Prospects in Classical and Quantum Gravity

From Theory to Phenomenology

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Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another university for the award of any other degree. The work in this thesis has been completed in collaboration with Xavier Calmet, Ibere Kuntz, Basem El Menoufi, Djuna Croon, Marcelo Gleiser, Boris Latos and Chen Sun and is comprised of the following papers:

- D.Croon, M.Gleiser, S.Mohapatra and C.Sun, "Gravitational Radiation Background from Boson Star Binaries", published in Phys.Lett. B783, 158 (2018)

  I have collaboratively worked on all aspects of the paper. The plots were handled by Dr Djuna Croon, while the calculations, writings (I was the lead in the introduction and conclusion) and checking were done collaboratively.

- Xavier Calmet, Iberê Kuntz, Sonali Mohapatra, "Gravitational Waves in Effective Quantum Gravity", published in Eur.Phys.J. C76 (2016) no.8, 425.

  I have collaboratively worked on all aspects of the paper. Initial calculations were done by myself and Dr Kuntz, under the supervision of Dr Calmet. Writing up was done by Dr Calmet, rechecked by the whole team.

- X.Calmet, B.K.El-Menoufi, B.Latosh and S.Mohapatra, "Gravitational Radiation in Quantum Gravity", published in Eur. Phys. J. C78, no. 9, 780 (2018)

  I have collaboratively worked on all aspects of the paper. I was the calculational lead on all sections of the paper, especially 7.3 and 7.5 under the guidance of Dr Basem El-Menoufi who guided and rechecked my calculations. Section 7.4 was the work of Dr El-Menoufi which I re-checked .Writing up was done by Dr El Menoufi and myself, rechecked and corrected by Dr Calmet.

- Basem El-Menoufi, Sonali Mohapatra, "What can Black Holes tell us about the UV and IR?", submitted to PRD.

  I have collaboratively worked on all aspects of the paper. I was the calculational lead, rechecked by Dr Basem El-Menoufi. Dr El-Menoufi was the lead in Section 9.3, which was based on his previous work. Section 9.6 was calculated by me using Mathematica and the package *diffgeo*. All concepts were developed in collaboration. The first draft of the paper was written by me, further polished in collaboration.

I have led or corroborated all the original research presented in this thesis.

Signature:

Sonali Mohapatra

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The journey of this thesis did not just begin at the beginning of my PhD. It began in the heart of a 5 year old who dreamt of understanding the universe. Growing up following the footprints of giants and hoping to do more than bumble my way into the truth, I dreamt of one day sitting in a dusty office etching the painstaking work of many years of a PhD into a poetic thesis. Much has changed over the years. While today, the PhD system breaks spirits more often than rewards, rushes more than allows the free-flow of a true pursuit of the truth, it still remains a temple for the pursuit of the scientific method. This thesis has gone through many trials and tribulations starting from regret and sadness over a previous broken down supervisor-student relationship, to torn ligaments leading to multiple surgeries, to a fight for my mental health. I am grateful and glad to have survived the tumultuous ride.

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This thesis is dedicated to "coming into my own".

Prospects in Classical and Quantum Gravity - From Theory to Phenomenology

PhD Thesis Abstract

Sonali Mohapatra

General Relativity (GR) is a highly successful theory whose predictions are still being confirmed a hundred years later. However, despite its significant success, there still remain questions beyond the realm of its validity. The reconciliation of Standard Model (SM) and GR or Quantum Mechanics (QM) and GR point towards the need for a potential modification of GR or a consistent theory of quantum gravity (QG). The purpose of this thesis is to explore classical and quantum gravity in order to improve our understanding of different aspects of gravity, such as black holes (BHs), exotic compact objects (ECOs) like Boson Stars (BS) and gravitational waves (GW). We follow recent advancements in the field of Effective Quantum Gravity (EQG) by noticing that gravity naturally lends itself to an effective framework. The cut off of this effective theory is set to be the Planck mass since this is where UV effects are expected to take over. We focus on finding low energy quantum corrections to General Relativity by using the effective 1PI action and the modified gravity propagator. These include predictions of two new gravitational wave modes in addition to the usual classical GW mode predicted by GR. We investigate and make comments on whether these modes could have been produced by the events observed by LIGO and the energy scales in which these could be possibly produced. In the next project, we investigated whether there exists a correction to the quadrupole moment formula in GR to calculate the energy carried away by gravitational radiation. We apply the corrected formula to calculate the gravitational radiation produced in a binary black hole system in the effective quantum gravity formalism. We make comments regarding its regime of validity. While working in the field of gravitational waves, an interesting aside was modelling of Exotic Compact Objects such as Boson Stars which could also potentially act as black hole mimickers. We calculated analytically the gravitational radiation background produced by binary BS systems. We also commented on and put constraints on their possible detectability by LISA. Last but not the least, an important area in QG is the study of black hole thermodynamics. Corrections to the Bekenstein-Hawking area theorem have been calculated in various quantum gravity approaches and have been found to have a logarithmic form. In the last paper of this thesis, combining insights from Effective Quantum Gravity and Black Hole Thermodynamics, we motivate a generalised Area-Entropy law for black holes building upon the idea of an adiabatic invariant. This allows us to find interesting constraints on the number of fields in a consistent theory of quantum gravity. This work is particularly interesting because of its potential consequences in finding minimal extensions to the standard model and combining the standard model with a consistent theory of gravity.

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Chapter 1

Introduction

> It's hard to imagine a more fundamental and ubiquitous aspect of life on the Earth than gravity, but what if it's all an illusion, a sort of cosmic frill, or a side effect of something else going on at deeper levels of reality?
>
> *Erik Verlinde*

1.1 Prelude

History teaches us many things. And without looking back, it is easy to make the same mistakes again and again. A look at the history of physics makes us realize that it has had a long history of contradictions between empirically successful theories. But these contradictions are not necessarily a comedy of errors, they can often be placeholders for terrific opportunities for advancement of our knowledge. Reviewing the history of physics, we find several of the major jumps ahead are the results of efforts to resolve precisely such contradictions, for example, the discovery of universal gravitation by Newton by combining Galileo's parabolas with Kepler's ellipses. The discovery of special relativity by Einstein to reconcile the contradiction between mechanics and electrodynamics. Furthermore, the discovery that spacetime is curved, just 10 years later, again by Einstein in an effort to reconcile Newtonian gravitation with special relativity.

It might also be said that, today, we are standing at a very important juncture in the evolution of physics in the world. After the recent discovery of the Higgs boson [1, 2] and the more recent confirmation of gravitational waves [3], today, there is a clear demarcation between what we know about the world and what we do not know. What we know is encapsulated into three major theories:

- Quantum Mechanics, which is the general theoretical framework for describing dynamics.

- The $SU(3) \times SU(2) \times U(1)$ standard model of particle physics, which describes all matter we have so far observed directly

- General Relativity (GR), which describes gravity, space and time.

The success of these theories and the story of their many trials and tribulations is indeed *"romanchak"* as we say in India. In this thesis we will concentrate on the theory of general relativity. The theory of general relativity has completely changed our understanding of space and time. From being an esoteric theory which took people by surprise to becoming a theory whose myriad predictions have been accurately experimentally tested and proven over the past hundred years, the story of GR is one of the greatest success stories of modern physics. Some of its successes are as follows [4]:

- The anomalous precession of the perihelion of Mercury,
- The angle of deflection of light for a gravitational field,
- Gravitational redshift,
- Post-Newtonian tests,
- Gravitational lensing, (which also allowed us to conjecture the existence of dark matter),
- Shapiro time delay,
- Tests of the equivalence principle,
- Strong field tests,
- Cosmological tests,
- Gravitational waves.

Not only that, but the usage of GR has become ubiquitous in our day to day lives due to the usage of satellite data and GPS systems which have applications in various other fields. However, GR, among all its successes, points towards its own shortcomings. One of the most glaring instances of this is in the prediction of black holes: GR breaks down at the singularity. More everyday examples include 1) the discrepancy between the predicted and the observed galaxy rotation curves which necessitates dark matter models [5], [6] 2) dark energy, which has been conjectured as the reason behind the current acceleration of the universe, [7] 3) the cosmological constant problem [7] and 4) reconciliation of quantum mechanics and gravity which leads us to the problem of quantum gravity [8], which is perhaps the most challenging problem in theoretical physics today. Thus, some of the main things we do not know about the world can be summarized as:

- dark matter,
- dark energy
- unification,
- modification of gravity or quantum gravity.

In fact, physics today stands on the brink of another apparent contradiction. General Relativity predicts that space-time is curved and the world is deterministic and predictive, however quantum field theory [9] and the standard model of particle physics talk about how the world is formed by discrete quanta jumping over a flat space-time, governed by global (Poincare) and local symmetries. The question however remains, why, given the continuous nature of our world, there isn't yet a framework for unifying quantum mechanics and GR. Or in other words, what does the apparent contradiction of the Standard Model of particle physics with General Relativity teach us? Some of the contenders of quantum gravity in today's world are string theory [10], loop quantum gravity [11], and causal dynamical triangulations [12]. However, most of these theories pre-suppose a fundamental nature of space-time in the UV in an ad-hoc manner. Without a way to connect these with experiments, progress seems to be halted!

Developments in physics have always gone hand in hand with developments in mathematics. Thus we must celebrate that today, we have new mathematical tools at our disposal like never before. Notably, powerful techniques of effective field theories [13, 14] have been developed and we are slowly understanding that all of our theories can be understood better in this effective framework. Thus, an interesting question to ask is whether we can push the frontiers of GR without making ad-hoc assumptions about the nature of reality in the UV but rather build upon available hints to go forward. Or in other words, can the effective field theory technique be applied to gravity to make important and "true" quantum predictions in the low energy regime?

Another landmark which led to a major development in the 1960s was the contradiction between the ideas that black holes could absorb energy and still remain in zero temperature equilibrium (as was thought at the time). This was the reason for the birth of "black hole thermodynamics". Surprising parallels between the second law of thermodynamics and the horizon area of a black hole were discovered [15]. This led to the understanding that black holes have entropy and this entropy can be written in terms of the horizon area, which in turn led to the discovery that the area spectrum of a black hole is discrete. Black holes saturate the high energy regime of gravity and thus, are a laboratory between the UV and IR. Thus, black hole thermodynamics today allows us a tantalizing dream of yet another possible path to get better hints of quantum gravity.

I consider that all of the above are parts of a huge jigsaw puzzle which are slowly being revealed to us when we manage to ask the right questions. In the following chapters, I will review basic concepts of general relativity, solutions of general relativity, gravitational waves and effective quantum gravity to build up essential background for the published papers included in this thesis which explore some of these questions. The original contributions start at Chapter 6.

1.2 Summary and Outline

General Relativity (GR) is a highly successful theory whose predictions are still being confirmed a hundred years later. However, despite its significant success, there still remain questions beyond the realm of its validity. The apparent irreconcilability between Standard Model (SM) and GR or Quantum Mechanics (QM) and GR point towards the need for a potential modification of GR or a consistent theory of quantum gravity (QG). The purpose of this thesis is to explore classical and quantum gravity in order to improve our understanding of different aspects of gravity, such as black holes (BHs), exotic compact objects (ECOs) like Boson Stars (BS) and gravitational waves (GW). In Chapters (2) to (5), we review the basic concepts in GR, discussing solutions such as black holes and exotic compact objects. We then proceed to discuss gravitational waves followed by a review of the effective field theory formalism of quantum gravity. This is followed by a review of the formulation and importance of the concept of black hole entropy.

In Chapter (6), based on [16], which is the first paper of the thesis, we follow recent advancements in the field of Effective Quantum Gravity (EQG) by noticing that gravity naturally lends itself to an effective framework. The cut off of this effective theory is set to be the Planck mass since this is where UV effects are expected to take over. We focus on finding low energy quantum corrections to GR by using the effective 1PI action and the modified gravity propagator. These include predictions of two new gravitational wave modes in addition to the usual classical GW mode predicted by GR. We investigate and make comments on whether these modes could have been produced by the events observed by LIGO and the energy scales in which these could be possibly produced.

In the next paper, Chapter (7), based on [17], we investigated whether there exists a correction to the quadrupole moment formula in GR to calculate the energy carried away by gravitational radiation. We apply the corrected formula to calculate the gravitational radiation produced in a binary black hole system in the effective quantum gravity formalism. We make comments regarding its regime of validity.

While working in the field of gravitational waves and black holes, an interesting aside was modelling of Exotic Compact Objects such as Boson Stars which could also potentially act as black hole mimickers. In Chapter (8), based on [18], we calculated analytically the gravitational radiation background produced by binary BS systems. We also commented on and put constraints on their possible detectability by LISA.

Last but not the least, an important area in QG is the study of black hole thermodynamics. Corrections to the Bekenstein-Hawking area theorem have been calculated in various quantum gravity approaches and have been found to have a logarithmic form. In the last paper of this thesis, Chapter (9), combining insights from Effective Quantum Gravity and Black Hole Thermodynamics, we motivate a generalised Area-Entropy law for black holes building upon the idea of an adiabatic invariant. This allows us to find interesting constraints on the number of fields in a consistent theory of quantum gravity. This work is particularly interesting because of its potential consequences in finding minimal extensions to the standard model and combining the standard model with a consistent theory of gravity.

We draw conclusions and discuss future directions in Chapter (10). A few of the important calculations which could not be included in chapters directly without causing a rift in the space-time continuum, have been placed in the Appendices (A, B and C). All paper acknowledgements are placed in Appendix (D).

Chapter 2

General Relativity: A Brief Review

> Macroscopic objects, as we see them all around us, are governed by a variety of forces, derived from a variety of approximations to a variety of physical theories. In contrast, the only elements in the construction of black holes are our basic concepts of space and time. They are, thus, almost by definition, the most perfect macroscopic objects there are in the universe.
>
> *Subrahmanyan Chandrasekhar*

General Relativity, which is Einstein's theory of space, time and gravity, is undoubtedly one of the most beautiful physical theories in existence. However, the skills required to understand the complicated mathematics might make many undergraduates frustrated. Even though the importance and relevance of aesthetics in physics is controversial and is up for debate, it cannot be denied that beautiful and exciting solutions of GR such as stars and black holes draw in students and keep them excited with the promise of understanding these fantastical objects one day.

There is an unbridled joy in finally coming to the end of a course of GR or working on the mathematical nature of these objects during one's PhD and finally knowing how to derive them. Such is the joy I have felt for the last four years and in this section, we review the geometrical formulation of the general theory of relativity.

The basic idea of GR is simple. While most forces of nature are fields defined on space-time, gravity *is* the geometry of space-time. It is based on the principle of general covariance which means that physical laws are unchangeable under general coordinate transformations. Here, spacetime is a four-dimensional Pseudo-Riemannian manifold $(\mathcal{M}, g_{\mu\nu})$ composed of a differentiable manifold $\mathcal{M}$ and a metric $g_{\mu\nu}$. The two together define our space-time. Points $p \in \mathcal{M}$ are dubbed events. Anything which has mass or energy (which are equivalent) curves this space-time. Test particles, being free from external forces, free fall on the spacetime along these curves. Their trajectories are the shortest distance on this curved space going from one point to the other given by geodesics:

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds} = 0, \tag{2.1}$$

where

$$\Gamma^\rho{}_{\mu\nu} = \frac{g^{\rho\sigma}}{2}\left(\frac{\partial g_{\nu\sigma}}{\partial x^\mu} + \frac{\partial g_{\sigma\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right) \tag{2.2}$$

are the Christoffel symbols of the Levi-Civita connection ∇. Massive particles follow time-like geodesics, while massless particles such as photons follow null geodesics. In principle, we can also define space-like geodesics, and particles which follow such geodesics could propagate at speeds which are superluminal but such particles (tachyons) would be unphysical. We might wonder what happens to field theory requirements such as scalar products on such a curved space, this can be understood by studying the motion (parallel transport) of scalars, vectors and tensors under general covariance. This is given by a covariant derivative:

$$D_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\mu\alpha} A^\alpha. \tag{2.3}$$

The above is the action on a contravariant vector field, this can be generalised to tensors of any rank. An important idea to touch upon here is the equivalence principle. Note that the geodesic Equation (2.1) is independent of the particle's mass. This demonstrates the equivalence principle: all particles undergo the same acceleration in the presence of a gravitational field, independently of their masses.

To understand gravity is to understand "curvature". The Riemann curvature tensor is given by

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}. \tag{2.4}$$

The Ricci tensor can be derived from the Reimann by "contracting" the first and third indices $R_{\sigma\nu} = g^{\rho\mu}R_{\rho\sigma\mu\nu}$. Contracting the remaining indices of the Ricci tensor, leads to the Ricci scalar or the curvature scalar: $R = g^{\mu\nu}R_{\mu\nu}$. These curvature measures define the transformation of vectors, volume elements and so on as they are parallel transported along a curved manifold. For an exhaustive review of the physical interpretations of these various curvatures, see [19]. In the Lagrangian formalism, we can write the Einstein-Hilbert action as

$$S = \int \mathrm{d}^4x\sqrt{-g}\frac{1}{16\pi G}(R - \Lambda) + S_m \tag{2.5}$$

which is the most general action containing up to two derivatives of the metric, guaranteeing that the field equation is second order in the metric and Λ is the cosmological constant. Just like any other field theory, the terms in the action must follow the symmetry group of GR which is the group of diffeomorphisms. Two different descriptions which are connected by a diffeomorphism describe the same physical reality. Thus, each term must be diffeomorphism invariant. Many alternative theories of GR, or higher derivative gravity theories rely on adding various other higher order diffeomorphism invariant terms to the action Equation (2.5).

Varying this action, Equation (2.5), with respect to the metric, (setting $\Lambda = 0$ for simplicity) we find the Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2.6}$$

which govern how the metric $g_{\mu\nu}$ responds to energy and momentum by using $T_{\mu\nu}$, the stress-energy or energy-momentum tensor. All throughout this thesis, we are using units such that the speed of light is $c = 1$. Solutions to Equation (2.6) lead to predictions of general relativity. Often, we are interested in solutions of Equation (2.6) in the vacuum where the energy-meomentum tensor vanishes, in that case, we see that Equation (2.6) reduces to:

$$R_{\mu\nu} = 0. \tag{2.7}$$

Let us note here, that the Christophel symbol or the affine connection, Equation (2.2), involves one derivative acting on the metric; the Ricci scalar, Ricci tensor and the Riemann tensor on the other hand, involve two. This will come in handy later when we write down the EFT of gravity in Chapter 4. As a sanity check, it can be seen that we recover Newtonian gravity as the weak-field limit of GR as can be found in any standard textbook. In the following sections, we will focus on exploring different vacuum and non-vacuum solutions to Equations (2.6) and (2.5) which are relevant for this thesis.

2.1 Vacuum Solutions

Einstein's equations are a system of coupled non-linear differential equations and in general constructing solutions of such systems is hard unless we make certain assumptions. Luckily, we can make certain assumptions in order to find exact solutions which can closely approximate what we want to describe. In the following section, we will explore *stationary vacuum solutions* to GR since making these assumptions simplify our problem. Some of these solutions include the Schwarzschild space-time, the de-Sitter space-time, the Minkowski space-time, the Kerr vacuum and so on.

2.2 The Schwarzschild Solution

The curiosity to know the secrets of what is out there in the night sky has motivated philosophers for a long time. With the advent of the "Renaissance" period, the efforts to apply logic, rationality and observation to build up the scientific enterprise took this inquiry further and probably led to the creation of the first astronomers and cosmologists who studied the mysteries of stars and planets. The theory of gravity developed hand in hand with these investigations and the evolution of our collective knowledge tracking the shoulders of giants such as Descartes, Copernicus, Galileo, Newton, Hilbert, Riemann, Noether and Einstein. Most celestial objects can be mathematically approximated to spherically symmetric gravitational fields. Thus, the next relevant question is: What does GR predict regarding the gravitational field around spherically symmetric objects such as stars and planets? This will describe the geometry of the space-time in the exterior of such objects.

We will assume a vacuum space-time:

$$R_{\mu\nu} = 0 \tag{2.8}$$

outside the star or spherical object. We will assume the source to be unrotating and unevolving, thus, we will investigate a static and stationary solution. Such a solution is called the Schwarzschild metric and is given by:

$$ds^2 = -\left(1 - \frac{R_s}{r}\right) dt^2 + \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{2.9}$$

where $d\Omega$ is the metric on a unit two-sphere,

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{2.10}$$

The constant R_s is called the Schwarzschild radius. By calculating the weak field limit, we find that

$$g_{tt} = -\left(1 - \frac{2GM}{r}\right) \tag{2.11}$$

The Schwarzchild metric should reduce to the weak-field limit when $r \gg 2GM$ and thus,

$$R_s = 2GM \tag{2.12}$$

which also allows us to identify the parameter M as the Newtonian mass of the gravitating object (though some care must be taken in this). Thus, the full Schwarzschild metric can be written explicitly as:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{2.13}$$

2.2.1 Singularities

From the form of Equation (2.13), we can see that the metric diverges both at $r = 0$ and $r = 2GM$. However, we must take care to understand which are "real" infinities as opposed to a breakdown of the coordinate system. Since scalars are coordinate independent, if they go to infinity, it signals a true singularity of the curvature. The Ricci scalar is the simplest such scalar but there are higher order ones as well. One of such scalars that we can calculate is the Kretschmann invariant, which is given by

$$K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{48G^2M^2}{r^6} \tag{2.14}$$

for the Schwarzschild metric. We can see that $r = 0$ is a true curvature singularity whereas $r = 2GM$ is just the event horizon.

2.2.2 The Schwarzschild Black Hole

The Schwarzschild solution describes the space-time around any spherically symmetric object such as stars, planets and satellites or even the observable universe. However, in the case of stars, the Schwarzschild radius is much smaller than the radius of the star, for example, the Schwarzschild radius of the Sun is approximately 3.0 km, whereas Earth's is only about 9 mm (0.35 in) and the Moon's is about 0.1 mm. The observable universe's mass has a Schwarzschild radius of approximately 13.7 billion light-years. Thus, these objects are static in the exterior but they do not have any singularities or horizons.

However, there are certain objects in the sky whose Schwarzschild radius is equal or greater than their radius. These objects seem to rip a "hole" in the space-time continuum due to a huge curvature and do not allow even light to escape out of the event horizon, thus looking black. Hence the name: *black holes*. A black hole is described by the Schwarzschild metric in its totality. However, the Schwarzschild solution is an idealized solution (without any energy-momentum anywhere in the universe), real black holes would of course, be approximations. There are various other static black hole solutions to the Einstein's equations, such as the Kerr, Kerr-Newman and the Reissner-Nordström metric.

There can be various stationary vacuum solutions to the Einstein's Equations such as this, but keeping in mind their relevance for this thesis, let us proceed to study a bit regarding a few non-vacuum solutions of GR.

