Investigations into alternative theories of cosmology

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Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another University for the award of any other degree.

The work in this thesis has been completed in collaboration with my supervisors David Seery and Christian Byrnes, and is comprised of the following papers:

Michaela G. Lawrence and David Seery. "Ultraviolet sensitivity of the cosmological sequester", published in APS Physical review D, **103**, 064018 (2021). arXiv:2007.07058 [hep-th]. The two-loop diagram given in the left-hand part of Fig. 1 in the paper (Fig. 2.1 in this thesis) was calculated by myself. Details of the calculation were not included in the published paper however we give details here, in Appendix A. The first draft of this paper was written by me, although it was significantly edited by David Seery.

Michaela G. Lawrence, David Seery and Christian T. Byrnes. "Constraints on a cubic Galileon disformally coupled to standard model matter", submitted to JCAP. arXiv:2007.16052 [gr-qc]. The first draft of this paper was written by me but was completed with the help of my collaborators.

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Summary

General relativity and the cosmological concordance model (ΛCDM) successfully explain a myriad of observations. The perihelion precession of Mercury; the polarization of the cosmic microwave background; the observed baryon acoustic oscillations feature in the matter power spectrum; and the statistics of weak gravitational lensing all contribute to the vindication of the theories. However, these theories fall short in some areas. One of these shortfalls is the explanation of the magnitude of the cosmological constant; another is the tension between late-time and early-time measures of the Hubble constant. The purpose of this thesis is to investigate alternative theories of gravity and cosmology and the possible rectification of their shortcomings.

Kaloper & Padilla's sequester theory decouples the cosmological constant from vacuum loops containing Standard Model particles. The sequestering mechanism is the result of an additional global term, representing a kind of matter which does not gravitate, as other contributions to the action do. This results in a very small effective cosmological constant which is proportional to the historic average of locally excited matter. We gauge the success of the theory by calculating the form of the divergence coming from two-loop Standard Model scalar vacuum diagrams, determining the level of detuning the theory suffers.

We also investigate the consequences of adding an additional component to the universe, cubic Galileons, and coupling them disformally to Standard Model matter. This is not an attempt to solve the cosmological constant problem, the cosmological constant remains in the action and the cubic Galileon exists as a sub-dominant contribution to the total energy density. We calculate the analogue of Maxwell's equations and use the Wentzel-Kramerss-Brillouin (WKB) approximation to find the wave equation for light and in turn, the small discrepancy in the speeds of light and gravity in this theory. Additionally, we also calculate constraints coming from observations of the integrated Sachs Wolfe (ISW) effect, and the phase velocity of light. These are compiled along with already established constraints coming from collider experiments.

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Notation

Throughout this thesis we use natural units such that $c = \hbar = 1$. For example, in these units we write the reduced Planck mass as,

$$M_{\mathrm{P}} = \sqrt{\frac{\hbar c}{8\pi G_{\mathrm{N}}}} \approx 4.341 \times 10^{-9}\mathrm{kg} = 2.435 \times 10^{18}\mathrm{GeV}, \tag{1}$$

where G_{N} is Newton's gravitational constant. We will use the mostly positive signature for the metric,

$$\mathrm{d}s^2 = \eta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2. \tag{2}$$

We denote a partial derivative as ∂_μ and a covariant derivative as ∇_μ where,

$$\nabla_\alpha T^\beta_\rho = \partial_\alpha T^\beta_\rho + \Gamma^\beta_{\sigma\alpha} T^\sigma_\rho - \Gamma^\sigma_{\rho\alpha} T^\beta_\sigma. \tag{3}$$

The affine connection is defined as,

$$\Gamma^\alpha_{\beta\mu} = \frac{1}{2} g^{\alpha\nu} \left(\partial_\beta g_{\nu\mu} + \partial_\mu g_{\beta\nu} - \partial_\nu g_{\beta\mu}\right). \tag{4}$$

The Riemann tensor is,

$$R_{\mu\alpha\beta}{}^{\sigma} = \partial_\alpha \Gamma^\sigma_{\beta\mu} - \partial_\beta \Gamma^\sigma_{\alpha\mu} + \Gamma^\sigma_{\alpha\nu}\Gamma^\nu_{\beta\mu} - \Gamma^\sigma_{\beta\nu}\Gamma^\nu_{\alpha\mu}. \tag{5}$$

The Ricci tensor is the Riemann tensor contracted on the second and fourth indices,

$$R_{\beta\mu} = R_{\beta\alpha\mu}{}^{\alpha}. \tag{6}$$

The following commutation relation is associated with the Riemann tensor,

$$\left[\nabla_\alpha, \nabla_\beta\right] A_\mu = -R_{\alpha\beta}{}^{\nu}{}_{\mu} A_\nu = R_{\alpha\beta\mu}{}^{\nu} A_\nu. \tag{7}$$

The energy-momentum tensor can be written as,

$$T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta + p g_{\alpha\beta}. \tag{8}$$

The cosmological constant plays an important role in this thesis. It it usually denoted by Λ. In Section 1.3 Λ is also used to denote a hard cutoff scale. In Chapter 4 Λ_{C} is used to denote the cosmological constant and Λ is used to denote the scale at which cubic Galileon terms become non-negligible.

Chapter 1

Introduction

In this thesis we address two theories and there effects on our understanding of cosmology. The first is the sequester theory, which attempts to remove the cosmological constant problem without changing other observations. The theory is examined in how it is affected by a particular class of quantum corrections.

The second theory that is examined proposes an additional component of the Universe that comes in the form of a cubic Galileon scalar disformally coupled to matter. This is tested against observations of: the differences in speeds of light and gravity; ATLAS constraints; and the integrated Sachs Wolfe (ISW) effect.

Setting the scene for these examinations, this chapter will begin with a brief review of general relativity (GR), including a description of some of its successes and shortcomings. The weak and strong equivalence principles, corner stones of GR are defined. We discuss the current state of cosmological observations of the accelerated expansion of the Universe. Finally, we explicitly state the cosmological constant problem, and give an introduction to the sequester theory of gravity, which is a proposed solution.

1.1 General Relativity

Einstein's theory of general relativity Einstein (1916, 1917) (GR), formulated over 100 years ago has consistently held up as the best theory of gravity to date. GR unifies Newton's gravity with special relativity, accurately describes the perihelion precession of Mercury Einstein (1916); Park et al. (2017) and predicted the existence of recently discovered gravitational waves Abbott et al. (2016a,b).

The equivalence principle as well as combining Newton's gravity and special relativity, guided the development of GR. The equivalence *principle* is formulated as separate equivalence *principles* today.

The weak and strong equivalence principles are cornerstones of Einstein's theory of general relativity. An observer who is freely falling in a static, homogeneous gravitational field does not feel any gravitational forces.

> **The weak equivalence principle** says that the gravitational force exerted on a particle of mass m is proportional to m. The gravitational force is not different for different *kinds* of matter, it is always $F = mg$, where g is the acceleration due to gravity. The titanium-platinum Eötvös parameter characterizing the relative difference in their free-fall accelerations, has been found to be the same to at least one part in 10^{15} Touboul et al. (2017).

> **The strong equivalence principle** extends the weak equivalence principle to all the laws of physics, gravitational and otherwise. The outcome of any local experiment (gravitational or not) in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime. The strong equivalence principle enforces that Newton's gravitational constant G_N is the same everywhere and constant over the life of the Universe. Any cosmological evidence for the existence of a fifth force would break the strong equivalence principle. The best limit on the validity of the strong equivalence principle under strong-field conditions was obtained with a pulsar in a triple stellar system, PSR J0337+1715. The extreme difference in binding energy between neutron stars and white dwarfs allows for precision tests of the strong equivalence principle via the technique of pulsar timing. It was found that found that any acceleration difference between the neutron star and inner white dwarf is less than one part in 10^5 Voisin et al. (2020).

Einstein's theory of general relativity (including the cosmological constant) satisfies the strong equivalence principle. In general, alternative theories of gravity break the strong equivalence principle. The weak equivalence principle means the trajectory of a point mass in a gravitational field depends only on its initial position and velocity, and is independent of its composition and structure. This is true even for virtual particles in the vacuum. We shall see later in this thesis that this can lead to problems.

The Einstein-Hilbert action, including a bare cosmological constant Λ_b and minimally[1] coupled to matter is,

$$S_{\rm EH} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - \Lambda_{\rm b}\right] + S_{\rm m}(g^{\mu\nu}, \Psi_i), \tag{1.1}$$

where, R is the Ricci scalar and $M_{\rm P}$ is the Planck mass. For now, we will not specify the action for matter but simply write as $S_{\rm m}$, where Ψ_i denotes all fields that couple minimally to the metric, understanding that this could include all types of matter, baryons, dark matter and radiation.

ΛCDM is a theory of cosmology that assumes general relativity as the theory of gravity and the constituents of the Universe as mostly cosmological constant $\Lambda_{\rm tot}$ and cold dark matter (CDM). Cold dark matter is matter that behaves non-relativistically (cold) and interacts very weakly with ordinary baryonic matter and electromagnetic radiation (dark). There are a wide variety of possible dark matter candidates (see Garrett and Duda (2011) for a review). The term "baryonic" is used here to refer to all objects made of atoms, ions and electrons, such as stars and planets. The cosmological constant is the "dark energy" in the ΛCDM theory, meaning it is responsible for the accelerating expansion of the universe.

The relative energy densities today are measured as being roughly 69% cosmological constant, 31% non-relativistic matter, mostly CDM, with baryons only making up around 5% of the total energy density of the Universe today. There is also a very small amount of radiation today Aghanim et al. (2018). ΛCDM is discussed further in the next section, however we now return to the Einstein-Hilbert action.

Varying the Einstein-Hilbert action $S_{\rm EH}$ with respect to the inverse metric $g^{\mu\nu}$, then rearranging, yields Einstein's field equations,

$$M_{\rm P}^2 G_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}\Lambda_{\rm b}. \tag{1.2}$$

$G_{\mu\nu}$ is the Einstein tensor and is equal to $R_{\mu\nu} - Rg_{\mu\nu}/2$ and is divergence-free due to the Bianchi identity $\nabla_\mu G^\mu{}_\nu = 0$. $T_{\mu\nu}$ is the energy-momentum tensor,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}. \tag{1.3}$$

Suspending the inclusion of the cosmological constant in Einstein's field equations one can see that matter, $T_{\mu\nu}$ is affecting spacetime $G_{\mu\nu}$ and vice versa.

[1] Minimal coupling here means that the matter is minimally coupled to gravity. That is to say that the only coupling of the matter fields Ψ_i to gravity is through the Lorentz invariant measure $\sqrt{-g}\,\mathrm{d}^4x$, where $g = \det g_{\mu\nu}$. Derivatives of the matter fields are accompanied by connection terms $\Gamma^\sigma_{\mu\nu}$ because one must use gauge covariant derivatives.

The total cosmological constant $\Lambda_{\rm tot}$ which sources the curvature of the Universe is a combination of (at least) two components [2] the bare cosmological constant Λ_b which can be seen explicitly in Equation 1.1, is a classical contribution, it is compatible with both general covarience and a conserved energy-momentum tensor (as has already been shown). $\Lambda_{\rm b}$ can be thought of as a free parameter, a priori we have no reason to discard it. The second contribution comes as a result of quantum field theory. $\rho_{\rm vac}$ corresponds to the energy stored in the vacuum and so is part of the energy momentum tensor $T^{\mu}_{\ \nu} = \delta^{\mu}{}_{\nu}\rho_{\rm vac}$. These two components are indistinguishable to cosmological probes so any cosmological observations can only constrain the linear combination $\Lambda_{\rm tot} = \Lambda_{\rm b} + \rho_{\rm vac}$.

1.2 Cosmology in ΛCDM

The perihelion precession of Mercury can be considered the first observational confirmation of General Relativity Einstein (1916). Now over one hundred years on we have a myriad of other observations which support general relativity and ΛCDM. In this thesis we concern ourselves with just a few of those observations coming from cosmology.

The Universe is expanding at an accelerating rate and this has been confirmed by a number of different experiments and observations of both late-universe and early-universe phenomena. The first evidence for the accelerated expansion came from measuring the brightness and redshift of type 1a supernovae Riess et al. (1998); Perlmutter et al. (1999). More recently, evidence has also come (indirectly) from early-time observations such as the CMB power spectrum Hinshaw et al. (2013); Hou et al. (2014).

The Hubble constant $H_0 \sim 70$ km s^{-1} Mpc^{-1} is a measure of the accelerating expansion of the Universe. At this value a galaxy one megaparsec (around 3.3×10^6 light-years) away, that is moving with the Hubble flow only, is travelling away from us at 70 km s^{-1}. If we imagine a galaxy further away, at two megaparsecs, this galaxy is travelling away from us at 140 km s^{-1}.

Assuming ΛCDM, the Hubble constant can be indirectly determined using CMB observations. By plotting the CMB power spectra one can read-off the sound horizon of the photo-baryonic fluid at last scattering by measuring the distance between peaks. Using

[2] The qualification "at least" is included here because we know of at least two contributions to the total comsological constant, $\Lambda_{\rm tot} = \Lambda_{\rm b} + \rho_{\rm vac}$, the bare cosmological constant and the vacuum energy density. In Chapter 2 of this thesis we break the vacuum energy density contribution down further into two components, giving us $\Lambda_{\rm tot} = \Lambda_{\rm b} + \Lambda_{\rm IR} + \Lambda_{\rm UV}$. This is covered in more detail in Chapter 2 but one can consider $\Lambda_{\rm IR}$ to be the contributions to the vacuum energy density coming from known physics and $\Lambda_{\rm UV}$ is an incalculable contribution from unknown physics lying above the standard model.

this as a standard ruler the distance to the last scattering surface can be inferred using knowledge of how the Universe has expanded between the time of last scattering and today. The value of the Hubble constant inferred using this method is 67.4 ± 0.5 km s^{-1} Mpc^{-1} Aghanim et al. (2018).

Determining H_0 using baryon acoustic oscillation measurements [3], another early-time probe, bears some similarities with inferring H_0 from the CMB Addison et al. (2018); Abbott et al. (2018). Using these methods the Hubble constant is measured to be 66.98 ± 1.18 km s^{-1} Mpc^{-1} and 67.77 ± 1.30 km s^{-1} Mpc^{-1} respectively.

Measuring H_0 with local experiments is more direct and normally relies on standard candles as opposed to standard rulers. Type 1a supernovae are standardizable (more so in combination with Cepheid observations), therefore we can determine a type 1a supernova's distance from us by measuring their apparent magnitude. The speed at which a type 1a supernovae travels away from us can be determined using their redshift Riess et al. (2019), in this case the Hubble constant is calculated to be 74.03 ± 1.42 km s^{-1} Mpc^{-1}. Other local measurements of H_0 include observations of time delays of multiply-imaged quasars Birrer et al. (2019), which deduces a Hubble constant of $72.5^{+2.1}_{-2.3}$ km s^{-1} Mpc^{-1}.

This list of probes of the accelerated expansion is far from exhaustive. However it is already apparent that different measurements are not compatible. Late-time observations which are cosmology independent compared to the early time value from the CMB, which assume ΛCDM, exhibit a roughly 10% discrepancy at a few standard deviations. Addressing the Hubble tension is not a major focus of this thesis but it has been a great interest to many in the cosmology community in recent years. Some believe that the error comes from the early universe probes (wrongly) assuming ΛCDM in order to infer a subsequently incorrect Hubble rate (see Di Valentino et al. (2019) and references therein). It seems there may be something wrong with the current understanding of the Universe and specifically with dark energy.

The cause of the acceleration is often referred to as dark energy. The cosmological constant is often referred to as the *simplest* theory of dark energy because it is a constant in spacetime. The cosmological constant does not redshift but every other form of matter or energy will. It is therefore inevitable that a cosmological constant will dominate the energy-density of the Universe at late times. A cosmological constant dominated universe is one undergoing a de Sitter phase (exponential expansion). The best explanation we have

[3] Abbott et al. (2018) measures the Hubble constant by making supernova measurements using the inverse distance ladder method based on baryon acoustic oscillations. Addison et al. (2018) inferred a value of the Hubble constant from the local distance ladder and from Planck CMB data.

so far for accelerating expansion is a cosmological constant with energy density roughly $\Lambda_{\text{tot}} \sim (\text{meV})^4$ [4].

