as given. The research design is based on a model of participatory action research, which is socio-political, participative, collaborative, emancipatory and recursive in nature, and aims to bring about desirable social change. Careful consideration is given to the credibility, transferability, dependability and confirmability of the research findings, and particular attention is paid to the role of the researcher in facilitating the research group. Data was collected primarily from meetings of the research group and a series of semi-structured empathetic interviews conducted with each teacher researcher. Audio-recordings were transcribed and condensed before being coded inductively and analysed through a thematic approach, using the constant comparative method to draw out meaning from the data. A case study approach was used as a means to capture and report the stories of how teacher researchers' thinking and classroom practice evolve and develop over the course of the project.

Findings from the project demonstrate how the five teacher researchers, through their involvement in the project, begin to question seriously and rethink previously held views about the nature of mathematics, their own relationship with the subject and notions of

mathematical ability. They exhibit a growing belief that the development of students' mathematical understanding and awareness of social justice issues are inextricably linked, rather than separate objectives. These changes in epistemologies appear to have an impact on teacher researchers' classroom practice and their students' dispositions towards learning mathematics. There is evidence that making mathematics more relevant and meaningful leads to raised levels of student engagement, and that focusing on how mathematics can be used to understand real-life issues and to construct an argument for change leads to increased student agency. The collaborative and participative nature of the research group shows how a mutually supportive environment can be created which promotes the self-efficacy of teacher researchers in addressing issues of social justice in their mathematics classrooms.

By relating the findings back to the underlying theories, conclusions are drawn of relevance to practitioners and researchers in the field of mathematics education. These relate to the relationship between teacher epistemologies and teaching approaches, the relevance and purpose of the school mathematics curriculum and the potential of participatory action research as a model of professional development which has a strong impact on classroom practice and promotes teachers' genuine engagement in and with research.

Key words

Mathematics education, social justice, participatory action research

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Chapter 1: Introduction

When people outside the world of mathematics education ask me what my research is about, I notice them raising their eyebrows, when I tell them the title of my thesis. This is normally followed by a slightly awkward discussion as I struggle to convince them of the link between mathematics and social justice. Such a link has become increasingly apparent to me during the fifteen years I have spent teaching mathematics in secondary schools and the ten years I have spent in other roles related to mathematics education. However, there is a widely held perception amongst the general public that mathematics is a neutral and value-free subject with, consequently, little relationship to issues of social justice. This stark contrast, between my personal beliefs about mathematics education and the public perception of school mathematics, partly explains my interest in this field of study and why I consider it to be one of critical importance.

This study focuses on working with a group of secondary mathematics teachers in developing their classroom practice. The primary research question is:

> *How can a commitment towards 'education for social justice' amongst mathematics teachers be translated into pedagogy and classroom practices which promote such aims?*

There are two aspects to this inquiry. The first is to formulate and develop a conceptualisation for such pedagogy and to consider how it is experienced by the learners themselves. Hence, the following two subsidiary research questions are posed:

> *What might mathematics education for social justice look like in practice in the secondary mathematics classroom?*
>
> *What impact might teaching mathematics for social justice have on students?*

The second aspect of the inquiry is to explore the processes through which such a development in classroom practice can take place. Hence, the following two subsidiary research questions are posed:

> *How can an awareness and understanding of issues of social justice be developed amongst mathematics teachers?*
>
> *What are the opportunities and constraints for teaching mathematics for social justice in schools in England?*

In Chapter 2, I reflect in detail on my experiences of learning and teaching mathematics, and how these further explain my research interest in social justice. I explore what lies behind the public perception of mathematics described above and why so many people exhibit feelings of anxiety towards the subject. I relate these considerations to alternative epistemologies of

mathematics and ideologies of mathematics education, and my positioning in relation to mathematics pedagogy. Through this process I arrive at a conceptualisation of teaching mathematics for social justice, which I use as a starting point for this study.

In Chapter 3, I reflect on my varied experiences of working with mathematics teachers and how these have influenced my positioning in relation to research methodology. I relate these considerations to research literature focusing on effecting change in mathematics teachers' practice. I discuss issues of trustworthiness in relation to a critical model of participatory action research and the framework I adopt for analysing data.

In the remainder of this introductory chapter, I explain why I believe this study has the potential to make a substantial and original contribution to knowledge and understanding in the field. In Section 1.1, I argue that, whilst there is an abundance of research articulating theories around teaching mathematics for social justice, there is little research evidence demonstrating how these theories can be translated into classroom practice in schools in England. In Section 1.2, I review the limited research in this field that is available.

1.1 Translating theory into practice

There is an abundance of research literature devoted to mathematics education and social justice, including several edited books (Burton, 2003; Sriraman, 2008), and special issues of journals (Gates & Jorgensen, 2009; Strutchens, et al., 2012; D'Ambrosio, et al., 2013), containing numerous contributions from a multitude of authors. Gates and Jorgensen (2009, p. 161) highlight the *"considerable interest in social justice around the world"* and how it is central to the policies of many governments and international organisations. Despite the particular interest in social justice within mathematics education, perhaps reflecting the subject's privileged position within the curriculum (see Chapter 2), Gates and Jorgensen (2009) highlight how there are relatively few articles, with a social justice focus, submitted to mathematics teacher education journals.

Many studies on mathematics education and social justice are theoretical or philosophical in nature, whilst others have a focus beyond formal school settings. There are relatively few studies which concentrate on the classroom practice of mathematics teachers in schools, and many of these are based on research carried out in the United States (US). Whilst much can be learnt from studies based in other countries, conditions and contexts in schools may be significantly different to those in schools in England. In the US, for example, some schools have been established as self-designated 'social justice' schools (Gutstein, 2006; Gregson, 2013), and there is no legally-binding national curriculum, although most states have adopted the

voluntary 'Common Core State Standards for Mathematics' introduced in 2010 (Wright, 2012). There is a notable lack of research into classroom practice, relating to social justice, of mathematics teachers in England.

Bishop (1998, p. 33) highlights what he sees as *"researchers' difficulties of relating ideas from research with the practice of teaching and learning mathematics"*, which he attributes to the adoption of business-style management in schools and the increasing involvement of politicians in the education agenda. He describes how research often fails to take sufficient account of everyday classroom situations, contexts and constraints faced by teachers. Skovsmose (2011) argues that the vast majority of classroom-based research takes place in 'prototypical mathematics classrooms', i.e. the most affluent classrooms, in which social justice issues may be less obvious and teachers are faced with less challenging circumstances. More generally, Apple (2006, p. ix) highlights the growth in 'critical' educational studies, focusing on complex power relations, but describes these as being mainly 'from the balcony', i.e. *"they are not sufficiently linked to the concrete realities of teachers' and students' lives"*.

I turn my attention now to reviewing the field, i.e. summarising those studies which do include a significant focus on classroom practice in relation to teaching mathematics for social justice.

1.2 A review of the field

Boaler (2008) reports on the 'Railside Project', a study of a high school in California (US) which adopted a collaborative, problem-solving approach to teaching mathematics based on 'Complex Instruction'. Boaler's in-depth analysis of a large number of lesson observations, interviews and questionnaires, suggests that this approach fostered more positive attitudes towards mathematics, and generally higher levels of mathematical attainment amongst students, than in comparable schools. She argues that the approach led to higher levels of 'relational equity', i.e. raising the status of students who would otherwise be less confident in learning mathematics, and to students taking more responsibility for the learning of others. She notes, however, that the mathematics teachers exhibited exceptional commitment and highly developed pedagogical expertise, untypical of most teachers in schools in California with similar levels of socio-economic deprivation (Wright, 2012).

The 'Railside Project' built on Boaler's (1998) earlier study of two schools in England, which employed contrasting approaches to teaching mathematics. In the first school, which used a teacher-led, text-book based approach to learning, students tended to develop procedural understanding, which was of little use when faced with having to use mathematics in unfamiliar situations. In the second school, which used an open-ended, investigative and

project-based approach, students tended to develop greater conceptual understanding. This prepared them better for school assessments, as well as using the mathematics they had learned in non-school settings.

Sebba et al. (2012) report how the 'Complex Instruction' approach was subsequently trialled with mathematics teachers in six schools in England through the 'REALMS Project'. They argue that the increases in students' mathematical attainment which they observed were associated with the development of participating teachers' pedagogy in ways that promoted students' reasoning and problem-solving skills, and greater depth of understanding. The two mathematics departments which experienced the most positive impact, on the attainment and attitudes of students, shared a commitment to mixed-ability teaching and enjoyed the support of the school's management in basing their entire mathematics curriculum on 'Complex Instruction'. Both schools were located in areas with relatively high levels of socio-economic deprivation and participating mathematics teachers included key members of the department who acted as drivers of change.

Gutstein (2007) reports on his own experiences, over a two year period, of teaching a series of 'real world projects' to students in one grade 7/8 class in an urban school in Chicago (US), in which almost all students were Latino/a and from low-income families. During this time, he combined the role of academic and teacher, opting to teach only this one particular class. Gutstein's (2006) pedagogical model of 'reading and writing the world with mathematics' is based on Feire's (1974) notion of 'education for critical consciousness'. He argues that his teaching approach, based around projects of direct relevance to students' own situations and experiences, developed students' critical understanding of mathematics, as well as their sense of agency.

Brown (2009) reports on his own experiences of teaching a Year 7 class in an urban primary school in Australia. He draws on ideas from 'Complex Instruction' (Boaler, 2008), i.e. adopting collaborative, problem-solving teaching approaches and developing shared responsibility for understanding, and Gutstein's (2006) ideas around promoting awareness of socio-political issues and agency amongst students. He describes how these approaches led to the development of a learning community, in which students valued each other's contributions and explanations, and which promoted the development of convincing mathematical arguments. He highlights how the use of learning journals made students more aware of their relationships with mathematics, by fostering their *"awareness of the 'self' acting as a mathematician"* (Brown, 2009, p. 182).

Gregson (2013) focuses on equitable mathematics teaching in her study of an urban 'social justice' school in the US. She reports the case of one teacher, Katherine, who was influenced by previous engagement with the ideas of Gutstein (2006), and in whose class she acted as a 'participant observer'. Katherine had been instrumental in establishing the school and played a key leadership role in the mathematics department. The study describes how Katherine negotiated tensions and dilemmas in teaching mathematics for social justice. The first of these was to prepare students for high-stakes mathematics tests whilst, at the same time, teaching them meaningful mathematics to prepare them for their future lives. The second was to foster students' independence whilst, at the same time, promoting and encouraging collaborative learning.

Planas and Civil (2009) report on an action research project involving experienced mathematics teachers, in low-income schools in Barcelona (Spain), who were invited to join the 'Critical Mathematics Education Group'. The group provided a *"model of professional development based on the involvement of the teachers as co-researchers of their local contexts and practices"* (Planas & Civil, 2009, p. 391). Planas was a participant in the research and also facilitated the group, whilst Civil acted as a consultant, having previously set up a similar group in the US. All eight teachers had been assigned to teach special classes of immigrant students, who were taught separately and followed a different curriculum. The group adopted collaborative, problem-posing and problem-solving pedagogical approaches, in drawing on, and acknowledging, the cultural experiences of the students, to generate more meaningful mathematical knowledge. Teachers developed a greater appreciation of how cultural diversity could provide opportunities for, rather than obstacles to, learning mathematics.

Esmonde and Caswell (2010) report on the work of the 'Radical Math Study Group', established in an urban, multi-lingual elementary school in Toronto (Canada), identified as underachieving in literacy and numeracy. The group, consisting of two university researchers acting as facilitators, school district staff and five mathematics teachers, aimed to develop alternatives to rote learning of procedural methods, which was previously relied upon, and to make connections between the community and the school mathematics curriculum. The group developed three classroom projects, drawing on the students' cultural and community knowledge, and with a social justice focus. The projects were successful in raising the engagement of students, and their families, with school mathematics, and developing students' competences in working collaboratively and independently on mathematical inquiries.

Jaworski (2006) reports on a project, in which she worked collaboratively with a group of mathematics teachers in schools in England, in order to address the gap she acknowledges between theory and classroom practice (see Section 1.1). She describes the establishment of an 'inquiry community', which aimed to challenge the status quo by promoting teachers' critical reflection on previously existing classroom practices. However, the principle she adhered to, of encouraging teachers to choose their own questions to research, meant that there is limited explicit focus on social justice in the study.

Cotton (2013b) reports on a project he carried out with trainee teachers in England in their final year of study, in which he encouraged them to question and reflect upon their own learning. He describes a change in their subject knowledge as they began to see questioning, exploring, and justifying as integral parts of mathematics. He argues that the development of caring and respectful relationships with students demonstrated the *"beginning of a 'preoccupation' with critical mathematics education"* (Cotton, 2013b, p. 93). He highlights the tensions between this 'preoccupation' and the necessity to conform to teachers' standards in order to gain qualified teacher status.

Bartell (2013, p. 132) maintains that *"virtually no research exists about in-service mathematics teachers learning to teach for social justice"*. She reports on a study, in the US, of a fifteen week graduate course which she taught to eight secondary mathematics teachers. Teachers were encouraged to develop their conceptualisations of teaching mathematics for social justice, through discussion and analysis of readings. Subsequently, a 'lesson study' model was employed, in which teachers worked collaboratively to design a lesson, observe each other teaching, meet immediately afterwards to evaluate, then revise and re-teach the same lesson. Bartell concludes that, whilst the course was successful in engaging participants with teaching mathematics for social justice, its short-term nature, and the focus of the 'lesson study' model on a single lesson, limited the longer-term and sustained development of their classroom practice.

1.3 Concluding remarks

The review of the field in Section 1.2 highlights the limited research published on translating theoretical ideas, on teaching mathematics for social justice, into classroom practice. Many of the studies (Boaler, 1998; 2008; Gregson, 2013) focus on untypical classrooms of teachers with a long history of engagement with social justice, and whose classroom practice is well established. Other studies (Gutstein, 2007; Brown, 2009) focus on teaching carried out by the researchers themselves. Whilst providing valuable insight into what teaching mathematics for

social justice might look like in practice, these studies do not address the challenge of how to widen the use of such pedagogies and enable other mathematics teachers to engage with them.

Several of the studies (Jaworski, 2006; Planas & Civil, 2009; Esmonde & Caswell, 2010; Sebba, et al., 2012) concentrate on the professional development of teachers, through establishing groups of teachers undertaking research into their own practice in collaboration with academic researchers. However, these studies tend to focus on specific aspects of teaching mathematics for social justice, such as recognising and drawing on the cultural experiences of learners, or adopting collaborative, problem-solving approaches to learning. Whilst providing valuable insight into the process of collaborative research between teachers and academics in the field, they do not address how these research processes relate to a wider conceptualisation of teaching mathematics for social justice.

The remaining two studies, whilst embracing a wider conceptualisation of teaching mathematics for social justice, do not address the longer-term development of classroom practice. The first study concentrates on working with trainee teachers (Cotton, 2013b), whilst the second involves teachers enrolled on a short-term graduate study programme (Bartell, 2013). Only four of the studies focus on classroom practice in England. However, in contrast to the tendency to concentrate on 'prototypal mathematics classrooms' (Skovsmose, 2011), all of the studies include schools with relatively high proportions of students from lower-income families.

I conclude that there is limited published research evidence available, which focuses on working collaboratively with teachers, particularly those in schools in England, to research and develop their own practice in relation to a wider conceptualisation of teaching mathematics for social justice. I argue that this study, therefore, has the potential to make a substantial and original contribution to knowledge and understanding in this field. I draw substantially on several of the studies that are available, in particular Boaler (2008) and Gutstein (2007), in developing my pedagogical positioning in Chapter 2, and Planas and Civil (2009) and Jaworski (2006), in developing my methodological positioning in Chapter 3.

Chapter 2: Pedagogical considerations

In this chapter, I develop the pedagogical framework, relating to teaching mathematics for social justice, on which this study is based (see Section 2.8). In developing this framework, I consider the nature of mathematics (Section 2.1) and how epistemologies of mathematics influence pedagogical perspectives and ideologies of mathematics education (Section 2.4). I explore two phenomena, related to the nature of mathematics, which I believe are peculiar and unique to the subject (Section 2.2). The first is the strong emotional response towards mathematics from many people, often characterised by feelings of anxiety towards, and alienation from, the subject. The second is the privileged position that mathematics holds within the curriculum, serving as a gatekeeper qualification, opening the door to future opportunities for learners.

There are two related theoretical perspectives which are particularly influential in my pedagogical positioning, which I discuss in detail. The first is the argument that mathematics contributes significantly to the role schools play in reproducing inequity (Section 2.6), and the second is a 'critical mathematics education' which offers a challenge to this status quo (Section 2.7).

As discussed further in Chapter 3, I consider reflexivity of the researcher as vitally important in ensuring trustworthiness of the research findings. Therefore, in establishing my pedagogical positioning in relation to the theories I draw upon, I also consider how my perspective is shaped by my own life experiences, in particular those as a learner (Section 2.3) and as a teacher (Section 2.5) of mathematics.

2.1 The nature of mathematics

Ernest (1991) describes two contradictory epistemologies of mathematics, the 'absolutist' and the 'fallibilist' views. The absolutist view is based on the assumption that all mathematical knowledge is derived from a series of unquestionable truths, or 'axioms', using a system of logic and deduction. Ernest argues that an absolutist paradigm has dominated the development of mathematics for over 2000 years, resulting in the commonly-held perception of mathematics as value-free and unrelated to human affairs, and higher status being afforded to formal proofs as a means of justifying mathematical knowledge. He highlights how the absolutist view of mathematics is being increasingly challenged by philosophers, and some mathematicians, who argue, from a fallibilist perspective, that mathematics is constructed by humans out of a need to make sense of the world around them, and that the axioms themselves are human constructs. Fallibilists contend that mathematical knowledge originates

through a process of inquiry, conjecture and peer scrutiny, regardless of whether it is subsequently proved formally. This view is reinforced when theorems, accepted for centuries as proved, are suddenly discovered to be fallible.

An absolutist view of mathematics is inadequate to explain the social, cultural and political nature of mathematics. Bishop (1979) describes how undergraduate students in Papua New Guinea struggled to interpret two-dimensional representations of three-dimensional shapes, because they didn't understand what he considered to be a Western concept of space, based on objective measurement. Bishop's work highlights the culturally specific nature of mathematics and the danger of assuming it is based on a universal language which can be understood across different cultures.

D'Ambrosio (2006) argues that a form of mathematics, dominant in Europe in the 16th and 17th centuries (albeit with influences from early Indian and Islamic civilisations), was imposed upon the rest of the world through the process of colonisation. Schools were established in the colonies, with the aim of subordinating indigenous cultures, and this 'academic' mathematics displaced other forms of mathematics previously practised by indigenous populations. The political nature of mathematics is also apparent in the way that new content is determined by powerful groups, such as business and commerce, who have the financial resources necessary to fund research in areas which serve their interests. D'Ambrosio (2008, p. 38) claims that, in this way, mathematics has contributed towards *"the technological, industrial, military, economic and political complexes"* which are responsible for *"the growing crises threatening humanity"*.

An absolutist view of mathematics also fails to take account of differences in achievement in, and engagement with, school mathematics amongst different groups within society (see Section 2.6). Increasing concerns over these differences prompted a 'social turn' in mathematics education in the mid 1980's, with a move by researchers towards explaining differences in achievement and engagement using social, rather than cognitive, theories (Lerman, 2000). Boaler (2000) argues that: *"Mathematics education is practiced within a social and political domain, and egalitarian achievement practices may depend on greater acknowledgement of that fact."* Bishop (1999, p. 1) describes the claim that mathematics is value-free as *"a myth which has been exploded in the last two decades"*.

Bernstein (2000) highlights how school mathematics is a 'recontextualisation' of mathematics. He argues that, since the school mathematics curriculum is determined by those working within mathematics education, with limited understanding of the work of real mathematicians,

there is a strong 'classification', or distinction, between school mathematics and mathematics in real life. Similarly, Boaler (2009) argues that students' experiences of school mathematics are impoverished because they are so far removed from the work of real mathematicians. This might explain why, despite the growing appreciation amongst mathematics education researchers, over the past thirty years, of the social, cultural and political nature of mathematics, the general public still continue to perceive the subject as neutral and value-free. For the vast majority of people, the primary experience of mathematics is gained through learning it at school. Their response to any mathematics they might subsequently come across, in real life or employment, is therefore mediated by the perception of the subject gained through their school experiences.

It might be assumed that the relatively small number of people who have studied the subject at degree level, or for whom a substantial part of their employment involves mathematics, might have reflected more deeply on the nature of mathematics and their own perceptions of the subject. However, a smaller-scale study I carried out four years ago, involving a series of interviews with colleagues, suggests this is not the case. It became apparent that even teachers with mathematics or mathematics-related degrees, and some with significant prior experience in mathematics-related employment, hadn't given these matters serious consideration (Wright, 2013).

2.2 Some peculiarities of school mathematics

A rough calculation suggests that adults in England, by the age of 16, will have spent in the region of 2000 hours learning mathematics at school, which is more than one sixth of the total time spent in lessons. Yet the vast majority will never even have considered questions relating to the nature of mathematics, such as: 'What is mathematics?' and 'Why do we spend so much time learning it?' The fact that people are willing to spend so much time learning a subject, without questioning its nature and purpose, suggests they appreciate the important role it plays as a gatekeeper qualification.

Black et al. (2009) highlight the role that school mathematics plays as a 'critical filter', enabling those who are successful to gain greater access to further education courses and more highly-paid employment. In her analysis of the link between education and future earnings, Wolf (2002) highlights how students choosing Advanced Level mathematics go on to earn around 10 per cent more than those choosing a combination of subjects not including mathematics. The question is whether this demonstrates, as Wolf suggests, that mathematics provides skills of

economic value to employers, or in other words, whether the role that mathematics plays as a critical filter is justified.

Lerman (2000, p. 21) argues that the privileged position of mathematics in the curriculum can be attributed to the historical adherence to the outdated notion of absolutism:

> *"Mathematics has stood as exemplar of truth and rationality since ancient times, giving it a unique status in most world cultures and intellectual communities. That status may account for mathematics being seen as a marker of general intellectual capacity rather than simply aptitude in mathematics."*

D'Ambrosio (2008) argues that the belief that success in mathematics can be used as a valid measure of general intelligence is a dangerous misperception that can be damaging to learners. Associating success in mathematics with general intelligence results in mathematical ability being viewed as static or fixed, rather than dynamic or incremental. In which case, it appears to make sense to place students with 'higher ability' in separate groups from those with 'lower ability', and teach them a more demanding curriculum. Thus, notions of fixed mathematical ability can be seen as contributing towards the dominant discourse in schools, where grouping by ability, or 'setting', in mathematics is considered to be the norm in secondary schools, despite the lack of research evidence suggesting it is effective (Winbourne, 2009), and there is a strong assumption that *"students with different levels of 'ability' require differentiated curricula"* (Morgan, 2009, p. 104).

The assessment of school mathematics has become increasingly high-stakes and, in 2006, a new measure of performance of schools in England was introduced, in which General Certificate of Secondary Education (GCSE) mathematics results, along with English, were given particular importance. High-stakes mathematics assessment, compounded by the introduction in schools of new accountability measures in the 1990's, has been blamed for the adoption of pedagogies based on 'teaching to the test', with increased focus on factual recall and procedural understanding:

> *"The current high stakes assessment system, where institutions are more accountable for results than for the mathematical understanding of their students, has a detrimental effect on the ability of young people to apply mathematics … some areas of mathematics which are more difficult to assess, such as problem-solving, reasoning and communication, are not given sufficient teaching time and are often replaced in the classroom by teaching routines and procedures necessary to pass the test." (ACME, 2011, p. 3)*

The perpetuation of such teaching approaches, viewed by mathematics learners as uninteresting and lacking relevance and meaning (Mukhopadhyay & Greer, 2008), has contributed towards a significant level of alienation from the subject amongst children and

adults. Nardi and Steward (2003) describe the 'quiet disaffection' of a large proportion of students as a result of mathematics teaching which they characterise as being boring, irrelevant, passive, learning rules without understanding their rationale, offering little opportunity for collaboration, and ignoring individual needs. They describe how mathematics is presented as a demanding subject in which the possibility of success is limited to those considered to have innate mathematical ability. Brown (1999) highlights how the vast majority of students in England, above the age of 12, are taught mathematics in ability groups, resulting in many students considering themselves as failures from an early age. Hodgen and Marks (2009, p. 31) describe how *"lower attaining students receive a largely remedial (and boring) curriculum, and most students regard themselves as weak mathematically."*

Black et al. (2009) argue that the common belief that ability is innate means that students who identify themselves closely with mathematics are those who see themselves as having 'special' mathematical ability. However, even successful learners of mathematics can become alienated from the subject because they don't wish to *"author their identities as passive receivers of knowledge"* (Boaler & Green, 2000). Because of the disengaging way mathematics is often taught, and the perceived failure that many students experience, it is not surprising that a large number of people distance themselves from it and that *"mathematical incompetence remains socially acceptable"* (NCETM, 2008, p. 5).

2.3 My experiences as a learner of mathematics

Throughout my own learning of mathematics, up until the end of my degree course, I can't recall ever considering, or being asked to consider, the nature of mathematics or why I spent so much time learning it. I had no appreciation of the social, cultural or political nature of the subject, or its privileged position in the curriculum and status in society. If I had been questioned about its nature, I have no doubt I would have repeated uncritically general assumptions made in the public discourse surrounding mathematics, i.e. it is about logic, proof and being right or wrong (see Section 2.1).

I do not have much recollection of learning mathematics at primary school, other than working through problems on my own, or in a small group, from text books. At secondary school (a former grammar school which had recently turned comprehensive), although mathematics was the subject in which I was most academically successful, English was the subject I enjoyed most. English was taught in a way which I found engaging, through the reading of modern classic novels, followed by group and whole class discussions around contemporary issues which they focused on, such as social inequality, criminality and politics. Mathematics, on the

other hand, was taught in a more traditional way, with examples explained on the board to the whole class, before we were directed to the text book for practice exercises.

Despite not having a particular liking for the subject, I considered myself to be a successful learner of mathematics, based on better performances than others in tests. This was despite not really putting in a great deal of effort, which led me to consider myself as naturally 'talented' at mathematics. At the age of sixteen, this made it easier for teachers and parents to persuade me to drop English in favour of mathematics at Advanced Level, and to convince me that this would provide me with greater opportunities in future life. Unfortunately, my school had a very archaic timetable arrangement that wouldn't allow me to take both English and mathematics, and I still regret not insisting at the time that I be allowed to study English.

I was attracted to university as a means of becoming more independent and discovering new horizons in life, rather than out of a strong desire to study further. I chose a mathematics degree as I believed this would give me the greatest chance of success, with least effort, enabling me to make the most of other opportunities university life could offer. I didn't enjoy studying mathematics at university, partly because the increased level of challenge meant that, for the first time, I found myself beyond my comfort zone. It was only an (ultimately) 'fortunate' sporting injury to my back which enabled me to gain a reasonable grade, since for the last six months leading up to my final examinations, I could do little else apart from lie flat and study. By the end of my degree, despite having studied mathematics over a period of sixteen years, I had still not considered the nature, purpose, position or status of the subject.

Luckily my mathematics learning didn't stop there. During my initial teacher education programme, for the first time in my life, I was encouraged by university tutors to reflect upon the nature of mathematics and my eyes were opened to its value-laden nature. I began to see mathematics as a human construct with subject content being determined by the interests of powerful groups in society. One example of this was looking back on the third year options which I had chosen during my degree course, including 'information theory' and 'coding theory'. I had assumed at the time that these were obscure branches of pure mathematics with no apparent applications to real life. However, on reflection, I began to appreciate that these areas of mathematics, rather than being politically neutral, had been developed primarily to serve the demands of banks and the military for secure financial transactions and communications. I was made aware of the multicultural nature of mathematics and how some ideas, for example Pascal's Triangle and Pythagoras' Theorem, whilst originating from non-Western cultures, in these cases China and India, had been appropriated by, and re-named after, European mathematicians.

I was also given the opportunity, for the first time, to engage with mathematical investigations and inquiry-based approaches to learning. This helped me to develop an appreciation of just how enjoyable, relevant and empowering learning mathematics could be. By the end of my teacher training course, my view of mathematics had changed significantly, and I had developed what I now recognise as a fallibilist epistemology of mathematics (Ernest, 1991).

2.4 The purpose of mathematics education

I have often asked mathematics teachers, and those training to become mathematics teachers, what they consider to be the main purposes of mathematics education. Invariably, I receive a wide range of responses including developing financial literacy, numeracy and problem-solving skills required in everyday life, other subject areas and in employment, appreciating the beauty of mathematics, and acquiring logical thinking and deductive reasoning. This wide range of aims is reflected in the introductory statement from the recently introduced 'mathematics programmes of study' in England (DFE, 2013).

A consensus has grown amongst the mathematics education community, over the past thirty years, that a more relevant and engaging mathematics curriculum is needed, with a greater focus on open-ended tasks, problem-solving and conceptual understanding (Cockcroft, 1982; ACME, 2011; OFSTED, 2012). Yet many mathematics classrooms continue to witness teaching approaches focused on the transmission of procedural skills in a way that disengages and alienates learners (see Section 2.2). This consensus contrasts with tensions and conflict between different political interest groups, resulting in the introduction of educational policies and reforms by successive UK governments which appear to contradict the aims of mathematics education as articulated in official curriculum documents.

Much of the contention in the development of mathematics curricula centres around conflicting views of the nature of mathematics (see Section 2.1), in particular, the status that is afforded to mathematical processes and applications, as opposed to content and procedures (Dowling & Noss, 1990). Such differences are apparent in a comparison of the previous and current Key Stage 3 (age 11-14) mathematics curricula in England. The previous 'mathematics programmes of study' (QCA, 2007) were presented in four sections: 'key concepts' (including communicating effectively, posing questions, developing arguments, critical understanding), 'key processes' (relating to stages of problem solving and the data handling cycle), 'range and content', and 'curriculum opportunities' (including working collaboratively, open tasks, contexts beyond the school). The three sections representing processes and applications accounted for six pages of the document, whilst the 'range and content' covered only two

pages. The new programmes of study (DFE, 2013) contain a section entitled 'working mathematically' covering just over one page, which includes developing fluency, reasoning mathematically, and solving problems. This is followed by five pages of 'subject content'.

Whilst the 'working mathematically' section of the new programmes of study reflects many of the changes called for by the mathematics education community, the renewed emphasis on subject content will do little to improve the way mathematics is commonly taught in schools. Michael Gove (Secretary of State for Education at the time) made clear, in his foreword to a report which was used to justify the new curriculum (Oates, 2010), how he believed *"identifying the crucial concepts and ideas that each year group should learn"* was far more important than providing guidance for teachers on mathematical processes and applications, which he described as *"vague generic statements of little value"*.

Ernest (1991) proposes a typology of five mathematics education ideologies, which help to explain tensions and conflicts that arise in the development of school mathematics curricula. He describes an 'ideology' as *"an overall, value-rich philosophy or world-view, a broad inter-locking system of ideas and beliefs"* (Ernest, 1991, p. 6). These ideologies include 'industrial trainer', 'technological pragmatist', 'old humanist', 'progressive educator' and 'public educator'. He associates the first four ideologies with an absolutist epistemology of mathematics, claiming that only a 'public educator' ideology is associated with a fallibilist epistemology (see Section 2.1). It is worth emphasising that, whilst Ernest's typology provides a useful classification framework, *"individual educators are not located wholly, exclusively, or unproblematically within one of these ideologies"* (Povey, 2003, p. 57).

Ernest describes the 'old humanist' ideology, more prevalent amongst mathematicians than mathematics educators, as aiming to preserve the abstract and rigorous nature of mathematics, for example by promoting the importance of formal proof within the curriculum (Andrews, 2012). An old humanist ideology is more concerned with promoting the intellectual development of a small elite, who are likely to go on to study mathematics at university, whilst advocating a more basic mathematics education for other students (Borovik, 2014). Its influence can be seen in the abandonment of coursework and modular examinations and in the introduction of more 'rigorous' and 'robust' GCSE's from September 2016 to be assessed exclusively through terminal examination papers (EDEXCEL, 2014).

The 'industrial trainer' ideology was influential amongst the 'new right' politicians who introduced neo-liberal reforms in the 1980's, which still impact on current educational policies such as the use of league tables to compare schools' performance and the promotion of

setting in mathematics classrooms. It promotes the marketisation of education, and advocates extending practices common in business and industry, such as promoting competition and accountability, to schools (Ball, 2013). Its influence can also be seen in the introduction of academies and free schools, acting outside the constraints of democratic control of local authorities, whose role is often replaced by individuals from the business world willing to act as sponsors.

The 'technological pragmatist' ideology, common amongst representatives of industry and commerce, emphasises the importance of mathematics skills for the economy, which is apparent in politicians' apparent unquestioning belief that more spending on mathematics education will inevitably lead to higher levels of economic growth (Wolf, 2002). It promotes the teaching of mathematics skills seen as useful in the workplace and the use of technology, including calculators, in mathematics teaching (Cockcroft, 1982). Its influence can be seen in the introduction of functional skills into the GCSE mathematics examinations from 2010, although these are set to disappear from the new GCSE in 2016.

The 'progressive educator' ideology, relatively common amongst teachers and educationists, promotes the acquisition of mathematical skills which are appropriate to the needs of the learner. It views the primary purpose of education as nurturing the development of the individual and is evident in the promotion of practical activities and personal exploration (Cockcroft, 1982). It was prevalent in schools, particularly in the primary sector, up until the 1980's, until politicians began to take more of an interest in influencing educational policy.

The 'public educator' ideology, common amongst mathematics education researchers (see Section 1.1), is primarily concerned with issues of equity and social justice in mathematics education. It promotes a critical understanding of mathematics and aims to use mathematics as a means of promoting democratic citizenship. Despite being reflected in the aspirational goals of many official curriculum documents, it has had little influence on educational policy in schools (Wright, 2012).

Conflict between different ideologies often results in policy makers focusing too much on countering the arguments of other interest groups, rather than considering what is best for mathematics learners. This is exemplified by the Math Wars, which raged in the US from the 1980's, between 'traditionalists' advocating 'conservative' teaching approaches, characterised by authoritarian, transmission-based modes of teaching, and 'reformers' advocating 'progressive' teaching approaches, characterised by collaborative, problem-solving pedagogies (Schoenfeld, 2004). The 'traditionalists' were influenced by old humanist and industrial trainer

ideologies, whilst the 'reformers' were influenced by technological pragmatist and progressive educator, and to a lesser extent public educator, ideologies (Ernest, 2004). Parallels can be drawn between similar, albeit less overt, tensions in the development of the mathematics curriculum in England (Wright, 2012).