2.3 Non-Vacuum Solutions

As seen above, the Schwarzschild metric is one of the simplest vacuum stationary static solutions to Einstein's equations. One of the simplest exact non-vacuum cosmological solutions to GR is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric which describes a homogeneous, isotropic, expanding (or otherwise, contracting) universe. However, apart from black holes and cosmological solutions, GR can be used to describe many other compact objects such as normal stars, white dwarfs, quasars, pulsars, binary star systems, galaxies, neutron stars (NS) so on. In real life, stars are not static. They evolve, and they might not necessarily be stationary solutions as well. The full solutions often need the full power of Numerical Relativity to compute. The discovery of Gravitational Waves by LIGO [20] has revived interest in such compact objects once again [21–23].

However, they have also opened up the possibility of searching for other "exotic" compact objects in the sky.

2.4 Exotic Compact Objects

Some physicists[1] consider Exotic Compact Objects (ECOs) to be "just a catchy name for several models that have been proposed as alternative to black holes". This is because unlike classical Black Holes, which have a horizon as well as singularity which makes them incompatible with quantum theory as well as difficult to model, these ECOs are huge massive bodies which bend light around them similar to black holes without having a horizon.

What makes ECOs, especially some models of Boson Stars, attractive is that they can be self gravitating, without any other signatures other than gravitational waves, with a radius very close to the Schwarzchild radius, which makes them "black hole mimickers". Other models are motivated by the search for dark matter. With the discovery of the Higgs Boson, theoreticians predict other "dark" scalars which might be present in the universe and making up the bulk of its mass in the form of dark matter. Under certain conditions, these scalars could also coalesce to form "Dark Stars" which would then have gravitational wave signatures.

The advent of gravitational wave cosmology has opened up the possibility of searching for these kind of stars. For an exhaustive review of the different types of ECOs and their possible signatures, refer to [24].

[1] Source: the Gravity Room (`http://thegravityroom.blogspot.com/2017/03/can-exotic-compact-objects-exist.html`)

2.4.1 Boson Stars

Boson Stars (BS) are one of the simplest kind of exotic compact objects that can be hypothesized to be found in our universe. As is evident from their name, they are composed of elementary particles, bosons.

With the success of the inflationary cosmology [25] and the discovery of the Higgs boson [1] [2] as a fundamental scalar in our universe, there has been a recent flurry of activity on whether there could be other scalars in the universe which are self-interacting. Some of these scalars may be the constituent of dark matter in the universe and be present in dark matter clusters. If the density of these clusters attains a certain minimum level, these bosons could form a Bose-Einstein condensate to form compact stars.

In the simplest case scenario, we can imagine stars formed out of one dark scalar, which does not have any SM interaction and thus, would not have any other radiation channel other than GW. These are known as mini boson stars. In the absence of Pauli's exclusion principle (which applies only to fermions), which prevents neutron stars from collapsing, such a system would have to be supported against gravitational collapse by a repulsive self interaction. The star is formed purely by gravitational interactions between the scalars. The Lagrangian for such a Einstein-Klein-Gordon system is given by:

$$\mathcal{L} = \sqrt{-g}\left[|\partial\phi|^2 - m^2\phi^2 - \frac{1}{2}\lambda|\phi|^4\right], \tag{2.15}$$

where ϕ is a complex scalar field carrying a global $U(1)$ charge. This is the case we consider in Chapter (8). Of course, this scenario can be extended to include more complex interactions and multiple (real or complex) scalars or even a mixture of different species of particles. For an exhaustive review of such stars, the reader is directed towards [24].

Chapter 3

Gravitational Waves

> It will give us ears to the Universe where before we've only had eyes.
>
> *Karsten Danzmann, LIGO*

Gravitational waves, in short, are tiny perturbations of the metric that propagate in spacetime, stretching it and causing observable effects on test particles. They remain one of the major predictions of general relativity 100 years after their discovery. With the recent first ever direct gravitational wave detection by LIGO [3] from the merger of two inspiralling black holes and many more events after that, the field of gravitational wave astronomy has burst wide open. In this thesis, in Chapter (3), we use the basic recipe of calculating gravitational modes in GR and extend it to the effective field theory formalism to find two new GW modes. In Chapter (7), we calculate the correction to the quadrupole moment formalism in the effective field theory framework and the correction to the total radiation carried away by those two extra modes. We make comments on their possible detectability. In Chapter (8), we calculate the total gravitational background radiation due to inspiralling binary Boson Star mergers in the sky, if they exist, and put bounds on the parameters based on their possible detectability by LISA. To that end, in this chapter, we will review the basic mathematics behind gravitational wave calculations. (see e.g. [26] for an extensive review on the subject).

As a starting point, let us split the space-time metric into a background metric and fluctuations as follows:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \ \ |h_{\mu\nu}| << 1 \tag{3.1}$$

In the above, $h_{\mu\nu}$ are considered as the gravitational wave degrees of freedom which propagate as fluctuations on the background metric. The resulting theory is known as the linearised theory. To keep this review simple, we start with the flat background case and use the Minkowski metric as the background. Plugging Equation (3.1) into Equation (2.6) leads to

$$\Box\bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^{\rho}\partial^{\sigma}\bar{h}_{\rho\sigma} - \partial^{\rho}\partial_{\nu}\bar{h}_{\mu\rho} - \partial^{\rho}\partial_{\mu}\bar{h}_{\nu\rho} = 16\pi G T_{\mu\nu}, \tag{3.2}$$

to first order in h. In the above, we have made the following field redefinition: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, where $h = \eta^{\mu\nu}h_{\mu\nu}$. Using the invariance under diffeomorphisms, which is the assumed underlying symmetry in GR, one can choose the harmonic gauge

$$\partial^{\nu}\bar{h}_{\mu\nu} = 0. \tag{3.3}$$

With this choice of gauge, (3.2) reduces to a wave equation:

$$\Box\bar{h}_{\mu\nu} = 16\pi G T_{\mu\nu}. \tag{3.4}$$

Thus, we can conclude that $\bar{h}_{\mu\nu}$ behaves as a wave. Note that, from (3.3) and (3.4), one finds the conservation of the energy-momentum tensor

$$\partial^{\nu}T_{\mu\nu} = 0. \tag{3.5}$$

To calculate the vacuum solution far away from any source, the propagation of gravitational waves or the interaction of the gravitational waves with detectors, we are interested in the region away from the source and thus, we put $T_{\mu\nu} = 0$ and then we have a wave equation given by:

$$\Box\bar{h}_{\mu\nu} = 0, \tag{3.6}$$

Outside the source, it can be shown that Equation (3.3) does not fix the gauge completely. In particular we can greatly simplify the above equation by fixing a gauge which leads to vanishing trace: $h = 0$. More explicitly, this leads us to fix another set of constraints:

$$\boxed{h^{0\mu} = 0; \ h^{i}_{i} = 0; \text{ and } d^{j}h_{ij} = 0}. \tag{3.7}$$

It can be shown that h^{00} is the static part of the gravitational wave, and since $h^{0\mu}$ vanishes, this leaves us only with the spatial part which is the time-dependant part and which determines the propagation of the wave. This defines the fixing of the "transverse-traceless" (TT) gauge. Note that this is not valid inside the source since $\Box\bar{h}_{\mu\nu} \neq 0$. In the TT gauge, the solution is

$$\Box h_{ij}^{TT} = 0 \tag{3.8}$$

and the solution h_{ij}^{TT} can be expanded in the plane wave basis. For any given symmetric tensor,S_{ij}, its transverse-traceless part is defined as:

$$S_{ij}^{TT} = \Lambda_{ij,kl} S_{kl} \tag{3.9}$$

and thus we can construct

$$h_{ij}^{TT} = \Lambda_{ij,kl} h_{kl}, \tag{3.10}$$

where $\Lambda_{ij,kl}$ is a projector operator known as the Lambda tensor defined as

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl}, \tag{3.11}$$

and the tensor

$$P_{ij}(\hat{n}) = \delta_{ij} - n_i n_j \tag{3.12}$$

is symmetric, transverse (i.e $n^i P_{ij}(\hat{n} = 0)$, is a projector (i.e $P_{ik}P_{kj} = P_{ij}$) and its trace is $P_{ii} = 2$. In the next section, working in the TT gauge, we will investigate how to calculate the power carried in gravitational waves which reaches the earth by using the quadrupole formula.

3.1 Gravitational Radiation

A lot can be learnt about gravitational radiation by studying electromagnetic (EM) radiation from accelerating charges. The only difference is that unlike EM radiation, gravitational radiation expansion starts from the quadrupole moment rather than the dipole moment. For an exhaustive review on gravitational radiation, see [26] or a brief primer of gravitational radiation in gravitational waves used by LIGO, see [27]. The starting point in linearised theory is Equation (3.4)

$$\Box\bar{h}_{\mu\nu} = 16\pi G T_{\mu\nu}. \tag{3.13}$$

Because this is linear in $h_{\mu\nu}$, this can be solved using the method of green's functions. If $G(x-x')$ is a solution of the equation

$$\Box_x G(x-x') = \delta^4(x-x'), \tag{3.14}$$

then, inserting cs back, the solution to Equation (3.4) becomes,

$$\bar{h}_{\mu\nu}(x) = \frac{16\pi G}{c^4}\int d^4x' G(x-x')T_{\mu\nu}(x') \tag{3.15}$$

Using the retarded green's function as the appropriate solution for a radiation problem, we find that Equation (3.15) becomes:

$$\bar{h}_{\mu\nu}(t,x) = \frac{4G}{c^4}\int d^3x' \frac{1}{x-x'} T_{\mu\nu}(t - \frac{|x-x'|}{c}, x') \tag{3.16}$$

Since outside the source, we will operate in the TT gauge we can project the above in the TT gauge. In the low velocity regime, $v << c$ or $\omega_s d << c$, where ω_s is the typical velocity of the radiation, we performing the multipole expansion and find that the traceless transverse part of the gravitational strain, h_{ij}^{TT} can be written as follows:

$$h_{ij}^{TT} = \frac{2}{c^4}\frac{G}{d_L}\frac{d^2 \mathrm{Q}_{ij}^{TT}}{dt^2} \tag{3.17}$$

where d_L is the luminosity distance from the source and Q_{ij}^{TT} is the traceless mass quadrupole moment. From now on, we will drop the superscript TT and use the normal tensor notation to mean traceless transverse. Here,

$$\begin{aligned} \mathrm{Q}_{ij}^{TT} \equiv \mathrm{Q}_{ij} &= M^{ij} - \frac{1}{3}\delta^{ij}M_{kk} \\ &= \int d^3x \rho(t,x)(x^i x^j - \frac{1}{3}r^2\delta^{ij}), \end{aligned} \tag{3.18}$$

where to lowest order in v/c, ρ becomes the mass density. Now, we can find the rate at which energy is carried away by these gravitational waves in terms of the above as:

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{c^3}{16\pi G}\int \dot{h}^2 dS = \frac{1}{5}\frac{G}{c^5}\sum_{i,j=1}^{3}\frac{d^3\mathrm{Q}_{ij}}{dt^3}\frac{d^3\mathrm{Q}_{ij}}{dt^3}, \tag{3.19}$$

$$\text{where}|\dot{h}|^2 = \textstyle\sum_{i,j=1}^{3} \frac{dh_{ij}}{dt}\frac{dh_{ij}}{dt}$$

the integral is over a sphere at radius d_L (contributing a factor $4\pi d_L^2$), and the quantity on the right-hand side must be averaged over (say) one orbit. Depending on the system, we can introduce a mass distribution through the quadrupole moment and then integrate over it. The above energy rate is the rate of energy drain from the orbital energy, so equating

$$\frac{dE_{\mathrm{orb}}}{dt} = -\frac{dE_{\mathrm{GW}}}{dt} \tag{3.20}$$

would allow us to find the expression for the frequency of the gravitational waves emitted. As an example, let us examine the system of binary mass system rotating around each other where we will assume that both masses are equal.

3.1.1 Example

In this example, we will investigate a binary system of two masses as shown in Fig. (3.1.1) rotating around each other and calculate the amount of gravitational radiation emitted by the system due to the loss of energy from its orbital motion. To consider the simplest picture, let us consider the example in [27] where the masses of the two point objects are given by m_1 and m_2 and for simplicity, we set $m_1 = m_2 = m$. $r = r_1 + r_2$ is the distance between the two objects.

The quadrupole moment of the mass distribution is

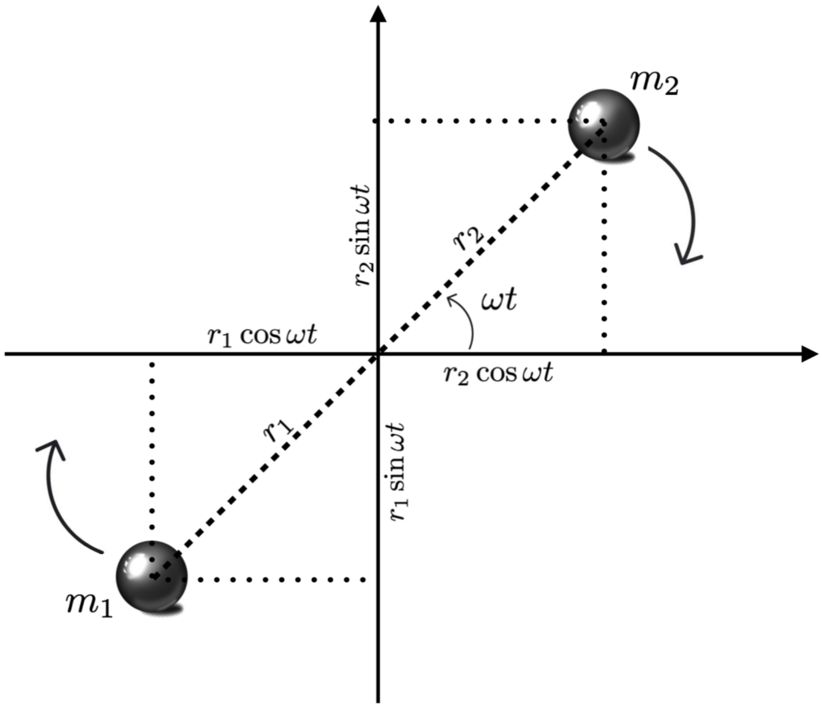


Figure 3.1: A two-body system, m_1 and m_2 orbiting the x-y plane around their center of mass. This image was reproduced following the example in [27].

$$Q_{ij} = \int d^3x\rho(x)\left(x_ix_j - \frac{1}{3}r^2\delta_{ij}\right) \tag{3.21}$$

$$= \sum_{A\in\{1,2\}} m_A \begin{pmatrix} \frac{2}{3}x_A^2 - \frac{1}{3}y_A^2 & x_Ay_A & 0 \\ x_Ay_A & \frac{2}{3}y_A^2 - \frac{1}{3}x_A^2 & 0 \\ 0 & 0 & -\frac{1}{3}r_A^2 \end{pmatrix} \tag{3.22}$$

where x_A and y_A are the projections on the x and$y$ axis respectively for r_A where $A \in 1, 2$ and for each object,

$$Q_{ij}^A(t) = \frac{m_A r_A^2}{2} I_{ij} \tag{3.23}$$

where $I_{xx} = \cos(2\omega t) + \frac{1}{3}$, $I_{yy} = \frac{1}{3} - \cos(2\omega t)$, $I_{xy} = I_{yx} = \sin(2\omega t)$ and $I_{zz} = -\frac{2}{3}$. Combining all of this we will get,

$$Q_{ij}(t) = \frac{1}{2}\mu r^2 I_{ij} \tag{3.24}$$

where μ is the reduced mass of the system. This allows us to calculate the gravitational wave luminosity as:

$$\frac{d}{dt}E_{\mathrm{GW}} = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6. \tag{3.25}$$

Using Equation (3.20), plugging in $E_{\mathrm{orb}} = -GM\mu/2r$ and using Kepler's third law, $r^3 = GM/\omega^2$ and the derivative $\dot{r} = -\frac{2}{3}r\dot{\omega}/\omega$, we can obtain:

$$\dot{\omega}^3 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mu^3 M^2 \tag{3.26}$$

and we can now see the evolution of the inspiral. In Chapter (7), we will use the same approach to calculate the modified quadrupole moment formula for a system of binary black holes in the effective field theory formalism.

Chapter 4

Effective Quantum Gravity

> The bedrock nature of space and time and the unification of cosmos and quantum are surely among science's great 'open frontiers'. These are parts of the intellectual map where we're still groping for the truth - where, in the fashion of ancient cartographers, we must still inscribe 'here be dragons'.
>
> *Martin Rees*

4.1 Finding the Effective Action

With the development of Quantum Field Theory (QFT) techniques, there were efforts to quantize gravity. These efforts can be traced back to 1930 to Rosenfeld [28] and [29]. A concise review of early efforts to quantize gravitation can be found here [30]. To general disappointment, it was found that GR is non-renormalizable. In particular, GR without matter is non-renormalizable at the second loop level [31], while with matter fields it cannot be renormalized at the one-loop level [32]. Using the path integral formalism in GR, it was shown that even a simple model describing a real massless scalar field ϕ interacting with GR is non-renormalizable [33]. Another attempt to renormalize gravity was taken by Stelle [34], where he studied an action with curvature squared terms such as:

$$S_{\text{stelle}} = -\int d^4x \left[\frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}^2\right] \tag{4.1}$$

where $\kappa = \sqrt{32\pi G}$. He found that even though this is formally renormalizable, it is plagued by a spin-2 ghost. Thus, this led us to interpret that the *quantum* theory of gravity was non-renormalizable. In general, efforts to write gravity as a field theory which can be renormalized have led physicists to encounter a number of problems. Due to the dimensionful coupling constant of gravity, the renormalization procedure generates an infinite number of counter terms in the gravitational action. Thus, there are an infinite number of coefficients associated with these counter terms which must be fixed empirically. This is impossible and makes quantum gravity non-predictive and non-falsifiable.

Although, quantum gravity has proven to be much more difficult to pin down than theoreticians might have hoped for, recent advancements in the regime of Effective Field Theories [13, 14] have been hopeful. While we know very little regarding the nature of gravity in the ultraviolet regime, we can safely assume that gravity can be treated as an EFT because nature, in general, allows for a decoupling of scales. This is the same reason why we can describe the macroscopic flow of water using the Navier-Stokes equations without having to always rely on the microscopic theory describing the behavior of the molecules. Starting with Weinberg in [35], the developments in effective field theory techniques helped us to revamp our understanding of renormalization: If a theory is an effective field theory and is non-renormalizable, then it is not fundamentally different from a renormalizable theory. All it means is that the theory is not sensitive to more microscopic scales which have been integrated out. The success of effective field theory techniques in the standard model have been tested extensively [36, 37]. This hints towards the fact that we can use the EFT procedure in gravity as well to decouple the low energy IR effects from the UV physics, especially since the UV theory is safely hidden behind such a high cut-off, namely, the Planck scale.

The lesson of EFT for gravity is that even though a renormalizable theory of the metric tensor that is invariant under general coordinate transformation is not possible, we must not despair of applying quantum field theory to gravity in order to probe long-range, low-energy IR effects. It might be well that in an effective field theory of matter and gravitation, the standard model and GR are the lowest order terms. Various efforts to consider specific terms and different effective actions have been taken in [38, 39]. However, the application of this point of view to long range properties of gravitation has been most thoroughly developed by Donoghue, El-Menoufi and their collaborators in recent years [40–42].

Let us now review the quantization procedure in order to investigate the treatment of gravity as an EFT. In order to write the effective Lagrangian for gravity, we start with the most general Lagrangian which includes all terms including higher order terms which retain the symmetry of the theory, which in this case is diffeomorphism invariance. Thus, the action becomes:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + ...\right]. \quad (4.2)$$

The Einstein-Hilbert term is the least suppressed term in the action. We see, in the above, divergences appearing at one-loop order, for example, are proportional to R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. As usual in an EFT framework, these can be renormalized by the inclusion of counterterms to the Lagrangian [32]. Thus our renormalized lagrangian to one-loop order can be written as

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Lambda + \bar{c}_1 R^2 + \bar{c}_2 R_{\mu\nu}R^{\mu\nu} + \bar{c}_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right]. \quad (4.3)$$

The coefficients $\bar{c}_i$ are chosen so that divergences at one-loop order turn out to be finite. Note that they are bare constants rather than observables now. We can further simplify this action by invoking the Gauss-Bonet theorem in 4-dimensions:

$$\delta \int \mathrm{d}^4x\sqrt{-g}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) = 0. \quad (4.4)$$

where the left side of the above equation is a topological invariant. We can use the above to eliminate $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ and absorbing it in the values of $\bar{c}_1$ and $\bar{c}_2$. Thus, the last term in Equation (4.3) can be ignored and we can proceed to quantize the action using the background field method [43]. Whatever be our background metric, $\bar{g}_{\mu\nu}$, we can perturb $g_{\mu\nu}$ around it

$$g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \quad (4.5)$$

where $h_{\mu\nu}$ are quantum fluctuations over the background metric. The raising and lowering of the indices is done using the background metric, so we can expand the contravariant metric tensor in orders of G ($\kappa = \sqrt{32\pi G}$) as follows:

$$g^{\mu\nu} = \bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}g_{\alpha\beta} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\alpha}h^{\nu\alpha} + ... \tag{4.6}$$

Similarly, we can expand the integration measure for the action as follows:

$$\sqrt{-g} = 1 + \frac{1}{2}\kappa h - \frac{\kappa^2}{4}h_{\alpha\beta}P^{\alpha\beta,\mu\nu}h_{\mu\nu}, \tag{4.7}$$

where $h = h^{\mu}_{\mu}$ and

$$P^{\alpha\beta,\mu\nu} = \eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu} \tag{4.8}$$

Thus, the EFT action Equation (4.3) can be seen as a conventional expansion in orders of the reduced Planck mass, $M_P = 1/\sqrt{8\pi G}$ or inverse orders of κ. In the low-energy regime, the terms which scale as one power of curvature scale as p^2 since they have two derivatives acting on the metric where p is some momentum scale, $p \ll M_P$. The curvature squared terms in Equation (4.3) scale as p^4, these would always be dominated by the p^2 terms in the low energy scale. Using this Stelle found tiny effects associated with the R term which led us to find extremely poor bounds on $c_{1,2} < 10^{74}$ [34]. As we have discussed before, the source of gravity in the Einstein Hilbert Action, Equation (2.5), is the stress energy tensor, $T_{\mu\nu}$ on the right hand side, which is how we can couple matter with gravity. In the simplest case,

$$S_{\text{matter}} = \int d^4x\sqrt{g}\left(\frac{g^{\mu\nu}}{2}\partial_\nu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2 - \xi RF(\phi)\right), \tag{4.9}$$

where, $F(\phi)$ is a function of the scalar field ϕ and we can set the non-minimal coupling ξ to zero for the minimal case. In the above,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\partial}{\partial g_{\mu\nu}}\mathcal{L}_{\text{matter}}. \tag{4.10}$$

The Feynman path integral formalism gives us a recipe to write the quantum version of a theory using the following functional:

$$Z(g_{\mu\nu}) = \int \mathcal{D}(gravity)\mathcal{D}\Phi e^{-i(S_{\text{true}}[g]+S_m[\Phi])}. \tag{4.11}$$

The Planck scale is where it is widely believed that GR breaks down. Thus, if we integrate out the quantum fluctuations $h_{\mu\nu}$ from Equation (4.11) above the Planck scale, we find

$$e^{-i\Gamma} = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\Phi\, e^{-i(S_{\text{eff}}[\bar{g}+h]+S_m[\Phi])}, \tag{4.12}$$

where S_m is the action of the matter sector and Φ represents a set of arbitrary matter fields (not necessarily scalar fields). Γ, in the above is the quantum effective action which describes quantum gravitational phenomena. We can use this to investigate the phenomenology of quantum gravity at low energies (below the Planck scale). As expected, the general result is quite cumbersome even at leading order, containing several terms that contribute equally [44]. However, in this thesis, we will only consider the limit of massless or very light fields in the minimal coupling limit. The outcome is neat enough in this limit and defining the renormalization scale, μ, the effective action reads [40, 43, 45, 46]:

$$\begin{aligned}\Gamma = \int \mathrm{d}^4x\sqrt{-g}\Bigg(&\frac{R}{16\pi G} - \Lambda + c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\\ &- \alpha R\ln\left(-\frac{\Box}{\mu^2}\right)R - \beta R_{\mu\nu}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu} - \gamma R_{\mu\nu\rho\sigma}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma}\Bigg).\end{aligned} \tag{4.13}$$

where c_is are the Wilson coefficients and α, β, γ are the predictions of the EFT.