This is not to say that the cosmological constant is a perfect theory of dark energy. We have already discussed the Hubble tension, however there is another missing piece in our understanding of the Universe: explaining the observed value of the cosmological constant, this is discussed in detail in the next section.

1.3 The Cosmological Constant Problem

Here, we show that the observed energy density of the vacuum Λ_{tot}, is at least 56 orders of magnitude smaller than known contributions to it Weinberg (1989). The vacuum energy density can be understood in terms of Feynman bubble diagrams and that is how we explain it here.

Exactly how the calculation of these quantum contributions to the vacuum is done does not matter. Much the same answer is achieved whatever kind of quantum field we choose to calculate it for (including for a massive scalar field). For illustration, we perform the calculation for ϕ^4 theory, a simple scalar theory with a non-trivial self interaction term.

1.3.1 An example calculation of a vacuum energy

We will begin by following a argument made by Sakharov Sakharov (1967) which says that:

A quantum fluctuation (or vacuum state fluctuation or vacuum fluctuation) is the temporary random change in the amount of energy in a point in space, as prescribed by Heisenberg's uncertainty principle. The uncertainty principle states the uncertainty in energy and time can be related by,

$$\Delta E \Delta t \geqslant \frac{1}{2}\hbar. \tag{1.4}$$

This means that pairs of virtual particles with energy ΔE and lifetime shorter than Δt are continually created and annihilated in empty space Heisenberg (1927). The vacuum is not empty.

The second part of Sakharov's argument develops this further: the energy-momentum tensor of a field placed in the vacuum state must be given by,

$$\langle 0|T_{\mu\nu}|0\rangle \equiv \langle T_{\mu\nu}\rangle = -\rho_{\text{vac}} g_{\mu\nu}, \tag{1.5}$$

[4] Although ΛCDM does not explain the Hubble tension, some alternative theories of dark energy might possibly explain the discrepancy, including the Galileon ghost condensate Peirone et al. (2019).

where ρ_{vac} is the constant energy density of the vacuum. This equation is valid for all fields in the Universe. This can be proved in more than one way but we present the quantum mechanical argument here.

1. In Minkowski spacetime the only invariant tensor is $\eta_{\mu\nu}$.
2. The vacuum state must be the same for all observers and therefore, $\langle T_{\mu\nu}\rangle \propto \eta_{\mu\nu}$.
3. Therefore, in a curved spacetime we can write, $\langle T_{\mu\nu}\rangle = -\rho_{\text{vac}}(t,\vec{x})g_{\mu\nu}$.
4. The energy-momentum tensor must be conserved therefore, $\rho_{\text{vac}}(t,\vec{x}) = \rho_{\text{vac}} = \text{const}$.

continuing Sakarov's argument, if the form of the energy-momentum tensor is as above and we assume the weak equivalence principle holds for all matter - and therefore includes zero point fluctuations. Then we must conclude that zero point fluctuations gravitate.

We have accepted that the vacuum gravitates. The rest of this section will then be dedicate to the question of: how much? This can be answered in many ways. Here, we have chosen to formulate this question in terms of Feynman diagrams.

Even more specifically, we will calculate the amplitude of a two-loop bubble diagram for a real, self-interacting scalar field. We did not *need* to do this. As we stated at the beginning of this section much the same answer is achieved whatever kind of field, and for however many loops, we choose to calculate the vacuum energy density for. So why did we make this choice? We could have performed a simpler calculation, choosing to calculate a one-loop diagram. The two-loop diagram we have chosen is slightly more realistic because it is the first example where we see a non-trivial interaction.

The reader might ask then why not have a look at the full complexity of the problem? Instead of demonstrating the calculation for a scalar field, demonstrate instead two different calculations: one calculation for a bosonic field and the other a fermionic field. The honest answer to this question is that we could have done this. Pedagogical calculations of this kind can be found in Martin (2012).

How much does the vacuum gravitate?

There are different ways to calculate the effect of these fluctuations. We do it here with perturbative quantum field theory[5] The Lagrangian density for a real, massive scalar field ϕ with a quartic interaction is,

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4. \tag{1.6}$$

[5]To calculate the effect of the quantum fluctuations on the vacuum we will perform the calculation perturbatively, order by order in a series expansion in the number of virtual particles participating in the interaction.

The vacuum is affected by quantum fluctuations just like everything else in the Universe. Quantum fluctuations of the vacuum change the matter field [6]. Because of the weak equivalence principle, this changed matter distribution will also result in a changed gravitational field.

Assuming that these quantum fluctuations do gravitate, we calculate here what these quantum fluctuations of the ϕ field do to the matter distribution in the Universe. We begin by defining a new action, the effective action, a modified expression for the action which takes into account quantum-mechanical corrections. The equations of motion for the vacuum expectation value of the field can be derived from the requirement that the effective action be stationary. The effective action is denoted here by S_{eff} and the following equation holds,

$$e^{iS_{\text{eff}}} = \left\langle e^{iS_\phi + iS_{\text{other}}} \right\rangle_\phi . \tag{1.7}$$

$\left\langle e^{iS_\phi + iS_{\text{other}}} \right\rangle$ is the average over the generating function. The generating function's partial derivatives "generate" the differential equations that determine the system's dynamics. We are averaging over the ϕ fluctuations. It is important to note that the quantum effective action S_{eff}, won't depend on the field ϕ explicitly because of the averaging of fluctuations around $\phi = 0$. S_{other} comes from shifting the background field.

To calculate the effective action, we need only to compute the connected diagrams[7] from the action and average over those. So the first contribution to S_{eff} is,

$$iS_{\text{eff}} \supseteq \left\langle i \int d^4x \frac{\lambda}{4!} \phi^4 \right\rangle_\phi . \tag{1.8}$$

We perform the calculation in the interaction picture Negele and Orland (1988). So,

$$S_{\text{eff}} \supseteq \frac{\lambda}{4!} \int d^4x \left\langle \phi^4 \right\rangle_\phi , \tag{1.9}$$

where ϕ^4 is at position x. To calculate a quantum contribution to the effective action we will evaluate the contribution coming from the diagram in Figure 1.1. This is two copies

[6] Presumably, the gravitational field responds to the matter field after accounting for quantum fluctuations. The Lamb shift is an example of how quantum fluctuations do indeed change measured results Lamb and Retherford (1947); Bethe (1947). The Lamb shift not only affects the inertial energy of an atom, but also its gravitational energy (see previous discussion of the weak equivalence principle). Without a quantum gravity theory we cannot know for certain how the gravitational field responds to the matter field.

[7] A connected Feynman diagram is one which every part of the diagram is connected to at least one external line. Non-connected diagrams do not contribute to the effective action and this can be seen by shifting the vacuum from $|0\rangle$ to $|\Omega\rangle$.

of the propagator at coincidence,

$$\left\langle \phi^4 \right\rangle_\phi = -\frac{\lambda}{4!} \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{1}{k^2+m^2} \frac{1}{q^2+m^2}. \tag{1.10}$$

Recall, that this will not explicitly depend on the field ϕ. We can also see from the Einstein-Hilbert action, Equation 1.1, that a contribution to the effective action that does not depend on any fields will behave like the cosmological constant term, so we can conclude that the net effect of the field fluctuations will behave like a cosmological constant.

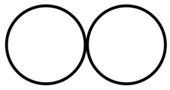

Figure 1.1: The vacuum bubble diagram that we calculate the amplitude for. It contributes to $\langle 0| S_{\text{eff}} |0\rangle$.

To simplify things we begin by calculating a single integral, we will label this D [8],

$$D = i \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{k^2+m^2-i\epsilon}. \tag{1.11}$$

If the integral were to be evaluated at this stage it would diverge as $k^4/k^2 = k^2$. The physical reason for this is because this integral is averaging over all the quantum fluctuations in the theory. It is true that the fluctuations get smaller when you go to higher energy. At very high energies the contribution from the fluctuations falls off, scaling as $1/k^2$ but, the number of fluctuations of wave number k grows, scaling as k^4.

Instead, we begin the calculation by performing a Wick rotation. Ignoring the $i\epsilon$ factors for a minute, there are singularities in the integrand when $k^2 = -m^2$. One can avoid the singularities in the integrand by changing from Lorentzian coordinates to Euclidian spherical polar coordinates. We imagine k_0 is a complex variable. The integration over d^4k includes $\mathrm{d}k_0$ integrated from negative to positive infinity. We find the singularities in the integrand as a function of k_0. The singularities of the integrand are at,

$$-k_0^2 + \vec{k}^2 + m^2 - i\epsilon = 0. \tag{1.12}$$

We can rearrange this to find the values of k_0,

$$k_0 = \pm\sqrt{\vec{k}^2 + m^2 - i\epsilon}. \tag{1.13}$$

We make a binomial expansion in powers of ϵ and only keep the lowest order. Writing $\vec{k}^2 + m^2 = E^2(\vec{k})$, the energy of the on-shell momentum particle k is,

$$k_0 = \pm E(\vec{k})\left(1 - i\frac{\epsilon}{E(\vec{k})}\right)^{1/2}. \tag{1.14}$$

[8] The $i\epsilon$ prescription comes from correct specification of the vacuum state Weinberg et al. (1995).

We define $\epsilon' = \epsilon\ E(\vec{k})$ and rewrite k_0,

$$k_0 = \pm E(\vec{k})\Big(1 - \frac{i}{2}\epsilon' + \dots\Big). \tag{1.15}$$

The positive solution of k_0 is on the positive side of the singularity and vice versa.

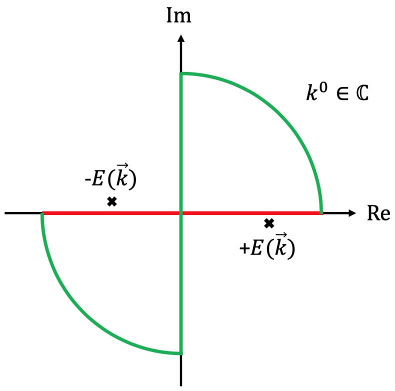

Figure 1.2: Integral diagram where the original contour is in red. Singularities are located at the crosses, just off the real axis.

The effect of the $i\epsilon$ factor has been to move the singularities just slightly off the axis. The purpose of this exercise was to thread a contour across the real axis without intercepting either of the singularities. We take $\epsilon \to 0$ later in the calculation.

Now, we can rotate the contour. The contour begins by tracing along the real axis, just missing both singularities. The contour is completed by following a quarter of a circle in the top right quadrant, coming down the imaginary axis, before finally following a quarter of a circle in the bottom left quadrant. No singularities are enclosed in the contour. Cauchy's theorem tells us that the integral over the closed contour must be zero,

$$\oint = \int_{\rightarrow} + \int_{\text{top arc}} + \int_{\downarrow} + \int_{\text{bot arc}} = 0.$$

The contributions coming from the arcs of the circle are zero so we need only evaluate the integral on the real axis and the imaginary axis. They must sum to 0,

$$\int_{\rightarrow} = \int_{\downarrow}.$$

This is Wick rotation. One can rotate the integral over the real axis in such a way that you avoid intercepting singularities, finally integrating on the imaginary axis. The integral can now be written,

$$D = i \int \frac{\mathrm{d}^3 k}{(2\pi)^4} i \mathrm{d}k_E \frac{1}{k_E^2 + \vec{k}^2 + m^2 - i\epsilon}, \tag{1.16}$$

where k_E is Euclidian and runs from negative to positive infinity. We can now drop the $i\epsilon$ term. This leaves a a four-dimensional integral in polar coordinates with a fairly simple integrand.

Renormalization is the process of factoring out the high energy part of the integral - the part where we assume quantum field theory is no longer an accurate description of physics. Followed by replacing this with something that we take from measurement. We will now factorise this integral into one part that we trust quantum field theory is accurate and another where we cannot make this assertion. There are various methods of doing this, they are called regularisation schemes. The two regularisation schemes that are most commonly used are hard cutoff and dimensional regularisation. The reason for exploring both in this thesis is because we wish to show that a cosmological constant problem exists whichever regularisation scheme you choose.

Hard Cutoff

We do trust quantum field theory to be an accurate description of physics up to around the TeV scale because it makes accurate predictions at the LHC. Presumably there is some high energy theory (quantum gravity) which will describe what goes on at higher energies.

For the hard cut off regularisation scheme one must decide at which energy scale to stop trusting quantum field theory. We call this energy scale the hard cutoff, Λ. We first perform the angular integration.

$$D = -1 \int_{k=0}^{k=\Lambda} \frac{2\pi^2}{\Gamma(2)} \frac{\kappa^3 d\kappa}{(2\pi)^4} \frac{1}{\kappa^2 + m^2}, \tag{1.17}$$

where $\Gamma(2) = 1$. It is possible to evaluate this directly,

$$D = -\frac{2\pi^2}{2(2\pi)^4} \left\{ \Lambda^2 + m^2 \ln \frac{m^2}{m^2 + \Lambda^2} \right\}. \tag{1.18}$$

Its not quite clear what this tells us yet. This regularisation scheme on the surface may appear somewhat crude. We began with an integral up to $k = \infty$. We knew when we began the calculation that quantum field theory is probably not an accurate description of physics up to infinite energies. Here, we have lowered the limit of integration to the point at which we stop trusting quantum field theory.

Presumably there exists a quantum gravity theory which describes physics at higher energies. Physicists have provided possible candidates but we certainly do not have a consensus. Here, we assume that the quantum gravitational contribution to the effective action (and in turn the total cosmological constant) is zero or at least negligible compared to the quantum field theory contribution we have calculated in this section (we revisit this assumption when concluding the calculation). We will see when compared with dimensional regularisation although applying a hard cutoff appears crude it yields consistent results.

Dimensional regularisation

Here, we demonstrate renormalization via dimensional regularisation where the high energy region of your integral (where we assume quantum field theory will not be an accurate description of the physics) is treated in a different way to the rest of the integral[9]. We have included this here to show that dimensional regularization cannot save us from the cosmological constant problem. One could think of the reason for the appearance of infinities as there being to many high energy fluctuations k^4 for the rate at which they are falling off $1/k^2$. In dimensional regularisation we change the amount of volume that there is at high energy making the integral converge. Instead of performing the integral in four dimensions it is done in $d = 4 - \omega$ dimensions. At the end of the calculation we take $d \to 4$,

$$D = -1 \int \frac{2}{\Gamma(2-\omega)} (4\pi)^{\omega-2} \frac{\kappa^{3-2\omega} d\kappa}{\kappa^2 + m^2}, \tag{1.19}$$

we are changing the number of high energy fluctuations but we are not changing the rate at which they fall off, they still scale as $(\kappa^2 + m^2)^{-1}$. We shall see that performing the calculation for small ω yields a convergent answer. Following this,

$$D = -1 \frac{2}{\Gamma(2-\omega)} (4\pi)^{\omega-2} \int \frac{\kappa^{3-2\omega} d\kappa}{\kappa^2 + m^2}. \tag{1.20}$$

We see that there is still a divergence at $\omega \to 0$. This integral like other dimensionaly-regularized integrals can be done in terms of Γ-functions using standard results,

$$D = -\frac{1}{2} (m^2)^{-1+2-\omega} \frac{\Gamma(2-\omega)\Gamma(1-2+\omega)}{\Gamma(1)}, \tag{1.21}$$

simplifying this,

$$D = -1(4\pi)^{\omega-2}(m^2)^{1-\omega}\Gamma(-1+\omega). \tag{1.22}$$

We Taylor expand each part of this in ω,

$$D = -1(4\pi)^{\omega-2}(m^2)^{1-\omega}\Gamma(-1+\omega) = -(4\pi)^{-2} m^2 \left\{ \frac{1}{\omega} - \gamma + \ln 4\pi - \ln m^2 + 1 + \dots \right\}, \tag{1.23}$$

where γ is the Euler-Mascheroni constant. This means the single integral after using dimensional regularisation is,

$$D = -\frac{m^2}{(4\pi)^2} \left\{ \frac{1}{\omega} + \ln 4\pi - \gamma - \ln m^2 + \dots \right\}. \tag{1.24}$$

[9] Dimensional regularization is used in Chapter 2 of this thesis.