Conflicts between different ideologies of mathematics education, which distract from the real needs of learners, help to explain why the curriculum continues to lack meaning and relevance for many students despite consistent calls for change. Gates (2006) highlights how teachers' underlying ideologies and predispositions, dependent upon their upbringing and social experiences, largely determine their positioning in relation to pedagogical beliefs and management styles. He argues that these ideologies lead to contradictions between teachers' beliefs and practice, for example when 'oppositional ideologies', such as those underlying a commitment towards collaborative, problem-solving pedagogies, come into conflict with the 'dominant ideologies' they encounter in schools. These contradictions shed further light on why a 'teacher-centred' approach persists in mathematics classrooms, despite *"a widespread belief in some notion of good mathematics teaching"* which is very different (Gates, 2006, p. 349).

Before exploring further (in Section 2.6) the underlying reasons for the apparent disparity between teaching approaches advocated by the mathematics education community and those commonly employed in schools, I reflect on my own experiences of becoming a teacher and how these relate to my ideology of mathematics education.

2.5 My experiences as a teacher of mathematics

Looking back at my experiences at university, there was almost an inevitability that I would choose to become a teacher. Both of my parents were teachers, my father a lecturer of applied mathematics at a polytechnic, and my mother a teacher of literacy at an adult education centre. Despite this, I had never considered becoming a teacher myself before going to university. Indeed, I had given very little thought to my future career prior to university, other than showing an interest at school in mechanical or civil engineering. I saw going to university as a goal in its own right, having witnessed my three older sisters using it as an opportunity to enjoy themselves and become more independent.

Whilst I certainly made the most of what university life had to offer, particularly the sport and social life, I became increasingly disillusioned and frustrated with the position of tradition and privilege in the university that I attended. I had chosen to apply to one of the two most prestigious universities in the country, not because I was particularly ambitious, but rather

because the success I had demonstrated in school mathematics meant that there was parental encouragement for me to do so. When I was offered a place, I accepted it without really giving it much thought, because I felt that's what anyone in my position would be expected to do.

I had been acutely aware of being one of the most privileged children in my primary school, one of the few whose family lived in a privately-owned house, rather than a council house. In contrast, I found myself one of the least privileged students at university, with most students coming from independent schools and wealthier families. I resented many of the traditions of university life, such as wearing gowns for formal dinner sittings, which I rarely attended, and could not understand why so many other students seemed to enjoy these traditions or engage with them so readily. I began to appreciate how many students had gained a place at the university because of their privileged upbringing and schooling, rather than because of any superior personal qualities which they, or others, might assume they possessed.

This sense of injustice explains my first political action, as part of a campaign run by the students' union and supported by the university, to visit comprehensive schools in my home town, speak to sixth-formers, and try and persuade more of them to consider applying to the two most prestigious universities in future. Approaching the end of my studies, the only career option that appealed to me, where I could make use of my mathematics degree, was teaching, which I began to see as an opportunity to redress disadvantage and injustice within society.

I chose a teacher training programme which placed particular emphasis on issues of equal opportunities, multiculturalism and the political nature of education, as this seemed to fit in with my growing interest in issues of power and privilege. As well as developing a fallibilist epistemology of mathematics (see Section 2.3), I began to appreciate how decisions regarding the school mathematics curriculum and how it should be taught, for example whether to teach in mixed-ability groups or set students according to ability, were dependent upon ideological positions of policy makers and those in positions of power. By the end of my initial teacher education, I had developed, primarily, a public educator ideology of mathematics education, with some elements of progressive educator (see Section 2.4).

I began my teaching career in 1987, in an inner-city school in London, at a time when the promotion of progressive approaches to teaching mathematics, including collaborative, problem-solving pedagogies and mixed-ability teaching, was prevalent (Cockcroft, 1982). By the time I left my last teaching post in 2011, I had witnessed a massive shift in the discourse around mathematics teaching in schools, accompanied by ever-increasing levels of scrutiny of

teachers and high-stakes testing. I witnessed first-hand the renewed focus on traditional, teacher-led pedagogies and steady growth of setting students by ability (See Section 2.2).

What characterised my professional interests throughout this period, including two breaks in teaching as a curriculum developer and as a mathematics consultant, was my concern for the alienation and disengagement of large numbers of students that I witnessed. It became increasingly apparent to me that this alienation was greatest amongst students from disadvantaged backgrounds, who were over-represented in the schools in which I chose to teach. My growing belief in the use of collaborative, problem-solving pedagogies, and mixed-ability teaching, as a way of addressing this alienation, led me to seek out schools in which to work, where there were opportunities to engage with and develop these ideas further. I also developed several classroom resources designed to make the mathematics curriculum more relevant and meaningful for students, including a series of activities focusing on issues of equity, social justice and human rights (Wright, 2004).

2.6 School mathematics and the reproduction of inequity

Despite increasing awareness of the social and political nature of school mathematics, and the growing consensus that a more relevant and engaging mathematics curriculum is needed, much mathematics teaching in England remains teacher-led, procedural and lacking in relevance (see Section 2.4). Noyes (2012) highlights how the prevalence of 'performative', teacher-centred pedagogies causes an alarming decline in students' attitudes towards mathematics as they progress through secondary school, resulting in relatively low participation rates in post-compulsory mathematics.

Of particular concern are the high levels of inequality in achievement and participation in school mathematics amongst students from different genders, ethnic backgrounds and social classes. Whilst girls perform at similar levels to boys in GCSE mathematics, this hides their significantly lower participation rates in post-compulsory mathematics education, and the correlation between social class and achievement in school mathematics remains as strong as ever (Noyes, 2009; Boaler, et al., 2011). There is evidence that more open-ended approaches to teaching mathematics, which promote collaborative problem solving amongst mixed-ability groups of students, lead to more equitable outcomes and encourage greater participation in higher level mathematics amongst both boys and girls (Boaler, 2008; Boaler, et al., 2011). Unfortunately, such approaches are the exception, rather than the rule, in many mathematics classrooms.

Since school mathematics acts as a 'critical filter', in determining future education and employment opportunities (see Section 2.2), these differences in achievement and participation in mathematics inevitably lead to the perpetuation of social inequities. Bourdieu's theory of 'reproduction' regards one of the school's primary functions as reproducing social order, and maintaining unequal power relations between different classes and groups in society (Bourdieu & Passeron, 1990). He argues that school mathematics plays a vital role in achieving this goal:

> *"Often with a psychological brutality that nothing can attenuate, the school institution lays down its final judgements and its verdicts, from which there is no appeal, ranking all students in a unique hierarchy of all forms of excellence, nowadays dominated by a single discipline, mathematics." (Bourdieu, 1998, p. 28)*

Bourdieu's theory of reproduction develops the notion of 'cultural capital', i.e. those cultural and social resources which are recognised and valued by the school (Jorgensen, et al., 2014). He argues that children from middle-class families acquire greater levels of cultural capital than children from working-class families because of their upbringing. This leads to higher levels of academic attainment and participation amongst middle-class children, and lower expectations of working-class children amongst teachers, most of whom are themselves from middle-class backgrounds (Lerman & Zevenbergen, 2004).

The main function of the school is to reproduce social inequity, which it does through concealing the power relations which exist between different groups and falsely attributing academic success to giftedness or merit (Bourdieu & Passeron, 1990). It is such notions of mathematical ability which lead to the prevalence of setting in mathematics in schools in England (see Section 2.2). In this way the school claims to provide equality of opportunity, whereas, in reality, *"only those children who come already endowed with [linguistic] capital are in a position to make the most of the opportunities schools purport to 'offer' equitably to all children"* (Noyes, 2008, p. 55).

Bourdieu argues that a person's 'habitus', i.e. their *"system of schemes of thought, perception, appreciation and action"* (Bourdieu & Passeron, 1990, p. 40), tends to reproduce the same conditions of which it is a product. In particular, those who are successful under the current system see no reason why it should be changed. This is reflected in the tendency of mathematics teachers to adopt pedagogies similar to those which they themselves experienced as (successful) learners, partially explaining the persistence of traditional, teacher-led approaches to teaching mathematics (see Section 2.4). Bourdieu argues that the notion of 'habitus' underlies *"the production of the most durable academic and social differences"* (ibid., p. 16), helping to explain why members of oppressed groups exhibit behaviours and make

choices that contribute towards the reproduction of their own oppression, a process Bourdieu refers to as 'symbolic violence' (Jorgensen, et al., 2014). These behaviours include attributing their lack of academic success to their own personal deficits and opting out of mathematics once it becomes non-compulsory (Mendick, 2003).

Bourdieu's analysis provides useful insight into how schooling contributes towards cultural and social reproduction, and why mathematical attainment remains so strongly correlated to social class, more so than to ethnicity or gender (Noyes, 2008). It leads to the conclusion that the under-achievement of students from marginalised backgrounds is a 'systemic' problem, rather than blaming this failure on *"individual deficiencies on the part of particular pupils and parents"* (Jorgensen, et al., 2014). It is, however, rather fatalistic in its approach, suggesting that any pedagogic action will inevitably tend towards such reproduction. The theory poses a paradox, for any teacher wishing to disrupt this cycle of social reproduction, in convincing students that:

> *"Either you believe I'm not lying when I tell you education is violence and my teaching is legitimate, so you can't believe me; or you believe I'm lying and my teaching is legitimate, so you still can't believe what I say when I tell you it is violence" (Bourdieu & Passeron, 1990, p. 12).*

A dependence on Bourdieu's ideas alone might therefore question the feasibility of teaching mathematics in a way that challenges existing power relations: *"Although Bourdieu's tools offer a convincing theorisation of the way things are …, they are not so useful in generating emancipatory pathways"* (Noyes, 2008, p. 64).

Bernstein's (2000) theory of 'pedagogic discourse' builds upon Bourdieu's theory of social reproduction, outlining how the school creates a 'mythological discourse' in which failure is attributed to inborn characteristics related to cognitive, affective or cultural deficits. However, he argues that this process is intentional, rather than an essential function of schooling: *"Some social groups are aware that schooling is not neutral, that it presupposes familial power both material and discursive, and that such groups use this knowledge to improve their children's pedagogic progress"* (ibid., p. xxiii).

Bernstein (2000) views mathematics as based on a strong 'classification', resulting in a dislocation with other subject areas and the development of specialised rules for communication and behaviour. He also views mathematics as involving strong 'framing', in which the teacher acts as a transmitter of knowledge and exerts a large amount of control over the discursive and social order. Academic success, in such a strongly classified and framed subject as mathematics, depends upon students' ability to decipher the 'rules of the game' in

the classroom. In particular, students need to be able to follow the 'recognition rules', i.e. identify relevant meanings from tasks that are set, and 'realisation rules', i.e. come up with legitimate and appropriate responses or actions.

Children from middle-class backgrounds tend to acquire such recognition and realisation rules through their upbringing, enabling them to read mathematical tasks and respond appropriately (Lerman, 2009). Children from working-class backgrounds, on the other hand, experience difficulty in responding to so-called 'realistic' test items, which are often rather contrived and bear little resemblance to real life contexts (Cooper & Dunne, 2000). They find it difficult to distinguish between the context of the task in relation to the mathematics classroom and a literal interpretation of the task in relation to the real life context. Cooper and Dunne (2000, p. 200) highlight the extent to which school mathematics assessments systematically disadvantage working-class children and question what it is that such assessments are intended to measure:

> *"Is it primarily children's 'mathematical' knowledge and understanding per se, or is it primarily their capacity to negotiate the boundary between the 'mathematical' and the 'real' as part of the process of discovering the test designer's intention for the item?"*

Bernstein (2000) argues that stronger framing results in more explicit realisation rules for children to follow. However, where framing is weaker, middle-class children are generally more able to create their own framing and thus make more effective use of realisation rules. He outlines two contrasting modes of teaching, the 'competence' and 'performance' models. Performance models, which dominate all levels of education and are common in mathematics classrooms (see Section 2.4), are based on strong classification and framing and low levels of autonomy. In contrast, competence models are based on weak classification and framing, promote higher levels of autonomy for learners and teachers, and are often viewed as empowering, either cognitively, culturally or politically.

A danger, highlighted in Bernstein's analysis, is that competence models of teaching, whilst appearing to be more emancipatory than performance models, might actually disadvantage students from working-class backgrounds. Lubianski (2000, cited in Lerman & Zevenbergen, 2004) reports how the 'reform' competence model of teaching mathematics in the US, based on inquiry-based classrooms and more relevant and meaningful contexts, disadvantaged working-class students and some ethnic-minority students. It would be easy to conclude that, in order to address inequities, performance models of teaching mathematics should be adopted based on stronger framing.

Lerman and Zevenbergen (2004) highlight how working-class students are already exposed to higher levels of pedagogic control, rote learning, closed questions and tasks, whilst middle-class students are more likely to experience a richer problem-solving approach to learning mathematics. Rather than advocating performance models of teaching, they suggest that *"some work needs to be done, both theoretically and practically, to mitigate the effects of invisible pedagogies - such as through modifying the strength of framing"* (p. 37). They argue that teachers, who tend to have expectations based on middle-class values, generally lack understanding and awareness of how students' responses might depend on their social background. Hence, students' misrecognition of implicit classroom rules and norms are often wrongly interpreted as misbehaviour and non-compliance, resulting in lower expectations and differential treatment.

Teachers therefore need to be made more aware of the effects of students' backgrounds on their ability to decipher recognition and realisation rules. Another strategy for helping more students to succeed in school mathematics, whilst providing a curriculum that is more relevant, meaningful and engaging, is to make the 'rules of the game' more explicit. This requires encouraging students and teachers to reflect upon the invisible processes, already outlined in this section, which are at work in the mathematics classroom, and which contribute towards the reproduction of social inequities.

2.7 Critical mathematics education

School mathematics currently alienates many students, particularly those from lower-income families, who tend to receive a predominantly functional mathematics education (see Section 2.2). Skovsmose (2011, p. 9) describes an 'exercise paradigm', frequently evident in mathematics lessons, in which students complete a series of almost identical, closed questions, cultivating a *"prescription readiness, which prepares the students for participating in work processes where a careful following of step by step instructions without any question is essential"*. Gutstein (2006, p. 10) argues that such a disempowering mathematics education reflects the need of capitalist economies for *"an ever-growing army of low-skilled, compliant, docile, pleasant, obedient service workers"*.

A 'critical mathematics education' offers an alternative perspective on teaching mathematics which challenges this status quo. It resonates with Ernest's (1991) public educator ideology of mathematics education (see Section 2.4), and draws on Freire's notion of 'critical education', which encompasses raising consciousness, the emancipation of learners, and the development of critical citizenship (Skovsmose, 2011). It is based on a belief that education can and should

tackle pressing issues currently facing our society, including the sustainability of the planet, human rights abuses and growing inequality (Cotton, 2013b). It contends that mathematics can play a critical role in restructuring society and it recognises students as *"key participants in the social movements of the day"* (Gutstein, 2006, p. 221).

Freire developed his theory of 'education for critical consciousness' through *"teaching adults how to read in relation to awakening their consciousness"* (Freire, 1974, p. 43). Frankenstein (1983, p. 325) argues that Freire's ideas provide useful insight into the practice of teaching mathematics in schools:

> *"Applying Freire's theory to mathematics education directs our attention to how most current uses of mathematics support hegemonic ideologies, how mathematics education also reinforces hegemonic ideologies, and how critical mathematics education can develop critical understanding and lead to critical action."*

Skovsmose (2011) highlights how it is possible for mathematics education to be both empowering (in a pragmatic sense) and disempowering (in a socio-political sense) at the same time, i.e. through enabling a learner to gain employment, whilst promoting the 'prescription readiness' referred to earlier. He argues that the main preoccupations of critical mathematics education should be with reflecting 'on', 'with' and 'through' mathematics. Learners should be encouraged to reflect through mathematics, by participating in 'landscapes of investigation', in which they are engaged in meaningful mathematical inquiries, making their own decisions, posing questions and interacting and communicating with other learners. They should reflect with mathematics, by carrying out mathematical inquiries which lead to a deeper understanding of a range of social, cultural, political and economic situations. Learners should also reflect on mathematics, by considering the nature and position of the subject, and how it can be used to make decisions and legitimise actions affecting themselves and others.

D'Ambrosio (2006, p. 31) contends that the dominance of 'academic' mathematics (see Section 2.1) has led to most school mathematics curricula becoming *"uninteresting, obsolete and useless"* for many learners. Whilst arguing that equal status should be afforded to different cultural forms of mathematics, he also recognises that some academic mathematics is essential for playing an active role in society, and that excluding learners from all academic mathematics would be disempowering. Similarly, Skovsmose (2011) warns that focusing on making mathematics more relevant to learners' backgrounds risks limiting their future opportunities and life experiences. By reflecting 'on', 'with' and 'through' mathematics, he argues that learners are more likely to realise their 'foregrounds', i.e. make the most of the opportunities provided by their social, political, cultural and economic situations.

Gutstein (2006) argues that acquiring 'mathematical power', i.e. the confidence to engage in complex mathematical tasks, through a curriculum emphasising communication, reasoning and problem-solving, is a necessary, but not sufficient, condition for the empowerment of learners. He draws upon Freire's notion of 'praxis', i.e. *"the connection of reflections and action"* (ibid., p. 28), to develop a framework for teaching mathematics for social justice which aims to prepare students to *"investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts"* (ibid., p. 4). Through 'reading and writing the world with mathematics', students use mathematics to develop their understanding of power relations, inequity, discrimination, and how these relate to their own lives and wider society. This fosters their sense of social agency and self-efficacy, i.e. their belief that they can influence and change society. Gutstein emphasises how the success of this approach depends upon students being willing participants, and fundamentally changing their orientation towards mathematics, and upon teachers appreciating the socio-political nature of mathematics and developing meaningful relationships with students.

2.8 Teaching mathematics for social justice

Before developing my own conceptualisation of teaching mathematics for social justice, it is important to establish the meaning assigned to 'social justice' in this study. The term implies a desirable phenomenon, after all, nobody would want to be seen to advocate 'social injustice'. This might explain why it has been embraced by various groups, each interpreting it in different ways depending on their own interests. For example, the Centre for Social Justice is a self-proclaimed 'independent think-tank', established in 2004 by former Conservative Party leader, Iain Duncan Smith. It views 'social justice' in terms of individuals' deficits which, it argues, create poverty and thus cause a drain on the economy. Its work focuses on 'social breakdown', i.e. *"family breakdown, educational failure, economic dependency and worklessness, addiction to drugs and alcohol, severe personal debt"* (Centre for Social Justice, 2014). It sees the voluntary and community sector as playing a leading role in addressing these issues and argues that the welfare state is responsible for entrenching dependency and trapping people in low-income areas. From this perspective on 'social justice', no thought is given to the need to restructure society or to challenge the current socio-economic and political systems.

Gutstein (2006, p. 8) warns against associating 'social justice' with economic needs and argues for a *"social justice agenda that instead places the material, social, psychological, spiritual, and emotional needs of human beings, as well as other species and the planet, before capital's*

needs". The Royal Society views 'social justice' as implying *"fairness and mutual obligation in society: that we are responsible for one another, and that we should ensure that all have equal chances to succeed in life"*, whilst recognising that the choice of method for redistributing life opportunities remains highly contentious (Royal Society for the Encouragement of Arts, Manufactures and Commerce, 2014). This focus on leading a fulfilling life, regardless of background, is echoed in Rawls' (1972, cited in Cotton, 2013a, p.74) concept of a new 'socially just' society in which *"no matter what position or role we are now placed in, we will be able to live life to the full"*.

These ideas are reflected in Bernstein's (2000) conditions for democracy in schools, which include the notions of individual, social and political 'pedagogic rights'. These rights include: 'enhancement', i.e. *"the right to be more personally, more intellectually, more socially, more materially [and] the right to the means of critical understanding and to new possibilities"*; social, intellectual, cultural and personal 'inclusion', whilst recognising the right to remain autonomous; and "*the right to participate in the construction, maintenance and transformation of order"* (p. xx).

Cotton (2013a, p. 74) describes a mathematics education in a 'socially just' world as one in which *"you would be happy for a child you love to learn in any classroom in any school, and to exchange places with any child in any classroom in any school"*. Gutstein (2006, p. 22) argues that mathematics educators have tended to focus too much on addressing issues of equity within mathematics, rather than on the need to restructure schools and society, and that the fundamental aim of teaching mathematics for 'social justice' should be *"liberation from oppression"*.

In developing my own framework for 'teaching mathematics for social justice', I draw, in particular, on Bernstein's (2000) conditions for democracy in schools, Gutstein's (2006) notion of 'reading and writing the world with mathematics', and Skovsmose's (2011) idea of reflecting 'on', 'with' and 'through' mathematics. I argue for an alternative approach to teaching mathematics that challenges, rather than contributes towards, the reproduction of inequities, and the perpetuation of privilege, within society. Such an approach should provide more engaging, relevant and meaningful experiences for mathematics learners, and greater opportunities for disadvantaged students to realise their foregrounds.

The characteristics of the approach to 'teaching mathematics for social justice' adopted in this study are summarised below:

1) Employ collaborative, discursive, problem-solving and problem-posing pedagogies which promote the engagement of learners with mathematics;
2) Recognise and draw upon learners' real-life experiences in order to emphasise the cultural relevance of mathematics;
3) Promote mathematical inquiries that enable learners to develop greater understanding of their social, cultural, political and economic situations;
4) Facilitate mathematical investigations that develop learners' agency, enabling them to take part in social action and realise their foregrounds;
5) Develop a critical understanding of the nature of mathematics and its position and status within education and society.

2.9 Concluding remarks

In this chapter, I have outlined the theoretical perspectives, and my own life experiences, which have shaped my pedagogical positioning in relation to school mathematics. In so doing, I have established the pedagogical framework, relating to teaching mathematics for social justice, on which this study is based. In the next chapter, I turn my attention to methodological considerations which inform my research design.

Chapter 3: Methodological considerations

In this chapter, I reflect on my experiences as a mathematics teacher educator and consider how these have influenced my ontological, epistemological and methodological positions underlying this study. I discuss the research literature highlighting the disparity between current mathematics education research and teachers' classroom practice, and explain my adoption of a participatory action research methodology. I outline how my critical methodological stance necessitates giving careful consideration to my role as researcher and I develop a framework for ensuring trustworthiness of the research findings. Finally, I outline the framework I use for analysing data.

3.1 My experiences as a mathematics teacher educator

I consider myself fortunate to have had so many opportunities, during my career in mathematics education, to work collaboratively with teachers, focusing on classroom practice. As well as the satisfaction gained from contributing towards developing practice across a wide range of classrooms, I have benefited from observing others, discussing ideas and adding to my own practice. As a head of mathematics, I was responsible for the development of classroom practice within the department in which I worked. As a curriculum developer, I involved teachers in all stages of the development of innovative teaching approaches, resulting in the publishing of new resources. As a mathematics consultant, I worked with schools and departments to address their professional development needs and promote effective teaching. As a school-based mentor, and more recently university-based tutor, I have helped trainee teachers learn how to develop their teaching practice in relation to research evidence on effective teaching and learning.

Over the course of my career, which included 15 years as a classroom teacher between 1987 and 2011, I have developed a growing commitment towards the use of collaborative, problem-solving pedagogies, and mixed-ability teaching (see Section 2.5). Throughout my career, I have found that most experienced and beginning teachers have been highly receptive to engaging with such pedagogies, and to exploring ways of making mathematics more relevant and meaningful, as advocated in official documentation (QCA, 2007; OFSTED, 2008). When I began teaching in secondary schools in 1987, investigative learning and mixed-ability teaching were strongly endorsed by the ILEA (Inner London Education Authority), a directly elected body that was abolished by the Conservative government in 1990. The SMILE Mathematics project, established by ILEA, encouraged all mathematics teachers to become involved in curriculum

development. I, along with many other teachers in Inner London schools, became involved in developing teaching ideas and resources, designed to support investigative learning and mixed-ability teaching, which were published and used widely in schools.

Since 1987, I have witnessed a dramatic change in the discourse in schools, resulting from the increasing marketization of education and performativity in schools (Ball, 2013). Whilst teachers are still just as interested in engaging with collaborative, problem-solving pedagogies, the current climate in schools, with target setting linked directly to students' attainment in tests, acts to constrain the extent to which these ideas are put into practice. Very few mathematics teachers are now involved in curriculum development and performance management structures promote the adoption of practices, policies and schemes of work decided by others. Mixed-ability teaching in London secondary schools has been replaced, almost universally (with a few notable exceptions), with setting by ability in mathematics classrooms (see Section 2.2).

As the discourse in schools has changed, my strong beliefs about mathematics education have increasingly brought me into conflict with others. As a head of mathematics, I argued strongly in defence of mixed-ability teaching against a head teacher who was adamant on introducing rigid setting based on regular testing. My approaches towards management and the professional development of colleagues have also brought me into conflict with others. A commitment towards collaborative decision making, and encouraging teachers to reflect critically on their own practice in order to identify their own areas for development, has not always gone down well with senior managers. The same head teacher referred to above criticised me for not keeping enough 'distance' from colleagues in my department. Whilst working as a local authority mathematics consultant, I remember being shocked, during my induction training, when I was warned against the dangers of 'collusion' with teachers. What was meant by this became clear when I was criticised by a National Strategy regional director for not adopting a directive enough approach, for example by allowing too much discussion amongst teachers during training sessions and adapting the resources to what I saw as the needs of the participants, rather than sticking rigidly to the script provided.

I was reassured, on becoming a mathematics teacher educator, to find that trainee teachers are more than willing to engage with collaborative, problem-solving mathematics pedagogies. As a tutor, I have maintained a collaborative and empathetic teaching approach, encouraging trainee teachers to reflect critically on different approaches, relating these to their own experiences of education and theories of teaching and learning, i.e. focusing on *"[not] 'how to do', but rather 'how to think' about the teaching and learning of mathematics"* (Noyes, 2007,

p. x). However, it is a source of frustration that, once trainee teachers become qualified, the excessive workload they experience, and the pressures of target setting and performance management within schools, mean they have little time to continue to think about and develop these ideas further.

My experiences of learning and teaching mathematics (see Sections 2.3 and 2.5) influenced my development of a 'fallibilist' epistemology of mathematics and a mainly 'public educator' ideology of mathematics education. I would describe my general ideology as one which recognises and values the strengths and competences of other people, rather than seeking to exert my own authority over them. This reflects my management style, which I would characterise as recognising, utilising and building upon the resources and expertise of colleagues, rather than imposing my own way of doing things.

I consider my general ideology to be significant in shaping my methodological positioning in relation to my research. I seek to adopt a methodological stance that builds upon my extensive involvement in working with mathematics teachers to develop their practice, through building genuinely participative and collaborative working relationships. A predisposition towards equity has led me to reject tradition and privilege, which I consider to be entirely human constructs, related to the interests of those in positions of power, rather than natural phenomena. A growing interest in social justice has led to me adopt an epistemology in which understanding power relations between individuals and different groups in society is essential for giving meaning to social reality.

3.2 Towards a research methodology

My ontological starting point is that social reality is constructed rather than being an objective truth. I consider privilege, equity and social justice to be critical to the study of mathematics education. I am aware that each of these phenomena is a conceptualisation and, as such, can mean very different things to different people. I view mathematics as a human construct, created out of a need to make sense of the world (see Section 2.1). I regard it as value-laden, rather than neutral, its contents, often the subject of contention, being determined by those in positions of power, for example through control over research funding. I believe mathematics education to be a fundamentally social and political practice: *"Mathematics education is a covert battleground in which the discourses of different practitioner and professional groups compete for dominance"* (Ernest, 2004, p. 82).

My epistemology is based on an interpretive view, recognising the need to explore power relations and interactions between different actors, including teachers, students, researchers

and policy-makers. I aim to explore the socio-political nature of mathematics and school mathematics, and to construct meaning from teachers' beliefs, conceptualisations and reflections on their practice. I am interested in students' responses, and how these reveal changes in their disposition towards mathematics, rather than concentrating on measurable outcomes from specific interventions. Drawing on ideas from phenomenology, I believe that concepts such as inequity and social justice cannot be fully understood without experiencing them, necessitating the construction of knowledge through action (Ladkin, 2005). My data is qualitative, rather than quantitative, in nature, focusing on the lived experiences of all those involved in the research. I consider knowledge to be subjective and reject the notion that research can be neutral or that the researcher plays an impartial or dispassionate role: *"There are no objective observations, only observations socially situated in the worlds of – and between – the observer and the observed"* (Denzin & Lincoln, 2008, p. 29).

In developing my methodological stance, I draw heavily upon critical theory. I consider the primary aim of my research to be bringing about desirable social change, through challenging social injustice and inequities associated with teaching mathematics. I view power relationships and ideologies as important considerations when adopting particular approaches to researching into mathematics education (Valero, 2004). I attach importance to establishing a research methodology that has resonance with the focus of my research, i.e. the development of classroom practice relating to teaching mathematics for social justice. I pay close attention to power relationships between all participants in the research, recognising my own role, as researcher, in constructing knowledge and meaning.

My current role, as initial teacher education tutor, provides me with privileged access to teachers, who have completed the programme, as potential research participants. My methodological stance requires careful consideration be given to power relationships that might exist, or have previously existed, between myself and these teachers (Drake & Heath, 2011). In some respects, I might be perceived by research participants as an 'insider', through my recent teaching experience and self-identification as a member of the mathematics education community, whilst in other respects, I might be viewed as an 'outsider', i.e. as the university-based researcher. Humphrey (2007, p. 21) describes how having such an *"insider-outsider status"* provides a valuable opportunity to *"mobilize both insider wisdom and outsider research"* to construct meaning from the research situation. Levin (2012, p. 143) highlights the importance of the role of 'friendly outsider', ensuring that decisions about actions are taken collaboratively with participants, whilst at the same time taking responsibility for producing an

academically rigorous text which is *"able to withstand the scrutiny of a critical reader without relenting to excuses like 'trust me – I have been there'"*.

3.3 Educational research and practice

In order to help explain the development of my own methodological positioning, I explore further the reasons behind the lack of mathematics education research focusing on teachers' development of their own practice in relation to teaching mathematics for social justice. Bishop (1998) argues that business-oriented policies, and the interference of politicians in decision-making in education, have resulted in research failing to take account of mathematics teachers' perspectives, or the situations they face in the classroom. The traditional 'centre-periphery' model of research relies on an initial study, showing limited concern for practice, packaged as a programme to be disseminated and implemented across schools. Much of this research is ineffective in changing classroom practice because it focuses on learning and the curriculum, and ignores institutional contexts and constraints.

There has been recent criticism, in the US and England, of educational research, particularly qualitative research, claiming it is of a poor quality, lacks relevance and is inaccessible to practitioners (Gough, 2004). This coincides with the growth of the 'evidence-based practice' movement, which calls for teachers to incorporate research findings into their classroom practice, and promotes systematic reviews and 'randomised control trials' (Oakley, 2006). An apparent reluctance of teachers to engage with research evidence is attributed to pressure of time, lack of skills necessary to access research, or to a resistance to change which is embedded in teachers' strongly-held beliefs and cultures (Sebba, 2004).

Critics of evidence-based practice associate it with an undermining of teachers' professionalism, through the adoption of management practices from the private sector, satisfying political demands for 'public accountability'. They argue that, whilst some teachers may be consulted in the formulation of research questions, the majority of teachers are expected merely to implement, without question, recommendations for changes in practice, based on the conclusions of research. These recommendations are used as targets, against which teachers' performance is measured (Hammersley, 2004). Hence, teachers' reluctance to engage with research findings is attributed to distrust of new initiatives, which they see as promoting a political agenda or as a tacit means of monitoring performance (Thomas, 2004).

Winch et al. (2013) warn against relying on the adoption of 'what works' protocols and argue for a greater focus on developing teachers' capacity to critically reflect on their own practice in relation to research evidence. Cordingley (2013) highlights the importance of inquiry-oriented

practice, facilitated by external experts, in promoting effective professional development amongst teachers. She argues that engagement in collaborative inquiry, promoting dialogue and peer support, can encourage teachers to take risks and to explore, not just what works, but why things work and in which contexts. Leat et al. (2014) argue that current research is often too focussed on school effectiveness, and questions about the purpose of the curriculum and how it should be taught are often discouraged. They underline the distinction between teachers engaging 'with' research, as a 'body of knowledge', and engaging 'in' research, as a 'professional learning process' or 'social practice', arguing that teachers benefit greatly when they do both. Teachers report that engaging in research provides opportunities to revisit values they have lost sight of due to the demanding nature of the profession.

Leat et al. (2014) warn about the danger that a more critical understanding of education, gained through teachers engaging in research, can lead to conflict with school managers. They also highlight how teachers' involvement is often overlooked in the reports written by university-based researchers. Cotton (2009, p. 1) underlines how teachers' and students' voices are often marginalised and *"allowed in only so long as they offer sound bites that sit neatly in the researcher's preferred story"*.

Graven and Lerman (2003) outline how 'communities of practice' are becoming increasingly popular in researching mathematics teachers' learning as part of their professional development. They emphasise the distinction between communities of practice in education, where the role of the teacher is central in maximising learning, from those in 'apprenticeship' contexts, where teaching is not considered necessary for learning to take place. Jaworski (2006) highlights the danger of applying Wenger's (1998) 'apprenticeship' model to education, in which learning is conceptualised as developing identity, and a sense of belonging, through participation in a community of practice. She argues that the process of 'alignment', i.e. *"individual members aligning themselves with conditions or characteristics of the practice"* (Jaworski, 2006, p. 190), results in the perpetuation of the 'normal desirable state', a situation in which routines and norms are established in order to avoid conflict and aggravation. In the context of mathematics teaching, this contributes towards the reproduction of existing practice, ignoring the need for a more relevant and engaging curriculum (see Section 2.2).

Jaworski (2006, p. 191) advocates the alternative model of a 'community of inquiry', in order to achieve 'critical alignment', which includes *"some sense of teachers critiquing and trying to develop, improve or enhance the status quo, alongside enculturation into existing social norms"*. A community of inquiry differs from a community of practice in that it encourages critical understanding, explicitly challenges the status quo, and develops meta-cognitive

awareness, leading to increased agency. Jaworski (2006) highlights how external support and stimulus, provided by university-based researchers, can be vital in establishing and sustaining communities of inquiry. Jackson and Temperley (2007) emphasise the pivotal role played by external partners in facilitating 'networked learning communities', which comprise teachers from a number of schools, thus building on and extending collaborative inquiry practices established in individual schools. Such external partners provide access to theory, research and practice from a knowledge base that is not constrained by institutional parameters.

3.4 Participatory action research

My ontological and epistemological positions (see Section 3.2) lead me towards adopting a 'participatory action research' methodology, which resonates with my aim of working collaboratively with teachers, as active participants 'in' research, in order to bring about desirable social change.