4.2 Non-Local Effective Quantum Gravity

In the massless or very light limit, non-localities are expected to show up as massless fields mediating long-range interactions. In fact, the quantum action in this case is given by Equation (4.13) and can be written as

$$\Gamma = \Gamma_L + \Gamma_{NL}, \tag{4.14}$$

where the local part reads

$$\Gamma_L = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \Lambda + c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right), \tag{4.15}$$

and the non-local part reads

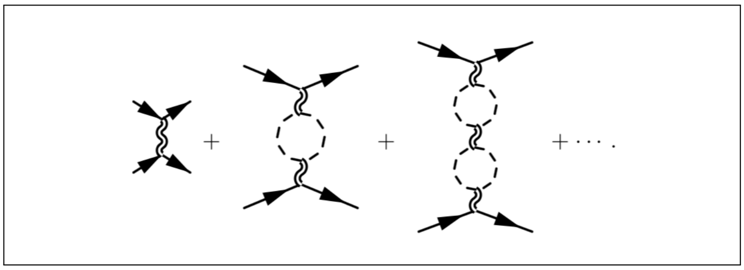
$$-\Gamma_{NL} = \int \mathrm{d}^4x\sqrt{-g}\left[\alpha R\ln\left(-\frac{\Box}{\mu^2}\right)R + \beta R_{\mu\nu}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu}\right. \tag{4.16}$$

$$\left. + \gamma R_{\mu\nu\rho\sigma}\ln\left(-\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma}\right]. \tag{4.17}$$

The non-local terms appear in the action due to resummation of matter loops in the graviton propagator:

$$= \frac{C_{\mu\nu\alpha\beta}}{k^2\left(1 - \frac{NGk^2}{120\pi}\log\left(-\frac{k^2}{\mu^2}\right)\right)}.$$

Here k is the external momentum and $C_{\mu\nu\alpha\beta}$ is the Weyl tensor. We can sum an infinite series of matter loops in the large N limit, where $N = N_s + 3N_f + 12N_V$ (N_s is the number of scalar fields, N_f is the number of Dirac fermions, N_V is the number of vector fields). The same resummation can also be performed at the level of a virtual graviton exchange. Therefore scattering amplitudes $a, b \to a', b'$ generate non-local interaction terms that should be taken into account at the level of an effective action:



The log operator is defined as

$$\log\frac{-\Box}{\mu^2} = \int_0^\infty \mathrm{d}s \left(\frac{1}{\mu^2 + s} - G(x, x', \sqrt{s})\right), \tag{4.18}$$

where $G(x, x'; \sqrt{s})$ is the Green's function of

$$(-\Box + k^2)G(x, x'; k) = \delta^4(x - x') \tag{4.19}$$

with proper boundary conditions. The non-local action represents the infrared part of quantum gravity. The coefficients α, β, γ, are determined completely by the effective field theory by integrating out the matter fields Φ and specifying the number of light fields in the theory with their respective spins as shown in Table 4.1, as opposed to the Wilson coefficients c_i in the local action which need to be determined empirically. The total contribution to each coefficient is given by simply summing the contribution from each matter species. For example, for N_s minimally coupled scalars and N_f fermions, we have

$$\alpha = \frac{5}{11520\pi^2}N_s - \frac{5}{11520\pi^2}N_f. \tag{4.20}$$

In the table, the results are shown for a scalar whose coupling is $\xi R\phi^2$, i.e $F(\phi) = \phi^2$.

	α	β	γ
real scalar	$5(6\xi - 1)^2/(11520\pi^2)$	$-2/(11520\pi^2)$	$2/(11520\pi^2)$
Dirac spinor	$-5/(11520\pi^2)$	$8/(11520\pi^2)$	$7/(11520\pi^2)$
vector	$-50/(11520\pi^2)$	$176/(11520\pi^2)$	$-26/(11520\pi^2)$
graviton	$430/(11520\pi^2)$	$-1444/(11520\pi^2)$	$424/(11520\pi^2)$

Table 4.1: Values of the coefficients α, β, γ for each spin taken from [40]. Results are shown for a scalar whose coupling is $\xi R\phi^2$. Each value must be multiplied by the number of fields of its category. The total value of each coefficient is given by adding up all contributions. See Equation (4.20) for an example.

For minimally coupled scalars, $\xi = 0$. And so, we can see that the non-local part is highly interesting, since it is completely independent of the UV completion and yet the coefficients α, β, γ are true quantum predictions.

The local action, on the other hand, as known, represents the high energy portion of quantum gravity. The Wilson coefficients, c_i, are renomalised parameters which contain information about the UV and can only be determined empirically. They depend on the renormalization scale μ in such a way that they cancel the μ-dependence of the non-local logarithm operator. Thus, the total effective action Γ is independent of μ. The renormalization group equation is

$$\beta_i = \mu \frac{d}{d\mu} c_i, \tag{4.21}$$

where $\beta_1 = -2\alpha$, $\beta_2 = -2\beta$ and so on are the beta functions, thus the running of c_i can also be obtained straightforwardly from Table 4.1. The relation between the beta functions of c_i and the coefficients α, β, γ is expected because the resultant action Γ must be independent of μ as explained above.

To conclude, while a consistent full theory of quantum gravity or experimental proof is still elusive, we can definitely use standard techniques which were not available before, to now quantize general relativity as an effective field theory. This is a conservative approach which leads to consistent quantum gravity phenomenology without having to invoke ad-hoc assumptions regarding the nature of the UV. The basic results of such an implementation of EFT techniques, namely, quantum corrections to the Newton's potential and scattering amplitudes have been well studied [47–49]. However most of the results were obtained for an effective model without nonlocal terms. Moreover, most applications of EFT techniques are limited to particle systems, while the effective action should also provide an adequate description of astrophysical phenomena such as gravitational wave production. In Chapters (6) and (7), we calculate the gravitational wave spectrum for the effective action as well as the gravitational wave radiation by a binary rotating mass system respectively as an effort towards advancing quantum gravity phenomenology.

Chapter 5

Entropy of Black Holes

> Think of the universe as a box of scrabble letters. There is only one way to have the letters arranged to spell out the Gettysburg Address, but an astronomical number of ways to have them spell nonsense. Shake the box and it will tend toward nonsense, disorder will increase and information will be lost as the letters shuffle toward their most probable configurations. Could this be gravity?
>
> *Elwood Smith,*
> *A Scientist Takes on Gravity*

The second law of thermodynamics states that "changes to a closed thermodynamic system is always in the direction of increasing entropy". Before the 1960s, it was a consensus that black holes are not thermodynamical objects since they were supposed to be zero temperature objects always in equilibrium. The following is a conversation by Wheeler in a famous tea session with Jacob Bekenstein as mentioned in his 1998 book *"Geons, Black Holes and Quantum Foams: A life in Physics"* [50]:

"*The idea that a black hole has no entropy troubled me, but I didn't see any escape* *from this conclusion. In a joking mood one day in my office, I remarked to Jacob Bekenstein that I always feel like a criminal when I put a cup of hot tea next to a glass of iced tea and then let the two come to a common temperature. My crime, I said to Jacob, echoes down to the end of time, for there is no way to erase or undo it. But let a black hole swim by and let me drop the hot tea and the cold tea into it. Then is not all evidence of my crime erased forever? This remark was all that Jacob needed. He took it seriously and went away to think about it.*"

The fact that black holes were thermodynamic systems as well was realized in the early 60s. Penrose, along with Floyd later [51, 52] showed that energy can be extracted from black holes by the Penrose process. Christodoulou, 1970 [53] proved that in any process, the irreducible mass of a black hole (which is proportional to the surface area) cannot decrease. Hawking, in 1971 [54], independently proved a general theorem that the horizon area can only ever increase. This is definitely reminiscent of the second law of thermodynamics and especially, entropy. In this chapter, we will review the basics of black hole thermodynamics based on Bekenstein's earliest works [55, 56] and arrive towards a precise definition of black hole entropy. We recommend [57] and [56] as a good review for anyone starting off in the field of black hole thermodynamics.

To start with, let us reflect on the meaning of entropy and whether it makes sense to associate entropy with black holes. So, what does entropy really mean? In information theoretic terms, entropy is the sense of inaccessibility of information about the internal configuration of a certain system. Thus, it is also a measurement of the ambiguity or the multiplicity of the many possible internal configurations of a system which gives rise to a particular macro-state. The stipulation that the entropy of a system can only ever increase, selects a preferred direction of time. If a system has g_n number of microstates, then the entropy of a system is related to it by:

$$g_n = \exp S. \tag{5.1}$$

Let us now investigate the analogies between black hole physics and thermodynamic/entropic systems.

- Firstly, just as a thermodynamic system in equilibrium can be completely described macroscopically by a few thermodynamic parameters, so a bare black hole system in equilibrium can be described by only three parameters: mass, charge and angular momentum.

- Secondly, black holes are mysterious due to the apparent loss of information. Any object, be it light or matter that goes into a black hole, loses all the information associated with it, in so far as an exterior observer is concerned. In short, information is lost. Entropy is the measure of missing information, thus, it makes sense to describe the loss of information into black holes by an associated entropy.

- Thirdly, we have already touched upon the increase of surface area of a black hole which is analogous to the increase of thermodynamic entropy of a system. Additionally, Hawking [54] also showed that when two black holes merge, the final surface area cannot be less than the sum of the individual surface areas.

5.1 The Area Entropy Law

The analogy between entropy and black hole horizon area led Bekenstein [55] to propose that the entropy of a black hole (S_{bh}) must be a monotonically increasing function of its horizon area, A:

$$S_{bh} \propto f(A). \tag{5.2}$$

There are various ways of thinking about black hole entropy. One way looking at it is the uncertainty of the internal configuration of a black hole which might have been formed by various initial conditions. But as the black hole evolves, the effects of the initial conditions are being washed out. For example, consider three black holes which have the same mass, charge and angular momentum (at equilibrium or late times). One of them might have been formed by the collapse of a normal star, the second by the collapse of a neutron star and the third by the collapse of a geon [58]. The measure of entropy for all three would be the same which would encapsulate the same uncertainty. This loss of information by the slow washing out of initial conditions would be reflected as a gradual increase in the entropy of each black hole.

But entropy is dimensionless (in natural units), so it was necessary to divide the horizon area by a universal constant with dimensions of area, and for this only one candidate presented itself: the (tiny) squared Planck length, $\hbar G/c^3$, which is equal to about (10^{-33}cm^2). Bekenstein remarked that the appearance of $\hbar$ in the entropy "is not totally unexpected", since "the underlying states of any system are always quantum in nature", and he proposed the black hole entropy formula

$$S_{bh} = \eta A/(\hbar G/c^3) = \frac{\ln k}{\alpha}\frac{A}{4l_P^2}, \tag{5.3}$$

where k and α are natural numbers and η is a dimensionless proportionality constant. According to Bekenstein, when a black hole absorbs one particle, at least one bit of information is lost regarding the existence of the particle. This bit of information must be at least equal to $\ln 2$. Thus, there must a minimal increase of the horizon area which might allow us to calculate the exact value of η. We will explore this and the ramifications of a minimal area increase for the quantization of the horizon area in the next section.

5.2 Adiabatic Invariant

The similarity of a black hole horizon area to the concept of an adiabatic invariant in mechanics has been central to the development of black hole thermodynamics. Thus, what is an adiabatic invariant?

Let us assume that there is a system which is governed by a Hamiltonian, $H(p, q, \lambda)$ which depends on a time dependant parameter $\lambda(t)$. Let us assume that T is the longest timescale of all of the internal motions of this system. Any quantity $A(p, q)$ which changes very little during the time T when H accumulates a non-negligible significant total change is said to be an adiabatic invariant. Ehrenfest [59] showed that for any quasiperiodic system, all Jacobi action integrals of the form $A = \oint p\, dq$ are adiabatic invariants. He further generalized his insight as follows: "any classical adiabatic invariant (action integral or not) corresponds to a quantum entity with discrete spectrum."

5.3 Black Hole Area Quantization

For classical extremal Kerr-Newman black holes, the constraint equation is given by:

$$M^2 = Q^2 + G^2 + J^2/M^2 \tag{5.4}$$

Assuming that the black hole is parametrised only by a few quantum numbers: mass M, spin angular momentum J, magnetic monopole G and charge Q and promoting these quantities to mutually commuting operators in the quantization process, we see that we can readily obtain the quantum mass spectrum:

$$M_{qgj} = M_P \left[q^2e^2/2 + g^2\hbar^2/8e^2 + \sqrt{(q^2e^2/2 + g^2\hbar^2/8e^2)^2 + j(j+1)} \right]^{1/2} . \tag{5.5}$$

For non-extremal classical black holes, the quantization process was not so straightforward. It was based on the insight that the horizon area was an adiabatic invariant [56].

Christodoulou and Ruffini [60], asked the question "Can the assimilation of a point particle by a Kerr black hole be made reversibly in the sense that all changes of the black hole are undone by the absorption of a suitable second particle?" This was a good question, because it was known by then that the area of a black hole cannot decrease, thus, any process which increased it, must be irreversible. A reversible process has to keep the area unchanged. They found that when a classical point particle is absorbed by the black hole from the turning point in its trajectory around the black hole, it leaves the horizon area unchanged, i.e. $\Delta A = 0$. Thus, this must be a slow, adiabatic and reversible process. The fact that the horizon area was found to be an adiabatic invariant points to the fact that in the quantum theory, A must have a discrete spectrum by virture of Ehrenfest's theorem. In order to show that, Bekenstein [56] replaced this classical point particle by a quantum particle (with its center of mass following a classical trajectory) with mass m_q, and incorporated the uncertainty principle to give the particle a radius of b. He showed that this leads to a minimal increase in horizon area of the black hole:

$$\Delta A_{\min} = 8\pi m_q b \tag{5.6}$$

By plugging in $b = \iota\hbar/m_q$ using quantum theory (ι is a number of order unity), we derive a universal minimal area increase of the horizon of the black hole as:

$$\Delta A_{\min} = 8\pi\iota\hbar = \alpha l_P^2 \tag{5.7}$$

Thus, we see that as soon as one allows quantum nuance to the problem, we end up with a minimal increase in area. This might suggest that this $(\Delta A)_{\min}$ corresponds to the spacing between eigenvalues of $\hat{A}$ in the quantum theory. And so, it was straightforwardly conjectured that the area spectrum of a non-extremal black hole would look like:

$$a_n = \alpha l_P^2 (n + \eta),\ \eta > -1,\ n = 1, 2, 3.... \tag{5.8}$$

where the condition on η excludes non-positive area eigenvalues.

5.4 The value of α

According to how Bekenstein thought, the quantization of horizon area in equal steps brings to mind a horizon formed by patches of equal area αl_P^2 which get added one at a time. In quantum theory degrees of freedom independently manifest distinct states. He used an argument calculating the total number of area states to come to the expression Equation (5.3), to calculate that $\alpha = 4 \ln k$.

However, Mukhanov's alternate route [61] might be easier to understand. He started off with the accepted formula for the black hole area entropy relation. In the spirit of the Boltzmann-Einstein formula, he views $\exp{(S_{BH})}$ as the degeneracy of the particular area eigenvalue because $\exp{(S_{BH})}$ quantifies the number of microstates of the black hole that correspond to a particular external macrostate. Since black hole entropy is determined by thermodynamic arguments only up to an additive constant, one writes, in this approach,

$$S_{BH} = A/4l_P^2 + \text{const.} \tag{5.9}$$

Substitution of the area eigenvalues from Equation (5.8) gives the degeneracy corresponding to the nth area eigenvalue as,

$$g_n = \exp{(\frac{a_n}{4l_P^2})} + \text{const.} \tag{5.10}$$

which allows us to see that if $g_n \in \mathbb{N}$ for every n, which it should be, then

$$\alpha = 4 \ln\{2, 3, 4....\}. \tag{5.11}$$

Here, $\alpha = 4 \ln 2$, corresponding to $g_1 = 2$ or a doubly degenerate ground state, seems to be the most obvious choice in order to get rid of all "ugly" constants in the area-entropy relation as can be seen from [56]. Recent papers by Kleban and Foit [62] have proposed gravitational wave experiments in order to test for the value of α.

5.5 Black Hole Area Spectrum in Quantum Gravity

These physical intuition based results have been translated into robust calculations in various different quantum gravity schemes. Since the proposal of the uniform area spectrum in 1975, many string theoretic calculations starting from Kogan (1986) [63], followed by Maggiore [64] and Lousto [65] have recovered this form. Many other canonical quantum gravity treatments of a shell or ball collapsing into a dust, for example, by Dolgov and Khriplovich [66] obtain results which correspond to a discrete yet non-uniform spacing, in the case of Berezin [67], the levels are infinitely degenerate, while Schiffer and Peleg [68] recover the uniform area spectrum. Loop Quantum Gravity treatment by Barreira, Carfora and Rovelli [69] and by Krasnov [70] leads to a discrete spectrum of complex form and highly non-uniform spacing for the black hole area.

Thus, we see that this particular area is rife with contradictory conclusions. Even in theories which recover a uniform area spectrum, there is generally no consensus on the spacing between the levels. This might hint towards the fact that either 1) none of the existing formal schemes of quantum gravity is as yet a quantum theory of gravity or 2) we do not yet have a general enough result for the black hole area spectrum or 3) both of the above. In Chapter (9) of this thesis, we will explore the correction to the black hole area-entropy law as calculated by El-Menoufi [71] using the effective quantum gravity formalism and use this to give interesting constraints on the number of light fields in a consistent theory of quantum gravity.

Chapter 6

Gravitational Waves in Effective Quantum Gravity

The Planck satellite may detect the imprint of the gravitational waves predicted by inflation. This would be quantum gravity written across the sky.

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In this short paper, we investigate quantum gravitational effects on Einstein's equations using effective field theory techniques. We consider the leading order quantum gravitational correction to the wave equation. Besides the usual massless mode, we find a pair of modes with complex masses. These massive particles have a width and could thus lead to a damping of gravitational waves if excited in violent astrophysical processes producing gravitational waves such as e.g. black hole mergers. We discuss the consequences for gravitational wave events such as GW 150914 recently observed by the Advanced LIGO collaboration.

6.1 Introduction

The recent discovery of gravitational waves by the Advanced LIGO collaboration [3] marks the beginning of a new era in astronomy which could shed some new light on our universe revealing its darkest elements that do not interact with electromagnetic radiations. This discovery could also lead to some new insights in theoretical physics. In this short paper, we study the leading effect of quantum gravity on gravitational waves using effective field theory techniques. While the discovery of a theory of quantum gravity might still be far away, it is possible to use effective field theory techniques to make actual predictions in quantum gravity. Assuming that diffeomorphism invariance is the correct symmetry of quantum gravity at the Planck scale and assuming that we know the field content below the Planck scale, we can write down an effective action for any theory of quantum gravity. This effective theory, dubbed Effective Quantum Gravity, is valid up to energies close to the Planck mass. It is obtained by linearizing general relativity around a chosen background. The massless graviton is described by a massless spin 2 tensor which is quantized using the standard quantum field theoretical procedure. It is well known that this theory is non-renormalizable, but divergences can be absorbed into the Wilson coefficients of higher dimensional operators compatible with diffeomorphism invariance. The difference with a standard renormalizable theory resides in the fact that an infinite number of measurements are necessary to determine the action to all orders. Nevertheless, Effective Quantum Gravity enables some predictions which are model independent and which therefore represent true tests of quantum gravity, whatever the underlying theory might be.

We will first investigate quantum gravitational corrections to linearized Einstein's equations. Solving these equations, we show that besides the usual solution that corresponds to the propagation of the massless graviton, there are solutions corresponding to massive degrees of freedom. If these massive degrees of freedom are excited during violent astrophysical processes a sizable fraction of the energy released by such processes could be emitted into this modes. We shall show that the corresponding gravitational wave is damped and that the energy of the wave could thus dissipate. We then study whether the recent discovery of gravitational waves by the Advanced LIGO collaboration [3] could lead to a test of quantum gravity.

6.2 The Modified Propagator

Given a matter Lagrangian coupled to general relativity with N_s scalar degrees of freedom, N_f fermions and N_V vectors one can calculate the graviton vacuum polarization in the large $N = N_s + 3N_f + 12N_V$ limit with keeping NG_N, where G_N is Newton's constant, small. Since we are interested in energies below $M_\star$ which is the energy scale at which the effective theory breaks down, we do not need to consider the graviton self-interactions which are suppressed by powers of $1/N$ in comparison to the matter loops. Note that $M_\star$ is a dynamical quantity and does not necessarily corresponds to the usual reduced Planck mass of order 10^{18} GeV (see e.g. [125]). The divergence in this diagram can be isolated using dimensional regularization and absorbed in the coefficient of R^2 and $R_{\mu\nu}R^{\mu\nu}$. An infinite series of vacuum polarization diagrams contributing to the graviton propagator can be resummed in the large N limit. This procedure leads to a resummed graviton propagator given by [126]

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_Nq^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)} \tag{6.1}$$

with $L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu/q^2$ and where μ is the renormalization scale. This resummed propagator is the source of interesting acausal and non-local effects which have just started to be investigated [40, 126–130]. Here we shall focus on how these quantum gravity effects affect gravitational waves.

