Gravitational theories beyond General Relativity

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Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another University for the award of any other degree.

The work in this thesis has been completed in collaboration with Xavier Calmet, Ian Moss, Sonali Mohapatra, and is comprised of the following papers:

- Xavier Calmet, Iberê Kuntz, 'Higgs Starobinsky Inflation', published in Eur.Phys.J. C76 (2016) no.5, 289.

- Xavier Calmet, Iberê Kuntz, Sonali Mohapatra, 'Gravitational Waves in Effective Quantum Gravity', published in Eur.Phys.J. C76 (2016) no.8, 425.

- Xavier Calmet, Iberê Kuntz, Ian G. Moss, 'Non-Minimal Coupling of the Higgs Boson to Curvature in an Inflationary Universe', published in Found.Phys. 48 (2018) no.1, 110-120.

- Xavier Calmet, Iberê Kuntz, 'What is modified gravity and how to differentiate it from particle dark matter?', published in Eur.Phys.J. C77 (2017) no.2, 132.

- Iberê Kuntz, 'Quantum Corrections to the Gravitational Backreaction', published in Eur.Phys.J. C78 (2018) no.1, 3.

I have led or corroborated all the original research presented in this thesis.

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Summary

Despite the success of general relativity in explaining classical gravitational phenomena, several problems at the interface between gravitation and high energy physics remain open to date. The purpose of this thesis is to explore classical and quantum gravity in order to improve our understanding of different aspects of gravity, such as dark matter, gravitational waves and inflation. We focus on the class of higher derivative gravity theories as they naturally arise after the quantization of general relativity in the framework of effective field theory.

The inclusion of higher order curvature invariants to the action always come in the form of new degrees of freedom. From this perspective, we introduce a new formalism to classify gravitational theories based on their degrees of freedom and, in light of this classification, we argue that dark matter is no different from modified gravity.

Additional degrees of freedom appearing in the quantum gravitational action also affect the behaviour of gravitational waves. We show that gravitational waves are damped due to quantum degrees of freedom and we investigate the backreaction of these modes. The implications for gravitational wave events, such as the ones recently observed by the Advanced LIGO collaboration, are also discussed.

The early universe can also be studied in this framework. We show how inflation can be accommodated in this formalism via the generation of the Ricci scalar squared, which is triggered by quantum effects due to the non-minimal coupling of the Higgs boson to gravity, avoiding instability issues associated with Higgs inflation. We argue that the non-minimal coupling of the Higgs to the curvature could also solve the vacuum instability issue by producing a large effective mass for the Higgs, which quickly drives the Higgs field back to the electroweak vacuum during inflation.

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Chapter 1

Introduction

1.1 Prelude

Over the course of the past hundred years, general relativity has survived every single experimental test. It was able to explain with high accuracy the anomalous precession of the perihelion of Mercury, which had previously disagreed with the predictions from Newton's gravity. It also correctly predicted the value for the light bending, which was twice the value predicted by the Newtonian theory. Other observations, spanning both classical [Einstein, 1916] and modern [Dicke, 1959, Schiff, 1960] tests, such as the gravitational redshift, post-Newtonian tests, gravitational lensing, Shapiro time delay, tests of the equivalence principle, strong field tests, cosmological tests, have all favoured general relativity (see [Will, 2014] for a review). This list goes on and on and, by the time of the writing of this thesis, no experiment has ever measured any deviation from general relativity. In fact, recent observations of gravitational waves by the LIGO collaboration have only reinforced how successful general relativity turns out to be [Abbott et al., 2016].

Given the triumph of general relativity, why should we look into modifying it then? Because, as in any other scientific theory, general relativity has its limitations and it is supposed to be taken seriously only within its domain of validity. As Newtonian physics once faced its own limitations, proving itself useless in relativistic and quantum scales for example, general relativity fails tremendously in certain scales. Of course, given the substantial number of evidence, no one questions the validity of general relativity within its scope, in the same way that no one doubts that Newtonian mechanics can be used to study ballistics. It is thanks to this decoupling of scales that we are able to do physics. This is, in fact, the core of effective field theory and the very reason why we can make progress in science.

Although it is not yet clear at what scale general relativity breaks down, there are possible indications that ask for new physics. The discrepancy between the observed and the theoretical galaxy rotation curves (see Figure 1.1) [Rubin and Ford, 1970, Rubin et al., 1980], for example, cannot be accounted for by either general relativity or the standard model of particle physics, indicating that one of these theories must be incomplete. Dark matter has been postulated as a new type of particle that could account for such discrepancy. Current data from the CMB, interpreted in the ΛCDM (Lambda Cold Dark Matter) framework, shows that our universe is made up of approximately 95.1% of an unknown type of energy, where dark matter constitutes 26.8%, dark energy 68.3% and ordinary matter only 4.9% [Ade et al., 2016]. However, the same observations can be interpreted in a context where general relativity is modified, without the need of postulating new particles. Examples include the tensor-vector-scalar gravity (TeVeS) [Bekenstein, 2004], the scalar-tensor-vector gravity (STVG) [Moffat, 2006, Brownstein and Moffat, 2006b, Brownstein and Moffat, 2006a] and $f(R)$ theories [Buchdahl, 1970, Capozziello et al., 2004, Katsuragawa and Matsuzaki, 2017]. TeVeS is a modification of general relativity obtained by the inclusion of new fields to the gravitational sector. It has become popular because it reproduces MOND — a classical modification of Newton's law — in the non-relativistic regime [Milgrom, 1983a, Milgrom, 1983b, Milgrom, 1983c]. STVG (also known as MOG) was developed via the inclusion of new fields and by promoting some constants of the theory, including the Newton's constant, to scalar fields. As pointed out in [Capozziello and De Laurentis, 2012], the class of theories $f(R)$ where the Einstein-Hilbert action is replaced by a generic function of the Ricci scalar R can also shed new light into the dark matter problem.

Other indications for new physics beyond general relativity also come from late-time cosmology. Dark energy has been hypothesized in order to account for the current acceleration of the universe [Peebles and Ratra, 2003, Carroll, 2001]. The simple addition of a cosmological constant, which is the most economical explanation, leads to other problems, mainly because most quantum field theories predict

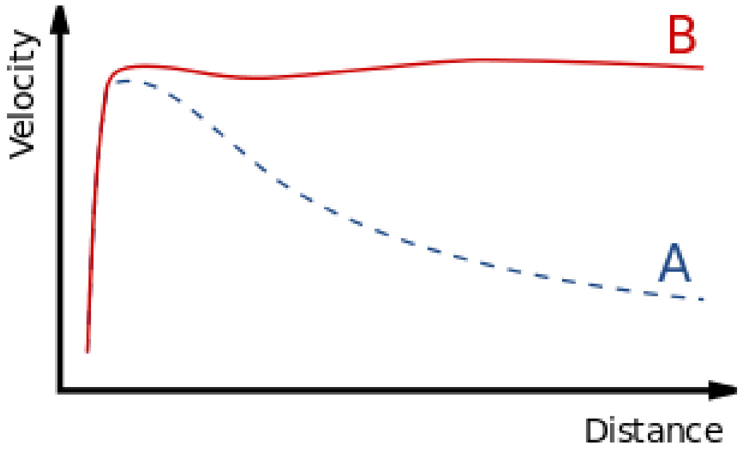

Figure 1.1: Discrepancy between predicted (A) and observed galaxy rotation curves (B). ©PhilHibbs/Wikimedia Commons/CC-BY-SA-2.0-UK.

a cosmological constant that is more than 100 orders of magnitude larger than the measured value [Adler et al., 1995]. Alternative explanations, such as the inclusion of scalar fields (known as quintessence) [Ratra and Peebles, 1988, Caldwell et al., 1998], are still very popular, but no evidence in its favour has been found so far. Another option would be to modify the gravitational sector in order to explain the accelerated expansion of today's universe.

The inflationary paradigm, initially developed to solve some inconsistencies of the Big Bang cosmology, might also necessitate physics beyond general relativity. In the simplest scenario, a new scalar field dubbed the inflaton is required to produce an exponential expansion of the early universe, resulting in the isotropic, homogeneous and flat universe that we observe today [Linde, 1982, Albrecht and Steinhardt, 1982]. Successful models include the Higgs inflation [Bezrukov and Shaposhnikov, 2008], where the scalar field is described by the Higgs boson, and Starobinsky inflation [Starobinsky, 1980], whose inflaton is hidden in the modification $f(R) = R + R^2$ of general relativity. See Section 1.3.1 for a brief review of inflation.

Lastly, there is the problem of quantum gravity, which is perhaps the most challenging problem in theoretical physics. Even though gravity is the oldest of the forces and the only one that is part of everyone's daily lives, it is still the only one lacking a full quantum treatment. Attempts to quantize gravity have led to numerous difficulties over the years, with partial success obtained only in the low-energy regime. While we are still far away from finding the right theory that could describe quantum gravity at, in principle, any energy scale, theoretical advances in the low-energy regime suggests that general relativity must be modified even below the Planck scale ['t Hooft and Veltman, 1974, Stelle, 1977, Stelle, 1978]. The renormalization procedure needed to make quantum general relativity finite at every loop order forces higher-derivative curvature invariants to appear in the action. We will discuss the quantization of general relativity in more detail in Section 1.3.2.

In the following sections, we will review basic concepts of general relativity, modified gravity and quantum gravity that will be important in the next chapters. The original contributions start at Chapter 2.

1.2 General relativity

In this section, we review the geometrical formulation of the general theory of relativity. One postulates that the spacetime is a four-dimensional Pseudo-Riemannian manifold $(\mathcal{M}, g_{\mu\nu})$ composed of a differentiable manifold $\mathcal{M}$ and a metric $g_{\mu\nu}$. Points $p \in \mathcal{M}$ are dubbed events. Test particles, being free from external forces, "free fall" along the spacetime. In a curved manifold, the trajectory of such particles are given by geodesics:

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds} = 0, \tag{1.1}$$

where

$$\Gamma^\rho{}_{\mu\nu} = \frac{g^{\rho\sigma}}{2}\left(\frac{\partial g_{\nu\sigma}}{\partial x^\mu} + \frac{\partial g_{\sigma\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right) \tag{1.2}$$

are the Christoffel symbols of the Levi-Civita connection and x^μ are local coordinates. Geodesics followed by massive particles are assumed to be time-like, whereas massless particles, e.g. photons, move along null-like geodesics. Particles that move along space-like geodesics are unphysical as they propagate at superluminal speeds. Such particles are named tachyons. Note that the geodesic equation (1.1) is independent of the particle's mass. This is exactly the equivalence principle: all particles undergo the same acceleration in the presence of a gravitational field, independently of their masses.

The Riemann tensor contains information about the curvature of the spacetime. In coordinates, it is given by

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho{}_{\nu\sigma} - \partial_\nu\Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\sigma} - \Gamma^\rho{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\sigma}. \tag{1.3}$$

Contracting the first and third indices of the Riemann tensor, one finds the Ricci tensor $R_{\sigma\nu} = g^{\rho\mu} R_{\rho\sigma\mu\nu}$. Contracting the remaining indices of the Ricci tensor, leads to the Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$.

The dynamics of the gravitational field is described by the Einstein's field equation, which reads

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.4}$$

where $T_{\mu\nu}$ is the stress-energy tensor of the matter fields. We are using units such that the speed of light is $c = 1$. Equation (1.4) describes the dynamical evolution of the metric $g_{\mu\nu}$, warping and bending spacetime according to the dynamical changes of the matter fields represented by $T_{\mu\nu}$. It is precisely the solutions of (1.4) that have led to the plethora of interesting and successful predictions of general relativity. Observe that Equation (1.4) cannot be proven from first principles. It was initially obtained by trial and error in an attempt to find a relation between curvature (geometry) and energy (physics).

However, one can adopt a variational approach whose field equations (1.4) could be deduced from. The Einstein-Hilbert action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \frac{1}{16\pi G} R + S_m \tag{1.5}$$

is the most general action containing up to two derivatives of the metric, guaranteeing that the field equation contains up to second orders of the metric. The variation of this action with respect to the metric field leads to (1.4). Needless to say, Equations (1.4) and (1.5) are equivalent and have the same physical information. Whether we start from the field equation or from the Lagrangian is just a matter of choice. They offer complementary advantages that can be used accordingly to the problem at hand.

Both the field equation (1.4) and the action (1.5) have an interesting feature. If $\phi : M \to M$ is a diffeomorphism of the spacetime M and $g_{\mu\nu}$ is a solution of (1.4) in the presence of a matter field ψ, then $\phi_* g_{\mu\nu}$ is also a solution of (1.4) in the presence of the matter field $\phi_* \psi$, where ϕ_* denotes the pushforward by ϕ. That is to say that the group of diffeomorphisms is a symmetry group of general relativity in the very same way that $U(1)$ is the symmetry group of electrodynamics. Note that, analogously to gauge theories, the invariance under diffeomorphisms is not a symmetry of the real world as it does not connect two different physical realities to the same description. It is rather a mathematical redundancy that connects two different descriptions to the same physical reality. Therefore, one cannot use such transformations to generate new solutions, but one can exploit this freedom to ease calculations. As we will see in Section 1.3.2, however, the importance of the diffeomorphism group is not restricted to easing calculations. It is rather a fundamental principle that guides us on how to look for new physics.

1.2.1 Cosmology

Cosmology is the study of the universe on very large scales. In these scales, one can employ the Copernican principle, which states that the universe is homogeneous (the metric is the same for all points in spacetime) and isotropic (every direction looks the same) on cosmological scales. This is, in fact, what has been observed in the CMB despite very small fluctuations (see Figure 1.2). The description of a

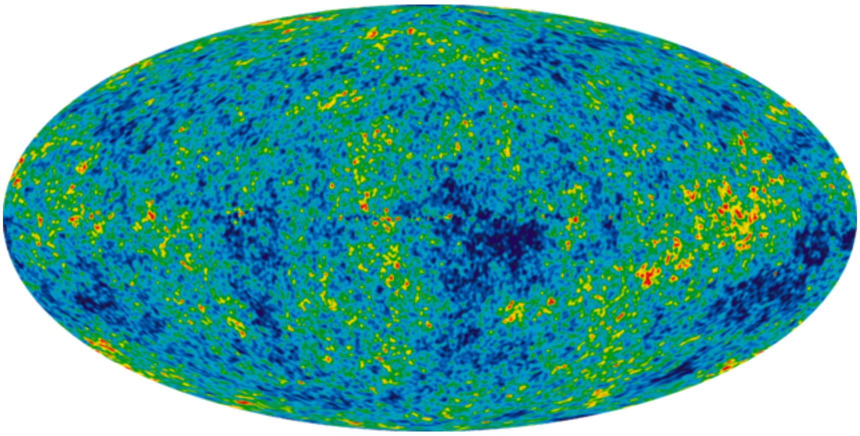

Figure 1.2: All-sky mollweide map of CMB obtained by the WMAP experiment. This image shows a temperature range of ±200 microKelvin [Bennett et al., 2003].

homogeneous and isotropic manifold is given by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{1.6}$$

where $a(t)$ is the scale factor that characterizes the relative size of spacelike hypersurfaces Σ at different times. The curvature parameter k is $+1$ for closed universes, 0 for flat universes and -1 for open universes. In this subsection we will adopt units such that $8\pi G = 1$.

For the ansatz (1.6), the dynamical evolution of the universe is dictated by the scale factor $a(t)$. Its functional form can be found by solving Einstein's equations (1.4) with the input (1.6). Let us assume that the universe is dominated by a perfect fluid with an energy-momentum tensor given by

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu + p g_{\mu\nu}, \tag{1.7}$$

where $u^\mu = \frac{dx^\mu}{d\tau}$ is the 4-velocity vector field of the fluid, p is the fluid's pressure and ρ is its energy density. Then Einstein's equation for an FLRW metric becomes

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2}, \tag{1.8}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p), \tag{1.9}$$

where overdots stand for derivative with respect to time t and H is the Hubble parameter. Equations (1.8) and (1.9) are known as Friedmann equations and they describe together the entire structure and evolution of an isotropic and homogeneous universe.

Friedmann equations (1.8) and (1.9) can be combined into the continuity equation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0, \tag{1.10}$$

which may also be written as

$$\frac{d \ln \rho}{d \ln a} = -3H(1 + \omega) \tag{1.11}$$

for the equation of state

$$\omega = \frac{p}{\rho}. \tag{1.12}$$

Integrating Equation (1.11) and using Equation (1.8) leads to the solution for the scale factor:

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+\omega)}}, & \omega \neq -1, \\ e^{Ht}, & \omega = -1. \end{cases} \tag{1.13}$$

This shows that the qualitative behavior of the cosmological evolution depends crucially on the equation of state ω. This fact will be further explored when studying inflation in Section 1.3.1, where we will be looking for fluids that violate the strong energy condition $1 + 3\omega > 0$.

1.2.2 Gravitational waves

Gravitational waves are one of the main predictions of general relativity (see e.g. [Maggiore, 2007] for an extensive review on the subject). They are tiny perturbations of the metric that propagate in spacetime, stretching it and causing observable effects on test particles. The first direct observation was made only in September 2015 by the LIGO collaboration [Abbott et al., 2016] and is considered by many the beginning of a new era in astronomy.

To study gravitational waves, one has to split the metric into a background metric and fluctuations that will be interpreted as the gravitational waves themselves. As a first approximation, we consider gravitational waves propagating in a Minkowski spacetime and we write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{1.14}$$

Plugging (1.14) into (1.4) leads to

$$\Box\bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\rho\partial^\sigma\bar{h}_{\rho\sigma} - \partial^\rho\partial_\nu\bar{h}_{\mu\rho} - \partial^\rho\partial_\mu\bar{h}_{\nu\rho} = 16\pi G T_{\mu\nu}, \tag{1.15}$$

where we have made the field redefinition $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$. We can now use the invariance under diffeomorphisms discussed above to simplify Equation (1.15). In the linear regime (1.14), a diffeomorphism locally becomes

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x). \tag{1.16}$$

Consequently, under (1.16) the field $h_{\mu\nu}$ transforms as

$$h_{\mu\nu}(x) \to h'_{\mu\nu}(x') = h_{\mu\nu}(x) - (\partial_\mu\xi_\nu + \partial_\nu\xi_\mu). \tag{1.17}$$

We can now take advantage of the freedom to choose ξ_μ to simplify (1.15). In fact, one can choose the harmonic gauge

$$\partial^\nu\bar{h}_{\mu\nu} = 0. \tag{1.18}$$

With this choice of gauge, (1.15) becomes

$$\Box\bar{h}_{\mu\nu} = 16\pi G T_{\mu\nu}, \tag{1.19}$$

which is the classical equation of a wave. We conclude that the perturbation of the metric $\bar{h}_{\mu\nu}$ behaves as a wave. Note that, from (1.18) and (1.19), one finds the conservation of the energy-momentum tensor

$$\partial^\nu T_{\mu\nu} = 0. \tag{1.20}$$

Equation (1.20) might seem contradictory as if the energy-momentum conservation holds, then there is no gravitational wave being emitted. This happens because in the linear regime around Minkowski the coupling between gravitational wave and matter is of higher order. It also illustrates that linear gravitational waves cannot carry their own sources, a fact that is also known in electrodynamics where linear eletromagnetic waves are not able to carry electric charges.

To find the energy and momentum carried away by gravitational waves, we must go beyond the linear order in $h_{\mu\nu}$ and figure out the contribution of gravitational waves to the curvature of spacetime. We can no longer use the Minkowski background for this because, otherwise, we would exclude from the beginning the possibility that gravitational waves curve the background. Thus, now we write

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{1.21}$$

where $\bar{g}_{\mu\nu}$ is a dynamical background metric. However, a problem immediately arises as there is no canonical way of defining what part of $g_{\mu\nu}$ is the background and what is the fluctuation. One could, in principle, shift x-dependent terms from $\bar{g}_{\mu\nu}$ to $h_{\mu\nu}$ and vice-versa.

A natural separation between background and fluctuations occurs when there is a clear distinction between their typical scales. Suppose the typical length scale of $\bar{g}_{\mu\nu}$ is its curvature radius L and the length scale of $h_{\mu\nu}$ is its reduced wavelength. If we assume that

$$\frac{\lambda}{L} \ll 1, \tag{1.22}$$

then $h_{\mu\nu}$ has the physical meaning of ripples in the background described by $\bar{g}_{\mu\nu}$. Note that now there are two small parameters: $h = O(|h_{\mu\nu}|)$ and $\epsilon = \lambda/L$. We first expand the equations of motion up to second order in h and then we project out the modes with a short wavelength, i.e. $\epsilon \ll 1$. The simplest way to perform this projection is by averaging over spacetime volume of size d such that $\lambda \ll d \ll L$. In this way, modes with a long wavelength of order L remain unaffected, because they are roughly constant over the volume used for averaging, while modes with a short wavelength of order λ average out because they oscillate very fast.

The separation of gravity into background and fluctuations allows one to expand metric-dependent quantities as

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + O(h^3), \tag{1.23}$$

where the bar quantities are calculated with respect to the background and the rest depends only on the fluctuation. The superscript (n) is used to indicate the order in h of the underlying term. The resulting Einstein's field equations, after expanding in h and averaging out rapid-oscillating modes, then become

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = 8\pi G(\bar{T}_{\mu\nu} + t_{\mu\nu}), \tag{1.24}$$

where

$$t_{\mu\nu} = \frac{-1}{8\pi G}\left\langle R^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(2)}\right\rangle \tag{1.25}$$

is the energy-momentum contribution from gravitational waves. The brackets in (1.25) denote an average over spacetime, which is responsible for taking only the long-wavelength modes. As it can be seen, the energy and momentum of gravitational waves result from the second order fluctuations of the metric as we had pointed out previously. When the gravitational waves are far away from the source (e.g. at the detector's vicinity), one can further simplify (1.25) by employing the limit of a flat background, imposing the TT gauge

$$h = 0, \quad h_{0\mu} = 0 \tag{1.26}$$

together with the equation of motion $\Box h_{\mu\nu} = 0$. Note that even after choosing the harmonic gauge (1.18), there is still a residual invariance left, which allows us to choose the TT gauge (1.26). In this situation, we find

$$t_{\mu\nu} = \frac{1}{32\pi G}\left\langle \partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta}\right\rangle. \tag{1.27}$$

Observe that, from the covariant conservation of the Einstein tensor

$$\bar{G}_{\mu\nu} = \bar{R}_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R} \tag{1.28}$$

with respect to the background connection ∇, one finds from Equation (1.24) that

$$\nabla^\mu(\bar{T}_{\mu\nu} + t_{\mu\nu}) = 0, \tag{1.29}$$

which shows that there is an exchange of energy and momentum between matter sources and the gravitational waves.