This does not immediately resemble the hard cutoff result Equation 1.18. This is because we have attempted two different ways of separating the integral into the trustworthy and untrustworthy parts. All that we have done up to this point is break the integral into two (in two different ways). Renormalisation is the process of breaking the integral into two, keeping the trustworthy part and rewriting the untrustworthy part in terms of observations. The untrustworthy part of the integral diverges like Λ and $1/\omega$ for the hard cut-off and dimensional regularisation methods respectively. There are standard prescriptions for dealing with the divergencies in dimensional regularisation. Here, we use modified minimal subtraction $\overline{\mathrm{MS}}$ Bardeen et al. (1978).

Modified minimal subtraction $\overline{\mathrm{MS}}$ amounts to dropping the terms proportional to $1/\omega$, $\ln 4\pi$ and $-\gamma$. Modified minimal subtraction is a standard way to manage high energy parts of the integration, the result is,

$$D = -1\frac{m^2}{16\pi^2}\ln\frac{m^2}{\mu^2} + \ldots \tag{1.25}$$

where μ has dimensions of [M] so that the total argument of the logarithum is dimensionless. This may be considered the simplest possible answer.

For the hard cut-off there is not really an analogous procedure. However, the integral can be broken up into different parts that represent the high-momentum and the low-momentum,

$$D = -\frac{\Lambda^2}{16\pi^2} - \frac{m^2}{16\pi^2}\ln\frac{m^2}{m^2+\Lambda^2}. \tag{1.26}$$

Now, suppose one takes a measurement at high momentum in such a way that we can replace Λ in the hard cut-off case with something tame. On dimensional grounds this would be a mass scale. Even if this is done, there is one remaining problem, the logarithm also depends on Λ. Therefore the logarithm is also sensitive to the result of the integral at very high momentum. We manage this with another scale, a mass scale μ, that replaces $m^2 + \Lambda^2$.

$$D = -1\left(M^2 + \frac{m^2}{16\pi^2}\ln\frac{m^2}{\mu^2} + \ldots\right), \tag{1.27}$$

where M is the threshold correction taken from the aforementioned measurement. The cut-off dependence has been repackaged in terms of what was taken from observation and the constant that comes along with it.

Now we see that the log terms match, in particular the coefficient of the log term in both cases is the same. It is not immediately obvious, but by comparison the high momentum part will be an additive constant. The term proportional to the logarithm in the $\overline{\mathrm{MS}}$ case is not allowed dimensionally. As before we rewrite this in terms of something taken from measurement, ensuring the dimensions work out correctly.

So the final formula we get is valid in either the hard cut-off or dimensional regularisation $\overline{\mathrm{MS}}$ schemes,

$$S_{\mathrm{eff}} \supseteq \frac{\lambda}{4!}\int \mathrm{d}^4x\left(M^2+\frac{m^2}{16\pi^2}\ln\frac{m^2}{\mu^2}+\dots\right)^2 \tag{1.28}$$

We now interpret this result. No matter what kind of matter fields this is calculated for, no matter how many loops in the calculation, the result is always a variation of what we have here.

Firstly, the scale of the answer is determined by either the threshold correction M or the mass scale m. There are three contributions, which scale as M^4, M^2m^2 and m^4. It is worthwhile keeping in mind what we thought each of these scales corresponded to: the part that we know quantum field theory is an accurate description of physics and the part where we assume some quantum gravity theory takes over. When cosmologists take a measurement of the cosmological constant, we measure *all* of the contributions to this operator from *all* of the loops involving *all* of the standard model species. The sum total of the contributions is of order,

$$(10^{-3}\mathrm{eV})^4 = M^4, M^2m^2, m^4.$$

So there are *two* cosmological constant problems. One from quantum field theory and another from quantum gravity. The version of the cosmological constant problem based on the quantum field theory says that there exists a term m^4 of this magnitude [10] for each of the standard model particles in the vacuum energy density. The vacuum energy contribution we calculated here was for a scalar field, but the argument we have made is a dimensional one. If we had performed the calculation for a fermion the contribution to the vacuum energy density would still be $\sim m^4$.

We can conclude that there ought to be contributions to the vacuum from standard model particles. The largest contribution comes from the top quark, which has a mass $m_{\mathrm{t}}^4 \sim (170\mathrm{GeV})^4$ Zyla et al. (2020). This results in an unpleasant surprise of order,

$$\frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{obs}}} \sim \left(\frac{170\mathrm{GeV}}{10^{-3}\mathrm{eV}}\right)^4 \sim 10^{56}. \tag{1.29}$$

We computed a two-loop calculation here because we worked with ϕ^4 theory because we wanted to demonstrate how the cosmological constant problem manifests in a theory with non-trivial interactions. The theory we chose has a four point vertex. Given we now see how this theory contributes to the vacuum energy at this scale we could consider going

[10] The factor m^4 is multiplied by a coupling constant. Most dimensionless coupling constants are of order one, some are as small as $\mathcal{O}(10^{-3})$.

to higher-loop order. One might even hope that the next-to-leading order may cancel the contribution of the leading order. Unfortunately, this is not the case. Furthermore, the higher-order loop contributions cannot even be considered much smaller than the leading order contribution. At three loops the contribution is essentially the same. This can be seen on dimensional grounds (there is nothing different about the propagators) and this is true for higher-order loops. It is true that there will be more factors of the coupling constant λ but as we have said before these are generally of order one. So, we can see that the convergence of the sum of all contributions to the vacuum energy density will be roughly m^4.

Often the cosmological constant problem is described as a discrepancy of 120 orders of magnitude, not 56 as we have found here. *Why is this?* The size of this discrepancy comes from the assumption that there is a larger contribution to the vacuum energy density, that scales with the Planck mass to the power 4,

$$\frac{M_{\rm Pl}^4}{\rho_{\rm obs}} \sim \left(\frac{10^{18}\text{GeV}}{10^{-3}\text{eV}}\right)^4 \sim 10^{120}. \tag{1.30}$$

It is not possible to know for sure that there is a contribution to the vacuum energy density of this scale. This is separate to the cosmological constant problem coming from quantum field theory, this problem concerns the unknown theory of quantum gravity. Most physicists do not expect that current formulations of quantum field theory are valid at these energies, one should expect some higher energy theory to take over at these scales. However, we have already seen that there is cosmological constant problem of 56 orders of magnitude, coming from well understood quantum field theory.

There are a great number of suggestions for the possible theory of quantum gravity (for a review see Schulz (2014)), the truth is that physics at these high energies are unknown, it may not even be accurately described by a field theory. Presumably somewhere at or below the Planck mass a field theory description becomes applicable. We might also postulate that perhaps at an inflationary scale or the (grand unified theory) GUT scale ($\sim 10^{16}$GeV) there might exist particles with this kind of mass increasing the size of the known cosmological constant problem.

1.3.2 The radiative stability problem

We have seen that the cosmological constant problem comes from quantum fluctuations in the vacuum. Calculating the vacuum energy density to higher precision by: changing the cut-off scale, adding more heavy particles, trusting quantum field theory to a higher energy, or by calculating to higher-loop order will all result in additional terms of order M^4.

Previously, we used renormalization to remove an infinite contribution to our calculation of the two-loop vacuum diagram. We could possibly use the tool of renormalization to remove these large but finite contributions of M^4. There is a problem though that we would have to renormalize every-time we changed the cutoff[11]. We are going to refer to this problem that exists with both perturbative and non-perturbative (Wilsonian) methods of quantum field theory as the radiative stability problem.

Recall that all cosmological probes of the cosmological constant cannot distinguish between the vacuum energy and the bare cosmological constant. What is actually measured by cosmologists is the linear sum of these two $\Lambda_{\text{tot}} = \Lambda + \rho_{\text{vac}}$. We know that the observed cosmological constant is around $(\text{meV})^4$. We also know that ρ_{vac} is at least $(170\text{GeV})^4$. Therefore, the bare cosmological constant must be a number of the same magnitude as ρ_{vac} and cancel exactly to 56 decimal places with ρ_{vac}, leaving a remainder which is the observed cosmological constant. This is the cosmological constant problem.

The radiative stability problem says, that every time the precision of your calculation is changed, a completely different value for ρ_{vac} is found. The value will still be roughly the same order of magnitude but the bare cosmological constant will have to be changed, so that it now cancels this new value for ρ_{vac} exactly to 56 decimal places with a remainder consistent with observations. The tuning of the bare cosmological constant required to match observation is unstable against any change in the effective description, be it changing the loop order in perturbation theory, or changing the cutoff.

1.4 Einstein and Jordan frames

Scalar-tensor theories of gravity can be written in more than one way. The Lagrangian in scalar-tensor theory can be expressed in the Jordan frame in which the scalar field or some function of it multiplies the Ricci scalar, or in the Einstein frame in which the Ricci scalar is not multiplied by the scalar field. It is possible to move between the two frames by means of a transformation. In this section we will transform an example action from the Einstein to the Jordan frame to illustrate the usefulness of transforming an action in

[11] A pedagogical explanation of the cosmological constant problem and the sequester theory is given in Padilla (2015). In these notes what we have called the "radiative stability problem" here is called the "comsological constant problem". The need to repeatedly fine tune whenever the higher loop corrections are included reflects the cosmological constant's sensitivity to the details of the (unknown) high energy physics

this way [12].

There are many examples in the literature of transforming gravitational theories from the Jordan frame to the Einstein frame (see Faraoni et al. (1999); Flanagan (2004)). There are less examples in the literature of this the other way around. This is because many alternative gravity theories are formulated by using a conformal transformation mapping the Jordan frame to the Einstein frame.

The Einstein-Hilbert action minimally coupled to a scalar-tensor Lagrangian $\mathcal{L}_{\rm ST}$, written in the Einstein frame, is,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \mathcal{L}_{\rm ST}(g^{\mu\nu}, \phi, \nabla\phi, ...)\right] + S_{\rm m}(g^{\mu\nu}, \Psi_i). \tag{1.31}$$

When this action is varied with respect to the inverse metric, the resulting equations of motion will always result in the *Einstein* tensor (see Equation (1.2)) summed with other parts of the theory. The Einstein frame description of scalar tensor theories is particularly useful in the context of gravitational lensing Faraoni and Gunzig (1998, 1999) for deducing the distribution of matter.

The Jordan frame is especially useful for looking at alternative couplings. Here, we illustrate the differences between the frames. Suppose that the matter action is not minimally coupled to the scalar field ϕ as in Equation 1.31 but instead conformally coupled. We will also specify our scalar tensor Lagrangian to be a massive scalar with a linear kinetic term. We can write this action in the Einstein frame,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2\right] + S_{\rm m}(\tilde{g}^{\mu\nu}, \Psi_i), \tag{1.32}$$

where,

$$\tilde{g}_{\mu\nu} = C^2(\phi)g_{\mu\nu}, \tag{1.33}$$

and therefore, $S_{\rm m}(\tilde{g}^{\mu\nu}, \Psi_i) = S_{\rm m}(C^{-2}(\phi)g^{\mu\nu}, \Psi_i)$. When written in the Einstein frame, as it is here, the conformal coupling is seen in the matter sector of the action. This non-minimal coupling between the scalar field ϕ and the matter fields Ψ_i can be seen as a fifth fundamental force. This means test particles do not follow the geodesics of $g_{\mu\nu}$ but of $\tilde{g}_{\mu\nu}$ instead. The coupling also means that the scalar field ϕ can exchange energy

[12] The sequester theory was first written in the Einstein frame Kaloper and Padilla (2014a), later a Jordan frame realisation of the theory D'Amico et al. (2017) inspired further development of the theory. The theory explored in Chapter 4 of this thesis is first written in the Einstein frame before it is transformed into the Jordan frame. The context of these two points is the reason that this section of the thesis transforms an example scalar tensor theory from the Jordan frame to the Einstein frame and not the other way around, as is common in the literature.

with the standard model sector and so it is no longer true that the energy-momentum tensor is covariantly conserved, instead the combination $T^{\mu\nu}_{\text{total}} = T^{\mu\nu}_{\text{m}} + T^{\mu\nu}_{\phi}$ is conserved, $\nabla_\mu T^{\mu\nu}_{\text{total}} = 0$.

With a change of frame we could move where we see the coupling, whilst still expressing a physically equivalent model. To express the action in terms of $\tilde{g}$ we will make use of Equation 1.33, as well as the following,

$$g^{\mu\nu} = C^2(\phi)\tilde{g}^{\mu\nu}, \tag{1.34}$$

$$\sqrt{-g} = C^{-4}(\phi)\sqrt{-\tilde{g}}, \tag{1.35}$$

$$R = C^2(\phi)\tilde{R} + 6C(\phi)\tilde{\Box}C(\phi) - 12\tilde{g}^{\alpha\beta}\tilde{\nabla}_\alpha C(\phi)\tilde{\nabla}_\beta C(\phi). \tag{1.36}$$

We can now express everything in terms of the tilded frame[13],

$$\begin{aligned} S = \int d^4x\sqrt{-\tilde{g}}\Big[&\frac{M_{\text{P}}^2}{2C^2(\phi)}(\tilde{R} + 6C(\phi)^{-1}\tilde{\Box}C(\phi) - 12C^{-2}(\phi)\tilde{g}^{\alpha\beta}\tilde{\nabla}_\alpha C(\phi)\tilde{\nabla}_\beta C(\phi)) \\ &- C^{-2}(\phi)g^{\tilde{\mu}\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi - \frac{1}{2C^4(\phi)}m^2\phi^2\Big] + S_{\text{m}}(\tilde{g}^{\mu\nu}, \Psi_i). \end{aligned} \tag{1.37}$$

Here, ϕ couples non-minimally to curvature (see the coefficient of the Ricci tensor depends on ϕ) but matter couples directly (minimally) to the Jordan frame metric[14]. Test particles follow the geodesics of the tilded, Jordan frame metric. And the energy-momentum tensor is conserved as follows, $\tilde{\nabla}_\mu \tilde{T}^{\mu\nu}_{\text{m}} = 0$.

Changing between the Einstein and Jordan frames is a powerful tool that we use throughout the rest of this thesis.

1.5 Introduction to the sequester

How might the physics community solve the cosmological constant problem? A symmetry, that prevents threshold corrections piling up, was originally thought to be a possible solution Zumino (1975). However, a symmetry solution has been unsuccessful in this endeavour Weinberg (1989).

[13]We have written covariant derivatives with tildes here as they are constructed using the metric $\tilde{g}_{\mu\nu}$. However, for a scalar ϕ it is true that the covariant derivative is equal to the partial derivative $\nabla_\alpha\phi = \partial_\alpha\phi$. Because the partial derivative is not itself constructed from the metric, we know that $\tilde{\partial}_\alpha\phi = \partial_\alpha\phi$.

[14]One could also rescale the scalar field and its mass, $\phi = C\tilde{\phi}$ and $m = C\tilde{m}$, see Dabrowski et al. (2009). If we had chosen the conformal factor $C(\phi)$ to be a constant and not depend on the field ϕ or any spacetime component, then making the aforementioned transformations of the scalar field and mass would yield a scalar field $\tilde{\phi}$ that is minimally coupled to the metric $\tilde{g}$. Although the scalar field ϕ would still be non-minimally coupled to $\tilde{g}$.

Dark energy theories such as Quintessence and Chameleons (inflation at late times) generally assume that there is no cosmological constant and that this comes about by some unknown mechanism. Dark energy is instead sourced by a scalar field (in the particular cases of Quintessence and Chameleons the potential energy of the scalar field is the source of the dark energy).