6.3 Gravitational Waves in QG

From the resummed graviton propagator in momentum space, we can directly read off the classical field equation for the spin 2 gravitational wave in momentum space

$$2q^2\left(1 - \frac{NG_Nq^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right) = 0. \tag{6.2}$$

This equation has three solutions [129]:

$$\begin{aligned} q_1^2 &= 0, \\ q_2^2 &= \frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi}{\mu^2 NG_N}\right)}, \\ q_3^2 &= (q_2^2)^*, \end{aligned} \tag{6.3}$$

where W is the Lambert function. The complex pole corresponds to a new massive degree of freedom with a complex mass (i.e. they have a width [129]). The general wave solution is thus of the form

$$h^{\mu\nu}(x) = a_1^{\mu\nu} \exp(-iq_{1\alpha}x^\alpha) + a_2^{\mu\nu} \exp(-iq_{2\alpha}x^\alpha) + a_3^{\mu\nu} \exp(-iq^\star_{2\alpha}x^\alpha). \tag{6.4}$$

We therefore have three degrees of freedom which can be excited in gravitational processes leading to the emission of gravitational waves. Note that our solution is linear, non-linearities in gravitational waves (see e.g. [131]) have been investigated and are as expected very small.

The position of the complex pole depends on the number of fields in the model. In the standard model of particle physics, one has $N_s = 4$, $N_f = 45$, and $N_V = 12$. We thus find $N = 283$ and the pair of complex poles at $(7 - 3i) \times 10^{18}$ GeV and $(7 + 3i) \times 10^{18}$ GeV. Note that the pole q_3^2 corresponds to a particle which has an incorrect sign between the squared mass and the width term. We shall not investigate this Lee-Wick pole further and assume that this potential problem is cured by strong gravitational interactions. The renormalization scale needs to be adjusted to match the number of particles included in the model. Indeed, to a good approximation the real part of the complex pole is of the order of

$$|\mathrm{Re}\ q_2| \sim \sqrt{\frac{120\pi}{NG_N}} \tag{6.5}$$

which corresponds to the energy scale $M_\star$ at which the effective theory breaks down. Indeed, the complex pole will lead to acausal effects and it is thus a signal of strong quantum gravitational effects which cannot be described within the realm of the effective theory. We should thus pick our renormalization scale μ of the order of $M_\star \sim |\mathrm{Re}\ q_2|$. We have

$$q_2^2 \approx \pm \frac{1}{G_N N} \frac{120\pi}{W(-1)} \approx \mp (0.17 + 0.71\ i) \frac{120\pi}{G_N N}, \tag{6.6}$$

and we thus find the mass of the complex pole:

$$m_2 = (0.53 - 0.67\ i) \sqrt{\frac{120\pi}{G_N N}}. \tag{6.7}$$

As emphasized before, the mass of this object depends on the number of fields in the theory. The corresponding wave has a frequency:

$$\begin{aligned} w_2 &= q_2^0 = \pm\sqrt{\vec{q}_2.\vec{q}_2 + (0.17 + 0.71\ i)\frac{120\pi}{G_N N}} \qquad (6.8)\\ &= \pm\left(\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} + \vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}}\right.\\ &\left. +i\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} - \vec{q}_2.\vec{q}_2 - 0.17\frac{120\pi}{G_N N}}\right). \end{aligned}$$

6.4 Bounds on the Mass

The imaginary part of the complex pole will lead to a damping of the component of the gravitational wave corresponding to that mode. The complex poles are gravitationally coupled to matter, we must thus assume that the massive modes are produced at the same rate as the usual massless graviton mode if this is allowed kinematically. During an astrophysical event leading to gravitational waves, some of the energy will be emitted into these massive modes which will decay rather quickly because of their large decay width. The possible damping of the gravitational wave implies that care should be taken when relating the energy of the gravitational wave observed on earth to that of the astrophysical event as some of this energy could have been dissipated away as the wave travels towards earth.

The idea that gravitational waves could experience some damping has been considered before [132], however it is well known that the graviton cannot split into many gravitons, even at the quantum level [133], if there was such an effect it would have to be at the non-perturbative level [134]. In our case, the massless mode is not damped, there is thus no contradiction with the work of [133]. Also, as emphasized before the dispersion relation of the massless mode of the gravitational wave is not affected, we do not violate any essential symmetry such as Lorentz invariance. This is in contrast to the model presented in [135].

6.5 Production of the Massive Modes

Since the complex poles couple with the same coupling to matter as the usual massless graviton, we can think of them as a massive graviton although strictly speaking these objects have two polarizations only in contrast to massive gravitons that have five. This idea has been applied in the context of $F(R)$ gravity [136] (see also [137, 138] for earlier works on gravitational waves in $F(R)$ gravity). We shall assume that these massive modes can be excited during the merger of two black holes. As a rough approximation, we shall assume that all the energy released during the merger is emitted into these modes. Given this assumption, we can use the limit derived by the LIGO collaboration on a graviton mass. We know that $m_g < 1.2 \times 10^{-22}$ eV and we can thus get a limit:

$$\sqrt{\mathrm{Re}\left(\frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}\right)} < 1.2 \times 10^{-22}\ \mathrm{eV} \tag{6.9}$$

we thus obtain a lower bound on N: $N > 4 \times 10^{102}$ if all the energy of the merger was carried away by massive modes. Clearly, this is not realistic as the massless mode will be excited. However, it implies that if the massive modes are produced, they will only arrive on earth if their masses are smaller than 1.2×10^{-22} eV. Waves corresponding to more massive poles will be damped before reaching earth. We shall see that there are tighter bounds on the mass of these objects coming from Eötvös type pendulum experiments.

At this stage, we need to discuss which modes can be produced during the two black holes merger that led to the gravitational wave observed by the LIGO collaboration. The LIGO collaboration estimates that the gravitational wave GW150914 is produced by the coalescence of two black holes: the black holes follow an inspiral orbit before merging and subsequently going through a final black hole ringdown. Over 0.2 s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum [3]. The typical energy of the gravitational wave is of the order of 150 Hz or 6×10^{-13} eV. In other words, if the gravitational wave had been emitted in the massive mode, they could not have been heavier than 6×10^{-22} GeV. However, this shows that it is perfectly conceivable that a sizable number of massive gravitons with $m_g < 1.2 \times 10^{-22}$ eV could have been produced.

Let us now revisit the bound on the number of fields N and thus the new complex pole using Eötvös type pendulum experiments looking for deviations of the Newtonian $1/r$ potential. The resummed graviton propagator discussed above can be represented by the effective operator

$$\frac{N}{2304\pi^2} R \log\left(\frac{\Box}{\mu^2}\right) R \tag{6.10}$$

where R is the Ricci scalar. As explained above the log term will be a contribution of order 1, this operator is thus very similar to the more familiar cR^2 term studied by Stelle long ago. The current bound on the Wilson coefficient of c is $c < 10^{61}$ [34, 139, 140]. We can translate this bound into a bound on N: $N < 2 \times 10^{65}$. This implies that the mass of the complex pole must be larger than 5×10^{-13}GeV. This bound, although very weak, is more constraining than the one we have obtained from the graviton mass by 37 orders of magnitude.

6.6 Conclusions

In this short paper we have investigated quantum gravitational effects in gravitational waves using conservative effective theory methods which are model independent. We found that quantum gravity leads to new poles in the propagator of the graviton besides the usual massless pole. These new states are massive and couple gravitationally to matter. If kinematically allowed, they would thus be produced in roughly the same amount as the usual massless mode in energetic astrophysical events. A sizable amount of the energy produced in astrophysical events could thus be carried away by massive modes which would decay and lead to a damping of this component of the gravitational wave. While our back-of-the-envelope calculation indicates that the energy released in the merger recently observed by LIGO was unlikely to be high enough to produce such modes, one should be careful in extrapolating the amount of energy of astrophysical events from the energy of the gravitational wave observed on earth. This effect could be particularly important for primordial gravitational waves if the scale of inflation is in the region of 10^{16} GeV, i.e. within a few orders of magnitude of the Planck scale.

Chapter 7

Gravitational Radiation in Quantum Gravity

Energy is liberated matter, matter is energy waiting to happen.

Bill Bryson

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The effective field theory of quantum gravity generically predicts non-locality to be present in the effective action, which results from the low-energy propagation of gravitons and massless matter. Working to second order in gravitational curvature, we reconsider the effects of quantum gravity on the gravitational radiation emitted from a binary system. In particular, we calculate for the first time the leading order quantum gravitational correction to the classical quadrupole radiation formula which appears at second order in Newton's constant.

7.1 Introduction

The aim of this work is to extend the study of quantum gravitational corrections to gravitational radiation initiated in [72, 73] using effective theory techniques to treat quantum gravity in a model independent way. In previous papers [72, 73] the authors focused on the production of new massive modes present in the effective action [129]. We expand on the previous analyses and calculate for the first time the genuine quantum gravitational correction to the quadrupole radiation formula first developed by Einstein. While the effect is way too small to be observable by the current gravitational wave observatories and thus has no impact for the recent gravitational wave observations [3, 74], our work offers a proof of principle that genuine calculations within quantum gravity at energies below the Planck mass are possible, even though we do not yet have a fully satisfactory ultra-violet complete theory of quantum gravity.

We follow the approach introduced by Weinberg [75] in the 70's and further developed by others [47, 76, 77]. The main benefit of the effective theory approach is its ability to separate out low-energy dynamics from the unknown ultra-violet physics associated with the completion of quantum gravity. Quantum general relativity has indeed a poor ultra-violet behavior, i.e. it is non-renormalizable, yet the unknown physics is solely encoded in the Wilson coefficients of the most general diffeomorphism invariant *local* Lagrangian. When the Wilson coefficients are measured, any observable computed in the effective theory is completely determined to any desired accuracy in the effective field theory expansion. More interesting are the contributions induced by long-distance propagation of massless (light) degrees of freedom. The latter comprise reliable and parameter-free, and thus model independent, predictions of quantum gravity since, by the very nature of the effective field theory, any ultra-violet completion must reproduce these results at low energies.

In this paper we revisit the long-distance limit of quantum gravity and the signatures thereof on the gravitational radiation emitted from binary systems. As we shall describe below, quantum corrections are encoded in a covariant effective action organized as an expansion in gravitational curvatures. Moreover, low-energy quantum effects manifest in the effective action via a covariant set of non-local operators. The three phases of the binary evolution will be affected by quantum corrections. Thanks to advances in infrared quantum gravity [40, 45, 46, 76–78], we could in principle determine the modified fate of each phase since the effective action retains the non-linear structure of the field equations. Nevertheless, to obtain analytic insight we only focus on the leading quantum corrections to the quadrupole radiation of general relativity. It is important to keep in mind that the initial stage of a coalescence process is the only part one can study with analytical tools.

We shall define two schemes to treat quantum corrections. The first is *non-perturbative*, in the sense that higher-derivative terms in the equations of motion are considered on the same footing as those of general relativity. We focus on the massive spin-2 sector and show that the propagator has a multi-sheet complex structure [79], which arises due to the logarithmic non-analyticity in the equations of motion. The imaginary part of the complex poles causes the massive spin-2 field to exhibit a Yukawa suppression in the far-field region. The second treatment is *perturbative* and aligns naturally with the power-counting of the effective theory. Namely, we look for small corrections to the lowest-order general relativity result, i.e. quadrupole radiation, and solve the equations of motion by iteration. This is the genuine quantum gravitational correction discussed early and the main new result of this paper. In the latter scheme, the correction to the spin-2 sector is a traveling wave at the speed of light, but the amplitude falls off faster than $1/r$.

Before we proceed, it is crucial to describe the physical content of our results. All our analysis is performed on the linear weak-field level, but general relativity and the associated quantum corrections are inherently non-linear. This distinction is crucial when one deviates from pure general relativity. Indeed, it was shown in [80] that an eternal Schwarzschild black hole is a solution to the *full* non-linear quantum corrected theory. On the contrary and due to the breakdown of Birkhoff's theorem, the gravitational field around a non-vacuum source such as a star receives a genuine quantum correction [80]. Hence, all our results will only pertain to the inspiraling phase of mergers where the gravitational radiation is sourced by horizonless objects such as neutron stars or black holes if we think of them as objects which are not vacuum solutions but rather astrophysical objects which are still experiencing gravitational collapse [81].

The paper is organized as follows. In Section 7.2 we start with a brief review of the effective theory and write down the non-local corrections we shall investigate. Section 7.3 is devoted to a quick survey of the radiation problem in local quadratic gravity. Section 7.4 and 7.5 treat the non-local corrections in the two different schemes described above. We conclude in Section 7.6. A careful derivation of the non-local kernel used in Section 7.5 is laid out in an appendix.

7.2 The non-local quantum corrections

The effective field theory treatment of quantum gravity is by now very well understood. The initial incarnation of the effective field theory was designed mainly to compute scattering amplitudes in flat space. For example, graviton-graviton scattering can be obtained to any desired accuracy in the counting parameter of the effective theory, i.e. $(GE^2)^n$ where E is the center-of-mass energy of the process. At lowest-order $\mathcal{O}(GE^2)$, one extracts vertices from the Einstein-Hilbert action and computes tree-level diagrams. At order $\mathcal{O}(GE^2)^2$, one-loop diagrams appear and the ultra-violet divergences renormalize the Wilson coefficients of the quadratic curvature action. The framework is readily extended to include matter fields. In summary, the action of the effective theory, accurate to order $(GE^2)^2$, reads[1]

$$S_{\text{EFT}} = \int_{\mathcal{M}} \left(\frac{R}{16\pi G} + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \mathcal{L}_m \right) \quad . \tag{7.1}$$

To complete the effective field theory program, a measurement of the Wilson coefficients is required as per usual with any ultra-violet-sensitive quantity in quantum field theory. Unfortunately, such experimental input is not available in our case and one might question if the effective field theory is able to make any predictions. It was the point of view developed in [47] where it is shown that there exist a class of quantum corrections that comprise reliable signatures of quantum gravity. The latter appear as *finite* non-analytic functions in loop processes and arise directly from the low-energy propagation of virtual massless quanta. As such, these corrections are purely of infra-red origin modifying the long-distance dynamics of gravitation. A

[1]Notice that in writing this action we have employed the Gauss-Bonnet identity to get rid of the Riemann squared invariant. We also dropped a total derivative, $\Box R$, that does not provide a non-trivial Feynman rule. Also note that the power counting in $\mathcal{L}_m$ depends on the mass of the matter field.

prime example is the correction to the non-relativistic Newtonian potential energy [82]

$$V_{\mathrm{N}}(r) = -\frac{Gm_1m_2}{r}\left(1 + \frac{3G(m_1+m_2)}{r} + \frac{41}{10\pi^2}\frac{l_P^2}{r^2}\right) \quad . \tag{7.2}$$

Moving ahead of scattering amplitudes, one inquires about the structure of long-distance quantum effects in the effective action. A substantial body of work has been devoted to construct the effective action of quantum gravity that encapsulates such quantum corrections. We refer the interested reader to the following articles and references therein [40, 45, 46, 76–78]. Here, we merely quote the leading operators in the non-local curvature expansion

$$\Gamma^{(2)}_{\mathrm{NL}} = -\int_{\mathcal{M}}\left[\alpha R \ln\left(\frac{\Box}{\mu^2}\right) R + \beta R_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}\ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu\alpha\beta}\right] , \tag{7.3}$$

where $\Box := g^{\mu\nu}\nabla_\mu\nabla_\nu$. The precise values of the coefficients depend on the spin of the massless particle that runs in the loop and are listed in table (7.1). Non-local effective actions open the door to (re)-examine plenty of questions in gravitational physics. In this paper, we shall focus on the effect of Eq. (7.3) on the production of gravitational radiation from binary systems.

7.3 Production of gravitational waves: local theory

As explained in [72, 73], quantum gravity contains two massive wave solutions on top of the usual massless mode of general relativity. We review the results presented in [72, 73] in preparation for calculation of the leading order quantum gravitational correction to the classical quadrupole formula. To streamline the discussion, we shall focus in this section on the local quadratic theory, i.e. Eq. (7.1). Analyzing the latter, albeit simple in nature, aids in drawing interesting parallels and contrasts when we discuss non-locality in the next section. We only consider a simple system where the two masses move in a perfectly circular orbit.

The equations of motion are easily obtained by linearizing the field equations of

	α	β	γ
Scalar	$5(6\xi - 1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	250	-244	424

Table 7.1: Coefficients for different fields. Note that these coefficients have been derived by many different authors, see e.g. [40, 47, 76, 77, 83–87]. All numbers should be divided by $11520\pi^2$. Here, ξ denotes the value of the non-minimal coupling for a scalar theory. All these coefficients including those for the graviton are gauge invariant. It is well known that one needs to be careful with the graviton self-interaction diagrams and that the coefficients α and β can be gauge dependent, see [88], if the effective action is defined in a naive way. For example, the numbers $\alpha = 430/(11520\pi^2)$ and $\beta = -1444/(11520\pi^2)$ for the graviton quoted in [40] are obtained using the Feynman gauge. However, there is a well-established procedure to derive a unique effective action which leads to gauge independent results [76, 77]. Here we are quoting the values of α and β for the graviton obtained using this formalism as it guaranties the gauge independence of observables.

Eq. (7.1)

$$\Box\bar{h}_{\mu\nu} - \kappa^2\Box\Big[\Big(c_1 + \frac{c_2}{2} + c_3\Big)\,\partial_\mu\partial_\nu\bar{h} - \Big(c_1 + \frac{c_2}{2} + c_3\Big)\,\eta_{\mu\nu}\Box\bar{h} + \Big(\frac{c_2}{2} + 2c_3\Big)\,\Box\bar{h}_{\mu\nu}\Big] = -16\pi G T_{\mu\nu} \tag{7.4}$$

where $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ is the trace-reduced tensor, $\kappa^2 = 32\pi G$ and we employed the harmonic gauge. It is more convenient to perform our calculation using the trace-reduced tensor, and only at the end obtain $h_{\mu\nu}$ by subtracting off the trace. Since the pioneering work of Stelle [34], it became quite common to dispense with the higher-derivative structure of the theory by introducing massive modes in the equations of motion. These extra modes decouple from the massless spin-2 mode. Working in momentum-space, we get

$$\bar{\mathcal{O}}_{\mu\nu}{}^{\alpha\beta}\,\bar{h}_{\alpha\beta}(k) = -16\pi G T_{\mu\nu}(k) \tag{7.5}$$

where

$$\begin{aligned}\bar{\mathcal{O}}_{\mu\nu}{}^{\alpha\beta} \;=\; & -\frac{k^2}{2}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu) \\ & -\kappa^2\left[\Big(c_1 + \frac{c_2}{2} + c_3\Big)(k^2 k_\mu k_\nu \eta^{\alpha\beta} - k^4\eta_{\mu\nu}\eta^{\alpha\beta}) + \Big(\frac{c_2}{2} + 2c_3\Big)\frac{k^4}{2}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu)\right] \;.\end{aligned} \tag{7.6}$$

Revealing the massive modes requires that we project out the spin-2 and spin-0 parts of the symmetric operator

$$\mathcal{P}^{(2)\alpha\beta}_{\mu\nu} = \frac{1}{2}\left(\theta^\alpha_\mu\theta^\beta_\nu + \theta^\alpha_\nu\theta^\beta_\mu\right) - \frac{1}{3}\theta_{\mu\nu}\theta^{\alpha\beta}, \quad \mathcal{P}^{(0)\alpha\beta}_{\mu\nu} = \frac{1}{3}\theta_{\mu\nu}\theta^{\alpha\beta}, \tag{7.7}$$

where $\theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu/k^2$. In harmonic gauge, we have $k^\mu\bar{h}_{\mu\nu} = 0$ and so Eq. (7.6) is easily rewritten as

$$\bar{\mathcal{O}}_{\mu\nu}{}^{\alpha\beta} = -k^2\left(1 + \kappa^2\Big(\frac{c_2}{2} + 2c_3\Big)k^2\right)\mathcal{P}^{(2)\alpha\beta}_{\mu\nu} - k^2\left(1 + \kappa^2\left(-3c_1 - c_2 - c_3\right)k^2\right)\mathcal{P}^{(0)\alpha\beta}_{\mu\nu} \;. \tag{7.8}$$

Inverting the operator yields the propagator in momentum-space

$$\bar{\mathcal{D}}_{\mu\nu}{}^{\alpha\beta} = -\frac{\left(\mathcal{P}^{(2)\alpha\beta}_{\mu\nu} + \mathcal{P}^{(0)\alpha\beta}_{\mu\nu}\right)}{k^2} + \frac{\mathcal{P}^{(2)\alpha\beta}_{\mu\nu}}{k^2 - m_2^2} + \frac{\mathcal{P}^{(0)\alpha\beta}_{\mu\nu}}{k^2 - m_0^2} \tag{7.9}$$

where we have used partial fractions to identify the masses of the spin-2 and spin-0 sectors

$$m_2^2 = \frac{M_P^2}{2(-c_2 - 4c_3)}, \quad m_0^2 = \frac{M_P^2}{4(3c_1 + c_2 + c_3)} \;. \tag{7.10}$$

We stress again that Eq. (7.9) is the propagator for $\bar{h}_{\mu\nu}$. For completeness, we can easily obtain the appropriate propagator for $h_{\mu\nu}$ by subtracting the trace of Eq. (7.9)

$$\begin{aligned}
\mathcal{D}_{\mu\nu}{}^{\alpha\beta} &= \bar{\mathcal{D}}_{\mu\nu}{}^{\alpha\beta} - \frac{1}{2}\eta_{\mu\nu}\eta^{\gamma\lambda}\bar{\mathcal{D}}_{\gamma\lambda}{}^{\alpha\beta} \\
&= -\frac{\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \eta_{\mu\nu}\eta^{\alpha\beta}}{2k^2} + \frac{\mathcal{P}^{(2)\alpha\beta}_{\mu\nu}}{k^2 - m_2^2} - \frac{\mathcal{P}^{(0)\alpha\beta}_{\mu\nu}}{2(k^2 - m_0^2)} \qquad (7.11)
\end{aligned}$$

which is the known result derived by Stelle [34]. As emphasized, the extension of general relativity including the terms quadratic in curvature contains three mass eigentstates: a massless mode with spin-2 and two massive modes with respectively spin 2 and 0. The massive spin-2 mode is formally a ghost while the massive spin-0 mode is healthy. However, as already explained in details in [73, 89], the massive spin-2, although it is formally a ghost, does not lead to any pathology. The effective action contains only classical fields, as the fluctuations of the graviton have been integrated out. The massive field with spin-2 can simply be seen as a field that couples with minus the Planck scale to the stress-energy tensor. It is a nothing but a repulsive force. Notice also here that either (or both) of m_0 and m_2 could be tachyonic depending on the exact values of the Wilson coefficients. In this section, we proceed under the assumption that the masses are real.