Torrance (2004, p. 199) argues that action research, based on a collaborative approach between academics and teachers, generates research data which is *"crucial to developing an understanding of theory-in-practice"*, providing an alternative notion of evidence-based practice. Atweh (2004) describes 'participatory action research' as a model of action research which is socio-political, participative, collaborative, emancipatory, critical and recursive in nature. As well as producing knowledge through its findings, participatory action research promotes Freire's concept of *"conscientization"* of participants and a *"more profound understanding of the situation"* (Reason, 1994, p. 328).

Skovsmose and Borba (2004, p. 209) advocate a 'critical research' model of participatory action research, which shares a *"research-resonance within critical mathematics education"*. The critical research model, on which my research design is based, views research as being carried out 'with', rather than 'on', teachers and students, recognising them as participants, rather than 'research objects'. It aims to uncover 'how' and 'why' a situation could be different and, in explicitly challenging the status quo, echoes Jaworski's (2006) aim of seeking 'critical alignment' (see Section 3.3).

In developing my research design, I draw upon two particular examples of collaborative and participatory action research studies in education (see Chapter 4). These studies focus on working with teachers in order to develop formative assessment in primary classrooms in England (Torrance & Pryor, 2001), and to develop mathematical teaching activities and approaches of benefit to immigrant students in Barcelona, Spain (Planas & Civil, 2009). Both studies emphasise the important role of the researcher in establishing and facilitating a

research group of teachers, promoting collaboration and sharing of ideas amongst group members, taking responsibility for the collation and analysis of data and reporting research findings. They also highlight how initial input from the researcher is pivotal in developing teachers' understanding of the theoretical framework underlying the research, an essential prerequisite for them to critically appraise their own practice in relation to this framework and generate new knowledge with the potential to contribute towards meaningful change.

3.5 Questions of validity or trustworthiness

I anticipate that both my pedagogical and methodological positions will be criticised by some for being 'ideological'. I acknowledge that my ideology has indeed shaped my positioning (see Section 3.2). However, I would contend that, by denying the socio-political nature of mathematics, or that of research, others are adopting just as strong an ideological stance as my own. I predict resistance to my positioning from those who adopt a more positivist stance, who maintain it is only possible to discover the objective truth by eliminating all forms of bias within the research process, and that *"the promotion of some practical or political cause"* endangers the primary goal of research as producing knowledge (Hammersley & Gomm, 1997, p. 5.4). Other mathematics education researchers, adopting socio-political perspectives, have experienced fierce resistance to their findings. Boaler's (2008) 'Railside Project', for example, which reports the success of a collaborative problem-solving approach to teaching mathematics in a school in the US, prompted a particularly aggressive reaction from 'traditionalists' in the Math Wars (see Section 2.4). In a constructive critique of her study, I outline how I believe she chose to pre-empt such an attack by including a quasi-statistical element in her research, which detracted from the qualitative case study she was presenting (Wright, 2012).

Whilst positivist and post-positivist paradigms remain influential, the number of researchers advocating interpretive paradigms is growing and such paradigms have gained at least comparable legitimacy (Lincoln & Guba, 2003). However, there has recently been a backlash against qualitative research from a *"reemergent scientism"*, promoted by US Government policy, which advocates a reliance on 'randomized controlled experiments' (Denzin & Lincoln, 2008, p. 11). The privileging of such research has also become apparent in UK Government policy with funding prioritised, through the Education Endowment Fund and the National College for Teaching and Leadership, for randomised control trials (BERA, 2014).

Criticism of my role in the research process is also likely from those adopting poststructuralist perspectives, who would argue that there are no such things as universal truths, rather there

are only situated truths within a discourse (MacLure, 2003). From such perspectives, my assertion, that the current situation in mathematics classrooms should not be taken for granted, is seen as a normative assumption. Similarly, the aim of my research, in bringing about change for the better by challenging inequities and power relations within mathematics education, would be seen as failing to take into account the argument that both the powerful and the powerless are complicit in the construction of power relations. I would argue, from a critical perspective, that the value of my research is that it seeks to develop theories which challenge social injustice, rather than merely describing or accounting for its existence. By questioning the notion of empowerment, poststructuralists are in danger of, at best, seeking to explain the status quo and, at worst, providing an excuse for doing nothing about it.

Whilst not wishing to fall into the trap of pandering to possible criticisms from others, I appreciate the need to defend my research findings against others with contrasting methodological perspectives. This is particularly important given that action research and research from a critical perspective are currently *"under-explored"* and *"under-represented"* in mathematics education research journals and handbooks (Vithal, 2004, p. 229). This is most likely because they are perceived, by many who are influential in educational research, as *"unscientific"*, *"exploratory"* and *"subjective"*, due to the lack of impartiality of the researcher (Denzin & Lincoln, 2008, p. 10).

I therefore give careful consideration to questions of 'validity' or 'trustworthiness' in my research, recognising that such notions may be interpreted and applied very differently from alternative perspectives. Many researchers advocating action research question the 'validity' of more orthodox research methodologies on the basis that they alienate human subjects and are exploitative, i.e. they preserve the powerful position of dominant cultures *"through monopolizing the development and use of knowledge to the disadvantage of the communities in which the research takes place"* (Reason, 1994, p. 328). They accept the partiality of action research which *"rejects the notion of an objective, value-free approach to knowledge generation in favor of an explicitly political, socially engaged, and democratic practice"* (Brydon-Miller, et al., 2003, p. 13).

Lincoln and Guba (2003) propose alternatives to the notion of 'validity', commonly used by positivist researchers. They argue that the concepts of 'authenticity' and 'trustworthiness' are more appropriate for ensuring the rigour of both the application of method, and the interpretation of findings, within qualitative research. I draw upon these alternative concepts, in developing my own framework for ensuring trustworthiness in my research, which I detail below.

3.6 Framework for ensuring trustworthiness

Shenton (2004, p. 64) proposes four criteria, drawn from Lincoln and Guba's (2003) framework, which can be used to ensure trustworthiness in qualitative studies (in preference to notions of validity and reliability favoured by positivist researchers):

a) credibility (in preference to internal validity);
b) transferability (in preference to external validity/generalizability);
c) dependability (in preference to reliability);
d) confirmability (in preference to objectivity).

Shenton specifies provisions that can be made by researchers to promote **credibility**, i.e. confidence that the phenomena being studied have been accurately represented, which can be subdivided into the five broad groups outlined below.

Credibility of the research processes: This can be established through deriving appropriate research methods from those that have been used successfully in comparable projects in the past. Criteria for credibility, established by action researchers, include the extent to which the research processes are participatory, collaborative, relevant, and most importantly, result in positive social change (Brydon-Miller, et al., 2003).

Credibility of the researcher: This can be established by making explicit the background and previous experiences of the researcher and the development of the researcher's thinking during the project. Having accepted that knowing is from a perspective, 'critical subjectivity' involves developing awareness of, and articulating, that perspective (Reason, 1994). From my acceptance of the partiality of my position as researcher, 'reflexivity', i.e. *"the process of reflecting critically on the self as researcher"* (Lincoln & Guba, 2003, p. 283), and critical subjectivity are essential in establishing credibility. Trustworthiness is dependent upon accounting for 'located perspective', i.e. *"the extent that we can simultaneously consider our subjectivity from a 'distance'"* (Ladkin, 2005, p. 123). Failing to do so can result in self-deception or reluctance to consider experiences which challenge initial perspectives (Reason, 1994). Maintaining a reflective journal can provide transparency by enabling the researcher to make clear how experiences, values and developing perspectives influence the research process (Ortlipp, 2008). Perspectives can also be made transparent through *"autoethnobiography"*, i.e. telling the story of the construction of our own identities and ideologies relating to the research, which rests on the assumption that *"the truth of the self is integral to the truth of the study"* (Humphrey, 2007, p. 22).

Credibility of the relationships between researcher and participants: This depends upon building the familiarity of the researcher with the culture of the research participants' organisations through *"prolonged engagement"* (Shenton, 2004, p. 65). Rapport and trust need to be established in order that participants are frank and honest in their answers to questions, rather than feeling pressured to give an answer that they believe the researcher is expecting. The risk of failing to give adequate consideration to the relationship between researcher and participants can undermine the basis of action research as an emancipatory process: *"A self-critical account that situates the researcher at the centre of the text can perpetuate the dominance our emancipatory intentions hope to fight."* (Humphries, 1997, p. 4.10). Power relations between myself and the research participants are considered further in Section 4.5.

Credibility of the data: This can be established through *"triangulation"* of information (Shenton, 2004, p. 65), i.e. by using different methods for collecting data relating to the same phenomenon. Other provisions that can be made are the use of *"iterative questioning"* (ibid., p. 67), i.e. covering the same ground with rephrased questions in order to establish consistency or contradictions in responses, and *"member checks"* (ibid., p. 68), i.e. presenting back data to research participants to check that transcripts, inferences and emerging theories agree with what they meant to say.

Credibility of the findings: This can be established by relating the findings to existing theories and previous research findings, and by opening up the research design and findings to scrutiny by peers and those overseeing the project. Whilst being critical of positivist notions of impartial knowledge, action researchers recognise the importance of engaging in debate with fellow academics advocating more conventional methodological stances (Brydon-Miller, et al., 2003; Levin, 2012). Presenting findings to other researchers from similar and contrasting research perspectives enables the researcher to consider *"alternative explanations"* (Levin, 2012, p. 143). Consideration of alternative interpretations of the data from different perspectives should be seen as lending credibility to the findings, rather than as a weakness, provided that sufficient consideration is given to these different perspectives:

> *"When the readers' different perspectives on a text are made explicit, the different analyses should also become comprehensible. Subjectivity in this sense of multiple perspectival interpretations will then not be a weakness, but testify to the fruitfulness and the vigor of interview research." (Kvale & Brinkmann, 2009, p. 213)*

To promote **transferability**, the researcher must provide sufficient contextual information in the research report to enable the reader to make an informed judgement about the extent to which the findings can be related to his or her own situation. One way of doing this is by providing *"thick description of the phenomenon under scrutiny"* (Shenton, 2004, p. 69), which should include detailed information on who was included in the study, how they were selected and methods used for data collection and analysis. It is important that the reflexivity and the critical subjectivity of the researcher (see above) are made clear and transparent in the reporting of the research. Also of relevance is the provision of contextual details relating to the research setting and the backgrounds of the researcher and research participants.

In order to achieve **dependability**, enough detail about the research process needs to be made available by the researcher to enable the study to be repeated by any future researcher who may wish to do so. The research report should include detailed descriptions of the *"research design and its implementation, … the operational detail of data gathering … and reflective appraisal of the project"* (Shenton, 2004, p. 71). Considering alternative explanations and interpretations of the data from different perspectives is important here, because the perspective of a future researcher may differ from that of the original researcher.

To establish **confirmability**, the researcher needs to convince the reader that the research findings are derived from *"the experiences and ideas of the informants, rather than the characteristics and preferences of the researcher"* (Shenton, 2004, p. 72). Many of the provisions for promoting credibility are relevant here including triangulation of information, reflexivity and critical subjectivity of the researcher. The characteristics of the research design and processes need to be reviewed carefully and periodically to ensure that the research is genuinely 'participatory' and 'collaborative' in nature.

These eight criteria detailed above constitute the framework I use to ensure the trustworthiness of my research. The table below summarises how the framework is applied to aspects of my research design (see Chapter 4).

Table 1: Application of framework for ensuring trustworthiness:

Criteria for ensuring trustworthiness:	**Aspects of the research design:**
Credibility of the research processes	• Processes are derived from those used successfully in comparable research projects, e.g. empathetic interviewing, participatory action research, methods drawn from grounded theory. • Secondary analysis/categorisation of the data focusing on research processes.
Credibility of the researcher	• Use of 'autoethnobiography' (see Sections 2.3, 2.5, 3.1). • 'Reflexivity' through maintaining a reflective journal and 'code log'.
Credibility of the relationships between researcher and participants	• 'Prolongued engagement' through selecting those I have worked with in the past to become teacher researchers. • Previous experience of working collaboratively with teachers. • Previous knowledge of, and engagement with, teaching mathematics for social justice. • Transparency and openness regarding the aims and processes of the research. • 'Empathetic' approach to interviewing.
Credibility of the data	• 'Triangulation' through data from research group meetings, interviews, short reports, student surveys and research journal field notes (both mine and those of teacher researchers). • 'Iterative questioning' through following up responses, made during research group meetings, in interviews. • 'Member checks' through presenting analysis of data back to teacher researchers for verification and further comment.
Credibility of the findings	• 'Plugging in' the data to theories (see Section 3.7). • Presenting and discussing findings regularly to/with other researchers at informal meetings, seminars, conferences and 'Special Interest Group' meetings. • Considering alternative interpretations of data analysis from different perspectives.
Transferability	• 'Thick description' of context through 'autoethnobiography', providing details of research design and detailed case study of teacher researchers, research group and research model.
Dependability	• Providing details of research design, in particular methods of data collection and analysis. • Reflective evaluation of the research project.
Confirmability	• 'Triangulation' and 'reflexivity'.

3.7 Framework for analysing data

My research data is qualitative in nature as I seek to construct, through interaction and dialogue (Kvale & Brinkmann, 2009), the stories of teachers' participation in the research project and the development over time of their conceptualisations of teaching mathematics for social justice and related classroom practices. These stories are captured through the use of semi-structured interviews, which are more suited to a desire to understand rather than to

explain (Fontana & Frey, 2008). I adopt an empathetic approach towards interviewing, for example by revealing my own feelings and opinions in order to build trust between myself and research participants, enabling a more meaningful representation of interviewees' views to emerge (ibid.). I seek to maintain a caring and considerate approach during interviews, demonstrating sensitivity towards the interviewees and establishing an encouraging environment, as these conditions have a significant influence on the outcomes (Dunne, et al., 2005).

My intention is to analyse and report research participants' experiences as a "*readable public story*", rather than to carry out a "*detailed linguistic or conversational analysis*" (Kvale & Brinkmann, 2009, p. 181). With these aims in mind, I consider it most appropriate to transcribe interviews and research group meetings using a literary style by, for example, ignoring pauses, fillers, intonations and colloquialisms during conversations. The resulting 'unfocused transcriptions' outline "*the basic 'intended meaning' of a recording of speech or action without attempting to represent its detailed contextual or interactional characteristics*" (Gibson & Brown, 2009, p. 116). Whilst such an approach minimises the amount of time required for the transcribing process, it is important to recognise that all transcriptions are "*impoverished, non-contextualized renderings of live interview conversations*" (Kvale & Brinkmann, 2009, p. 178).

I adopt a thematic analysis approach to analysing the transcripts, making use of *"meaning condensation"* and *"meaning interpretation"* (Kvale & Brinkmann, 2009, p. 197). The text is first of all reduced and broken down into units of meaning, for which preliminary themes are drafted. These are then compared across different units of meaning to create wider themes, which are then reported, through relating them to the research questions and theoretical framework underlying the research, in order to generate meaning (ibid.).

I incorporate methods drawn from 'grounded theory', described by Gibson and Brown (2009, p. 26) as "*the process of developing theory through analysis, rather than using analysis to test preformulated theories*", in my thematic analysis. Such methods are consistent with my critical research methodology in that, whilst I assert that current practice should not be taken as given, there is no pre-existing hypothesis on how to translate a commitment towards teaching mathematics for social justice into practice. My initial conceptualisation (see Section 2.8) offers a starting point for envisioning a more desirable alternative, however this conceptualisation is expected to develop during the course of the research and there is no pre-determined notion of what it might ultimately look like. Thus theories and hypotheses are free to emerge through the research project, albeit with an initial theoretical framework informing and guiding the initial action research cycle.

I make use of the 'constant comparative method' from grounded theory which Gibson and Brown (2009, p. 28) describe as "*comparing findings or observation with other instances in which those findings might be applicable*" and involving three stages: "C*reating categories, properties and theoretical relations; Solidifying the theory; Writing*". Theoretical relations are expressed through hypotheses, which are relationships between categories and properties. The theory and its components (categories, properties and hypotheses) are then solidified, or firmed up, by removing non-relevant properties and categories and continuing the analysis until theory saturation is achieved, i.e. further analysis "*comprises nothing new in the form of properties but simply reaffirms what is already known*" (ibid., p.28).

Categorisation is used in order to enable the emergence and development of themes from the data. Since my intention is to reconstruct the stories of research participants in a way that will provide meaning, I am wary of using rigid and simplistic coding, that can be easily quantified. Such a reductionist approach, whilst allowing easy comparison of large amounts of data, may lead to an impoverishment of the stories of the research participants which I am aiming to report: *"By creating a generalized 'set' of data that speaks to a range of participants' experiences, researchers lose focus on the particularities of the cases being examined"* (Gibson & Brown, 2009, p. 128). Hence codes and categories are used only to facilitate the exploration of *"commonalities"*, *"differences"* and *"relationships"* (ibid., p. 129) between emerging themes, by enabling easy comparison between inter-related units of meaning. Such comparisons take into account the context of each unit of meaning belonging to a particular category, where necessary returning to the original text and audio-recordings, and may lead to new readings of the data not apparent when units of meaning are considered in isolation (ibid.).

Bergstrom (2012) argues that inductive coding, in which the codes are derived from the data, is more useful for generating meaning in thematic analysis associated with design-based research, than deductive coding, in which the codes are derived from the initial theory. I consider an inductive approach to data analysis more useful for analysing and reporting research participants' experiences within the participatory action research model I am adopting, which shares with design-based research the characteristic of *"working with iterative cycles for developing both theory and practice equally"* (ibid., p.25). However, I consider a deductive approach to coding more appropriate for evaluating the credibility of research processes, since criteria for establishing such credibility have already been articulated by action researchers (see Section 3.6).

Since my research methodology rests on the collaborative construction of knowledge, data analysis includes iterative processes, in which initial findings are presented back to teacher

researchers for comment. This is intended to promote discussion during interviews and meetings of the research group, thus generating further data related to the findings. Presenting findings to other university-based academics, not involved in the project, also provides opportunities to consider alternative interpretations of the data from various theoretical perspectives within mathematics education. Jackson and Mazzei (2012, p. viii) outline the need to avoid the "*simplistic treatment of data and data analysis in qualitative research that … reduce complicated and conflicting voices and data to thematic "chunks" that can be interpreted free of context and circumstance*". As an alternative they suggest "*plugging in*" the data to the texts of theorists whose work underlies the research. This process, characterised by "*reading-the-data-while-thinking-the-theory*" (ibid., p.4) allows new analytical questions to emerge that can give new meaning to the data. Findings from the research project are therefore related back to theories underlying my pedagogical positioning, including those of Bourdieu, Bernstein, Boaler, Ernest, Freire, Gutstein and Skovsmose (see Chapter 2), as well as those theories underlying my methodological positioning discussed in this chapter. In this way, the data is interrogated in relation to the "*self-understanding*" of teacher researchers, the "*critical commonsense understanding*" of the wider mathematics education community, as well as the "*theoretical understanding*" derived from the theories underlying the research (Kvale & Brinkmann, 2009, p. 214).

3.8 Concluding remarks

In this chapter, I have outlined how my own experiences as a mathematics teacher educator have influenced my methodological positioning in relation to this research study. I have used this positioning to explain my choice of the 'critical research' model (Skovsmose & Borba, 2004) of participatory action research as the basis for my research design. I have given careful consideration to my role as researcher and to issues of validity and trustworthiness of research. I have developed a framework for ensuring the trustworthiness of my research and outlined how this has been applied to the research design. Finally, I have discussed and explained the framework for analysing data which I have adopted. In the next chapter, I describe in more detail the research design and the data analysis processes employed in this study.

Chapter 4: Research design

In this chapter, I describe in detail my research design, which is based upon the critical research model of participatory action research (Skovsmose & Borba, 2004), and takes into account the methodological considerations discussed in the previous chapter. I outline the establishment of a research group, consisting of five teacher researchers and myself, the schedule of group meetings, and the structure of the action research cycles. I describe the data collection methods employed, including semi-structured interviews and student surveys designed by the teacher researchers themselves. I detail the data analysis processes, highlighting how initial findings were presented back to teacher researchers for comment. Finally, I highlight ethical considerations relating to the research design.

4.1 The critical research model

The critical research model (Skovsmose & Borba, 2004, p. 214), built around action research cycles, rests upon the assumption that the 'current situation' needs to be changed for the better by addressing "*possibilities that can be imagined and alternatives that can be realised*". The 'current situation' (CS), in this case, is existing practice in relation to mathematics teaching in schools and the contribution it makes to reproducing inequity and injustice in society (see Chapter 2). The 'imagined situation' (IS) is an alternative vision of what could be, in this case, initially, my conceptualisation of teaching mathematics for social justice. The 'arranged situation' (AS) represents an attempt to put some aspect of the imagined situation into practice, bearing in mind the reality and constraints of the current situation.

There are three processes which are integral to the critical research model (see Figure 1 below):

- 'Pedagogical imagination' (PI) involves developing a critical understanding of the current situation, acknowledging that this situation should not be taken as a given and exploring possible alternatives. Ideas might originate from previous research findings, theories and philosophies of education, practical knowledge of teachers, or the process of cooperation and negotiation between researcher and teachers.
- 'Practical organisation' (PO) involves cooperation between the researcher, teachers and others, for example students and administrators, in organising an arranged situation. This might mean negotiating a situation taking into account the realities and constraints of the current situation.

- 'Explorative reasoning' (ER) involves analysing the arranged situation in order to better understand the imagined situation. Whilst the arranged situation and imagined situation are not the same, analysing the arranged situation can help to draw conclusions about the feasibility of the imagined situation.

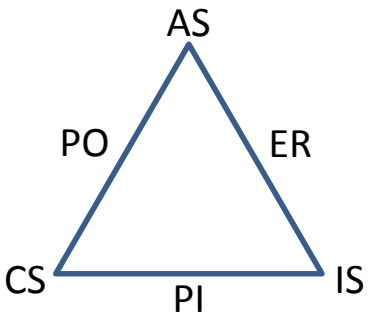

Figure 1: ***"Model of critical research indicating which processes such research might include"***
Source: Skovsmose and Borba (2004, p. 216)

Figure 2 below illustrates the critical research model spanning three cycles, for example the trajectory CS_1, CS_2, CS_3, … represents the development of the current situation.

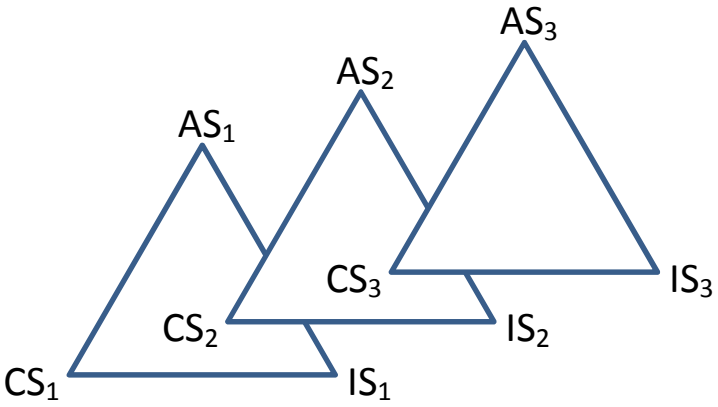

Figure 2: ***"Model of critical research indicating what transformations such research might be associated with"***
Source: Skovsmose and Borba (2004, p. 221)

4.2 The research group and critical research cycles

In June 2013, I contacted approximately 120 mathematics teachers with an invitation to take part in the research project, along with an information leaflet and a consent form. All of these teachers were completing their first year as newly qualified teachers, having the previous year completed the same initial teacher education programme on which I had worked as a tutor. I

anticipated that constituting a group from teachers I had previously worked with would make it easier and quicker to build working relationships. I also expected teachers at this stage of their careers to be more idealistic, enthusiastic, self-reflective and open to innovation and change than those with more experience and firmly-established practice.

The information leaflet (included as Appendix 9) provided details of the aims of, and background to, the research project, and the time commitment expected from participants, including attendance at seven research group meetings, three interviews, participation in three action research cycles, keeping a research journal and writing a short report at the end of the project. It also included the potential benefits of taking part, including the professional development opportunities afforded by the project, and the qualification criteria for inclusion in the research group, i.e. a personal commitment towards teaching mathematics for social justice and the agreement of the school's head teacher to participate. The consent form (included as Appendix 10) is discussed further in Section 4.5.

My initial target was to recruit between 8 and 10 teachers, with the expectation that at least half of these would complete the project. In the event, expressions of interest in the project were received from six teachers with all-but-one returning the completed consent forms. A research group was established, consisting of myself, as 'university-based researcher', along with five mathematics teachers, referred to as 'teacher researchers'. Four of the teacher researchers completed the project, with the fifth teacher researcher resigning from her teaching post part-way through the year.

The first meeting of the research group, largely facilitated by me as researcher, was held in July 2013. The main purpose of the meeting was to raise teacher researchers' awareness of the theoretical frameworks underlying the research project, recognised by Torrance and Pryor (2001) and Planas and Civil (2009) as a necessary precondition for effecting meaningful change in the classroom. The meeting provided an opportunity for me to present theories relating to my initial conceptualisation of teaching mathematics for social justice (see Chapter 2) and for teacher researchers to reflect on their own classroom practice in relation to this (CS_1). Through discussing, and developing a critical understanding of, this practice (PI), teacher researchers began to articulate what a desirable alternative might look like (IS_1). The meeting also provided an opportunity to discuss the critical research model and some of the research methods used in the project including the use of research journals, interviews and data collection.

Subsequently, two meetings of the research group were held during each of three action research cycles, spanning the 2013-14 academic year, a 'mostly planning' meeting at the

beginning, and a 'mostly reflection' meeting at the end of each cycle. During these meetings I continued to play a facilitative role and to make significant inputs, for example by providing examples of existing resources (Wright, 2004; Gutstein & Peterson, 2005) to stimulate ideas for classroom activities. The 'mostly planning' meetings provided opportunities for teacher researchers to design and plan activities and approaches to try out in the classroom (AS_1, AS_2, AS_3), taking into account the desired outcomes and the practical constraints they faced (PO). The 'mostly reflection' meetings focused on evaluating and discussing the extent to which these activities and approaches were successful in relation to the desired outcomes (ER). These reflections led to teacher researchers beginning to develop their own conceptualisations of teaching mathematics for social justice (IS_2, IS_3), accompanied by changes in their classroom practice (CS_2, CS_3), during the second and third research cycles.

The following are examples of the focus of discussions during research group meetings:

- Reflecting on the development of ideas and thinking relating to the project (prompted by notes made in research journals).
- Discussing the rationale for, and processes involved in, the critical research model.
- Discussing research articles and inputs provided by me or teacher researchers.
- Discussing opportunities and constraints relating to translating theories on teaching mathematics for social justice into practice.
- Engaging with examples of relevant classroom activities proposed by me or teacher researchers and considering how to adapt these for use in the classroom.
- Planning and designing teaching ideas and approaches to be tried out in the classroom.
- Discussing, negotiating and agreeing on methods to be used to collect student level data.
- Presentations by teacher researchers of their evaluations of activities tried out in the classroom, prompted by students' feedback and examples of students' work, followed by general discussion.
- Presentations by me of initial findings from data analysis, followed by general discussion.

4.3 Semi-structured interviews and data collection

In addition to the research group meetings, three individual interviews were held between myself (as interviewer) and each teacher researcher (as interviewee), immediately after the initial research group meeting (in July 2013), after the fourth meeting (in February 2014) and at the end of the project (in July 2014). The interviews were conducted in interviewees'

schools, to help them relax, to minimise demands on their time, and to help build relationships of trust between myself and teacher researchers. Whilst the same set of initial questions (shown below) were asked to all interviewees, follow-up questions (some of which are shown as bullet points below), consistent with an empathetic approach to interviewing (Fontana & Frey, 2008), were employed. These were individually tailored to explore responses in more detail.

Initial questions for interview 1:

a) What does teaching mathematics for social justice mean to you?

b) How do you think social justice relates to your current classroom practice?

c) What do you think teaching mathematics for social justice might look like in an ideal world?

d) Where does your commitment to teaching mathematics for social justice come from?

e) What do you hope to get out of your participation in this project?

Initial questions for interview 2:

a) *Comment on how valid you find the analysis and interpretation of your data for the first part of the project as presented on the sheet. [A thematic analysis of each individual teacher researcher's data up until meeting 3 was provided in advance.]*

- *[Follow up questions prompted teacher researchers to consider each of the four initial themes emerging from the analysis.]*

b) Tell me a bit more about the first classroom activity that you tried.

- *In particular, how did it relate to TMSJ (teaching mathematics for social justice) and how did the students respond?*

c) How will you approach the second classroom activity (or activities)?

- *Which of the three activities agreed at meeting 4 will you try and how will you approach them differently to cycle 1?*

d) How has your thinking on TMSJ developed since the start of the project?

- *How has your classroom practice developed?*
- *What do you see as the opportunities and constraints for TMSJ?*

Initial questions for interview 3:

a) What does teaching mathematics for social justice mean to you now?

- *How have your views changed since the beginning of the project?*
- *What factors do you think have most strongly influenced your views on TMSJ?*

b) How do you think your classroom practice has changed, since the beginning of the project, in relation to social justice?

- *What factors do you think have most strongly influenced the development of your classroom practice?*
- *What impact do you think this has had on your students?*

c) *How do you think you have benefited from your participation in this project?*

- *What did you like most about the project?*
- *What would you change about the project?*

In order to stimulate reflection and discussion of classroom interventions and research processes during meetings, teacher researchers were asked to keep research journals (provided at the first meeting). They were also encouraged to video parts of their lessons and to present these during meetings, although only one of them actually did this. It was also suggested that teacher researchers might wish to observe each other teaching and provide feedback on each other's lessons during the meetings. Whilst there was initial enthusiasm for this idea, a lack of funding to facilitate visits to other schools meant that it didn't happen. All of the research group meetings and the interviews were audio-recorded, and these recordings generated most of the data used to narrate the stories of teacher researchers' participation in the project.

In order for students' voices to be heard, I considered it important that data was also collected from students involved in the project. Bearing in mind the collaborative nature of the research and teachers' in-depth knowledge of the classroom situation, I felt it most appropriate for teacher researchers to collect data on students' experiences, and to decide themselves what form this data collection should take. My role in this decision-making process was limited to facilitating discussions, drawing on my own research knowledge, suggesting possible data collection tools (including conducting interviews or focus groups, carrying out surveys, collecting students' work), and ensuring consistency so that meaningful comparisons might be made across different classrooms and schools.

Following a discussion on the collection of student-level data at meeting 2, it was decided that carrying out a student survey at the end of the classroom intervention was most appropriate for exploring students' attitudes and dispositions towards mathematics. Teacher researchers agreed to administer an anonymous survey during the first research cycle, which asked students to write down their maths group and sex, and responses to these two questions:

- *How do you feel about maths?*
- *What do you think about the maths we did today?*

The survey was reviewed by the research group at meeting 4, with a general consensus that the wording wasn't clear enough for students to fully appreciate the aims of the survey or the

distinction between the two questions, i.e. between exploring students' general dispositions towards mathematics and whether they felt any differently as a result of the classroom intervention. A new protocol was agreed on how to introduce the survey during cycles 2 and 3, making the rationale for the two questions explicit (see Appendix 5). It was also decided to amend the wording of the first question to:

- *How do you feel about maths in general?*

The responses from the student survey were used by teacher researchers to reflect upon and evaluate their classroom interventions during meetings 3, 5 and 7. These were collected at the end of each meeting and extracts were used to provide additional insight, and students' perspectives, when narrating the stories of teacher researchers' involvement in the project.

At the end of the project, teacher researchers were each asked to write a short report (approximately two A4 pages) on their experiences of being part of the research group, and the impact they believe it had on them personally, on their classroom practice, and on their students. This provided additional data on evaluating the critical research methods and processes, from the individual perspectives of the teacher researchers. The reports also provided an opportunity for teacher researchers to summarise their experiences, providing a useful personal record of their professional development, and facilitating the sharing of ideas from the project with other teachers.

Throughout the duration of the research project, I kept my own research journal to ensure reflexivity and to enable me to make explicit, during the reporting of the study, my own perspective and the lens through which the data was interpreted. I recorded my immediate thoughts following meetings, interviews, and presentations of initial findings at conferences and seminars. I subsequently recorded my reflections on, and development of, these thoughts, and how these informed the planning of future events. Notes from my research journal proved useful in evaluating the success of the critical research methods and processes, in particular the vital role I played within the research project.

4.4 Data analysis and presentation of initial findings

In this section I detail how I applied the framework for analysing data (refer to Section 3.7) to the data from the research project. I began the analysis of each audio recording by playing it through, without pausing, whilst writing down my immediate thoughts in my research journal, in order to gain an overall picture. I then carefully transcribed each recording using a literary style. Whilst extremely time-consuming, I found that the transcribing process was an important step in me becoming familiar with the data. I included regular times in the

transcripts to make it easier to refer back to the audio recordings, which I did when reading and re-reading the transcripts during the subsequent data analysis.

The meaning within the transcripts was condensed by breaking down the text into units of meaning and summarising the meaning of each unit using descriptive text. Each unit was then assigned a category, generated through an inductive process, relating to its meaning. The unit was also assigned a property, giving a more detailed indication of the nature of the statement being made, together with a crude measure, on a scale of 1 to 5, of the extent to which the statement was a positive assertion of that property. During the coding process, where necessary, initial units of meaning were further broken down into smaller units of meaning, so that only one category and property were assigned to each unit. A code log was kept to record decisions taken regarding the creation of, and subsequent changes to, categories and properties (included as Appendix 1).

An extract from the analysis of Rebecca's second interview transcript is included, in Appendix 3, as a further illustration of the meaning condensation and coding processes. Following the breaking down of the text into units of meaning, the ninth unit in this extract was:

> *R: I think that's what [Brian] was saying wasn't it? It's quite easy to define injustice, than what social justice. I don't really know what social justice means.*

The meaning of this statement was condensed as:

> *R: Easier to define social justice by defining injustice. Limited thinking around TMSJ before project.*

This unit of meaning was assigned the code 'PreEng T2'. 'PreEng' indicates that it relates to the category 'Previous engagement with TMSJ issues', 'T' indicates the property 'Large amount of thought given', and '2' indicates that the statement is a mostly negative assertion of this property (see code log in Appendix 1), i.e. Rebecca had previously given relatively little thought to issues of teaching mathematics for social justice. The twelfth unit of meaning in the extract illustrates how some initial units of meaning were subsequently broken down into smaller units, in this case 12.1 and 12.2, so that different codes could be assigned to each.

The 'constant comparative method' was then employed to re-read and analyse the data by exploring and comparing units of meaning sharing similar codes. This enabled four initial themes to emerge (see Section 5.1), which were subsequently used as a framework for further data analysis. The process of 'plugging in' the data was used to relate these initial themes back to the theories underlying the research (see Chapters 2 and 3) to enable further themes and theories to emerge.