The anthropic solution to the cosmological constant problem says that the bare cosmological constant does in-fact cancel the vacuum energy density to the large number of decimal places leaving behind the observed cosmological constant that we see today. Given an infinite number of possible universes all with different values of the bare cosmological constant. Only universes with a small Λ_{tot} such as the one we live in are reasonably capable of structure formation and in turn supporting intelligent life. [15]. Anthropic arguments gained credibility with the development of string theory (see Linde (2017); Bull et al. (2016) for details).

A holistic theory of gravity would not only satisfy observations and solve the cosmological constant problem it would also be UV complete (make accurate predictions at high energies or short distances). Making a UV complete or quantum theory of gravity is not the focus of this thesis. We can assume that general relativity is an effective theory of gravity, the low energy limit of some higher energy theory Donoghue (1994). In a similar way Newtonian gravity is a low energy limit of general relativity.

There are many different ways to attack the cosmological constant problem, this thesis focuses on just one possible solution, the sequester. The sequester breaks the weak equivalence principle with a global modification to gravity at infinite wavelength. In the sequester theory there exists a new kind of matter that does not gravitate where the vacuum energy density flows to.

When cosmologists measure the cosmological constant (of which one contribution comes from the vacuum energy density) what is actually measured is the curvature of the universe and the ratios of the different components in it. We believe quantum effects generate large vacuum energies, but cosmologists measure very small curvatures. The sequester theory attempts to break this direct link between the vacuum energy and the curvatures measured in cosmology. This is done without affecting other instances of quantum corrections which are responsible for other observations such as the Lamb shift discussed earlier. One could think of this as breaking the weak equivalence principle but only for the vacuum energy density.

[15] This is a very brief summary of the anthropic solution to the cosmological constant problem. For a much more rigorous explanation see Banks et al. (2001) and for historical context see Weinberg (1987).

We can think of the cosmological constant as the gravitational source of longest (infinite) wavelength. For these reasons there have been more than one attempt to modify General Relativity at very large distances. The basic idea behind degravitation is to have gravity become suppressed beyond some large scale L, so that ultra long wavelength sources like the cosmological constant no longer gravitate Arkani-Hamed et al. (2002); Dvali et al. (2007). Other theories that modify gravity on large finite scales exist e.g. the fat graviton theory Sundrum (2004b,a).

The sequester theory modifies gravity only on infinite wavelengths, a global modification of gravity. Locally the sequester theory mimics general relativity so cannot be constrained using fifth force tests. The sequestering scenario Kaloper and Padilla (2014a,b, 2015); Kaloper et al. (2016a); Kaloper and Padilla (2017) aims to resolve the problem of radiative instability of the vacuum energy.

The basic idea of sequestering the vacuum energy is that the bare cosmological constant Λ [16] is promoted to a global dynamical variable. Λ is still a spacetime constant, but we can vary over it in the action. An additional term σ is added to the standard Einstein-Hilbert Lagrangian at the most global scale,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \lambda^4\mathcal{L}_m\left(\lambda^{-2}g^{\mu\nu}, \Psi\right) - \Lambda\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right), \tag{1.38}$$

where, μ is some mass scale. This is done with the aim of coupling the global dynamical variable Λ to the standard model in such a way that we can remove the radiative stability problem. The precise form of the sequestering function σ (which does not couple to the metric and in turn gravity) is determined by phenomenology, and the global parameter λ fixes the hierarchy between matter scales and the Planck mass.

We will see in the next Chapter that the sequester mechanism successfully cancels the vacuum energy at each and every order in the expansion and leaves behind a very small residual cosmological constant that depends on the historic average of locally excited matter and not on the vacuum energy. This residual cosmological constant is calculated approximately by Kaloper & Padilla to be smaller by a few orders of magnitude than the vacuum energy density we observe today Kaloper and Padilla (2014a) and so an extra dark energy is required to explain this.

What is the physical mechanism behind this? One can think of the sequester as the vacuum energy flowing to another kind of matter which does not gravitate, breaking the weak equivalence principle. The rest of the matter in the Universe remains untouched and so all other physics apart from that which relates to the vacuum energy remains the

[16] From here on the bare cosmological constant will no longer be referred to as Λ_b but simply Λ

same. To a person wishing to solve the radiative stability problem this sounds fantastic. Although there is a caveat, that we require a UV complete theory to properly explain this new mysterious type of matter which the vacuum energy flows to and which breaks the weak equivalence principle.

In the next Chapter we will explore the sequester in detail. We do not concern ourselves with the nature of the UV complete theory. Instead we examine whether different realisations of the sequester Kaloper and Padilla (2014a,b); Kaloper et al. (2016b); Kaloper and Padilla (2017) provide radiative stability for all kinds of vacuum energy loops, including loops containing gravitons.

Chapter 2

Ultraviolet sensitivity of the cosmological sequester

Abstract

We revisit the "sequester" proposal of Kaloper, Padilla and collaborators, in which the amplitude of the cosmological constant is decoupled from large contributions due to loops containing Standard Model particles. We review the different formulations of the model that have appeared in the literature, and estimate the importance of a particular class of quantum corrections—those that dress the interaction between the "rigid" scalars and infrared properties of the spacetime such as its 4-volume and integrated curvature. In formulations that do not adequately sequester graviton loops we argue that dressing of these interactions causes further failures of complete sequestration. We estimate the size of the effect and find that it is typically smaller than the cosmological term directly induced by loops containing a single virtual graviton. Meanwhile, in the most developed formulation of the scenario (where a rigid scalar couples to the Gauss–Bonnet density), this dressing can be absorbed into a rescaling of the rigid fields and is therefore harmless.

2.1 Introduction

It is nearly 40 years since the cosmological constant problem was first stated clearly Wilczek (1984); Weinberg (1989). (For the earlier history, see Ref. Rugh and Zinkernagel (2002); Straumann (2002).) Despite immense efforts over the intervening decades, it remains the most enigmatic component of the concordance cosmological model. The problem is simple to state. Observation requires the cosmological constant Λ to dominate the present Hubble rate, and therefore $3H_0^2M_{\mathrm{P}}^2 \approx \Lambda$. The measured value of H_0 gives an estimate

$\Lambda \approx 10^{-12}\,\text{eV}^4$. Meanwhile quantum contributions to Λ from Standard Model particles are much larger. Why, then, is the measured value so small?

The case for 'fine tuning'.—The operational meaning of Λ is less clear than other quantities that are known to receive large quantum corrections, such as the running couplings that appear in scattering amplitudes, because it couples only at wavenumber zero where scattering does not occur. Nevertheless, like any low-energy constant, Λ presumably can be divided into an incalculable ultraviolet contribution Λ_{UV} from unknown physics lying above the Standard Model, and an infrared contribution Λ_{IR} generated by quantum corrections with Standard Model particles running in the loops. We expect $\Lambda_{\text{IR}} \sim m_{\text{t}}$ from loop diagrams containing the top quark, which is the heaviest Standard Model particle.[17]

It follows that $\Lambda = \Lambda_{\text{UV}} + \Lambda_{\text{IR}}$ should be of order $m_{\text{t}}^4 \sim (175\,\text{GeV})^4$ or larger unless Λ_{UV} is accurately balanced to cancel large contributions from Λ_{IR}^4. The measurement $\Lambda \sim 10^{-12}\,\text{eV}^4$ apparently implies that Λ_{UV} is balanced so that cancellation occurs to roughly 56 decimal places.[18] The scales that contribute to Λ_{UV} and Λ_{IR} are very different, so there is no reason why Λ_{UV} should be related to Standard Model energies. This makes it unlikely that cancellation happens by accident.

Unless new physics changes the relationship between Λ, Λ_{IR} and Λ_{UV}, the most plausible conclusion is that whatever determines Λ_{UV} must be constrained by some principle forcing Λ to be nearly zero. Such a principle would strongly violate decoupling, because it would make the Wilson coefficients of the low-energy action into highly sensitive functions of the ultraviolet boundary condition. The apparent tuning we observe would be a consequence of this exquisite sensitivity.

It is certainly possible that the correct resolution of the cosmological constant problem involves a failure of decoupling along these lines. Unfortunately, modern ideas in particle

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[17] It is often said that the low-energy calculation yields $\Lambda_{\text{IR}} \sim M_{\text{P}}^4$, but this is not justified. Although the vacuum energy computed with a hard momentum cutoff is quartically divergent, it must be remembered that *cutoffs do not track dependence on heavy masses* Burgess and London (1993). The low-energy theory cannot yield a trustworthy dependence on any mass scale heavier than it contains itself, and the heaviest mass described by the effective Lagrangian for the Standard Model is the top mass m_{t}. See also §4.2 of Ref. Lyth (2011), and Refs. Sloth (2010); Maggiore (2011); Mangano (2010), which show explicitly that the quartically divergent terms cannot be interpreted as a dark energy component.

If there *is* a contribution to the cosmological constant of order $\sim M_{\text{P}}^4$, it comes from Λ_{UV} and not Λ_{IR}. For example, this might happen if local field theory remains valid all the way up to the Planck scale, and the low-energy gravitational force is generated by integrating out one or more particles of mass $\sim M_{\text{P}}$. But the outcome could be different if local field theory fails as a good approximation to Nature at some much lower scale.

[18] The large number of decimal places required is because cancellation has to occur in $\Lambda_{\text{UV}} + \Lambda_{\text{IR}}$.

physics have not yielded any candidate principle that could be responsible for the smallness of Λ. Moreover, the failure of decoupling makes robust low-energy model building difficult. For these reasons it is now more common to look for an alternative resolution.

Overview of this paper.—In this paper we revisit the "sequester" proposal of Kaloper & Padilla Kaloper and Padilla (2014a,b); Kaloper et al. (2016b); Kaloper and Padilla (2017). This is a concrete scenario for new physics that changes the argument given above by removing ("sequestering") the low-energy contribution from all Standard Model particles. The outcome is that the observed cosmological constant Λ would be set by Λ_{UV}, unless there are further contributions from new unsequestered sectors.

By itself the sequester (or at least its simplest versions) would not explain the observed magnitude of Λ.[19] Even if all matter species participate in sequestration, there would still be a puzzle if we expect Λ_{UV} to receive contributions larger than $10^{-3}\,\mathrm{eV}$. This might be the case, for example, if low-energy Einstein gravity is an effective description generated by integrating out one or more Planck-mass particles. The advantage of the sequester is that the small observed value no longer requires cancellations between Λ_{UV} and Λ_{IR}. We express this by saying that its value is *technically natural* within the Standard Model. Whether or not it is technically natural with respect to the ultraviolet model is a question that can be resolved only when that theory is specified.

The status of arguments based on technical naturalness has been called into question following the discovery of a Higgs particle at $M \sim 125\,\mathrm{GeV}$ without new accompanying particles Richter (2006); Hossenfelder (2018). In the formulation we are using, "naturalness" has a clear meaning in terms of sensitivity—or lack of it—to large corrections between widely separated scales Giudice (2013); Williams (2015); Giudice (2019).[20] This is not merely an aesthetic choice, and accordingly Nature may or may not be "natural" in our sense. Nevertheless, it is reasonable to expect this concept of naturalness to be a useful guide because experience has shown that the vast majority of physical phenomena *do* decouple in this way.

Clearly we should not be satisfied with a theory in which Λ is made technically natural

[19] In §2.3.4 we will see that the most developed version of the sequester would absorb Λ_{UV} in addition to Λ_{IR}, at the cost of introducing a new cosmological-like term associated with an unknown scale μ. See Eqs. (2.33) and (2.34). Therefore, no matter what strategy we choose, it seems that one can not arrive at an unambiguous prediction for the observed value of Λ.

[20] This is a broader definition than the original concept of technical naturalness due to 't Hooft 't Hooft (1980). 't Hooft's criterion that a small parameter y is natural if the symmetry of the theory is enlarged in the limit $y \to 0$ is a sufficient but not necessary condition for widely separated scales to decouple in this sense.

at the expense of other low-energy constants that receive large ultraviolet corrections. If this occurs we have not removed the problem, but merely translated it from one low-energy sector to another. In this paper we aim to apply this test to the sequester model.

Synopsis.—Two principal variants of the sequester have been discussed in the literature. In the first version, one works in the Einstein frame and couples the sequestered sectors to a conformally rescaled metric. This version was introduced in Refs. Kaloper and Padilla (2014a,b); see Ref. Padilla (2015) for a pedagogical description. We describe it as the 'Einstein frame model'. The conformal rescaling dynamically adjusts mass scales in the sequestered sector relative to the fixed Planck scale. A global constraint couples the cosmological term to this conformal factor, allowing it to absorb contributions from pure matter loops. In this version, loops involving virtual gravitons are known to reintroduce unsequestered corrections to the observed Λ. We discuss this model and the degree to which it ameliorates ultraviolet sensitivity of the cosmological constant in §2.2.

A second variant was introduced in Refs. Kaloper et al. (2016b); Kaloper and Padilla (2017). In this version one works in the Jordan frame and there is no auxiliary rescaled metric. There are two global constraints, the first of which couples the gravitational scale to the mean Ricci curvature of spacetime. The second couples the cosmological constant to the total spacetime volume and a physical mass scale μ, which is *a priori* unknown. The is the 'Jordan frame model.' In this version one can adjust the way in which the global constraints couple to spacetime curvature so that loops of virtual gravitons are also absorbed. This version of the sequester and its ultraviolet properties are discussed in §2.3. We conclude in §2.4.

Notation.—We work in units where $c = \hbar = 1$. The (reduced) Planck mass is $M_{\mathrm{P}} \equiv (8\pi G)^{-1/2} = 2.435 \times 10^{18}$ GeV. We express the cosmological constant in terms of an energy scale Λ with engineering dimension $[\mathrm{M}]^4$. The corresponding "cosmological term" in the Einstein equations is Λ. We generally frame our calculations in Minkowski space to avoid unneeded complexities associated with curved spacetime; because ultraviolet properties do not depend on these curvature scales, this procedure does not forfeit any essential generality.

2.2 Einstein frame model

2.2.1 The sequester action

In this section we briefly review the sequester mechanism in Einstein frame Kaloper and Padilla (2014a,b), and discuss its ultraviolet sensitivity. The gravitational action is written in terms of the Einstein-frame metric $g_{\mu\nu}$. Sequestration of one or more matter sectors is achieved by coupling them to a conformally rescaled ('Jordan frame') metric $\tilde{g}_{\mu\nu} = \lambda^2 g_{\mu\nu}$, viz.

$$\begin{aligned} S = \int \mathrm{d}^4x\, \sqrt{-g}\bigg(\frac{M_{\mathrm{P}}^2}{2} R(g) - \Lambda + \Lambda_{\mathrm{UV}} - \lambda^4 \mathcal{L}_{\mathrm{m}}(\tilde{g}^{\mu\nu}, \Psi) \bigg) \\ + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right). \end{aligned} \tag{2.1}$$

If multiple sectors are to be sequestered their actions should appear additively. In Eq. (2.1), $R(g) = g^{\mu\nu}R_{\mu\nu}(g)$ is the Ricci scalar constructed using the Einstein-frame metric $g_{\mu\nu}$, $\mathcal{L}_{\mathrm{m}}$ is a matter Lagrangian density, and Ψ schematically stands for the different species of sequestered matter fields. We assume these to be the Standard Model fields. The low energy contribution to the cosmological constant, Λ_{IR}, does not appear in Eq. (2.1) explicitly. It is generated by the infrared part of loop corrections to $\mathcal{L}_{\mathrm{m}}$. The bare cosmological constant (if there is one), plus any contributions from unsequestered sectors that have been integrated out to produce (2.1), are included in Λ_{UV}.

The quantity Λ is no longer the combination $\Lambda_{\mathrm{UV}} + \Lambda_{\mathrm{IR}}$, but is rather a new field that can loosely be regarded as a counterterm for Λ_{IR}. In particular, although it participates in the path integral, we take Λ to have no local degrees of freedom. It is determined classically by extremization of the action. The dimensionless conformal rescaling λ is taken to be a field of the same kind.