Given the manifest decoupling of the modes, the solution to Eq. (7.4) is the direct sum of the three sectors. One can switch back to position-space and write down the solution for the trace-reduced metric perturbations, making sure to define the propagators with retarded boundary conditions

$$\begin{aligned}
\bar{h}_{\mu\nu} &= 16\pi G \int d^4x'\, G^{\text{ret.}}(x - x'; 0) T_{\mu\nu}(x') \qquad (7.12) \\
&\quad -16\pi G \int d^4x'\, G^{\text{ret.}}(x - x'; m_2)\mathcal{P}^{(2)\alpha\beta}_{\mu\nu} T_{\alpha\beta}(x') \\
&\quad -16\pi G \int d^4x'\, G^{\text{ret.}}(x - x'; m_0)\mathcal{P}^{(0)\alpha\beta}_{\mu\nu} T_{\alpha\beta}(x').
\end{aligned}$$

Note that the general relativity solution is given by

$$\bar{h}^{\text{GR}}_{\mu\nu} := 16\pi G \int d^4x'\, G^{\text{ret.}}(x - x'; 0) T_{\mu\nu}(x') \qquad (7.13)$$

It is important to realize that the two new terms are of the same order in G as the usual solution from general relativity. These are not corrections to general relativity solutions. There are simply additional classical modes present in the action. We stress that each of these terms is a solution to their partial differential equations which are fully decoupled. We write them as a direct sum for convenience, but the reader should not get confused.

We consider our source to be a simple binary system and set the origin of the coordinates to coincide with the center-of-mass of the system

$$T_{\mu\nu} = \sum_{i=1}^{2} M_i \dot{x}_\mu \dot{x}_\nu \, \delta^{(3)} \left(\vec{x} - \vec{X}_i(\tau)\right) \tag{7.14}$$

where a dot denotes a derivative with respect to proper time, τ, and $\vec{X}_i$ is the trajectory of the mass. In the slow-velocity limit, proper time coincides with coordinate time to lowest order in velocity. We notice first that the spin-0 mode couples to the trace of the energy-momentum tensor, which is time-independent for a binary system in circular orbit. Focusing on the massive spin-2 sector, we are interested in the leading behavior in the far-zone ($|\vec{x} - \vec{x}'| \approx |\vec{x}| := r$). It suffices to solve for the spatial components, i.e. $\bar{h}_{ij}$, the other metric perturbations are determined using the harmonic gauge condition. With this set-up, Eq. (7.12) becomes[2]

$$\bar{h}_{ij} = \bar{h}_{ij}^{\mathrm{GR}} - 16\pi G \int d\omega \, e^{-i\omega t} I_{ij}(\omega) \int \frac{k^2 dk d\Omega_k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{(\omega + i\epsilon)^2 - k^2 - m_2^2} \tag{7.15}$$

where

$$I_{ij}(\omega) = -\frac{1}{2}\mu(d\omega_s)^2 \begin{pmatrix} \delta(\omega + 2\omega_s) + \delta(\omega - 2\omega_s) & -i(\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s)) & 0 \\ -i(\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s)) & -\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s) & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{7.16}$$

In the above, μ is the reduced mass of the binary, d is the orbital separation and ω_s is the orbital frequency. In Eq. (7.15), notice most importantly the $i\epsilon$ prescription is due to the retarded boundary conditions. The angular integrals in Eq. (7.15) are readily done, and the final integral over the spatial momentum depends crucially on the size of the mass compared to the orbital frequency. In the complex k-plane, the poles are situated at

$$k_\pm = \pm\sqrt{\omega^2 - m_2^2} \pm \mathrm{sgn}(\omega) \, i\epsilon \quad . \tag{7.17}$$

[2]In writing Eq. (7.15) we ignored all terms proportional to the trace of the energy-momentum tensor, which is time independent for a binary in circular orbit.

One notices two features of the above expression. First, the poles are real (imaginary) if the mass is smaller (greater) than the frequency. Second, if the poles are real then the sign of the frequency is important in moving the poles off the real axis, which is paramount in obtaining a proper propagating wave. After a careful computation we find

$$\bar{h}_{ij}(t,r) = \bar{h}^{\mathrm{GR}}_{ij} - 4G\frac{\mu(d\omega_s)^2}{r}\left[\theta(m_2 - 2\omega_s)e^{-\sqrt{m_2^2-4\omega_s^2}r}Q_{ij}(t,0;0) + \theta(2\omega_s - m_2)Q_{ij}(t,r;m_2^2)\right] \tag{7.18}$$

where we defined

$$Q_{ij}(t,r;m^2) = \begin{pmatrix} \cos\left(2\omega_s\left(t-\sqrt{1-(m/2\omega_s)^2}r\right)\right) & \sin\left(2\omega_s\left(t-\sqrt{1-(m/2\omega_s)^2}r\right)\right) & 0 \\ \sin\left(2\omega_s\left(t-\sqrt{1-(m/2\omega_s)^2}r\right)\right) & -\cos\left(2\omega_s\left(t-\sqrt{1-(m/2\omega_s)^2}r\right)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{7.19}$$

The remaining integrals can now easily be performed. We find

$$h_{ij}(t,r) = h^{\mathrm{GR}}_{ij} - 4G\frac{\mu(d\omega_s)^2}{r}\left[\theta(m_2 - 2\omega_s)e^{-\sqrt{m_2^2-4\omega_s^2}r}Q_{ij}(t,0;0) + \theta(2\omega_s - m_2)Q_{ij}(t,r;m_2^2)\right] , \tag{7.20}$$

in the far zone, where

$$h^{\mathrm{GR}}_{ij} := 4G\frac{\mu(d\omega_s)^2}{r}Q_{ij}(t,r;0) \quad . \tag{7.21}$$

The explicit calculations are shown in Appendix A.

Comments about the above result are in place:

- The second term has the opposite sign in comparison to that of general relativity, which signifies the repulsive nature of the massive spin-2 sector. This mode is *classically* healthy because it carries positive-definite energy. To compute the radiated power, one simply has to construct the energy-momentum tensor from the Lagrangian of the theory. Since the different modes are decoupled [34], the total energy-momentum tensor is likewise decoupled. The latter is quadratic in the field variables and so obviously the negative sign in the massive spin-2 solution does not affect the positivity of the energy.

- Eq. (7.18) contains two parts. If the mass is large compared to the characteristic frequency of the system, the result is a standing wave due to the Yukawa suppression. Hence, formally no energy is transmitted to infinity. The traveling wave portion has outgoing spherical wave-fronts and is viable only if the frequency is large enough to excite the massive mode.

- The $i\epsilon$ prescription is crucial to obtain a solution that represents a traveling wave: the position of the poles changes when the frequency flips from $\omega = 2\omega_s$ to $\omega = -2\omega_s$. This takes place consistently such that all exponential factors arrange correctly and yield sinusoidal functions propagating at the correct speed appropriate for a massive wave.

- The wave is sub-luminal and has a group velocity $v_g(\omega) = \sqrt{1-(m_2/\omega)^2}$, which is readily identified from the dispersion relation $k(\omega) = \omega\sqrt{1-(m/\omega)^2}$. This is precisely the relativistic velocity of a free massive particle.

- For completeness, we can easily compute the total emitted power. We use the fact that the total energy-momentum tensor is the direct sum of the three modes and notice that the energy-momentum tensor of a massive spin-2 theory is identical to that of general relativity[3]. To lowest order in the mass, we have the rate of energy loss

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{32G\mu^2 d^4\omega_s^6}{5}\left(1+\theta(2\omega_s - m_2)\right) + \mathcal{O}\left(\frac{m_2}{\omega_s}\right) . \tag{7.22}$$

where, as explained in [73] where this equation was first derived, the first term is the power lost in the massless gravitational mode while the second term represents the power lost in the massive spin-2 mode.

7.4 Quantum non-locality: Non-perturbative treatment

We now include the non-local higher curvature corrections in the equations of motion. Adding the non-local corrections, we find (in harmonic gauge)

$$\Box\bar{h}_{\mu\nu} - \kappa^2\Box\Big[\left(c_1(\mu) + \frac{c_2(\mu)}{2} + c_3(\mu)\right)(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box)\bar{h} - \left(\alpha + \frac{\beta}{2} + \gamma\right)\mathfrak{L}(\bar{h})_{,\mu\nu}$$
$$+\left(\alpha + \frac{\beta}{2} + \gamma\right)\eta_{\mu\nu}\Box\mathfrak{L}(\bar{h}) + \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right)\Box\bar{h}_{\mu\nu} - \left(\frac{\beta}{2} + 2\gamma\right)\Box\mathfrak{L}(\bar{h}_{\mu\nu})\Big] = -16\pi G T_{\mu\nu} \tag{7.23}$$

[3]Notice that this is true in general, i.e. not necessarily requiring the Pauli-Fierz tuning.

where

$$\mathfrak{L}(f) := \int d^4x' \mathfrak{L}(x-x') \, f(x'), \quad \mathfrak{L}(x-x') = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} \ln\left(\frac{-k^2}{\mu^2}\right) \quad . \tag{7.24}$$

The non-local function, $\mathfrak{L}(x-x')$, must be supplemented by a boundary condition to be well-defined. We impose retarded boundary conditions by sending $k^0 \to k^0 + i\epsilon$ inside the logarithm; see the discussion in the appendix. The exact form of $\mathfrak{L}(x-x')$ is derived in Appendix (**??**), nevertheless, we will not need such an expression in this section. In fact, we wish to treat the higher-derivative terms along the same lines of the last section. We refer to this treatment as *non-perturbative*, and so we transform Eq. (7.23) to momentum-space and obtain the non-analytic operator

$$\begin{aligned} \bar{\mathcal{O}}_{\mu\nu}{}^{\alpha\beta} = &-k^2 \left(1 + \kappa^2 \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) k^2 - \kappa^2 \left(\frac{\beta}{2} + 2\gamma\right) k^2 \ln\left(\frac{-k^2}{\mu^2}\right)\right) \mathcal{P}^{(2)\alpha\beta}_{\mu\nu} \\ &- k^2 \left(1 + \kappa^2 \left(-3c_1(\mu) - c_2(\mu) - c_3(\mu)\right) k^2 - \kappa^2 \left(-3\alpha - \beta - \gamma\right) k^2 \ln\left(\frac{-k^2}{\mu^2}\right)\right) \mathcal{P}^{(0)\alpha\beta}_{\mu\nu} \end{aligned} \tag{7.25}$$

whose propagator is readily constructed

$$\begin{aligned} \bar{\mathcal{D}}_{\mu\nu}{}^{\alpha\beta} = &\frac{\mathcal{P}^{(2)\alpha\beta}_{\mu\nu}}{-k^2 \left(1 + \kappa^2 \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) k^2 - \kappa^2 \left(\beta/2 + 2\gamma\right) k^2 \ln\left(-k^2/\mu^2\right)\right)} \\ &+ \frac{\mathcal{P}^{(0)\alpha\beta}_{\mu\nu}}{-k^2 \left(1 + \kappa^2 \left(-3c_1(\mu) - c_2(\mu) - c_3(\mu)\right) k^2 - \kappa^2 \left(-3\alpha - \beta - \gamma\right) k^2 \ln\left(-k^2/\mu^2\right)\right)} \quad . \end{aligned} \tag{7.26}$$

We decompose the trace-reduced metric perturbations (in harmonic gauge) as follows[4]

$$\bar{h}_{\mu\nu} = \bar{h}^{(2)}_{\mu\nu} + \bar{h}^{(0)}_{\mu\nu}, \quad \bar{h}^{(2)}_{\mu\nu} := \mathcal{P}^{(2)\alpha\beta}_{\mu\nu} \bar{h}_{\alpha\beta}, \quad \bar{h}^{(0)}_{\mu\nu} := \mathcal{P}^{(0)\alpha\beta}_{\mu\nu} \bar{h}_{\alpha\beta} \quad . \tag{7.27}$$

We focus on the spin-2 sector and separate out the general relativity piece by rewriting the denominator in Eq. (7.26)

$$\begin{aligned} &\frac{1}{k^2 \left(1 + \kappa^2 \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) k^2 - \kappa^2 \left(\beta/2 + 2\gamma\right) k^2 \ln\left(-k^2/\mu^2\right)\right)} \\ &= \frac{1}{k^2} - \frac{\kappa^2 \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) - \kappa^2 (\beta/2 + 2\gamma) \ln\left(-k^2/\mu^2\right)}{\left(1 + \kappa^2 \left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) k^2 - \kappa^2 \left(\beta/2 + 2\gamma\right) k^2 \ln\left(-k^2/\mu^2\right)\right)} \quad . \end{aligned} \tag{7.28}$$

[4]Note that the sum $\mathcal{P}^{(2)} + \mathcal{P}^{(0)} = \mathbb{1}$ when it acts on symmetric tensors satisfying the harmonic gauge.

This way the spin-2 sector reads

$$\bar{h}^{(2)}_{ij}(\omega,\vec{x}) = \bar{h}^{(2)\mathrm{GR}}_{ij}(\omega,\vec{x}) + \bar{h}^{(2)\mathrm{m}}_{ij}(\omega,\vec{x}), \tag{7.29}$$

where the massive spin-2 piece is now transparent. Working in the far-zone, we have

$$\begin{aligned}\bar{h}^{(2)\mathrm{m}}_{ij}(\omega,\vec{x}) &= -(16\pi G\kappa^2) I_{ij}(\omega) \times \\ &\int \frac{k^2 dk d\Omega_k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \frac{\left(\frac{c_2(\mu)}{2} + 2c_3(\mu)\right) - (\beta/2+2\gamma)\ln\left(-k^2/\mu^2\right)}{\left(1+\kappa^2\left(\frac{c_2(\mu)}{2}+2c_3(\mu)\right)k^2 - \kappa^2(\beta/2+2\gamma)k^2\ln\left(-k^2/\mu^2\right)\right)},\end{aligned} \tag{7.30}$$

where $I_{ij}(\omega)$ is given in Eq. (7.16) and we work temporarily in a mixed frequency-position representation. Compared to Eq. (7.15), we observe that the non-analyticity has turned the denominator into a transcendental function which is infinitely-valued. A careful investigation of the latter is essential to understand the physical content of the result. The angular integrals in Eq. (7.30) are readily performed

$$\begin{aligned}&\bar{h}^{(2)\mathrm{m}}_{ij}(\omega,\vec{x}) = (16\pi G\kappa^2) I_{ij}(\omega)\left(\frac{1}{8\pi^2 r}\right) \times \\ &\frac{d}{dr}\int_{-\infty}^{\infty} dk \frac{(e^{ikr}+e^{-ikr})\left[\left(\frac{c_2(\mu)}{2}+2c_3(\mu)\right) - (\beta/2+2\gamma)\ln\left((k^2-\omega^2)/\mu^2\right)\right]}{\left(1+\kappa^2\left(\frac{c_2(\mu)}{2}+2c_3(\mu)\right)(\omega^2-k^2) - \kappa^2(\beta/2+2\gamma)(\omega^2-k^2)\ln\left((k^2-\omega^2)/\mu^2\right)\right)} ,\end{aligned} \tag{7.31}$$

where it is understood that $\omega \to \omega + i\epsilon$ in the integrand to enforce retarded boundary conditions. Similar to the previous section, we evaluate the above integral in the complex plane. The situation here is rather complicated because the logarithm is infinitely-valued. This causes the integrand in Eq. (7.31) to possess infinitely many poles that appear on the various Riemann sheets of the logarithm. The values of the poles are compactly encoded in the Lambert-W function [89, 129]

$$\omega^2 - k^2 = m_2^2 := \frac{1}{\kappa^2(\beta/2+2\gamma) W\left(-\frac{2\exp\left(\frac{-c_2(\mu)-4c_3(\mu)}{\beta+4\gamma}\right)}{\kappa^2\mu^2(\beta+4\gamma)}\right)} . \tag{7.32}$$

This reproduces the result obtained in [73]. We see from table (7.1) that the combination $(\beta/2+2\gamma)$ is positive-definite for all massless particles, and thus the argument of the Lambert-W function in Eq. (7.32) is negative-definite.

We will comment on the pole structure of Eq. (7.31) as we proceed, but for now it suffices to pick a Riemann sheet in order to evaluate the integral. On each sheet, there is a single complex pole given any choice of the ultra-violet data, i.e.

the Wilson coefficients and the renormalization scale [79]. Let us treat in detail the integral involving the positive exponential in Eq. (7.31), where our choice of the branch cut and integration contour is shown in Fig. (7.1). Clearly, a generally complex solution to Eq. (7.32) introduces two poles which are mirror images of each other. Let us define two quantities

$$\Omega := \omega^2 - \Re m_2^2, \quad \zeta := \Im m_2^2 - \epsilon\,\mathrm{sgn}(\omega) \ . \tag{7.33}$$

Notice that the sign of both Ω and ζ is not fixed at this stage. A direct computation yields

$$k_\pm = \begin{cases} \pm\sqrt{\tfrac{1}{2}(\Omega^2+\zeta^2)^{1/2}+\tfrac{1}{2}\Omega} \mp i\,\mathrm{sgn}(\zeta)\sqrt{\tfrac{1}{2}(\Omega^2+\zeta^2)^{1/2}-\tfrac{1}{2}\Omega} \ , & \Omega > 0 \\ \pm\sqrt{\tfrac{1}{2}(\Omega^2+\zeta^2)^{1/2}-\tfrac{1}{2}|\Omega|} \mp i\,\mathrm{sgn}(\zeta)\sqrt{\tfrac{1}{2}(\Omega^2+\zeta^2)^{1/2}+\tfrac{1}{2}|\Omega|} \ , & \Omega < 0 \end{cases} \tag{7.34}$$

Compared to Eq. (7.17), we notice the important difference that the retarded $i\epsilon$-prescription does not play role in placing the poles because $\Im m_2^2$ is non-zero. For definiteness, let us focus on the case when Ω is positive. Since we close the contour in the upper-half-plane (cf. Fig. (7.1)), we only pick poles with positive imaginary part, and hence the contribution to the metric perturbations is Yukawa-suppressed. The same conclusion applies to the integral involving e^{-ikr} as we close the contour in the lower-half-plane. The discontinuity across the branch cut cancel out in the final result and we are left with only the contribution from the residues.

$$\begin{aligned} \bar{h}_{ij}^{(2)\mathrm{m}}(t,\vec{x}) = -4G\frac{\mu(d\omega_s)^2}{r}&\frac{\frac{c_2(\mu)}{2}+2c_3(\mu)-(\beta/2+2\gamma)\ln\left(-m_2^2/\mu^2\right)}{\frac{c_2(\mu)}{2}+2c_3(\mu)-(\beta/2+2\gamma)\ln\left(-m_2^2/\mu^2\right)+(\beta/2+2\gamma)}\times \\ &\exp\left(-r\sqrt{\frac{1}{2}(\Omega_s^2+\zeta^2)^{1/2}-\frac{1}{2}\Omega_s}\right)\exp\left(-ir\,\mathrm{sgn}(\zeta)\sqrt{\frac{1}{2}(\Omega_s^2+\zeta^2)^{1/2}+\frac{1}{2}\Omega_s}\right)\times \\ &Q_{ij}(t,0;0) \ , \end{aligned} \tag{7.35}$$

where $\Omega_s := (2\omega_s)^2 - \Re m_2^2$. We immediately observe a problem with the above result, namely that the solution does not represent a propagating wave although $\Omega > 0$. Looking back at the local theory, we immediately realize that the reason for this is that the placement of the poles is not controlled by the sign of ω because $\Im m_2^2$ is non-zero. Moreover, the limit to the local theory ($\zeta \to 0$) does not exist given the structure of Eq. (7.35).

In order to remedy this situation, we devise a new prescription for the poles in lieu of Eq. (7.34). We first observe that the solutions to Eq. (7.32) come in conjugate pairs which appear on the mirror-symmetric Riemann sheets of the logarithm [79]. Since one is free to pick a Riemann sheet on which to carry the contour integral, we demand the choice of the sheet to follow from the sign of the frequency. More precisely, let us say we picked a particular sheet and carried the integral for $\omega = 2\omega_s$, then the integral with $\omega = -2\omega_s$ is to be evaluated on the mirror-symmetric sheet. We can summarize this prescription by staying on a single sheet but modifying equation Eq. (7.34) to read

$$k_\pm = \begin{cases} \pm\sqrt{\frac{1}{2}(\Omega^2+\zeta^2)^{1/2}+\frac{1}{2}\Omega} \mp i\,\mathrm{sgn}(\omega)\sqrt{\frac{1}{2}(\Omega^2+\zeta^2)^{1/2}-\frac{1}{2}\Omega} \;, & \Omega > 0 \\ \pm\sqrt{\frac{1}{2}(\Omega^2+\zeta^2)^{1/2}-\frac{1}{2}|\Omega|} \mp i\,\mathrm{sgn}(\omega)\sqrt{\frac{1}{2}(\Omega^2+\zeta^2)^{1/2}+\frac{1}{2}|\Omega|} \;, & \Omega < 0 \end{cases} \tag{7.36}$$

This prescription elegantly yields the desired behavior we are after. Let us also take the limit that the Wilson coefficients are large compared to (β,γ)[5], hence we arrive at the radiation field

$$\bar{h}_{ij}^{(2)\mathrm{m}}(t,\vec{x}) = -4G\frac{\mu(d\omega_s)^2}{r}\exp\left(-r\sqrt{\frac{1}{2}(\Omega_s^2+\zeta^2)^{1/2}-\frac{1}{2}\Omega_s}\right)Q_{ij}\left(t,r;m_{\text{eff}}^2\right) \quad , \tag{7.37}$$

where the effective mass of the wave is

$$m_{\text{eff}}^2 := (2\omega_s)^2 - \frac{1}{2}(\Omega_s^2+\zeta^2)^{1/2} - \frac{1}{2}\Omega_s \quad . \tag{7.38}$$

Eqs. (7.37) and (7.38) furnish the main results of our analysis in this section. Although we obtained Eq. (7.37) for $\Omega_s > 0$, the corresponding result for $\Omega_s < 0$ could readily be obtained using Eq. (7.36). Thanks to our new prescription in Eq. (7.36), the limit to the local theory ($\Im m_2^2 \to 0$) exists and is manifest in our final result. As expected, Eq. (7.37) represents a massive spherical wave albeit the amplitude is Yukawa suppressed due to the unavoidable imaginary part of the poles. Most importantly, the effective mass in Eq. (7.38) determines the speed of propagation of the wave. Finally, it is important to note that we did not place any restrictions

[5]This limit gets rid of the prefactor appearing on the first line of Eq. (7.35). Therefore, strictly speaking Eq. (7.37) is correct up to corrections $\mathcal{O}\left((\beta+4\gamma)/(c_2+4c_3)\right)$.

regarding the signs and values of $\Re m_2^2$ and $\Im m_2^2$. From a phenomenological standpoint, it is crucial that the wave is sub-luminal, i.e. a positive-definite $m_{\rm eff}^2$, which requires

$$0 < \Re m_2^2 \leq (2\omega_s)^2, \qquad \frac{\sqrt{\frac{1}{2}(\Omega_s^2+\zeta^2)^{1/2}+\frac{1}{2}\Omega_s}}{2\omega_s} \leq 1 \ . \tag{7.39}$$

The calculation of the emitted power is complicated by the fact that the mass of the massive spin-2 field is now complex due to the non-local part of the action. A complex mass implies that this field has a width [129] and a width cannot be implemented in a simple way in the Lagrangian. The calculation of the energy-momentum tensor $T_{\mu\nu}$ required to calculate the emitted power of a binary system into that mode is thus more complicated than in the local theory case. A standard way to introduce a width in a Lagrangian consists in including the interactions between the particle under consideration and its decaying product. It is clear that in the case, it will be an high order effect since we are working at second order in curvature and we can thus ignore the imaginary part of the mass. We thus recover the energy loss calculated in the previous section

$$\frac{dE_{\rm GW}}{dt} = \frac{32G\mu^2 d^4\omega_s^6}{5}\left(1+\theta(2\omega_s - \Re m_2)\right) + \mathcal{O}\left(\frac{\Re m_2}{\omega_s}\right) \ . \tag{7.40}$$

where as before the first term is the power lost in the massless gravitational mode while the second term represents the power lost in the massive spin-2 mode [73]. This result was derived in [73].