In order to ensure credibility of the data analysis process through 'member checks' (see Section 3.6), and to generate further data, I presented initial findings and themes emerging from the data analysis back to teacher researchers for their comment. General findings from the analysis of the first set of interviews were presented back to all five teacher researchers during meeting 4, and findings from an analysis of each individual teacher researcher's data (up until meeting 3) were presented back during the second interview. In order to ensure I considered alternative interpretations of the data, I also presented my initial findings periodically to other researchers on my doctoral programme, to a mathematics education 'special interest group' at the university in which I worked, and to other academics at various research seminars and conferences I attended.

A secondary thematic analysis of the transcripts was carried out, this time focusing on the research processes. A new set of categories was assigned to those units of meaning for which such coding was considered appropriate. These categories were derived deductively from the key processes of the 'critical research' design (see Section 4.1), and from characteristics considered important for ensuring the trustworthiness of participatory action research (see Section 3.6). The code log for this secondary data analysis is included as Appendix 2. The constant comparative method was then applied to this secondary categorisation in the same way as before.

The responses from the student surveys, together with the teacher researchers' final reports, were used to triangulate the data collected from meetings and interviews. Extracts from student survey responses were also used to enrich the reporting of the stories of teacher researchers' involvement in the project through giving a voice to students (see Parts 1, 2, 3 in Chapter 5).

4.5 Ethical considerations

Because of the collaborative and participative nature of the research, it was clear that teacher researchers would need to commit a significant amount of their time and energy to the project. Such a commitment is particularly demanding given the pressures teachers face from ever-increasing levels of performativity and accountability in schools (Torrance, 2004). Before informed consent was sought, I therefore felt it imperative to specify the time commitments expected from teacher researchers over the course of the project, and to emphasise the importance of a shared interest in teaching mathematics for social justice. However, whilst making significant demands on time, the research project also offered significant benefits for participants, including the rich professional development opportunities provided through

belonging to a research group. This was seen as particularly beneficial for teachers nearing the completion of their first year as newly qualified teachers, after which they would no longer receive the additional support provided by school-based mentors. The expected time commitments and potential benefits of the research project were both included in the information leaflet sent to potential participants (see Appendix 9).

There are several risks and ethical issues, associated with participatory action research, that were considered when planning the research design. Firstly, the collaborative and participative nature of the research necessarily means that the process is unpredictable, messy and difficult to manage (Smith, et al., 2010). There is a possibility that participants in the research project might reject my own analysis of the situation, or that raised levels of consciousness amongst participants could lead to raised expectations which cannot be fulfilled through the project or that lead to conflict with school managers (Todhunter, 2001). For this reason, the aims of the research and the research design were made clear to participants in the information leaflet, as was the fact that the initial conceptualisation of teaching mathematics for social justice was based firmly upon a significant body of evidence from research findings.

In order to ensure 'informed consent', the consent form (included as Appendix 10) required participants to sign statements confirming they had read the information leaflet and understood the voluntary nature of their participation, the expectations they would face as teacher researchers, and the confidentiality and availability of data. Bearing in mind the relative inexperience of those invited to take part in the project, wherever possible, opportunities were sought to engage with, and elicit support from, colleagues and managers of teacher researchers in schools. Consent of the head teacher was a pre-condition for participation in the project and this was included on the consent form. Before signing the consent form, head teachers were required to read the information leaflet, which emphasised the potential benefits of participation for the school, including the development of classroom practices which address the alienation and disengagement of students and promote mathematical reasoning and problem-solving skills.

It was made clear to teacher researchers that they might request, immediately after a research group meeting or interview, that particular comment(s), they themselves had made, be excluded from the transcript. Permission was sought from teacher researchers for open access to be provided, at a later date, to transcripts from research group meetings and interviews, and short reports. I made clear my intention to develop a future strategy for making the data available on-line, subject to password protection and to the confidentiality provisions outlined below, should such access be a requirement for the publication of reports or articles, based on

this research project, in research journals. However, open access will not be provided to student-level data collected by teacher researchers or to lists of pseudonyms linked to names of research participants.

In order to ensure security of data, all files were stored only in private vaults on secure USB flash drives, which were password protected and encrypted. To ensure confidentiality, all names within the data collected, including those of teacher researchers, students, other participants in the research, schools or any other person or institution that might lead to the revealing of the identities of those taking part in the research project, were replaced with pseudonyms. These pseudonyms were also used for file names and in all data analysis, including the initial findings presented back to the teacher researchers and others. The same pseudonyms are used in this thesis and they will also be used in any future report, based on this research project, which I author. It was agreed with teacher researchers that they would also use pseudonyms in any reports that they write in future, unless they choose to reveal their own identity, in which case they would continue to use pseudonyms for all other research participants. Lists of pseudonyms linked to names of research participants will be stored separately from the research data under the same level of security.

My methodological stance required me to pay careful attention to any power relations that might exist between me and the research participants. In my role of teacher educator, I had previously worked with all five teacher researchers during their initial teacher education. This included running a programme of six subject-focused training days, which they attended. I had acted as subject tutor to four of the teacher researchers, which involved marking and grading academic assignments, visiting them in their schools, carrying out lesson observations and providing formative feedback. Whilst the lesson observations were not graded, trainee teachers were aware that my final recommendation would affect their overall grade and whether they attained qualified teacher status.

In order to address issues relating to previously-existing power relations, I made clear in the information leaflet, and during early meetings of the research group, that the research project was related to my own academic study and was distinct from my work as a teacher educator. This message was reinforced by incorporating the University of Sussex logo on the information leaflet and consent form, rather than those of the university in which I worked. It also helped that almost a full year had elapsed since teacher researchers had completed their PGCE qualification, during which I had little or no contact with them. This acted as a natural break making it easier for me to establish different relationships with teacher researchers. However,

I was still aware of the need to be sensitive to my relationships with teacher researchers and to ensure new relationships were based on collaboration, mutual respect and trust.

The collaborative and participative nature of the research project meant that data collection was an evolving process, with the methods for collecting student-level data by teacher researchers dependent upon negotiations and agreement at early research group meetings. Therefore, ethical approval for conducting the research was sought and obtained from the University of Sussex in two stages. Initially, in April 2013, I sought ethical approval to establish the research group and to carry out interviews with teacher researchers. The only research data collected at this stage were my audio recordings of research group meetings and interviews. In November 2013, following agreement with teacher researchers that data would be collected from students through them administering a survey, a second ethical approval application was made to enable this data to be collected. Since I was not directly interacting with, or collecting data from students, both ethical approval applications were considered as 'low risk'.

4.6 Concluding remarks

In this chapter I have described in detail my research design, data analysis processes, and ethical considerations in carrying out the research. In the next chapter I present in detail the research findings in the form of a case study, which tells the stories of individual teacher researchers' involvement with the research project, the development of the research group as a whole and the processes and characteristics of the research model.

Chapter 5: Research findings

In this chapter, I report the findings of my research. In Section 5.1, I describe four themes that emerged from the main thematic analysis of the data, which were utilised in subsequent analysis. In Section 5.2, I explain the rationale behind my decision to report the findings as a case study in five sections (5.3 to 5.7), the first four of which are reported using these themes. Sections 5.3, 5.4 and 5.5 recount the individual stories of teacher researchers involved in the research project. Section 5.6 reports on the story of the research group as a whole, making reference to the participation of all five teacher researchers and my role in the research. Section 5.7 reports on the application of the critical research model (Skovsmose & Borba, 2004), drawing on the secondary thematic analysis, and focusing primarily on the credibility of the research processes (see Section 4.4).

5.1 Four emerging themes

Four significant themes emerged from applying the thematic analysis (see Section 4.4) to the data from the first set of interviews. These four themes were used in the thematic analysis of data from the remaining meetings and interviews, and are used for reporting the research findings in the first four sections of the case study described below.

The four themes were:

Theme 1: Changing epistemologies of mathematics

Discussions during research group meetings frequently focused attention on teacher researchers' epistemologies of mathematics, students' perceptions of the subject, and how these related to pedagogical approaches. The development of teacher researchers' thinking and classroom practice appeared to be closely related to their own relationships with mathematics and views on the legitimacy of addressing social justice issues in the mathematics classroom.

Theme 2: Developing student agency

Teacher researchers appeared to share a commitment towards changing society for the better, citing this as a significant influence on their choices to become teachers and their evolving teacher identities. They increasingly regarded promoting student agency, in various manifestations, as key in developing their practice, alongside making mathematics more relevant, meaningful and engaging for students.

Theme 3: Collaborative nature of research group

Teacher researchers valued the professional development opportunity that working collaboratively with colleagues from different schools, and engaging with research theory, provided. They appreciated how the research group provided the mutual support necessary to overcome constraints and try out alternative approaches.

Theme 4: Dominant discourses on ability and attainment

Teacher researchers recognised that the current exam-focused culture in schools, together with increasing workload and levels of scrutiny of teachers, posed significant constraints on realising the aims of the project. They increasingly questioned their own notions of mathematical ability, and considered how to help all students develop the skills required for achieving success in mathematics.

Figure 3 below provides a useful representation of how the four emerging themes, represented by the four horizontal layers of the cuboid, were used in the analysis of data over the duration of the project. The six vertical slices represent the members of the research group, and the ten cross-sections represent the chronological development of project, with the first meeting in the foreground, and the final interview in the background.

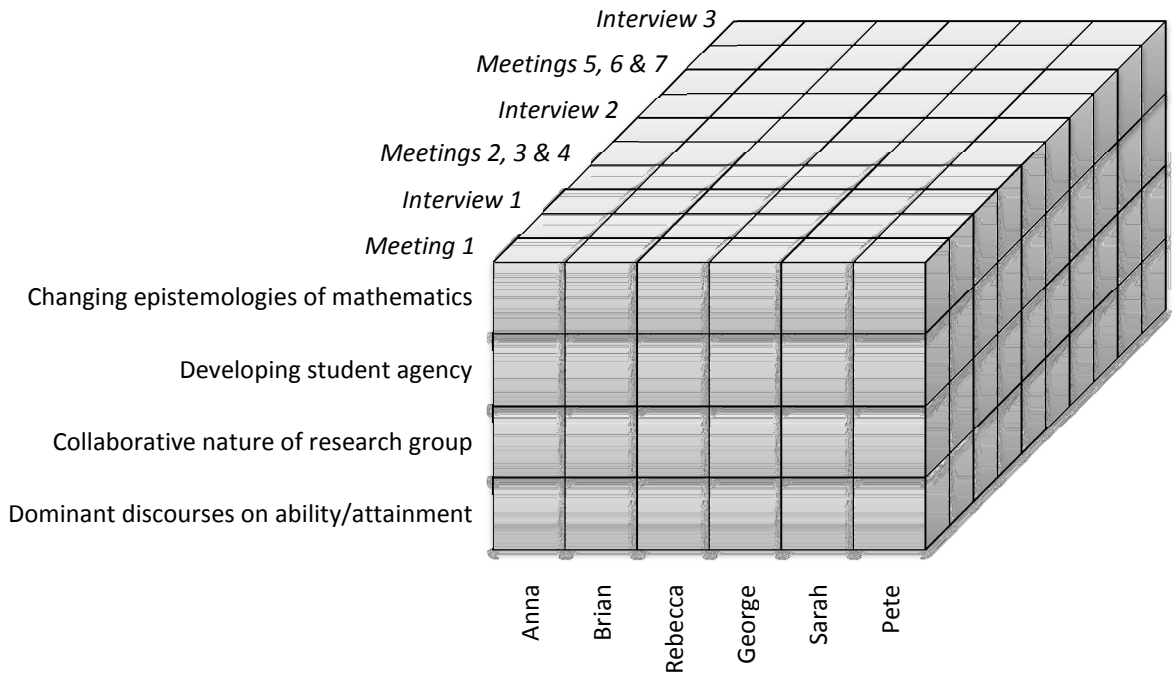

Figure 3: Use of the four emerging themes in the data analysis

5.2 The case study

A case study approach is used to narrate the stories of the teacher researchers' involvement in the project, and the development of their thinking and classroom practice. The case being studied in this instance is the 'Teaching Mathematics for Social Justice' research group, which was established using the 'critical research' model of participatory action research (see Section 4.2). The case study is presented in five sections (5.3 to 5.7).

All names reported in the case study, including those of the teacher researchers and their schools, have been replaced by pseudonyms, in order to protect anonymity. Background information about schools, in the introduction to each of the first four sections of the case study, is taken from the most recent Ofsted report available, although these are not referenced, again to protect anonymity. A brief description of each activity, tried out by teacher researchers as part of the project, is included in Appendix 4.

In Sections 5.3, 5.4 and 5.5, I recount the individual stories of three of the teacher researchers, Anna, Brian and Rebecca. They were chosen because they each attended all seven meetings of the research group, submitted all student survey responses and completed a final report. Amongst the research group, they also represent a broad range of experiences, social backgrounds, degree qualifications and levels of previous engagement with social justice issues. In terms of promoting the transferability of the research findings (see Section 3.6), it is anticipated that aspect of the case study will therefore enable a wide range of mathematics teachers, interested in teaching mathematics for social justice, to identify closely with teacher researchers' stories. Quotes from teacher researchers and students serve to strengthen the confirmability of the research findings (see Section 3.6), by demonstrating how these are derived from the experiences of the research participants themselves. Quotes from the teacher researchers are referenced to units of meaning, which were numbered sequentially, in the transcript of each meeting and interview.

Section 5.6 tells the story of the development of the research group as a whole, drawing on analysis of data from all five teacher researchers, including George and Sarah, and myself. It focuses on commonalities in experiences, in relation to the four overall themes that emerged from the analysis (see Section 5.1). It reports on relationships between members of the research group, the functioning of the group and the development of a group identity. It highlights how the evolution of the research group influenced, and in turn was influenced by, the development in thinking and practice of individual teacher researchers. It is anticipated

that this aspect of the case study will be of particular interest to those working collaboratively with groups of teachers to critically reflect on, and develop, classroom practice.

Section 5.7 reports on the important role the critical research model played in producing reliable and trustworthy research findings. It draws on the secondary thematic analysis, based on a different set of categories derived from the key processes of the critical research design, i.e. pedagogical imagination, practical organisation and explorative reasoning, and characteristics of participatory action research, i.e. the extent to which the research is participatory, collaborative, relevant, and results in positive social change (see Section 4.4). These processes and characteristics are used as themes in the report, which focuses on the credibility of the research processes (see Section 3.6). It is anticipated that this aspect of the case study will be of particular interest to those working collaboratively with teachers in researching into effective change in classroom practice.

5.3 Anna's story

Anna taught mathematics at St. Francis' Church of England School, a relatively small girls' secondary comprehensive school situated in Inner London. A large number of students came from single-parent or low-income families and the proportion eligible for free school meals was more than twice the national average. A majority of students were from minority ethnic backgrounds, including many of African heritage, and a higher than average proportion of students had learning difficulties. The school was popular and over-subscribed. The achievement of students was considered outstanding, with over 80% attaining five or more GCSEs, at grade C or above, including English and mathematics.

Theme 1: Changing epistemologies of mathematics

Anna was a successful learner of mathematics, choosing to study it at Advanced Level because of its status as a gatekeeper qualification. She pursued her interest in humanities by studying for a psychology degree, through which she developed an awareness of social issues. She only began to appreciate the value-laden nature of mathematics, and how ignorant she had been of this beforehand, when training to be a teacher. This was when she began to recognise the difference in attainment between different groups of students as a social justice issue.

> *I always remember this session we did where they were saying 'What is maths?' And I was like 'What is maths? I've just joined this teacher training course to teach maths and I don't really know what it is'. (Anna, Interview 2, #22)*

Through her involvement with the project, Anna developed a greater appreciation of the socially-constructed nature of mathematics, and how ideologies of mathematics education

relate to views of mathematics. She argued that teachers mustn't avoid politicising the classroom, through fear of forcing their views on others, because it was important to counteract negative messages about mathematics students received from elsewhere.

From the start of the project, Anna saw mathematics as a legitimate forum for raising awareness of social justice issues. Initially, she was concerned about how much 'concrete' mathematics was being learned whilst focusing on social justice issues, which she saw as enriching the mathematics classroom and a break from preparing students for tests. Later, she began to see social justice and mathematics teaching as inextricably linked. The project convinced her that providing opportunities for students to revisit mathematics skills, and apply them to real life contexts, rather than learning them in isolation, led to longer-term retention. She observed how learning about social justice issues could help students develop mathematical understanding, for example looking at representations of how the money paid for a Fair Trade chocolate bar was distributed helped students see the link between percentages and the hundred square. She realised students didn't understand this concept well, despite having 'covered' the skills previously.

> *I like the lesson because I learnt about fair trade and now I kind of understand decimals, fractions and percentages. (Student in Anna's Year 7 set 4 of 5, survey response to Fair Trade activity)*

Anna increasingly recognised the need to incorporate social justice issues into schemes of work and make the links with mathematics clearer, for example the Wealth Distribution activity directly linked percentages and proportion to fairness, when considering inequality in earnings. She viewed extended projects, which incorporated learning and immediately applying mathematics skills, as a way of enabling such links to be more easily made. However, she highlighted that students didn't always make use of the mathematics skills they learnt, for example omitting to use stratified sampling when planning a questionnaire. Sometimes students didn't believe they were learning mathematics at all, particularly when applying skills in which they were already proficient.

> *It was fun. The presenting was fun and enjoyable. It was okay, but it wasn't really relevant to maths. (Student in Anna's Year 9 top set, survey response to Making a Change Project)*

Anna believed that, by providing students with real life contextualised problems which required the application of specific procedures, they would gain an appreciation of the value and purpose of learning abstract mathematics. She recognised the need for real life contexts to be genuine and convincing, rather than over-simplistic. She argued that projects needed to

be planned carefully, with a clear rationale for the development of applicable mathematical skills, at an appropriate level of challenge for students.

> *And the kids are going to have to be coming to me and saying 'Oh we've got these numbers but how do we represent this?' And it's their questions that are going to be the need for them to learn the stuff, as opposed to me saying 'this is something we're learning, go away and do it'. That's the dream. (Anna, Interview 2, #105)*

Whilst still recognising a place for teaching discrete mathematics skills, Anna argued that students needed to be re-oriented towards an alternative view of mathematics, from Reception onwards, with a greater appreciation of how mathematics skills can be applied to real life contexts. She believed that the activities tried out during the project could help students develop such an appreciation. She argued, however, that for this to happen, others' views of mathematics would need to be challenged, for example senior managers would need to recognise the value of learning how to apply a skill, as much as learning the skill in the first place.

Theme 2: Developing student agency

Anna attributed her concerns for social justice and equity to her family background. She was brought up by her mother, who experienced a difficult childhood, dropping out of school and becoming a cleaner and groom for rich families. Anna developed an awareness of unfairness and exploitation through witnessing her mother being treated badly, despite working so hard. She began to realise the education system was not a meritocracy in which the most intelligent and hard-working children necessarily succeeded.

> *People say 'Well, we've got a good education system, you know, we live in a country where you can get wherever you want'. Well actually, people can't, because of the barriers. (Anna, Interview 2, #16)*

Anna's desire, to address inequity and help disadvantaged children overcome barriers to learning, was her motivation for becoming a teacher. She saw the project as providing an opportunity to refocus on these aims.

> *I had become very much caught up in school life, and getting through each half term's test, losing sight of it really. The research project was an opportunity to refocus then, and it still is. It's sort of confirming my desire to teach in this way. (Anna, Interview 2, #35)*

Anna's initial focus was on exploring ways of using mathematics to develop understanding of social, political, economic and cultural issues. She designed and tried out a project on Wealth Distribution and was pleased with the extent to which students engaged with the notion of fairness, and related this to their own experiences. However, students who encountered

difficulties in accessing the mathematics, for example those struggling to divide proportionally, found it difficult to fully grasp the concept of wealth distribution. In general, she found students keen to discuss social justice issues. The Fair Trade activity prompted a lively debate about why the government charges so much tax, why supermarkets earn more from Fair Trade products, and why cocoa farmers are paid so little.

> *Today shows us about the unfairness farmers get and how we can help them by using fair trade. (Student in Anna's Year 7 set 4 of 5, survey response to Fair Trade activity)*

Anna was pleased with the positive response to the activities from students, who began to demonstrate higher levels of motivation and engagement, making lessons more enjoyable and satisfying to teach. The behaviour of difficult classes improved significantly and she noted that one student in particular, who normally exhibited considerable behavioural difficulties, transformed into a 'dream student' for the duration of the Wealth Distribution project.

> *I tried a few things with my bottom set and their motivation has just been so high in those particular lessons that I've had to very rarely like tell them to get on with things or to do things. (Anna, Interview 3, #42)*

Anna recognised how adopting open, collaborative approaches to learning enabled students to develop agency. Over the course of the project, she began to appreciate the importance of allowing students to choose issues that they felt were of interest to them, rather than deciding for them. Students were excited and inspired by the Making a Change Project, appreciating the opportunity to choose their own issue to research and to use mathematics to construct their own argument. One group, for example, chose to investigate whether teachers exhibited favouritism towards certain students. Presenting their findings, in an assembly, to the whole year group made students feel they were being given a voice.

> *They were all so passionate about the things they were presenting about, was the key thing, and the fact that they got to actually tell everyone what they found out. (Anna, Interview 3, #47)*

Anna was generally pleased with the way students were willing to use mathematics to justify their arguments. However, she felt that not all students appreciated how to construct a strongly mathematical argument, or to make a sensible suggestion for change, and these were areas where she would need to provide more guidance in future. There were other occasions when students became so absorbed by the social issues that they ignored the mathematics completely, for example when assuming population growth would just tail off with no apparent mathematical justification.

> *I enjoyed this project and I enjoyed using maths to back up my points. (Student in Anna's Year 9 top set, survey response to Making a Change Project)*

The Making a Change Project was particularly successful in raising the confidence and self-esteem of lower-attaining students. Anna's Year 8 bottom set engaged passionately with the activity and produced presentations in which the mathematics content rivalled those of higher sets, one group's presentation being voted by other students as the best in the year group. This made her more optimistic that making mathematics more relevant and meaningful enables a wider range of students to engage with, and understand, the subject.

Anna believed that mathematics was essential for students to understand and explain the world around them, and to function and participate fully in society. She began to appreciate the need for students to become more aware of their own situation, its limitations, what they can change, and how to go about changing it. She found that talking to students about their real-life situations helped strengthen relationships of trust, which had a positive knock-on effect in all mathematics lessons. However, she recognised that, in relating mathematics to students' lives, potentially difficult questions, such as the level of support provided to people not working at all, needed to be dealt with sensitively, particularly with a class in which some students' parents were on benefit or low income.

> *It makes them feel like you're interested in what they're going to do and what their hopes are. (Anna, Interview 3, #51)*

Theme 3: Collaborative nature of research group

Anna felt she had already established herself as an effective classroom teacher and saw the project as an opportunity to develop her teaching practice in a direction she was comfortable with.

> *I think the whole project is, for me, about developing myself as a practitioner, and in a way that I'd like to develop. (Anna, Interview 1, #12)*

Anna valued the opportunity of working collaboratively with other teacher researchers to generate ideas. Whilst she hadn't thought very much about teaching mathematics for social justice beforehand, discussions at early meetings reassured her that the project resonated with her own educational ideology and motivation for becoming a teacher.

> *I'm quite interested in learning from the other teachers on the programme, because they're obviously all doing this because they have an interest in social justice … I need a deeper understanding of how maths can be used in social justice issues, and hopefully I'll be able to learn from other people, and from just trying things out, where these things fit in. (Anna, Interview 1, #64)*

Anna really enjoyed the research group meetings, especially time spent on joint planning and evaluation of lessons. She felt that being exposed to theories, and hearing other people's perspectives when discussing these in meetings, influenced her own thinking and approach to planning the curriculum. She felt the project had a significant impact on her classroom practice, much more so than other professional development she had experienced, however she still considered this to be a 'work in progress'.

> *It's been the most impactful CPD, in my opinion, that I've had this year, because it's sustained … I've actually seen the impact of this project on the children and the lessons that I teach, whereas very often with CPD, it's one afternoon, you go away and come back, and it goes out of your head like that. (Anna, Interview 3, #64)*

Anna recognised that lessons, incorporating activities similar to those from the project, could take much longer to plan, an important consideration given teachers' excessive workloads. She therefore welcomed the opportunity to plan collaboratively and share ideas with other teacher researchers, which she believed resulted in higher quality teaching resources, as well as saving time. This was particularly true during the second and third cycles, when it was decided everyone should try out the same set of activities.

> *Through sharing resources when we all have a clear focus, and we've all agreed on what we're trying to achieve, you cut through all the rubbish, really. And the discussions give you fresh ideas you might not have thought about. (Anna, Interview 3, #62)*

Through reporting back on the project at department meetings, Anna began to convince other colleagues in her school of the relevance of social justice issues to mathematics teaching, and to encourage them to try out some of the activities. She persuaded the whole department to carry out the Making a Change Project simultaneously, with year 8's, which facilitated the swapping of questionnaires between classes, and enabled presentations to be made to the whole year group during the final lesson. She prompted the head of department to consider adopting many of the ideas from the project in a redesigned scheme of work, including extended projects, applications to real-life contexts, and replacing some tests with group presentations. She expressed a determination, in her future role as a curriculum coordinator in another school, to persuade others to take on board ideas from the project.

Theme 4: Dominant discourses on ability and attainment

Before joining the project, Anna believed the best way to address inequities in education was to concentrate on raising the attainment of disadvantaged students in her school, providing them with opportunities they would not otherwise have.

> *I've chosen to teach in a school where it's classed as a challenging school, because the kids stereotypically wouldn't be expected to achieve very much. … So I think, in the sense of bringing about social justice through education, I'm involved in that just through being at this school. (Anna, Interview 1, #21)*

Initially, Anna was concerned that bringing social justice issues into the mathematics classroom might conflict with this aim, particularly when she realised how time-consuming this was, compared to more traditional ways of teaching. The need to complete the scheme of work, to ensure her students performed as well as those from other classes in regular half-termly tests, limited the time she was willing to devote to incorporating social justice issues.

> *I'm still very much passionate about my pupils getting grades, because they need these grades, more than other kids need grades, because they're going to be fighting against kids who've been to grammar schools, who have parents who can pay for them to have internships, all these different things. They're not going to have all those opportunities, so they need their grades. (Anna, Interview 1, #51)*

However, over the course of the project, Anna started to believe that teaching mathematics in a more relevant and meaningful way would lead to longer-term improvements in students' mathematical understanding. She recognised that her initial concerns resulted from pressures to demonstrate short-term progress of students, which she began to see as conflicting with good teaching. She remained reluctant to try activities with her Year 11 class, because of proximity of the exams and the pressure she felt to help them get C grades, although she recognised that this conflicted with the aims of the project.

> *I know it's a hideous approach to have but, at the moment, I'm just trialling it with Key Stage 3, because I feel, in the long run, my Key Stage 3 will really benefit from it. (Anna, Interview 2, #62)*

Anna recognised the difficulty in persuading senior managers of the benefits of teaching mathematics for social justice, since these are long-term and difficult to measure, whereas managers wanted to see evidence of short-term progress. She believed that close monitoring and pressure to teach to the tests might dissuade some teachers from embracing the ideas.

> *Whilst other teachers would probably love to do this stuff, they don't have the confidence that the children would get the grades that they need to get in half term tests. That is the single main constraint, I think. And also, unless teachers have read up on this way of teaching, they won't necessarily trust that it will get long-term results either. They'll just think that it's a very risky strategy. (Anna, Interview 2, #140)*

Anna believed that being trusted, and given the freedom to try things out with her classes, enabled her to set broader learning objectives, such as students deciding on a change they would like to see made, rather than focusing on specific mathematical content skills. There were still occasions, however, when she felt unable to spend as much time as she would like

on a social justice issue, because of the pressure to cover all of the content students needed for an upcoming test. She speculated that, in a school with less flexibility, the project would entail more risk, and she would feel obliged to teach shorter activities, woven more closely into the scheme of work.

Anna believed that using group work and focusing on conceptual understanding were particularly important for students in lower sets, who often lacked confidence and self-esteem. However, she also recognised that students in her Year 8 bottom set required more scaffolding when tackling open-ended activities. She viewed the main difference between students in higher and lower sets in terms of their behaviour and attitude, and was concerned that setting resulted in students, who didn't like mathematics, being clustered together in one class. She began to increasingly question the desirability of setting, and showed interest in other schools which achieved excellent results without setting. She believed mixed-ability classes would most benefit students who would otherwise be in bottom sets, whose generally weaker language skills would be strengthened through increased opportunity to engage in discussion with more articulate students. She suggested that, were she to achieve her ambition of becoming a head of department, she would aim to introduce mixed-ability classes.

> *Some people just get maths and are really good at it, and it just knocks your confidence when you don't understand and people are bragging about their seven A's. Overall, I find maths hard because the numbers for me just get muddled up on the page and it's really confusing. But it doesn't help when people say you're bad at it because then you suddenly give up maths and don't try. (Student in Anna's Year 8 bottom set, survey response to Wealth Distribution activity)*

5.4 Brian's story

Brian taught mostly mathematics, and some citizenship, at Oak Academy (part of the Forest Federation of academies), a much larger than average secondary school, which became an academy in 2007. The school was ethnically diverse and the proportion of students with a first language other than English was well above average. The proportion of students eligible for free school meals was above average, although the number of students with special educational needs was low. The school was popular and over-subscribed, drawing students from a wide area across inner-city London. The achievement of students was considered outstanding, with over 80% of students attaining five or more GCSEs, at grade C or above, including English and mathematics. The school was mixed, although in mathematics, girls and boys were taught in separate classes.

Theme 1: Changing epistemologies of mathematics

Brian had been interested in social justice issues before becoming a teacher, as demonstrated by his involvement with a global poverty charity and his choice to study for a geography degree. He described his motivation for becoming a teacher as a desire to help develop positive character and resilience amongst students, and address injustice and inequity in society. He didn't have a particularly close relationship with mathematics, although he recognised its importance as a gatekeeper qualification. Initially, he viewed helping students in disadvantaged schools to attain good grades in mathematics as a means of challenging injustice.

> *I believe that one of the key reasons I teach is to develop kids' characters, to make them into confident, resilient, hopefully joyful individuals. (Brian, Interview 1, #42)*

Brian regarded mathematics as being everywhere in real life, and having extensive links to other subjects. He had already incorporated some social justice issues into his mathematics lessons in order to address a perceived lack of awareness amongst students and the general public. He believed using group work, discussion and problem-solving approaches made mathematics more fun and relevant for learners, as well as making teaching more equitable through the sharing of knowledge between lower and higher-attaining students. He saw the project as a way of building on existing practice and his involvement reaffirmed his belief in these pedagogies.

> *I think things such as trying to give them a bit of agency and choice in lessons, things like encouraging them to work together in groups … have been things that I've done more of because, as part of the project, I've found them to be helpful and useful. (Brian, Interview 3, #41)*

Brian grew increasingly aware of the importance of applying mathematics skills to real situations, rather than developing these skills in isolation. He recognised the desirability of linking social justice issues, such as global inequality, more closely to mathematics skills from the scheme of work, in this case cumulative frequency. He endeavoured to identify meaningful and realistic contexts to enhance the learning of all mathematics topics, including algebra, which is often taught in an abstract way.

> *It's given me the confidence to step off the scheme of work treadmill, of getting through different topics or chapters, and actually saying: 'Well, these topics, say cumulative frequency, or percentages, I can fit these within a project on something to do with these kids' world, or to do with our world as a whole'. (Brian, Interview 3, #30)*

As Brian began to incorporate these ideas more regularly, students increasingly saw the inclusion of social justice issues as a normal and legitimate aspect of learning mathematics, questioning its purpose less often. Many students developed a broader view of mathematics

as something that was more applicable to real life, rather than as a collection of unrelated topics that they periodically revisited. However, others remained happy to learn mathematical procedures without any explicit purpose.

> *The maths we did today was interesting as it was not a theoretical thing, it tackled a real life issue that plagues the world. (Student in Brian's Year 8 set 2 of 4, survey response to Water Availability project)*

Brian described one of the most important benefits of the project as the closer relationships it helped him to build with students. Doing activities that allowed students to express themselves more freely enabled him to talk informally with them about issues they faced in their day-to-day lives, as well as their experiences of, and feelings towards mathematics. This helped him appreciate the extent to which students weren't getting a fair deal from the way mathematics is commonly taught, a situation he believed would get worse with the pressure, from the new mathematics curriculum, to cram in even more content. Establishing trust made it easier for him to convince students to engage with open-ended and creative tasks in mathematics.

Brian became more aware, through the project, of different perspectives on mathematics education, including how many politicians and economists see its primary aim as contributing towards economic growth. He recognised how some influential mathematicians emphasised the importance of learning about rigorous proof, particularly for students studying mathematics at university, contrasting with his own emphasis on developing mathematical proficiency required in future life. He observed how some teachers in his school, whilst keen to include social justice issues in their lessons, neglected the relationship between meaningful context and mathematical understanding, which he increasingly regarded as important.

Theme 2: Developing student agency

Brian observed how making the mathematics more accessible, for example by representing visually how the money paid for a chocolate bar is distributed, enabled more students to engage with issues such as Fair Trade. He highlighted how, through the project, students gained insight into social justice issues. For example, after tackling the Election activity and exploring how different voting systems led to different outcomes in elections, they raised the question of who chose the voting system.

> *The maths today made me realise how the simplest maths can change lives. (Student in Brian's Year 10 top set, survey response to Fair Trade activity)*

Brian described how providing more meaningful activities, through the project, led to greater enjoyment of, and improved attitudes towards, mathematics amongst students, particularly those normally exhibiting more challenging behaviour and participating less in lessons.

> *I liked what we did today because it was something totally different. We learn more about the world like this, while using maths. (Student in Brian's Year 8 set 2 of 4, survey response to Election activity)*

Over the course of the project, Brian developed an increasingly critical view of the existing education system, strengthening his belief that schools perpetuated inequity. He believed that making students more aware of this situation, and how it might disadvantage them, was necessary to prevent their own exploitation in later life. He began to advocate the importance of students developing their own opinions, based on independence of thought, critical understanding and the strength of arguments, in order to create a more just society. He argued that mathematics could play an important role in achieving these aims, and countering the false arguments students were exposed to regularly in the media.

> *If you do want to see the world improved on a big scale internationally, that also has to be done at a local level. And for that to happen, individuals have to be able to enjoy and engage with the world, and that's quite a difficult task if they're not skilled up. (Brian, Interview 3, #13)*

Brian observed, during the Water Availability activity, how groups of students managed to develop arguments for the allocation of scarce water resources, using statistical data provided, and present these to other groups. He was particularly pleased with how students were willing to change their views after listening to the arguments of other groups. However, during another statistical activity looking at how tabloid headlines, such as 'immigrants swamping the country', were exaggerated, he was disappointed with how students appeared to retain their previously held stereotyped views.