The global term σ is a function of Λ and λ in the specific dimensionless combination $\Lambda/(\lambda\mu)^4$. Critically, it does *not* couple to either the Einstein- or Jordan-fame metric, and therefore does not source the global gravitational field. The scale μ has dimension [M], but its precise meaning depends on the definition of σ. We will discuss its significance in more detail in §2.3.2 below. Finally, for reasons to be explained below, we should take σ to be an odd function of its argument. It is otherwise assumed to be an arbitrary smooth function.

The rigidity of Λ and λ is unusual, but can be given a local, microscopic basis in terms of integrals of a four-form flux F_4 over spacetime Kaloper et al. (2016b). Such a flux is a top-order form in $d = 4$ dimensions and therefore acts as a volume form in the integral $\int F_4$. In particular, this integral can be written without requiring a metric. Borrowing terminology from thermodynamics, we describe terms such as $\int F_4$ that do not scale with $g_{\mu\nu}$ as *intensive*. Ordinary contributions to the action such as $\int(\star 1)$ are conversely *extensive*. Notice that if σ does not scale at least with the coordinate volume of spacetime, this violates Hawking's suggestion that the action should be additive over cobordant regions [21] in order for quantum gravitational amplitudes to superpose correctly Hawking (1979).

2.2.2 Low-energy phenomenology

We now consider low-energy solutions to (2.1). First, notice that the matter contribution in (2.1) can be written[22]

$$S_{\mathrm{m}} \equiv -\int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\,\mathcal{L}_{\mathrm{m}}(\tilde{g}^{\mu\nu},\Psi). \tag{2.2}$$

Therefore it is clear that the matter fields Ψ are minimally coupled to the Jordan-frame metric $\tilde{g}_{\mu\nu}$. By taking $\tilde{g}_{\mu\nu}$ to be flat up to corrections from the Newtonian potential, it follows that predictions for laboratory measurements in a weak gravitational field will match those of the unsequestered Standard Model.

We conclude that the masses and other properties of the Standard Model reported by the Particle Data Group Tanabashi et al. (2018) are those measured in $\tilde{g}_{\mu\nu}$. We denote these experimental scales with a tilde, viz. $\tilde{M}_Z$, $\tilde{m}_{\mathrm{t}}$. They are related to scales measured in the metric $g_{\mu\nu}$ by a conformal rescaling $\tilde{M}_Z \to M_Z = \lambda\tilde{M}_Z$.

Sequestering low-energy loops.—Extremization of (2.1) with respect to Λ and λ yields,

$$\frac{\sigma'}{(\lambda\mu)^4} = \int \mathrm{d}^4x\,\sqrt{-g}, \tag{2.3a}$$

$$4\frac{\Lambda}{(\lambda\mu)^4}\sigma' = \int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\,\tilde{T}^{\mu}{}_{\mu} = \int \mathrm{d}^4x\,\sqrt{-g}\,T^{\mu}{}_{\mu}, \tag{2.3b}$$

where a prime $'$ denotes differentiation of σ with respect to its argument, and the Jordan-frame energy–momentum tensor $\tilde{T}_{\mu\nu}$ measured with respect to $\tilde{g}^{\mu\nu}$ is defined by

$$\tilde{T}_{\mu\nu} \equiv -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta S_{\mathrm{m}}}{\delta\tilde{g}^{\mu\nu}}. \tag{2.4}$$

A similar definition applies for the Einstein-frame energy-momentum tensor $T_{\mu\nu}$, which is measured with respect to $g^{\mu\nu}$. The two definitions are related by $\tilde{T}_{\mu\nu} = \lambda^{-2}T_{\mu\nu}$. We

[21] Two compact boundaryless manifolds M and N are cobordant if there exists a compact manifold with boundary W such that $\partial W = M - N$. In other words two manifolds are cobordant if their disjoint union bounds a manifold.

[22] We take this as the definition of the matter action S_{m}.

assume $\sigma' \neq 0$ at the extremum. To allow consistent solutions with $\Lambda < 0$ but $\lambda > 0$ we require $\sigma'(x)$ to be even, and hence $\sigma(x)$ must be odd, as stated above.

Taking the ratio of Eqs. (2.3b) and (2.3a) yields a constraint for Λ,

$$\Lambda = \frac{1}{4}\langle\!\langle T^{\mu}{}_{\mu}\rangle\!\rangle, \tag{2.5}$$

where $\langle\!\langle \cdots \rangle\!\rangle$ denotes *spacetime* averaging in the metric $g_{\mu\nu}$, i.e. $\langle\!\langle Q\rangle\!\rangle \equiv \int \mathrm{d}^4x\,\sqrt{-g}\,Q/\int \mathrm{d}^4x\,\sqrt{-g}$. Since we assume σ is differentiable, Eq. (2.3a) requires the volume of spacetime to be finite if we wish to avoid $\lambda = 0$. (This would conformally rescale all masses in the sequestered sector to zero.) It follows that the spacetime average $\langle\!\langle Q\rangle\!\rangle$ can be defined, even if it is difficult to evaluate in practice.

The Einstein equations that follow from (2.1) are

$$\begin{aligned} M_{\mathrm{P}}^2 G_{\mu\nu} &= T_{\mu\nu} - (\Lambda - \Lambda_{\mathrm{UV}})g_{\mu\nu} \\ &= T_{\mu\nu} - \frac{1}{4}\langle\!\langle T^{\mu}{}_{\mu}\rangle\!\rangle g_{\mu\nu} + \Lambda_{\mathrm{UV}}^4 g_{\mu\nu}, \end{aligned} \tag{2.6}$$

where $G_{\mu\nu}(g) = R_{\mu\nu}(g) - R(g)g_{\mu\nu}/2$ is the usual Einstein tensor constructed from $g_{\mu\nu}$. As explained above, the σ term in the action does not couple to $g_{\mu\nu}$ and therefore does not source a long-wavelength gravitational field. Diffeomorphism invariance guarantees that any matter loops renormalize the cosmological term in $\mathcal{L}_{\mathrm{m}}$ measured using $\tilde{g}_{\mu\nu}$ (see Fig. 2.2), and therefore

$$\tilde{T}_{\mu\nu} = \tilde{\Lambda}_{\mathrm{IR}}\tilde{g}_{\mu\nu} + \tilde{\tau}_{\mu\nu}(\tilde{g}, \Psi, \cdots), \tag{2.7}$$

where the ‘subtracted’ energy–momentum tensor $\tilde{\tau}_{\mu\nu}(\tilde{g}, \Psi, \ldots)$ vanishes outside matter[23]. We have added a tilde to Λ_{IR} to indicate that it is built from scales such as $\tilde{m}_{\mathrm{t}}$ measured in a homogeneous gravitational field. When expressed in terms of $T_{\mu\nu}$ we obtain

$$T_{\mu\nu} = \lambda^4\tilde{\Lambda}_{\mathrm{IR}}g_{\mu\nu} + \lambda^2\tilde{\tau}_{\mu\nu}(\tilde{g}, \Psi, \ldots), \tag{2.8}$$

It follows that the effective Einstein equation can be written

$$M_{\mathrm{P}}^2 G_{\mu\nu} = \tau_{\mu\nu} - \frac{1}{4}\langle\!\langle \tau^{\rho}{}_{\rho}\rangle\!\rangle g_{\mu\nu} + \Lambda_{\mathrm{UV}} g_{\mu\nu}. \tag{2.9}$$

The conclusion is that, in the Einstein equation, the low-energy contribution Λ_{IR} is removed to all orders in the loop expansion of $\mathcal{L}_{\mathrm{m}}$.

What has been achieved?—To reiterate, this does not “solve” the cosmological constant problem because we still have no means to estimate Λ_{UV}. Depending on the ultraviolet

[23] $\tilde{\tau}_{\mu\nu}$ are, by definition, local excitations about the vacuum and so are zero where there is only vacuum and no other matter. See Padilla (2015).

model, it may be large. But since an estimate of Λ_{UV} was never the aim of the sequester, this criticism is unfair. Instead, what has been achieved is that *if* Λ_{UV} *can somehow be made small, its impact on the global spacetime geometry is not destabilized by loops at much lower scales.*

Loosely speaking, this analysis shows that the sequester is not a 'field theory' mechanism, in the sense that the properties of loops are unmodified in the ultraviolet. Rather, we have added a new form of matter Λ that is constrained by its field equation to cancel the portion of the vacuum energy sourced by matter loops. Ordinarily this would be of no benefit, because the energy density associated with Λ would itself gravitate. As explained above, the special feature of the action for Λ is that its σ part does not source any gravitational field. Heuristically, this allows us to 'degravitate' or 'sequester' the vacuum energy by storing it in σ. When stored in this way the matter loops are gravitationally inert.

After vacuum loops have been sequestered, the effective source term for the gravitational field is the subtracted energy–momentum tensor $\tau_{\mu\nu}$ computed in the metric $g_{\mu\nu}$, together with a correction from its spacetime volume average $\langle\!\langle \tau^{\rho}{}_{\rho} \rangle\!\rangle$. The size of this correction was estimated in Refs. Kaloper and Padilla (2014a,b), who considered a model in which the unsequestered contribution Λ_{UV} was set to zero.

Nevertheless, there is something surprising about this outcome. We are still working in the context of local field theory, with its characteristic poor control of ultraviolet effects. Where has the original ultraviolet sensitivity of the cosmological term gone? The sequester *does* contain a new physical ingredient, in the form of the σ-term that is shielded from gravity. However, we have not introduced a new physical principle that forces Λ to capture the entirety of Λ_{IR} in this non-gravitating sector. Therefore one might worry that quantum corrections 'detune' the dynamical equation for Λ, preventing complete sequestration of Λ_{IR} and reintroducing the low-energy cosmological term.

Radiative corrections.—Indeed, when discussing any proposed solution to the cosmological constant problem it is *never* sufficient to work at tree level. Like any naturalness problem, the cosmological constant problem is intrinsically quantum mechanical because it is only in a quantum theory that loop corrections generate direct correlations between widely separated scales. The conclusion is that radiative corrections must be included before we can judge the merits of any particular proposal.

A subset of relevant corrections were considered in Refs. Kaloper and Padilla (2014a,b); Kaloper et al. (2016b); Kaloper and Padilla (2017). First, these authors considered a symmetry $\lambda \to \Omega\lambda$, $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$, $\Lambda \to \Omega^4 \Lambda$ under which Eq. (2.1) is invariant. They argued this was sufficient to guarantee that, to all orders in matter loops, Λ_{IR} would couple in the same way as the tree-level vacuum energy. In our presentation this symmetry is implied by coupling $\mathcal{L}_{\text{m}}$ to the Jordan-frame metric $\tilde{g}_{\mu\nu}$ but Λ to the Einstein-frame metric $g_{\mu\nu}$. The loop-level behaviour of Λ_{IR} then follows from diffeomorphism invariance with respect to $\tilde{g}_{\mu\nu}$. We will give a pedestrian proof of these properties in §2.2.3 below, based on analysis of Feynman diagrams. As we show there, like all global symmetries, this one is broken by coupling to gravity.

Second, Refs. Kaloper and Padilla (2014a,b); Kaloper et al. (2016b); Kaloper and Padilla (2017) studied the symmetry $\Lambda \to \Lambda + \lambda^4 \nu^4$, $\mathcal{L}_{\text{m}} \to \mathcal{L}_{\text{m}} - \nu^4$ which they suggested would guarantee that Λ absorbed Λ_{IR} to all orders in matter loops. [That is, that Λ and $T_{\mu\nu}$ would appear additively in the Einstein equation as in Eq. (2.6).] This last symmetry is not in fact a transformation of the fields that participate in the action, and is not respected by quantum corrections.

2.2.3 Ultraviolet sensitivity in Einstein frame

This list does not exhaust the loop corrections to Eq. (2.1). In particular, the analysis of Refs. Kaloper and Padilla (2014a,b); Kaloper et al. (2016b); Kaloper and Padilla (2017) leaves open the issue of (i) corrections to the intensive global function σ that "stores" the unwanted large loop terms; and (ii) corrections to the extensive interaction $\int \mathrm{d}^4x \sqrt{-g}\,\Lambda$. To study corrections to σ would require a microscopic theory that explains how the flux F_4 is supported. The sequester proposal does not aim to provide such a description. (For recent attempts to describe an ultraviolet completion of this kind, see Refs. Padilla (2019); Bordin et al. (2020); El-Menoufi et al. (2019); Sobral-Blanco and Lombriser (2020); Alexander et al. (2020).) We comment on this in §2.4. On the other hand, the interaction term couples to spin-2 excitations of the metric $g_{\mu\nu}$, and will therefore be "dressed" by loops containing off-shell quanta associated with these excitations [24] (cf. Ref. Oda (2017a)). This is a model independent effect, in the sense that it does not depend on the microscopic origin of F_4. In this section we aim to enumerate these corrections and quantify their impact.

Loops respect diffeomorphism invariance.—First, we pause to prove the property stated above, that pure matter loops generate a cosmological term scaling as λ^4 to all orders in the loop expansion. This follows from diffeomorphism invariance with respect to $\tilde{g}_{\mu\nu}$, but

[24]Expanding 2.1 perturbativly in excitations of the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ etc. will mean that we have factors of h multiplying parts of the matter Lagrangian and so we see an interaction. So we have an interaction between spin-2 quanta and standard model particles. In fact any standard model loops will be accompanied by graviton loops with Λ insertions. See 2.1

can also be proved by direct analysis of Feynman diagrams. The results will assist us in an analysis of corrections to the Λ coupling, to be given below.

Consider any operator in $\mathcal{L}_{\mathrm{m}}$ formed from a monomial of n_b bosonic fields and n_f fermionic fields. After replacing measured mass scales $\tilde{M}$ by their conformally rescaled equivalents $M = \lambda\tilde{M}$, and performing the same replacement $k = \lambda\tilde{k}$ for momenta, it can be checked that such an operator scales like $\lambda^{n_b+3n_f/2}$. Meanwhile, a boson propagator scales like λ^{-2} whereas a fermion propagator scales like λ^{-3}. Therefore a diagram containing I_b internal boson lines, E_b external boson lines, I_f internal fermion lines, and E_f external fermion lines will scale like λ^D, where

$$D = -2I_b - 2E_b - 3I_f - 3E_f + \sum_i N_i\Big(n_{b,i} + \frac{3}{2}n_{f,i}\Big). \tag{2.10}$$

N_i is the number of vertices of type i, each of which contains $n_{b,i}$ bosonic fields and $n_{f,i}$ fermionic fields.

Each diagram must satisfy the topological identity $2I + E = \sum_k N_k n_k$, where now I denotes the total number of internal lines (whether bosons or fermions), E denotes the total number of external lines, N_k denotes the number of vertices of type k, and each type-k vertex connects n_k lines. To translate to an operator in the effective action we should amputate external lines. Applying the identity separately to the bosonic and fermionic components of the amputated diagram, it follows that the effective operator will scale like $\lambda^{D_{\mathrm{amp}}}$, where

$$D_{\mathrm{amp}} = E_b + \frac{3}{2}E_f. \tag{2.11}$$

(This analysis applies even if the bosonic and fermionic components are disconnected, provided the assignment of internal and external lines is the one appropriate for the entire diagram.) No matter how complex the diagram, Eq. (2.11) involves only the total number of amputated bosonic and fermionic lines. Such a diagram will renormalize operators that are polynomial in E_b bosonic fields and E_f fermionic fields. The λ-dependence of this renormalization will be $\lambda^{E_b+3E_f/2}$, the same as we deduced above for unrenormalized operators in $\mathcal{L}_{\mathrm{m}}$. The conclusion, as has already been stated, is that pure matter loops preserve the λ-dependence of the coupling in Eq. (2.1).[25]

Stability of global constraint.—Next, we argue that detuning the dynamical equation for Λ can prevent complete sequestration. Specifically, to obtain complete cancellation in the

[25]Recall that this scaling applies *after* conformal redefinition of the masses. From inspection of Eq. (2.1), one might expect the cosmological term generated by matter to scale as $\lambda^4\tilde{M}_{\mathrm{SM}}^4$, where $\tilde{M}_{\mathrm{SM}}$ is some characteristic Standard Model scale. This does not conflict with (2.11) for $E_b = E_f = 0$ because after rescaling $M_{\mathrm{SM}} = \lambda\tilde{M}_{\mathrm{SM}}$ the cosmological term scales as λ^0 as claimed.