1.3 Modified gravity

In this section, we review some models of modified gravity that are relevant for this thesis. We start by discussing Lovelock's theorem, which limits the theories one can construct from the metric tensor alone. We then introduce modifications of general relativity in light of Lovelock's result. For a complete review of modified gravity, see [Clifton et al., 2012].

Suppose that the gravitational action contains only the metric field $g_{\mu\nu}$ and its derivatives up to second order. Then, varying the action

$$S = \int \mathrm{d}^4 x \mathcal{L}(g_{\mu\nu}) \tag{1.30}$$

leads to the Euler-Lagrange expression

$$E^{\mu\nu} = \frac{d}{dx^\rho}\left[\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho}} - \frac{d}{dx^\lambda}\left(\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho\lambda}}\right)\right] - \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} \tag{1.31}$$

and the Euler-Lagrange equation $E_{\mu\nu} = 0$. Lovelock's theorem [Lovelock, 1971, Lovelock, 1972] states that the only possible second-order Euler-Lagrange expression obtainable in a four-dimensional space from the action (1.30) is

$$E^{\mu\nu} = \alpha\sqrt{-g}\left[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right] + \lambda\sqrt{-g}g^{\mu\nu}, \tag{1.32}$$

where α and λ are constants. Therefore, any four-dimensional gravitational action involving only the metric and its derivatives of up to second order leads inevitably to Einstein's equations with or without a cosmological constant.

As a corollary, modifying general relativity requires evading one of the hypotheses of Lovelock's theorem, which includes:

- Considering fields other than the metric;
- Allowing for higher derivatives of the metric;
- Giving up locality;
- Increasing the number of spacetime dimensions;
- Considering other mathematical structures rather than Riemannian manifolds.

In this thesis, we consider the first three of these, focusing mainly on higher derivatives of the metric. As we will see, these three types of modifications are related to each other and they all show up as part of the same formalism.

Let us consider some examples of models that differ from general relativity. The scalar-tensor theories of gravity, whose typical example is Brans-Dicke theory [Brans and Dicke, 1961], is a modification of general relativity that contains an additional scalar field ϕ coupled to the Ricci scalar:

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(\phi R - \frac{\omega(\phi)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\Lambda(\phi)\right), \tag{1.33}$$

where ω is an arbitrary function and Λ is a ϕ-dependent generalization of the cosmological constant. An important feature of this theory is that under a conformal transformation

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\Omega(x)}g_{\mu\nu}, \tag{1.34}$$

where $\Omega(x) = -\frac{1}{2}\ln\phi$, (1.33) can be transformed into general relativity minimally coupled to a scalar field. Performing the transformation (1.34) in the action (1.33) leads to

$$S_E = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left(\frac{1}{16\pi}\tilde{R} - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\psi)\right), \tag{1.35}$$

where ψ is defined by

$$\frac{\partial\Omega}{\partial\psi} = \sqrt{\frac{4\pi}{3+2\omega}}$$

and

$$V(\psi) = \frac{1}{8\pi}e^{4\Omega}\Lambda$$

is the potential of ψ. Here the objects with the tilde are calculated with the transformed metric $\tilde{g}_{\mu\nu}$. The subscript in S_E stands for Einstein frame, a typical nomenclature used in the literature to refer to the action with the transformed metric $\tilde{g}_{\mu\nu}$, as opposed to the Jordan frame, which refers to the action with the original metric $g_{\mu\nu}$. Therefore, Equation (1.35) shows that in the Einstein frame the theory becomes the same as general relativity in the presence of the scalar field ψ, which is minimally coupled to gravity through the Jacobian $\sqrt{-\tilde{g}}$. This hidden scalar field is sometimes called scalaron.

There are also theories whose gravitational sector includes other types of fields other than scalars, such as the bimetric theories, tensor-vector-scalar theories (also known as TeVeS) and scalar-tensor-vector theories (not to be confused with TeVeS) [Clifton et al., 2012].

But instead of considering new explicit fields, we can simply consider higher order derivatives in the field equations as opposed to the second order differential equation of general relativity. For example, the class of models described by $f(R)$ [Sotiriou and Faraoni, 2010, De Felice and Tsujikawa, 2010], i.e.

$$S = \int \mathrm{d}^4x\sqrt{-g}f(R), \tag{1.36}$$

allows for arbitrary powers of the Ricci scalar and, consequentely, it produces terms with higher derivatives in the equations of motion. It is important to stress, however, that these theories are equivalent to Brans-Dicke theory (1.33). In fact, let $V(\phi)$ be the Legendre transform of $f(R)$ such that $\phi = f'(R)$ and $R = V'(\phi)$. Then, under a Legendre transformation of (1.36), one obtains the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\phi R - V(\phi)\right), \tag{1.37}$$

which looks exactly like Equation (1.33) with a potential $V(\phi)$ and $\omega = 0$. By extension, according to (1.35), $f(R)$ is also equivalent to general relativity with a scalar field. This is the nature of the aforementioned relation between additional fields and higher derivative terms. We will see in Chapter 2 that this idea, in fact, extends to more general theories.

An important example of this kind of theory is

$$f(R) = R + \bar{b}_1R^2, \tag{1.38}$$

known as Starobinsky gravity [Starobinsky, 1980]. This theory successfully explains cosmological inflation by assuming that the inflaton is the scalaron itself. We will see more details of this particular modification in the next subsection.

When considering higher derivatives of the metric, the Ricci scalar is not the only curvature invariant available. Inspired by the renormalization procedure after the quantization of general relativity, other curvatures invariants, such as $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, have become equally important ['t Hooft and Veltman, 1974, Stelle, 1977]. In fact, they are all invariant under the diffeomorphism group and, therefore, should be all considered together. Equation (1.38) then becomes

$$\mathcal{L} = \frac{1}{16\pi G}R + \bar{b}_1R^2 + \bar{b}_2R_{\mu\nu}R^{\mu\nu} + \bar{b}_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{1.39}$$

Classically, these terms lead to modifications of the Newton's potential that give rise to Yukawa interactions as shown by Stelle [Stelle, 1978]. More importantly, these terms are counterterms that renormalize the quantum gravitational interaction at one-loop order. We will see more on the quantization of the gravitational field in Section 1.3.2.

1.3.1 Inflation

Inflation is a period of exponential expansion of the early universe that is believed to have taken place just 10^{-34} seconds after the Big Bang. First put forward to explain the absence of magnetic monopoles in the universe, inflation later turned out to resolve many other long-standing problems in Big Bang cosmology (see [Baumann, 2011] for a review).

The conventional Big Bang theory requires very finely-tuned initial conditions to allow the universe to evolve to its current state. Inflation serves as a bridge between the today's universe and the Big Bang without the need of fine-tuning. Particularly, it explains why the universe we observe is so homogeneous, isotropic and flat.

The comoving particle horizon, i.e. the maximum distance that a light ray can travel between the instants 0 and t, for a universe dominated by a fluid with equation of state $\omega = \frac{p}{\rho}$ is

$$\tau \propto a(t)^{\frac{1}{2}(1+3\omega)}, \tag{1.40}$$

where $a(t)$ is the scale factor of the FLRW universe (1.6). Note that the qualitative behaviour of the comoving horizon τ depends on the sign of $1+3\omega$. Fluids satisfying the strong energy condition

$$1 + 3\omega > 0, \tag{1.41}$$

such as matter and radiation dominated universes, would produce a comoving horizon that increases monotonically with time, implying that the regions of the universe entering the horizon today had been far outside the horizon during the CMB decoupling. This leads to the conclusion that causally disjoint patches of the universe yielded a very homogeneous pattern at the CMB, a clear contradiction known as the horizon problem.

Combining Friedmann equation (1.8) with the continuity equation (1.10), one finds

$$\frac{d\Omega}{d\log a} = (1+3\omega)\Omega(\Omega-1), \tag{1.42}$$

where

$$\Omega = \frac{\rho}{\rho_c}. \tag{1.43}$$

The critical energy density $\rho_c = 3H^2$, H being the Hubble constant, is the energy density required for a flat universe. The differential equation (1.42) makes clear that $\Omega = 1$ is an unstable fixed point if the strong energy condition (1.41) is satisfied, thus requiring a finely-tuning initial condition in order to produce a flat universe.

The origin of both the horizon and the flatness problems seem to be related to the strong energy condition. This suggests that a simple solution can be found by violating the relation (1.41), which necessarily requires the fluid pressure to be negative

$$p < -\frac{1}{3}\rho. \tag{1.44}$$

From Friedmann equation (1.9), one can also see that (1.44) is equivalent to an accelerated expansion

$$\frac{d^2a}{dt^2} > 0. \tag{1.45}$$

Equation (1.44) can be satisfied by a nearly constant energy density ρ. The simplest way to do this is by adopting a scalar field — the inflaton — whose potential is sufficiently flat so that the field can slowly roll down the hill (see Figure 1.3), producing a roughly constant energy density. For this reason, this type of model is known as slow-roll inflation.

To see how this process occurs, let us consider a generic scalar field ϕ minimally coupled to gravity:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right), \tag{1.46}$$

where $V(\phi)$ denotes the potential of the field ϕ. The energy-momentum tensor for the scalar field is given by

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right), \tag{1.47}$$

where S_ϕ the scalar field action. It follows from (1.47) that the energy density and

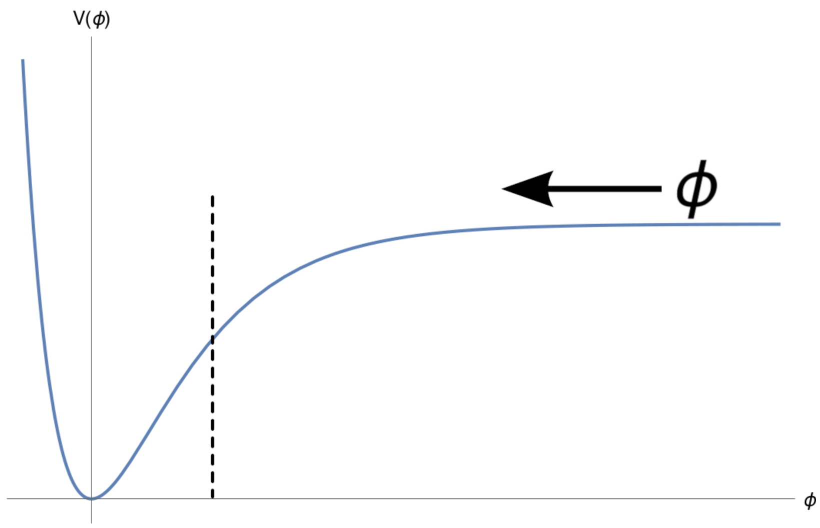

Figure 1.3: Illustration of slow-roll inflation. The inflaton starts out at the top of the hill and slowly rolls down to smaller values during inflation. The vertical dashed line represents the end of inflation.

the pressure of ϕ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1.48}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{1.49}$$

respectively. The resulting equation of state is

$$\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi} + V(\phi)}. \tag{1.50}$$

Therefore, a scalar field ϕ is able to produce inflation if the potential energy $V(\phi)$ dominates over the kinetic energy $\frac{1}{2}\dot{\phi}^2$. In this case, the equation of state becomes $\omega_\phi \approx -1$, which satisfies the condition (1.44).

Now we only need to find a specific description for the scalar field whose potential has the required form described above. Among the sea of models that one can find in the literature, Higgs and Starobinsky inflation stand out as they are both favoured by the CMB constraints [Akrami et al., 2018]. In the former, the inflaton is given by the Higgs field, which is coupled non-minimally to the Ricci scalar [Bezrukov and Shaposhnikov, 2008]:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M^2}{2}R + \xi\mathcal{H}^\dagger\mathcal{H}R\right), \tag{1.51}$$

where M is a mass parameter that contributes to the Planck mass $M_p = (8\pi G)^{-1/2}$, $\mathcal{H}$ is the $SU(2)$ scalar doublet which reads

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{1.52}$$

in the unitary gauge, where v is the vacuum expectation value and h denotes the Higgs boson. It is possible to get rid of the non-minimal coupling to gravity by transforming the theory (1.51) to the Einstein frame via the transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega = 1 + \frac{\xi h^2}{M_p^2}. \tag{1.53}$$

This leads to a non-canonical kinetic term for the Higgs field that can be canonically normalized by a field redefinition of the form

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_p^2}{\Omega^4}}. \tag{1.54}$$

Then, the action in the Einstein frame reads

$$S_E^{\text{Higgs}} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left(\frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) \right), \tag{1.55}$$

where

$$U(\chi) = \frac{1}{\Omega(\chi)^4} \frac{\lambda}{4} \left(h(\chi)^2 - v^2 \right)^2. \tag{1.56}$$

For small field values $h \approx \chi$ and $\Omega^2 \approx 1$, thus the potential has the well-known Mexican hat shape of the initial Higgs field h. On the other hand, for large field values of $\chi \gg \sqrt{6} M_p$, one finds [Bezrukov and Shaposhnikov, 2008]

$$h \approx \frac{M_p}{\xi} \exp \frac{\chi}{\sqrt{6} M_p} \tag{1.57}$$

and

$$U(\chi) = \frac{\lambda M_p^4}{4\xi^2} \left(1 + \exp \frac{-2\chi}{\sqrt{6} M_p} \right)^{-2}. \tag{1.58}$$

Hence, the potential $U(\chi)$ is exponentially flat and has the plateau similar to Figure 1.3, making slow-roll inflation possible.

Starobinsky inflation, on the other hand, is described by the scalaron of Starobinsky gravity (1.38). In the Einstein frame, the theory takes the form [Starobinsky, 1980]

$$S_E^{\text{Starobinsky}} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left(\frac{1}{16\pi G} \tilde{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right), \tag{1.59}$$

where

$$V(\phi)=\frac{M_p^4}{\alpha}\left(1-\exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}\right)\right)^2. \qquad (1.60)$$

Thus, it also produces the flatness in the potential for high values of the field ϕ.

1.3.2 Quantum gravity

Although little is known about quantum gravity in the ultraviolet regime, many advances have been achieved in recent years using effective field theory techniques to study low energy quantum gravitational effects [Buchbinder et al., 1992, Vilkovisky, 1992]. The popular belief that general relativity cannot be quantized is, at best, incomplete and precedes all modern knowledge of quantum field theories. This misconception is commonly associated with the fact that the renormalization procedure generates an infinite number of counterterms in the gravitational action. The coefficient of each counterterm is free and must be fixed by observations, thus indicating that the theory loses its predictive power and becomes unfalsifiable. However, just a small set of free parameters shows up at low energies since high order terms are suppressed by inverse powers of the Planck mass $M_p \sim 10^{19}$GeV. The high value of M_p is what makes classical general relativity so successful and quantum effects so difficult to probe experimentally.

Divergences appearing at one-loop order, for example, are proportional to R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, and can be renormalized by the inclusion of counterterms to the Lagrangian ['t Hooft and Veltman, 1974]:

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G}R-\Lambda+\bar{b}_1R^2+\bar{b}_2R_{\mu\nu}R^{\mu\nu}+\bar{b}_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right]. \qquad (1.61)$$

The coefficients $\bar{b}_i$ are bare constants and not observables. They are chosen so that divergences at one-loop order turn out to be finite. The curvature squared terms are not all independent due to a topological restriction that occurs only in four dimensions. This relation goes by the name of Gauss-Bonnet theorem and states that the integral of $G=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$ over a compact oriented manifold $\mathcal{M}$ is related to the Euler characteristic $\chi(\mathcal{M})$, thus the integral itself is a topological invariant and its variation results in:

$$\delta\int \mathrm{d}^4x\sqrt{-g}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2\right)=0. \qquad (1.62)$$

One can then eliminate one of the invariants in terms of the others. We choose to eliminate $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ and, therefore, we can simply ignore the last term in (1.61).

This theory can be quantized using the background field method [Barvinsky and Vilkovisky, 1985]. We perturb the metric $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ and integrate out the fluctuations $h_{\mu\nu}$ using the Feynman path integral formalism:

$$e^{-\Gamma} = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\Phi\, e^{-(S[g+h]+S_m[\Phi])}, \tag{1.63}$$

where S_m is the action of matter sector and Φ represents a set of arbitrary matter fields (not necessarily scalar fields). The quantum effective action Γ describes quantum gravitational phenomena and can be used to investigate the phenomenology of quantum gravity at low energies (below the Planck scale). As expected, the general result is quite cumbersome even at the leading order, containing several terms that contribute equally [Codello and Jain, 2016]. However, if one considers only the limit of massless or very light fields, the outcome turn out to be very neat. In this limit, non-localities are expected to show up as massless fields mediate long-range interactions. In fact, the quantum action in this case is given by [Barvinsky and Vilkovisky, 1987, Barvinsky and Vilkovisky, 1985, Barvinsky and Vilkovisky, 1990, Donoghue and El-Menoufi, 2014]

$$\Gamma = \Gamma_L + \Gamma_{NL}, \tag{1.64}$$

where the local part reads

$$\Gamma_L = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \Lambda + b_1(\mu)R^2 + b_2(\mu)R_{\mu\nu}R^{\mu\nu}\right), \tag{1.65}$$

and the non-local one reads

$$-\Gamma_{NL} = \int \mathrm{d}^4x\sqrt{-g}\Bigg[c_1 R\ln\left(-\frac{\Box}{\mu^2}\right)R + c_2 R_{\mu\nu}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu} \tag{1.66}$$

$$+ c_3 R_{\mu\nu\rho\sigma}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma}\Bigg]. \tag{1.67}$$

The log operator is defined as

$$\ln\frac{-\Box}{\mu^2} = \int_0^\infty \mathrm{d}s\left(\frac{1}{\mu^2+s} - G(x,x',\sqrt{s})\right), \tag{1.68}$$

where $G(x,x';\sqrt{s})$ is the Green's function of

$$(-\Box + k^2)G(x,x';k) = \delta^4(x-x') \tag{1.69}$$

with proper boundary conditions. The non-local piece represents the infrared portion of quantum gravity and, as such, it is completely independent of the UV completion. In fact, the coefficients c_i are genuine predictions of the quantum theory of gravity. They are determined once the matter fields Φ that are integrated out in Equation (1.63) and their respective spins are specified; see Table 1.1. The total contribution to each coefficient is given by simply summing the contribution from each matter species. For example, for N_s minimally coupled scalars ($\xi = 0$) and N_f fermions, we have

$$c_1 = \frac{5}{11520\pi^2} N_s - \frac{5}{11520\pi^2} N_f. \tag{1.70}$$

The local action, on the other hand, represents the high energy portion of quantum

	c_1	c_2	c_3
real scalar	$5(6\xi - 1)^2/(11520\pi^2)$	$-2/(11520\pi^2)$	$2/(11520\pi^2)$
Dirac spinor	$-5/(11520\pi^2)$	$8/(11520\pi^2)$	$7/(11520\pi^2)$
vector	$-50/(11520\pi^2)$	$176/(11520\pi^2)$	$-26/(11520\pi^2)$
graviton	$430/(11520\pi^2)$	$-1444/(11520\pi^2)$	$424/(11520\pi^2)$

Table 1.1: Values of the coefficients c_i for each spin (ξ is the non-minimal coupling coefficient of scalars to gravity) [Donoghue and El-Menoufi, 2014]. Each value must be multiplied by the number of fields of its category. The total value of each coefficient is given by adding up all contributions. See Equation (1.70) for an example.

gravity. As a result, the coefficients b_i cannot be determined from first principles. They are renormalized parameters which must be fixed by observations (as opposed to $\bar{b}_i$ that are not observables). They depend on the renormalization scale μ in such a way that they cancel the μ-dependence of the non-local logarithm operator. Thus, the total effective action Γ is independent of μ. The renormalization group equation is

$$\mu \partial_\mu b_i = \beta_i, \tag{1.71}$$

where $\beta_i = -2c_i$ are the beta functions, thus the running of b_i can also be obtained straightforwardly from Table 1.1. The relation between the beta functions of b_i and the coefficients c_i is expected because the resultant action Γ must be independent of μ as explained above.

While we are yet far away from being able to probe quantum gravity experimentally, the above shows that we can use standard techniques from quantum field theory to quantize general relativity. Needless to say, this is the most conservative approach of all. If we insist that general relativity and quantum field theory are correct descriptions of our world below the Planck scale — and as far as observations go, this is indeed true —, we can then quantize general relativity in the very same way that the other interactions are quantized. As a result, we can make genuine and model-independent predictions of quantum gravity without appealing to *ad hoc* hypotheses.