7.5 Quantum non-locality: perturbative treatment

While in the previous sections we studied effects at order G, i.e., the effects of the same strength as that of the standard general relativity gravitational wave solution, we now turn our attention to genuine quantum gravitational corrections to the general relativity wave solution which appear at order G^2. These corrections are the analogue of the long-distance corrections to the Newtonian potential, i.e. Eq. (7.2), that have been derived in [82, 90]. To this aim, we look for a solution to Eq. (7.23) perturbatively close to general relativity

$$\bar{h}_{\mu\nu} = \bar{h}_{\mu\nu}^{\rm GR} + \mathfrak{h}_{\mu\nu} \tag{7.41}$$

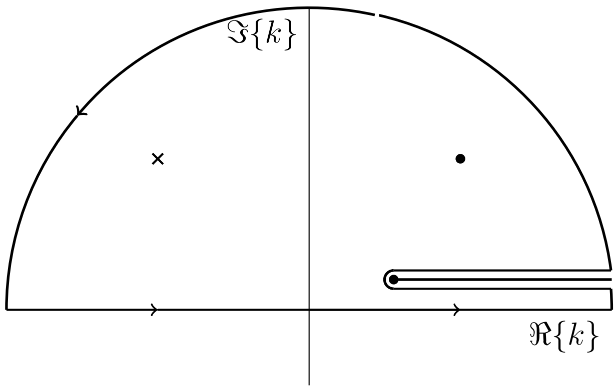

Figure 7.1: This figure shows our choice of integration contour in the complex k-plane, which is relevant for the integral involving the positive exponential factor in Eq. (7.31). The horizontal line denotes the branch-cut in the upper-half-plane. The cross (dot) denotes the relevant pole if $\mathrm{sgn}(\zeta)$ is positive (negative).

where $\mathfrak{h}_{\mu\nu}$ comprises a *long-distance* correction to general relativity. Plugging this ansatz back in the equations of motion yields

$$\Box\mathfrak{h}_{ij} - \kappa^2\left(\frac{c_2(\mu)}{2} - 2c_3(\mu)\right)\Box^2\bar{h}_{ij}^{\mathrm{GR}} + \kappa^2\left(\frac{\beta}{2} + 2\gamma\right)\Box^2\mathfrak{L}(\bar{h}_{ij}^{\mathrm{GR}}) = 0 \quad , \tag{7.42}$$

where we have used the leading-order equation $\Box\bar{h}_{\mu\nu}^{\mathrm{GR}} = -16\pi G T_{\mu\nu}$. In our current approach the local pieces drop out, i.e. the middle term in Eq. (7.42), because away from the source we have that $\Box\bar{h}_{\mu\nu}^{\mathrm{GR}} = 0$. For the general relativity solution, we use the quadrupole formula

$$\bar{h}_{ij}^{\mathrm{GR}} = 4G\frac{\mu(d\omega_s)^2}{r}Q_{ij}(t,r;0) \quad . \tag{7.43}$$

We can simplify Eq. (7.42) if we commute one factor of the d'Alembertian past the logarithm in Eq. (7.42). The homogenous solution of $\mathfrak{h}_{\mu\nu}$ is set to zero and so we end up with

$$\mathfrak{h}_{ij} = \frac{\kappa^4}{4}\left(\beta + 4\gamma\right)\mu(d\omega_s)^2\,\mathfrak{L}(\delta^{(3)}(\vec{x})Q_{ij}(t,r;0)) \quad . \tag{7.44}$$

At this point, the exact expression of $\mathfrak{L}(x - x')$ derived in Eq. (C.7) is employed. The integral is quite involved, but we find it instructive to show some details that help illuminate the properties of the non-local distribution. Let us focus on a single component of the correction, say $\mathfrak{h}_{xx}$. The delta function allows us to integrate freely over spatial coordinates

$$\mathfrak{h}_{xx} = \frac{\kappa^4 (\beta+4\gamma)\,\mu(d\omega_s)^2}{4} \times \lim_{\delta\to 0}\int dt' \left[\frac{i}{\pi^2}\left(\frac{\Theta(t-t')\Theta((t-t')^2-r^2)}{((t-t')^2-r^2+i\delta)^2} - \frac{\Theta(t-t')\Theta((t-t')^2-r^2)}{((t-t')^2-r^2-i\delta)^2}\right)\right]\cos(2\omega_s t'). \tag{7.45}$$

Now the remaining integral is readily performed in the complex plane. Writing the cosine function in terms of complex exponentials, we close the contour appropriately. The step function $\Theta(t-t')$ picks up the causal pole and one ends up with manifestly real solutions

$$\mathfrak{h}_{xx} = -\mathfrak{h}_{yy} = \frac{\kappa^4 (\beta+4\gamma)\,\mu(d\omega_s)^2}{8\pi r^2}\left(2\omega_s\sin(2\omega_s t_r) - \frac{1}{r}\cos(2\omega_s t_r)\right), \tag{7.46}$$

$$\mathfrak{h}_{xy} = \mathfrak{h}_{yx} = -\frac{\kappa^4 (\beta+4\gamma)\,\mu(d\omega_s)^2}{8\pi r^2}\left(\frac{1}{r}\sin(2\omega_s t_r) + 2\omega_s\cos(2\omega_s t_r)\right), \tag{7.47}$$

where $t_r := t - r$ is the retarded time. As we advertised, the above result represents a traveling massless wave, but with the far-field falling faster than the typical $1/r$ behavior of general relativity. A final comment is in place: the corrections in Eq. (7.46) do not affect the radiated power since the field falls off faster than $1/r$. Since we are working perturbatively in G, the rate of energy loss is to be computed using the same expression in general relativity. Clearly as the power is obtained by averaging the energy flux over a sphere situated at infinity, any component in the wave solution that decays faster than $1/r$ does not contribute to the emitted power. This is not surprising, as here, the only degree of freedom involved that can carry energy is the massless spin-2 mode of general relativity. While the emitted power into massless gravitational waves is not corrected by quantum gravity at order G^2, the strain which is given by

$$h(t) = D^{\mu\nu}\bar{h}_{\mu\nu} = D^{\mu\nu}\bar{h}^{\mathrm{GR}}_{\mu\nu} + D^{\mu\nu}\mathfrak{h}_{\mu\nu}, \tag{7.48}$$

where $D^{\mu\nu}$ is the detector tensor, receives a quantum gravitational correction at this order.

7.6 Conclusions

In this paper we worked within the effective theory approach to quantum gravity which enables model independent calculations at energies below the Planck mass.

The long-distance limit of quantum gravity is well described by the effective field theory framework. The advances in infrared quantum gravity opens the door to investigate a wide variety of gravitational observables. Using these now well established techniques, we reconsidered the question of quantum gravitational corrections to the emission of gravitational waves by a astrophysical binary system.

In this work we focused on the gravitational waveform emitted by a binary system during the inspiral phase. For completeness, we first revisited the production of massive spin-2 modes predicted by quantum gravity. We have then calculated the leading order quantum gravitational correction to the classical quadrupole radiation formula which appears at second order in Newton's constant. This is a genuine quantum gravitational prediction which is model independent. Clearly this is a small effect which is unlikely to be relevant for any foreseeable gravitational wave experiment. However, this result is important as it demonstrates that quantum gravitational calculations are possible when using well established effective field theoretical techniques. This prediction of quantum gravity is model independent. As expected, the emitted power into massless gravitational waves is not corrected by quantum gravity at order G^2. However, we have found that the strain receives a quantum gravitational correction at order G^2.

Chapter 8

Gravitational Radiation Background from Boson Star Binaries

Every time you accelerate - say by jumping up and down - you're generating gravitational waves.

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We calculate the gravitational radiation background generated from boson star binaries formed in locally dense clusters with formation rate tracked by the regular star formation rate. We compute how the the frequency window in gravitational waves is affected by the boson field mass and repulsive self-coupling, anticipating constraints from EPTA and LISA. We also comment on the possible detectability of these binaries.

8.1 Introduction

The recent detection of gravitational waves (GW) by LIGO and VIRGO have opened up a new window for our understanding of the physical properties of the universe [3]. Probing the energy density of the stochastic Gravitational Wave Background (GRB) formed by the superposition of a large number of individual gravitational wave merger events is a long term goal of the next generation of GW detectors. It is thus of great interest to investigate different potential sources of GRBs and how to distinguish between their potential observational signatures. In this letter, we compute the GRB of an important class of hypothetical objects, merging binaries of Exotic Compact Objects (ECOs) composed of self-interacting scalar field configurations known as boson stars (BSs). Such objects were first proposed in the late 1960s [92] and further studied in the 1980s and 1990s [21, 93–95], but are now experiencing a revival due to their potential role as dark matter candidates [24] and as remnants of early universe physics [96]. The gravitational wave production from individual events of the merger of two boson stars has been studied in [97] and [22], for example. A preliminary estimate of the GRB in boson-star binary mergers was given in [98].

The success of inflationary cosmology [25] and the discovery of the Higgs Boson [1] [2] have opened up the possibility that different self-interacting scalar fields might exist in nature. The presence of such fundamental scalar fields in the early universe, maybe in dark matter clusters, could have led to their condensation into self-gravitating compact objects [99–101]. It is quite remarkable that for a repulsive self-interaction $\lambda|\phi|^4$ and a scalar field mass m, such objects have masses $M_{\rm BS} \sim \sqrt{\lambda} M_{\rm Pl}^3/m^2$, which, for $m/\lambda^{1/4} \sim m_p$, where m_p is the proton mass, are parametrically equivalent to the Chandrasekhar mass [102].

Indeed, even a free, massive scalar field can generate a self-gravitating object, supported against gravitational collapse solely by quantum uncertainty [92]. This distinguishes them from fermionic compact objects such as neutron stars (NS) and white dwarfs, which are prevented from collapse due to degeneracy pressure [103]. Another key difference, important observationally to distinguish the two classes of compact objects, is that the simplest BSs do not radiate electromagnetically.

Given the uncertainty in the details of BS formation, and to provide a more general analysis, we assume here that BSs were formed at a rate that tracks the regular star formation rate, in locally-dense dark matter clusters. We will thus adopt this initial range of redshifts as a benchmark for our analysis. Our results can be extended to arbitrarily large redshifts.

As with their fermionic counterparts, BSs have a critical maximum mass against central density beyond which they are unstable to gravitational collapse into black holes (BHs) [93, 106]. In this paper, we treat the two stars in the binary BS system as having the same maximum mass and radius, which leads to the two objects having the same compactness, defined as $C = G_N M/R$. The GRB is typically characterized by the dimensionless quantity $\Omega_{\rm GW}(f)$, the contribution in gravitational radiation in units of the critical density in a frequency window f and $f + \delta f$ to the total energy-density of the universe in a Hubble time. By studying their gravitational imprints, we hope to gain insight on the properties of these exotic objects, expanding the results of [98] and bringing them closer to current and planned observations.

8.2 Boson Star properties

8.2.1 Isolated Boson Stars

Very light bosons could form a Bose-Einstein condensate (BEC) in the early or late universe through various mechanisms [99–101]. Such objects are macroscopic quantum states that are prevented from collapsing gravitationally by the Heisenberg uncertainty principle in the non-interacting [92] and attractive self-interaction case [99], or, in another possibility, through a repulsive self-interaction that could balance gravity's attraction [102]. In this Letter, we study an Einstein-Klein-Gordon system with the following Lagrangian,

$$\mathcal{L} = \sqrt{-g}\left[|(\partial\phi)|^2 - m^2|\phi|^2 - \frac{1}{2}\lambda|\phi|^4\right], \tag{8.1}$$

where ϕ is a complex scalar field carrying a global $U(1)$. Real scalar fields can also form gravitationally-bound states, but these are time-dependent and have different properties [107]. Colpi *et al* showed that the maximum mass of a spherically-symmetric BS with repulsive self-interaction is given by [102]

$$M_*^{max} \sim \frac{0.22\, M_p^2\, \alpha^{1/2}}{m} \approx \frac{0.06\sqrt{\lambda}M_p^3}{m^2}, \tag{8.2}$$

where the rescaled coupling α is defined as $\alpha \equiv \lambda\, M_p^2/(4\pi\, m^2)$. For a boson star with a repulsive self-interaction, the radius can be estimated to be

$$R_* \sim \frac{\sqrt{\lambda}}{\sqrt{G_N} m^2}. \tag{8.3}$$

The compactness of boson stars is discussed in many references such as [24, 108]. We note that the compactness and mass of the stars are especially relevant for binary GW events. Different formation mechanisms have been discussed in Refs. [99–101]. However, since we are focussing here on the gravitational radiation background, we need not worry about specific formation mechanisms that lead to highly compact BSs. We will assume they exist and compute their contribution to the GRB. We also note that if one assumes the complex scalar ϕ to be responsible for the dark matter in the Bullet Cluster, Ref. [109] shows that the constraint on the dark matter cross section [110–112] can be translated into a bound on the boson's self-coupling, because the relative velocity of the Bullet Cluster is higher than the sound speed of the condensate. The translated bound on the self-interaction strength is

$$\lambda \lesssim 10^{-11} \left(\frac{m}{\mathrm{eV}}\right)^{3/2}. \tag{8.4}$$

A modest lower bound on the self-interaction can be found from the gravitational wave speed as it propagates through the DM halo, as given by [113].

We note in passing that Ref. [109] shows that BEC requires light scalars $m < 1$eV. However, the bound is based on the inter-particle spacing estimated from the average density of dark matter in the Universe. Since in the absence of a fundamental theory the exact formation process of boson stars remain unclear, we consider the possibility of their formation due to a large local density fluctuation. Therefore, we do not worry about the bound on the scalar mass. In what follows, we saturate the Bullet Cluster bound and parametrize the boson star mass effectively as

$$M_* = x M_*^{max} = 3.1 \times 10^{11}\, x \left(\frac{\mathrm{eV}}{m}\right)^{5/4} M_\odot, \tag{8.5}$$

where x is the fraction between boson star mass and the maximum stable mass, and the radius will be given by,

$$R_* = y \frac{\sqrt{\lambda}}{\sqrt{G_N} m^2} = 1.1 \times 10^{7}\, y \left(\frac{\mathrm{eV}}{m}\right)^{5/4} R_\odot, \tag{8.6}$$

where y is the fraction or multiple of the star radius from Eq. (8.3). From Eqs. 8.5 and 8.6 we obtain the compactness of these boson stars as

$$C_* = \frac{G_N M_*}{C_*} = 0.06 \times \left(\frac{x}{y}\right). \tag{8.7}$$

8.2.2 Boson Star Binaries

We briefly describe the properties of boson star binaries that are relevant for the calculation of gravitational radiation. In what follows, we assume a conservative model for the estimation of the binary formation rate, which tracks the star formation rate (SFR) of luminous stars. Empirically, the luminous star-formation rate can be parametrized as a function of redshift z and stellar mass M [114], in units of $\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ as

$$\mathrm{SFR}(z, M) = \mathrm{SFR}_0 \left(\frac{M_\odot}{M}\right) \frac{a\, e^{b(z-z_m)}}{a - b + b\, e^{a(z-z_m)}} \,. \tag{8.8}$$

The parameters SFR_0, z_m, a, and b are all determined by fitting to observations such as gamma-ray burst rates and the galaxy luminosity function. We adopt the fit from gamma-ray bursts from [115]. We further parameterize the efficiency of the binary boson star formation as a fraction of $\mathrm{SFR}(z, M)$, denoted as $f_{\mathrm{BBS}} \leq 1$. We stress that this effective parametrization does not assume a specific boson star formation mechanism nor a similarity between that and luminous star formation. The boson star binary formation rate is, for a boson star of mass M_* and formation redshift z_f,

$$R_{\mathrm{BBS}}(z_f, M_*) = f_{\mathrm{BBS}} \times \mathrm{SFR}(z_f, M_*). \tag{8.9}$$

Since we do not need all of the binaries to survive today to leave their gravitational radiation imprint, we calculate the merger rate at redshift z, which is mainly determined by the binary formation rate at redshift z_f. On the other hand, the larger the binary separation at formation, the less likely they would have successfully merged, due to gravitational perturbations from other sources. Following Ref. [20], we use an appropriately normalized weight function $p(\Delta t)$ to account for the merger efficiency, where Δt is the time delay from formation of the binary to coalescence,

$$R_m(t, M_*, f_{\mathrm{BBS}}) = \int_{\Delta t_{min}}^{\Delta t_{max}} R_{\mathrm{BBS}}(t - \Delta t, M_*)\, p(\Delta t)\, d\Delta t. \tag{8.10}$$

Here, Δt_{min} is the minimum time between formation and coalescence, and Δt_{max} is determined by the maximum initial separation which allows for binary formation. As we will see below, the result is not sensitive to the precise choice of Δt_{max}. We will comment on a suitable Δt_{min} for this integral in the following section. We relate redshift to cosmic time with the approximate formula from Ref. [116],

$$t(z) = \frac{2/H_0}{1 + (z+1)^2}, \tag{8.11}$$

where H_0 is the Hubble constant today. Next, let us estimate $p(\Delta t)$. For a pair of stars A and B, their initial separation a defines a sphere inside which the number of stars is $N(a) = \rho\pi a^3/6$. Assuming that the chance of any pair of stars forming a binary is roughly the same inside the sphere, the probability that stars A and B are bounded is

$$p(a) = \begin{pmatrix} N(a) \\ \\ 2 \end{pmatrix}^{-1} = \frac{2}{N(a)(N(a)-1)} \propto a^{-6}. \tag{8.12}$$

This simple model captures the sharp decrease in the binary population as the pair separation increases. We note that the difficulty for binaries with initial large separation to form is not from perturbations that rip the two stars apart. Instead, the many 'inbetweeners' are likely to form binaries with each of the two stars separately. Since gravitational radiation is the only channel for energy release, and since most of the initial binding and inspiraling process can be described by Newtonian dynamics, we use the merging time as in Ref. [117],

$$\Delta t \sim a^4. \tag{8.13}$$

This gives a weight function $p(\Delta t) \sim 1/\Delta t^{3/2}$. [1] This weight function also implies that the result is not sensitive to Δt_{max} and the precise determination of the initial separation. The boson star formation rate and merger rate are shown in Fig. 8.1. As one can see, the merger rate is not very sensitive to Δt_{min}. The magnitude of the merger rate is controlled by f_{BBS}, which will be constrained together with their mass and radius.

[1] Note that this differs from Ref. [20, 118], where a *fiducial* model is used and the weight function for NSs is chosen to be $p(\Delta t) \sim 1/\Delta t$. For a study of different delay models, please refer to Refs. [119–122].

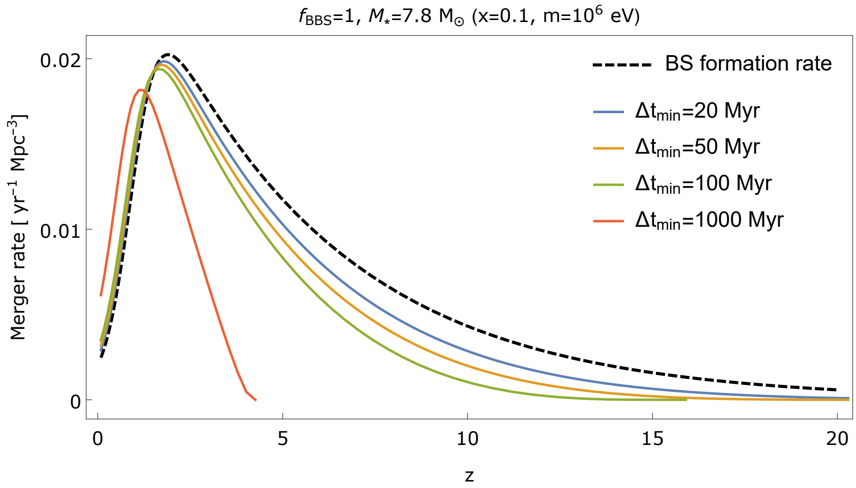

Figure 8.1: (Colored plot online.) We take the shape of the regular star formation rate (dashed) from Ref. [115] using the gamma ray burst fit therein ($\nu = 0.16, z_m = 1.9, a = 2.76, b = 2.56$), and assume that the boson star formation tracks the regular star formation with efficiency $f_{\rm BBS}$. We compare it with the merger rate (solid) calculated using Eq. (8.10). It is observed that Δt_{min}, the minimum delay between formation and merging, has a small effect on the result as long as the delay is comparable to NS mergers (20 Myr) [20] and BH mergers (50 Myr) [118]. The benchmarks in Fig. 8.2 correspond to Δt_{min} ranging from 10^{-12} Myr to 26 Myr.