> *We've worked out this calculation that should completely debunk all these headlines, and instead the kids were still like 'They're stealing our jobs'. (Brian, Interview 2, #51)*

Brian introduced the Making a Change Project, in the third cycle, by getting students to compare mathematical statements, such as 'One in five people go to bed hungry each night', with non-mathematical statements, such as 'There are lots of people in the world who are hungry'. This enabled students to appreciate for themselves the power of using mathematics to construct a convincing argument. Students were further encouraged to develop independent learning skills, and an appreciation of how mathematics can be relevant to their lives, by choosing their own issues to research.

> *It was good because we got to learn about maths in the context of life, so we have no excuse for 'Why do we need this? (Student in Brian's Year 9 top set, survey response to Making a Change Project)*

During the project, Brian began to recognise how the students who struggled to manage their own behaviour and appreciate how this affected other people, tended to be those who also had difficulty coping with open-ended tasks and engaging with social justice issues such as fairness. Whilst students needed to think critically, he argued that it was equally important for them to develop the skills necessary to engage with social situations, involving a degree of compliance with social norms. He strengthened his belief that students should reflect on their emotional responses towards mathematics, and appreciate more why they might need to persevere with procedures seemingly unrelated to their lives. He emphasised the need to help all students develop the personal and social skills required to take advantage of learning opportunities and be successful in mathematics.

> *I think that they've made more progress because abstract, or things that don't seem relevant, are now relevant in terms of playing the game. (Brian, Interview 3, #51)*

Theme 3: Collaborative nature of research group

Brian believed strongly that educational disadvantage and global inequity were entirely unnatural phenomena, resulting inevitably from the existing economic system, which could and should be challenged. He was motivated, initially, by a desire to help students understand and change the world.

> *They need to know what's going on in the world, and if they're not going to get that from someone else, I may as well do it as a starter in my lesson. (Brian, Interview 1, #78)*

Brian saw the research group as providing an opportunity to discuss and debate theories underlying the project, in a relaxed and non-threatening environment. The initial conceptualisation of teaching mathematics for social justice, presented at the first meeting, challenged what he later recognised as his previous narrow perspective. Discussions within the group helped him to develop his own thinking, deepening and broadening his understanding of the functions of schools and education, in relation to social justice. He began to appreciate how building a better world, and developing skills necessary to engage at a local level, both relate to promoting understanding of fairness and justice.

> *Some of the arguments we've had have been really enjoyable, and helped me think things through a lot better. (Brian, Interview 3, #57)*

Brian was keen to engage with research on social justice issues, as well as with research methods and processes. He was already interested in pursuing these interests through further study and the project encouraged him to apply for, and secure, a place on a Masters programme, which he was due to start immediately after the project.

> *I'd like to see if I could be involved in the research in the longer term, writing papers, seeing whether it's an area I think could add value to or not. (Brian, Interview 1, #98)*

Brian particularly appreciated working collaboratively with teacher researchers from other schools, allowing him to focus on best practice, rather than worrying about how lessons would be judged against performance criteria used in his school. He described sharing ideas as the best thing about the project, making it easier to locate resources, and spread the burden of planning, which was time and energy-consuming. He believed the mutually supportive environment, within the research group, gave him the confidence to try things out that he would otherwise have been wary of doing. He felt belonging to the group enhanced his interest in social justice issues, giving legitimacy to his desire to challenge the existing system.

> *And it's also provided that additional incentive to do it, and to take the risk, because you know that you're going to be asked to talk about it. But also you know you're going to be allowed to talk about it in a way that says that messing up doesn't matter. (Brian, Interview 3, #32)*

Brian viewed the project as effective professional development, due to its sustained nature, although it was hard to separate the impact on his teaching from the rapid learning he believed was inevitable at such an early stage of his career. This was in contrast to what he saw as inadequate support provided to newly qualified teachers, who were generally overloaded with lessons, rather than being allowed to become good teachers.

> *One of the things that really appeals to me is that I can engage with other high quality teachers in other schools that are trying creative things. (Brian, Interview 1, #102)*

As he grew in confidence, Brian began to develop ideas collaboratively with other teachers in his department who, he realised, were willing to engage with SMSC (Spiritual, Moral, Social and Cultural aspects of learning), for which he was the link teacher. He recognised that, because Ofsted required SMSC to be addressed in all subjects, it was useful for justifying the inclusion of social justice issues in mathematics lessons to others, including senior managers. He later disseminated ideas from the project more widely, by providing training sessions for the whole school and across the Forest federation. He noticed that teachers' fear and reluctance to take on board new ideas could be overcome by demonstrating their benefit and

value. When something new was tried, and other teachers got to hear how well it worked, the idea tended to spread very quickly.

Theme 4: Dominant discourses on ability and attainment

Brian believed high levels of scrutiny in his school made teachers, particularly those new to the profession, more risk-averse and less innovative in their practice, although he thought this was less of a problem in his department. Monitoring was also problematic, as students' progress was often judged by what was easy to measure, for example the number of pages of work completed, rather than how much learning had taken place. He argued that assessing the developing understanding of social justice issues, which involved qualitative changes in attitude, was much harder than measuring progress in mathematics, for example by comparing responses to closed questions before and after a lesson.

> *I think it makes you less likely to take risks with your classes. If you know that there's a chance that someone pops in, you're more likely to do lots of very average lessons, than one lesson that could blow up in your face or it could go amazingly, because you know that you'd be judged on that one lesson. (Brian, Interview 1, #94)*

Brian highlighted how pressure to complete the scheme of work, in order to prepare students for tests, combined with a lack of time, were significant constraints on developing teaching ideas and using extended projects. He felt frustrated that his additional responsibilities, including mentoring other teachers, meant he had less time to develop his own practice, although he believed this enabled him to have a wider impact. He managed to navigate these constraints successfully, for example by linking social justice issues more closely to upcoming topics in the scheme of work. He regarded the time spent at research group meetings as worthwhile, because it enabled the sharing of resources.

> *As always I think the biggest constraint is time, and the biggest worry is exams and observations. (Brian, Interview 2, #152)*

Brian observed that, whilst there was a generally positive response to the activities, the increase in enjoyment and engagement amongst the highest-attaining students was less noticeable. He attributed this to the satisfaction these students felt from getting most questions correct, whilst seeing others getting them wrong. Alternative teaching approaches, with more discussion and less emphasis on right and wrong answers, might therefore be perceived as challenging the basis of their success. He also found his Year 11 class less enthusiastic about including social justice issues in mathematics lessons, blaming this on their ingrained views of mathematics, resulting from longer exposure to traditional teaching approaches.

> *I think, if you are at the top end of the top set, you've put your hat on the fact that you get things right, and as soon as in maths it's no longer about you getting the right numerical answer, you suddenly feel like things are not under your control any more, and you're not top dog any more. (Brian, Interview 2, #108)*

Brian increasingly recognised that students in lower sets could potentially benefit most from the project, since their weaker mathematical skills were more likely to lead to them being disadvantaged or exploited in future. However, he appreciated that they generally struggled, and produced less work, when given more open-ended tasks, resulting in a tendency to structure learning more for these students, using shorter, closed tasks. Whilst enabling students to experience immediate progress, and helping improve their behaviour, this tendency limited opportunities for them to develop independence and critical thought. He saw it as less risky to try out new ideas with higher-attaining students, who generally had greater intrinsic motivation and were more likely to respond positively.

> *And I think another constraint with some classes is definitely behaviour. You tend to do the nicest projects with the nicest kids, unfortunately. (Brian, Interview 1, #46)*

Brian became increasingly aware of the power of education to either maintain or challenge the existing social order, although he recognised the limited influence that he could have as an individual teacher. A desire to work at a strategic level, in order to have more influence on institutional change, was a factor in his decision to pursue a Masters degree in global governance.

> *The reality is that one of the few places you have the ability to really affect how social reproduction occurs, and how you re-shape the next generation, is through education. (Brian, Interview 3, #74)*

Brian increasingly believed setting students according to ability contributed towards the widening gap in attainment during Key Stages 3 and 4, and that mixed-ability teaching would help catalyse discussions in all mathematics classes. However, he recognised that the popular view of mathematics, as centred on calculations and procedures, meant many teachers saw setting as making life easier, by narrowing the range of attainment in each class. He doubted schools would move away from setting in mathematics, as long their success continued to be judged by how many students attained grade C or above, thus focusing resources on a narrow range of achievement. Despite efforts to convince them otherwise, he was frustrated that most students continued to believe success in mathematics was down to ability, rather than effort, which was highlighted by a survey his Year 7 and 8 classes carried out on each other. This made him realise how much work needed to be done to challenge students' well-established views on ability.

> *Maths makes me confident because I am quite good at it, in my opinion. (Student in Brian's Year 10 top set, survey response to Fair Trade activity)*

5.5 Rebecca's story

Rebecca taught mathematics at Ash Academy (part of the Forest Federation of academies), a mixed secondary school in Outer London which opened as an academy in 2009. Students came from a diverse range of ethnic heritages and approximately one third spoke English as an additional language, although most were fluent in English. The proportion of students eligible for free school meals was more than twice the national average and the proportion with a statement of special educational needs was well above average. The school had a relatively stable student population and was oversubscribed. Achievement was well above average with over 70% of students attaining five or more GCSEs, at grade C or above, including English and mathematics.

Theme 1: Changing epistemologies of mathematics

Rebecca developed a close relationship with mathematics as a child and there were always high expectations on her to do well. At school, she viewed mathematics as value-free, and was attracted to its abstract nature and the way it was possible to get an exact answer. Being successful, she was happy to study mathematics for its own sake, and resented her teacher's attempts to demonstrate its applicability to science experiments. She studied mathematics at degree level and wouldn't have felt comfortable teaching any other subject. She only realised the importance of convincing students of its relevance when she became a teacher.

> *That was always the appeal, rather than actually thinking about why it would be useful, which I don't think I thought about until I was leaving university. (Rebecca, Interview 1, #38)*

Rebecca's motivation for joining the project included being intrigued and curious to find out more about the relationship between teaching mathematics and social justice. She admitted knowing little about this beforehand, describing how her department had been perplexed when asked to incorporate SMSC into the schemes of work.

> *When I saw your first email, first of all I had to google it because I didn't have a clue what you were going on about. But it's just never occurred to me to try and teach maths in that sort of way. I'd never heard of teaching maths for social justice before. (Rebecca, Meeting 1, #61)*

Through the project, Rebecca became more aware of her own perspective on mathematics, and how this differed to others in her department and the research group. She described having to make a conscious effort, for the sake of her students, to link mathematics to the real

world. The first meeting prompted her to question her previous assumptions about mathematics and, for the first time, to consider its value-laden nature. She later acknowledged the need for students to reflect on its nature, in order to challenge ingrained attitudes towards the subject.

> *You were talking about the nature of maths at the beginning, that's definitely changed the way that I've thought. (Rebecca, Interview 3, #102)*

Rebecca increasingly recognised how using real life examples helped develop mathematical understanding, and encouraged students to look for patterns and generalise, rather than manipulating numbers without considering their meaning. Using male and female earnings to exemplify a dual bar chart, for example, promoted students' engagement with the mathematical properties of the graph. She argued that students would be more likely to understand procedures, and how they might use them in later life, if their purpose was made clearer, for example how the random generation of numbers could be used to produce a representative sample.

> *I think that the lesson on Fair trade was very good to know how much farmers get from growing cocoa. It has helped me a lot about percentages. (Student in Rebecca's Year 9 set 3 of 4, survey response to Fair Trade activity)*

Rebecca became increasingly comfortable using real life cases to enrich mathematics lessons, for example the woman wrongly convicted of murdering her two children, who actually died from cot death, based on inaccurate conditional probabilities. She appreciated the need to avoid contrived contexts, commonly used in school mathematics, such as favourite colours. She felt the project provided structure, giving her the confidence to plan around broader objectives, such as using mathematics to support an argument, rather than focusing on narrow content skills.

> *And I think a lot of that kind of 'Yes I can actually teach something useful that's not just proportionality' has come through having to think about it in this project. (Rebecca, Interview 3, #41)*

Whilst Rebecca increasingly viewed social justice as a legitimate focus in mathematics, she acknowledged that most lessons she taught remained skills-focused. She found it difficult to focus on social justice issues and mathematical skills at the same time, noticing that students enjoyed discussing the issues, but showed reluctance to relate them to the mathematics. She remained concerned that the mathematics in some activities, such as Election, was too easy, and that the open-ended nature of statistical projects made it difficult to ensure students collected data appropriate for the procedures and methods she might want them to learn.

> *Even the Making a Change Project, there was obviously quite a lot of maths in it, but there wasn't a mathematical objective that they were learning, necessarily, through doing it, like how to draw a bar chart, or whatever. (Rebecca, Interview 3, #83)*

Theme 2: Developing student agency

Rebecca believed students needed to develop mathematical skills to make sense of the growing amount of information available, helping them to avoid being misled by others. These beliefs resonated with theoretical ideas she encountered in the project, in particular, using mathematics as a means of making sense of the world and empowering students by developing their agency.

> *Unless you actually have some kind of understanding of how to look at statistics, and how to look at the information that's given, and when to question it, and, you know, 'What's reliable and what isn't?' then you don't have a hope. (Rebecca, Interview 1, #76)*

Rebecca was motivated by students' apparent lack of awareness to try out activities aimed at developing an understanding of issues such as equality and fairness. She was pleased with the extent to which her students engaged with these activities, and she described really enjoying teaching them herself. She highlighted how the students, whose engagement increased the most, were not necessarily those who were normally confident in mathematics.

> *In general I do not enjoy maths as I think I'm not very good at it. Today I enjoyed the maths lesson as I enjoyed finding out about fair trade and I liked seeing all the statistics of the money different people make from a bar of chocolate. (Student in Rebecca's Year 9 set 3 of 4, survey response to Fair Trade activity)*

The Fair Trade activity prompted heated debate amongst students about why cocoa farmers got so little (4 per cent) from the money spent on a Fair Trade bar of chocolate, prompting most students to insist they wouldn't buy Fair Trade again in future. Whilst appreciating students didn't necessarily have to share her views, Rebecca was frustrated that they ignored what she felt the mathematics was highlighting, i.e. that the farmers earned eight times as much as normal.

> *I think maths today was good as it's showing actual statistics which has made me think 'fair trade' isn't fair. (Student in Rebecca's Year 9 set 3 of 4, survey response to Fair Trade activity)*

Rebecca was already convinced of the merits of student-led learning and believed that current mathematics teaching was generally too directed. She believed students should understand the rationale for what they were learning, identify which procedures to use to solve problems, and decide how to interpret results for themselves.

> *But it's not someone standing up and saying 'Today we're going to learn this thing', it's kind of 'Oh, you need this? Well, look, here's my method'. (Rebecca, Interview 1, #46)*

Rebecca was surprised that even higher-attaining students struggled when it came to using statistics to support an argument, a skill she considered important. The first activity she tried was an attempt to develop the agency of students in her class by getting them to collect data from other students, and using it to argue for a change that they would like to see made in the school.

> *I think the agency thing was definitely something I hadn't considered at the start. Like, I saw it more as applying maths to different situations, rather than using maths to actually change something. (Rebecca, Interview 3, #8)*

Rebecca initially felt very negative about the activity, frustrated by logistical problems she encountered getting groups to design questionnaires, and complete letters arguing for their change, on time. She felt many of the changes sought, such as amending the school rules on body piercing, were unrealistic, which meant that none of the requests were agreed to. However, despite her own disappointment, most students enjoyed the activity, particularly choosing their own issue, designing their own questionnaires, and circulating these via registers.

> *It was the logistics that really got me, rather than anything fundamentally wrong with the idea. (Rebecca, Interview 2, #20)*

After further reflection, Rebecca concluded that she had given students too much independence in one go and that she needed to provide more structure by breaking the activity down into shorter tasks. She believed the activity resulted in students developing strategies for seeking change, including collecting data to support their argument, despite their requests not being granted. Developing student agency remained central to her thinking and, during the third cycle, she refined her ideas through the design of the Making a Change Project, which her students enjoyed even more. This time around, with clearer guidance, students made more realistic requests and used mathematics more effectively to support their arguments.

> *I liked the presentation as I got to do something that I felt strongly about. It gave me a chance to express how I feel, also including maths to support my presentation. (Student in Rebecca's Year 9 set 3 of 4, survey response to Making a Change Project)*

Theme 3: Collaborative nature of research group

Rebecca saw the project as a welcome opportunity to develop her own practice, in an area she knew little about, through working collaboratively with other teachers. By joining the project, she appreciated that her fundamental assumptions about mathematics were liable to be questioned, but she was keen to challenge herself and to critically reflect on her existing practice.

> *I guess my interest in it is way more just 'I want to be able to do this and I can't do it at the moment', rather than 'This is a big thing in society that needs to change'. (Rebecca, Interview 1, #79)*

Rebecca described the research group meetings as her favourite aspect of the project, particularly when they focused on the joint planning and evaluation of lessons. She enjoyed sharing ideas with others in the group, especially in areas in which it was more difficult to think up activities and there were fewer resources available. She felt that the group's decision, in the second and third cycles, for all teacher researchers to try out the same activities, made it easier to relate to each other's experiences. She believed that sharing experiences and jointly evaluating activities, with other teacher researchers, significantly influenced the development of her own practice. She regarded meeting with teachers from other schools as particularly valuable, since most of her department had followed the same initial teacher education programme, taught in only one school, and hence tended to think and teach in a similar way.

> *Actually seeing what other people have tried out, I found a lot more useful than things that necessarily I came up with myself. (Rebecca, Interview 3, #46)*

Rebecca believed that she wouldn't have had the confidence to try out the activities on her own, without the support provided by the research group. Having initially felt very negative after trying the first activity, she described being pleasantly surprised by the encouragement she received when feeding back on her experiences at the third meeting. The responses from other teacher researchers helped her to appreciate the activity's value, and that the difficulties encountered were inevitable teething problems associated with introducing a new way of working.

> *It is quite useful having that kind of, I don't know, support almost and being able to just tell someone exactly what happened and have their, kind of, outside view on it. (Rebecca, Interview 2, #41)*

Rebecca felt that she hadn't significantly engaged with educational research before the project, partly because she had studied for a mathematics degree in which reading articles was not a requirement. In contrast, she was keen to read the research articles I circulated, and showed a great deal of interest in the initial thematic analyses of the interviews and meetings,

that I presented to the group. She was aware of her own developing understanding of research processes, as well as theories underlying the project. She described how designing and trying out activities in the classroom helped her make sense of these theories, which in turn made her a better teacher.

> *This is my only experience of any kind of research … I have learnt an awful lot about the process, as opposed to just what we're researching. (Rebecca, Meeting 4, #68)*

Rebecca described colleagues' initial scepticism about the project, particularly when the first activity she tried didn't go according to plan. However, as she grew in confidence, she began to share ideas, gradually convincing others of the relevance of social justice issues to teaching mathematics. She provided a training session for her department, showcasing ideas from the project, following which colleagues showed increasing enthusiasm for trying out the activities. She wrote a project on nutrition, which was incorporated into the Year 7 scheme of work.

> *I think the Making a Change Project has been really successful. I think that one's going to stick around our school. (Rebecca, Interview 3, #50)*

The positive feedback received from colleagues underlined for Rebecca the benefits of the project, encouraging her to try out more ideas and develop her own practice further. She found that others began to take on board the ideas and develop similar resources, for example one colleague was prompted to write a project on heart transplants.

> *Success has bred more success, because if they've seen a lesson go well, then they want to teach it, and then their lesson goes well, and then it sort of spreads. (Rebecca, Interview 3, #57)*

Rebecca identified future opportunities for disseminating ideas through training days within her school and across the Forest Federation. She believed that making the resources from the project, and the experiences of the research group, more widely available would encourage teachers further afield to take on board the ideas.

Theme 4: Dominant discourses on ability and attainment

Rebecca was aware of the high levels of monitoring and scrutiny in her school, but believed this didn't necessarily discourage creativity in teaching. Students had high expectations that lessons would be interactive and engaging.

> *But, at the same time actually, we have a lot of monitoring and scrutiny. Like, we have windows in every class and people wander up and down, and they will come into your lesson … But actually, I think we do have a lot of risk-taking and stuff. I don't think it has to be stifling. (Rebecca, Meeting 2, #34)*

Rebecca recognised that the focus on preparing students for exams meant substantial pressure, from managers and students, particularly those in their final year, to complete schemes of work. For this reason, she tended to try activities, even in Key Stage 3, with classes that had already completed the work for the term. This avoided students complaining that they had not been prepared, as well as others, for the tests. When incorporating social justice issues, she felt the need to justify what mathematics skills were being learnt, to allay students' concerns that the content of the lesson wouldn't prepare them for tests.

> *I think the maths we did today was easy because we had to write a letter, so we did a minimal amount of maths. (Student in Rebecca's Year 9 set 3 of 4, survey response to initial Making a Change activity)*

Rebecca felt increasing pressure, towards the end of the year, to complete the schemes of work, partly due to these being rewritten with more content. She expressed concern that there was only sufficient time to teach mathematical procedures, and not enough time for students to learn how to apply them adequately. There was some flexibility to try out new ideas, but teachers were then expected to catch up with the scheme of work at a later date. Because of the time-consuming nature of teaching social justice issues, she began to appreciate the need to link these more closely to specific mathematics skills.

> *I do think I feel under more pressure to get through all the material. I am struggling a bit on that front, which means that any social justice activity has to be very specifically linked to something, a mathematical skill that is not going to be taught in any other way. (Rebecca, Interview 2, #119)*

Rebecca felt that some activities with broader aims, such as the Making a Change Project, didn't fit easily into the scheme of work, and many teachers only used them at the end of term when they had no other resources to use. Some teachers tailored this activity towards particular mathematical skills, identified as weaknesses, such as drawing pie charts or collecting primary data, insisting all students included these in their presentations. She hoped the new national curriculum would provide more opportunities for teaching reasoning skills, assessed through presentations and extended writing, making it easier to teach ideas from the project.

At the start of the project, Rebecca believed that helping students to appreciate links within mathematics was more appropriate for students in higher sets, whilst revealing the complexities of mathematics might confuse students in lower sets. However, during the project, she began to reassess the desirability of setting students by attainment, recognising that this meant concentrating students with poor behaviour and attitudes towards mathematics together in bottom sets, thus hindering their progress even further. She reflected

on her experiences of teaching a mixed-ability Year 7 class, before they were placed in sets, realising how much she had enjoyed it, and noticing less difference between students than she had expected.

Rebecca considered poor behaviour to be one of the biggest barriers to trying out new approaches, for example she abandoned the Election activity, first time around with Year 8's (set 3 of 4), because they started shouting at each other when she asked them to work in groups. She recognised her tendency to resort to teaching poorly-behaved classes in a more structured way, appreciating that this conflicted with the aims of the project.

> *I know the way that I teach classes that are badly behaved is so structured, to make up for the fact that they can't be left to their own devices for five minutes. … That kind of approach doesn't really lend itself necessarily to an extended open activity, where they actually get to think more deeply about the things that are involved. (Rebecca, Interview 3, #69)*

Rebecca tried out most of the activities from the project with the same Year 9 class (set 3 of 4), which she described as her favourite and best-behaved class. Even though many students in this group were disappointed with their current grades, she felt the positive relationships she had established meant they were more willing to accept her trying out alternative approaches. It appeared to her that the more activities they did, the better they responded, partly because they felt special that she had chosen to try the activities with them.

5.6 A study of the research group

In this section, I report on the development of the research group as a whole, including the functioning of the group, the relationships between its members, and the development of a group identity. The study of the research group focuses on commonalities between the experiences of all five teacher researchers, in relation to the four themes emerging from the thematic analysis. Background information is provided for Sarah, George, and the schools in which they taught, to supplement that provided for Anna, Brian and Rebecca, in sections 5.3, 5.4 and 5.5 respectively.

Sarah taught mostly mathematics, and some history, at St. Francis' Church of England School, the same school in which Anna taught. She resigned her post at the school in March 2014, and hence was only involved in the first and second cycles. She was present at five of the seven research group meetings and I interviewed her twice, at the start of the project and just before she left the school.

George taught mathematics at Bishop Godfrey Catholic School in Inner London, a federation between a boys' school and a girls' school, each smaller than an average secondary school. The

proportion of students with a statement of special educational needs was above average, and for those eligible for free school meals, was well above average. The vast majority of students came from minority ethnic backgrounds, the largest groups being of Black African and Black Caribbean heritage. The proportion of students who spoke English as an additional language was above average. The achievement of the boys' and girls' schools were both above average, with over 60% and 70%, respectively, of students attaining five or more GCSEs at grade C or above, including English and mathematics. George was unable to attend the fourth research group meeting.

All five teacher researchers had been through the same school-based initial teacher education programme, on which I had also been a subject tutor. They all completed their first year as newly qualified teachers in July 2013, hence for most of the project, were in their second year as qualified teachers.

Theme 1: Changing epistemologies of mathematics

Whilst at school, most teacher researchers had viewed mathematics as being almost exclusively content-focused, for example Anna regarded it as mainly about calculations and algebra. Sarah was unusual in that she had experienced some problem-solving at an early age, although this was more down to the influence of her father, who was a primary school teacher, than her school. All five teacher researchers considered themselves to be successful learners of mathematics.

All five teacher researchers described how their epistemologies of mathematics only changed after they left school, as they began to appreciate its value-laden and socially constructed nature. They highlighted how the most significant changes took place during their initial teacher education programme and through their involvement with the project. During research group meetings, through reflecting on the nature and position of mathematics, and engaging with the underlying theories, they began to rethink their epistemologies and became steadily more aware of their own perspectives on mathematics teaching, and how these contrasted with those of other people.

Discussions at the first meeting suggested Anna, Brian and Sarah were already aware of the status mathematics occupied as a gatekeeper qualification, indeed they cited this as a reason for deciding to teach the subject. However, the idea of making the nature of mathematics more explicit to students was new to all teacher researchers except Brian, who had previously held some discussions with his students on their perceptions of mathematics and how they felt towards it.

As teacher researchers tried out activities during the project, their changing views of mathematics were generally accompanied by a reassessment of their thinking about teaching social justice issues. They began to shift, from considering this as a way of enriching mathematics lessons, towards seeing it as an essential and legitimate part of teaching mathematics. They began to appreciate how applying mathematical skills to social justice contexts could promote students' mathematical understanding, whilst making the subject more relevant and meaningful.

Teacher researchers, particularly Anna, Brian and Rebecca, adopted an increasingly critical stance towards conventional approaches to teaching mathematics, strengthening their views of these as lacking relevance and meaning, and focusing narrowly on procedural understanding. This was reinforced by feedback collected from students, indicating that there was a common perception of mathematics as boring and pointless. However, they felt that learning discrete mathematics skills was still important, and that this should be complemented by, rather than replaced by, tackling social justice issues.

Brian and Rebecca described how incorporating social justice issues impacted significantly on students' perceptions of mathematics, particularly for those completing several activities. Rebecca highlighted how her students began to appreciate that mathematics could involve extended writing. There were, however, occasions when students' deeply ingrained views of mathematics became apparent. Some of Sarah's students, because of the large amount of writing they did, felt they hadn't learnt any mathematics. A minority of students expressed concern that, by focusing on social justice issues, they were not studying 'real' or 'proper' mathematics, as some of George's students referred to it.

There was growing appreciation amongst teacher researchers of the need to establish a stronger link between social justice issues and mathematics skills. They realised how important it was to identify mathematical skills at an appropriate level of challenge for students. If these were too easy, students wouldn't recognise that mathematical learning had taken place, and if too difficult, students wouldn't be able to engage with the social justice issues.

Theme 2: Developing student agency

Teacher researchers were, to varying extents, motivated by a desire to change society for the better, Anna, Brian and George citing this as a primary reason for becoming teachers. Initially, all five saw raising the attainment of disadvantaged students as an important way of addressing inequity. Brian and George also saw teaching as a means of developing general awareness of social justice issues, in order to facilitate future change in society.

There was early consensus amongst teacher researchers that boundaries between mathematics and other subjects were too rigid, and that it was legitimate to explore social justice issues in mathematics lessons. They identified the need to make mathematics more realistic and meaningful so that, by appreciating the rationale for learning specific mathematical procedures, students would feel motivated, rather than compelled, to learn. All five teacher researchers advocated employing progressive teaching pedagogies, including student-led learning, group work, discussion and open-ended problem solving.

Initially, teacher researchers generally appeared most comfortable with the idea of students using mathematics to make sense of social justice issues and how these related to their lives. During the first cycle, issues tackled in mathematics lessons included public misperceptions of the extent of benefit fraud and the ethnic and religious make-up of the UK population (by Anna), changes in the levels of global inequality (by Brian), water usage and sustainability (by George), and exploring data about lifestyle (by Sarah). During the second and third cycles, students also explored Fair Trade, average incomes, global inequality and voting systems.

During the first cycle, Rebecca was the only teacher researcher to focus primarily on developing student agency. Whilst she initially felt frustrated with her attempts to encourage students to work independently, her ideas were embraced with enthusiasm by the research group. This stimulated growing interest in the notion of student agency amongst all teacher researchers, who had given little thought to this prior to the project. This culminated in the design of the Making a Change Project, in which students used mathematics in order to develop and support their arguments, which became a focus for all five teacher researchers' attention during the third cycle.

George was concerned that agency, on its own, was not necessarily desirable, as students would only become positive agents of change if they also developed open-mindedness and sensitivity towards social justice issues. On the other hand, he emphasised how expecting students to come up with a conclusion, or develop an opinion, considered agreeable by the teacher, such as advocating the purchase of Fair Trade products, conflicted with the aim of developing agency. Rebecca outlined the importance of mathematics teachers acknowledging they held strong opinions on some issues, without seeking to impose them, so that students appreciated the relevance of mathematics.

All five teacher researchers emphasised the generally positive response from students to the project activities. They observed much higher levels of engagement with mathematics, more genuine interest in the issues, and greater enjoyment of learning. This was particularly

noticeable amongst students who were normally poorly motivated, and badly behaved, in mathematics lessons. These observations concurred with the feedback from students, who referred to mathematics, in general, as being boring and irrelevant, whilst describing the project activities as being different, and helping them to see how mathematics could be more useful in real life.

Theme 3: Collaborative nature of research group

Brian, George and Rebecca acknowledged how the opportunity of belonging to a collaborative research group attracted them to the project. The invitation came at an ideal time, when all five teacher researchers were just completing their first year as newly qualified teachers, and were starting to think about the direction they would like their practice to develop. There were notable differences, within the research group, in previous engagement with social justice issues, with Rebecca beginning to explore the concepts for the first time, whilst Brian and George had been active in organisations, with a social justice focus, for a number of years. All five teacher researchers, however, readily acknowledged the limits of their own understanding and were keen to learn more.

There was consensus amongst teacher researchers that discussing ideas, comparing experiences, and the joint planning, teaching and evaluation of activities, impacted considerably on their thinking and classroom practice. Meeting with teachers from different schools was seen as particularly beneficial, as it exposed them all to a wider range of ideas and perspectives. Members of the research group already knew each other, which made it easier to establish working relationships within the group. They acknowledged the pivotal role I played, as university-based researcher, in raising their awareness of research processes and providing a structure for developing ideas. The theories I presented challenged their preconceptions, and fostered deeper understanding, and a broader vision, of teaching mathematics for social justice.

The mutually supportive character of the research group played an important role in encouraging teacher researchers to take risks and overcome the challenges and constraints they faced. This was exemplified in the way the group encouraged and reassured Rebecca, when she appeared disheartened by problems she encountered during the first activity. This support ensured the value of her initial idea was recognised, and its potential realised, through the design of the Making a Change Project in the third cycle. George and Brian highlighted how being part of the group provided an additional incentive to try out the activities.

All teacher researchers, but particularly Anna and Brian, described how the project provided a model of professional development which was considerably more effective than others they had experienced. The project's focus on relating theories to practice, and its long-term and sustained nature, were seen as key factors explaining the positive impact it had on their classroom practice. The project was seen to develop both mathematics-related and generic teaching skills, for example Brian described how it helped him cultivate closer relationships with students, and George claimed it helped him manage group work and discussions more effectively.

There was a general willingness to engage with the research methodology and theories underlying the project and an enthusiasm to read and discuss research articles I circulated. Anna, Brian and Rebecca reviewed relevant teaching resources, and presented these to others during meetings. All five teacher researchers collaborated over the design of the student survey, during the second meeting, which they subsequently reviewed and amended in order to make it more effective.

Over the course of the project, teacher researchers' confidence grew to the extent that they began to encourage other teachers in their schools to take on board the ideas. They began to recognise what George described as 'the multiplier effect', by witnessing how news of the positive impact the activities had on students spread quickly across their departments, resulting in a rapid growth in interest in the project. Anna, George and Rebecca persuaded their departments to use some of the activities with a whole year group. Anna, Brian and Rebecca ran training sessions for their departments, focusing on ideas from the project. All teacher researchers agreed that collating ideas from the project, as a structured resource, would encourage other teachers to take them on board.

Theme 4: Dominant discourses on ability and attainment

All five teacher researchers followed a school-based initial teacher education programme, which emphasised addressing educational inequity primarily through raising the attainment of students in disadvantaged schools. This helped to explain the initial concerns of teacher researchers, particularly Anna, that the project might conflict with such aims. However, the positive impact of the project, on students' engagement, enjoyment, and understanding of mathematics, provided reassurance that there was no conflict between teaching mathematics for social justice and raising mathematical attainment.

There was growing appreciation amongst teacher researchers that a narrow focus on raising students' attainment, whilst ignoring structural inequities, was counter-productive. Brian and

George argued that high levels of scrutiny and monitoring led to low-risk teaching and mainly procedural understanding, conflicting with demands of higher education and employers for more creative, independent thinkers.

There was general agreement amongst teacher researchers that the research group enabled them to overcome many of the constraints they faced in bringing social justice issues into the mathematics classroom. Pressure to complete schemes of work was alleviated by the growing awareness of links between mathematical skills and social justice issues, resulting from discussions at meetings, which enabled them to more easily incorporate these into lessons. They believed that sharing ideas and resources compensated for the additional time and energy required to plan lessons incorporating social justice issues.

Anna, Brian and Rebecca expressed an initial preference for trying out ideas with students in higher sets, who they felt were generally better behaved and more positively disposed towards learning mathematics. They began to recognise, however, that alternative teaching approaches were equally important for students in lower sets, who were those most commonly failed by the existing system. Over time, they realised that the benefits of the project were most apparent amongst lower-attaining students, whose engagement and enjoyment generally increased more than those of other students. They noticed that resistance to contextualised and discursive approaches to learning mathematics was most likely amongst higher-attaining students. This was attributed to such students associating their success in mathematics with well-established conventional teaching approaches, thus perceiving any change as a possible threat to their continued success.