1.4 Outline of this thesis

This thesis contains a collection of published work that was completed as part of my doctoral degree, which concerns modifications of gravity and its implications to gravitational waves, inflation and dark matter. It is organized as follows:

- In Chapter 2, based on [Calmet and Kuntz, 2017], we set up a new formalism to classify gravitational theories based on their degrees of freedom and how they interact with the matter sector. We argue that every modification of the action performed by the inclusion of additional curvature invariants inevitably leads to new degrees of freedom. This can be seen by diagonalizing the action, either via field redefinitions or through the linearization process around a given background, and further canonically normalizing it. A particular example of this is the well-known equivalence of $f(R)$ and general relativity minimally coupled to a scalar field. We also give less obvious examples where invariants such as $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are also present. As an application, we consider the dark matter problem and we show that particle dark matter models and modified gravity models are actually equivalent as they are both based on new degrees of freedom.

- Chapter 3 is based on [Calmet et al., 2016] and we study gravitational waves from the effective field theory perspective. We show that one-loop quantum corrections lead to modifications of the analytic structure of the graviton propagator, yielding the so-called dressed propagator for the graviton. The dressed propagator contains additional complex poles, thus effectively leading to new propagating degrees of freedom. The real part is interpreted as the mass of the modes and the imaginary part is interpreted as their width. We study the consequences of these additional degrees of freedom for gravitational waves. Particularly, we show that gravitational waves become damped when quantum gravitational effects are taken into account. The consequences for gravitational wave events, such as GW 150914, recently observed by the Advanced LIGO collaboration, are discussed.

- The effect on the energy of gravitational waves due to one-loop corrections are studied in Chapter 4, which contains [Kuntz, 2018]. By performing the short-wave formalism, we separate the modes with a long wavelength from the ones with a short wavelength. The former contains information about the contribution of gravitational waves to the spacetime curvature, while the latter affects the propagation of gravitational waves in curved spacetimes. The energy-momentum tensor $t_{\mu\nu}$ of gravitational waves is then calculated, thus showing how quantum effects contribute to the backreaction of gravitational waves. The trace of the effective energy-momentum tensor is shown to be non-vanishing and, hence, it contributes to the cosmological constant. The first bound on the amplitude of the massive mode is found by comparing the gravitational wave energy density $\rho = t_{00}$ with LIGO's data. In addition, we show that the propagation of gravitational waves in curved spacetimes can be obtained by covariantization of the gravitational wave equation in flat spacetime, i.e. by simply replacing $\eta_{\mu\nu}$ and ∂_μ by $\bar{g}_{\mu\nu}$ and ∇_μ, respectively.

- Chapter 5, which is composed by [Calmet and Kuntz, 2016], is designated to investigate an interesting interplay between Higgs and Starobinsky inflation. We show that Starobinsky inflation, based on the modification $f(R) = R + \alpha R^2$ of general relativity, can be generated by quantum effects due to the non-minimal coupling of the Higgs to gravity. After quantization of the gravitational action, the coefficient α acquires a dependence on the coefficient ξ of the coupling between the Higgs and the Ricci scalar. For large values of ξ, one obtains the required value for α so that Starobinsky inflation can take place. This formalism avoids instability issues caused by large values of the Higgs boson as the scalaron in the Starobinsky model is the only field required to take large values in the early universe.

- In Chapter 6 [Calmet et al., 2018], we study the instability problem in a more general setting, i.e. when the inflaton is not restricted to be the Higgs field. In these cases, even though inflation is not driven by the Higgs, the direct coupling between the Higgs and the curvature could still cause problems during and after inflation as claimed in [Herranen et al., 2014, Herranen et al., 2015]. We argue that, after canonically normalizing the Higgs field, an interaction between the inflationary potential and the Higgs is induced. This interaction produces a large effective mass for the Higgs, which quickly drives the Higgs boson back to the electroweak vacuum during inflation, thus stabilizing the false vacuum.

- Lastly, we draw the conclusions and discuss future directions in Chapter 7.

Chapter 2

What is modified gravity and how can we differentiate it from particle dark matter?

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An obvious criterion to classify theories of modified gravity is to identify their gravitational degrees of freedom and their coupling to the metric and the matter sector. Using this simple idea, we show that any theory which depends on the curvature invariants is equivalent to general relativity in the presence of new fields that are gravitationally coupled to the energy-momentum tensor. We show that they can be shifted into a new energy-momentum tensor. There is no a priori reason to identify these new fields as gravitational degrees of freedom or matter fields. This leads to an equivalence between dark matter particles gravitationally coupled to the standard model fields and modified gravity theories designed to account for the dark matter phenomenon. Due to this ambiguity, it is impossible to differentiate experimentally between these theories and any attempt to do so should be classified as a mere interpretation of the same phenomenon.

2.1 Introduction

General relativity and the standard model of particle physics have both been extremely successful in describing our universe both on cosmological scales as well as on microscopic scales. Despite this amazing success, some observations cannot be explained by these otherwise extremely successful models. For example, the cosmic microwave background, the rotation curves of galaxies or the bullet cluster to quote a few [Hooper, 2010], suggest that there is a new form of matter that does not shine in the electromagnetic spectrum. Dark matter is not accounted for by either general relativity or the standard model of particle physics [1]. While a large fraction of the high energy community is convinced that dark matter should be described by yet undiscovered new particles, it remains an open question whether this phenomenon requires a modification of the standard model or of general relativity. Here we want to raise a slightly different question namely whether the distinction between modified gravity or new particles is always clear. We will show that this is not always the case.

Models of modified gravity are attractive given the frustrating success of the standard model at surviving its confrontation with the data of the Large Hadron Collider. Modified theories of gravity have been developed in the hope of finding solutions to the dark matter or dark energy questions. All sorts of theories have been proposed in order to address these problems. Among them, we can find higher derivative gravity theories (e.g. $f(R)$), the scalar-tensor theories (e.g. Brans-Dicke), the non-metric theories (e.g. Einstein-Cartan theory), just to cite a few, see [Clifton et al., 2012] for a substantial review.

In the context of quantum field theories, fields are just dummy variables as the action is formulated as a path integral over all field configurations. This implies a reparametrization invariance of field theories. In gravitational theories (see e.g. [Calmet and Yang, 2013]), this corresponds simply to the freedom to pick a specific frame to define one's model. The reparametrization invariance makes it difficult to differentiate between the plethora of models as depending on which field variables

[1] One should note though that the possibility of Planck mass quantum black holes remnants [Chen et al., 2015, Calmet, 2015] is not excluded, but it is difficult to find an inflationary model that produces them at the end of inflation

are picked, the very same model could appear to be very different in two different frames. One of the aims of this article is to apply a very simple and obvious criterion to classify gravitational theories. The idea is to identify their gravitational degrees of freedom by looking at the poles in the field equations and carefully identifying the coupling of these poles to the metric and the energy-momentum tensor (matter sector). This enables one to unambiguously compare two gravitational models. Some work in this direction was done in the past [Magnano, 1995], but the focus was given to the different action principles, namely the metric, metric-affine and affine formalisms. Here we present a broader approach which can be applied to any kind of theory independently of its action principle.

In this paper, we aim to propose a general framework where gravitational theories can be compared to each other so that we are able to classify them into different classes of physically equivalent theories. The classification method will be presented in Section 2.2 together with some examples. In Section 2.3 we apply these ideas to the dark matter problem and show that the distinction between modified gravity or dark matter as a new particle is not always so clear. In particular, we show that any theory which depends on the curvature invariants is equivalent to general relativity in the presence of new fields that are gravitationally coupled to the energy-momentum tensor. We show that they can be shifted into a new energy-momentum tensor. Modified dark matter is thus equivalent to new degrees of freedom (i.e. particles) that are coupled gravitationally to regular matter. We then make the conclusions in Section 2.4.

2.2 Classification of extended theories of gravity

Fields in a quantum field theory are dummy variables. The same applies to the metric in a gravitational theory. Therefore two apparently very different gravitational theories can actually turn out to be mathematically equivalent when expressed in the correct variables. A famous example is the $f(R)$ theory:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} f(R) + \mathcal{L}_M \right) \tag{2.1}$$

where $f(R)$ is a polynomial of the Ricci scalar. When mapping the theory from the Jordan to the Einstein frame it becomes obvious that $f(R)$ is equivalent to usual general relativity with a scalar field that is gravitationally coupled to matter. Indeed, it is well known that after a Legendre transformation followed by a conformal rescaling $\tilde{g}_{\mu\nu} = f'(R)g_{\mu\nu}$, $f(R)$ theory can be put in the form [De Felice and Tsujikawa, 2010]

$$\begin{aligned} S &= \int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{16\pi G}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \\ &\quad + \int d^4x\sqrt{-\tilde{g}}F^{-2}(\phi)\mathcal{L}_M(F^{-1}(\phi)\tilde{g}_{\mu\nu}, \psi_M), \end{aligned} \tag{2.2}$$

where

$$\phi \equiv \sqrt{\frac{3}{16\pi G}}\log F, \tag{2.3}$$

$$F(\phi) \equiv f'(R(\phi)). \tag{2.4}$$

Hence all the matter fields acquire a universal coupling to a new scalar field ϕ through the factor $F^{-1}(\phi)$. Massless gauge bosons are exceptions since their Lagrangians are invariant under the metric rescaling. This simple example demonstrates that, despite the apparent simplicity of $f(R)$ which naively seems to only depend on the metric $g_{\mu\nu}$, the theory also contains an extra scalar degree of freedom.

This well-known example can be generalized to any gravitational theory. A general gravitational theory, assuming that it is a metric theory, will have at least one metric tensor (if it is to have general relativity in some limit) and fields of different spins. We will assume that this theory can be described by an action $S = S[\phi^1_{\alpha_1}, \ldots, \phi^n_{\alpha_n}]$, where $\phi^i_{\alpha_i}$ are the fields and α_i represents generically the number of indices, i.e. the type of the field (e.g. scalar, tensor, etc). The coupling of the gravitational degrees of freedom to matter $\mathcal{L}_\mathcal{M}$ needs to be specified. An algorithm to classify gravitational theories, in the sense of comparing two gravitational theories, can be designed as follows.

1) The first step then is to find all of the gravitational degrees of freedom of each theory.

2) Verify how these degrees of freedom couple to the metric tensor, to the matter degrees of freedom as well as to themselves.

The first step might sound obvious if what we have in mind are theories with a canonical Lagrangian. However, this is not the case for gravitational theories where degrees of freedom are hidden in terms in the action with a higher number of derivatives (higher than two) acting on the metric as we have seen in the previous example. The identification of the degrees of freedom can be done as usual by linearizing the equations of motion around a fixed background $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$, identifying the full propagator $\mathcal{P}_{\alpha\beta\mu\nu}$:

$$\mathcal{D}_{\alpha\beta\mu\nu}h^{\mu\nu} = T_{\alpha\beta} \implies \mathcal{P}_{\alpha\beta\mu\nu} = \mathcal{D}^{-1}_{\alpha\beta\mu\nu}, \tag{2.5}$$

where $\mathcal{D}_{\alpha\beta\mu\nu}$ is the modified wave operator. The position of the poles will reveal the different degrees of freedom hidden in a potentially clumsy choice of variables. These degrees of freedom can be made explicit in the action, in some cases after the kinetic terms have been canonically normalized.

Having identified the degrees of freedom of the theories, we are left with the task of classifying their dynamics. For this purpose, there are two different approaches: one can either apply suitable transformations on the fields on the level of the Lagrangian in order to try to map one theory to another or one can proceed by calculating straightforwardly the equations of motion of each of them and then checking if they match in the end. It has to be stressed that both approaches lead to the same outcome and therefore we can conveniently choose how to proceed accordingly to the theory at hand.

In our previous example, we have shown that equation (2.2) implies that $f(R)$ theories can be described by a scalar field minimally coupled to general relativity. This means that $f(R)$ is formally equivalent to general relativity in the presence of a scalar field. Indeed, both theories have the same degrees of freedom and their actions can be mapped into each other by field redefinitions. As can be seen from (2.2), it is just a matter of choice whether the new scalar field ϕ belongs to the gravity sector or to the matter sector.

The same reasoning can be used for more general theories where it is also possible to identify new degrees of freedom besides the metric and the scalar of Equation (2.2). In fact, an additional massive spin-2 is present in the generic theory $f(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})$ [Magnano and Sokolowski, 2003, Nunez and Solganik, 2005, Chiba, 2005]. As this is an important example for our considerations, we will now reproduce this well known fact using the results of [Hindawi et al., 1996].

Consider the theory

$$
\begin{aligned}
S = \frac{1}{2\kappa^2} \int \mathrm{d}^4x\sqrt{-g}\Big[R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} \\
+ \mathcal{L}_M(g_{\mu\nu}, \phi_\alpha)\Big], \qquad (2.6) \\
= \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}\left[R + \frac{1}{6{m_0}^2}R^2 - \frac{1}{2{m_2}^2}C^2 + \mathcal{L}_M(g_{\mu\nu}, \phi_\alpha)\right],
\end{aligned}
$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, ${m_0}^{-2} = 6\alpha+2\beta+2\gamma$ and ${m_2}^{-2} = -\beta-4\gamma$. The matter sector is represented by $\mathcal{L}_M(g_{\mu\nu}, \phi_\alpha)$, where ϕ_α denotes a set of arbitrary fields of any spin, but for the sake of the argument we will ignore the matter Lagrangian for a while. Now we introduce an auxiliary scalar field λ:

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}\left[R + \frac{1}{6{m_0}^2}R^2 - \frac{1}{6{m_0}^2}(R - 3m_0^2\lambda)^2 - \frac{1}{2{m_2}^2}C^2\right] \qquad (2.7) \\
&= \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}\left[(1+\lambda)R - \frac{3}{2}m_0^2\lambda^2 - \frac{1}{2{m_2}^2}C^2\right] \\
&= \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}\left[e^\chi R - \frac{3}{2}m_0^2(e^\chi - 1)^2 - \frac{1}{2{m_2}^2}C^2\right].
\end{aligned}
$$

In the last line, we made the redefinition $\chi = \log(1+\lambda)$. The equation of motion for λ is algebraic and given by $R = 3m_0^2\lambda$. Substituting this back into the action gives the original theory back. Therefore, both theories are equivalent. Now we can perform a conformal transformation $\tilde{g}_{\mu\nu} = e^\chi g_{\mu\nu}$

$$
S = \frac{1}{2\kappa^2} \int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} - \tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2 - \tfrac{3}{2}{m_0}^2\left(1 - e^{-\chi}\right)^2 - \frac{1}{2{m_2}^2}\tilde{C}^2\right], \qquad (2.8)
$$

where we have used the fact that C^2 is invariant under conformal transformations. Now we can rewrite the above action as

$$
\begin{aligned}
S = \frac{1}{2\kappa^2} \int d^4x\sqrt{-\tilde{g}}\Big[\tilde{R} - \tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2 - \tfrac{3}{2}{m_0}^2\left(1 - e^{-\chi}\right)^2 \\
- \frac{1}{2m_2^2}\left(\tilde{R}_{\lambda\mu\nu\rho}\tilde{R}^{\lambda\mu\nu\rho} - 2\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + \tfrac{1}{3}\tilde{R}^2\right)\Big] \qquad (2.9) \\
= \frac{1}{2\kappa^2} \int d^4x\sqrt{-\tilde{g}}\Big[\tilde{R} - \tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2 - \tfrac{3}{2}{m_0}^2\left(1 - e^{-\chi}\right)^2 - \frac{1}{{m_2}^2}\left(\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} - \tfrac{1}{3}\tilde{R}^2\right) \\
- \frac{1}{2m_2^2}\left(\tilde{R}_{\lambda\mu\nu\rho}\tilde{R}^{\lambda\mu\nu\rho} - 4\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + \tilde{R}^2\right)\Big]. \qquad (2.10)
\end{aligned}
$$

Due to the Gauss-Bonnet theorem, the last term of the last line vanishes and we end up with

$$
\begin{aligned}
S = \frac{1}{2\kappa^2} \int d^4x\sqrt{-\tilde{g}}\Big[\tilde{R} - \tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2 - \tfrac{3}{2}{m_0}^2\left(1 - e^{-\chi}\right)^2 \\
- \frac{1}{{m_2}^2}\left(\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} - \tfrac{1}{3}\tilde{R}^2\right)\Big]. \qquad (2.11)
\end{aligned}
$$

We then add a auxiliary symmetric tensor field $\tilde{\pi}_{\mu\nu}$:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\Big[\tilde{R} - \tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2 - \tfrac{3}{2}{m_0}^2\left(1-e^{-\chi}\right)^2 - \tilde{G}_{\mu\nu}\tilde{\pi}^{\mu\nu} + \tfrac{1}{4}{m_2}^2\left(\tilde{\pi}_{\mu\nu}\tilde{\pi}^{\mu\nu} - \tilde{\pi}^2\right)\Big]. \tag{2.12}$$

where $\tilde{\pi} = \tilde{\pi}_{\mu\nu}\tilde{G}^{\mu\nu}$ and $\tilde{G}_{\mu\nu}$ is the Einstein tensor in the Einstein frame. The $\tilde{\pi}_{\mu\nu}$ equation of motion is

$$\tilde{G}_{\mu\nu} = \tfrac{1}{2}m_2^2\left(\tilde{\pi}_{\mu\nu} - \tilde{g}_{\mu\nu}\tilde{\pi}\right), \tag{2.13}$$

which can be written in the form

$$\tilde{\pi}_{\mu\nu} = 2{m_2}^{-2}\left(\tilde{R}_{\mu\nu} - \frac{1}{6}\tilde{g}_{\mu\nu}\tilde{R}\right). \tag{2.14}$$

Substituting this equation of motion back into the action (2.12) leads to the action (2.11), thus they are equivalent. Therefore, we have proven the equivalence between the actions (2.6) and (2.12). From action (2.12), we can see that our original theory is equivalent to general relativity in the presence of a canonical scalar field and a non-canonical symmetric rank-2 tensor field. It is tempting to say that $\tilde{\pi}_{\mu\nu}$ is a spin-2 field, but this is not obvious at this stage. So far, $\tilde{\pi}_{\mu\nu}$ describes 10 degrees of freedom, while a massive spin-2 describes only 5. In the simplest case of a free spin-2 field $\phi_{\mu\nu}$ on a flat spacetime, such field is described by the Pauli-Fierz action. The divergence and the trace of its equation of motion imply the conditions:

$$\partial^\mu\phi_{\mu\nu} = 0, \quad \phi = 0, \tag{2.15}$$

which constrains the number of degrees of freedom to 5. For a general spin-2 field though, the above conditions are no longer satisfied, but we can still find generalized conditions in order to reduce the number of degrees of freedom to 5. From the trace of the $\tilde{g}_{\mu\nu}$ equation of motion and from the divergence of the $\tilde{\pi}_{\mu\nu}$ equation of motion we find:

$$\tilde{\nabla}^\mu\left(\tilde{\pi}_{\mu\nu} - \tilde{g}_{\mu\nu}\tilde{\pi}\right) = 0, \tag{2.16}$$

$$\tilde{\pi} - {m_2}^{-2}\left[\left(\tilde{\nabla}\chi\right)^2 + 2{m_0}^2\left(1-e^{-\chi}\right)^2\right] = 0. \tag{2.17}$$

The above conditions give 5 constraints, thus reducing the number of degrees of freedom described by $\tilde{\pi}_{\mu\nu}$ to 5. Now $\tilde{\pi}_{\mu\nu}$ is a pure spin-2 field. Furthermore, if we linearize our theory, the above conditions give Pauli-Fierz conditions back and, therefore, $\tilde{\pi}_{\mu\nu}$ would produce a canonical spin-2 field. Thus, we managed to find a spin-2 field, even though it does not appear canonically in the Lagrangian.

To canonically normalize the field $\tilde{\pi}_{\mu\nu}$, we need to perform another transformation on the metric. We start by writing the Lagrangian (2.12) in the form

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\Bigg\{\Bigg[\Big(1+\tfrac{1}{2}\tilde{\pi}\Big)\tilde{g}^{\mu\nu}-\tilde{\pi}^{\mu\nu}\Bigg]\tilde{R}_{\mu\nu}+\tfrac{1}{4}{m_2}^2\Big(\tilde{\pi}_{\mu\nu}\tilde{\pi}^{\mu\nu}-\tilde{\pi}^2\Big) \\ & -\tfrac{3}{2}\left(\tilde{\nabla}\chi\right)^2-\tfrac{3}{2}{m_0}^2\Big(1-e^{-\chi}\Big)^2\Bigg]\Bigg\}. \end{aligned} \tag{2.18}$$

To get a canonical Einstein-Hilbert term, we need to redefine the metric as

$$\sqrt{-\bar{g}}\bar{g}^{\mu\nu}=\sqrt{-\tilde{g}}\left[\left(1+\tfrac{1}{2}\tilde{\pi}\right)\tilde{g}^{\mu\nu}-\tilde{\pi}^{\mu\nu}\right], \tag{2.19}$$

which leads to the transformations

$$\bar{g}^{\mu\nu} \;=\; (\det A)^{-1/2}\tilde{g}^{\mu\lambda}A^{\nu}_{\lambda} \tag{2.20}$$

$$A^{\nu}_{\lambda} \;=\; (1+\tfrac{1}{2}\phi)\delta^{\nu}_{\lambda}-\phi^{\nu}_{\lambda}. \tag{2.21}$$

We have introduced the new notation $\phi^{\nu}_{\mu}=\tilde{\pi}^{\nu}_{\mu}$ to emphasize that the indices of $\phi_{\mu\nu}$ are raised and lowered using $\bar{g}_{\mu\nu}$, while the indices of $\tilde{\pi}_{\mu\nu}$ were raised and lowered using $\tilde{g}_{\mu\nu}$. Therefore, in the new variables the Lagrangian reads

$$\begin{aligned} S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}\Bigg[& \bar{R}-\tfrac{3}{2}\left(A^{-1}(\phi_{\sigma\tau})\right)_{\mu}{}^{\nu}\,\bar{\nabla}^{\mu}\chi\bar{\nabla}_{\nu}\chi-\tfrac{3}{2}\left(\det A(\phi_{\sigma\tau})\right)^{-1/2}\left(1-e^{-\chi}\right)^2 \\ & -\bar{g}^{\mu\nu}\left(C^{\lambda}{}_{\mu\rho}(\phi_{\sigma\tau})C^{\rho}{}_{\nu\lambda}(\phi_{\sigma\tau})-C^{\lambda}{}_{\mu\nu}(\phi_{\sigma\tau})C^{\rho}{}_{\rho\lambda}(\phi_{\sigma\tau})\right) \\ & +\tfrac{1}{4}{m_2}^2\left(\det A(\phi_{\sigma\tau})\right)^{-1/2}\left(\phi_{\mu\nu}\phi^{\mu\nu}-\phi^2\right)\Bigg], \end{aligned} \tag{2.22}$$

where

$$C^{\lambda}{}_{\mu\nu}=\tfrac{1}{2}(\tilde{g}^{-1})^{\lambda\rho}(\bar{\nabla}_{\mu}\tilde{g}_{\nu\rho}+\bar{\nabla}_{\nu}\tilde{g}_{\mu\rho}-\bar{\nabla}_{\rho}\tilde{g}_{\mu\nu}). \tag{2.23}$$

Due to the transformation (2.20), the metric $\tilde{g}=\tilde{g}(\phi_{\mu\nu})$ now depends on the spin-2 field. Thus the spin-2 kinetic term appears explicitly in the action through $C^{\lambda}{}_{\mu\nu}$.