Einstein equation, the factor of 4 that appears on the far left of Eq. (2.3b) is required to match a factor of 4 from the trace of the metric in $T^{\mu}{}_{\mu}$. Even a small mismatch of these factors will leave a residual low-energy cosmological term in Eq. (2.6).

While the 4 from the trace $\delta^{\mu}{}_{\mu}$ cannot be modified by ultraviolet effects, the other factor of 4 is a consequence of the power λ^{-4} appearing in the combination $\Lambda/(\lambda\mu)^4$ that enters the global function σ. We will argue below that this factor can be renormalized by ultraviolet effects. It follows that Eq. (2.1) may receive significant corrections from high energies, and therefore fails the test for naturalness in the sense we have defined.

How sensitive is the successful operation of the sequester to the precise factor 4 in Eq. (2.3b)? If it is replaced by $4(1+\alpha)$, the analogue of the sequestered Einstein equation (2.9) becomes

$$\begin{aligned} M_{\mathrm{P}}^2 G_{\mu\nu} = {} & \frac{\alpha}{1+\alpha}\lambda^4 \tilde{\Lambda}_{\mathrm{IR}} g_{\mu\nu} + \tau_{\mu\nu} \\ & - \frac{1}{4(1+\alpha)} \langle\!\langle \tau^{\mu}{}_{\mu} \rangle\!\rangle g_{\mu\nu} + \Lambda_{\mathrm{UV}} g_{\mu\nu}. \end{aligned} \tag{2.12}$$

As expected, there is now incomplete cancellation of Λ_{IR}. To estimate the magnitude of the residual cosmological term requires a numerical estimate for λ. In a finite universe, Eq. (2.3a) yields

$$\lambda \sim \sigma' \frac{H_{\mathrm{age}}}{\mu}. \tag{2.13}$$

The mass scale H_{age} was introduced in Ref. Kaloper and Padilla (2014b) and specifies the lifetime of the universe. This roughly determines the spacetime volume,

$$\frac{1}{H_{\mathrm{age}}^4} \sim \int \mathrm{d}^4 x \, \sqrt{-g}. \tag{2.14}$$

Clearly $H_{\mathrm{age}} < H_0 \sim 10^{-33}\,\mathrm{eV}$.

Ref. Kaloper and Padilla (2014b) suggested that σ should be engineered to obtain $\lambda = \mathrm{O}(1)$. In this case, the effective gravitating cosmological constant is roughly $\alpha\Lambda_{\mathrm{IR}} \sim \alpha\tilde{\Lambda}_{\mathrm{IR}} \sim \alpha\tilde{m}_{\mathrm{t}}^4$, assuming $|\alpha| \ll 1$. With these estimates, $|\alpha|$ must inherit the tuning to 56 decimal places that was previously required for the combination $\Lambda_{\mathrm{UV}} + \Lambda_{\mathrm{IR}}$. If λ is made smaller then α can be relaxed accordingly, but this scenario encounters other difficulties Kaloper and Padilla (2014b).

Extensive corrections to the Λ coupling.—Let us now estimate the model-independent corrections to the extensive coupling $-\Lambda V$, where $V = \int \mathrm{d}^4 x \, \sqrt{-g}$.

First, consider the two-loop correction that appears in the left-hand diagram of Fig. 2.1. Regarded as a contribution to the quantum effective action, this contains a single insertion of a Λ vertex which is "bridged" to a pure Standard Model loop by a pair of spin-2 excitations. [26] This diagram is part of a larger class of diagrams, represented by the right-hand part of Fig. 2.1, in which an arbitrary number of Λ insertions are bridged to a Standard Model sub-diagram (of arbitrary complexity) by graviton lines.

(These diagrams are not the only sources of renormalization for the Λ coupling. We could equally well consider diagrams in which the Λ insertions are embedded within the Standard Model sub-diagram. For our purpose, it suffices to consider only a sub-class of possible renormalizations.)

According to the analysis given above, the λ dependence of this Standard Model sub-diagram can be computed from Eq. (2.10). This time we are not amputating external lines, so the scaling is $\lambda^{D_{\rm sub}}$ where $D_{\rm sub} = -E_b - 3E_f/2$. Because the sub-diagram connects to the ring of Λ insertions via graviton lines (which do not scale with λ) we have $E_b = E_f = 0$. Meanwhile, counting the number of Λ insertions and a factor $M_{\rm P}^{-2}$ for each graviton propagator, we conclude that such a diagram produces an operator $\mathcal{O}$ in the quantum effective action of the form

$$\begin{aligned}\mathcal{O}_n &= \frac{c_n}{(2\pi)^{4(L+1)}} M_{\rm SM}^4 \frac{\Lambda}{M_{\rm P}^4}\left(\frac{\Lambda}{M_{\rm P}^2 M_{\rm SM}^2}\right)^{n-1} \\ &= \frac{c_n}{(2\pi)^{4(L+1)}} \Lambda^n M_{\rm P}^{-2(n+1)} M_{\rm SM}^{6-2n},\end{aligned} \tag{2.15}$$

where L counts the number of loops in the Standard Model sub-diagram and c_n is a Wilson coefficient that can be taken to be of order unity. The scale $M_{\rm SM}$ represents a typical Standard Model mass. After replacing $M_{\rm SM}$ by its experimentally-measurable counterpart $\tilde{M}_{\rm SM} \sim$ TeV, it follows that $\mathcal{O}$ scales like λ^{6-2n}.

To validate Eq. (2.15) we have evaluated the explicit two-loop diagram given in the left-hand part of Fig. 2.1, for which $n = 1$. Using dimensional regularization as the ultraviolet regulator, this yields the expected scaling

$$\mathcal{O}_1 = \frac{c_1}{(2\pi)^8} \frac{\lambda^4 \tilde{M}_{\rm SM}^4}{M_{\rm P}^4} \Lambda. \tag{2.16}$$

The factor $(2\pi)^{-8}$ is included from the measure on the loop integrals. Eq. (2.16) will be the leading correction provided $|\Lambda| \lesssim (M_{\rm P} M_{\rm SM})^2$. This will generally be the case where Λ is dynamically constrained to sequester a loop contribution of order $M_{\rm SM}^4$. The field

[26] Some details of the calculation of this diagram's amplitude can be found, in Appendix A

equations corrected by $\mathcal{O}_1$ are

$$(1-\lambda^4\epsilon)\frac{\sigma'}{(\lambda\mu)^4} = \int \mathrm{d}^4x\,\sqrt{-g}, \tag{2.17a}$$

$$4(1+\lambda^4\epsilon)\frac{\Lambda}{(\lambda\mu)^4}\sigma' = \int \mathrm{d}^4x\,\sqrt{-g}\,T^{\mu}{}_{\mu}, \tag{2.17b}$$

$$M_{\mathrm{P}}^2 G_{\mu\nu} = T_{\mu\nu} - (1+\lambda^4\epsilon)\Lambda g_{\mu\nu} + \Lambda_{\mathrm{UV}} g_{\mu\nu}, \tag{2.17c}$$

where $\epsilon \equiv (2\pi)^{-8} c_1 (\tilde{M}_{\mathrm{SM}}/M_{\mathrm{P}})^4 \ll 1$ and we have dropped terms of $\mathrm{O}(\epsilon^2)$. After eliminating Λ, the Einstein equation can be written, still up to $\mathrm{O}(\epsilon)$ [cf. (2.12)],

$$M_{\mathrm{P}}^2 G_{\mu\nu} = \Big((\lambda^4\epsilon)\Lambda_{\mathrm{IR}} + \Lambda_{\mathrm{UV}}\Big) g_{ab} + \tau_{\mu\nu} - \frac{1}{4}\langle\!\langle \tau^{\rho}{}_{\rho} \rangle\!\rangle. \tag{2.18}$$

We have omitted $\mathrm{O}(\epsilon)$ corrections if they merely perturb existing terms of order unity. The conclusion is that $\mathcal{O}_1$ corrects Eqs. (2.17a)–(2.17b) differently, and therefore renormalizes the relative factor 4 between their left-hand sides.

As in the analysis leading to Eq. (2.12), this $\mathcal{O}_1$-corrected factor no longer cancels the exact 4 coming from $\delta^{\mu}{}_{\mu}$, leaving a residual loop term in Eq. (2.18). This outcome is practically inevitable. In this formulation of the sequester, one is attempting to balance a protected topological quantity $\delta^{\mu}{}_{\mu}$ against the properties of a class of unprotected Lagrangian operators. One is immune from ultraviolet effects but the other is not, making the balance extremely delicate.

Size of residual loop-level term.—How significant is this effect? Taking λ of order unity and $\tilde{M}_{\mathrm{SM}}$ of order 1 TeV makes ϵ of order 10^{-68}, or possibly as large as 10^{-62} if we omit the loop-counting factor $(2\pi)^{-8}$ on the assumption it is partially cancelled by combinatorial factors. Meanwhile if $\tilde{\Lambda}_{\mathrm{IR}}$ is also of order 1 TeV then the residual loop-sourced cosmological term in (2.18) is of order $(10^{-4}\,\mathrm{eV})^4$ to $(10^{-5}\,\mathrm{eV})^4$, or $(10^{-4}\,\mathrm{eV})^4$ to $(10^{-3}\,\mathrm{eV})^4$ if the loop-counting factor is omitted. This is on the boundary of being acceptable given current observational constraints.

The outcome is that whether the Einstein-frame model can survive ultraviolet corrections to the extensive coupling is model-dependent. Assuming the sequestered sector to be the Standard Model gives a barely acceptable phenomenology, with success or failure largely dependent on whether λ is larger or smaller than unity.

Alternatively, if the sequestered sector contains particles that are heavier than the Standard Model—for example, perhaps from a higher-lying supersymmetric sector—then the model is unlikely to survive unless λ is significantly smaller than unity. If the heaviest sequestered mass scale is even 10 TeV then the residual cosmological constant is already in excess of the observed value.

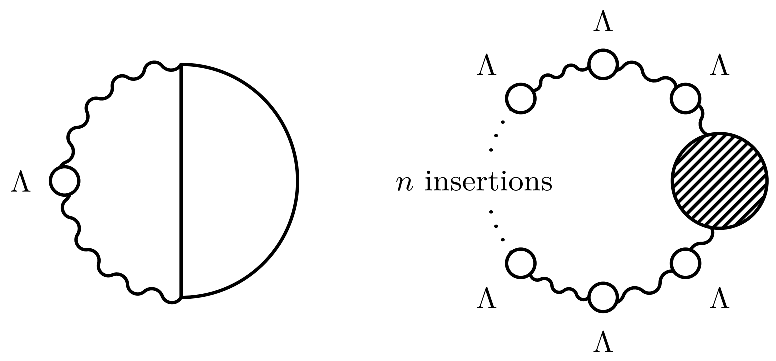

Figure 2.1: Loops renormalizing the coupling of Λ to the spacetime volume. **Left**: loop with single insertion of Λ vertex, represented by the open circle. Wiggly lines represent spin-2 excitations of the metric $g_{\mu\nu}$; solid lines represent Standard Model fields. This diagram renormalizes the coefficient of Λ in σ. **Right**: loop containing n insertions of Λ. This diagram renormalizes the coefficient of Λ^n in σ. The shaded circle represents any Standard Model sub-diagram. The left-hand diagram is a particularly simple example of the class represented by the right-hand diagram.

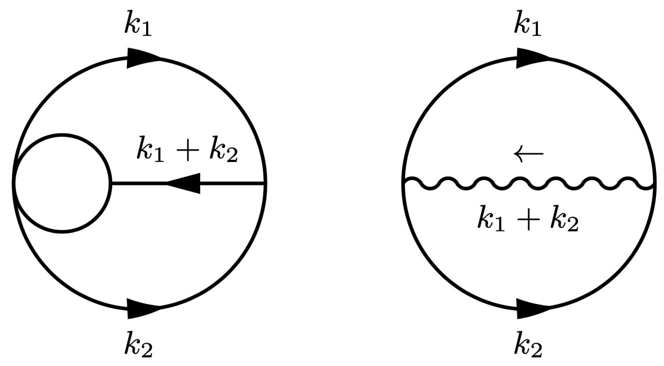

Figure 2.2: Diagrams contributing to renormalization of the cosmological constant in the Einstein frame. Solid lines represent generic Standard Model particles, and wiggly lines represent spin-2 excitations of the metric $g_{\mu\nu}$. **Left**: pure Standard Model loop. This diagram scales like λ^4 when expressed in terms of experimentally-measured mass scales, and is captured by the sequester. **Right**: mixed Standard Model and graviton loop. Because the Einstein-frame graviton propagator is proportional to the hard scale M_{P}^{-2}, this diagram must scale like λ^6 rather than λ^4. As explained in the main text, this implies it is not captured by the sequester in Einstein frame.

2.3 Jordan frame model

2.3.1 Graviton loop corrections

The radiative corrections described in §2.2.3 above involved loop diagrams containing virtual Einstein-frame gravitons that dress the Λ coupling to the spacetime volume. There is a further class of diagrams of this type that are significant in the Einstein frame. These are loop diagrams containing virtual gravitons that contribute to the low-energy cosmological constant. As explained in Ref. Kaloper and Padilla (2017), and as we will review below, these contributions escape the sequester.

Consider the left-hand diagram of Fig. 2.2, which is a loop diagram containing only matter fields. As explained in the discussion leading to Eq. (2.11), this diagram has $E_b = E_f = 0$ external lines and therefore scales like λ^0 multiplied by M_{SM}^4. Expressed in terms of the mass measured in a homogeneous gravitational field this is $\propto \lambda^4 \tilde{M}_{\mathrm{SM}}$.

Diagrams containing virtual gravitons.—Now consider the right-hand diagram of Fig. 2.2. In addition to matter fields (represented by the solid lines), this contains an internal graviton (represented by the wiggly line). Each graviton propagator is proportional to the fixed Planck scale M_{P}^{-2} with no conformal rescaling. It follows on dimensional grounds that a renormalization of the cosmological term with zero external lines, any number of internal matter lines, and n_g internal graviton lines, will scale as $M_{\mathrm{SM}}^{4+2n_g}/M_{\mathrm{P}}^{2n_g} \propto \lambda^{4+2n_g}$.[27] If $n_g \neq 0$ these diagrams do not preserve the λ dependence of Eq. (2.1) Kaloper and Padilla (2017).

2.3.2 Jordan-frame formulation

To solve this, Ref. Kaloper and Padilla (2017) proposed an alternative description of the sequester that we review below. It is based on[28] a reformulation of Eq. (2.1) in the Jordan frame Kaloper et al. (2016b). We wish to analyse the ultraviolet properties of this formulation separately, so we discuss it here before going on to consider the problem of capturing diagrams containing virtual gravitons.

In Eq. (2.1) the gravitational action is built from $g_{\mu\nu}$, but matter couples to $\tilde{g}_{\mu\nu}$. The conformal factor between $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ adjusts the importance of the matter action $\mathcal{L}_{\mathrm{m}}$ relative to the fixed Einstein term $R(g)$. Alternatively, one can build the action solely

[27] Such scalings are possibly modified by powers of logarithms, but we drop these unless they are dominant.