8.3 Gravitational Waves from Boson Stars

8.3.1 Gravitational Waves from Single Binaries

In this section we consider the gravitational wave emission from a binary merger of two boson stars, focussing only on the gravitational interaction between the stars. The most important contribution to the stochastic background comes from the inspiral phase of the binary mergers. In this stage, the calculation can be done analytically. The system can be approximated by a pair of purely self-gravitating point masses emitting mostly gravitational quadrupole radiation. The radiation power is

$$P = \frac{32}{5} G_N \mu^2 \omega^6 r^4. \tag{8.14}$$

Solving the dissipation equation $P = -\dot{E}$ gives us the characteristic $f(t) \sim t^{-3/8}$ relation, and the radius as a function of t, with t being the time before coalescence,

$$f(t) = \frac{5^{3/8}}{8\pi}(G_N m_c)^{-5/8} t^{-3/8},$$
$$r(t) = \left(\frac{256}{5} G_N^3 (M_A + M_B) M_A M_B\right)^{1/4} t^{1/4}, \tag{8.15}$$

where m_c is the chirp mass given by $m_c = \frac{(M_A M_B)^{3/5}}{(M_A+M_B)^{1/5}}$, with M_A, M_B being the masses of the two stars. This approximation holds until the binary evolves beyond its innermost stable circular orbit (ISCO). Inside the ISCO, tidal effects need to be taken into account, and the post-Newtonian expansion breaks down. The frequency of the ISCO is given by [24]

$$f_{\rm ISCO} = \frac{C_*^{3/2}}{3^{3/2}\,\pi\, G_N(M_1 + M_2)}, \tag{8.16}$$

which is a function of the compactness of the stars defined in Eq. 8.7. For boson stars with a fraction x of the maximum mass (8.2), and a fraction or multiple y of the radius (8.3),

$$f_{\rm ISCO} \approx \frac{m^2\sqrt{G_N}}{6\sqrt{6}\pi^{5/4}\sqrt{\lambda}}\sqrt{\frac{x}{y^3}} \approx 2.02 \times 10^{-15}\ \text{Hz}\sqrt{\frac{x}{y^3}}\sqrt{\frac{1}{\lambda}}\left(\frac{m}{\text{eV}}\right)^2. \tag{8.17}$$

If we saturate the Bullet Cluster bound as in Eq. 8.4, $f_{\rm ISCO}$ scales as $\sim m^{5/4}$.

$$f_{\rm ISCO} \approx 6.4 \times 10^{-10}\text{Hz}\left(\sqrt{\frac{x}{y^3}}\right)\left(\frac{m}{\text{eV}}\right)^{5/4}. \tag{8.18}$$

Of course, if we choose not to saturate the Bullet Cluster bound we have a much broader window of models to probe, something that can be easily done.

We will estimate Δt_{min} in (8.10) based on the following argument: if the boson star binary is formed at an initial distance inside the ISCO, the binary will not experience an inspiral phase. Therefore we choose Δt_{min} to correspond to t_{ISCO}, the time between entering the ISCO and coalescence. In what follows, we sum up the contributions from individual mergers to get the total gravitational radiation energy density. When we do the summation, we use f_{ISCO} as the cut off frequency for each binary to guarantee the calculation based on quadrupole radiation is valid.

8.3.2 Gravitational Radiation Energy Density

The energy spectrum of the gravitational radiation from boson stars is defined as,

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}, \tag{8.19}$$

where ρ_{GW} is the energy density of the gravitational wave in that frequency range and ρ_c is the critical energy density. Following [118], this can be written using the merger rate per unit of comoving volume per source time $R_m(z, M_*)$, and the differential energy emitted by a single source dE/df_s as,

$$\begin{aligned}
\Omega_{\rm GW}(f, M_*, f_{BBS}) &= \frac{f}{\rho_c H_0}\int_0^{z_{max}} \frac{R_m(z, M_*, f_{\rm BBS})}{(1+z)\sqrt{\Omega_M(1+z)^3+\Omega_\Lambda}}\frac{dE}{df_s}\, dz \\
&= f^{2/3} f_{\rm BBS}\left(\frac{M_*}{M_\odot}\right)^{2/3}\left(\frac{\pi^{2/3}G_N^{2/3}M_\odot^{5/3}}{2^{1/3}3\rho_c H_0}\right) \\
&\int_0^{z_{max}} \frac{R_m(z, M_\odot, 1)}{(1+z)^{4/3}\sqrt{\Omega_M(1+z)^3+\Omega_\Lambda}}\, dz \\
&= 1.03\times 10^{-6}\, f_{\rm BBS}\, x^{2/3}\left(\frac{f}{10^{-4}\ \rm Hz}\right)^{2/3}\left(\frac{\rm MeV}{m}\right)^{5/6},
\end{aligned} \tag{8.20}$$

where we have used $f_s = (1+z)f$ (explicitly shown in B) for the emitted (source) frequency, and

$$\frac{dE}{df_s} = \frac{\pi^{2/3}}{3}G_N^{2/3}m_c^{5/3}f_s^{-1/3}. \tag{8.21}$$

$f_{\rm ISCO}$ works as a cut-off at the high end of the spectrum, which is shown in Eq. (8.18). The spectrum is shown in Fig.8.2 for several benchmark scenarios. In this plot, the fraction of the radius (8.3) is taken as $y = 1$. It is seen that the signal may be within reach of the next generation of gravitational wave interferometer experiments, and pulsar timing arrays. Also, we observe that the high end of the frequency band, determined by $f_{\rm ISCO}$, is proportional to $m^{5/4}$, if we saturate the Bullet Cluster bound, which indicates that boson stars consisting of heavy scalars are more likely to be probed by gravitational wave experiments. We show in Fig. 8.3 the bound on binary formation efficiency $f_{\rm BBS}$, star mass, and star radius based on LISA for a few benchmarks of scalar mass.

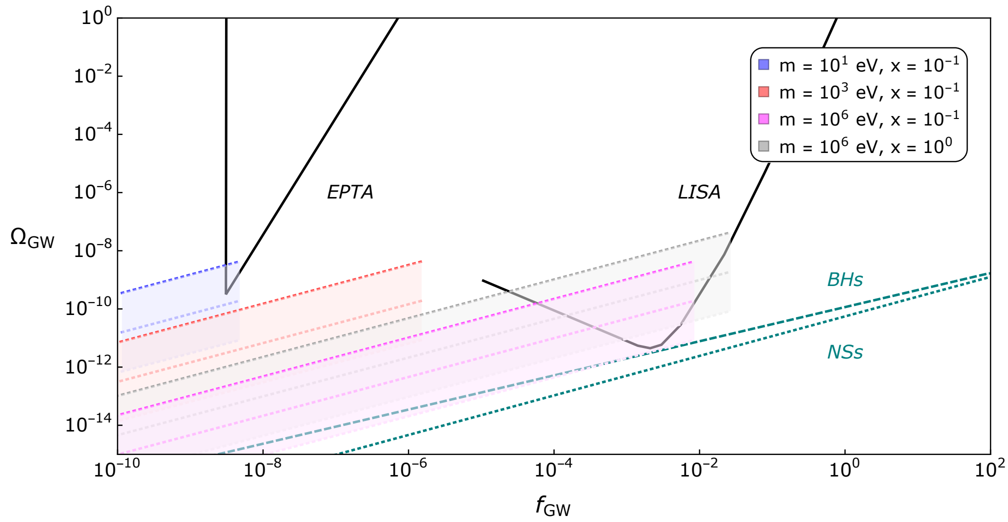

Figure 8.2: (Colored plot online.) Plot of (8.20). Here the fraction of the maximum boson star mass (8.2) is taken conservatively to be $x = 10^{-1}$, and the fraction or multiple of the radius (8.3) is taken as $y = 1$. The self-coupling λ has been chosen to saturate the Bullet Cluster constraint (8.4). From (8.5), for $m_b \sim$ keV (MeV), and $x = 0.1$, the mass of the star is $M_* \sim 10^6\, M_\odot$ $(10^3 M_\odot)$. The upper, lower, and middle lines are chosen for $f_{\rm BBS} = 1/2$, $f_{\rm BBS} = 10^{-3}$, and their geometric mean, respectively. Also shown are the EPTA [123] and the LISA [124] exclusion prospects, and the expected backgrounds due to Binary BHs and NSs [20].

8.4 Discussion

As shown in Fig. 8.2, the gravitational signal from binaries of stars made of light bosons fall within the reach of the next generation of gravitational wave detectors and pulsar timing arrays. Failure to detect such spectra can be interpreted as a bound on the boson star parameters, as illustrated in Fig. (8.3). Such a bound can in turn be translated to bounds on the boson mass and self-coupling, once a specific formation scenario is assumed.

The most important contribution to the boson star binary spectrum comes from the inspiral phase, which peaks at $f_{\rm ISCO}$, the frequency corresponding to the innermost stable orbit. This peak frequency (8.16) is a function of the compactness of the boson stars, which depends on the scalar mass and self-coupling. This is to be compared with objects of which the compactness is known [24]. The compactness of a BH is 1/2, whereas realistic assumptions on the EOS for NSs would put them

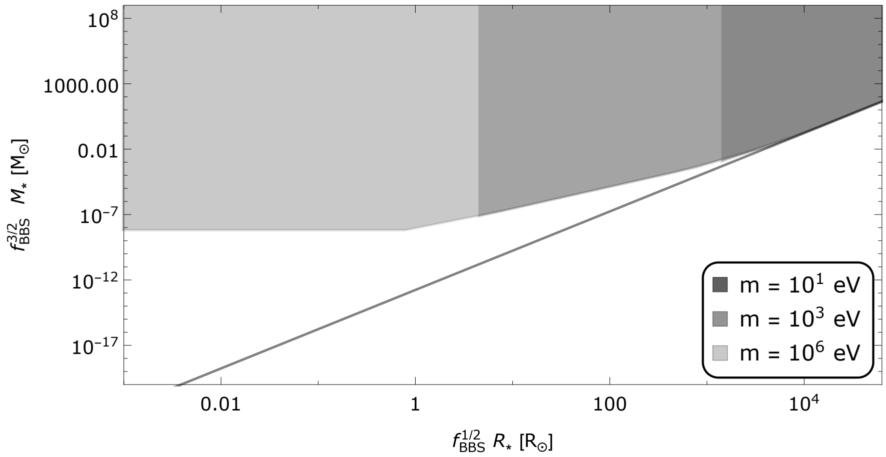

Figure 8.3: The bound on boson star parameters based on LISA. The gray region can be constrained by LISA. We take three benchmarks, with $m = 10$ eV (the darkest region), $m = 10^3$ eV (the both darker and the darkest region), $m = 10^6$ eV (all colored area). The straight line is derived by setting $f_{ISCO} = 10^{-5}$ Hz, which is the lower end of LISA's sensitivity band. In this plot, LISA is not sensitive to the region below the straight line.

in the range $0.13 \lesssim C \lesssim 0.23$. For the boson stars considered here, the compactness saturates at $C \leq 0.16$, so close to the lower range of NSs and below that of BHs. We also note that BS mergers are not accompanied by electromagnetic signatures.

In this paper we have only considered the gravitational interaction between boson stars in a binary. Numerical studies of mini-boson stars [97] and solitonic boson stars [22] suggest that the scalar contact interactions may play an important role in the late evolution of the merger. We leave the study of such effects in our model (8.1) for future work.

It is important to distinguish the stochastic background from boson stars from that due to more conventional binaries, such as BHs and NSs. Such a comparison relies on three main features. The stochastic spectrum is characterized by the fractional energy density $\Omega_{\rm GW}(f)$ and the frequency band f. As is shown in equation (8.20), $\Omega_{\rm GW}(f)$ can be written as a function of the formation rate (parametrized by $f_{\rm BBS}$) and the mass of the boson stars (as a function of x and m). A fundamental difference is that boson star masses (8.2) can take on a wide range of values, from that of NSs to that of supermassive BHs. Boson stars with a mass that falls outside the range typical for NSs and BHs are particularly interesting observationally.

This corresponds to relatively heavy bosons, with $m \sim 10^5\sqrt{x}\,\mathrm{eV}$. Also, a more exotic formation scenario than the one considered here may distinguish the boson star signal. For example, by considering redshifts different than the ones that track ordinary star formation. We leave the analysis of how these parameters impact the boson star stochastic background for future work.

Chapter 9

What can Black Holes tell us about the UV and IR?

What we observe as material bodies and forces are nothing but shapes and vibrations in the structure of space.

Erwin Schrodinger

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Combining insights from both the effective field theory of quantum gravity and black hole thermodynamics, we derive two novel consistency relations to be satisfied by any quantum theory of gravity. First, we show that a particular combination of the number of massless (light) fields in the theory must take integer values. Second, we show that, once the massless spectrum is fixed, the Wilson coefficient of the Kretschmann scalar in the low-energy effective theory is fully determined by the logarithm of a single natural number.

9.1 Introduction

The link between black holes and their intrinsic thermodynamical behavior is perhaps *the* key to a consistent theory of quantum gravity. The underlying quantum degrees of freedom of a black hole must necessarily account for its entropy [163–166]. Although currently we are far from describing black hole "micro-states", powerful insights were revealed relying on effective field theory lore. Indeed, Hawking radiation was discovered by considering the quantum dynamics of matter fields in a fixed black hole geometry [167].

Donoghue [47, 48] demonstrated that quantum gravity, at distances large compared to the Planck length, is well described by an effective field theory (EFT). It is remarkable that gravity lends itself naturally to the EFT framework. All the unknown physics coming from the ultraviolet (UV) is encoded solely in the Wilson coefficients of the most general diffeomorphism-invariant action. Similar to any UV-sensitive quantity in quantum field theory, the Wilson coefficients can only be determined empirically. More importantly, long-distance quantum effects furnish a set of reliable and parameter-free predictions of the EFT, as they emerge from the low-energy portion of loops containing massless (light) degrees of freedom.

Recently, some work has been done to adapt and utilize the EFT framework to study quantum aspects of black hole thermodynamics in the context of Euclidean quantum gravity [141, 142]. In particular, it was shown that the long-distance contribution to the partition function is captured by covariant non-local operators. This is an interesting development which, in particular, allows us to quantify a set of quantum corrections to the various thermodynamic relations governing black holes. Notably, it was shown in [141] that the non-local operators are responsible for generating the logarithmic correction to the Bekenstein-Hawking entropy of Schwarzschild black hole.

The advent of gravitational wave astronomy [3] has revived experimental efforts in testing black hole thermodynamics [62, 168]. In this letter we aim to utilize the structure of the logarithmic contribution to the entropy, obtained from the effective theory, to derive two consistency relations that hold for any theory (model) of quantum gravity. This is achieved by invoking Bekenstein's conjecture [169] that the area of Schwarzschild black hole has a discrete spectrum in any quantum theory of gravity. In essence, the universality of the EFT and black hole thermodynamics are the two main motivations behind such relations.

The first relation sets a constraint on the number of massless (light) fields coupled to gravity. This is indeed remarkable because to an effective field theorist, gravity can generally couple to any number of massless fields. The second relation constrains the Wilson coefficient of the Kretschmann scalar, measured at an arbitrary scale, to be determined in terms of a single natural number. This is striking given that experimental bounds are exceedingly weak [139] on the Wilson coefficients of quadratic gravity. We will now briefly review the basic ingredients necessary for the derivation of the consistency conditions which will follow.

9.2 Black Hole Area Quantization

The proposal that the area of a black hole is quantized relies on the observation that the area behaves *classically* as an adiabatic invariant. The initial evidence for this conjecture came from the work of Christodoulou and Ruffini [53, 170]. In particular, they showed that the area of a non-extremal black hole does not change in the process of absorbing a classical point particle if the capture takes place at the turning point of the particle's orbit. The implied reversibility of this process hints towards the adiabatic invariance of the horizon area [56]. Ehrenfest's hypothesis [56, 59] states that any quantity which, classically, is an adiabatic invariant is quantized in the quantum theory. If one then accepts Bekenstein's conjecture, it follows that the area is quantized in any consistent theory of quantum gravity.

Now the natural question is, what does this quantized area spectrum look like? Ascribing quantum mechanical uncertainty to the captured particle, it is straightforward to show that there exists a minimal increase in the horizon area [55, 56]. This motivates the following spectrum [56, 152],

$$\mathcal{A}_n = \gamma_0 l_p^2 n, \quad n = 1, 2, 3... , \tag{9.1}$$

where γ_0 is some number that will be discussed later. For some exhaustive reviews on this topic, the reader is directed towards [56, 57]. The discrete nature of the area spectrum has also been found in quantum gravity approaches such as string theory [63], and loop quantum gravity [171, 172]. Nevertheless, there is no consensus regarding the uniformity of the spacing. In Loop Quantum Gravity [143], in particular, the area shows a highly non-uniform spectrum. In this paper, remaining agnostic to the spacing, we use a generalized quantization rule for the area as our starting point [144]. This will allow us not to dwell on any particular model of quantum gravity.

9.3 EFT and black hole Thermodynamics

Hawking and Gibbons pioneered a consistent approach to study the thermal properties of black holes [173]. For a gravitational system at finite temperature, the partition function of the canonical ensemble reads

$$Z(\beta) = \int \mathcal{D}\Psi \mathcal{D}g \, e^{-\mathcal{S}_E - \mathcal{S}_\partial} \quad . \tag{9.2}$$

where Ψ stands for all matter fields coupled to gravity, $\mathcal{S}_E$ is the Euclidean action and $\mathcal{S}_\partial$ denotes the Hawking-Gibbons-York boundary action [174, 175]. As is customary from finite temperature field theory, the integral extends over (anti)-periodic field configurations for bosons (fermions). For gravity the prescription is to sum over positive-definite metrics with a fixed induced metric on the boundary. In the canonical ensemble, the boundary geometry is flat space on $\mathbb{S}_1 \times \mathbb{R}^3$ with the circumference of the time circle given by β.

In a semi-classical evaluation of the partition function, the Euclidean section of the black hole appears as a saddle point. In [141], it was shown how to apply the techniques of the EFT of quantum gravity to compute the partition function, Eq. (9.2). As we alluded to in the introduction, the partition function contains non-local operators which encapsulate the long distance dynamics. The result simplifies if the background geometry is a Kerr-Schild spacetime [141], which is the case for Schwarzschild black hole. At one loop, or more precisely at next-to-leading order in the EFT expansion, the partition function of Schwarzschild black hole in 4D is obtained by a Wick rotation of the effective action [142]

$$\ln Z = \Gamma_{\text{local}}[\bar{g}] + \Gamma_{\ln}[\bar{g}] - \mathcal{S}_\partial \quad , \tag{9.3}$$

where $\bar{g}$ denotes the background Kerr-Schild spacetime. First, we have

$$\Gamma_{\rm local}[\bar{g};\mu] = \int d^4x \Big[\frac{M_P^2}{2}R + c_1^r(\mu)R^2 + c_2^r(\mu)R_{\mu\nu}R^{\mu\nu} + c_3^r(\mu)R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + c_4^r(\mu)\Delta R + \mathcal{O}(R^3)\Big] \ . \tag{9.4}$$

In the above, μ is the scale of dimensional regularization, Δ is the 4D flat Laplacian on $\mathbb{R}^3 \times S^1$, and let us note that the c_i^r are renormalized Wilson coefficients. Second, the non-local portion reads

$$\Gamma_{\ln}[\bar{g}] = -\int d^4x \left[\alpha R \ln\left(\frac{-\Delta}{\mu^2}\right) R + \beta R_{\mu\nu}\ln\left(\frac{-\Delta}{\mu^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}\ln\left(\frac{-\Delta}{\mu^2}\right) R^{\mu\nu\alpha\beta} + \Theta \ln\left(\frac{-\Delta}{\mu^2}\right)\Delta R\right] \ , \tag{9.5}$$

where the coefficients are finite numbers born out of the calculation and depend on the spin of the massless field [141]. The action in Eq. (9.5) is expressed in quasi-local form, and we truncated the partition function at second order in the curvature expansion. The possible effects of the higher curvature operators on the thermodynamics were thoroughly discussed in [142]. Of most importance to our analysis is the invariance of the partition function under the renormalization group flow. Explicitly, the beta function of the Kretchman scalar coefficient is

$$\beta_{c_3} = -2\gamma \ . \tag{9.6}$$

The logarithmic *form factor* in Eq. (9.5), $\ln(-\Delta/\mu^2)$ at first glance, is a very complicated object. Indeed, the form factor is an integration kernel that must be evaluated in position space and takes the form of a distribution [142]

$$\mathcal{L}(\vec{x}-\vec{x}') = -\frac{1}{2\pi}\lim_{\epsilon\to 0}\left[\mathcal{P}\left(\frac{1}{|\vec{x}-\vec{x}'|^3}\right) + 4\pi\left(\ln(\mu\epsilon) + \gamma_E - 1\right)\delta^3(\vec{x})\right] \ , \tag{9.7}$$

where $\mathcal{P}$ stands for principal value. With the kernel in hand, a direct evaluation of Eq. (9.5) is possible [142]

$$\ln Z(\beta) = -\frac{\beta^2}{16\pi G} + \left[64\pi^2 c_3^r(\mu) + 2\,\Xi\ln(\mu\beta)\right] \ , \tag{9.8}$$

where $\beta = 8\pi GM$ and Ξ counts the number of light fields minimally coupled to gravity

$$\Xi = \frac{1}{180}\left(2N_S + \frac{7}{2}N_F - 26N_V + 424\right). \tag{9.9}$$

In the above, N_S, N_F and N_V are the number of scalars, Weyl-fermions and vectors in our theory, and the number 424 is the contribution due to pure gravity. Using the partition function one can immediately recover the logarithmic correction to Bekenstein-Hawking entropy [141, 176–187].

$$S_{\rm bh} = \frac{\mathcal{A}}{4G} + \left(64\pi^2 c_3^r(\mu) + \Xi \ln\left(\mu^2 \mathcal{A}\right)\right) \quad , \tag{9.10}$$

where $\mathcal{A} = 16\pi(GM)^2$ is the horizon area of a Schwarzschild black hole. The above structure reveals the power of the EFT, in particular, there is a clear separation between the short distance and long-distance dynamics. All the unknown ultraviolet physics is encoded in the renormalized Wilson coefficient, while the logarithm of the area emerges from the infrared structure of the theory.

Furthermore, we can manifest the invariance of Eq. (9.10) under the renormalization group by using dimensional transmutation. The constant $c_3^r(\mu)$ is traded off for a dimensionful quantity, $\mathcal{A}_{\rm QG}$, as follows

$$c_3^r(\mu) = -\frac{\Xi}{64\pi^2}\ln\left(\mu^2\mathcal{A}_{\rm QG}\right) \quad , \tag{9.11}$$

and thus Eq. (9.10) becomes

$$S_{\rm bh} = S_{\rm BH} + \Xi\,\ln\left(\frac{\mathcal{A}}{\mathcal{A}_{\rm QG}}\right) \quad . \tag{9.12}$$

where $S_{\rm BH} = \mathcal{A}/4G$. We finally note that $\mathcal{A}_{\rm QG}$ is intrinsically tied to the UV completion of quantum gravity. The constant c_3^r, and thus $\mathcal{A}_{\rm QG}$, are in principle determined either by matching onto the full theory at some scale or using experimental input.

9.4 Constraints on the UV and IR

Given area quantization, there are two quantities in Eq. (9.12), namely Ξ and $\mathcal{A}_{\rm QG}$, that we can constrain. This is remarkable since they both have different origins. On the one hand, from the low-energy standpoint, the unknown scale $\mathcal{A}_{\rm QG}$

is UV-sensitive and can only be determined empirically. On the other hand, Ξ only knows about the IR as it enumerates the spectrum of light fields coupled to gravity.