Poor behaviour remained a major constraint on trying new ideas, particularly for George and Rebecca, who appeared less comfortable using discussion and group work with challenging classes. All teacher researchers, to some extent, professed a tendency to be more structured in their teaching of lower sets, by breaking activities down into shorter closed tasks, whilst acknowledging that this conflicted with the aims of the project.

Teacher researchers, particularly Brian and Rebecca, viewed building relationships of trust with students as important for convincing them of the benefits of adopting alternative approaches to learning. Where such trust existed, students responded better to the activities, which helped to further strengthen the relationships. They believed this process might help them challenge students' ingrained views of mathematics, which had become well-established through experiencing many years of conventional teaching approaches.

At the start of the project, teacher researchers tended not to challenge the notion of rigid setting by ability, prevalent in all four schools, although George acknowledged that teachers' views of ability, and their expectations of students, should be considered as problematic. However, Anna, Brian, Rebecca and Sarah began to increasingly question the benefits of setting, blaming it for widening differences in mathematical attainment. They recognised that concentrating students, who lacked confidence and a disposition towards learning, in lower sets would limit, even further, their potential for future achievement in mathematics.

5.7 A study of the research model

In this section, I report on the important role the critical research model played in producing reliable and trustworthy research findings. The secondary thematic analysis highlighted how the key processes of the critical research model (see Section 4.1) were integral to the functioning of the research group. It also demonstrated that the characteristics of participatory action research, seen as necessary for ensuring trustworthiness (see Section 3.6), were evident in the project. These processes and characteristics are used as themes in reporting this aspect of the case study, and are summarised in figure 4 below.

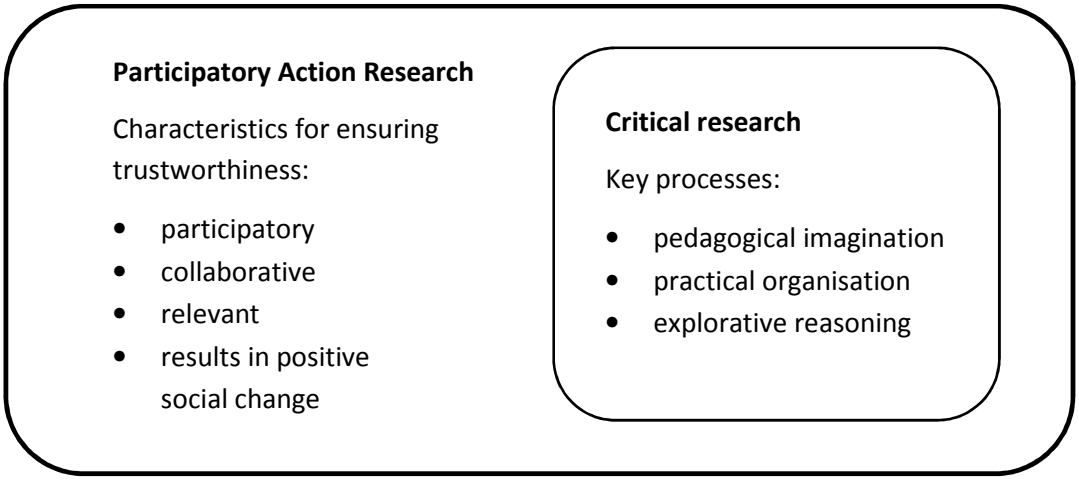

Figure 4: Characteristics and processes of the 'critical research' model

Pedagogical imagination

At the first research group meeting, I presented an initial conceptualisation of teaching mathematics for social justice (see Chapter 2), which proffered alternative approaches to existing practice in schools, and highlighted theories underlying these alternatives. This prompted a great deal of discussion amongst teacher researchers, as they related this conceptualisation to their own practice, and to other theoretical frameworks which they drew upon, such as that of 'participative action' outlined by George. The theories played a

significant role in informing teacher researchers' actions in the classroom, for example Rebecca highlighted how Gutstein and Peterson (2005) inspired her thinking in the design of the Making a Change Project, and Brian referred to Bourdieu's ideas (Bourdieu & Passeron, 1990) in developing his thinking around enabling all students to succeed in school mathematics.

Teacher researchers developed a critical understanding of current practice in schools, by relating the theories to their own experiences. Their discussions focused on apparent contradictions in educational policy. They outlined, for example, how the government highlighted the importance of mathematics education for producing the creative thinkers and problem-solvers needed to generate economic growth, whilst at the same time promoting traditional teaching approaches resulting in a less relevant curriculum and procedural understanding. There was growing consensus that existing practices needed to change, to enable students to engage more with mathematics, to develop the collaborative and independent skills needed to prepare them for an ever-changing world, and to avoid exploitation in future life.

Discussions amongst teacher researchers led to refinements to alternative approaches advocated in the initial conceptualisation. George, for example, proposed breaking down barriers to learning by making the purpose of learning mathematics more explicit to students, whilst Sarah suggested encouraging students to hypothesise and think for themselves, so that they are able to apply what they've learnt, rather than focusing on knowledge without agency.

Practical organisation

There was considerable discussion at the second meeting around identifying constraints on teaching mathematics for social justice. Teacher researchers highlighted how regular tests, combined with close monitoring of students' scores, resulted in pressure to complete schemes of work, so as not to disadvantage students in their own classes. During subsequent meetings, teacher researchers developed a better understanding of these constraints, through discussing their experiences of trying out, and evaluating, classroom activities. They began to identify less immediately obvious constraints, such as their tendency to provide more structured learning for weaker students, making it more difficult for them to develop independent study skills.

Teacher researchers demonstrated a willingness to explore ways of overcoming these constraints, in order to demonstrate the viability of desirable alternatives to existing practice. Anna argued that group work helped to address issues of low confidence and self-esteem amongst students in lower sets, whilst Brian emphasised the importance of linking ideas to

forthcoming topics in the scheme of work. There was discussion around taking advantage of the obligation on schools to address cross-curricular themes, such as SMSC, to justify bringing social justice issues into mathematics lessons.

The second, fourth and sixth meetings focused primarily on the joint planning of classroom activities to be tried during the first, second and third cycles, respectively. During the first cycle, teacher researchers tried out their own ideas, based on discussions at the meetings. However, during the second and third cycles, they agreed to all try the same activities, which facilitated the collaborative planning and evaluation of activities. During the fourth meeting, Anna, Brian and Rebecca presented ideas from three books (Wright, 2004; Gutstein & Peterson, 2005; Coles, et al., 2013), which I had identified as relevant to the project and had asked them to read beforehand. I presented ideas from another two similar books (Smith & Armstrong, 2003; Carel Press, 2013). After lengthy discussion, the group decided on three activities, which were based on ideas from these books.

Explorative reasoning

The third, fifth and seventh meetings focused primarily on evaluating the activities tried out during the first, second and third cycles, respectively. At each meeting, teacher researchers were invited, in turn, to evaluate each activity they tried, by presenting a summary of what they did, whether it achieved its aims, what impact it had on students, and what teacher researchers had learnt from doing it. They used students' responses from the surveys, and notes made in their research journals, to provide evidence to support their evaluations. At the third meeting, Anna also included some video clips of her students reflecting on the Wealth Distribution activity.

Following each presentation, the other teacher researchers were invited to comment and to ask questions. The ensuing discussions enabled teacher researchers to relate the evaluations back to the theories, and to consider the extent to which they resonated with the aims of the project, for example by considering how the activities contributed towards developing student agency. It also led to reflections on the feasibility of the initial conceptualisation, for example during the third meeting, there was discussion of the need for new approaches to be tried several times, and ideas refined, before the benefits became apparent.

Through these evaluations, teacher researchers were able to identify common experiences, which provided further insight into issues relating to teaching mathematics for social justice. A discussion at the fifth meeting, for example, prompted by survey responses from Brian's

students, led to the recognition that higher-attaining students tended to be most resistant to ideas from the project, which all teacher researchers had observed to some extent.

Participatory research

Time was allocated, at the first meeting, for teacher researchers to read and discuss an introductory chapter of a book (Gutstein & Peterson, 2005), which I had identified as relating theories underlying the research to ideas for classroom activities. I also presented, and discussed with teacher researchers, the methodology of participatory action research, and the key processes and features of the critical research model (see Section 4.1).

At the second meeting, I presented my initial findings from the first set of interviews, explaining in detail the methods employed in the thematic analysis of the data (see Section 4.4). Teacher researchers showed considerable interest in this analysis, and in ensuing discussions, were keen to relate the findings back to previous discussions around evidence-informed practice, monitoring and scrutiny. I presented further findings, from my ongoing thematic analysis, during the second set of interviews. In all cases, there was agreement from teacher researchers that my interpretation of the data was accurate.

Teacher researchers embraced the opportunity to decide for themselves what form the collection of data, on the impact of the activities on students, should take. This was discussed in detail at the second meeting, at which they agreed to conduct a survey immediately after each activity, asking students to contrast their general feelings towards mathematics with their thoughts about the activity they had just completed. Teacher researchers reviewed the design of the survey at subsequent meetings, deciding to amend the wording slightly and agreeing a protocol for administering the survey.

Collaborative research

I made clear, at the first meeting, my intention that the research group should be a genuine collaboration between ‘teacher researchers’ and me, as ‘university-based researcher’. I encouraged them to let me know if they thought anything I did during the project conflicted with this aim. I paid particular attention to transparency, keeping teacher researchers informed about agendas for the meetings, and my rationale behind them, and inviting them to suggest their own agenda items. I made clear to them, well in advance of each interview, the nature and purpose of the initial questions I would be asking.

I viewed my role largely as a facilitator of the research group, which included organising the meetings and encouraging the sharing of ideas amongst teacher researchers. After each meeting, I circulated notes summarising the discussions, particularly those relating to the

planning of each activity, checking their accuracy against the audio-recording. I created an on-line folder, accessible to all group members, which made it easier to share resources with each other. During the third cycle, I set up a website that enabled students' presentations, from the Making a Change Project, to be collated and viewed by students in all four schools.

I was careful to maintain a balance between encouraging teacher researchers to take the initiative, and using my expertise to provide necessary inputs to advance the work of the research group. One occasion when I decided a more direct stimulus from me was necessary was after the fourth meeting, when I judged that discussions amongst teacher researchers had not adequately prepared them to try out activities agreed for the second cycle. George and Sarah, for example, had not been able to attend the planning part of the meeting. I therefore included a question, in the second interviews, on how they intended to approach teaching these activities. This was designed to generate further discussion and help them to refine their plans. I also asked them not to try the activities until after these interviews.

During the research group meetings, teacher researchers offered each other encouragement and support, for example by helping to identify what went well during each activity. They also made use of meetings to seek advice and ideas from each other on how to tackle particular problems or constraints they had encountered.

Relevant research

Before the fourth meeting, I circulated a draft version of a conference paper (Wright, 2014), which reported initial findings from the thematic analysis of data from the first three meetings and the first interviews, relating these findings back to the theories underlying the project. The paper prompted lengthy discussion at the meeting which, according to teacher researchers, helped them to make sense of their own developing practice, in relation to the theories. Brian was particularly interested in how the notion of schools reproducing inequity through privileging students with cultural capital (Bourdieu & Passeron, 1990) related to an apparent contradiction, between his desire to promote critical thinking amongst students, and the need for them to comply with certain social norms in order to achieve success in mathematics.

By the end of the project, teacher researchers recognised that, by engaging with theories when planning and evaluating classroom activities, they had begun to appreciate the relevance of these theories to their own developing practice. In her third interview, Anna described how she had progressively gained a clearer understanding of how the theories presented at the first meeting played out in the classroom. At the final meeting, George maintained that the alternative vision of teaching mathematics, offered by the project, was essential for

challenging existing practice, given its many shortcomings highlighted by teacher researchers during discussions.

Research resulting in positive social change

All teacher researchers believed the project had a significant impact on their classroom practice, raising levels of student engagement with, and enjoyment of, mathematics. The activities had helped students to develop their understanding of social justice issues, their appreciation of how mathematics related to real life and how it could be used to argue for change in the world. Anna described how she had become more confident to teach mathematics lessons based on broader objectives, whilst Rebecca outlined how she had started to use more meaningful contexts routinely in her lessons. Involvement in the project enabled teacher researchers to develop their practice in directions which resonated with their teacher identities. In the third interview, for example, Brian described how the aims of the project had become the 'motivation and engine' behind his teaching.

Teacher researchers appeared to strengthen their belief that existing practice needed to be changed and began to recognise the nature of school mathematics as a social justice issue. At the fourth meeting, Anna argued that students were exposed to so many negative messages related to learning mathematics, and if teachers didn't challenge these, they were in effect endorsing them. Brian increasingly challenged the notion that education was a meritocracy, and argued that, unless teachers encouraged students to develop critical understanding, public opinion would continue to be dictated by those with the loudest voice, rather than the strongest argument.

The development of teacher researchers' self-efficacy was demonstrated by the impact their involvement in the project had, not only on their own practice, but on the practice across their departments. Anna highlighted, in the final meeting, how encouraging colleagues to try out ideas from the project had led to a greater appreciation within her department of the need to provide meaningful context, and a willingness to replace some tests with student presentations. She was determined to try out ideas from the project in her new school, and had considered strategies for doing this in a situation where there was likely to be less flexibility.

The strongest indication that teacher researchers believed that they had generated new and relevant knowledge was the enthusiasm they demonstrated for disseminating ideas and resources from the project to other teachers within their own schools and further afield. There was considerable discussion at the final meeting about how they could share their experiences

from the project more widely, and promote the research group as an effective model of sustained professional development.

5.8 Concluding remarks

In this chapter, I have reported the findings from the research project in the form of a case study, which tells the stories of individual teacher researchers' involvement with the project, the development of the research group as a whole and the processes and characteristics of the research model. In the next chapter, in line with the framework for analysing data (see Section 3.7), I summarise how the research findings were related back to the theories underlying the research, in order to generate new theories and knowledge. The processes of analysing the data, reporting the findings and relating the findings back to the theories, were closely integrated in an overall structure that was iterative in nature. For example, the reporting of initial findings to teacher researchers, and how these related to theories, generated additional data, which were fed back into the thematic analysis. However, for clarity, I have reported separately the two processes of summarising the research findings (in Chapter 5) and relating these back to the theories (in Chapter 6).

Chapter 6: Relating findings to theories

In this chapter, I relate the research findings back to the theories, which inform my pedagogical and methodological positioning (see Chapters 2 and 3 respectively), in order to generate new theories and knowledge relating to the focus of my research. In Sections 6.1 to 6.4, I use the four emerging themes (see Section 5.1) as a structure for summarising how the findings from the main thematic analysis of the data relate back to the theories. In Section 6.5, I relate the findings from the secondary thematic analysis back to the theories, focusing on the key processes and characteristics of the research model (see Section 5.7).

6.1 Changing epistemologies of mathematics

Ernest (1991) argues that there is a strong association between a fallibilist view of mathematics and a public educator ideology of mathematics education, with teachers' epistemologies of mathematics exerting a strong influence on their favoured teaching approaches. However, findings from this research project suggest that relationships between epistemologies of mathematics, ideologies of mathematics education and favoured teaching approaches are less clear-cut.

All five teacher researchers, at the start of the project, professed a commitment to teaching mathematics for social justice, which in many ways is analogous to Ernest's public educator ideology. This commitment appeared to have been an important consideration in their decisions to become teachers. Yet only after becoming teachers did they begin to consider, for the first time, questions about the nature and purpose of mathematics. They professed to having been in a contented state of ignorance in this regard throughout their schooling, and for Rebecca, throughout her mathematics degree. Anna described a sense of shock on realising that she had started training to be a teacher without previously giving these questions any serious consideration. Even after becoming teachers, they acknowledged giving much less thought to the nature of mathematics than they did during the project. Over the course of the project, teacher researchers' views of mathematics changed considerably, as they began to more fully appreciate its socially-constructed and value-laden nature. They experienced a shift in their epistemologies towards the fallibilist end of the spectrum, albeit from different starting points, and their views of mathematics remain very much under review and revision. This would suggest that their epistemologies of mathematics were not a significant factor in the development of their public educator ideologies.

Anna and Brian attributed their desire to address inequity to ideological positions arising from their experiences outside of school. Rebecca's motivation for joining the project was a concern

for her students, who she recognised were not all willing to study mathematics just for the sake of it, as she had done. She acknowledged having a very abstract mathematical outlook and appeared to relish the opportunity to challenge her own thinking. She was intrigued by a project that she knew very little about, but which she believed would question the fundamental principles underlying how she thought about mathematics. Despite her close relationship with mathematics, her concern for the welfare of her students was strong enough to make her reappraise this relationship. This suggests that a public educator ideology, manifested through a concern for issues of equity and social justice, made teacher researchers more predisposed towards questioning the nature of mathematics, and hence adopting a more fallibilist epistemology. This represents a reversal of Ernest's (1991) proposed causal relationship between epistemologies of mathematics and philosophies of education. The findings concur with Gates' (2006) view, that teachers' underlying ideologies, dependent upon their previous experiences, largely determine their pedagogical beliefs.

At school, all five teacher researchers had been very successful learners of mathematics. They had all been happy to study the subject without considering its relevance or purpose, and had accepted its abstract nature, and the instrumental way in which it was taught, without question. Anna and Brian had chosen to teach mathematics, despite studying for psychology and geography degrees, because they recognised its importance as a gatekeeper qualification. However, they had never previously considered why it occupied such a prominent place in the curriculum. Bourdieu and Passeron's (1990) notion of primary dispositions towards learning suggests that teachers are predisposed towards adopting the same teaching approaches they experienced themselves as learners, especially if they experience success. Teacher researchers' enthusiasm for progressive pedagogies, which they were exposed to during their initial teacher education, indicates that they have shown a willingness to disrupt this tendency, suggesting a disposition towards questioning the way they were taught themselves.

Despite their willingness to question the nature of mathematics and the way they were taught, teacher researchers, as school students, were not given opportunities to do so. My own experiences suggest this is not uncommon. Boaler (2009) highlights how questions such as 'What is mathematics?' and 'Why do we learn it?' are rarely considered by learners. Skovsmose (2011) argues that, through reflecting on the nature of mathematics, learners might develop a more critical understanding of mathematics, and be less inclined to believe the common myth that mathematics is a neutral and value-free subject. However, there are many people who, having given a great deal of consideration to its nature, still persist with an absolutist view of mathematics, and see progressive pedagogies as a threat to maintaining its

rigour. These include leading 'traditionalists' in the Math Wars that raged in the US from the 1990's (Wright, 2012).

Over the course of the project, Anna, Brian and Rebecca transformed their thinking around teaching mathematics for social justice. They shifted from viewing mathematics as a medium for raising awareness of social justice issues, which might in turn enrich lessons, towards appreciating the crucial link between teaching mathematics and addressing social justice concerns. They observed that, by making mathematics more meaningful, and by applying skills to real situations rather than learning them in isolation, the activities helped students develop a deeper understanding of specific mathematical concepts. They began to appreciate the desirability of linking social justice issues closely to mathematical skills, at an appropriate level of challenge, within the scheme of work. This resonates with Freire's (1974) notion of developing genuine understanding, through promoting a critical awareness amongst learners, of their own situations, and how these relate to their studies.

Anna, Brian and Rebecca concurred that students should be encouraged to reflect upon their views of mathematics and the rationale behind alternative approaches proposed for teaching the subject. There was evidence from the project that, through engaging with the activities, students' attitudes towards mathematics changed significantly, as they developed a growing appreciation of its relevance and purpose. However, there was initial reluctance amongst some students to accept that applying mathematical methods to social justice issues counted as 'real' mathematics, perhaps reflecting the extent to which their views had been shaped by previous teaching. Anna warned against shying away from challenging existing views of mathematics since, by doing so, teachers would be reinforcing the negative image of mathematics students commonly encountered elsewhere. Brian was the only teacher researcher to try activities primarily aimed at challenging students' views of mathematics, although he was disappointed that attitudes towards mathematical ability remained largely unchanged. Despite his attempts to encourage students to view success in mathematics as down to effort, rather than innate ability, highlighted by Askew et al. (1997) as an important factor in raising mathematical achievement, most students retained a strongly-entrenched belief in the latter.

6.2 Developing student agency

All teacher researchers observed a significant increase in students' engagement with mathematics as a result of the activities tried out through the project, particularly amongst students who previously lacked confidence, or were poorly behaved in lessons. Feedback from students suggested this was because they could more easily see its purpose and relevance to their current and future lives. This contrasted with students' general views of mathematics, which they often described as being boring and pointless, echoing the 'quiet disaffection' Nardi and Steward (2003) observed amongst mathematics students.

Involvement in the project reaffirmed teacher researchers' strong beliefs in progressive pedagogies, including open-ended, collaborative, problem-solving and student-led approaches to learning. Such teaching approaches were frequently cited by students as reasons why they enjoyed the activities more than routine mathematics lessons. Over the course of the project, teacher researchers increasingly recognised the adoption of progressive pedagogies as a social justice concern, for example Brian, who became familiar with Boaler's (2008) research on complex instruction, began to advocate group work as promoting equity in the classroom. This concurs with Gutstein's (2006) assertion that progressive pedagogies are a necessary pre-condition for teaching mathematics for social justice.

The idea of developing students' agency had a significant impact on the teacher researcher's thinking and practice, although it was sometimes difficult to separate this from other aspects of the initial conceptualisation of teaching mathematics for social justice. Having given it little thought before the project, developing students' agency became an important focus for teacher researchers during the third cycle. They demonstrated increasing enthusiasm for three aspects relating to student agency, described by Skovsmose (2011) as reflecting 'with' mathematics, reflecting 'through' mathematics and reflecting 'on' mathematics.

Reflecting 'with' mathematics was evident in many of the activities, which aimed to use mathematics to raise awareness of social justice issues, including Fair Trade, inequality and voting systems (see Appendix 4). Being able to understand and explain the increasingly complex world around them, was seen as essential for students to be able to participate fully in society and avoid being exploited. The Making a Change Project developed these ideas further, by helping students to understand a particular social justice issue of their choice, then encouraging them to use mathematics to argue for a change they would like to see made. Using mathematics to better understand a relevant issue, whilst simultaneously making the subject more meaningful and developing mathematical understanding, echoes Gutstein's

(2006) aim of encouraging students to 'read and write the world with mathematics'. Whilst students didn't experience their suggestions for changes being acted upon, as happened with some of Gutstein's (2007) 'real world projects', teacher researchers maintained that growing appreciation of how to go about arguing for a change reflected students' developing agency.

Teacher researchers encouraged students to reflect 'through' mathematics by enabling them to make decisions relating to their own learning. As part of the Making a Change Project, students were encouraged to choose issues they considered relevant, and which they felt passionately about. They were then encouraged to work independently, in groups, researching their chosen issue, designing questionnaires to collect data, and developing their own argument to present to others. Feedback indicated that students welcomed opportunities to decide for themselves which mathematical procedures to apply, and to present their own mathematical arguments and interpretations. Working in this way also helped students appreciate the rationale and the need for learning particular mathematics skills.

One issue that arose, in trying out activities that promoted independent, student-led learning, was that some students found it more difficult than others to cope with the additional responsibility this entailed. Teacher researchers noticed how these students tended to be the same students who found it difficult to comply with the expected norms of behaviour, and were commonly placed in lower sets. In other words, they lacked the 'cultural capital' (Bourdieu & Passeron, 1990) required to be successful in mathematics. Bernstein (2000) highlights how students from disadvantaged backgrounds are often further disadvantaged by less structured teaching approaches, in which the 'rules of the game' are even more unclear. Their lack of cultural capital means they are less able to recognise what is expected of them and turn this into realising success. He stresses, however, that this does not imply teachers should avoid less structured teaching approaches, but rather that they should make the 'rules of the game' more explicit and transparent.

Through his involvement with the project, Brian recognised the importance of helping students to identify and develop the social skills required to be successful in mathematics. He described how this became increasingly influential in the development of his classroom practice. There was growing appreciation, amongst teacher researchers, of the challenge involved in helping all students develop independent learning skills, particularly in classes, predominantly lower sets, where low confidence and poor behaviour meant teachers were inclined to provide more structure and support for students. However, there was broad agreement on the importance of overcoming this challenge in order to address issues of equity and social justice within mathematics classrooms. Anna described how, by maintaining a balance between providing

appropriate support and guidance, and encouraging students to work independently, students in a bottom set outperformed many students in higher sets, in presenting strong and coherent arguments at the end of the Making a Change Project. Brian highlighted how he achieved a similar balance when, during his starter to the same activity, he enabled students to appreciate for themselves the value of using mathematics to support an argument, by providing contrasting examples.

Teacher researchers exhibited growing beliefs that students should be encouraged to reflect on the nature (see Section 6.1) and position of mathematics. Over the course of the project, Anna and Brian, in particular, adopted a more critical view of school mathematics and a growing conviction that students should appreciate that success in mathematics was not based entirely on merit. They argued that, by understanding their own situation and how they might be disadvantaged in learning mathematics, students were less likely to become complicit in their own exploitation, i.e. they might avoid what Bourdieu and Passeron (1990) describe as 'symbolic violence'.

Another issue relating to student agency was the danger of 'agency without enlightenment', as George called it. He argued that agency, on its own, does not necessarily bring about change for the better, and a degree of open-mindedness and sensitivity towards social justice issues is also required. This raises the question of whether teaching mathematics for social justice should encourage students to adopt particular opinions about issues or encourage them to engage with the issues and form their own opinions. This dilemma became apparent during the Fair Trade activity (see Appendix 4) when teacher researchers became concerned when several students concluded that Fair Trade wasn't actually 'fair' after all, and therefore they shouldn't buy Fair Trade products in future. Such views might be considered to reflect the argument that the whole system of global trade is grossly unfair, and that Fair Trade, whilst slightly less unfair, merely provides legitimacy to this system. The crucial outcome here was that, by appreciating how farmers receive only 4 per cent of the price paid for a Fair Trade bar of chocolate, albeit eight times more than for other bars, students demonstrated a deeper understanding of Fair Trade, enabling them to make better informed decisions in future.

Freire (1974) would argue that teacher researchers, by exposing students to activities such as those described above, have enabled them to move, from a position of 'naïve' awareness, towards one of 'critical' awareness, through genuine and authentic education based on dialogue, rather than transmission of knowledge. This perspective on education resonates with the critical methodology underlying the research project, in which an assumption is made that the current situation should not be taken as given. Over the course of the project, teacher

researchers strengthened their belief that the existing system of mathematics education is unfair and unjust, and needs to be changed. They articulated the view that education can play a critical role in challenging inequities and injustices within society, thus creating a better world. From this critical perspective, the teacher researchers' roles, and my role as university-based researcher, resonate with a 'radical' desire to empower others to transform their world by becoming more aware of, and reflecting upon, their own situation, rather than superimposing solutions upon them (Freire, 1974).

6.3 Collaborative nature of research group

Teacher researchers described how working collaboratively in a group, sharing ideas and experiences with colleagues from other schools, and jointly planning, teaching and evaluating activities, had a significant impact on their thinking and classroom practice, as well as being a thoroughly enjoyable experience. The mutual support and sense of common purpose within the research group encouraged them to take risks, overcome constraints, and try out alternative teaching approaches and ideas. There was general acknowledgement that the role I played in the group was crucial in promoting collaboration and facilitating the sharing of ideas, and that drawing on my theoretical knowledge helped teacher researchers to challenge their previous views and assumptions about teaching mathematics. Jaworski (2006) highlights how such a role is essential for the establishment of a 'community of inquiry' in which the cultivation of critical understanding, and meta-cognitive awareness of the research participants, serves to challenge, rather than perpetuate, the status quo.

Teacher researchers described how the initial conceptualisation of teaching mathematics for social justice, that I presented, and the discussions which followed, helped them to develop their thinking and to broaden their perspectives. This resonates with the aims of the participatory action research models (Torrance & Pryor, 2001; Skovsmose & Borba, 2004; Planas & Civil, 2009) which informed the research design of this project. The initial input of the researcher is seen as essential for encouraging participants to critically appraise their own practice in relation to the theories, in order to bring about effective change. Teacher researchers described how the theories informed their planning and evaluation of the activities, which in turn, through reflection, helped them to make better sense of the theories. This suggests the project facilitated a genuine interaction between theory and practice (Torrance, 2004), a process Freire (1974) refers to as 'praxis', leading to a better understanding of their situations amongst teacher researchers (Reason, 1994).

Teacher researchers demonstrated their willingness to engage in research, through reading, discussing and presenting to each other, research articles relating to the theories underlying the project. They showed an interest in the research design and data analysis methods employed in the project, which I regularly shared with the group. They took responsibility for the design of the student survey, which they administered to students, using the responses as part of their presentations at research group meetings. Over the course of the project, they recognised that their engagement in research had led to a greater understanding of research processes. Their experiences of the project were in stark contrast to their previous limited, or complete lack of, engagement with research. The apparent willingness of teacher researchers to engage in this research project challenges the notion that teachers are naturally resistant to change (Sebba, 2004). Instead, it suggests that teachers' lack of engagement with research can be attributed to the constraints they face in the classroom, which can be overcome through the adoption of collaborative and participatory research methodologies.

Teacher researchers described the project as providing much more effective professional development than they had otherwise experienced. They attributed this to its sustained nature, its focus on relating theories to classroom practice, and the positive impact it had on them and their students. They described professional development generally available to new teachers as inadequate and ineffective, with very few opportunities to work collaboratively with teachers in other schools. Leat et al. (2014) highlight how engagement 'in' research, as well as 'with' research, can be a very positive learning experience for teachers, leading to them adopting a more critical stance on issues such as the curriculum. Whilst they highlight how this can potentially bring teachers into conflict with colleagues and managers, this was not the experience of the teacher researchers in this project.

Teacher researchers described how the project gave them the confidence to encourage others in their schools, and more widely, to try out the ideas, and to influence departmental policy. They experienced a wave of interest in the project, with ideas adopted by other teachers in their schools, as news spread about the positive impact they had on students. There was enthusiasm amongst teacher researchers for collating the ideas in a resource that could be shared with other teachers who wished to develop their practice in a similar direction. Thus the work of the research group, comprising teacher researchers from a number of schools, stimulated further interest in, and inquiry into, teaching mathematics for social justice within the departments and schools in which the teacher researchers were based. This echoes Jackson and Timperley's (2007) findings that 'networked learning communities' can build upon and promote collaborative inquiry and learning cultures across schools.

6.4 Dominant discourses on ability and attainment

Teacher researchers acknowledged how, at the start of the project, their perspectives on teaching mathematics for social justice focused mainly on raising attainment and aspirations of students from lower socio-economic backgrounds. They had chosen to teach in schools with a large proportion of students from such backgrounds and were aware of the gatekeeper role that mathematics played. They hoped therefore to challenge inequity by raising the mathematical attainment of their own students. Bourdieu and Passeron (1990), however, warn that counter-examples, which demonstrate how some students from lower socio-economic backgrounds buck the trend and attain success, merely give legitimacy to the education system, perpetuating the myth that it is based on a meritocracy, and disguising its true purpose of reproducing inequities and hierarchies in society.

Over the course of the project, teacher researchers' perspectives broadened as they began to appreciate how structural inequities disadvantage students from lower socio-economic backgrounds. They developed greater awareness of the constraints they faced as mathematics teachers, i.e. the exam-oriented curriculum, pressure to get through the scheme of work, generally high workloads, high levels of monitoring of students' progress and scrutiny of their own performance. They recognised how these constraints encouraged risk-averse teaching and a focus on teaching mathematical procedures in isolation, in which it is relatively easy to measure progress. They realised that this approach to learning, referred to by Skovsmose (2011) as the 'exercise paradigm', resulted in students adopting generally negative attitudes towards mathematics, describing it in their feedback as boring and irrelevant. They believed that being part of the research group helped them overcome many of these constraints and to develop alternative teaching approaches. However, they continued to navigate around these constraints with caution, for example Anna steered clear of trying out activities with her Year 11 examination class.

Teacher researchers began to realise that, as long as mathematics was generally perceived as boring and lacking in meaning and purpose, students who appreciated the value of learning mathematics for its own sake would be more motivated to learn. They noticed that those students unable to conform to the expected norms of behaviour were the same students who lacked the skills necessary to be successful in mathematics. When adopting progressive teaching approaches, and making mathematics more meaningful and purposeful, teacher researchers observed the most significant increases in engagement and understanding amongst students who had previously been disaffected, often those in lower sets. Ironically, poor behaviour, more common amongst disaffected students and in lower sets, was cited as

one of the biggest disincentives to adopting progressive teaching approaches. Thus, through their own lack of motivation and poor behaviour, students lacking in cultural capital are complicit in their own failure in mathematics. Bourdieu and Passeron (1990) refer to this process as 'symbolic violence'. They argue that children from middle-class families are more likely to acquire cultural capital, with their experiences from a very young age instilling in them a disposition towards learning. Hence this symbolic violence contributes towards the reproduction of inequities. Teacher researchers noted that the least enthusiasm for alternative teaching approaches generally came from the highest attaining students, whose interests were best served by maintaining the status quo.

Teacher researchers drew attention to the apparent contradiction between the desirability of promoting critical and conceptual understanding of mathematics, and ensuring students develop the cultural capital they need to succeed under the current assessment regime. Brian highlighted how, in order to 'play the game' (Bourdieu & Passeron, 1990), students need to demonstrate compliance by adopting the social norms they perceive as necessary for success. Since achievement in mathematics continues to be associated with procedural fluency and instrumental understanding, such a conflict is therefore likely to persist. George claimed that schools spent much of their time stifling the creativity of students and promoting compliance, a process described by Skovsmose (2011) as cultivating a 'prescription readiness'. This resonates with Bernstein's (2000) argument that pedagogy is influenced primarily by a need to control rather than to learn, with the 'regulative discourse', i.e. the 'rules of social order', always dominant over the 'instructional discourse', i.e. the 'rules of discursive order'.

Within the current school system, there is a delicate balance to be maintained, by mathematics teachers wishing to promote social justice, between promoting critical understanding and developing cultural capital. Teacher researchers recognised the importance of promoting discursive and open-ended learning with lower-attaining students, whilst providing the additional support required for them to develop the skills needed for these approaches. Brian argued that making the current situation more explicit to students, for example the need to learn seemingly irrelevant procedures in order to succeed, would help to achieve this balance by promoting purpose and agency at the same time. Teacher researchers recognised that the success of such strategies depended upon building relationships of trust with students, which were seen as necessary for students to buy into the project's alternative views of mathematics and teaching approaches. This concurs with Bernstein's (2000) view that an education based on principles of social justice requires teachers to develop 'relational'

authority, based on negotiation with students, rather than 'positional' authority, which exploits power relations existing between teachers and students.