In the presence of external matter the argument goes in the same way, except that after performing the transformations the matter Lagrangian becomes

$\mathcal{L}_M(e^{-\chi}\tilde{g}_{\mu\nu}(\phi_{\sigma\tau}),\phi_\alpha)$ and the action reads

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}\Big[\bar{R}-\tfrac{3}{2}\left(A^{-1}(\phi_{\sigma\tau})\right)_\mu{}^\nu\,\bar{\nabla}^\mu\chi\bar{\nabla}_\nu\chi-\tfrac{3}{2}\left(\mathrm{det}A(\phi_{\sigma\tau})\right)^{-1/2}\left(1-e^{-\chi}\right)^2$$
$$-\bar{g}^{\mu\nu}\left(C^\lambda{}_{\mu\rho}(\phi_{\sigma\tau})C^\rho{}_{\nu\lambda}(\phi_{\sigma\tau})-C^\lambda{}_{\mu\nu}(\phi_{\sigma\tau})C^\rho{}_{\rho\lambda}(\phi_{\sigma\tau})\right) \tag{2.24}$$
$$+\tfrac{1}{4}{m_2}^2\left(\mathrm{det}A(\phi_{\sigma\tau})\right)^{-1/2}\left(\phi_{\mu\nu}\phi^{\mu\nu}-\phi^2\right)+\bar{\mathcal{L}}_M(e^{-\chi}\tilde{g}_{\mu\nu}(\phi_{\sigma\tau}),\phi_\alpha)\Big].$$

where

$$\bar{\mathcal{L}}_M=e^{-2\chi}(\det A(\phi_{\mu\nu}))^{-1/2}\mathcal{L}_M. \tag{2.25}$$

We see that, in general, external matter couples minimally to the usual graviton through the Jacobian $\sqrt{-\bar{g}}$ and non-minimally to the fields χ and $\phi_{\mu\nu}$.

In the following, we will calculate explicitly the coupling between external matter and the additional degrees of freedom χ and $\phi_{\mu\nu}$. Consider a matter Lagrangian being composed of a scalar, a vector and a spinor field:

$$\mathcal{L}_M=\mathcal{L}_0+\mathcal{L}_1+\mathcal{L}_{1/2}, \tag{2.26}$$

where

$$\mathcal{L}_0 = \tfrac{1}{2}\nabla_\mu\sigma\nabla^\mu\sigma \tag{2.27}$$
$$\mathcal{L}_1 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.28}$$
$$\mathcal{L}_{1/2} = i\bar{\psi}\partial\!\!\!/\psi. \tag{2.29}$$

After transforming the metric to $\bar{g}_{\mu\nu}$ (i.e., $g_{\mu\nu}\to\tilde{g}_{\mu\nu}\to\bar{g}_{\mu\nu}$), we get

$$\bar{\mathcal{L}}_0 = \tfrac{1}{2}e^{-\chi}(A^{-1})_\alpha{}^\nu\bar{g}^{\alpha\mu}\nabla_\mu\sigma\nabla_\nu\sigma, \tag{2.30}$$
$$\bar{\mathcal{L}}_1 = -\tfrac{1}{4}(\det A)^{1/2}(A^{-1})_\rho{}^\mu(A^{-1})_\lambda{}^\nu\bar{g}^{\rho\alpha}\bar{g}^{\lambda\beta}F_{\mu\nu}F_{\alpha\beta}, \tag{2.31}$$
$$\bar{\mathcal{L}}_{1/2} = e^{-\chi}(A^{-1})_\alpha{}^\nu i\bar{\psi}\bar{g}^{\alpha\mu}\gamma_\mu\partial_\nu\psi, \tag{2.32}$$

and $\bar{\mathcal{L}}_M=\bar{\mathcal{L}}_0+\bar{\mathcal{L}}_1+\bar{\mathcal{L}}_{1/2}$. One can also consider interaction terms, namely the Yukawa interaction and the gauge interactions for spinor-vector fields and scalar-vector fields and study how the are affected by the metric redefinition:

$$\mathcal{L}_{\mathrm{Yukawa}} = -g\bar{\psi}\phi\psi, \tag{2.33}$$
$$\mathcal{L}_0 = \tfrac{1}{2}(D_\mu\sigma)^\dagger(D^\mu\sigma)=\frac{1}{2}\nabla_\mu\sigma\nabla^\mu\sigma+e^2A_\mu A^\mu\sigma^2, \tag{2.34}$$
$$\mathcal{L}_{1/2} = i\bar{\psi}D\!\!\!\!/\,\psi=i\bar{\psi}\gamma^\mu\nabla_\mu\psi-eA_\mu\bar{\psi}\gamma^\mu\psi, \tag{2.35}$$

where $D_\mu = \nabla_\mu + ieA_\mu$. After transforming the metric to $\bar{g}_{\mu\nu}$ (i.e., $g_{\mu\nu} \to \tilde{g}_{\mu\nu} \to \bar{g}_{\mu\nu}$), one finds

$$\bar{\mathcal{L}}_{\text{Yukawa}} = -e^{-2\chi}(\det A)^{-1/2} g\bar{\psi}\phi\psi, \tag{2.36}$$

$$\bar{\mathcal{L}}_0 = \tfrac{1}{2}e^{-\chi}(A^{-1})_\alpha{}^\nu \bar{g}^{\alpha\mu}(\nabla_\mu\sigma\nabla_\nu\sigma + e^2 A_\mu A_\nu \sigma^2), \tag{2.37}$$

$$\bar{\mathcal{L}}_{1/2} = e^{-\chi}(A^{-1})_\alpha{}^\nu \bar{g}^{\alpha\mu}(i\bar{\psi}\gamma_\mu\partial_\nu\psi - eA_\mu\bar{\psi}\gamma_\nu\psi), \tag{2.38}$$

and $\bar{\mathcal{L}}_M = \bar{\mathcal{L}}_0 + \bar{\mathcal{L}}_1 + \bar{\mathcal{L}}_{1/2} + \bar{\mathcal{L}}_{\text{Yukawa}}$. We note that the massive spin-2 field couples to all matter fields of spin 0, 1/2 and 2 because of the matrix A. On the other hand, the scalar field χ does not couple to photons. The masses of the spin 0 and massive spin 2 gravitational fields can be tuned by adjusting the coefficients of the action. On the other hand, their interactions with matter fields, while not always universal, are fixed by the gravitational coupling constant. As usual, the massless graviton couples universally and gravitationally to matter fields.

2.3 Application to dark matter

As already emphasized, astrophysical and cosmological evidence for dark matter is overwhelming. Several explanations have been proposed to explain the dark matter phenomenon. These models are usually classified into two categories: modifications of Einstein's general relativity or modifications of the standard model in the form of new particles. The aim of this section is to point out that these two categories are not so different after all. In fact, every modified gravity model has new degrees of freedom besides the usual massless graviton.

The first attempt to explain galaxy rotation curves by a modification of Newtonian dynamics is due to Milgrom [Milgrom, 1983a]. While Milgrom's original proposal was non-relativistic and very phenomenological, more refined theories have been proposed later on, including Bekenstein's TeVeS theory [Bekenstein, 2004], Moffat's modified gravity (MOG) [Moffat, 2006] and Mannheim's conformal gravity [Mannheim, 2012], which are relativistic. While these theories seem to be able to explain the rotation curves of the galaxies (see e.g. [Famaey and McGaugh, 2012] for a recent MOND review where the observational successes are discussed in details), it is more difficult to imagine how they would explain the bullet cluster observations or the agreement of the CMB observation with the standard cosmological model ΛCMB which posits the existence of cold dark matter. We shall not dwell on the question of the viability of modified gravity as we may simply not yet have found the correct model. However, we merely point out that if such a theory exists, it will not be necessarily very different from a model involving particles as dark matter.

Indeed, whatever this realistic theory might be, it can be parameterized by a function $f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \phi_\alpha)$ modelled using effective theory techniques. Here R is the Ricci scalar, $R_{\mu\nu}$ is the Ricci tensor and ϕ_α denotes collectively any type of field that is also responsible for the gravitational interaction. In terms of effective field theory, any theory of modified gravity can be described by

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \phi_\alpha) + \int d^4x\sqrt{-g}\mathcal{L}_M \tag{2.39}$$

where G is Newton's constant. We are only assuming diffeomorphism invariance and the usual space-time and gauge symmetries for the matter content described by the Lagrangian $\mathcal{L}_M$. A successful model should lead to a modification of Newton's potential that fits, e.g., the galaxy rotation curves. It is not difficult to imagine that the standard Newtonian term $1/r$ would come from the usual massless spin-2 graviton exchange while the non-Newtonian terms would have to be generated by the new degrees of freedom. Clearly, it is not straightforward to come up with such a model, however, as mentioned before, there are a few known examples.

While it is obvious that new degrees of freedom are included when ϕ_α is added to the function f as in [Moffat, 2006], it is much less clear how they are identified when the theory is a function of the curvature invariants only as we stressed before. Hence we will restrict ourselves to the theory $f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$. From the arguments made at the end of Section 2.2, we know that this theory is equivalent to general relativity in the presence of a scalar field and of a massive spin-2 field. Therefore, there is no difference between introducing new particles and introducing modifications of gravity, which raises the question of whether it is possible to differentiate experimentally between models of modified gravity and particle dark matter. Nonetheless, since the massive spin-2 particle is a ghost, this result also suggests that a good dark matter model is very likely to be described either by an $f(R)$ theory and hence a scalar field.

Any modification of gravity that has general coordinate invariance as a symmetry can be reformulated, using appropriate variables, as usual general relativity accompanied by new degrees of freedom. We have seen that these new degrees of freedom may not couple universally to matter. Modified gravity can thus be seen as a model with new dark matter particles that are very weakly coupled to the standard model. These apparently very different models describe the same physics as their actions are related by simple variable transformations. This may provide a simple way for modified gravity proponents to explain bullet cluster experiments or the cosmic microwave background.

2.4 Conclusions

In this paper, we proposed a classification scheme for gravitational theories. In particular, we showed the equivalence between the broad class of theories $f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$ and general relativity in the presence of additional matter fields, namely a scalar and a massive spin-2 field. We have shown that these new degrees of freedom can be shifted into a redefined stress-energy tensor and that they will couple gravitationally to the matter fields introduced in the model. We conclude that any attempt to modify the Einstein-Hilbert action, preserving the underlying symmetry, leads to new degrees of freedom, i.e., new particles. In that sense, this is not different from including new matter fields by hand in the matter sector that are coupled gravitationally to the standard model matter fields. Assuming that models of modified gravity preserve diffeomorphism invariance, we have shown that they are equivalent to general relativity with new degrees of freedom coupled gravitationally to the fields of the standard model. From that point of view, there is a duality between models of modified gravity and particle physics models with new fields that are coupled gravitationally to the standard model.

These results may make it easier to analyse the physics of models of dark matter involving a modification of gravity and, in particular, the fact that they are dual to some very weakly coupled dark matter model could help to resolve the apparent conflict with bullet cluster observations.

While we focussed on dark matter in this paper as an application for the classification of extended theories of gravity we proposed, another obvious application would be to the physics of gravitational waves for which extended theories of gravity are also important, see e.g. [Capozziello et al., 2011, De Laurentis et al., 2016, Capozziello and Stabile, 2015, Calmet et al., 2016].

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Chapter 3

Gravitational Waves in Effective Quantum Gravity

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In this short paper, we investigate quantum gravitational effects on Einstein's equations using effective field theory techniques. We consider the leading order quantum gravitational correction to the wave equation. Besides the usual massless mode, we find a pair of modes with complex masses. These massive particles have a width and could thus lead to a damping of gravitational waves if excited in violent astrophysical processes producing gravitational waves such as e.g. black hole mergers. We discuss the consequences for gravitational wave events such as GW 150914 recently observed by the Advanced LIGO collaboration.

The recent discovery of gravitational waves by the Advanced LIGO collaboration [Abbott et al., 2016] marks the beginning of a new era in astronomy which could shed some new light on our universe revealing its darkest elements that do not interact with electromagnetic radiations. This discovery could also lead to some new insights in theoretical physics. In this short paper, we study the leading effect of quantum gravity on gravitational waves using effective field theory techniques. While the discovery of a theory of quantum gravity might still be far away, it is possible to use effective field theory techniques to make actual predictions in quantum gravity. Assuming that diffeomorphism invariance is the correct symmetry of quantum gravity at the Planck scale and assuming that we know the field content below the Planck scale, we can write down an effective action for any theory of quantum gravity. This effective theory, dubbed Effective Quantum Gravity, is valid up to energies close to the Planck mass. It is obtained by linearizing general relativity around a chosen background. The massless graviton is described by a massless spin 2 tensor which is quantized using the standard quantum field theoretical procedure. It is well known that this theory is non-renormalizable, but divergences can be absorbed into the Wilson coefficients of higher dimensional operators compatible with diffeomorphism invariance. The difference with a standard renormalizable theory resides in the fact that an infinite number of measurements are necessary to determine the action to all orders. Nevertheless, Effective Quantum Gravity enables some predictions which are model independent and which therefore represent true tests of quantum gravity, whatever the underlying theory might be.

We will first investigate quantum gravitational corrections to the linearized Einstein's equations. Solving these equations, we show that besides the usual solution that corresponds to the propagation of the massless graviton, there are solutions corresponding to massive degrees of freedom. If these massive degrees of freedom are excited during violent astrophysical processes a sizable fraction of the energy released by such processes could be emitted into these modes. We shall show that the corresponding gravitational wave is damped and that the energy of the wave could thus dissipate. We then study whether the recent discovery of gravitational waves by the Advanced LIGO collaboration [Abbott et al., 2016] could lead to a test of quantum gravity.

Given a matter Lagrangian coupled to general relativity with N_s scalar degrees of freedom, N_f fermions and N_V vectors one can calculate the graviton vacuum polarization in the large $N = N_s + 3N_f + 12N_V$ limit with keeping NG_N, where G_N is Newton's constant, small. Since we are interested in energies below $M_\star$ which is the energy scale at which the effective theory breaks down, we do not need to consider the graviton self-interactions which are suppressed by powers of $1/N$ in comparison to the matter loops. Note that $M_\star$ is a dynamical quantity and does not necessarily correspond to the usual reduced Planck mass of order 10^{18} GeV (see e.g. [Calmet, 2013]). The divergence in this diagram can be isolated using dimensional regularization and absorbed in the coefficient of R^2 and $R_{\mu\nu}R^{\mu\nu}$. An infinite series of vacuum polarization diagrams contributing to the graviton propagator can be resummed in the large N limit. This procedure leads to a resummed graviton propagator given by [Aydemir et al., 2012]

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)} \tag{3.1}$$

where $L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu/q^2$, q^μ is the 4-momentum, μ is the renormalization scale and the $i\varepsilon$ prescription is implicit. This resummed propagator is the source of interesting acausal and non-local effects which have just started to be investigated [Aydemir et al., 2012, Donoghue and El-Menoufi, 2014, Calmet and Casadio, 2014, Calmet et al., 2015, Calmet, 2014, Calmet and Casadio, 2015]. Here we shall focus on how these quantum gravity effects affect gravitational waves.

From the resummed graviton propagator in momentum space, we can directly read off the classical field equation for the spin 2 gravitational wave in momentum space

$$2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right) = 0. \tag{3.2}$$

This equation has three solutions [Calmet, 2014]:

$$\begin{aligned} q_1^2 &= 0, \\ q_2^2 &= \frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi}{\mu^2 N G_N}\right)}, \\ q_3^2 &= (q_2^2)^*, \end{aligned} \tag{3.3}$$

where W is the Lambert function. The complex pole corresponds to a new massive degree of freedom with a complex mass (i.e. they have a width [Calmet, 2014]). The general wave solution is thus of the form

$$h^{\mu\nu}(x) = a_1^{\mu\nu} \exp(-iq_{1\alpha}x^\alpha) + a_2^{\mu\nu} \exp(-iq_{2\alpha}x^\alpha) + a_3^{\mu\nu} \exp(-iq^\star_{2\alpha}x^\alpha), \tag{3.4}$$

where $a_i^{\mu\nu}$ are polarization tensors. We therefore have three degrees of freedom which can be excited in gravitational processes leading to the emission of gravitational waves. Note that our solution is linear, non-linearities in gravitational waves (see e.g. [Aldrovandi et al., 2010]) have been investigated and are as expected very small.

The position of the complex pole depends on the number of fields in the model. In the standard model of particle physics, one has $N_s = 4$, $N_f = 45$, and $N_V = 12$. We thus find $N = 283$ and the pair of complex poles at $(7 - 3i) \times 10^{18}$ GeV and $(7 + 3i) \times 10^{18}$ GeV. Note that the pole q_3^2 corresponds to a particle which has an incorrect sign between the squared mass and the width term. We shall not investigate this Lee-Wick pole further and assume that this potential problem is cured by strong gravitational interactions. The renormalization scale needs to be adjusted to match the number of particles included in the model. Indeed, to a good approximation the real part of the complex pole is of the order of

$$|\text{Re } q_2| \sim \sqrt{\frac{120\pi}{NG_N}} \tag{3.5}$$

which corresponds to the energy scale $M_\star$ at which the effective theory breaks down. Indeed, the complex pole will lead to acausal effects and it is thus a signal of strong quantum gravitational effects which cannot be described within the realm of the effective theory. We should thus pick our renormalization scale μ of the order of $M_\star \sim |\text{Re } q_2|$. We have

$$q_2^2 \approx \pm \frac{1}{G_N N} \frac{120\pi}{W(-1)} \approx \mp(0.17 + 0.71\ i) \frac{120\pi}{G_N N}, \tag{3.6}$$

and we thus find the mass of the complex pole:

$$m_2 = (0.53 - 0.67\ i) \sqrt{\frac{120\pi}{G_N N}}. \tag{3.7}$$

As emphasized before, the mass of this object depends on the number of fields in the theory. The corresponding wave has a frequency:

$$\begin{aligned} w_2 &= q_2^0 = \pm\sqrt{\vec{q}_2.\vec{q}_2 + (0.17 + 0.71\ i)\frac{120\pi}{G_N N}} \qquad (3.8) \\ &= \pm\left(\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} + \vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}}\right. \\ &\left. + i\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} - \vec{q}_2.\vec{q}_2 - 0.17\frac{120\pi}{G_N N}}\right). \end{aligned}$$

The imaginary part of the complex pole will lead to a damping of the component of the gravitational wave corresponding to that mode. The complex poles are gravitationally coupled to matter, we must thus assume that the massive modes are produced at the same rate as the usual massless graviton mode if this is allowed kinematically. During an astrophysical event leading to gravitational waves, some of the energy will be emitted into these massive modes which will decay rather quickly because of their large decay width. The possible damping of the gravitational wave implies that care should be taken when relating the energy of the gravitational wave observed on earth to that of the astrophysical event as some of this energy could have been dissipated away as the wave travels towards earth.

The idea that gravitational waves could experience some damping has been considered before [Jones and Singleton, 2015], however it is well known that the graviton cannot split into many gravitons, even at the quantum level [Fiore and Modanese, 1996], if there was such an effect it would have to be at the non-perturbative level [Efroimsky, 1994]. In our case, the massless mode is not damped, there is thus no contradiction with the work of [Fiore and Modanese, 1996]. Also, as emphasized before the dispersion relation of the massless mode of the gravitational wave is not affected, we do not violate any essential symmetry such as Lorentz invariance. This is in contrast to the model presented in [Arzano and Calcagni, 2016].

Since the complex poles couple with the same coupling to matter as the usual massless graviton, we can think of them as a massive graviton although strictly speaking these objects have two polarizations only in contrast to massive gravitons that have five. This idea has been applied in the context of $F(R)$ gravity [Vainio and Vilja, 2017] (see also [Bogdanos et al., 2010, Capozziello and Stabile, 2015] for earlier works on gravitational waves in $F(R)$ gravity). We shall assume that these massive modes can be excited during the merger of two black holes. As a rough approximation, we shall assume that all the energy released during the merger is emitted into these modes. Given this assumption, we can use the limit derived by the LIGO collaboration on a graviton mass. We know that $m_g < 1.2 \times 10^{-22}$ eV [Abbott et al., 2016] and we can thus get a limit:

$$\sqrt{\mathrm{Re}\left(\frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}\right)} < 1.2 \times 10^{-22}\ \mathrm{eV} \tag{3.9}$$

we thus obtain a lower bound on N: $N > 4 \times 10^{102}$ if all the energy of the merger was carried away by massive modes. Clearly, this is not realistic as the massless mode will be excited. However, it implies that if the massive modes are produced, they will only arrive on earth if their masses are smaller than 1.2×10^{-22} eV. Waves corresponding to more massive poles will be damped before reaching earth. We shall see that there are tighter bounds on the mass of these objects coming from Eötvös type pendulum experiments.