[28] "based on" is an important qualification here, as it is not simply a change of frame that is needed to get from one theory to the other a second flux is also needed to lead to the $\hat{\sigma}$ term.

from the Jordan-frame metric, leaving the relative importance of the Einstein term as a free parameter,

$$\begin{aligned} S = \int \mathrm{d}^4x \ \sqrt{-g}\bigg(\frac{\kappa^2}{2} R - \Lambda + \Lambda_{\mathrm{UV}} - \mathcal{L}_{\mathrm{m}} \bigg) \\ + \sigma\Big(\frac{\Lambda}{\mu^4}\Big) + \hat{\sigma}\Big(\frac{\kappa^2}{M_{\mathrm{P}}^2}\Big). \end{aligned} \tag{2.19}$$

This is the Jordan-frame formulation of the sequester. The gravitational coupling is set by κ, which is related to M_{P} by the global term $\hat{\sigma}$. As with σ, this should be a smooth function of its argument and is assumed to be produced by integration of a second flux, $\int \hat{F}_4$. The Einstein frame metric $g_{\mu\nu}$ does not appear.

Sequestration of low-energy loops.—The field equations that follow from (2.19) are

$$\kappa^2 G_{\mu\nu} = T_{\mu\nu} - \Big(\Lambda - \Lambda_{\mathrm{UV}}\Big) g_{\mu\nu}, \tag{2.20a}$$

$$\frac{1}{\mu^4}\sigma'\Big(\frac{\Lambda}{\mu^4}\Big) = \int \mathrm{d}^4x\, \sqrt{-g}, \tag{2.20b}$$

$$\frac{1}{M_{\mathrm{P}}^2}\hat{\sigma}'\Big(\frac{\kappa^2}{M_{\mathrm{P}}^2}\Big) = -\int \mathrm{d}^4x\, \sqrt{-g}\ \frac{R}{2}. \tag{2.20c}$$

Using the definition of spacetime average $\langle\!\langle \cdots \rangle\!\rangle$ given in §2.2.2, Eqs. (2.20b)–(2.20c) require

$$\langle\!\langle R \rangle\!\rangle = -2\frac{\mu^4}{M_{\mathrm{P}}^2}\frac{\hat{\sigma}'}{\sigma'}. \tag{2.21}$$

Meanwhile, tracing the Einstein equation (2.20a) and taking the spacetime average, we find

$$\langle\!\langle R \rangle\!\rangle = -\frac{1}{\kappa^2}\left[\langle\!\langle T^\mu{}_\mu \rangle\!\rangle - 4(\Lambda - \Lambda_{\mathrm{UV}})\right]. \tag{2.22}$$

Eqs. (2.21) and (2.22) must hold simultaneously, and therefore

$$\Lambda - \Lambda_{\mathrm{UV}} = \frac{1}{4}\langle\!\langle T^\mu{}_\mu \rangle\!\rangle - \frac{\mu^4}{2}\frac{\kappa^2}{M_{\mathrm{P}}^2}\frac{\hat{\sigma}'}{\sigma'}. \tag{2.23}$$

Finally, we replace Λ in the Einstein equation to obtain

$$\kappa^2 G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}\langle\!\langle T^\mu{}_\mu \rangle\!\rangle + \frac{\mu^4}{2}\frac{\kappa^2}{M_{\mathrm{P}}^2}\frac{\hat{\sigma}'}{\sigma'} g_{\mu\nu}. \tag{2.24}$$

Relation between the Einstein and Jordan frame.—The Einstein and Jordan frame formulations are related by a change of frame, and therefore must presumably be regarded as equivalent. This equivalence holds even up to quantum corrections provided one is sufficiently careful to include contributions from the transformation Jacobian; see, e.g. Ref. Brax et al. (2011). The key issue to be addressed is how ultraviolet modes enter each formulation, to be discussed in §2.2.3.

Before doing so, we enumerate the principal differences between the sequester phenomenology in Einstein frame and Jordan frame. First, in Jordan frame, not only the low-energy loop contribution Λ_{IR} is sequestered, but also the ultraviolet part Λ_{UV}. This happens because both sources for the cosmological term now couple to the Jordan-frame metric. The distinction between them is therefore arbitrary at the level of the Einstein equation. We will see below that this emerges from a more general conclusion, that fluctuations coupling to the Jordan-frame metric (including gravitons) are sequestered, whereas fluctuations coupling to the Einstein-frame metric are not.

Second, the critical factor of 1/4 in the combination $T_{\mu\nu} - \langle\!\langle T^{\mu}{}_{\mu} \rangle\!\rangle/4$ is *not ultraviolet sensitive*. In particular, it is no longer produced by balancing a topological invariant against the properties of a particular group of Lagrangian operators. Instead, the factor of 1/4 in Eqs. (2.20b) and (2.24) is also produced by a trace. Therefore it is not corrected by extensive renormalizations of the coupling of Λ to spacetime. We will consider below what is the effect of these renormalizations in the Jordan frame.

Third, the Jordan frame formulation generates a residual cosmological-like term. This is the last term in (2.24). Assuming $\kappa^2 \sim M_{\mathrm{P}}^2$ and $\sigma' \sim \hat{\sigma}' \sim \mathrm{O}(1)$, it yields a residual cosmological constant of order μ^4. Ref. Kaloper and Padilla (2017) argued that this contribution is at least radiatively stable because it arises from the intensive term σ, which does not couple either to g_{ab} or the matter fields in $\mathcal{L}_{\mathrm{m}}$. It is therefore uncorrected by matter and graviton loops. On the other hand, depending on its origin, σ might be susceptible to other loop corrections associated with unknown mass scales. If so, μ must apparently be associated with the *lowest* of these scales, because it is the most relevant terms involving Λ that dominate Eq. (2.22). (However, it should be remembered that μ does not have a precise meaning until we specify the typical size of Taylor coefficients in σ.)

This does not preclude the possibility that μ could typically be large. As with the cosmological constant itself this need not be fatal for the model, because we can always suppose that the renormalized value of μ is lower than its natural scale. If we choose to do so, however, then presumably we encounter a new naturalness problem in the Λ sector. In particular, Λ must become relevant at a very low energy scale $\sim (10^{-3}\,\mathrm{eV})^4$ to avoid an unwanted large contribution.

At the level of the effective theory (2.19) there is nothing further that can be said to set our expectations about the typical size of μ. To do so would require a detailed microscopic theory of the fluxes and how they are sourced. Such a theory could be used to compute corrections to the functions σ and $\hat{\sigma}$. In this connection, see Refs. Padilla (2019); El-Menoufi et al. (2019).

2.3.3 Ultraviolet dependence in Jordan frame

In Eq. (2.19) there will be extensive renormalizations of the coupling of κ and Λ to spacetime. Note that Λ couples to the spacetime volume, whereas κ^2 couples to the integrated curvature $\int \mathrm{d}^4x\,\sqrt{-g}\,R$. Renormalizations of the Λ coupling were considered above and are unchanged in this theory. Renormalization of the κ coupling will arise from diagrams analogous to those of Fig. 2.1, but with insertions of $\kappa^2 R$ rather than Λ. (As before, the class of diagrams shown on the right-hand side of Fig. 2.1 does not exhaust the contributions at a given order in $\kappa^2 R$, but they provide a representative class that is simple to study.) The leading effects can be summarized by the replacements

$$\frac{\kappa^2}{2}R \to \frac{\kappa^2}{2}(1+\alpha\epsilon)R, \tag{2.25a}$$

$$\Lambda \to (1+\beta\epsilon)\Lambda \tag{2.25b}$$

where α and β are O(1) Wilson coefficients, and ϵ is defined by

$$\epsilon \equiv \frac{1}{(2\pi)^8}\frac{M_{\mathrm{SM}}^4}{\kappa^4} \ll 1. \tag{2.26}$$

To account for renormalizations of the low-energy cosmological constant from diagrams including virtual gravitons, as in the right-hand diagram of Fig. 2.2, we include a representative term $\gamma M_{\mathrm{SM}}^6/\kappa^2$ in $\mathcal{L}_{\mathrm{m}}$, where γ is another O(1) coefficient. A contribution of this form will be generated by diagrams such as the right-hand side of Fig. 2.2 containing a single internal graviton line. It would typically be accompanied by contributions of higher order in κ^{-2} from diagrams containing two or more internal graviton lines, but if the scale M_{SM} of the sequestered sector is far below the Planck scale then the one-graviton diagram will be dominant. Notice that this term will contribute to the κ field equation. This is the origin of the mismatch that allows such contributions to escape complete sequestration.

After a short calculation, it follows that the effective Einstein equation in this model can be written, up to O(ϵ),

$$\begin{aligned}\kappa^2 G_{\mu\nu} = (1-\alpha\epsilon)T_{\mu\nu} - \Big(1-(\alpha+\beta)\epsilon\Big)\frac{\langle\!\langle T^\rho{}_\rho\rangle\!\rangle}{4}g_{\mu\nu} \\ + \beta\epsilon\Lambda_{\mathrm{UV}}g_{\mu\nu} - \frac{\gamma}{2}\frac{M_{\mathrm{SM}}^6}{\kappa^2}g_{\mu\nu} \\ + \frac{\mu^4}{2}\frac{\kappa^2}{M_{\mathrm{P}}^2}\frac{\hat{\sigma}'}{\sigma'}(1+\alpha\epsilon)g_{\mu\nu}.\end{aligned} \tag{2.27}$$

We can identify a number of effects. First, dressing of the κ^2 coupling (proportional to α) can be absorbed into a redefinition of the Planck scale. It does not cause de-tuning of the sequester. By comparison, we cannot simply absorb (2.25b) into a redefinition of Λ because of its κ dependence.

Second, dressing of the Λ coupling (proportional to β) is again responsible for breaking complete cancellation of the low-energy cosmological contribution between $T_{\mu\nu}$ and $\langle\!\langle T^{\mu}{}_{\mu} \rangle\!\rangle/4$. The residual cosmological constant will be of order $\epsilon\Lambda_{\mathrm{IR}} \sim \epsilon M_{\mathrm{SM}}^4$ and therefore of a similar size to the estimates for the Einstein frame given at the end of §2.2.3. For numerical values we refer to the discussion given there.

Third, the Λ dressing *also* causes inexact cancellation of the ultraviolet part Λ_{UV}. The leftover piece has exactly the same structure as the left-over low energy loop contribution in $T_{\mu\nu}$, again because there is no distinction between these terms at the level of the Einstein equation. In the remainder of this paper we shall drop explicit dependence on Λ_{UV} and include its contribution in $\mathcal{L}_{\mathrm{m}}$ if required. Finally, we clearly see the contribution of the right-hand diagram in Fig. 2.2; this produces the term proportional to $\gamma M_{\mathrm{SM}}^6/\kappa^2$ Kaloper and Padilla (2017).

2.3.4 Sequestering the graviton loops

Kaloper et al. observed that the troublesome γ term appears in Eq. (2.27) as a consequence of its appearance in the κ field equation Kaloper and Padilla (2017). If it could be removed from this field equation then terms of any order in κ^{-2} contained in $T_{\mu\nu}$ would be sequestered as part of the usual cancellation between $T_{\mu\nu}$ and $\langle\!\langle T^{\mu}{}_{\mu} \rangle\!\rangle/4$, at least in the absence of renormalizations to the Λ coupling to the volume of spacetime.

In turn, graviton-loop contributions to the low-energy cosmological term contribute to the κ field equation *only* because the graviton propagator carries a normalization of κ^{-2}. To decouple these contributions Kaloper et al. proposed the following formulation (which they described as 'omnia sequestra') Kaloper and Padilla (2017); Coltman et al. (2019)

$$\begin{aligned} S = \int \mathrm{d}^4x \, \sqrt{-g}\bigg(\frac{M_{\mathrm{P}}^2}{2} R + \theta R_{\mathrm{GB}} - \Lambda - \mathcal{L}_{\mathrm{m}} \bigg) \\ + \sigma\left(\frac{\Lambda}{\mu^4}\right) + \hat{\sigma}(\theta). \end{aligned} \tag{2.28}$$

Recall that we are now absorbing Λ_{UV}, if present, into $\mathcal{L}_{\mathrm{m}}$. The normalization of the Einstein term reverts to the fixed Planck scale M_{P}. Meanwhile we introduce the Gauss–Bonnet density R_{GB} coupled to a rigid scalar θ that replaces κ. The Gauss–Bonnet density is defined by

$$R_{\rm GB} \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}. \tag{2.29}$$

In four dimensions its integral is proportional to a topological invariant, the Euler characteristic $\chi(M)$ of the manifold M. Because it is topological (it integrates to a boundary term), it follows that $R_{\rm GB}$ does not modify the form of the graviton propagator or its self-interaction vertices. The conclusion is that each internal graviton line scales like $M_{\rm P}^{-2}$ and carries no θ dependence. Operators in the quantum effective action that are built from diagrams containing such lines do not perturb the field equation for θ.

Further, because of its topological character, the coefficient of the Gauss–Bonnet density is not renormalized. At the level of Feynman diagrams this follows because θ does not contribute to graviton vertices. Therefore there is no analogue of the diagrams in Fig. 2.1 for $R_{\rm GB}$. For the same reason, quantum corrections do not introduce θ-dependence in $\mathcal{L}_{\rm m}$ at any order in the loop expansion.

The coupling of Λ to the spacetime volume will still be dressed by graviton loops, yielding Eq. (2.25b) with the replacement $\kappa^2 \to M_{\rm P}^2$ in ϵ. However, unlike Eqs. (2.25b) and (2.26), there is now no obstruction to absorbing the loop correction into a redefinition of Λ. Accordingly we do not expect de-tuning of the sequester in this case.

To verify this expectation consider the field equations following from (2.28) with the leading loop correction to the Λ coupling included,

$$M_{\rm P}^2 G_{\mu\nu} = T_{\mu\nu} - (1+\alpha\epsilon)\Lambda g_{\mu\nu}, \tag{2.30a}$$

$$\frac{\sigma'}{\mu^4} = (1+\alpha\epsilon)\int \mathrm{d}^4x\,\sqrt{-g} \tag{2.30b}$$

$$\hat{\sigma}' = -\int \mathrm{d}^4x\,\sqrt{-g}\,R_{\rm GB} \tag{2.30c}$$

From Eqs. (2.30b)–(2.30c) we conclude

$$\frac{\hat{\sigma}'}{\sigma'}\mu^4 = -(1-\alpha\epsilon)\langle\!\langle R_{\rm GB}\rangle\!\rangle. \tag{2.31}$$

Because the Gauss–Bonnet density integrates to the Euler characteristic, up to a numerical factor, this is a relatively stringent condition on μ. Assuming the derivatives σ' and $\hat{\sigma}'$ are order unity[29] it roughly requires $\mu \sim H_{\rm age}$, where $H_{\rm age}$ is the quantity defined in (2.14). See also Ref. Coltman et al. (2019).

Meanwhile, the trace of the Einstein equations requires

$$R = \frac{4}{M_{\rm P}^2}(1+\alpha\epsilon)\Lambda - \frac{1}{M_{\rm P}^2}T^{\mu}{}_{\mu}. \tag{2.32}$$

[29]In our presentation, we are absorbing the integrated fluxes $\int F_4$, $\int \hat{F}_4$ into the definition of σ, $\hat{\sigma}$. If these factors are large they may modify conclusions based on dimensional analysis of (2.31).

As in the analyses given above, taking the spacetime expectation of this formula gives an expression for Λ in terms of $\langle\!\langle R \rangle\!\rangle$ and $\langle\!\langle T^{\mu}{}_{\mu} \rangle\!\rangle$. This expression should be used to eliminate Λ from the Einstein equation. Finally, expressing $\langle\!\langle R \rangle\!\rangle$ in terms of $\langle\!\langle R_{\mathrm{GB}} \rangle\!\rangle$ yields

$$M_{\mathrm{P}}^2 G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} \langle\!\langle T^{\rho}{}_{\rho} \rangle\!\rangle g_{\mu\nu} - L g_{\mu\nu}, \tag{2.33}$$

where L is defined by (cf. Eqs. (11)–(12) of Ref. Kaloper and Padilla (2017))

$$\begin{aligned} L^2 = \frac{3}{8} M_{\mathrm{P}}^4 \bigg(& \langle\!\langle R_{\mathrm{GB}} \rangle\!\rangle - \langle\!\langle W^2 \rangle\!\rangle + \frac{2}{M_{\mathrm{P}}^2} \langle\!\langle (T_{\mu\nu} - T g_{\mu\nu}/4)^2 \rangle\!\rangle \\ & - \frac{1}{6 M_{\mathrm{P}}^4} \Big[\langle\!\langle T^2 \rangle\!\rangle - \langle\!\langle T \rangle\!\rangle^2 \Big] \bigg), \end{aligned} \tag{2.34}$$

where $T = T^{\rho}{}_{\rho}$ and $W_{\mu\nu\rho\sigma}$ is the Weyl tensor derived from $g_{\mu\nu}$. This is exactly the result derived in Ref. Kaloper and Padilla (2017). As expected, dressing of the Λ coupling has no effect at the level of the effective Einstein equation. We conclude that extensive renormalizations of the coupling between Λ and the spacetime volume do *not* de-tune sequestration in the formulation (2.28).