Brian was uncomfortable with some of the theories, which he considered to be unduly pessimistic, in particular Bourdieu and Passeron's (1990) argument that teachers wishing to challenge the current situation, merely by becoming teachers, give legitimacy to the school system. He argued that people who do nothing about an unjust situation are just as culpable as those who actively promote it, echoing Freire's (1972) assertion that to do nothing about oppression is to side with the oppressors. Teacher researchers generally believed they had agency in challenging the current situation, viewing education as a means of promoting a more equitable and just society. Anna suggested it was the responsibility of teachers to provide the cultural capital that some students lacked, as they weren't likely to get it from anywhere else.

At the start of the project, it was apparent that none of the teacher researchers had questioned the legitimacy of setting, reflecting the dominant discourse in schools in which setting is commonly accepted as the norm in mathematics classrooms (Hardy, 2004) despite a lack of evidence to suggest it is effective (Winbourne, 2009). Exploring mixed-ability teaching was beyond the scope of the project, since this was not something teacher researchers had any control over. Despite this, over the course of the project, they adopted an increasingly critical stance towards setting, blaming it for widening the achievement gap between the lowest and highest-attaining students. These sentiments suggest they had developed a view of mathematical ability as incremental rather than fixed, contrary to prevailing attitudes amongst mathematics teachers (Morgan, 2009). By the end of the project, several teacher researchers had developed a strong preference for mixed-ability teaching over setting, for example Anna stated she would like to introduce mixed-ability mathematics classes, if she achieved her ambition of becoming a head of department.

Through their involvement in the project, teacher researchers demonstrated that, by critically reflecting on their practice, in relation to the theories underlying the project, they were able to develop views contrary to the dominant discourses on ability and attainment. This highlights the potential of the critical research model for transforming discourses in schools. My own experiences of teaching in various Inner London comprehensive schools from 1987, and visiting similar schools more recently as a teacher educator, suggest that it is possible for dominant discourses to change over time, and to vary from one school to another.

6.5 Processes and characteristics of the research model

The secondary thematic analysis demonstrated that the key processes of the critical research model (Skovsmose & Borba, 2004), i.e. 'pedagogical imagination', 'practical organisation' and 'explorative reasoning' (see Section 4.1), were used effectively in order to achieve successful outcomes for the project. It also demonstrated the extent to which the desirable characteristics of participatory action research, i.e. participatory, collaborative, relevant, resulting in positive social change (see Section 3.6), were evident in the research project.

Pedagogical imagination was apparent in the way teacher researchers engaged with the theoretical ideas that I presented, demonstrating a willingness to use these to interrogate and challenge their existing practice and thinking. By relating these theories to their own classroom practice, and discussing the implications with others in the research group, they developed a more critical understanding of the current situation, strengthened their beliefs that this situation needed to change, and generated alternative ideas to try out in the classroom.

Through sharing experiences of teaching, and relating these to the theories, teacher researchers increasingly recognised and developed an awareness of the constraints they faced, when attempting to put some of these alternative ideas into practice. Practical organisation was apparent in the way they were able to navigate these constraints through collaborative planning, sharing ideas and providing mutual encouragement and support. Through this cooperation, they were able to design a number of classroom activities, aimed at addressing various aspects of the initial conceptualisation of teaching mathematics for social justice, and taking account of the constraints.

The teacher researchers demonstrated explorative reasoning through presenting detailed evaluations of each activity they tried, drawing on evidence from the student surveys and their research journals. Each presentation generated a discussion amongst the group, which led to a deepening understanding of the current situation and the outcomes of each classroom intervention. This resulted in teacher researchers refining the strategies they used for subsequent activities, and drawing conclusions regarding the feasibility of the initial conceptualisation of teaching mathematics for social justice.

The adoption of the critical research model ensured that the research project was genuinely participatory and collaborative in nature, as advocated by proponents of participatory action research (Brydon-Miller, et al., 2003; Atweh, 2004). Teacher researchers periodically discussed the research methods and data analysis methods employed and were involved in important

elements of the design. They played an active role in the development of the research project, for example by collectively deciding which activities to try out in the classroom. My relationship with the teacher researchers was based on openness and transparency and I played very much a facilitative role. They described the mutually supportive character of the research group as a key factor in the success of the project.

The knowledge that was generated, through developing understanding of the relationship between theory and practice, and through my reporting of the initial findings, was viewed by teacher researchers as being highly relevant to their own situations (Reason, 1994; Levin, 2012). They described the significant impact of the project on their own thinking and classroom practice, and the wider impact it was beginning to have across each school. They also highlighted the impact of the project on students' engagement with mathematics, their appreciation of its purpose and relevance, and their development of mathematical understanding. Teacher researchers described how the project strengthened their commitment to challenge the current situation, and advanced their agency and self-efficacy in taking control over the direction and extent of changes in their classroom practice.

The short reports, written by Anna, Brian and Rebecca at the end of the project (included as Appendices 6, 7 and 8), provide compelling evidence of the perceived benefits of the project for the teacher researchers and the students involved. They describe the positive social change that occurred within teacher researchers' classrooms, proposed by Brydon-Miller et al. (2003) as a key indicator of the credibility of the research processes. The reports also reinforce the credibility of my research findings, as the final thoughts of the teacher researchers appear to concur with my findings from the thematic analysis of the data, for example they articulate clearly the significant impact of the project on teacher researchers' classroom practice and thinking.

6.6 Concluding remarks

This chapter has focused on relating the research findings back to the theories which informed my initial conceptualisation of teaching mathematics for social justice and my research methodology (see Chapters 2 and 3). By 'plugging in' the data to these theories (see Section 3.7), new analytical questions have emerged. A discussion of these questions has enabled me to make sense of the findings and to generate new theories and knowledge. These are summarised in the concluding chapter, in which I draw out implications for mathematics teaching, teacher education and research.

Chapter 7: Conclusions and implications

In this chapter, I draw conclusions based on the discussions in the previous chapter, in which I related the findings back to the theories underlying the research. I begin by reviewing the aims of the research and discussing how the main and subsidiary research questions have been addressed (Section 7.1). I then draw out implications of the research findings, which I report in three sections: implications for mathematics teaching (Section 7.2), implications for the development of mathematics teachers (Section 7.3), and implications for mathematics education research (Section 7.4). Since my research methodology makes little distinction between teachers and researchers, I anticipate that both groups, and others involved in mathematics education, will be interested in the research findings, particularly those who share my belief that the existing system of mathematics education is unjust and needs changing. I go on to argue how this research study makes a substantial and original contribution to knowledge and understanding in the field (Section 7.5). Finally, I provide a critical evaluation and reflection of the research project (Section 7.6).

7.1 A review of the aims of the research

The primary research question was:

> *How can a commitment towards 'education for social justice' amongst mathematics teachers be translated into pedagogy and classroom practices which promote such aims?*

The critical research design provided an effective model which enabled teacher researchers to collaboratively plan, teach and evaluate a series of activities. These were aimed at putting into practice aspects of an initial conceptualisation of teaching mathematics for social justice, which I presented. Through evaluating these activities, in relation to the initial conceptualisation, teacher researchers effected significant changes in their practice.

These changes included developing progressive pedagogies, establishing stronger links between awareness of social justice issues and mathematical understanding, enhancing student agency, and making mathematics more meaningful and relevant to students' lives. This addresses the first subsidiary research question:

> *What might mathematics education for social justice look like in practice in the secondary mathematics classroom?*

Through these alternative approaches, students developed more positive attitudes towards mathematics, significantly increasing their levels of engagement with, and enjoyment of, the subject, and beginning to appreciate its purpose. This was particularly noticeable for those

students previously disaffected or alienated from mathematics. This addresses the second subsidiary research question:

> *What impact might teaching mathematics for social justice have on students?*

These outcomes informed the conclusions drawn from the research project, relating to implications for mathematics teaching, which are further elaborated in Section 7.2.

As well as impacting on their classroom practice, the critical research design also provided an effective model for developing teacher researchers' thinking around teaching mathematics for social justice. The collaborative and participatory nature of the research group, and the engagement with theories underlying the aims of the project, contributed to teacher researchers reflecting critically on, and reviewing, their epistemologies of mathematics and their existing classroom practice. This addresses the third subsidiary research question:

> *How can an awareness and understanding of issues of social justice be developed amongst mathematics teachers?*

These outcomes informed the conclusions drawn from the research project, relating to implications for the development of mathematics teachers, which are further elaborated in Section 7.3.

The mutual support provided by the research group enabled teacher researchers to overcome the constraints they faced in translating ideas into practice, including excessive focus on exams and completing schemes of work, and high levels of monitoring and scrutiny. The positive impact of the project on teacher researchers' classroom practice, and more widely across their departments, highlighted the potential of the critical research model for promoting the aims of the project on a wider scale. This addresses the last subsidiary research question:

> *What are the opportunities and constraints for teaching mathematics for social justice in schools in England?*

These outcomes informed the conclusions drawn from the research project, relating to implications for mathematics education research, which are further elaborated in Section 7.4.

7.2 Implications for mathematics teaching

The initial conceptualisation of teaching mathematics for social justice (see Chapter 2) provided a valuable starting point for generating ideas for classroom activities. The research project demonstrated how mathematics can be used as a means of helping students to better understand issues of social justice, thus contributing towards building a better society, perceived by many teachers as an essential aim of education in general. At the same time, exploring such issues can make a significant contribution towards developing mathematical understanding. Deeper understanding of both the social justice issues and mathematical concepts is more likely to develop when there is a meaningful link between the two, as there is, for example, between Fair Trade and percentages.

Progressive pedagogies, based on discursive, collaborative, problem-solving approaches to learning, are a necessary, but not sufficient, condition for teaching mathematics for social justice. The project outlined how such pedagogies can be built upon, in order to promote student agency, by encouraging students to make their own decisions about which issues to explore and which mathematical procedures to apply. Through developing and presenting their own arguments, students appreciate how mathematics can be used, not only to better understand a situation, but also to argue for a change. Unfortunately, progressive pedagogies are discouraged by the existing mathematics curriculum, which cultivates unnecessary concern amongst the most highly attaining students that such approaches do not prepare them adequately for exams. In order to allow students to become independent and critical mathematics thinkers, consideration should be given to the nature of assessment currently used to measure success in school mathematics.

Making mathematics more meaningful, and more relevant to students' real life experiences, significantly raises levels of engagement with, and enjoyment of, the subject. Students begin to see it as having purpose and being more applicable to their future lives. Increasing engagement is particularly noticeable amongst lower-attaining and disaffected students, showing the potential of these approaches for closing the attainment gap between higher and lower-attaining students. These strategies can help reduce the number of learners who become alienated from mathematics and, in turn, foster a more positive attitude towards mathematics amongst the general public.

Initial resistance, apparent amongst some higher-attaining students, to the alternative teaching approaches developed in this project, can be attributed to the perceived threat posed to their continued success in the subject. It should be noted, however, that many higher-

attaining students choose not to continue studying the subject once it becomes non-compulsory. Teaching mathematics for social justice has the potential to encourage more higher-attaining students to study mathematics at advanced levels, by convincing them that they can continue to be successful in a subject, which has been made more engaging, relevant and meaningful.

The project provided insight into the extent to which cultural capital places some students at an advantage over others in mathematics classrooms. In order to challenge structural inequities within schools, the 'rules of the game' need to be made more explicit, to enable all students to develop the learning and social skills necessary for achieving success in mathematics. Progressive pedagogies should be adopted with all students, but particularly those lacking cultural capital, who are less predisposed towards, but have the most to gain from, discursive and open-ended approaches to learning. However, these students require careful induction into how to engage with such approaches more effectively. A fine balance must be maintained between providing appropriate guidance and support to students, and promoting independent learning, mathematical thinking and critical understanding.

The project highlighted how rare it is for students to be asked to reflect on the nature of mathematics, despite its privileged position in the school curriculum. Teachers of mathematics, even those who have studied the subject to degree level, are rarely encouraged to reflect on questions about the nature and position of mathematics before training to become teachers. This allows myths, such as mathematics being a value-free and neutral subject, and mathematical success being dependent upon innate ability, to be perpetuated from generation to generation, resulting in the continued alienation of a large proportion of society from the subject. Such myths need to be challenged by encouraging students to reflect on the nature of mathematics, and their own feelings towards the subject, by making this an explicit part of the mathematics curriculum.

Students should be made aware that success in mathematics is not always achieved on merit, for example, by encouraging them to explore data showing how mathematical attainment is associated with parental income. Enabling students, who are disadvantaged in learning mathematics, to better understand their own situation, can help them to overcome barriers to achieving success in a subject which, to a large extent, determines their future life opportunities. The project highlighted the importance of establishing relationships of trust between teacher and students, based on fostering mutual respect rather than exploiting positions of authority, before engaging in such discussions.

7.3 Implications for the development of mathematics teachers

The critical research design (see Chapter 4), on which this project was based, presents a highly effective model of professional development for teachers. The project demonstrated that the critical aspect of the design, which challenges the current situation and focuses on relating theories to practice, can have considerable impact on the development of teachers' thinking and classroom practice. The project proved attractive and intriguing to teachers, for whom addressing issues of equity and social justice were important factors in becoming a teacher. It demonstrated how, for such teachers, the critical research model enables them to re-engage with aims, which they are likely to have lost sight of in their first few years of teaching, thus becoming more comfortable with their evolving identities as teachers. High levels of interest shown by colleagues of teacher researchers suggest that many mathematics teachers in schools may think in this way.

The project highlighted the importance of teachers engaging with research as part of their professional development, and the crucial role played by an external partner in facilitating this and identifying relevant research. The initial conceptualisation of teaching mathematics for social justice (see Chapter 2) provides a valuable starting point for generating discussion amongst teachers on how theories relate to existing practice across schools. Critically appraising their own practice, in relation to these theories, ensures teachers try out activities in the classroom, which are informed by previous research findings. Through reflecting on the outcomes of these activities, in relation to the theories, teachers develop a more profound understanding of their own practice and how this relates to existing practice across schools. In so doing, they become more critically aware of the structural causes of inequity and injustice in mathematics education, including dominant discourses on ability and the privileging of cultural capital.

The project highlighted how mathematics teachers, with a strong concern for social justice, are favourably disposed towards reviewing their own epistemologies of mathematics and reflecting on their experiences as learners. In so doing, they are likely to be surprised, or even shocked, when realising how little consideration they have previously given to the nature and position of mathematics. Through reviewing their epistemologies of mathematics, and relating these to the theories, teachers are likely to strengthen their commitment to adopting progressive pedagogies, such as student-led, collaborative problem-solving approaches. This suggests that initiatives aimed at promoting the use of progressive pedagogies, through initial

teacher education or continuing professional development, would benefit from incorporating opportunities for teachers to reflect on their epistemologies of mathematics.

The project highlighted the importance of a collaborative, participatory and sustained approach to professional development. Through discussing theories, sharing ideas and experiences, jointly planning, teaching and evaluating activities, and providing mutual support, teachers are better able to overcome constraints, be more innovative, and to develop their thinking and classroom practice appreciably. Over a prolonged period of time, collaborative relationships are able to grow and become more effective, particularly when teachers are drawn from a number of different schools. Through actively participating in the design of their professional development, teachers display agency and self-efficacy in maintaining control over the direction and extent of changes in their classroom practice.

7.4 Implications for mathematics education research

The project demonstrated how the critical research design (see Chapter 4) offers a model of research that generates relevant knowledge and has the potential to bring about positive social change. Through systematic reflection on classroom practice and reference to theories underlying the research, new knowledge is produced which is transferable to other classroom situations. Research generated collaboratively with teachers, working in normal classroom situations, is more likely than other modes of research to be perceived as authentic by other teachers, and seen as relevant and applicable to their own situations. Undertaking more research based on this model therefore has the potential to significantly increase teachers' engagement both in, and with, mathematics education research.

The project demonstrated how the critical aspect of the design enables new knowledge to be generated in areas, such as teaching mathematics for social justice, which challenge existing dominant discourses within schools. By accepting that the current situation should not be taken as given, developing a critical understanding of practice, and exploring the feasibility of alternatives, new theories can be generated with the potential to address issues of injustice existing within mathematics education, schools and society. This distinguishes the critical research model from reflective practice, in which developments in classroom practice may occur, but these do not necessarily challenge the status quo or contribute towards positive social change.

The project highlighted the many constraints faced by mathematics teachers wishing to develop their classroom practice in ways which promote social justice. These include the exam-focused nature of the curriculum, pressure to complete schemes of work, high levels of

scrutiny, and the requirement to demonstrate short-term progress. The critical research model offers a means of overcoming these constraints, and empowering teachers to develop their practice by adopting alternative approaches to those which are common in most mathematics classrooms. The impact of the project across the departments in which the teacher researchers worked, suggests that this 'bottom-up' research model is scalable, and has the potential to influence mathematics education discourses across schools.

The critical research design offers a model of research which is participatory, collaborative, relevant, and results in positive social change. It demonstrates how qualitative research can generate findings exhibiting high levels of reliability and trustworthiness. Credibility of the research is ensured through reflexivity of the university-based researcher, establishing relationships built on mutual respect, trust and transparency, involving teacher researchers in the research design and collection of data, and relating the research findings closely to existing theories. Including sufficient detail of teacher researchers' situations, when recounting the stories of their involvement in the research, ensures transferability. Outlining details of the research design, including the establishment and operation of the research group, ensures dependability. Triangulating the research findings, by relating them to students' feedback and teacher researchers' own accounts of the project, ensures confirmability.

7.5 Contribution to knowledge and understanding in the field

I highlighted in Chapter 1 how relatively little research has been published, particularly in England, which focuses on working with teachers to research and develop their classroom practice in relation to theories promoting teaching mathematics for social justice. I therefore propose that this study makes an original contribution to knowledge and understanding in the field, by demonstrating how a university-based researcher can work collaboratively with a group of mathematics teachers, to achieve this aim.

The teacher researchers involved in this study shared a commitment towards teaching mathematics for social justice, as articulated in the initial conceptualisation included in the information leaflet (see Appendix 9). Whilst they differed in their social backgrounds and educational experiences, and in their previous engagement with social justice issues, they were all at a relatively early stage of their careers and shared an enthusiasm for learning. The study shows how mathematics teachers with these characteristics can work within not untypical classroom situations, to challenge the dominant discourses and constraints they face in schools. In so doing they develop agency and self-efficacy in relation to their own practice.

Starting from a broad conceptualisation of teaching mathematics for social justice, this study highlights the impact that researching and developing teachers' practice can have on students' engagement with, and critical understanding of, mathematics. It demonstrates how engagement by teachers, with theories underlying such a conceptualisation, enables them to develop a more profound understanding of the current situation, including structural causes of inequity within mathematics education, which impacts on their thinking and practice. The study draws attention to the processes which facilitate transformations in classroom practice, with researchers and practitioners acting as collaborative agents of change.

The lack of published research evidence, involving teachers as active participants in the research process, reflects the dominance of the traditional 'centre-periphery' model of research, and the distrust of collaborative and participatory research methodologies. Through the attention it pays to addressing issues of reliability and trustworthiness, this study provides a significant contribution to the field by challenging this state of affairs. It demonstrates how participatory action research, in particular the critical research model, can be systematic and rigorous, as well as generating relevant findings and promoting positive social change. The framework for analysing data, adopted in this study, highlights how research findings can be related back to the theories underlying the research, allowing new analytical questions to emerge. This iterative process gives new meaning to the data, enabling the stories of teachers' participation in the research to be narrated in a way that illustrates the development of theory-informed practice.

This study therefore contributes substantially to both pedagogical and methodological knowledge, and understanding, in the field. It does so by providing insight into what teaching mathematics for social justice might look like in practice, how it can be promoted through effective professional development, and by demonstrating how teachers can be involved in systematic collaborative inquiry, which generates reliable and trustworthy findings that challenge the status quo.

7.6 Critical reflection and evaluation

When I set out to establish a 'teaching mathematics for social justice' research group in June 2013, I was somewhat nervous and anxious about how the project might turn out. The collaborative and participatory nature of the critical research model meant that I had only limited control over the project's development and outcomes. Its success was largely dependent upon the commitment and participation of the teacher researchers. There were many events that could have easily threatened the existence or continuation of the research

group, including the possibilities that nobody would respond to my invitation to join, or that teacher researchers might leave before the end of the project, due to changing jobs, personal circumstances, the demands of teaching, or simply through losing interest. Because of the methods drawn from grounded theory used for data analysis, there was also the real possibility that no findings of any significance would emerge from the data.

However, the unpredictability of the research model proved to be its greatest strength, since it was the active role played by the teacher researchers that enabled them to develop agency and self-efficacy and to generate findings of relevance to other classroom practitioners. My fears and anxieties turned out to be unjustified as the commitment and enthusiasm of the teacher researchers were unwavering. Their levels of engagement, and the time and effort they contributed towards the project, surpassed my expectations. Anna, Brian and Rebecca, in particular, took their roles very seriously and accorded the project high priority. Their short reports (included as Appendices 6, 7, 8) highlight the extent to which they believed the project had a significant and beneficial impact on the development of their thinking and classroom practice.

I had anticipated that the research findings would confirm my initial thoughts, about the effectiveness of collaborative and participatory methods of inquiry, in bringing about desirable change in classroom practice. However, I had not anticipated how strong an impact the project would have on students' engagement with mathematics. Nor did I foresee the extent to which it would influence the development of teacher researchers' thinking, for example on Anna's notions of mathematical ability and Brian's views on cultural capital. The success of research projects such as this one, however, should not be taken for granted. It must be remembered that bottom-up reform is entirely dependent upon the will of practitioners to engage with, and play an active role in, the research.

Considerable care was taken to engage with issues of power relations within the research group, in particular those between myself and teacher researchers (see Section 4.5). It is not possible to say with any certainty that all such issues were resolved. However, the secondary thematic analysis, which focused on the key processes and characteristics of the research model, including the extent to which the research was participatory and collaborative, did not reveal any evidence that previously-existing power relations impacted upon the dynamics of the research group. No references were made at any point during meetings or interviews to historical situations, such as lesson observations or the grading of assignments, where I may have been perceived as making judgements about the performance or competence of student teachers. The break of one year between my previous encounters with teacher researchers, as

a teacher educator, together with my focus on openness and transparency within the research group, enabled new relationships to be established. However, these new relationships benefited from the trust, rapport and mutual respect I had already established with teacher researchers in my previous role. Despite making it clear to teacher researchers that they were able to request that particular comments made during meetings or interviews be excluded from the transcripts, no such requests were made. The research findings suggest that my role in the research group, whilst acknowledged as essential, was viewed by the teacher researchers as facilitative rather than regulatory. There is strong evidence to suggest that participation in the research group empowered teacher researchers to overcome constraints they had previously encountered in relation to teaching mathematics for social justice, and to take increasing control over the extent and direction of the development of their practice.

The five teachers who joined the project constituted only a small proportion (less than five per cent) of those invited to take part. This might suggest that the relevance of the findings of this study is limited to a small group of mathematics teachers who share the same characteristics as those who took part in the project (see Section 7.5). However, the positive outcomes teacher researchers achieved, through their participation in the project, generated substantial interest, and enthusiasm for trying out many of the ideas, amongst other teachers within their departments. This suggests that there is a wider group of mathematics teachers who might be interested in teaching mathematics for social justice, and developing their own thinking and practice around this concept, were it not for the substantial pressures they face in schools due to increasing levels of performativity, workload, and the focus on high-stakes exam results. A future study might therefore explore the extent to which a similar research group, based on a critical research model, might generate interest in teaching mathematics for social justice amongst this wider group of mathematics teachers, or act as a catalyst for turning such 'interest' into action.

This research project was small-scale and unfunded, which generated its own challenges, such as the necessity for teacher researchers to attend twilight meetings, often after a busy day of teaching. The dates of several of these meetings had to be changed at short notice, due to rearranged parents' evenings and industrial action taking place in schools and universities at the time. Fortunately, the commitment towards teaching mathematics for social justice, shared by all in the research group, meant the meetings went ahead regardless. The lack of funding also limited the opportunities for collaboration amongst teacher researchers, for example it was not possible to carry out peer visits, or observations, in each other's schools. It also limited the extent to which I could liaise with other university-based researchers,

regarding the research. However, the unfunded nature of the research also provided me with the freedom to focus on aspects of mathematics education for which I have a genuine passion.

I feel privileged to have witnessed first-hand the development of teacher researchers' thinking and classroom practice, relating to teaching mathematics for social justice. I have no doubt that having such insight, at an earlier stage of my own teaching career, would have enabled me to develop my own classroom practice in a way which more effectively challenged the social reproduction of inequity, and resonated more closely with my underlying ideology. My research methodology has encouraged me to reflect upon, and hence make more sense of, my own educational experiences, for example by helping me to better understand the reasons underlying past conflict with managers. The experience of facilitating this research project has reaffirmed my belief in the desirability of developing mutually respectful and collaborative relationships between researchers and participants, teachers and learners, tutors and tutees, managers and those being managed. The success of this project has encouraged me to continue to work with mathematics teachers, both experienced and those new to the profession, in reflecting on classroom practice and bringing about change which promotes equity and social justice.

7.7 Final remarks

This research project aimed to explore how a commitment towards education for social justice amongst mathematics teachers could be translated into pedagogy and classroom practices which promote such aims. The critical research model of participatory action research provided an effective means of realising these aims, enabling the project to have a considerable impact on the thinking and classroom practice of the teacher researchers, and the engagement and mathematical agency of their students.

The initial conceptualisation of teaching mathematics for social justice offers a useful starting point for generating an alternative vision of what mathematics teaching might look like. The critical research model provides a means through which this alternative vision might begin to be realised, whilst at the same time providing an effective model of professional development for teachers. It also provides a model of research that generates trustworthy knowledge, which is relevant and applicable to the classroom situation, and which has the potential to bring about positive social change.

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Appendices

Appendix 1: Code log for main data analysis

Code and Category	**Created**	**Properties*** TMSJ = Teaching Mathematics for Social Justice
BecTea = Reasons for becoming a maths teacher	03/10/2013 During coding of Sarah Int1	E = Desire to engage students in the subject. I = Desire to address injustices. S = Opportunity to engage with all sections of society. D = Helping students to develop as individuals.
DesCha = Desire to bring about change in society	22/09/2013 During coding of Rebecca Int1	A = Bring about social change through student agency. S = Address structural inequities in education, society. G = Important that everyone aware of what's going on in the world around them. C = Schools promote middle class norms and values. D = Demonstrate to others that TMSJ is a desirable and viable alternative. B = Background led to desire to challenge injustices. P = Develop students' cultural capital to promote achievement in maths. M = Mixed ability seen as desirable for raising maths achievement.
EmpMat = Empowerment through learning maths	22/09/2013 During coding of Rebecca Int1	A = Develop student agency. E = Empowerment encouraged elsewhere. S = Appreciate maths can be used to support an argument. D = Recognition of disempowering nature of school and school maths. U = Students need maths to be able to understand and interpret their world. N = Students need to appreciate the nature/status of maths. C = By comparing, communicating experiences with others, stories more meaningful. M = Students need to consider their own emotions towards maths. V = Students need to consider their own views of maths. R = Maths related to students' own experiences
EngMat = Engagement with maths	09/08/2014 During coding of Meetings 4/5/6/7	A = TMSJ approach re-engages alienated students. S = TMSJ approach raises engagement of all students. T = TMSJ approach raises enjoyment of teachers.
LegAre = Seeing maths as legitimate area for issues of SJ	22/09/2013 During coding of Rebecca Int1	E = Issues addressed elsewhere. R = Issues important for relationships in classroom. T = TMSJ issues need to be addressed more in maths. S = Students willing to accept SJ relevant to maths. M = School managers willing to accept SJ relevant to maths. N = Use TMSJ issues to enrich mathematical topics. C = Other teachers in school willing to accept SJ relevant to maths. U = TMSJ contributes towards mathematical understanding.

Code and Category	**Created**	**Properties*** TMSJ = Teaching Mathematics for Social Justice
NatMat = Consideration of nature of school maths	22/09/2013 During coding of Rebecca Int1	R = Should relate to real life issues. V = Consider maths as value-laden. G = Appreciation of maths as gatekeeper qualification. L = Appreciation of links between maths and other subjects. A = Students can't access problems as no focus on skills needed. E = Maths invokes anxiety and negative emotions in students/adults and alienates. U = Maths curriculum aimed at minority who go to university. C = Maths curriculum develops conceptual understanding. B = Maths seen as boring and irrelevant.
NotAbi = Notions of ability in maths	22/09/2013 During coding of Rebecca Int1	D = Higher ability/sets have a more positive disposition towards learning. L = Higher ability/sets more likely to see links. E = Lower ability/sets more likely to focus on getting through exam. A = It's considered acceptable to say you're not good at maths. I = It's assumed that being good at maths means you're generally intelligent. G = Differences in attitude to maths exist between boys/girls. S = Lower ability/sets have lower self-esteem, confidence, eloquence. P = Being good at maths seen as special/unusual by others. T = Esoteric maths more suitable for higher ability than lower ability.
PraCon = Practical constraints on TMSJ	03/10/2013 During coding of Sarah Int1	T = Time consuming to prepare teaching ideas. B = Bad behaviour of students significant disincentive. E = Teaching is exhausting and TMSJ needs creative energy. G = Government policy restrictive. S = High levels of scrutiny within school. P = Pressure to get through scheme of work. C = Conflict with aim of doing what's best for the students. L = TMSJ takes longer than expected compared to normal maths lessons. R = Large range of resources to choose from.

Code and Category	**Created**	**Properties*** TMSJ = Teaching Mathematics for Social Justice
ProDev = Interest in personal and professional development	22/09/2013 During coding of Rebecca Int1	A = Appreciate that TMSJ area to develop. D = Identified as development need within school/dept. N = Willingness to take on new ideas. G = Wish to improve own general practice. C = Challenge to see how students respond. R = Interest in learning more about research. W = Opportunity to work with other teachers. S = Project seen as providing more structure for thoughts. P = Project provides incentive to try out activities through agreed deadlines.
ProPed = TMSJ related to progressive pedagogies	23/09/2013 During coding of Rebecca Int1	C = Students need critical understanding of maths. S = Promote student-led learning. P = Promote project-based learning. K = Promote practical, kinaesthetic approach to learning maths. L = Promote problem-solving approach to learning. E = Promote enjoyment of learning maths. R = Establish relationships based on trust and mutual respect. G = Promote collaborative group work. T = Provide time to practise maths skills and consolidate
RatMat = Rationale for learning maths.	22/09/2013 During coding of Rebecca Int1	U = Students understand why they need to learn concepts and procedures. A = Easier to see in some areas of maths, e.g. Statistics, than others. W = Students need to consider why we need to learn maths.
RelMat = Relationship with mathematics	22/09/2013 During coding of Rebecca Int1	L = Love for maths. B = Appreciation of beauty of maths. S = Maths considered as special. O = Influenced by significant other. L = Lack of confidence in maths. M = Maths seen as medium through which teacher operates. U = Gained better understanding of own views of maths. C = Maths central to way of thinking.

*Note: Properties were assigned on a scale of 1 to 5 according to the extent to which they were satisfied. For example, a unit of meaning which indicated strongly that the reason for becoming teacher was a desire to engage students in the subject, would be coded as ‘BecTea E5’.

Appendix 2: Code log for secondary data analysis

Code and category	**Properties*** CS = Current Situation; IS = Imagined Situation; AS = Arranged Situation (see Section 4.1); TR = teacher researcher
Ped = Pedagogical Imagination (critical research process)	C = Critical understanding of CS/practice. A = Acknowledge that CS/practice should not be taken for granted. T = Possible alternatives to CS (AS) arise from theories. P = Possible alternatives to CS (AS) arise from practice. D = Possible alternatives to CS (AS) arise from discussion
Pra = Practical Organisation (critical research process)	C = Cooperation in organising AS. A = Acknowledge constraints of CS. N = Negotiate between desired outcomes and constraints.
Exp = Explorative Reasoning (critical research process)	E = Evaluation of success of AS based on evidence. A = Analysis of AS leads to better understanding of IS (TMSJ). D = Conclusions drawn about feasibility of IS (TMSJ).
Par = Participatory (action research characteristic)	T = TRs understand relationship between theory and practice. A = Agreement between my and TR's interpretation of data. R = TRs understand research processes and design. D = TRs feel involved in development of project.
Col = Collaborative (action research characteristic)	R = Collaborative relationships between TRs and me. T = Collaborative relationships between TRs and TRs. F = My role facilitates work of the group.
Rel = Relevant (action research characteristic)	F = Focus on TMSJ recognised as pertinent to general classroom practice. A = New knowledge developed has application to TR's practice.
Pos = Resulting in positive social change (action research characteristic)	C = TRs change their own practice. A = TRs develop agency - control direction of change in practice. E = TRs develop self-efficacy - control extent of change in practice. J = Project challenges unjust practices, discourses, power relations. U = Project develops new knowledge, theories and understanding.

*Note: Properties were assigned on a scale of 1 to 5 according to the extent to which they were satisfied. For example, a unit of meaning which indicated clearly a critical understanding of the current situation (current practice), thus demonstrating Pedagogical Imagination, would be coded as 'Ped C5'.

Appendix 3: Example of meaning condensation and coding process

Extract taken from the analysis of the first interview between Rebecca (R) and myself (P):

Int#	Ref#	?	Time	Dialogue	Meaning condensation	Categories + Properties
2R1	1	P	0.06	There's really five main questions I'm going to ask. I would think it would take about 30 or 40 minutes. Something like that. But, that's 'How long is a piece of string?' type thing. So the first question is … 'What does teaching maths for social justice mean to you?'		
2R1	2	R	0.26	OK. So like I say, I didn't really have any idea before this project.	Limited thought about TMSJ before project.	PreEng T1
2R1	3	R	0.36	I guess, using maths to teach big things about the world that maybe wouldn't normally fit into the maths curriculum, or indeed anywhere else in the curriculum. I really like that idea.	Bring global issues into the maths curriculum. Not taught elsewhere.	LegAre E1
2R1	4	P	0.51	What do you mean by 'big things'?		
2R1	5	R	0.54	Like, all the ideas, how do you empower people to believe that they can change the world around them?	Empower students and develop student agency.	EmpMat A3
2R1	6	R	1.06	Like, there's no subject, well I guess maybe they do that in citizenship, but it's not really at National Curriculum level for how empowered are you?	Empowerment not catered for in school curriculum.	EmpMat E2
2R1	7	R		So I quite like the idea that you could try and teach that through maths.	Teach empowerment through maths.	EmpMat E3
2R1	8	R	1.22	And I guess through a lot of projects and looking at really interesting problems that are actually real life problems, rather than maths problems dressed up as real life problems. But with a kind of emphasis on things that are actually problems, like inequality and injustices.	Real life problems in maths including equity, injustice. Pseudo-realistic problems in maths.	NatMat R3
2R1	9	R	1.45	I think that's what [Brian] was saying wasn't it? It's quite easy to define injustice, than what social justice. I don't really know what social justice means.	Easier to define social justice by defining injustice. Limited thinking around TMSJ before project.	PreEng T2
2R1	10	P	1.52	And I think that's quite a perceptive idea. It's easier to define it by what it's not, than what it is. Because it is really addressing injustice.		
2R1	11	P	2.16	So what might some of those real life projects be? What about that?		
2R1	12.1	R	2.20	So a couple of the articles and things that I was reading on-line were kind of looking at the breakdown of exam results by different races, and house prices in different areas. And just all these things where you're actually using real life data and you are looking for a problem.	Examples of real life projects focusing on social justice issues.	NatMat R5
2R1	12.2	R		You're not saying to the kids 'there is a pattern here', but you're kind of getting them to look and actually decide whether there is.	Develop agency in students through maths.	EmpMat A4

Notes:

- First column: 'Int#' represents the reference for the interview (or meeting). The reference here is 2R1 because it is the second stage of data collection (between meetings 1 and 2 which would be 1M1 and 3M2 respectively) and Rebecca's first interview.
- Second column: 'Ref#' represents the reference for the unit of meaning (numbered sequentially).