At this stage, we need to discuss which modes can be produced during the two black holes merger that led to the gravitational wave observed by the LIGO collaboration. The LIGO collaboration estimates that the gravitational wave GW150914 is produced by the coalescence of two black holes: the black holes follow an inspiral orbit before merging and subsequently going through a final black hole ringdown. Over 0.2 s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum [Abbott et al., 2016]. The typical energy of the gravitational wave is of the order of 150 Hz or 6×10^{-13} eV. In other words, if the gravitational wave had been emitted in the massive mode, they could not have been heavier than 6×10^{-22} GeV. However, this shows that it is perfectly conceivable that a sizable number of massive gravitons with $m_g < 1.2 \times 10^{-22}$ eV could have been produced.

Let us now revisit the bound on the number of fields N and thus the new complex pole using Eötvös type pendulum experiments looking for deviations of the Newtonian $1/r$ potential. The resummed graviton propagator discussed above can be represented by the effective operator

$$\frac{N}{2304\pi^2} R \log\left(\frac{\Box}{\mu^2}\right) R \tag{3.10}$$

where R is the Ricci scalar. As explained above the log term will be a contribution of order 1, this operator is thus very similar to the more familiar cR^2 term studied by Stelle long ago. The current bound on the Wilson coefficient of c is $c < 10^{61}$ [Hoyle et al., 2004, Stelle, 1978, Calmet et al., 2008]. We can translate this bound into a bound on N: $N < 2 \times 10^{65}$. This implies that the mass of the complex pole must be larger than 5×10^{-13}GeV. This bound, although very weak, is more constraining than the one we have obtained from the graviton mass by 37 orders of magnitude.

In this short paper we have investigated quantum gravitational effects in gravitational waves using conservative effective theory methods which are model independent. We found that quantum gravity leads to new poles in the propagator of the graviton besides the usual massless pole. These new states are massive and couple gravitationally to matter. If kinematically allowed, they would thus be produced in roughly the same amount as the usual massless mode in energetic astrophysical events. A sizable amount of the energy produced in astrophysical events could thus be carried away by massive modes which would decay and lead to a damping of this component of the gravitational wave. While our back-of-the-envelope calculation indicates that the energy released in the merger recently observed by LIGO was unlikely to be high enough to produce such modes, one should be careful in extrapolating the amount of energy of astrophysical events from the energy of the gravitational wave observed on earth. This effect could be particularly important for primordial gravitational waves if the scale of inflation is in the region of 10^{16} GeV, i.e. within a few orders of magnitude of the Planck scale.

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Chapter 4

Quantum Corrections to the Gravitational Backreaction

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Effective Field Theory techniques are used to study the leading order quantum corrections to the gravitational wave backreaction. The effective stress-energy tensor is calculated and it is shown that it has a non-vanishing trace that contributes to the cosmological constant. By comparing the result obtained with LIGO's data, the first bound on the amplitude of the massive mode is found: $\epsilon < 1.4 \times 10^{-33}$.

4.1 Introduction

The recent experimental discovery of gravitational waves (GWs) [Abbott et al., 2016] has marked a new era for both observational and theoretical physics. With the new coming data from LIGO and from future experiments like LISA, it will become possible to test modified gravity theories, establishing for which range of parameters these theories agree with observations. Particularly, it may be even possible to test Quantum Gravity in its low energy limit, even though a complete quantum theory for gravity remains one of the greatest problems in modern physics.

A natural observable to consider is the GW energy. As a non-linear phenomenon, gravity couples to itself and thus gravitates, which means that GWs — being a manifestation of gravity — produce a backreaction onto the spacetime. Hence, one should be able to find a stress-energy tensor for the GWs that accounts for this phenomenon. In the case of classical General Relativity (GR), such a stress-energy tensor is known:

$$t^{\mathrm{GR}}_{\mu\nu} = \frac{1}{32\pi G}\left\langle \partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta}\right\rangle, \tag{4.1}$$

where $h_{\mu\nu}$ are metric perturbations and the brackets denote an average over spacetime, which is responsible for taking only the long-wavelength modes; its precise definition will be explained later on. The GW stress-energy tensor has also been calculated for some other theories, including $f(R)$, Chern-Simons and higher-derivative gravity [Stein and Yunes, 2011, Preston, 2016, Preston and Morris, 2014, Saito and Ishibashi, 2013]. In [Berry and Gair, 2011], it was indicated how the parameters of an analytic $f(R)$ theory could be constrained by the measurement of the energy or momentum carried away by the GWs.

The phenomenology, however, is not the only motivation. An alternative for dark energy has been proposed based on the effective stress-energy tensor [Preston and Morris, 2014, Rasanen, 2010, Rasanen, 2004, Buchert and Rasanen, 2012]. Although this is not possible in GR because of the vanishing trace of $t^{\mathrm{GR}}_{\mu\nu}$, it was pointed out it could be possible in modified gravity theories. However, it was also found that in some models such as Starobinsky gravity, the effective stress-energy tensor could not be the only factor as it does not produce the right value for the cosmological constant [Preston and Morris, 2014]. We will show that the large contributions from the Standard Model cannot be canceled by the quantum gravitational effects, thus requiring the existence of another mechanism able to reconcile the discrepancy between theory and observation.

The purpose of this paper is, then, two-fold: we will establish new phenomenological bounds and discuss the possibility of generating a contribution to the cosmological constant in this framework. Effective Field Theory techniques will be used to calculate quantum contributions to the GW backreaction and to the wave equation in an arbitrary background. The short-wave formalism will be employed, consisting of an averaging procedure that separates the low-frequency modes from the high-frequency ones, in order to calculate the GW stress-energy tensor in quantum GR. These theoretical findings will be useful to constrain some of the parameters of Effective Quantum Gravity by the direct comparison with LIGO's observations. Furthermore, on the theoretical side, they give us new insights into gravity at the quantum level since this approach is model independent and, as such, leads to genuine predictions of Quantum Gravity.

This paper is organized as follows. In Section 4.2, we will review the main results of the Effective Field Theory approach applied to gravity. In Section 4.3, we use the short-wave formalism to calculate the leading order quantum corrections to the GW stress-energy tensor. The result allows us to constrain the amplitude of the massive mode present in Effective Quantum Gravity. In Section 4.4, we discuss the quantum corrections to the propagation of GWs and we show that the equation describing the propagation in curved spacetime can be obtained by performing a minimal coupling prescription to the equation in Minkowski space. We draw the conclusions in Section 4.5.

4.2 Effective quantum gravity

The quantum effective action of gravity up to quadratic order in curvature is given by [Donoghue and El-Menoufi, 2014]

$$\Gamma = \int \mathrm{d}^4x\sqrt{-g}\bigg(\frac{M_p^2}{2}R + b_1R^2 + b_2R_{\mu\nu}R^{\mu\nu} + c_1R\log\frac{-\Box}{\mu^2}R + c_2R_{\mu\nu}\log\frac{-\Box}{\mu^2}R^{\mu\nu} + c_3R_{\mu\nu\rho\sigma}\log\frac{-\Box}{\mu^2}R^{\mu\nu\rho\sigma}\bigg), \tag{4.2}$$

where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass, G is the Newton's constant, μ is the renormalization scale and the kernel R denotes the Riemann tensor and its contractions (Ricci tensor and Ricci scalar) depending on the number of indices it carries. The signature $(-+++)$ will be adopted. We set the bare cosmological constant to zero as it is not important to our considerations. The coefficients b_i are free parameters and must be fixed by observations, while the coefficients c_i are predictions of the infra-red theory and depend on the field content under consideration (see Table 1 in [Donoghue and El-Menoufi, 2014] for their precise values). The log operators are known to lead to acausal effects that need to be removed by resolving the non-local operator as

$$\log \frac{-\Box}{\mu^2} = \int_0^\infty \mathrm{d}s \left(\frac{1}{\mu^2 + s} - G(x, x', \sqrt{s}) \right), \tag{4.3}$$

where $G(x, x'; \sqrt{s})$ is a Green's function for

$$(-\Box + k^2) G(x, x'; k) = \delta^4(x - x'), \tag{4.4}$$

and imposing proper boundary conditions on $G(x, x'; k)$ so that the result respects causality. Moreover, in the weak field limit, the log terms are not independent due to the following relation (see [Preston, 2016]):

$$\delta \int \mathrm{d}^4x \sqrt{-g} \left(R_{\mu\nu\rho\sigma} \log \frac{-\Box}{\mu^2} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} \log \frac{-\Box}{\mu^2} R^{\mu\nu} + R \log \frac{-\Box}{\mu^2} R \right) \overset{\text{weak}}{=} 0. \tag{4.5}$$

This can also be seen by linearizing the field equations [Calmet et al., 2017a]. The log operators in the above expression certainly break the topological invariance given by the Gauss-Bonnet theorem. Nonetheless, such expression still provides a useful relation that can be used to simplify calculations in the weak field limit. Therefore, since we will be interested only in the weak field scenario, the last term in (4.2) will be eliminated in favour of the other two log terms, which translates into a shift of their coefficients:

$$c_1 \to \alpha \equiv c_1 - c_3, \tag{4.6}$$

$$c_2 \to \beta \equiv c_2 + 4c_3. \tag{4.7}$$

Hence, from now on, α will denote the coefficient of $R \log \frac{-\Box}{\mu^2} R$ and β the coefficient of $R_{\mu\nu} \log \frac{-\Box}{\mu^2} R^{\mu\nu}$. Note, however, that the last term in (4.2) will give independent contributions in the non-linear regime and, in particular, the background equations of motion (left-hand side of (4.20) below) will be changed, but none of this affects the right-hand side of (4.20).

The quantum action (4.2) yields the equations of motion (EOM)

$$G_{\mu\nu} + \Delta G^{L}_{\mu\nu} + \Delta G^{NL}_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{4.8}$$

where $\Delta G^{L}_{\mu\nu}$ denotes the local contribution to the modification of Einstein's tensor and $\Delta G^{NL}_{\mu\nu} = \Delta G^{\alpha}_{\mu\nu} + \Delta G^{\beta}_{\mu\nu}$ is the non-local one (due to the log operator), coming from the terms proportional to α and β, denoted by $\Delta G^{\alpha}_{\mu\nu}$ and $\Delta G^{\beta}_{\mu\nu}$, respectively. Here we will show only the calculation of the non-local part $\Delta G^{NL}_{\mu\nu}$ as the local contribution can be straightforwardly obtained from it. However, our final results will be completely general, including both local and non-local physics. The $\Delta G^{\alpha}_{\mu\nu}$ has been calculated in the literature [Codello and Jain, 2017]:

$$-\xi \Delta G^{\alpha}_{\mu\nu} = 2\left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R\right)\left(\log \frac{-\Box}{\mu^2} R\right) - 2\left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right)\left(\log \frac{-\Box}{\mu^2} R\right), \tag{4.9}$$

where $\xi = \frac{1}{16\pi G\alpha}$. Note that the integral term appearing in [Codello and Jain, 2017], which comes from the variation of the D'Alembert operator, is not present here. This is because in the weak field limit the variation of the D'Alembert operator leads to negligible contributions [Donoghue and El-Menoufi, 2015]. The other contribution to $\Delta G_{\mu\nu}$ is given by

$$\begin{aligned} \zeta \Delta G^{\beta}_{\mu\nu} = &-\frac{1}{2} g_{\mu\nu} R_{\rho\sigma} \log\left(\frac{-\Box}{\mu^2}\right) R^{\rho\sigma} + \Box \log\left(\frac{-\Box}{\mu^2}\right) R_{\mu\nu} + g_{\mu\nu} \nabla_\rho \nabla_\sigma \log\left(\frac{-\Box}{\mu^2}\right) R^{\rho\sigma} \\ &+ R^{\sigma}_{\mu} \log\left(\frac{-\Box}{\mu^2}\right) R_{\nu\sigma} + R^{\sigma}_{\nu} \log\left(\frac{-\Box}{\mu^2}\right) R_{\mu\sigma} \\ &- \nabla_\rho \nabla_\mu \log\left(\frac{-\Box}{\mu^2}\right) R^{\rho}_{\nu} - \nabla_\rho \nabla_\nu \log\left(\frac{-\Box}{\mu^2}\right) R^{\rho}_{\mu} \end{aligned} \tag{4.10}$$

where $\zeta = \frac{1}{16\pi G\beta}$.

4.3 Gravitational wave backreaction

The first step is to separate the fluctuations $h_{\mu\nu}$ (GWs) from the background geometry $\bar{g}_{\mu\nu}$, via $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. This separation is only meaningful in the limit where the GW wavelength λ is much smaller than the background radius L, i.e. $\lambda \ll L$, so that a clear distinction between background and GW can be made. As a first approximation, the background metric $\bar{g}_{\mu\nu}$ will be used to raise/lower indices as well as to build all the operators, e.g. $\Box = \bar{g}^{\mu\nu}\nabla_\mu\nabla_\nu$. The connection is also assumed to be compatible with $\bar{g}_{\mu\nu}$ instead of $g_{\mu\nu}$.

The separation of gravity into background and fluctuations allows one to expand the Ricci tensor as

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + O(h^3), \tag{4.11}$$

where the bar quantities are calculated with respect to the background and the rest depends only on the fluctuation. The superscript (n) is used to indicate the order in h of the underlying term. Naively, one could think that the EOM could be calculated order by order in h, giving no backreaction into the background. The problem is that there are two small parameters in the game, namely the fluctuation amplitude h and $\varepsilon \equiv \frac{\lambda}{L}$, so that one can compensate the other. Their relation is fixed by the EOM[1] and in the presence of external matter

$$h \ll \varepsilon \ll 1, \tag{4.12}$$

as can be seen from Equation (4.8).

To obtain the GW backreaction, one then needs to calculate the average of tensor fields over a region of length scale d, where $\lambda \ll d \ll L$. This makes the high-frequency modes go away due to their rapid oscillation, but leave the low modes intact. The subtle point is that there is no canonical way of summing tensors based on different points of a manifold. Here Isaacson's definition [Isaacson, 1968, Isaacson, 1967] of the average of a tensor will be used, which is based on the idea of parallel transporting the tensor field along geodesics from each spacetime position to a common point where its different values can be compared:

$$\langle A_{\mu\nu}(x)\rangle = \int j_\mu^{\alpha'}(x,x')j_\nu^{\beta'}(x,x')A_{\alpha'\beta'}(x')f(x,x')\sqrt{-\bar{g}(x')}d^4x', \tag{4.13}$$

where $j_\mu^{\alpha'}$ is the bivector of geodesic parallel displacement and $f(x,x')$ is a weight function that falls quickly and smoothly to zero when $|x-x'| > d$, such that

$$\int_{\text{all space}} f(x,x')\sqrt{-\bar{g}(x')}d^4x' = 1. \tag{4.14}$$

From this definition, the following rules can be proven [Stein and Yunes, 2011]:

[1] Note that $\bar{R}_{\mu\nu} \sim \frac{1}{L^2}$, $R^{(n)}_{\mu\nu} \sim \frac{h^n}{\lambda^2}$ and the contribution of GWs to the curvature is negligible compared to the contribution of matter sources.

- The average of an odd product of short-wavelength quantities vanishes.
- The derivative of a short-wavelength tensor averages to zero, e.g., $\left\langle \nabla_\mu T^\mu_{\alpha\beta} \right\rangle = 0$.
- As a corollary, integration by parts can be performed and one can flip derivatives, e.g., $\langle R^\mu_\alpha \nabla_\mu S_\beta \rangle = -\langle S_\beta \nabla_\mu R^\mu_\alpha \rangle$.

Therefore, to obtain the backreaction one has to calculate

$$\langle G_{\mu\nu} \rangle + \left\langle \Delta G^{NL}_{\mu\nu} \right\rangle = 8\pi G \langle T_{\mu\nu} \rangle \tag{4.15}$$

up to second order in h (higher orders are extremely small)[2]. Taking the average of Equation (4.9), gives

$$\begin{aligned} -\xi \left\langle \Delta G^\alpha_{\mu\nu} \right\rangle = 2 \left(\left\langle R_{\mu\nu} \log\left(\frac{-\Box}{\mu^2}\right) R \right\rangle - \frac{1}{4} \bar{g}_{\mu\nu} \left\langle R \log\left(\frac{-\Box}{\mu^2}\right) R \right\rangle \right) \\ - 2 \left\langle (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) \log\left(\frac{-\Box}{\mu^2}\right) R \right\rangle . \end{aligned} \tag{4.16}$$

It follows from the rules that

$$\left\langle R_{\mu\nu} \log\left(\frac{-\Box}{\mu^2}\right) R^{\mu\nu} \right\rangle = \bar{R}_{\mu\nu} \log\left(\frac{-\Box}{\mu^2}\right) \bar{R}^{\mu\nu} + \left\langle R^{(1)}_{\mu\nu} \log\left(\frac{-\Box}{\mu^2}\right) R^{(1)\mu\nu} \right\rangle , \tag{4.17}$$

since the average of linear terms in h vanishes. Cross terms (e.g. $\bar{R}R^{(2)}$) are absent as they are negligible due to the condition (4.12). In addition, the last line of Equation (4.16) has a global derivative so that the high-frequency contribution averages to zero.

The combination of Equations (4.16) and (4.17) results in

$$\begin{aligned} -\xi \left\langle \Delta G^\alpha_{\mu\nu} \right\rangle = 2 \left(\bar{R}_{\mu\nu} - \frac{1}{4} \bar{g}_{\mu\nu} \bar{R} \right) \log\left(\frac{-\Box}{\mu^2}\right) \bar{R} + 2 \Bigg(\left\langle R^{(1)}_{\mu\nu} \log\left(\frac{-\Box}{\mu^2}\right) R^{(1)} \right\rangle \\ - \frac{1}{4} \bar{g}_{\mu\nu} \left\langle R^{(1)} \log\left(\frac{-\Box}{\mu^2}\right) R^{(1)} \right\rangle \Bigg) \\ - 2(\nabla_\mu \nabla_\nu - \bar{g}_{\mu\nu} \Box) \log\left(\frac{-\Box}{\mu^2}\right) \bar{R}. \end{aligned} \tag{4.18}$$

[2]When performing the scalar-vector-tensor decomposition to second order in perturbation theory, one has to take into account the contributions from the coupling between scalar and tensor perturbations [Marozzi and Vacca, 2014]. These contributions are automatically being taken into account here as we are not decomposing the metric perturbation and everything is given in terms of the entire perturbation $h_{\mu\nu}$.

Similarly, taking the average of Equation (4.10) gives

$$
\begin{aligned}
\zeta \left\langle \Delta G^{\beta}_{\mu\nu} \right\rangle = & -\frac{1}{2}\bar{g}_{\mu\nu}\left(\bar{R}_{\rho\sigma}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho\sigma} + \left\langle R^{(1)}_{\rho\sigma}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)\rho\sigma}\right\rangle\right) \\
& + \Box\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\mu\nu} + \bar{g}_{\mu\nu}\nabla_\rho\nabla_\sigma\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho\sigma} \\
& + \bar{R}^{\sigma}_{\mu}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\nu\sigma} + \bar{R}^{\sigma}_{\nu}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\mu\sigma} + 2\left\langle R^{(1)\sigma}_{\mu}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)}_{\nu\sigma}\right\rangle \\
& - \nabla_\rho\nabla_\mu\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho}_{\nu} - \nabla_\rho\nabla_\nu\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho}_{\mu}. \qquad (4.19)
\end{aligned}
$$

Combining Equations (4.15), (4.18) and (4.19) leads to the background EOM

$$
\begin{aligned}
& \bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} - \frac{2}{\xi}\left[\left(\bar{R}_{\mu\nu} - \frac{1}{4}\bar{g}_{\mu\nu}\bar{R}\right)\log\left(\frac{-\Box}{\mu^2}\right)\bar{R} - \left(\nabla_\mu\nabla_\nu - \bar{g}_{\mu\nu}\Box\right)\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}\right] \\
& - \frac{1}{2\zeta}\bar{g}_{\mu\nu}\bar{R}_{\rho\sigma}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho\sigma} + \frac{1}{\zeta}\bar{R}^{\sigma}_{\mu}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\nu\sigma} + \frac{1}{\zeta}\bar{R}^{\sigma}_{\nu}\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\mu\sigma} \\
& + \frac{1}{\zeta}\Box\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}_{\mu\nu} + \frac{1}{\zeta}\bar{g}_{\mu\nu}\nabla_\rho\nabla_\sigma\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho\sigma} - \frac{1}{\zeta}\nabla_\rho\nabla_\mu\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho}_{\nu} \\
& - \frac{1}{\zeta}\nabla_\rho\nabla_\nu\log\left(\frac{-\Box}{\mu^2}\right)\bar{R}^{\rho}_{\mu} \\
& = 8\pi G(\langle T_{\mu\nu}\rangle + t^{GR}_{\mu\nu} + t^{NL}_{\mu\nu}), \qquad (4.20)
\end{aligned}
$$

where $t^{GR}_{\mu\nu}$ is the classical contribution to the GW stress-energy tensor:

$$
t^{GR}_{\mu\nu} = -\frac{1}{8\pi G}\left(\left\langle R^{(2)}_{\mu\nu}\right\rangle - \frac{1}{2}\bar{g}_{\mu\nu}\left\langle R^{(2)}\right\rangle\right) \qquad (4.21)
$$

and $t^{NL}_{\mu\nu}$ is the non-local one:

$$
\begin{aligned}
t^{NL}_{\mu\nu} = -\frac{1}{8\pi G}\Bigg[& -\frac{2}{\xi}\left(\left\langle R^{(1)}_{\mu\nu}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)}\right\rangle - \frac{1}{4}\bar{g}_{\mu\nu}\left\langle R^{(1)}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)}\right\rangle\right) \\
& + \frac{2}{\zeta}\left\langle R^{(1)\sigma}_{\mu}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)}_{\nu\sigma}\right\rangle - \frac{1}{2\zeta}\bar{g}_{\mu\nu}\left\langle R^{(1)}_{\rho\sigma}\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)\rho\sigma}\right\rangle\Bigg]. \qquad (4.22)
\end{aligned}
$$

Similarly, the local contribution is given by

$$
\begin{aligned}
t^{L}_{\mu\nu} = -\frac{1}{8\pi G}\Bigg[& -32\pi G b_1\left(\left\langle R^{(1)}_{\mu\nu}R^{(1)}\right\rangle - \frac{1}{4}\bar{g}_{\mu\nu}\left\langle R^{(1)}R^{(1)}\right\rangle\right) \\
& + 32\pi G b_2\left\langle R^{(1)\sigma}_{\mu}R^{(1)}_{\nu\sigma}\right\rangle - 8\pi G b_2\bar{g}_{\mu\nu}\left\langle R^{(1)}_{\rho\sigma}R^{(1)\rho\sigma}\right\rangle\Bigg]. \qquad (4.23)
\end{aligned}
$$

Therefore, the total GW stress-energy tensor is $t_{\mu\nu} = t^{GR}_{\mu\nu} + t^{L}_{\mu\nu} + t^{NL}_{\mu\nu}$.