We start by considering an adequate generalization to Bekenstien-Mukhanov quantization as given in [144]

$$\mathcal{A}_n = \left(\gamma_0 n + \gamma_1 n^\delta + \gamma_2 \ln n\right) l_{\mathrm{P}}^2, \quad n = 1, 2, 3, ... \tag{9.13}$$

where $\gamma_0, \gamma_1, \gamma_2$ and $(\delta > 0)$ are constants. Statistical mechanics tells us that the total number of micro-states accessible to the system is simply the exponential of the entropy

$$g_n \equiv \exp\left(S_{\mathrm{bh}}(n)\right) \ . \tag{9.14}$$

Plugging Eq. (9.12) into Eq. (9.14) yields

$$g_n \equiv \exp\left(\frac{\mathcal{A}_n}{4 l_{\mathrm{P}}^2}\right) \exp\left(\Xi \ln \frac{\mathcal{A}_n}{\mathcal{A}_{\mathrm{QG}}}\right) \ . \tag{9.15}$$

Demanding that g_n is a natural number ($\mathbb{N}$) for all n, we see that both exponentials in the above expression must individually [1] $\in \mathbb{N}$. Expanding $\mathcal{A}_n$ as in Eq. (9.13), the first exponential imposes the following conditions on the constants

$$\gamma_0 = 4 \ln k, \quad \gamma_1 = 4 \ln k_1, \quad \gamma_2 = 4 q_1 \ , \tag{9.16}$$

and

$$k, k_1, \delta \in \mathbb{N}/\{1\}, \quad q_1 \in \mathbb{N} \ . \tag{9.17}$$

Examining the second exponential, we find the condition

$$\left(\frac{\mathcal{A}_n}{\mathcal{A}_{\mathrm{QG}}}\right)^{\Xi} \in \mathbb{N}. \tag{9.18}$$

Using Eq. (9.13) in the above, we find

$$\left(\frac{(\gamma_0 n + \gamma_1 n^\delta + \gamma_2 \ln n) l_{\mathrm{P}}^2}{\mathcal{A}_{\mathrm{QG}}}\right)^{\Xi} \in \mathbb{N}, \ \forall n. \tag{9.19}$$

---

[1] Looking at Eq. (9.15), it is true that one might take each exponential to be a natural number upto a multiplicative n-independent constant. More explicitly, $\exp\left(\mathcal{A}_n/4 l_{\mathrm{P}}^2\right) = R_1 \cdot x$ and $\exp\left(\Xi \ln \mathcal{A}_n/\mathcal{A}_{\mathrm{QG}}\right) = R_2 \cdot 1/x$, where $(R_1, R_2) \in \mathbb{N}$. Nevertheless, the only consistent value of x which satisfies these relations for every n is 1.

First, it is impossible to satisfy the above condition, for each and every n, unless $\gamma_2 = 0$. Hence, Eq. (9.19) simplifies to

$$\left(\gamma_0 n \frac{l_{\rm P}^2}{\mathcal{A}_{\rm QG}}\right)^{\Xi} \left(1 + n^{\delta-1}\frac{\gamma_1}{\gamma_0}\right)^{\Xi} \in \mathbb{N},\ \forall n. \tag{9.20}$$

Demanding that the above condition is satisfied for each and every n, we have

$$\gamma_1 = \gamma_0 \times s, \quad s \in \mathbb{Z}^+ \quad , \tag{9.21}$$

in addition to a pair of remarkable constraints. First, the exponent Ξ must be a natural number

$$\frac{1}{180}(2N_S + \frac{7}{2}N_F - 26N_V + 424) = l, \quad l \in \mathbb{N} \quad . \tag{9.22}$$

Secondly, we have a constraint on the scale $\mathcal{A}_{\rm QG}$

$$\frac{\mathcal{A}_{\rm QG}}{l_{\rm P}^2} = \frac{\gamma_0}{m^{1/l}}, \quad m \in \mathbb{N} \quad . \tag{9.23}$$

It is not hard to saturate Eq. (9.22) if we do not restrict ourselves to the standard model massless spectrum. For example, we can minimally satisfy this relation with $N_S = 8, N_F = 0$ and $N_V = 10$, which yields $l = 1$. The apparent simplicity of Eqs. (9.22-9.23) is quite striking and shows the power of the effective theory framework. We stress that the validity of these constraints indeed relies on the conjecture that the horizon area is quantized.

9.5 Dimensional Transmutation & Constraints on c_3

We can now use Eq. (9.23) to derive a *quantization rule* for the Wilson coefficient, c_3^r, which is practically our measurable quantity. Fixing the scale to $\mu_\star$ in Eq. (9.11)

$$c_3^r(\mu_*) = -\frac{\Xi}{64\pi^2}\ln\left(\mu_*^2 \mathcal{A}_{\rm QG}\right) \quad , \tag{9.24}$$

we obtain

$$c_3^r(\mu_\star) = \frac{1}{64\pi^2}\left[\ln m - l\ln\,\gamma_0 - l\ln\left(\frac{\mu_\star^2}{8\pi M_{\rm P}^2}\right)\right] \quad . \tag{9.25}$$

The above relation is quite non-trivial, because it forces the value of the Wilson coefficient at any scale to take on a very special value which must be expressible in terms of a single natural number m, once γ_0 and l are fixed. This is, in essence, a quantization rule for the coefficient of the Kretschmann scalar in the low-energy limit of any UV model of quantum gravity. Although we can not make similar statements about the other constants in the action, we conjecture that similar relations extends to the rest of the constants in Eq. (9.4).

9.6 Field Counting

Another interesting feature of Eq. (9.24) is that the size of c_3^r depends crucially on IR data, given by the number of massless (light) fields in the theory. This direct mixing between the UV and IR is unexpected from the EFT perspective; in other words, black hole thermodynamics provides a portal linking the IR to the UV. But this raises the question: What counts as a light field in the vicinity of a black hole?

To determine this, we note that there are two mass scales in our partition function: the Planck mass, M_{P} and the mass of the black hole, M_{bh}. For a field, m, to be considered light and enter Ξ, the condition [41] is

$$\frac{1}{m^2}\int d^4x\sqrt{g}R_{\mu\nu\alpha\beta}\nabla^2 R^{\mu\nu\alpha\beta} \gg 1. \tag{9.26}$$

where $\nabla^2 = g^{\mu\nu}\nabla_\mu\nabla_\nu$. For the Schwarzschild black hole this translates into

$$\frac{m}{M_{\mathrm{P}}} \ll \frac{8\sqrt{3}\pi^2 M_{\mathrm{P}}}{M_{bh}} \quad . \tag{9.27}$$

Thus, the heavier the black hole, the lighter the field has to be. A quick back-of-the-envelope calculation tells us that for a solar mass black hole, only strictly massless particles would contribute to Ξ, if we restrict ourselves to the standard model. Of course, in this case Eq. (9.22) hints at the presence of massless particles beyond the standard model.

9.7 Outlook

Our results show good prospects, especially in two concrete respects. First, on the formal level, the quantization rules, Eq. (9.22) and Eq. (9.25), present us with low-energy theorems for models or theories of quantum gravity. It will be very interesting to investigate which approaches in the market have the potential to satisfy these conditions. Second, the insights we revealed have phenomenological consequences. For example, quantum corrections in the effective action leave imprints in gravitational wave observations [17], which one hopes to use to better constrain c_3^r, given the special form of Eq. (9.25).

Chapter 10

Conclusions

Gravity is a habit that is hard to shake off.

Terry Pratchet, Small Gods

In this thesis, we have studied classical and quantum extensions of general relativity and its applications to gravitational waves, black holes and exotic compact objects.

In Chapter, 2, we carefully introduced the reader to basic concepts of GR, including some of its solutions. Vacuum solutions such as the Schwarzschild Solution were discussed in detail while discussing its relevance to stars and especially, black holes. We also gave a short introduction to the non-vacuum solutions of GR while touching upon Exotic Compact Objects, especially, Boson Stars.

Chapter 3 reviewed the recipe of calculating gravitational waves from Einstein's equations around a flat background. We also discussed how to calculate the total power carried away as gravitational radiation from collision events through an example of a binary system of two massive bodies rotating around each other. This would pave the way to many calculations in Chapter 6 and Chapter 7.

In Chapter 4, we introduced the reader to the basic concepts in the formulation of an effective field theory of quantum gravity. In the limiting case of gravity coupled to only massless or light fields, the effective action was presented. We saw that the non-local part of the action is highly interesting. The coefficients of the non-local terms, unlike the Wilson coefficients are true "quantum" predictions of the theory and do not have to be empirically determined. Various phenomenological implications of such a framework were discussed.

In Chapter 5, we reviewed the birth of black hole thermodynamics historically and in a pedagogical manner. The parallels between the second law of thermodynamics and the black hole horizon area were examined. The relation between the entropy of the black hole and its horizon area as calculated by Bekenstein were derived using fundamental arguments. We also discussed the quantization of the horizon area using insights from Ehrenfest's theorem and its significance in quantum gravity. Finally, we investigated the Area-Entropy law and discussed quantum corrections to it as derived in various quantum gravity theories. This laid the groundwork for the final paper included in the thesis.

In Chapter 6 steps were taken to calculate the gravitational wave modes starting from the effective action laid out in Chapter 4. We followed the traditional recipe as discussed in Chapter 3 to find that Effective Quantum Gravity predicts two additional massive gravitational wave modes in addition to the usual classical massless mode in GR. These modes are complex conjugates of each other, which implies that one of them has the wrong signature and thus, might be a ghost. However, this could potentially be cured by strong gravity effects in the UV. The mode with the right sign is a highly damped oscillator which dies down exponentially fast. Back-of-the-envelope calculations helped us put bounds on the massive graviton mass. However, we concluded that these modes could not have been produced in events detected by LIGO till now. The energy of events in which these massive modes could be produced approximate inflationary scales $> 10^{17}$ GeV.

In Chapter 7, we consider the effect of quantum gravity on the gravitational radiation emitted by a binary black hole system. In particular, working to second order in curvature, we calculate for the first time the leading order quantum gravitational correction to the classical quadrupole formula which appears at second order in Newton's constant. Even though this is extremely small without any scope of experimental detection in the near future, we see that model-independent quantum predictions can be made using the EFT framework in gravity. At order G^2, the usual massless mode does not receive a correction, even though the strain does.

In Chapter 8, we calculate the total gravitational radiation background generated by binary boson star pairs that might be created in locally dense dark matter clusters. These bosons stars are formed of complex scalars having a repulsive self-interaction which stabilizes them against gravity. We found that the gravitational signal from binaries of stars made of light bosons fall within the reach of the next generation of gravitational wave detectors and pulsar timing arrays. In case of no detection by LISA, these can be translated into bounds on the mass and coupling of the bosons in our model.

In Chapter 9, we motivate a generalized area spectrum for a black hole. Using this area-spectrum, the Bekenstein-Hawking Area law and the demands from statistical physics, we calculate the number of microstates of a black hole in an effective quantum gravity framework. Interestingly, we find strict constraints on the number of light fields that can be coupled to gravity in a consistent theory of quantum gravity. Using dimensional transmutation, we also found an expression for the Wilson coefficient c_3 which can be completely fixed by a set of three natural numbers (or in fact, one, since the other two can be fixed from other considerations)! This is an important result as it shows us hints of the UV quantization in the Wilson coefficients and could also lead to tighter constraints on c_3. This paper is also very interesting because it proposes low energy theorems which a consistent quantum theory of gravity must follow. It also hints at minimal extensions of the standard model.

In this thesis, we focused on effective field theories of gravity as they arise in the low energy limit of a UV completion, thus allowing one to investigate gravitational phenomena in a model-independent manner. This is highly important since, currently we have no way of proving or falsifying one or more of the proposed UV completions of gravity. It may well be that in order to understand the true quantum nature of gravity, we might have to move beyond traditional ideas of space and time. However, this thesis stresses that EFT provides us a way to model-independently push the frontiers of quantum gravity phenomenology without making ad-hoc assumptions about the UV. This could very well pave the way forward by making some essential contact with quantum gravity experiments.

Appendix A

Quadrupole Moment Contour Integrals

In this appendix, we explicitly show the derivation of Equation 7.20. Equation. (7.15) was given by:

$$\bar{h}_{ij} = \bar{h}^{\mathrm{GR}}_{ij} - 16\pi G \int d\omega\, e^{-i\omega t} I_{ij}(\omega) \int \frac{k^2 dk d\Omega_k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{(\omega + i\epsilon)^2 - k^2 - m_2^2} \tag{A.1}$$

where

$$I_{ij}(\omega) = -\frac{1}{2}\mu(d\omega_s)^2 \begin{pmatrix} \delta(\omega + 2\omega_s) + \delta(\omega - 2\omega_s) & -i(\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s)) & 0 \\ -i(\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s)) & -\delta(\omega + 2\omega_s) - \delta(\omega - 2\omega_s) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

As we pointed out before, in the above, μ is the reduced mass of the binary, d is the orbital separation and ω_s is the orbital frequency. In Equation (7.15), notice most importantly the $i\epsilon$ prescription is due to the retarded boundary conditions. In the complex k-plane, the poles are situated at

$$k_{\pm} = \pm\sqrt{\omega^2 - m_2^2} \pm \mathrm{sgn}(\omega)\, i\epsilon \quad . \tag{A.3}$$

One notices two features of the above expression. First, the poles are real (imaginary) if the mass is smaller (greater) than the frequency. Second, if the poles are real then the sign of the frequency is important in moving the poles off the real axis, which is paramount in obtaining a proper propagating wave. Let us now write, Equation (A.1) as:

$$\bar{h}_{ij} = \bar{h}_{ij}^{\rm GR} - 16\pi G \int d\omega\, e^{-i\omega t} I_{ij}(\omega)\mathbb{A} \tag{A.4}$$

where

$$\begin{aligned}
\mathbb{A} &= \int \frac{k^2 dk d\Omega_k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{(\omega+i\epsilon)^2 - k^2 - m_2^2} \\
&= \int \frac{2\pi k^2 dk \sin\theta d\theta}{(2\pi)^3} \frac{e^{ikr\cos\theta}}{(\omega+i\epsilon)^2 - k^2 - m^2} \\
&= -\int\int_{u=-1}^{1} \frac{2\pi k^2 dk du}{(2\pi)^3} \frac{e^{ikru}}{(\omega+i\epsilon)^2 - k^2 - m^2} \\
&= i\int_0^\infty \frac{2\pi k^2 dk}{(2\pi)^3 kr} \frac{[e^{ikr} - e^{-ikr}]}{(k-k_)(k-k_+)} \\
&= \frac{i}{2}\int_{-\infty}^{\infty} \frac{1}{4\pi^2}\frac{kdk}{r}\Big[\frac{e^{ikr} - e^{-ikr}}{(k-k_)(k-k_+)}\Big] \\
&= \frac{i}{8\pi^2 r}\int_{-\infty}^{\infty} \frac{kdke^{ikr}}{(k-k_)(k-k_+)} - \frac{1}{8\pi^2 r}\int_{-\infty}^{\infty} \frac{kdke^{-ikr}}{(k-k_)(k-k_+)} \\
&= \mathbb{A}_1 + \mathbb{A}_2.
\end{aligned} \tag{A.5}$$

where $\mathbb{A}_1$ and $\mathbb{A}_2$ are two contour integrals. In the first case, $\mathbb{A}_1$, the poles are on the real axis and in the second case, the poles are on the imaginary axis. There can be two cases:

- Case I: $\omega > m_2 \Rightarrow \sqrt{\omega^2 - m_2^2} \in \mathbb{R}$,
- Case II: $\omega < m_2 \Rightarrow \sqrt{\omega^2 - m_2^2} \in \mathbb{I} = i\sqrt{m_2^2 - \omega^2}$.

For Case I, taking the contour in the upper half plane and closing the contour in the lower half plane in the second case and adding both as well as for both $\mathrm{Sgn}(\omega) = +$ and $\mathrm{Sgn}(\omega) = -$, we get

$$\begin{aligned}
\mathbb{A}_1 &= \frac{i}{8\pi^2 r} 2\pi i \mathrm{Res}(k_+) - \frac{i}{8\pi^2 r}(-2\pi i)\mathrm{Res}(k_-) \\
&= \frac{-1}{4\pi r} e^{i\sqrt{\omega^2 - m_2^2} r}\Theta(2\omega_2 - m_2)\Theta(\omega) - \frac{1}{4\pi r} e^{-i\sqrt{\omega^2 - m_2^2} r}\Theta(\omega_2 - m_2)\Theta(-\omega)
\end{aligned} \tag{A.6}$$

Similarly,

$$\mathbb{A}_2 = \frac{-1}{4\pi r} e^{-\sqrt{m^2 - \omega_1^2} r}\Theta(m_2 - 2\omega_1). \tag{A.7}$$

Adding both of the above and plugging in the expression for A.2, we get the expression

$$\bar{h}_{ij}(t,r) = \bar{h}_{ij}^{\mathrm{GR}} - 4G\frac{\mu(d\omega_s)^2}{r}\left[\theta(m_2 - 2\omega_s)e^{-\sqrt{m_2^2-4\omega_s^2}r}Q_{ij}(t,0;0) + \theta(2\omega_s - m_2)Q_{ij}(t,r;m_2^2)\right] \tag{A.8}$$

where we defined

$$Q_{ij}(t,r;m^2) = \begin{pmatrix} \cos\left(2\omega_s\left(t - \sqrt{1-(m/2\omega_s)^2}r\right)\right) & \sin\left(2\omega_s\left(t - \sqrt{1-(m/2\omega_s)^2}r\right)\right) & 0 \\ \sin\left(2\omega_s\left(t - \sqrt{1-(m/2\omega_s)^2}r\right)\right) & -\cos\left(2\omega_s\left(t - \sqrt{1-(m/2\omega_s)^2}r\right)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{A.9}$$

The remaining integrals can now easily be performed. We find

$$h_{ij}(t,r) = h_{ij}^{\mathrm{GR}} - 4G\frac{\mu(d\omega_s)^2}{r}\left[\theta(m_2 - 2\omega_s)e^{-\sqrt{m_2^2-4\omega_s^2}r}Q_{ij}(t,0;0) + \theta(2\omega_s - m_2)Q_{ij}(t,r;m_2^2)\right] , \tag{A.10}$$

in the far zone, where

$$h_{ij}^{\mathrm{GR}} := 4G\frac{\mu(d\omega_s)^2}{r}Q_{ij}(t,r;0) \quad . \tag{A.11}$$

Appendix B

Power Spectrum Calculation

In this appendix, we derive the power spectrum calculation explicitly to arrive at the result, Equation (8.21):

$$\begin{aligned}
\frac{dE}{df_s} &= \frac{dE}{dt}\frac{dt}{df_s} \\
&= \left(\frac{dE}{dt}\right) \times \left(\frac{df_s}{dt}\right)^{-1} \\
&= \left(\frac{32}{5}\frac{G}{c^5}\mu^2\omega^6 r^4\right)\left(\frac{5}{96}\pi^{-8/3}f^{-11/3}(Gm_c)^{-5/3}\right) \\
&= \frac{1}{3}(G\pi)^{2/3}m_c^{5/3}f^{-1/3} \qquad \text{(B.1)}
\end{aligned}$$

where $r = \left(\frac{GM}{\omega^2}\right)^{1/3}$ and $\omega = f\pi$ and we have set $c = 1$. $f_s = (1+z)f$ for the emitted (source) frequency, and dE/dt is the energy radiated by a binary system of two self gravitating point masses rotating around each other which we derived in the example in Chapter (3).

Appendix C

Non-Local Distribution Function

In this appendix, we derive the distribution $\mathfrak{L}(x-x')$ which formally reads

$$\mathfrak{L}(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} \log\left(\frac{-p^2}{\mu^2}\right) \quad . \tag{C.1}$$

As it stands, the above integral is meaningless without specifying a boundary condition. To ensure causality, we impose retarded boundary conditions by writing $p^0 \to p^0 + i\epsilon$. In fact, this is not an *ad hoc* prescription. It was explicitly shown in [40] that using the in-in formalism to compute the effective action automatically yields a causal non-local distribution. Although ref. [40] was concerned with the time-dependent case, the conclusion is clear that in-in field theory guarantees the causal behavior of the equations of motion. We start by expressing the logarithm as follows

$$\log\left(\frac{-p^2}{\mu^2}\right) = -\int_0^\infty dm^2 \left(\frac{1}{-p^2+m^2} - \frac{1}{\mu^2+m^2}\right) \quad . \tag{C.2}$$

Notice that each integral diverges separately in such a way that the sum is finite. We have to introduce an explicit regulator, thus when we plug back in Equation (C.1)

$$\mathfrak{L}(x-x') = \lim_{\delta\to 0}\left[\int_0^\infty dm^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} \frac{e^{-\delta\sqrt{\vec{p}^2+m^2}}}{(p^0+i\epsilon)^2 - \vec{p}^2 - m^2} - \delta^{(4)}(x-x')\ln(\delta\mu)^2\right] \quad . \tag{C.3}$$

As per usual, the integral over p^0 is readily performed and the poles are situated at

$$p^0 = \pm\sqrt{\vec{p}^2+m^2} - i\epsilon \tag{C.4}$$

which forces the integral to vanish if x and x' are spacelike separated as one desires. Hence,

$$\mathfrak{L}(x-x') = \Theta(t-t')\Theta((x-x')^2) \lim_{\delta\to 0} \int_0^\infty dm^2 \int \frac{d^3p}{(2\pi)^3} e^{+i\vec{p}\cdot(\vec{x}-\vec{x}')} e^{-\delta\omega_p} \frac{\sin(\omega_p \Delta t)}{-\omega_p} \tag{C.5}$$

where $\omega_p := \sqrt{\vec{p}^2 + m^2}$ and $\Delta t := t - t'$. Now the mass integral is easily done

$$\mathfrak{L}(x-x') = -\Theta(t-t')\Theta((x-x')^2) \lim_{\delta\to 0} \int \frac{d^3p}{(2\pi)^3} e^{+i\vec{p}\cdot(\vec{x}-\vec{x}')} \left(\frac{e^{ip(\Delta t + i\delta)}}{\Delta t + i\delta} + \frac{e^{-ip(\Delta t - i\delta)}}{\Delta t - i\delta} \right) . \tag{C.6}$$

The rest of the integral is elementary and yields a distribution, which is both Lorentz-invariant and retarded

$$\begin{aligned} \mathfrak{L}(x-x') &= \lim_{\delta\to 0} \left[\frac{i}{\pi^2} \left(\frac{\Theta(t-t')\Theta((x-x')^2)}{((t-t'+i\delta)^2 - (\vec{x}-\vec{x}')^2)^2} - \frac{\Theta(t-t')\Theta((x-x')^2)}{((t-t'-i\delta)^2 - (\vec{x}-\vec{x}')^2)^2} \right) \right. \\ &\qquad \left. - \delta^{(4)}(x-x') \ln(\delta\mu)^2 \right] . \end{aligned} \tag{C.7}$$

As we can see, this function has support only on the past light cone, which is as we expected. As a sanity check, this can also be seen to reduce to the cosmological expression found in [40, 91] when we integrate over d^3x.

Appendix D

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