Appendix 4: Brief description of activities

For cycle 1, teacher researchers designed their own activities to try out in classrooms. For cycles 2 and 3, all teacher researchers agreed on three activities to try out in their classrooms.

Activities from action research cycle 1:

Wealth in Britain (Anna): Students were asked to imagine that the population is divided into five groups. Each group is considered to be represented by a profession, e.g. doctor, cleaner. The group task was to decide how to divide up £100 fairly between the groups. Students were given 100 counters to facilitate this. They were encouraged to use percentages to describe their choices. They were then encouraged to compare their results with how wealth is shared out between the richest 20%, poorest 20%, etc. of the population in the UK. Students were first asked to predict how they thought the wealth was distributed, before being shown a video with the reality (richest 20% get 62% or £62, poorest 20% get 0.6% or 60p). They were encouraged to articulate ideas about fairness using percentages. Finally, students were asked to relate their findings to what they might earn in a day in different jobs, how much tax they would pay and the minimum wage.

Water Availability (Brian): The activity was based on the 'Water Availability' case study from the Bowland Maths resources (http://www.bowlandmaths.org.uk/). Students were given data about three countries, Turkey, Jordan and Algeria, including population, area and water resources. They were shown a fictitious clip from the 'World Water Board' asking them to decide which country should be given funding to develop their resources. Students were first asked to choose a country without any debate. Then they were prompted to discuss the data further and take into account compound measures and rates such as water per person. Students were provided with more detailed profiles on each country. A further vote was taken and students were asked to justify their choices. Finally, they were encouraged to reflect on the purpose of the activity, i.e. how mathematics, in particular data, can be used to make an objective decision.

Making a Change **- initial attempt** (Rebecca): Students were asked to identify something about the school they would like to change. They were then asked to collect data by carrying out a survey of students' opinions about their proposed change. They were encouraged to make use of stratified sampling which they had learnt during a previous lesson. Students were asked to design a questionnaire and choose a sample of students in the school to complete this, using class lists containing basic data. The questionnaires were distributed via registers. Completed questionnaires were analysed and students were asked to use the results to support their proposed changes. Finally, they were asked to write a letter to the person within the school they felt had the authority to consider their proposals. These letters were then sent to the person identified.

Activities from action research cycle 2:

Average Income: Students explored differences in average wages between men and women, and other different categorisations, using data from a variety of sources. They were encouraged to consider the differences in earnings and how these are hidden by mean incomes. They were asked to compare GDP per capita for rich and poor countries and consider how this measure hides inequality within the countries. Students then compared global inequality with inequality in the UK, relating this to the recent increase in soup kitchens, food banks and child poverty in the UK, and to the London Living Wage. They were encouraged to compare incomes in the UK, London and the immediate area around their school, using census data. Students were asked to consider how the mean and median are both used in different ways to measure average income, and how this relates to issues of social justice, e.g. the median was used to argue for a minimum wage, the mean hides extreme values and inequality.

Fair Trade: Students were asked to explore how the price paid for a Fair Trade product is divided up and what percentage goes to the producers. They were asked to predict what this might be, discuss what they thought it should be, and then relate these ideas to the actual situation. They made use of the data in the 'Fair Trade Chocolate' activity from 'Human Rights in the Curriculum: Mathematics' (Wright, 2004). They were encouraged to use this data to calculate the actual amount producers received from a chocolate bar by calculating percentages of given amounts. Particular issues relating to Fair Trade were discussed, e.g. why supermarkets make more (as a percentage) from a Fair Trade chocolate bar than a normal chocolate bar. Students were encouraged to choose appropriate graphs to represent the data. They were asked to consider the extent to which people would be willing to pay extra for Fair Trade products, e.g. by carrying out a survey. They were encouraged to use their findings to discuss the extent to which Fair Trade is actually fair.

Measuring Inequality: Students explored the use of Lorenz Curves and the Gini Coefficient to display and measure inequality. They were asked to look at wealth distribution within the US and the UK, represent these with Lorenz curves, and relate these to how the proportion of people living close to the poverty line might be hidden by a small minority that are very rich. They considered various definitions of poverty and how these have changed over time. Students explored how a Lorenz Curve can be used to display inequality within other data, such as earnings or pocket money of a group of people, which they generated themselves. They were encouraged to look at different methods of finding an irregular area (in order to calculate the Gini Coefficient), such as counting squares or calculating areas of trapeziums. They were also encouraged to explore alternative ways of representing inequality, e.g. the percentage of wealth owned by the richest 1% of the population. They made use of the data in the 'Global Inequality' activity from 'Human Rights in the Curriculum: Mathematics' (Wright, 2004).

Activities from action research cycle 3:

Election: Students explored different voting systems, focusing on the mathematics underlying them, and how these might lead to different outcomes in an election. They made use of the ballot papers in the 'Election' activity from 'Human Rights in the Curriculum: Mathematics' (Wright, 2004). They were then encouraged to vote on something they felt strongly about, e.g. deciding the class's 'favourite film', helping them to appreciate how relevant and powerful mathematics can be. Students were asked to discuss and justify which voting system should be used, i.e. 'Which is the fairest and why?' Having carried out an election, they were encouraged to reflect on the fairness of the outcome. They considered the possible use of tactical voting, e.g. you can split the vote under some methods by nominating a very similar film to the one you don't want to win. Students related their discussions to the election systems used in UK elections and were encouraged to consider whether alternative systems, e.g. AV voting, should be used instead. They were encouraged to consider other issues around voting, e.g. whether voting should be made compulsory (as in Australia).

Making a Change Project: Students worked together, in groups, to research a social justice issue of their choice. They were asked to choose an issue of interest to them, where they would like to see a change made to make the situation fairer. They were encouraged to choose an issue and a proposed change that might be achievable, and to use mathematics to support their argument. They were given a number of lessons in which to develop their argument and plan a presentation to the rest of the class. Students then made their presentations and the class voted for the best three presentations. The best three presentations from each school were then uploaded to a gallery so that they could be shared with students in the other three schools. Students from all four schools were then able to review all of the presentations, vote for their favourites, and provide feedback focusing on how successfully the groups had used mathematics to support their argument.

Nature of Maths: Students were encouraged to engage with a number of activities that prompted discussion related to the nature of mathematics and its position in society. These included researching data on the link between parents' income and mathematical attainment, the impact of different mathematics qualifications on future income, different mathematics qualifications required for different employment paths, and the mathematical skills actually used in various jobs. Students were encouraged to carry out surveys into attitudes towards mathematics, e.g. how useful adults found the mathematics they learnt at school and whether people believed that success in mathematics was more attributable to effort or ability. They were also asked to comment on various quotes about mathematics made by famous people in order to prompt discussion about the nature of mathematics.

Appendix 5: Student survey and protocol

Survey

Date:		Male or female:		Maths class:	
Please answer the two questions below as fully as you can …					
How do you feel about maths in general?					
What do you think about the maths we did today?					
Thank you for completing this survey!					

Protocol

(Agreed at meeting on 10th February 2014)

In order to help students to differentiate between the two questions, the words 'in general' should be added to the first question. So the questions are now:

- *How do you feel about maths in general?*
- *What do you think about the maths we did today?*

The survey should be conducted with students immediately after trying out the classroom activities.

The teacher researcher should make clear to students that the survey is for the purposes of a research project (experience has shown that this encourages students to take it more seriously) and therefore the response should make sense to somebody who doesn't know the class.

Students should be encouraged to work individually, i.e. to express their own views without being influenced by others.

There should be some introduction to the survey, i.e. the teacher researcher explains to students the rationale for each question, as follows:

- *The first question is meant to find out in general what you feel about maths, i.e. how did you feel about maths as a subject before doing the activity?*
- *The second question is meant to find out what you think of the task you've just done and whether it has changed your view of the topic or maths as a subject. In other words, how do you think differently about maths as a result of doing the activity?*

The introduction/explanation should take 5 minutes.

Then students should be allowed 5 minutes to complete the survey.

Appendix 6: Anna's short report

26th July 2014

Introduction

I anticipated that joining the TMSJ research project would be an interesting opportunity for my professional development beyond my NQT year. Teachers in the UK train, qualify, and then are left to stagnate somewhat with regards to CDP; in-school CPD is rarely consistent or tailored enough to sufficiently meet individual practitioners' specific development needs and out-of-school CPD is very hit and miss in terms of effectiveness and often difficult to 'sell' to SLT. I saw the project as a way to keep myself moving forward professionally and it has exceeded my expectations.

The first thing one is rightly encouraged to consider when evaluating any CPD is the impact on students' learning and people tend to look to the immediate attainment of current students as a way to quantify this. I feel this is inappropriate; impactful CPD should have far reaching outcomes that influence future learners as well as current. For me, while there were numerous clear benefits for current learners in my classroom, the true impact of being part of this project is something that will be seen in the long term.

Specifically, I believe participation in the project will impact the learning of my future students due to the following; the positive impact on my professional understanding of how maths can be taught in a way that creates beautiful learning opportunities which learners enjoy, the opportunity to develop my skills as a collaborative and reflective practitioner, and the (unanticipated) modification of my own conceptualisation on the nature of maths as a subject.

My professional understanding of how maths can be taught

Even prior to taking part in this project my 'gut instinct' told me that the way we generally approach maths education in the UK is not working; students tend to be rushed through topics, there is too much focus on testing and too many students do not enjoy maths. I understood all this and yet there was a conflict in my mind; the time taken to plan and deliver such learning opportunities conflicted with the time constraints faced in having to 'get students through tests' every half term.

What struck me through incorporating TMSJ activities into my lessons was primarily how engaged students became, particularly low-attaining students who had previously hated maths. But it was not just increased engagement; sharing ideas for resources meant that I was able to plan more cleverly, as I did not have to spend so long searching for suitable activities. Previously I would try and find something related to a social justice issue and crudely 'fit it in'. As the project progressed and I became more aware of the different ways social justice could be approached in the maths classroom, I began to find the space to interweave the learning of the maths into social justice issues; students were not just being inspired and engaged by maths, they were learning and, crucial to this, misconceptions were emerging that otherwise would have remained unidentified.

A particular example is of my year 7 mid to low attaining class. We had studied percentages for around 4 lessons at a level dictated by their KS2 attainment, but it was only when we looked at a lesson comparing the percentage of profit that goes to a farmer through buying fair trade

versus a regular product did I really see the holes in these learners' conceptual understanding of percentages. It served as an incredibly useful tool for identifying and addressing misconceptions which just would not have arisen had we stuck to the traditional 'cover one objective every lesson then move on' approach.

Because of my participation in the project I now see the kinds of activities we have been trialling as an essential component of what a secondary maths education should look like. I have been inspired to consider how I can interweave such activities into the schemes of work I develop and feel that the learning and love of maths my future students experience will be all the greater and richer because of it.

Developing as a collaborative and reflective practitioner

The project provided an opportunity to meet with maths teachers in other schools and share ideas and resources. This gave the primary benefit of reducing planning load leading to better planned learning sequences for students. Having the opportunity to hear about others' successes and struggles gave a sense of the bigger picture and deepened my own reflections on what I was hoping to achieve through participating in the project. I feel I have become better able to reflect on the learning in my classroom and more attuned to deciphering the difference between lessons that students enjoy and lessons where students are enjoying learning.

A crucial benefit for me personally is I feel empowered to take my future CPD into my own hands, equipped with the skills and confidence to drive my own development in directions I feel appropriate. So long as I am part of a network of teachers, through twitter or other social media, I will continue to explore and share ideas with other practitioners.

Beyond the research group, my participation in the project impacted on my department. Several teachers utilised project resources and the whole of year 8 took part in the final 'Making a Change' project. As well as increasing my confidence in my ability to produce and share resources that other teachers will engage with, a whole cohort of learners taking part in a meaningful and engaging project has also served to raise the profile of maths among learners and teachers across the school.

My own conceptualisation of maths as a subject

Before taking part in the project my understanding of maths as a subject and also of TMSJ was limited. The way the ideas for TMSJ were presented to us as teacher researchers was very open-ended, allowing for a lot of deep personal exploration as to what elements I agree with and find most useful.

In hindsight, I think I previously saw maths as combining three main elements; financial literacy (the bit all students need to know to function in society), statistics (coming from a psychology background I saw this as a separate bit of maths for conducting research) and abstract maths (i.e. algebra; the bit that no-one really needs to know but we teach it anyway in case they go on to do A level maths).

While my understanding of the nature of maths is still limited, I feel I have a broader understanding of how maths is an interconnected and rich subject, how maths is, put simply, a

human language designed to enable us to describe and represent patterns in the world around us.

Additionally, I feel I have developed an awareness of the value-laden nature of maths as a subject and also an understanding of how and why this ought to be shared with learners.

Maths is a living breathing subject and learners who are denied the opportunity to delve into the possibilities it provides for exploring and changing their own worlds are denied a vibrant and valid conceptualisation of the subject.

Conclusion

To conclude, being part of the TMSJ research group has been the most impactful form of CPD I have experienced over the past year and is something that I see as having a long-term impact on my teaching, my ability to keep moving myself forward as a reflective practitioner and my broader understanding of the subject that I teach.

I see my participation in the project over the last year as the start of my post-NQT professional development as I feel empowered to take a deep and long-term approach to making improvements to my teaching practice. The impact I hope to be sustained and far reaching.

Appendix 7: Brian's short report

25th July 2014

I engaged in this project as a Participant Researcher with the intention of seeing how my own interest in international justice could map onto my work as a teacher. It provided me with an opportunity to spend time with other teachers with relatively similar experiences (2/3 years into the profession) and with similar frustrations, which most frequently seemed to be the exam focus of our schools. Now, at the end of the project I am pleased to see that Teaching Mathematics for Social Justice has actually changed the underlying motivations behind my teaching. In particular whilst I came in with a narrow focus on helping the kids to engage in international issues such as Global Hunger or Child Mortality, I left with a deep-seated belief that even the way I conducted myself with students was a constituent of social justice. What I mean by this is that every interaction became an opportunity to help students to overcome the injustices they would focus in their lives as well as preparing them to help alleviate the injustices experienced by others. An example of this would be discussing the private school system with one of my classes. Outlining that the playing field was by no means even. In the words of a recent film, my pupils would be entitled to proclaim "the odds are never in our favour".

Through this project, in particular our discussions of the summarized TMSJ theories, I have come to see the privileged role that Mathematics plays in the way individuals are judged and categorized. Due to the project, I ended up discussing these features with students. We also discussed and evaluated the emotional baggage they brought with them to their mathematics lessons. This was particularly constructive with a low achieving (penultimate) set of girls who had already experienced 3 years of secondary school mathematics. Their emotions were overwhelmingly negative: anger, frustration, despair and general unhappiness. By working through these explicitly and highlighting the pre-eminence of Mathematics in society we were able to find much common ground and thus change the atmosphere in the class. It moved from a teacher attempting to get students to do what they want (e.g, staying on track with the scheme of work), to a teacher helping students to both beat the present (education) system and to be able to engage confidently with a world where much is predicated by mathematics (tax, high paying jobs, shares, etc). A key component of my approach was also to listen a lot (to all sorts of things within and outside the maths curriculum!) and speak encouragements frequently (vital where one of the key barriers was a lack of confidence). Sadly for me (and a few of the kids!) I was only with this class for a term but in that time both their approach to maths and their maths teacher (according to them and their previous teacher) drastically changed as they became part of a mutual project of advancement. A number of the students ended up moving to different sets and taking a lead role in those classes!

One of our discussion themes this year has been around the ability to reflect and evaluate ethics – that is saying whether things are right or wrong. We have seen this in some of the tasks such as evaluating fairtrade, inequality, voting systems and other things. I was interested by how hard some of the year 7s (levels 3 – 5) found these tasks. Evaluating the fairness of a situation was not an immediate response to the analysis conducted in a way that it was for my high achieving year 8s (levels 5 -7). I observed that those who found it harder to evaluate the fairness of ethics of a situation were also those who were generally worse behaved (if not in

my class then across the school). Whilst this is not comprehensive proof it got me thinking about the relation between these two things. That behaving in a way that is conducive to your own and others learning (a positive atmosphere in QTS speak) is connected to evaluating the ethics of your own actions – whether it is right or wrong. This in turn was connected to evaluating the ethics of a situation – for instance whether it is right or wrong that the top 1% own 40% of the wealth. Again, this is not comprehensive but an interesting thought I had as a result of the project. It leads onto the question of how 'evaluation' can be taught and to what extent and whether this would also lead to a change in personal behaviour.

A really enjoyable part of this project was giving students the chance to present an argument for something they wanted to see change. The response of the pupils to this task was so positive that I think it is something I would try to conduct in one way or another with all of my classes. When given a set of arguments pupils were quick to state that those containing statistical content were stronger than those lacking it. As a result with little encouragement other than some careful scaffolding, they produced presentations saturated in mathematical content. Producing their own arguments and listening to those of others led to many expressing surprise at the utility of mathematics. Some claimed they would never again ask, "what's the point?" or "where will we use is in real life?" but I hope they don't stick to this.

My approach to teaching and the content of what I teach has been significantly affected by my participation in this project. It has also had an impact on others in my Academy and outside it. I have run numerous mini-CPDs on the material here, sharing the ideas and activities under the auspice of 'SMSC' (social, moral, spiritual and cultural) development, which I have led for our Maths department. Three or four other teachers have ended up adding and developing their own resources including issues such as Asylum Seeking. Whilst most of these materials were in the form of using social justice issues as the examples used to aid the understanding of key concepts (e.g. percentages of decimals), they also went beyond, leading pupils to evaluate the ethics of these situations. The popularity of the activities received and the positive feedback from my lessons (from other teachers and pupils) has resulted in me delivering a number of CPD sessions to different parts of the [Forest] Federation. Particularly relevant was addressing NQTs from across the [Forest] Federation on the topic of SMSC development, which really revolved around helping teachers of all subjects evaluate how they teach their subject for social justice. Taking on board the theories surrounding TMSJ I asked similar questions around the privileged position some subjects hold, the inequities of society and other things to help others to engage in these themes in their own contexts. All of these sessions have received a positive reception and feedback with lots of teachers emphasizing that they didn't realize how easy it really was to include these elements into the lesson and how interesting it made what they were teaching.

By participating in this research project I have been able to grapple with academic notions in a manner relevant to my own teaching. Our discussions enabled me to crystalize my own thoughts on various issues through the encouragement and critique of my peers. Ruminating on the original conceptions of TMSJ for an entire year would have seemed daunting but it has actually been a perfect timescale and I believe it has and will be far more beneficial than just receiving a lecture on this issue in the form of a book, talk of seminar. For me I will miss the opportunity to have a parallel community of teachers with whom I can discuss my teaching who are separate from the specific school ethos within which I operate. It has provided an

opportunity for me to see which problems are specific to my school and area of teaching whilst also seeing which problems are systemic structural problems across the education system.

Appendix 8: Rebecca's short report

25th July 2014

I started this project with very little idea of what Teaching Maths for Social Justice meant or what it would look like. I think I have always taught maths in the way that I enjoyed learning it: emphasizing the connections between different ideas within the subject, but never worrying too much about real-life applications. At school I welcomed the absence of messy real-world data and experiments in maths lessons, and enjoyed the comfort of textbook questions that used only nice numbers to give rise to neat solutions. However, since "when will we ever need this?" is a frequent question in my lessons, I know that not all my students feel the same way! Part of my motivation for taking part in this project was a desire to make the maths I taught seem more relevant to my students. There was also a degree of curiosity (I don't think I had ever heard the phrase 'Social Justice' before so I had no idea how to teach maths for it) and the feeling that I should have some kind of CPD focus once my NQT year finished.

Being part of the research group has introduced me to a lot of new ideas – the one that sticks with me is the idea that maths is not value-free, because I remember it seeming so strange an idea when I first heard it during the meeting, but yet it made so much sense. I've always enjoyed learning new things, and I think the meetings were my favourite part of the project because I invariably arrived exhausted after rushing up from school, but left feeling like I'd learned something. The collaboration with people in other schools teaching similar things was a big morale boost as well as being incredibly useful. I think my enthusiasm would definitely have waned after the first few badly planned activities if I had been trying to do this on my own.

The project has not fundamentally changed the majority of my classroom practice – most of my lessons are very similar to the way I taught last year. However, it has had some impact, although not necessarily in the way I had expected it would. I think when I began I felt that my lessons needed more real-life examples and applications to make the maths seem relevant. Through the project, however, I have realised that maths can still be made relevant without every topic having a direct connection to the students' future lives and careers. In particular, projects aimed at developing students' agency can be much more powerful than telling students that plotting straight line graphs will help them solve optimisation problems or that vectors will be really useful to them in A level physics. It's only while writing this that I've realised that the Year 9 class on whom I tried out most of my TMSJ projects hardly ever ask why they have to learn something, even though this time last year several of them were in my Year 8 class and asked it almost every lesson. Perhaps this is evidence that they've learned something this year!

I found the joint planning cycles really helpful, particularly the Spring term cycle where we had looked at a resource in advance to try to get some ideas. The evaluation meetings were more interesting when we had all tried similar things and there was no need to explain how each activity worked. The resource-sharing that we did in the final cycle was also a huge help, both to cut down the time needed to scaffold the projects and also to see how other people were approaching them. One of the biggest drawbacks I found at the start of the project was that lessons took more time to prepare for than I realistically had, so most of them were not as well planned as they needed to be. This definitely made the first project I tried more difficult than it

should have been, and meant that I felt even more nervous about teaching it. I was always a little apprehensive before any TMSJ-related lessons, mainly because it was a departure from my normal style of teaching (maths, maths, maths) and I was worried the students would react badly to anything they saw as a change of routine – several of them would ask why there wasn't a level or grade in the title, for example. It was only the Year 9 class that I ever really felt relaxed with, partly because I had a better relationship with them anyway and also because they had tried enough activities that they stopped feeling so novel.

Towards the end of this project I have made much more of an effort to share TMSJ activities and ideas with my department, who have generally taken to it incredibly enthusiastically. I think I have become much more comfortable talking about TMSJ and creating resources than actually teaching them, though – almost everyone who has taken some of the resources I planned and used them with their own classes has enjoyed those lessons much more than I did originally. A lot of my concerns have been about behaviour. There are two classes in particular that I have really struggled with this year, and to prevent chaos breaking out my lessons have been very 'routine'. One class's behaviour deteriorated very rapidly during a lesson that featured discussion and group-work for more than a 5-minute burst. The other class surprised me more – their behaviour wasn't too bad during the Fairtrade lesson, but they hated having to learn about something that wouldn't come up on their GCSE. To them, 'relevant' means a past-paper question.

I was worried at points this year about how to fit TMSJ lessons into a fast-paced scheme of work, and projects always seemed like 'extras' for the end of term. I thought the solution to this would be to teach maths through social justice projects, thereby killing two birds with one stone, but I think the most successful projects have actually been the mathematically open-ended ones, where students have to select the maths that they can best use to solve the problem in front of them. This makes it much harder to hit a single curriculum objective but I think it makes the original problem seem much more real, and in turn makes the maths more relevant. From a curriculum point of view, this should fit in well with the new KS3 focus on mastery, because surely no one can claim to have mastered a topic without being able to spot where and when it might be useful. I have certainly come to see TMSJ projects as a much more valid use of lesson time than I did at the start.

I would like to embed TMSJ principles into my classroom practice beyond the occasional project, for example through the examples and questions I use in everyday lessons. I'm sure I thought this aspect would be much easier to put into practice than planning and teaching whole projects, as it just involves minor tweaks to lessons already planned. I remember reading an example (possibly in an article we read in the very first meeting) where a long multiplication example was written in terms of the average wage for a sweatshop worker – surely a much more meaningful question than calculating the cost of eight chocolate bars. However, in practice I have found coming up with ideas and finding enough information to write these kinds of questions is incredibly time-consuming, and so I inevitably revert to the contrived examples in the textbook.

For TMSJ to be developed further in my school I think it definitely needs a whole-department approach, with time allocated for projects in the scheme of work, joint planning to ensure that people have time to create good resources, and some kind of assessment of reasoning and

problem-solving skills so that even the most apathetic students realise that they are learning much more in maths lessons than will be tested on a GCSE paper. Thankfully, I think this will be possible in my school, especially since the collaborative, problem-solving nature of TMSJ makes it a good fit with the new curriculum.

I have really enjoyed being involved with this project. Thank you!

Appendix 9: Information leaflet

[University of Sussex logo]

RESEARCH PROJECT: INFORMATION FOR PARTICIPANTS AND SCHOOLS

PROJECT TITLE

Teaching mathematics for social justice: translating theories into practice

INVITATION TO BECOME A TEACHER RESEARCHER

I am inviting you to take part in the above research project as a teacher researcher. I am carrying out this research as part of my studies for a Doctorate in Education at the University of Sussex (School of Education and Social Work). The research is also being supported by my employer, the Institute of Education, University of London. The research is not being funded by any other organisation.

Before you decide whether or not to take part, it is important for you to understand why the research is being undertaken and what it will involve. Please take the time to read the following information carefully.

BACKGROUND AND AIMS OF THE PROJECT

There is growing consensus amongst teachers, teacher educators, university academics, school managers and school inspectors in England, that a more engaging and relevant mathematics curriculum is needed, with greater emphasis on problem-solving and development of conceptual understanding. Too much focus on factual recall and procedural understanding in mathematics classrooms has led to the disengagement and disaffection of a significant number of learners.

This research project aims to address this alienation of learners through developing a better understanding amongst students of the nature of mathematics and the reason for learning it, and an appreciation of how mathematics can be used to better understand the world around them. It aims to promote students' mathematical reasoning and problem-solving skills and to develop mathematical activities that relate to issues of social justice which will affect their lives, e.g. unequal distribution of resources, threats to the environment and fairness of voting systems.

This project, which draws on significant academic research findings relating to the above, aims to use participatory action research methods to explore how a commitment to teaching mathematics for social justice can be translated into pedagogy and classroom practices which promote such aims. The project will run from June 2013 until July 2014 and will involve the establishment of a group of 'teacher researchers' from different schools, who will develop their own understanding of teaching mathematics for social justice through meeting and sharing ideas for practical classroom activities and teaching strategies. The group will collaborate in planning, trying out and evaluating these ideas in their own classrooms and reflecting on the development of their classroom practice.

PARTICIPATION IN THE RESEARCH

I am inviting former mathematics Teach First participants and PGCE students who completed their PGCE year at the Institute of Education (University of London) in 2012 to participate as

teacher researchers in this project. Those taking part should have an interest and commitment towards teaching mathematics for social justice, and will need to obtain the agreement of their head teacher. It is anticipated the research group will include between 8 and 10 teacher researchers.

It is up to you to decide whether or not you wish to take part in the research project. If you do decide to take part, you should firstly seek the agreement of your head teacher by passing on the accompanying letter and a copy of this information leaflet. You should then sign and return to me the consent form (which also needs to be signed by your head teacher) by 21st June 2013. If you do decide to take part you are still free to withdraw at any time and without giving a reason (you should keep a copy of this leaflet for reference).

POTENTIAL BENEFITS OF TAKING PART

As a classroom practitioner, your understanding and experience of the classroom situation will help to generate valuable new knowledge on how to translate theories on teaching mathematics for social justice into practice. As well as contributing towards developing understanding in this area, there are many potential benefits to you as an individual. The project will provide a rich professional development opportunity for teacher researchers, through participation in a genuinely collaborative learning community, gaining insight into an aspect of mathematics teaching of personal interest, reflecting on and developing classroom practice and research skills.

Insights gained into the research process and engagement with research findings will also provide a foundation for teacher researchers to continue, beyond the lifetime of the project, to reflect upon and develop their classroom practice in light of their emerging theoretical understandings. Teacher researchers will be encouraged, if they so wish, to play an active role in the dissemination of the research findings, for example, through authoring or co-authoring, articles for professional journals. Taking part in the research project would also provide an ideal stepping stone for any teacher researcher interested in studying for a Masters qualification in the future.

TIME COMMITMENTS INVOLVED IN TAKING PART

As an experienced mathematics teacher myself, I appreciate the demanding nature of the job and the importance of prioritising the learning and welfare of your students. With this in mind, I have outlined in detail below the time you will need to commit to the research project if you decide to take part:

- You will need to attend a total of seven twilight meetings (e.g. 4-6pm) of the research group consisting of an initial meeting in June or July 2013, followed by two meetings per term during the 2013-14 academic year. These are likely to be held at the Institute of Education, University of London.
- I will interview each teacher researcher three times, at the start of, mid-way through, and at the end of the project. Each interview will last less than one hour and I will arrange these in your school at a time convenient to you.
- You will need to plan, teach and evaluate at least one classroom intervention per term.
- You will need to collect some data to assess the impact of the classroom interventions on your students.

- You will need to keep your own research journal with your thoughts and reflections on your involvement in the project.
- You will need to write up a short report (2 pages) at the end of the project focusing on your experiences and the impact of the project on your classroom practice.

COLLECTION OF DATA

The meetings of the research group and the individual interviews with teacher researchers will be audio-recorded. These recordings will be used as research data in order to explore how teacher researchers' commitments towards teaching mathematics for social justice can be translated into pedagogy and classroom practices which promote such aims. The short reports will also be used for this purpose.

The purpose of the research journals will be to encourage personal reflection and to record thoughts and experiences. This will help to prompt discussions at research group meetings and develop ideas for inclusion in the final reports. However, the research journals will not form part of the research data and will remain private to the teacher researchers.

Due to the collaborative nature of the project, discussions will be held within the research group before agreeing how to collect data on the impact of the project upon students' attitudes towards mathematics (e.g. interviews with groups of students, student surveys, collecting examples of students' work).

CONFIDENTIALITY AND SECURITY OF DATA

All data collected as part of the research project will (subject to legal limitations) be kept strictly confidential and will be stored on secure USB flash drives in private vaults which ensure files are password protected and encrypted. Teacher researchers may request, immediately following a research group meeting or interview, that particular comment(s) they themselves have made be deleted from the data (i.e. from transcripts of meetings and interviews).

Anonymity will be ensured by replacing, with pseudonyms, all names within the data collected, including those of teacher researchers, students, other participants in the research, schools or any other person or institution that might lead to the revealing of the identities of those taking part in the research project. These pseudonyms will also be used for file names, data analysis and the reporting of findings in my research thesis and any article subsequently written by myself or teacher researchers (with the exception of any teacher researchers who choose to reveal their own identities, but not those of other research participants, in any articles that they author themselves). Lists of pseudonyms linked to names of research participants will be stored separately from the research data under similar levels of security.

PUBLISHING OF RESEARCH FINDINGS

The results of this research project will be used in the writing of my thesis for my Doctorate in Education studies at the University of Sussex. This thesis will be available online through the university library. The results may also be used by me, or teacher researchers, to publish articles in academic and professional journals, outlining the findings of the research project.

Before publishing articles, some journals require that data from research projects be made available to other researchers. For this reason, I will be seeking permission from teacher researchers for data collected (transcripts rather than audio-recordings) from research group

meetings, interviews and short reports to be made available for such a purpose. As part of the project, I will develop a strategy for making the data available on-line, subject to password protection and to the confidentiality provisions outlined above. However, any student level data collected or lists of pseudonyms linked to names of research participants will not be made available to others.

ETHICAL APPROVAL FOR THE PROJECT

This research has been approved by the University of Sussex Social Sciences & Arts Cross-Schools Research Ethics Committee (C-REC). If you have any concerns regarding the way in which the research has been conducted, you should contact my supervisor, [name of supervisor], in the first instance.

[contact email address of supervisor]

CONTACT FOR FURTHER INFORMATION

Thank you for taking the time to read this information sheet.

If you require any further information regarding this research project, please feel free to contact me using the email below.

Pete Wright
Lecturer in Mathematics Education
Institute of Education, University of London
[contact address, telephone number and email address]

June 2013

Appendix 10: Consent form

[University of Sussex logo]

CONSENT FORM FOR RESEARCH PROJECT PARTICIPANTS

PROJECT TITLE: Teaching mathematics for social justice: translating theories into practice

Project Approval Reference:

I agree to take part in the above University of Sussex research project. I have read and understood the Information Leaflet, which I may keep for my records, and have had an opportunity to ask for further information if required. I understand that agreeing to take part means that I am willing to:

- Attend seven twilight meetings of the research group between June 2013 and July 2014;
- Participate in three interviews with the researcher during this period;
- Allow the meetings and interviews to be audio-taped;
- Plan, teach and evaluate at least one classroom intervention per term;
- Collect data to assess the impact of the classroom interventions on my students;
- Keep a personal research journal with my thoughts and reflections on my involvement in the project;
- Write a short report (2 pages) at the end of the project focusing on my experiences and the impact of the project on my classroom practice.

I understand that any information I provide is confidential, and that no information that I disclose will lead to the identification of any individual in the reports on the project, either by the researcher or by any other party.

I consent to the transcripts from audio recordings of meetings and interviews and the short reports written at the end of the project (all in anonymised and secure format) being made available to other researchers and interested professional parties if requested.

I understand that my participation is voluntary, that I can choose not to participate in part or all of the project, and that I can withdraw at any stage of the project without being penalised or disadvantaged in any way.

I consent to the processing of my personal information for the purposes of this research study. I understand that such information will be treated as strictly confidential and handled in accordance with the Data Protection Act 1998.

Name: (participant) **Signature:** (participant) **Date:**

Head teacher's agreement:

I agree to the participation of the above named teacher in the research project referred to above and outlined in the 'Letter to head teacher' and 'Information for participants and schools' leaflet.

Name: (head teacher) **Signature:** (head teacher)

School: **Date:**

Please return the completed consent form to Pete Wright by Friday 21st June 2013