At this point some comments are necessary. First of all, observe that the left-hand side of Equation (4.20) corresponds solely to the background effect, which we interpret as pure gravity. In fact, the left-hand side is exactly the same as in Equation (4.8) when $g_{\mu\nu}$ is replaced by $\bar{g}_{\mu\nu}$. The right-hand side represents the matter sector, as usual, but with the inclusion of the GW contribution. Such a contribution represents the most general stress-energy tensor to leading order, accounting for both classical and quantum effects. Note that, due to the diffeomorphism invariance of the theory, the total energy-momentum tensor is covariantly conserved

$$\nabla^{\mu}(T_{\mu\nu} + t_{\mu\nu}) = 0, \tag{4.24}$$

which shows that energy and momentum are exchanged between matter sources and GWs. Far away from the source, this gives the conservation of the GW energy-momentum tensor

$$\partial^{\mu} t_{\mu\nu} = 0. \tag{4.25}$$

Up to this point, no gauge conditions have been applied and $t_{\mu\nu}$ also accounts for spurious degrees of freedom. To eliminate them, we shall take the limit where the GW is far away from the source, so that the background is nearly flat and the linear EOM becomes [Calmet et al., 2017a]

$$\Box_{\eta} h_{\mu\nu} + 16\pi G \left[b_2 + \beta \log \left(\frac{-\Box_{\eta}}{\mu^2} \right) \right] \Box_{\eta}^2 h_{\mu\nu} = 0, \tag{4.26}$$

where $\Box_{\eta} = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}$ is the flat D'Alembert operator. Note the absence of the parameter α in Equation (4.26). This happens because α is proportional to terms depending on the trace h, which can be taken to be zero far away from the source. Using the gauge conditions $\partial^{\nu} h_{\mu\nu} = 0$ and $h = 0$ (only valid outside the source) together with Equation (4.26) in the definition of $t_{\mu\nu}$ gives

$$t_{\mu\nu} = \frac{1}{8\pi G} \left[\frac{1}{4} \left\langle \partial_{\mu} h_{\alpha\beta} \partial_{\nu} h^{\alpha\beta} \right\rangle + \frac{1}{2} \left\langle h_{\mu}^{\sigma} \Box_{\eta} h_{\nu\sigma} \right\rangle - \frac{1}{8} \eta_{\mu\nu} \left\langle h_{\rho\sigma} \Box_{\eta} h^{\rho\sigma} \right\rangle \right], \tag{4.27}$$

Comparing this to Equation (4.1), it is clearly seen that the first term in $t_{\mu\nu}$ corresponds to GR, while the other two come from quantum corrections. Observe that the log operators do not appear explicitly in Equation (4.27) as the gravitational field is on shell. This means that their contribution will only show up in the field solutions. For the same reason, the procedure (4.3) of imposing causality need not be pursued at this stage as the non-local effects are only reflected in the solutions for $h_{\mu\nu}$. The parameters b_2 and β now only appear in the mass m of $h_{\mu\nu}$.

The GW energy density is then

$$\rho \equiv t_{00} = \frac{1}{8\pi G}\left[\frac{1}{4}\left\langle \dot{h}_{\alpha\beta}\dot{h}^{\alpha\beta}\right\rangle + \frac{1}{2}\left\langle h_0^\alpha \Box_\eta h_{0\alpha}\right\rangle + \frac{1}{8}\left\langle h_{\rho\sigma}\Box_\eta h^{\rho\sigma}\right\rangle\right]. \tag{4.28}$$

As a concrete example, take a plane wave solution propagating in the z direction

$$h_{\mu\nu} = \epsilon_{\mu\nu}\cos(\omega t - kz). \tag{4.29}$$

Plugging this into Equation (4.28) gives

$$\rho = \frac{1}{16\pi G}\left[\frac{\epsilon^2\omega^2}{4} + \frac{1}{2}\left(\epsilon_0^\alpha \epsilon_{0\alpha} + \frac{\epsilon^2}{4}\right)\left(\omega^2 - k^2\right)\right], \tag{4.30}$$

where $\epsilon^2 = \epsilon_{\mu\nu}\epsilon^{\mu\nu}$. Therefore, modifications in the dispersion relations lead to measurable differences into the GW energy. In the case of the classical wave, i.e. $\omega^2 = k^2$, the second term vanishes identically, resulting in the classical energy as expected. In the most general case, there could be complex frequencies leading to damping as was shown in [Calmet et al., 2016, Calmet, 2014, Calmet and Kuntz, 2016, Calmet et al., 2017b]. In such case, Equation (4.30) can be straightforwardly generalized. Note that the second term in (4.30) is proportional to the particle's mass m and, therefore, is constant as any change in the frequency would be compensated by a change in the momentum. Dividing the constant term by the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$, where H_0 is the today's Hubble constant, leads to the frequency-independent gravitational wave density parameter Ω_0 which was constrained to be smaller than 1.7×10^{-7} by LIGO [Abbott et al., 2017]:

$$\Omega_0 = \frac{1}{12}\left(\epsilon_0^\alpha \epsilon_{0\alpha} + \frac{\epsilon^2}{4}\right)\frac{m^2}{H_0^2} < 1.7 \times 10^{-7}. \tag{4.31}$$

We remind the reader that the initial parameters b_2 and β appear only in terms of the mass m as the field $h_{\mu\nu}$ is on shell. Figure 4.1 shows the allowed region of the parameter space (m, ϵ). Using the lower bound on the mass of the complex pole[3] found in [Calmet et al., 2016], i.e. $m > 5 \times 10^{-13}$GeV, we can translate the above constraint to

$$\epsilon < 1.4 \times 10^{-33}. \tag{4.32}$$

[3]This conservative bound, and consequently the bound on ϵ, was obtained assuming all the energy of a merger goes into the complex mode. Naturally, this does not represent the real situation as the classical mode should also be produced. In a more careful analysis, we expect to get a better bound.

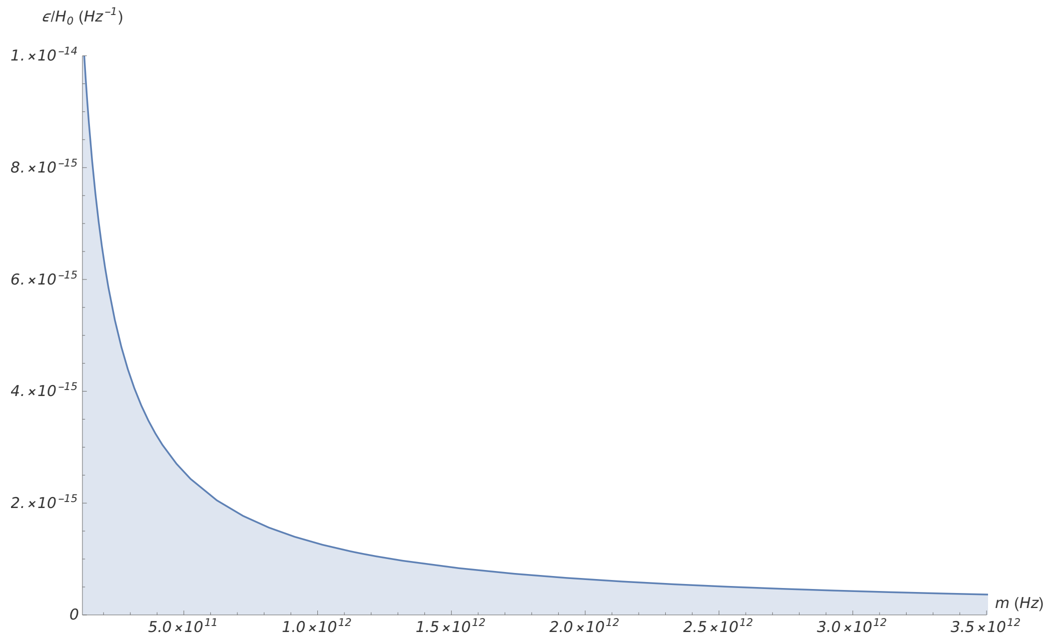

Figure 4.1: The blue area in the graph represents the allowed region of the parameter space (m, ϵ).

To the best of our knowledge, this is the first bound ever found on the amplitude of the massive mode. It is 12 orders of magnitude smaller than the strain sensibility of LIGO's interferometer, which can probe amplitudes up to $\sim 10^{-22}$ in the frequency range from 10 Hz to 10 kHz. Although it seems hopeless to reach such small distances, the Chongqing University detector (currently under development) will be able to probe amplitudes as small as 10^{-32} [Baker, 2009] in the high-frequency range 0.1–10 GHz, which is not far from the bound just found. Observe, however, that we have found an upper bound on ϵ and not a lower one, thus ϵ could be arbitrarily small and not be detectable by the Chongqing detector. Should the existence of these extra modes be confirmed in future experiments, this would be the first evidence for a massive mode.

As it was stressed before, the effective energy-momentum tensor may lead to a contribution to the accelerated expansion of today's universe if its trace is different from zero. The trace of the GW energy-momentum tensor (4.27) is non-vanishing:

$$t = -\frac{1}{32\pi G}\left\langle h_{\alpha\beta}\Box_\eta h^{\alpha\beta}\right\rangle \neq 0, \tag{4.33}$$

as the gravitational field now satisfies the modified EOM (4.26). Therefore, the energy-momentum tensor $t_{\mu\nu}$ can be split into a traceful and a traceless component

$$t_{\mu\nu} = t_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}t + \frac{1}{4}\eta_{\mu\nu}t \tag{4.34}$$

and the cosmological constant can be identified as

$$\Lambda \equiv \frac{1}{16}\left\langle h_{\alpha\beta}\Box_\eta h^{\alpha\beta}\right\rangle = \frac{1}{16}\epsilon^2 m^2, \tag{4.35}$$

where in the second equality the plane wave solution (4.29) was used. After taking the average, Λ depends very mildly on space and time. In fact, it is precisely constant across any region of length d and its variation only becomes appreciable in a region containing several lengths of size d. Therefore, for our purposes, we can safely neglect the spacetime dependence of the emergent cosmological constant Λ and consider it a constant indeed. Remember that, initially, the cosmological constant was set to zero. A non-zero initial or bare cosmological constant Λ_b would just be shifted by the Λ found above and the physical cosmological constant would be $\Lambda_{eff} \equiv \Lambda_b + \Lambda$. The important proposition here is that quantum gravitational waves give a non-zero contribution to the cosmological constant Λ_{eff}. In this scenario, the new gravitational interactions and degrees of freedom appearing in high energies are represented by non-local effects in the low-energy limit. The latter, combined with the local interactions, yields a gravitational energy-momentum tensor whose trace is non-vanishing and which contributes to the total cosmological constant.

4.4 Gravitational wave propagation

Up to now, only the physics of the low-frequency waves has been considered. For completeness, we shall turn our attention to the high-frequency ones, which will lead to the equation describing the GW propagation in curved spacetime. This is easily achieved by subtracting the background equation (4.20) from the total EOM (4.8)

$$G_{\mu\nu} + \Delta G_{\mu\nu} - \langle G_{\mu\nu} + \Delta G_{\mu\nu}\rangle = 8\pi G(T_{\mu\nu} - \langle T_{\mu\nu}\rangle), \tag{4.36}$$

where $\Delta G_{\mu\nu} = \Delta G^L_{\mu\nu} + \Delta G^{NL}_{\mu\nu}$. Ignoring the local part for a moment and keeping only the terms up to linear order in h and λ/L gives

$$R^{(1)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(1)} + \frac{2}{\xi}(\nabla_\mu\nabla_\nu - \bar{g}_{\mu\nu}\Box)\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)} + \frac{1}{\zeta}\Bigg[\Box\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)}_{\mu\nu} + \bar{g}_{\mu\nu}\nabla_\rho\nabla_\sigma\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)\rho\sigma} - \nabla_\rho\nabla_\mu\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)\rho}_\nu - \nabla_\rho\nabla_\nu\log\left(\frac{-\Box}{\mu^2}\right)R^{(1)\rho}_\mu\Bigg] = 0 \tag{4.37}$$

Outside the matter source, we can use the gauge $\nabla^\nu h_{\mu\nu} = 0$ together with $h = 0$, leading to

$$\Box h_{\mu\nu} + 16\pi G\beta\log\left(\frac{-\Box}{\mu^2}\right)\Box^2 h_{\mu\nu} = 0. \tag{4.38}$$

Analogously, including the local terms gives

$$\Box h_{\mu\nu} + 16\pi G\left[b_2 + \beta\log\left(\frac{-\Box}{\mu^2}\right)\right]\Box^2 h_{\mu\nu} = 0. \tag{4.39}$$

When deriving Equation (4.39), we made use of the commutation relation of covariant derivatives and we discarded terms proportional to the background curvature as they only contribute to higher orders in λ/L. Equation (4.39) describes the propagation of GWs in an arbitrary curved background in the absence of external matter, when the only source for a non-vanishing Ricci tensor is the GW energy-momentum tensor. The curvature terms do not appear as they provide no contribution to leading order. Therefore, the case where curvature is present can be obtained by applying a simple "minimal coupling" prescription to Equation (4.26) where spacetime is flat, that is, by performing the following substitution

$$\eta_{\mu\nu} \to \bar{g}_{\mu\nu}, \tag{4.40}$$

$$\partial_\mu \to \nabla_\mu. \tag{4.41}$$

Equations (4.20) and (4.39) together describe the entire classical and quantum process (to leading order) of the GW self-gravitation: small perturbations around spacetime change the curvature, which in turn modify the GW's trajectory and vice-versa.

4.5 Conclusions

We showed in this paper how to calculate the quantum corrections to the GW stress-energy tensor. The result shows that quantum effects promote the traceless tensor $t^{\rm GR}_{\mu\nu}$ to a traceful quantity that contributes to the current accelerated expansion of the universe. In addition, the energy density component acquires a dependence on modifications to the dispersion relation, indicating a useful observable to probe when looking for quantum gravitational effects. In fact, by using the latest LIGO's data, it was obtained a new upper bound on the amplitude of the massive mode. We also showed that the high-frequency mode equation led to a generalization of the wave equation (4.26) to arbitrary curved spacetimes (4.39). Such a generalization is important to the study of quantum GW solutions in cosmology and in the early universe where the spacetime was highly curved. Lastly, it must be stressed once again that these quantum contributions are model independent (since they are derived from an Effective Field Theory) and constitute actual predictions of Quantum Gravity, shedding new light on Quantum Gravity as a whole and giving us some hints of how a complete theory, if such a theory exists, should behave below the Planck scale.

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Chapter 5

Higgs Starobinsky Inflation

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In this paper we point out that Starobinky inflation could be induced by quantum effects due to a large non-minimal coupling of the Higgs boson to the Ricci scalar. The Higgs Starobinsky mechanism provides a solution to issues attached to large Higgs field values in the early universe which in a metastable universe would not be a viable option. We verify explicitly that these large quantum corrections do not destabilize Starobinsky's potential.

The idea that inflation may be due to degrees of freedom already present in the standard model of particle physics or quantum general relativity is extremely attractive and has received much attention in the recent years. In particular two models stand out for their simplicity and elegance. Higgs inflation [Bezrukov and Shaposhnikov, 2008, Barvinsky et al., 2008, Barvinsky et al., 2009] with a large non-minimal coupling of the Higgs boson H to the Ricci scalar ($\xi H^\dagger HR$) and Starobinsky's inflation model [Starobinsky, 1980] based on R^2 gravity are both minimalistic and perfectly compatible with the latest Planck data [Akrami et al., 2018].

These two models should not be considered as physics beyond the standard model but rather both operators $\xi H^\dagger HR$ and R^2 are expected to be generated when general relativity is coupled to the standard model of particle physics. We will come back to that point shortly. The aim of this paper is to point out an intriguing distinct possibility, namely that Starobinsky inflation is generated by quantum effects due to a large non-minimal coupling of the Higgs boson to the Ricci scalar. In that framework, we do not need to posit that the Higgs boson starts at a high field value in the early universe which would alleviate constraints coming from the requirement of having a stable Higgs potential even for large Higgs field values [Kobakhidze and Spencer-Smith, 2014, Degrassi et al., 2012, Bezrukov et al., 2012].

We shall now argue that both terms necessary for Higgs inflation or Starobinsky's model are naturally present when the standard model of particle physics is coupled to general relativity. While the quantization of general relativity remains one of the outstanding challenges of theoretical physics, it is possible to use effective field theory methods below the energy scale $M_\star$ at which quantum gravitational effects are expected to become large. The energy scale $M_\star$ is usually assumed to be of the order of the Planck scale $M_P = \sqrt{8\pi G_N}^{-1} = 2.4335 \times 10^{18}$ GeV, however recent work has shown that even in four space-time dimensions this energy scale is model dependent. At energies below $M_\star$, we can describe all of particle physics and cosmology with the following effective field theory (see e.g. [Codello and Jain, 2016, Donoghue and El-Menoufi, 2014, Birrell and Davies, 1984])

$$S = \int d^4x \sqrt{-g}\Bigg(\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R - \Lambda_C^4 + c_1 R^2 + c_2 C^2 + c_3 E + c_4 \Box R - L_{SM} - L_{DM} + O(M_\star^{-2})\Bigg) \tag{5.1}$$

where we have restricted our considerations to dimension four operators which are expected to dominate at least at low energies. Note that we are using the Weyl basis and the following notations: R stands for the Ricci scalar, $R^{\mu\nu}$ for the Ricci tensor, $E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$, $C^2 = E + 2R_{\mu\nu}R^{\mu\nu} - 2/3R^2$, the dimensionless ξ is the non-minimal coupling of the Higgs boson H to the Ricci scalar, the coefficients c_i are dimensionless free parameters, the cosmological constant Λ_C is of order of 10^{-3} eV, the Higgs boson vacuum expectation value, $v = 246$ GeV contributes to the value of the Planck scale

$$(M^2 + \xi v^2) = M_P^2\,, \tag{5.2}$$

L_{SM} contains all the usual standard model interactions (including mass terms for neutrinos) and finally L_{DM} describes the dark matter sector (this is the only part of the model which has not yet been tested experimentally). Submillimeter pendulum tests of Newton's law [Hoyle et al., 2004] lead to extremely weak limits on the parameters c_i. In the absence of accidental cancellations between these coefficients, they are constrained to be less than 10^{61} [Calmet et al., 2008]. The discovery of the Higgs boson and precision measurements of its couplings to fermions and bosons at the LHC can be used to set a limit on ξ. One finds that $|\xi| < 2.6 \times 10^{15}$ [Atkins and Calmet, 2013]. Clearly very little is known about the values of c_i and ξ.

Besides describing all of particle physics and late time cosmology, the action given in Eq. (5.1) can also describe inflation if some of its parameters take specific values and if some of its fields fulfil specific initial conditions in the early universe. This action, depending on the initial conditions, can describe either Higgs inflation if $\xi \sim 10^4$ and the Higgs field is chosen to take large values in the early universe or Starobinsky inflation if $c_1 \sim 10^9$ and the corresponding scalar extra degree of freedom (which can be made more visible by going to the Einstein frame) takes large values in the early universe.

If we assume that the Higgs field take small values in the early universe, Eq. (5.1) reduces to

$$S^J_{Starobinsky} = \int d^4x\sqrt{g}\frac{1}{2}\left(M_P^2 R + c_S R^2\right) \tag{5.3}$$

during inflation which in the Einstein frame gives

$$S^E_{Starobinsky} = \int d^4x\sqrt{g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{M_P^4}{c_S}\left(1-\exp\left(-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}\right)\right)^2\right) \tag{5.4}$$

We have assumed that the scalar degree of freedom σ hidden in R^2 takes large field values in the early universe. A successful prediction of the density perturbation $\delta\rho/\rho$ requires $c_S = 0.97 \times 10^9$ [Netto et al., 2016, Starobinsky, 1983]. On the other hand, if we assume that only the Higgs field takes large values in the early universe,the action (5.1) reduces to

$$\begin{aligned} S^J_{Higgs} &= \int d^4x\sqrt{-g}\left(\frac{M^2}{2}R + \xi_H H^\dagger H R - L_{SM}\right) \\ &= \int d^4x\sqrt{-g}\left(\frac{M^2+\xi_H h^2}{2}R - \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{\lambda}{4}(h^2-v^2)^2\right) + \cdots . \end{aligned} \tag{5.5}$$

In the Einstein frame, one obtains

$$S^E_{Higgs} = \int d^4x\sqrt{\hat{g}}\left(\frac{M_P^2}{2}\hat{R} - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + U(\chi) + \cdots\right) \tag{5.6}$$

with

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_H^2 h^2/M_P^2}{\Omega^4}} \tag{5.7}$$

where $\Omega^2 = 1 + \xi_H^2 h^2/M_P^2$ and

$$U(\chi) = \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}(h(\chi)^2 - v^2)^2. \tag{5.8}$$

A successful prediction of the density perturbation $\delta\rho/\rho$ requires $\xi_H = 1.8 \times 10^4$.

These two models are very attractive because they do not necessitate physics beyond the standard model. Furthermore, they are compatible with current cosmological observations which favour small tensor perturbations that so far have not been observed. It has actually been pointed out that both models are phenomenologically very similar [Bezrukov and Gorbunov, 2012, Salvio and Mazumdar, 2015]. However, while Starobinky's inflation model does not suffer from any obvious problem, it has recently been pointed out that in the case of Higgs inflation, which necessitates the Higgs field to take very large field values, our universe will not end up in the standard model Higgs vacuum if it is metastable as suggested by the latest measurement of the top quark mass, but rather in the real vacuum of the theory which does not correspond to the world we observe. In this paper, we point out that there is an alternative possibility. Namely when quantum corrections are taken into account, a large non-minimal coupling of the Higgs boson can generate Starobinsky inflation by generating a large coefficient for the coefficient of R^2 in the early universe. While the model corresponds to Starobinsky's model, the Higgs boson plays a fundamental role as it triggers inflation by generating a large coefficient for R^2.

The action given in Eq. (5.1) needs to be renormalized. We will work in dimensional regularization to avoid having to discuss the dependence of observables on the cutoff (this problem is due to the non-renormalizability of quantum gravity). We shall neglect the cosmological constant which is not important for inflation purposes. In that case, Newton's constant does not receive any correction to leading order. On the other hand, the coefficient c_1 of R^2 gets renormalized and one can define a renormalization group equation for this coupling constant. N_s scalar fields with a non-minimal coupling to the Ricci scalar ξ will lead to the following renormalization group equation [Codello and Jain, 2016, Donoghue and El-Menoufi, 2014, Birrell and Davies, 1984]

$$\mu \partial_\mu c_1(\mu) = \frac{(1-12\xi)^2}{1152\pi^2} N_s \tag{5.9}$$

to leading order (i.e. neglecting the graviton contribution which is suppressed by $1/\xi$), note that fermions and vector fields do not contribute to the renormalization of c_1 in the Weyl basis. The renormalization group equation can be easily integrated, one finds [Codello and Jain, 2016, Donoghue and El-Menoufi, 2014, Birrell and Davies, 1984]:

$$c_1(\mu_2) = c_1(\mu_1) + \frac{(1-12\xi)^2 N_s}{1152\pi^2} \log \frac{\mu_2}{\mu_1}. \tag{5.10}$$

The bounds on c_1 in today's universe are very weak as mentioned before. Even if $c_1(today)$ is of order unity, it would have been large in the early universe if the Higgs non-minimal coupling ξ is large. Indeed, we assume that inflation took place at some high energy scale e.g. $\mu \sim 10^{15}$ GeV, the log term is a factor of order 60 if we take the scale μ_1 of the order of the cosmological constant. A Higgs non-minimal coupling to the Ricci scalar of $\xi = 1.8 \times 10^4$ would lead to a coefficient $c_1 = 0.97 \times 10^9$ for R^2. Assuming that the scalar extra degree contained in R^2 took large field values in the early universe, a large non-minimal coupling of the Higgs boson to the Ricci scalar can trigger Starobinsky inflation even if the standard model vacuum is metastable as the Higgs boson itself does not roll down its potential during inflation. Inflation is due entirely to the R^2 but is triggered by the Higgs large non-minimal coupling.

Let us emphasize two important points. The first one is that $c_1 \sim 0.97 \times 10^9$ is fixed by the CMB constraint [Netto et al., 2016]. This parameter only takes such a large value at inflationary energy scales due to its renormalization group evolution. The second one is that we are neglecting the running of the Higgs boson non-minimal coupling to the Ricci scalar. However, this is a very good approximation. The leading contributions of the standard model to the beta-function of the non-minimal coupling are known [Buchbinder et al., 1992] :

$$\beta_\xi = \frac{6\xi + 1}{(4\pi)^2}\left[2\lambda + y_t^2 - \frac{3}{2}g^2 - \frac{1}{4}g'^2\right] \tag{5.11}$$

where λ is the self-interaction coupling of the Higgs boson, y_t is the top quark Yukawa coupling, g the SU(2) gauge coupling and g' the U(1) gauge coupling. Quantum gravitational corrections will be suppressed by powers of the Planck mass and can thus be safely ignored as long as we are at energies below the Planck mass.

One might worry that if the large non-minimal coupling of the Higgs boson triggers a large coefficient for the operator R^2, it might also generate new terms in the effective action which could destabilize the potential. The leading order effective action to the second order in the curvature expansion induced by scalar fields non-minimally coupled to gravity is known [Codello and Jain, 2016, Donoghue and El-Menoufi, 2014]:

$$S_{EFT} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R + \alpha R^2 + \beta R \log\frac{-\Box}{\mu^2}R + \gamma C^2 + \cdots\right). \tag{5.12}$$

Note that here we are neglecting the cosmological constant, $\alpha = c_1 \times 16\pi G$ and $\gamma = c_2 \times 16\pi G$ are renormalized coupling constants and we shall assume that c_2 is small at the scale of inflation, it is not sensitive to the Higgs boson's non-minimal coupling, while we have fixed the Higgs non-minimal coupling such that $c_1 = 0.97 \times 10^9$. The coefficient β is a prediction of the effective action and is given by $N_s(1 - 12\xi)^2/(2304\pi^2) \times 16\pi G$ [Donoghue and El-Menoufi, 2014] where N_s is the number of scalar field degrees of freedom in the model, in our case 4. The coefficient $N_s(1 - 12\xi)^2/(2304\pi^2)$ is indeed large and of the order of 7.8×10^6 and we have to check that the log-term does not lead to sizable contributions to the effective potential of the Starobinsky's field. Before verifying this explicitly, let us mention that the large non-minimal coupling between the Higgs boson and the Ricci scalar which is necessary to induce Starobinsky inflation does not lead to perturbative unitarity problems [Calmet and Casadio, 2014] (see Appendix A).

Note that the coefficients of E and of C^2 do not depend on the non-minimal coupling of the Higgs boson to the Ricci scalar. Furthermore in 4 dimensions, E does not contribute to the equations of motion. The coefficient of the term C^2 is assumed before renormalization to be of the same order as that of R^2, i.e. of order 1. However, after renormalization, the coefficient of R^2 is tuned to be very large and of the order of 10^9 while the coefficient of C^2 remains small compared to the renormalized coefficient of R^2. C^2 is thus negligible.

We shall treat the effective action (5.12) as a $F(R)$ gravity with $F(R) = R + \alpha R^2 + \beta R \log \frac{-\Box}{\mu^2} R$. There is a well established procedure to map such models from the Jordan frame to the Einstein frame, see e.g. [Sebastiani and Myrzakulov, 2015]. The potential for the inflaton in the Einstein frame is given by

$$V(\phi) = \frac{1}{2\kappa^2}\left(e^{\sqrt{\frac{2}{3}}\kappa\phi} R(\phi) - e^{2\sqrt{\frac{2}{3}}\kappa\phi} F(R(\phi))\right) \tag{5.13}$$

where $\kappa^2 = 8\pi G$ and $R(\phi)$ is a solution to the equation

$$\phi = -\sqrt{\frac{3}{2}}\frac{1}{\kappa}\log\frac{dF(R)}{dR}. \tag{5.14}$$

We can find a formal solution to this equation

$$R(\phi) = \frac{1}{2\alpha}\left(\frac{1}{1+\frac{\beta}{2\alpha}\log\left(\frac{-\Box}{\mu^2}\right)}\right)\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi} - 1\right). \tag{5.15}$$

This expression for $R(\phi)$ can be understood as a series in $\frac{\beta}{2\alpha}$ which is a small parameter:

$$R(\phi) = \frac{1}{2\alpha}\left(1 - \sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{\beta}{2\alpha}\log\left(\frac{-\Box}{\mu^2}\right)\right)^n\right)\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi} - 1\right). \tag{5.16}$$

where the log-term can be expressed using

$$\log\left(\frac{-\Box}{\mu^2}\right) = \int_0^\infty ds\left(\frac{1}{\mu^2+s} - \frac{1}{-\Box+s}\right). \tag{5.17}$$

The zeroth order term in $\frac{\beta}{2\alpha} \sim 4\times 10^{-3}$ corresponds to the usual Starobinsky solution:

$$R(\phi)^{(0)} = R(\phi)_{Starobinsky} = \frac{1}{2\alpha}\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi} - 1\right). \tag{5.18}$$

The series expansion will generate higher order terms corresponding to operators of the type $\exp(-\sqrt{\frac{2}{3}}\kappa\phi)(2/3\kappa^2\partial_\mu\phi\partial^\mu\phi - \sqrt{2/3}\kappa\Box\phi)$ and higher derivatives thereof. These new terms are however suppressed by powers of $\frac{\beta}{2\alpha}$ and can be safely ignored. It is easy to check that the log-term appearing in the $F(R)$ term of the potential (5.13) is also suppressed by $\frac{\beta}{2\alpha}$ compared to the usual Starobinsky's potential.

We conclude that the large quantum corrections induced by the large Higgs boson non-minimal coupling do not affect the flatness of Starobinsky's potential. Let us add a few remarks. The model discussed above is not a new model. Physics (including reheating or preheating and all of particle physics) is identical to that predicted in Starobinsky's model. We merely identify a new connection between the Higgs boson and inflation. As in the case of the standard Starobinsky model, a coupling $\phi^2 h^2$ will be generated. It is however suppressed by factors of m^2_{Higgs}/M_P^2 which is a small number, particle physics will thus not be affected and the Higgs boson behaves as the standard model Higgs boson. Furthermore, the Higgs field does not take large values in the early universe, we can thus safely ignore the term $H^\dagger HR$ when studying the inflationary potential. Note that there are subtleties when considering the equivalence of quantum corrections in different parameterizations/representations of the theory (i.e. when going from the Jordan frame to the Einstein frame). Here we are avoiding this problem: we renormalized the theory in the Jordan frame where the model is defined and then map the effective action to the Einstein frame. When proceeding this way, there are no ambiguities or risk to mix up the orders in perturbation theory and the expansion in the conformal factor (see e.g. [Calmet and Yang, 2013, Kamenshchik and Steinwachs, 2015, Vilkovisky, 1984]).

In this paper, we have identified a new connection between the Higgs boson and inflation. In the model envisaged here, the Higgs boson is not the inflaton but it generates inflation by creating a large Wilson coefficient for the R^2 operator and it is thus at the origin of Starobinsky's inflation. This mechanism is interesting as it does not require physics beyond the standard model. The Higgs boson does not need to take large field values in the early universe and we could thus be living in a metastable potential.

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Chapter 6

Non-Minimal Coupling of the Higgs Boson to Curvature in an Inflationary Universe

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In the absence of new physics around 10^{10} GeV, the electroweak vacuum is at best metastable. This represents a major challenge for high scale inflationary models as, during the early rapid expansion of the universe, it seems difficult to understand how the Higgs vacuum would not decay to the true lower vacuum of the theory with catastrophic consequences if inflation took place at a scale above 10^{10} GeV. In this paper, we show that the non-minimal coupling of the Higgs boson to curvature could solve this problem by generating a direct coupling of the Higgs boson to the inflationary potential thereby stabilizing the electroweak vacuum. For specific values of the Higgs field initial condition and of its non-minimal coupling, inflation can drive the Higgs field to the electroweak vacuum quickly during inflation.

The non-minimal coupling $\xi\phi^2 R$ of scalars (ϕ) to curvature R has attracted much attention in the recent years. Indeed, in four space-time dimensions, ξ is a dimensionless coupling constant and as such is likely to be a fundamental constant of nature. With the discovery of the Higgs boson, the only known fundamental scalar field so far observed, it became clear that this parameter is relevant and should be considered when coupling the standard model of particle physics to general relativity.

The value of the non-minimal coupling of the Higgs boson to curvature is a free parameter of the standard model of particle physics. There has been no direct measurement so far of this fundamental constant of nature. The discovery of the Higgs boson at the Large Hadron Collider at CERN and the fact that the Higgs boson behaves as expected in the standard model implies that the non-minimal coupling is smaller than 2.6×10^{15} [Atkins and Calmet, 2013]. This bound comes from the fact that for a large non-minimal coupling the Higgs boson would decouple from the standard model particles. We have little theoretical prejudice on the magnitude of this constant. Conformal invariance would require $\xi = 1/6$, but this symmetry is certainly not an exact symmetry of nature.

Assuming that the standard model is valid up to the Planck scale or some 10^{18} GeV, the early universe cosmology of the Higgs boson represents an interesting challenge. Given the mass of the Higgs boson which has been measured at 125 GeV and the current measurement of the top quark mass, the electroweak vacuum is at best metastable [Degrassi et al., 2012]. The implication of this metastability of the electroweak vacuum for the standard model coupled to an inflation sector has recently been discussed [Lebedev and Westphal, 2013]. Indeed, one finds that the Higgs quartic coupling which governs the shape of the Higgs potential for large field value turns negative at an energy scale $\Lambda \sim 10^{10} - 10^{14}$ GeV. The electroweak vacuum with the minimum at 246 GeV is not the ground state of the standard model, but rather there is a lower minimum to the left and our vacuum is only metastable. This is a problem in an inflationary universe.

In an expanding universe with Hubble scale H, the evolution of the Higgs boson h is given by

$$\ddot{h} + 3H\dot{h} + \frac{\partial V(h)}{\partial h} = 0 \tag{6.1}$$

where $V(h)$ is the potential of the scalar field. Even if one imposes as an initial condition at the start of our universe that the Higgs field starts at the origin, it will most likely be excited to higher field values during inflation. Indeed, because the mass of the Higgs boson is very small compared to the scale of inflation, it is essentially massless. Quantum fluctuations of the Higgs field will drive it away from the minimum of the potential. Its quantum fluctuations are of order the Hubble scale H. Thus, for $H > \Lambda$, it is likely that the Higgs will overshoot the barrier between the false vacuum in which our universe lives and the lower state true vacuum of the theory.

In [Lebedev and Westphal, 2013, Lebedev, 2012], it is shown that a direct coupling of the Higgs boson to the inflaton field can significantly affect this picture if this coupling makes the Higgs potential convex. This interaction between the inflaton and the Higgs boson drives the Higgs field to small values during inflation. This is closely related to an earlier claim [Espinosa et al., 2008] that the curvature coupling of the Higgs boson resembles an additional mass term $-\xi R$ in the Higgs potential and could stabilise the Higgs boson. We shall argue below this interpretation of the curvature term is not entirely correct, and in fact, the two mechanisms are closely related when carried out correctly. Assuming that there is no new physics between the weak scale and the scale of inflation, we shall derive a new prediction for the value of the non-minimal coupling of the Higgs boson to the Ricci scalar.

Before the discovery of the Higgs boson, cosmologists had already been investigating the non-minimal coupling of scalars to curvature. In inflationary cosmology one often deals with actions of the type

$$S_{scalar} = \int d^4x \, \sqrt{-g} \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \xi \phi^2 R \right), \tag{6.2}$$

where m is the mass of the scalar field ϕ. This coupling has been extensively studied, see e.g. [Chernikov and Tagirov, 1968, Callan et al., 1970, Frommert et al., 1999, Cervantes-Cota and Dehnen, 1995, Bezrukov and Shaposhnikov, 2008]. With the discovery of the Higgs boson, it became clear that this coupling was not only an exotic term that could be implemented in curved space-time but that this coupling is phenomenologically relevant.

Before deriving our prediction for the value of the non-minimal coupling of the Higgs boson to curvature, we need to address a common misconception which can be very important when discussing Higgs physics within the context of cosmology and very early universe physics. It is often argued that the non-minimal coupling which appears in Eq.(6.2) of a scalar field to curvature is identical to a contribution to the mass of the scalar field that is curvature dependent. We will prove that this is not strictly correct. We will then show that the non-minimal coupling of the Higgs boson to curvature does actually help to stabilize the Higgs potential, and furthermore, it can even drive the Higgs field towards the false vacuum from a Planck-scale initial value.

We shall first address the issue of the Higgs mass. If one naively varies the action for a scalar field ϕ containing the non-minimal coupling (6.2), one obtains the field equation

$$(\Box + m^2 - \xi R)\phi = 0, \tag{6.3}$$

and it is often argued that this term ξR is a curvature dependent mass term for the scalar field ϕ. In an FLRW background, the curvature drops from $R = 12H^2$ during inflation, with constant expansion rate H, to $R \approx 0$ in a radiation dominated era after inflation, which could lead to an overproduction of the Higgs boson after inflation [Herranen et al., 2015]. This argument is however incomplete. The problem is that the non-minimal coupling induces a mixing between the kinetic term of the scalar field and of the metric field. We will illustrate this point with the standard model of particle physics since this is the only model so far that contains a fundamental scalar field which has actually been discovered experimentally, however the same line of reasoning applies to any scalar field non-minimally coupled to curvature.

Starting with the standard model Lagrangian $\mathcal{L}_{SM}$, we have

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi \mathcal{H}^\dagger \mathcal{H} \right) R - (D^\mu \mathcal{H})^\dagger (D_\mu \mathcal{H}) - \mathcal{L}_{SM} \right] \tag{6.4}$$

where $\mathcal{H}$ is the SU(2) scalar doublet, we shall see that this is not actually the Higgs boson of the standard model. After electroweak symmetry breaking, the scalar boson gains a non-zero vacuum expectation value, $v = 246$ GeV, M and ξ are then fixed by the relation

$$(M^2 + \xi v^2) = M_P^2 \,. \tag{6.5}$$

The easiest way to see that $\mathcal{H}$ is not actually the Higgs boson is by doing a transformation to the Einstein frame [van der Bij, 1994, Zee, 1979, Minkowski, 1977]

$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, where $\Omega^2 = (M^2 + 2\xi\mathcal{H}^\dagger\mathcal{H})/M_P^2$. The action in the Einstein frame then reads

$$S = \int d^4x \sqrt{-\tilde{g}}\left[\frac{1}{2}M_P^2\tilde{\mathcal{R}} - \frac{3\xi^2}{M_P^2\Omega^4}\partial^\mu(\mathcal{H}^\dagger\mathcal{H})\partial_\mu(\mathcal{H}^\dagger\mathcal{H}) - \frac{1}{\Omega^2}(D^\mu\mathcal{H})^\dagger(D_\mu\mathcal{H}) - \frac{\mathcal{L}_{SM}}{\Omega^4}\right]. \tag{6.6}$$

Expanding around the Higgs boson's vacuum expectation value and specializing to unitary gauge, $\mathcal{H} = \frac{1}{\sqrt{2}}(0, \phi + v)^\top$, we see that in order to have a canonically normalized kinetic term for the physical Higgs boson we need to transform to a new field χ where

$$\frac{d\chi}{d\phi} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2v^2}{M_P^2\Omega^4}}\,. \tag{6.7}$$

Expanding $1/\Omega$, we see at leading order the field redefinition simply has the effect of a wave function renormalization of $\phi = \chi/\sqrt{1+\beta}$ where $\beta = 6\xi^2v^2/M_P^2$. Thus the canonically normalized scalar field, i.e., the true Higgs boson, does not have any special coupling to gravity and it couples like any other field to gravity in accordance with the equivalence principle.

This effect can also be seen in the Jordan frame action (6.4) as arising from a mixing between the kinetic terms of the Higgs and gravity sectors. After fully expanding the Higgs boson around its vacuum expectation value and also the metric around a fixed background, $g_{\mu\nu} = \bar{\gamma}_{\mu\nu} + h_{\mu\nu}$, we find a term proportional to $\xi v\phi\Box h^\mu_\mu$:

$$\begin{aligned}\mathcal{L}^{(2)} &= -\frac{M^2+\xi v^2}{8}(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial_\rho h^{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu\Box h^\nu_\nu \\ &\quad +\frac{1}{2}(\partial_\mu\phi)^2 + \xi v(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu})\phi\end{aligned} \tag{6.8}$$

After correctly diagonalizing the kinetic terms and canonically normalizing the Higgs field and graviton using

$$\phi = \chi/\sqrt{1+\beta} \tag{6.9}$$

$$h_{\mu\nu} = \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\bar{\gamma}_{\mu\nu}\chi. \tag{6.10}$$

We again find the physical Higgs boson gets renormalized by a factor $1/\sqrt{1+\beta}$.

These results demonstrate that the non-minimal coupling does not introduce stronger gravitational interactions for the Higgs boson once its field has been correctly canonically normalized. We stress that the underlying reason is that there is no violation of the equivalence principle. Our findings are in sharp contrast to the claims made in [Herranen et al., 2014]. The only valid bound to date on the non-minimal coupling of the Higgs boson to curvature is that obtained in [Atkins and Calmet, 2013], namely that its non-minimal coupling is smaller than 2.6×10^{15}. While the fact that we may be living in a metastable vacuum is problematic for the Higgs boson in an inflationary context, the non-minimal coupling of the Higgs boson to curvature does not create a new problem. On the contrary, we shall now show that this non-minimal coupling could solve the stability issue.

Let us now study the coupling of the Higgs boson to an inflationary potential $V_I(\sigma)$ that is induced by the mapping from the Jordan frame to the Einstein frame. Indeed, even if no direct coupling between the Higgs boson is assumed in the Jordan frame, it will be induced in the Einstein frame:

$$V_I(\sigma) \rightarrow \frac{V_I(\sigma\Omega)}{\Omega^4} = \frac{V_I(\sigma\Omega)}{\left(1 + \frac{2\xi v \phi(\chi) + \xi \phi(\chi)^2}{M_P^2}\right)^2}, \tag{6.11}$$

but bear in mind that the inflaton field σ does not have a canonically normalized kinetic term.

Let us first consider Higgs field values $v \ll \phi \ll M_P |\xi|^{-1/2}$. In that case, we see immediately that

$$\frac{V_I(\sigma\Omega)}{\Omega^4} \approx V_I(\sigma) \left(1 - 2\xi\phi^2/M_p^2\right). \tag{6.12}$$

A coupling between the inflaton and the Higgs field is induced by the transformation to the Einstein frame. Note that there is a priori no reason to exclude a coupling of the type $V_I \mathcal{H}^\dagger \mathcal{H}$ in the Jordan frame where the theory is defined. There could be cancelations between this coupling and that generated by the map to the Einstein frame. The magnitude of the coupling between the Higgs boson and the inflaton appearing in the mapped inflationary potential thus cannot be regarded as a prediction of the model. Let us ignore a potential direct inflaton-Higgs coupling for the time being and continue our investigation of the induced coupling. We will now show that a non-minimal coupling of the Higgs boson to curvature can solve some of the problems associated with Higgs cosmology within the standard model of particle physics.

In the early universe we need to consider large Higgs field values ($\phi \gg v$). As explained previously, even if one is willing to fine-tune the initial condition for the value of the Higgs field, it will experience quantum fluctuations of the order of the Hubble scale H. Unless the Hubble scale is much smaller than the energy scale at which the electroweak vacuum becomes unstable, the Higgs field is likely to swing into the lower true vacuum of the theory. A Higgs non-minimal coupling to the Ricci scalar could actually solve this problem since, as we will show, it will generate a direct coupling between the Higgs boson and the inflaton if the Jordan frame action contains an inflationary potential V_I.

It has been shown that a direct coupling between the Higgs boson and the inflaton can drive the Higgs field [Lebedev and Westphal, 2013] to the false electroweak vacuum quickly during inflation even if the Higgs field initial value is chosen to be large. There are basically three scenarios for the onset of inflation: the thermal initial state [Guth, 1981], ab initio creation [Vilenkin, 1983, Hawking and Moss, 1982] and the chaotic initial state [Linde, 1983, Linde, 1986]. The thermal initial state starting from a temperature just below the Planck scale would introduce thermal corrections to the Higgs potential preventing vacuum decay until the temperature fell to the inflationary de Sitter temperature, at which point it becomes a question of vacuum fluctuation as to whether the Higgs survives in the false vacuum. However, the consistency of the thermal equilibrium of the standard model fields when the Higgs takes a large value has not yet been verified. The ab initio creation is an attractive possibility, where the Higgs would nucleate at the top of the potential barrier. In this case also, stability depends on the size of vacuum fluctuations during inflation. The final possibility, the chaotic initial state, would have the Higgs field start out at arbitrarily large values. The most likely initial values would be larger than the instability scale Λ, preventing the Higgs field from entering the false vacuum. An anthropic argument could be applied to rule out these initial conditions, but we shall see that the non-minimal curvature coupling of the Higgs boson can force the Higgs into the false vacuum without anthropic considerations.

As we have seen, the Einstein frame potential is given by

$$V_E = \frac{V_I(\sigma) + V_\phi(\phi)}{(1 + \xi\kappa^2\phi^2)^2} \tag{6.13}$$

where $\kappa^2 = 8\pi G$. Note that V_E is the total potential in the Einstein frame and it accounts for both the inflaton potential in (6.11) and the Higgs potential. The inflationary expansion rate H_I is the expansion rate of the false vacuum,

$$H_I^2 = \frac{V_I(\sigma)}{3M_p^2}. \tag{6.14}$$

The most extreme chaotic initial condition, and the one relevant to eternal chaotic inflation, is one where V_E is close to the Planck scale. For an unstable Higgs potential V_ϕ, $V_E \sim M_p^4$ is only possible when $\xi < 0$, as shown in Fig. 6.1.

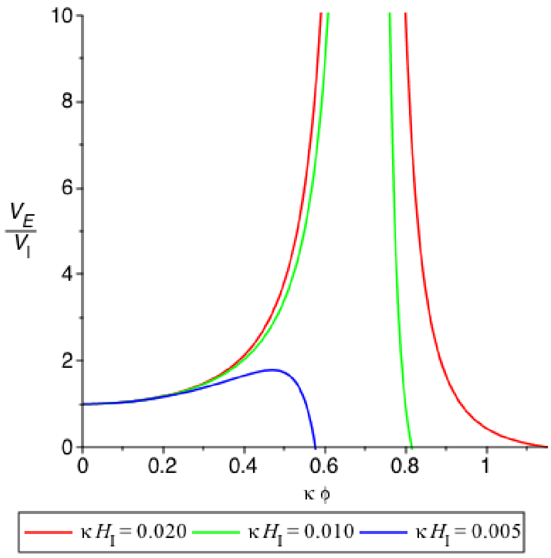

Figure 6.1: The Einstein frame Higgs potential $V_E(\phi)$ for different values of the false-vacuum inflation rate H_I for $\xi = -2$. The potential vanishes at $\phi = \phi_m$, and there is an asymptote at $\phi = \phi_c$. Consistency of the model (no ghosts) requires $\phi < \phi_c$. An initial condition $V_E \sim M_p^4$ can be achieved with the initial ϕ close to ϕ_c.

Let us denote by ϕ_m the value of the field at which the potential vanishes,

$$V_I(\sigma) + V_\phi(\phi_m) = 0. \tag{6.15}$$

Note that ϕ_m depends on H_I. The asymptote in the potential is at ϕ_c,

$$1 + \xi\phi_c^2/M_p^2 = 0. \tag{6.16}$$

Provided that $\phi_c < \phi_m$, then there is an initial value of ϕ close to ϕ_m at which $V_E \sim M_p^4$ (note that it has been shown in [Calmet and Casadio, 2014] that even with a large non-minimal coupling of the Higgs boson to curvature, the cutoff of the effective field theory can be as large as the Planck scale), since $\phi = \phi_c$ is an asymptote. If $\phi_c > \phi_m$, then there is no such value.

Starting from the initial value, the Higgs field evolves to small field values on a timescale comparable to the Hubble expansion rate. Unfortunately, we cannot simply expand the conformal factor in the denominator of the Einstein frame potential for all values of ξ. However, it is straightforward to see this effect from kinetic terms of the Higgs boson and of the inflaton. The kinetic terms for the Higgs and inflaton are multiplied by g_ϕ and g_σ respectively, where

$$g_\phi = \frac{1+\xi\kappa^2\phi^2+6\xi^2\kappa^2\phi^2}{(1+\xi\kappa^2\phi^2)^2}, \qquad g_\sigma = \frac{1}{(1+\xi\kappa^2\phi^2)^2} \tag{6.17}$$

Note that is it possible to use a canonically normalised Higgs field χ as we had done previously, but not both the Higgs and inflaton fields at the same time because the field space metric is curved.

The early evolution of the Higgs field is described by the equation

$$\ddot{\chi} + 3H\dot{\chi} + \frac{dV_E}{d\chi} = 0. \tag{6.18}$$

For the inflaton, one has

$$(g_\sigma\dot{\sigma})^{\cdot} + 3Hg_\sigma\dot{\sigma} + \frac{dV_E}{d\sigma} = 0, \tag{6.19}$$

while the expansion rate is given by

$$3H^2 = \kappa^2\left(\frac{1}{2}g_\sigma\dot{\sigma}^2 + \frac{1}{2}\dot{\chi}^2 + V_E\right). \tag{6.20}$$

The inflaton equation can also be written as

$$\ddot{\sigma} + \left(\frac{1}{g_\sigma}\frac{dg_\sigma}{d\chi}\right)\dot{\chi}\dot{\sigma} + 3H\dot{\sigma} + \frac{1}{g_\sigma}\frac{dV_E}{d\sigma} = 0. \tag{6.21}$$

Note that the second term in this equation is not considered in [Lebedev and Westphal, 2013]. For $\chi > M_p$, we have

$$V_E \approx (V_I + V_\phi)e^{\sqrt{8/3}\kappa(\chi-\chi_0)}, \qquad g_\sigma \approx e^{\sqrt{8/3}\kappa(\chi-\chi_0)}. \tag{6.22}$$

There is thus rapid evolution of χ and slow evolution of σ (assuming slow-roll conditions on V_I). Indeed, the inflaton evolves on a longer timescale than the Higgs field, leaving a gradual reduction in H_I, and also ϕ_m. Eventually, the potential evolves to $\phi_c > \phi_m$, but at all stages the Higgs field lies on the false vacuum side of the potential barrier. As long as the vacuum fluctuations do not cause quantum tunnelling, the Higgs field will enter the false vacuum.

The condition that $\phi_c < \phi_m$ implies limits on the curvature coupling ξ. In order to determine these limits we need to calculate ϕ_m from (6.15), and this requires an expression for the Higgs potential. For a standard model Higgs field, the large field Higgs potential in flat space is given by

$$V_\phi = \frac{1}{4}\lambda(\phi)\phi^4 \tag{6.23}$$

In curved space, the Higgs develops a mass of order H multiplied by Higgs couplings, but we can think of this as a radiative correction to ξ and regard ξ as the effective curvature coupling at the inflationary scale. Other curvature corrections to the Higgs potential may well be important, but for now these will be neglected.

The effective Higgs coupling $\lambda(\phi)$ vanishes at some large value of ϕ which we identify as the instability scale Λ. The value of Λ is very strongly dependent on the top quark mass, and currently all we can say is that it lies in the range $10^9 - 10^{18}$ GeV. Furthermore, adding additional particles to the standard model changes the instability scale (or removes the instability altogether). It is therefore convenient to give results treating Λ as a free parameter. In the range of Higgs field values where the potential barrier lies, we use an approximation to the running coupling given by

$$\lambda(\phi) \approx b\left\{\left(\ln\frac{\phi}{M_p}\right)^4 - \left(\ln\frac{\Lambda}{M_p}\right)^4\right\}, \tag{6.24}$$

with $b \approx 0.75 \times 10^{-7}$. This fits quite well to the renormalisation group calculations [Degrassi et al., 2012].

The plots in Fig. (2) show numerical results for the values of $-\xi$ which are lower bounds of the range which is consistent with chaotic initial conditions. Also shown by the dashed lines are the quantum bounds from the vacuum tunnelling rate $\exp(-8\pi^2\Delta V_E/3H_I^2) \sim O(1)$, where ΔV_E is the height of the potential barrier [Hawking and Moss, 1982]. (The quantum bound on $-\xi$ is lower than the one quoted in [Herranen et al., 2014], which we believe is due to our inclusion of the $8\pi^2/3$ factor.) The results show curves for different values of the false vacuum Hubble parameter, essentially corresponding to different initial values of the inflaton field through (6.14). We ought to expect that this initial Hubble parameter is close to the Planck scale. As advertised, a non-minimal coupling of the Higgs boson can drive the Higgs boson into the false vacuum of the standard model early on during

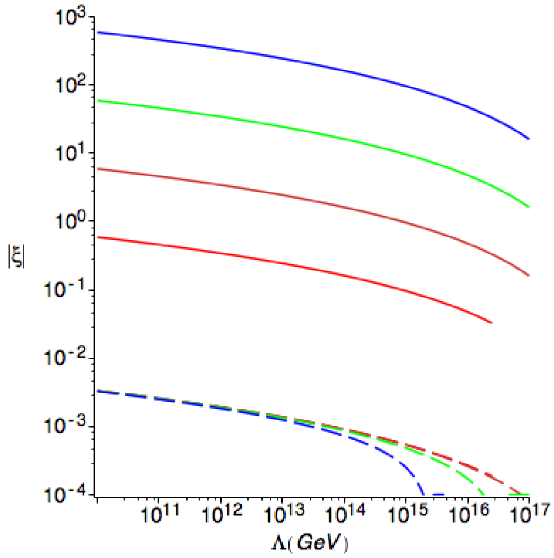

Figure 6.2: The lower bound on $-\xi$, where ξ is the curvature coupling, for consistent chaotic initial conditions on the Higgs field which will lead the Higgs into the false vacuum. The horizontal axis is the Higgs stability scale. The different curves from bottom to top are for the false vacuum Hubble parameter $0.1M_p$ to $10^{-4}M_p$. The dashed lines show the lower bound for quantum stability of the false vacuum.

inflation. Instead of being a source of problems, it can solve some of the issues associated with the cosmological evolution of the Higgs boson.

It is worth mentioning as well that our results also imply that the non-minimal coupling of the Higgs boson will not influence reheating as long as the Higgs field value is small during inflation. Reheating could be generated by a direct coupling of the Higgs boson to the inflaton via either couplings of the type $\sigma^2\mathcal{H}^\dagger\mathcal{H}$ or $\sigma\mathcal{H}^\dagger\mathcal{H}$. As usual right-handed neutrinos N could also play a role in reheating via a coupling $\bar{N}N\sigma$. However, none of these couplings will be significantly influenced by the conformal factor or the rescaling of the Higgs boson as long as one is considering small Higgs field values.

We have seen that a non-minimal coupling of the Higgs boson to the Ricci scalar does not generate new issues for Higgs boson physics in the early universe and that, on the contrary, there is a range of values for ξ for which the Higgs potential is stabilized thanks to the coupling of the Higgs boson to the inflaton generated by the non-minimal coupling of the Higgs boson to curvature. This becomes obvious when mapping the Jordan frame action to the Einstein frame. Finally, it has been shown in [Calmet and Casadio, 2014] that the non-minimal coupling ξ does not introduce a new scale below the Planck mass which finishes establishing our point that the standard model, if we add a non-minimal coupling to the Ricci scalar, could be valid up the Planck scale in an inflationary universe.

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Chapter 7

Conclusions

In this thesis, we have studied classical and quantum extensions of general relativity and their applications to gravitational waves, inflation and dark matter. We focused on effective field theories as they arise in the low energy limit of any UV completion, thus allowing one to investigate gravitational phenomena in a model-independent way.

In Chapter 2, which is [Calmet and Kuntz, 2017], we have shown that modifying the gravitational sector is not really different from modifying the matter sector. One unavoidably includes new degrees of freedom when the Einstein-Hilbert action is complemented with higher curvature invariants. Whether these new degrees of freedom belong to the matter or gravitational sector is just a matter of interpretation, thus not affecting the observables. We then used this equivalence to argue that dark matter could equally be described by a modified gravity model. It is important to stress that, by the time of writing, there is no generally accepted theory that explains the anomalous rotation galaxy curves. Nonetheless, whatever the theory for dark matter that turns out to be right, there will always exist a modified gravity equivalent version of it.

Then in Chapter 3, composed by [Calmet et al., 2016], we studied gravitational waves using the effective field theory approach to quantum gravity. As argued in Chapter 2, modifications of general relativity inevitably leads to new degrees of freedom. In quantum gravity, this is no different. We showed that new degrees of freedom appear in the form of complex poles in the dressed propagator of the graviton, i.e. the propagator containing one-loop quantum corrections. These new poles contribute to new modes of oscillation of gravitational waves and, because they are complex, they lead to a damping in gravitational waves. The damping forces the wave to lose energy to the environment, so it becomes crucial to take this effect into account when inferring the energy released during the merger of black holes. From the bound on the graviton mass found by LIGO, we could constrain the number of fields present in a fundamental theory of gravity.

In Chapter 4, which contains [Kuntz, 2018], we extended the study of gravitational waves and calculate the energy carried away by the complex modes. By employing the short-wave formalism, we were able to calculate the energy-momentum tensor of gravitational waves in quantum gravity. The energy density then follows directly from the energy-momentum tensor as usual. In addition to the term due to classical general relativity, another term that depends on modifications of the dispersion relation shows up. A direct comparison with the expression for the energy density with LIGO's data permits us to find the first constraint on the amplitude of the complex mode. We also showed how the gravitational wave equation in a flat spacetime can be generalized in a curved spacetime by a simple "minimal coupling" prescription.

In Chapter 5 we started the study of inflation via a new model proposed in [Calmet and Kuntz, 2016] which combines ideas from Higgs and Starobinsky inflation. We showed that Starobinsky gravity can naturally show up in the formalism of effective field theory. In fact, the square of the Ricci scalar is required for renormalization purposes. In addition, we showed that the coefficient of R^2 flows to the required value in the Starobinsky model when the coefficient of the non-minimal coupling between the Higgs boson and gravity is large. Hence, the Higgs boson is able to trigger Starobinsky inflation via its coupling to gravity. This avoids instability issues caused by large values of the Higgs boson as the scalaron in the Starobinsky model is the only field required to take large values in the early universe.

We continued the study of inflation in Chapter 6 through the non-minimal coupling of the Higgs boson to gravity [Calmet et al., 2018]. We showed that, after diagonalizing and canonically normalizing the action, the induced coupling between the inflationary potential and the Higgs is able to rapidly bring the Higgs field back to the false vacuum even when the scale of its fluctuation is higher than the potential barrier. Thus, the induced coupling between the Higgs and the inflaton's potential is able to stabilize the electroweak vacuum. We also considered the problem of quantum tunnelling that can happen between the false and true vacuum of the theory and we established bounds on the coefficient of the coupling between the Higgs and the curvature so that the Higgs boson remains in the electroweak vacuum.

Although this thesis has presented an important step forward in the field of modified gravity, many problems remain unaddressed. Particularly, there is still a plethora of models seeking elucidation of the dark sector, of inflation and of quantum gravity. In order to rule out some of them, more accurate data are necessary. Upcoming data from LIGO, LISA, Planck and other collaborations should help us on this matter. But in the meantime, while we wait for higher precision experiments, we should concentrate our efforts in theoretical and phenomenological aspects of the effective field theory of gravity as they are model-independent and, in principle, should correctly describe gravity all the way up to the Planck scale. Clearly, at the Planck scale the effective field theory breaks down and one must start worrying about possible UV completions. This is the greatest limitation of the formalism presented in this thesis as we cannot use it to study super-Planckian phenomena. In addition, the effective field theory approach does not address certain conceptual problems in quantum gravity, such as the problem of time. Nonetheless, it provides a systematic way of calculating observables and making falsifiable predictions.

We finish this thesis by indicating potential research directions:

- Can the effective field theory of gravity solve the problem of singularities? It is generally accepted that a quantum theory of gravity should be able to get rid of the singularities of general relativity. While we are still far from finding the UV completed theory for quantum gravity, quantum gravitational effects in the infrared could shed some new light on the problem.

- Black holes are known to cast shadows in their surroundings that are formed due to an extreme type of light bending, forcing photons to get in orbit around them. These shadows carry important information about the spacetime and have distinct phenomenological signatures that can be used to probe the differences among modified gravity theories and further constrain effective theories of gravity.

- General relativity is known to be plagued with pathologies such as traversable wormholes and closed timelike curves. If these solutions were real, they would allow for time travel backwards in time, which would violate causality. Can quantum gravity in the infrared rule out these possibilities?

- A natural extension of the formalism used to calculate the one-loop effective action of quantum gravity would be to consider the Palatini procedure, where the metric and the connection are seen as independent variables. In classical general relativity, varying with respect to the metric and to the connection separately still produces Einstein's equations. However, when quantum corrections are taken into account, this equivalence between the metric and Palatini formalisms no longer holds. The latter could lead to new insights on quantum gravity.

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Appendix A

Perturbative unitarity

It has been shown in [Calmet and Casadio, 2014] that a large non-minimal coupling of the Higgs to the Ricci scalar does not lead to a new physical scale. While perturbative unitarity appears to be naively violated at an energy scale of M_P/ξ, it can be shown by resumming an infinite series of one-loop diagrams in the large ξ and large N limits but keeping $\xi G_N N$ small that perturbative unitarity is restored (this phenomenon has been called self-healing by Donoghue). In this limit one finds

$$iD^{\alpha\beta\mu\nu}_{dressed} = -\frac{i}{2s}\frac{L^{\alpha\beta}L^{\mu\nu}}{\left(1 - \frac{sF_1(s)}{2}\right)}. \tag{A.1}$$

where $L^{\alpha\beta} = \eta^{\alpha\beta} - q^\alpha q^\beta/q^2$, $s = q^2$ and

$$F_1(q^2) = -\frac{1}{30\pi}N_s G_N(\bar{h})(1 + 10\xi + 30\xi^2)\log\left(\frac{-q^2}{\mu^2}\right). \tag{A.2}$$

The background dependent Newton's constant is given by

$$G_N(\bar{h}) = \frac{1}{8\pi(M^2 + \xi\bar{h}^2)}\ . \tag{A.3}$$

In the model described in this paper, one has $\bar{h} = v$. Note that $F_1(s)$ is negative, there is thus no physical pole in the propagator. The dressed amplitude in the large ξ and large N limits is given by

$$A_{dressed} = \frac{48\pi G_N(\bar{h})s\xi^2}{1 + \frac{2}{\pi}G_N(\bar{h})s\xi^2\log(-s/\mu^2)} \tag{A.4}$$

One easily verifies that the dressed amplitude of the partial-wave with angular momentum $J = 0$ fulfils

$$|a_0|^2 = \mathrm{Im}\,(a_0)\,, \tag{A.5}$$

where a_0 is the amplitude of the $J = 0$ partial wave. In other words, unitarity is restored within general relativity without any new physics or strong dynamics (we are keeping ξG_N small) and there is no new scale associated with the non-minimal coupling despite naive expectations. The cut-off of the effective theory is thus the usual Planck scale.