2.4 Conclusions

In this paper we have studied a class of radiative corrections to the sequester model proposed by Kaloper, Padilla and collaborators. Although the corrections we compute have previously been recognized, their effect has not been studied explicitly. The class of diagrams we study renormalize the couplings between the "rigid" scalar fields that are characteristic of the sequester scenario, and infrared properties of the spacetime such as its volume and integrated curvature.

In both the Einstein and Jordan frame formulations (given by Eqs. (2.1) and (2.19) in our notation), we find that these renormalizations disrupt complete sequestration of low-energy loop contributions. If the sequestered sector is the Standard Model, we find that these corrections very nearly produce an unacceptable cosmological term in excess of the observed value $\Lambda \sim (10^{-3}\,\mathrm{eV})^4$. Whether or not a particular realization of the scenario yields an acceptable phenomenology then depends on how the global function σ is engineered (and likewise for $\hat{\sigma}$ in the Jordan-frame formulation).

Alternatively, if the sequestered sector contains higher mass particles such as supersymmetric partners with masses in excess of 10 TeV, the residual cosmological term is likely to be fatal. The situation could possibly be saved if physical scales are significantly rescaled in the effective Einstein frame metric. This is easiest to see in the explicit Einstein-frame description, where masses are rescaled by the conformal factor λ. We can possibly arrange for this rescaling λ to be small, but such scenarios encounter other difficulties Kaloper and Padilla (2014b).

The simpler formulations of the sequester (those that do not invoke the Gauss–Bonnet density) are already known to "fail" in the sense that they do not capture contributions to the vacuum energy from diagrams that contain virtual gravitons. Although the renormalizations we have computed are related to these known failure modes, they are not the same. In most models the loop terms we compute are likely to be somewhat smaller, since the leading contribution involves two virtual gravitons and therefore scale as $(M_{\mathrm{SM}}/M_{\mathrm{P}})^4$. This should be compared to a single-graviton loop scaling as $(M_{\mathrm{SM}}/M_{\mathrm{P}})^2$ as in the left-hand diagram of Fig. 2.2.

We find that these renormalizations do not affect the most developed formulation of the sequester, given by Eq. (2.28) in our notation. In this formulation, dressing of the Λ interaction can be absorbed into a redefinition of Λ itself and is therefore harmless.

Whether or not one finds the sequester a plausible solution to the naturalness problem of the cosmological constant depends on whether we are prepared to accept its key ingredient—the introduction of non-gravitating sectors in the action that are shielded from gravity: they do not source gravitational fields, and they do not interact with gravitons. For related models utilising a similar premise see Refs. Oda (2017b); Carroll and Remmen (2017); Sobral-Blanco and Lombriser (2020); Bordin et al. (2020). This is the cost of entry for all versions of the sequester scenario. Once accepted, it is only necessary to arrange for the large low-energy loop contribution to be 'stored' in these non-gravitating sectors.

At the level of the effective actions used in this paper there is little more that can be said. In particular, we have not been able to apply "naturalness" arguments to the non-gravitating functions σ and $\hat{\sigma}$, because to do so would require specification of a microscopic theory that describes the fluxes F_4, $\hat{F}_4$ that project out local degrees of freedom from the rigid fields Λ, κ and θ (depending on the formulation in use). These non-gravitating sectors are the final repository for sequestered vacuum energy. If it is possible to build models in which these sectors have their own microscopic description, it would be very interesting to apply naturalness criteria to the model as a whole.

Chapter 3

Introduction to dynamical dark energy and alternative couplings

A large variety of dark energy and modified gravity models have been proposed to account for the present day accelerated expansion of the Universe (generally assuming all contributions to the cosmological constant are zero). Horndeski theory provides us with a useful tool to study this large theory space. In this section, we will first outline Horndeski theory, before discussing alternative couplings in the context of Horndeski theory.

3.1 Horndeski theories

In this subsection we outline the properties of Horndeski theories Horndeski (1974). Horndeski theory is the most general theory of gravity in four dimensions whose Lagrangian is constructed out of the metric tensor and a scalar field and leads to second order equations of motion and has no Ostrogradsky ghosts[30]. Horndeski theory provides a general framework that applies to most of the higher derivative scalar dark energy theories such as Galileons Nicolis et al. (2009), chameleons Khoury and Weltman (2004b,a), quintessence Ratra and Peebles (1988); Caldwell et al. (1998), k-essence Armendariz-Picon et al. (2000, 2001) and fab-4 Charmousis et al. (2012a,b); Copeland et al. (2012).

The Horndeski action (in its contemporary formulation) could also be referred to as

[30] The Ostrogradsky theorem states that any physical system with equations of motion having derivative order greater than two suffer from the linear instability, provided that the system is non-degenerate Ostrogradsky (1850). For a pedagogical explanation see Section 1.3 of Kobayashi (2019). It should be emphasized that the second-order (and no higher) equations of motion are not the necessary conditions for the absence of Ostrogradsky ghosts in theories with multiple fields, see Section 4 in Kobayashi (2019) for a review of beyond Horndeski theories.

the action of the four-dimensional generalized Galileon,

$$S\left[g_{\mu\nu},\phi\right]=\int \mathrm{d}^4x\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G_{\mathrm{N}}}\mathcal{L}_i\left[g_{\mu\nu},\phi\right]+\mathcal{L}_{\mathrm{m}}\left[g_{\mu\nu},\psi_M\right]\right], \tag{3.1}$$

with the $\mathcal{L}_i$ parameters being,

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi,X)\\
\mathcal{L}_3 &= G_3(\phi,X)\Box\phi\\
\mathcal{L}_4 &= G_4(\phi,X)R+G_{4,X}(\phi,X)\left[(\Box\phi)^2-\phi_{;\mu\nu}\phi^{;\mu\nu}\right]\\
\mathcal{L}_5 &= G_5(\phi,X)G_{\mu\nu}\phi^{;\mu\nu}-\frac{1}{6}G_{5,X}(\phi,X)\left[(\Box\phi)^3+2\phi^{\nu}_{;\mu}\phi^{\alpha}_{;\nu}\phi^{\mu}_{;\alpha}-3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right].
\end{aligned} \tag{3.2}$$

In general the factors G_2 to G_5 can be arbitrary functions of the scalar field ϕ and its linear kinetic term $X=-(1/2)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$. $\mathcal{L}_4$ and $\mathcal{L}_5$, are strongly constrained by the direct measurement of the speed of gravitational waves following GW170817, Lombriser and Taylor (2016); Bettoni et al. (2017); Creminelli and Vernizzi (2017a); Sakstein and Jain (2017a); Ezquiaga and Zumalacárregui (2017).

In the discussion above we have described the *generalised* Galileon, synonymous with Horndeski theory. Looking more closely at the G_4 term, if the coupling $G_4(\phi,X)$ is independent of ϕ, this corresponds to the covariant Galileon theory that respects the Galilean symmetry, $\partial_\mu\phi\rightarrow\partial_\mu\phi+b_\mu$ in Minkowski spacetime Nicolis et al. (2009).

3.2 Screening

A scalar field theory as dark energy modifies gravity on cosmological scales. However, the extra force mediated by the scalar degree of freedom ϕ must be "screened" on small scales (such as within the solar system) where general relativity has been tested to high precision. Three groups of the screening mechanisms have been proposed they occur when:

- ϕ is effectively massive in the vicinity of a source. This means that in terrestrial experiments, the large mass of the field suppresses its interaction with matter. More precisely the effective potential for ϕ depends on the local energy density through the coupling of ϕ to matter. This is known as the chameleon screening mechanism Khoury and Weltman (2004b,a).

- ϕ is effectively weakly coupled to the source by making the non-linear kinetic term $X\Box\phi$ large. This is Vainshtein screening Vainshtein (1972); Deffayet et al. (2002); Babichev and Deffayet (2013).

- The coupling of ϕ to matter is effectively suppressed by making the linear kinetic term X large. This is called symmetron screening Hinterbichler and Khoury (2010); Hinterbichler et al. (2011) or k-mouflage Babichev et al. (2009).

The Vainshtein mechanism is relevant to the Galileon theories, and below we will review this screening mechanism in the context of a cubic Galileon ϕ minimally coupled to gravity. Beginning with the action,

$$S = \int d^4x\sqrt{-g}\left(\frac{M_\mathrm{P}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{c_3}{\Lambda_3^3}\Box\phi g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right), \tag{3.3}$$

where Λ_3 is a constant with dimensions of mass and c_3 is a dimensionless constant. Varying this action with respect to the field ϕ gives the equations of motion for ϕ,

$$0 = \Box\phi + \frac{2c_3}{\Lambda_3^3}\left[-\nabla^\beta\nabla^\nu\phi\nabla_\beta\nabla_\nu\phi + (\Box\phi)^2 + [\nabla^\nu, \nabla_\alpha]\nabla^\alpha\phi\nabla_\nu\phi\right]. \tag{3.4}$$

It is convenient to use the commutation relation for covariant derivatives to write the final term as $R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ because we know that the curvature must be zero in a flat spacetime.

We will now assume that the scalar field changes only radially $\phi = \phi(r)$ and we will specify to the Minkowski metric in polar coordinates,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right). \tag{3.5}$$

Working in this metric and integrating (to turn the second order ODE into a first order ODE) the equation of motion gives,

$$0 = \frac{4c_3}{\Lambda_3^3}\left(\frac{\phi'}{r}\right)^2 + \left(\frac{\phi'}{r}\right) - cr^{-3} \tag{3.6}$$

where we have used $'$ to denote a derivative with respect to r. c is a constant of integration, normally determined by boundary conditions. We will leave c undetermined here as it does not change the illustration. We now solve for ϕ' using the quadratic formula,

$$\phi' = -\frac{r\Lambda_3^3}{8c_3} \pm \frac{\Lambda_3^{3/2}}{8c_3}\sqrt{\frac{16c_3c}{r} + r^2\Lambda_3^3}. \tag{3.7}$$

From this expression, one notices that in the limit of large c_3 (keeping all other quantities fixed) ϕ' scales like $1/c_3^{1/2}$ [31]. Therefore, we can write (taking the positive square root above),

$$\phi' = \left(\frac{\Lambda_3^3c}{4rc_3}\right)^{1/2} - \frac{\Lambda_3^3r}{8c_3} + \left(\frac{\Lambda_3^9r^5}{2^{12}cc_3^3}\right)^{1/2} + O\left(c_3^{-2}\right). \tag{3.8}$$

[31] Substituting the scalar field solution into the Einstein equations, the Galileon contributions are suppressed by powers of $1/c_3$, hence they can be neglected in the large c_3 limit. This suggests that we can analyse this system, in the large c_3 limit, by a perturbative expansion in powers of $1/c_3$.

The first term (which scales with $r^{-1/2}$) in the expansion arises from the nonlinear kinetic term. The second term (which scales with $-r$) in the expansion arises from the linear kinetic term. If we are to use a Galileon scalar field as the dark energy, it must also respect observations on small scales. We previously described the Vainshtein mechanism as a suppression of the coupling to matter, occurring when the non-linear kinetic term is dominant. Enforcing that the nonlinear kinetic term is dominant this leads to the following inequality for r,

$$r^3 < \frac{16c_3c}{\Lambda_3^3}, \tag{3.9}$$

we will label the corresponding radius at the boundary of this inequality $r_{\rm V} = (16c_3c)^{1/3}/\Lambda_3$. This means that the fifth force mediated by the Galileon field is suppressed at length scales smaller than $r_{\rm V}$. Therefore, one must tune the free parameters of the theory in such a way that this radius $r_{\rm V}$ is sufficiently large that the fifth force effects are not seen on small scales, for example within the solar system.

The purpose of the Vainshtein screening in this example was to screen the fifth force on short scales. One might also postulate that one could screen on cosmological time scales. Imagine a universe where the dark energy is entirely described by cubic Galileons. One could imagine tuning the coefficient of the nonlinear kinetic term $c_3\Lambda_3^{-3}$ in order to achieve a cosmological Vainshtein solution; a screening which universally suppresses fifth forces when the mean cosmological energy density is large, in order to achieve consistency with measurements of the matter dominated universe Chow and Khoury (2009); Burrage et al. (2017). We will revisit this idea in the next Chapter.

3.3 Couplings

Minimal couplings occur when scalar fields and other matter in the universe couple minimally to the metric $g_{\mu\nu}$. There are other ways in which species can couple to each other. We have explored one of those ways already in Section 1.4, the conformal coupling. There is another coupling worth examining, the disformal coupling. A conformally and disformally coupled metric can be expressed as,

$$\tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi, \tag{3.10}$$

where X is the standard kinetic term for the scalar field ϕ i.e. $X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$. The metric $\tilde{g}_{\mu\nu}$ is the most general metric that is a function of $g_{\mu\nu}$, ϕ and its first derivatives and which respects causality and the weak equivalence principle [32] Bekenstein (1992).

It is important for us to define exactly what we mean by respecting causality. We define it here as the metric formulation possessing well defined past and future light-cones. No closed causal curves exist Bruneton (2007); Bonvin et al. (2007)[33].

What does this metric mean physically? These transformations of the metric can be thought of as stretching of the metric and therefore they change physical scales. A purely conformal transformation, when $D = 0$ will stretch all spacetime dimensions the same amount. A disformal transformation is a slightly harder to interpret physically it is not really a scaling, since it is additive (it does not scale the metric that already exists, but rather adds new pieces). When $D \neq 0$, this will appear as stretching the direction of $\partial_\alpha \phi$ with a different factor from other directions. Shapes *dis*form under *dis*formal transformations. This means that Maxwell's equations are only invariant under Lorentz transformations in this metric if $D = 0$.

These transformations given in Equation 3.10, preserve the form of the Horndeski action. By this we mean that transforming a Horndeski action in this way simply changes the form of the functions, but ultimately they correspond to a theory in the Horndeski class Bettoni and Liberati (2013).

There is no reason to expect that a cubic Galileon does not conformally or disformally couple. In Burrage et al. (2017) it was found that this cosmological Vainshtein screening increases the allowable parameter space of the conformally coupled cubic Galileon. It was found that the cosmological Vainshtein mechanism keeps the background close to that of ΛCDM and suppresses Galileon forces until the energy density of the Universe is sufficiently low. However, this did not save the cubic Galileon (acting as the sole source of dark energy) being ruled out by integrated Sachs-Wolfe effect (ISW) observations Renk et al. (2017).

3.4 Screening coupled Galileons

Returning to the idea of Galileon dark energy, and couplings. In Karwan et al. (2016) a Galileon theory of gravity, where only the disformal coefficient is allowed to vary,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(\phi)\partial_\mu \phi \partial_\nu \phi, \tag{3.11}$$

[32] A coupling of this kind does indeed break the strong equivalence principle, we state the differences between the two equivalence principles in Subsection 1.1

[33] This is especially important to remember later in the thesis when we find that for a disformally coupled cubic Galileon gravity travels faster than electromagnetic waves, without violating causality.

is studied. It was found that for this theory, the Vainshtein mechanism is absent. In the sense that the strength of the fifth force that is sourced by the disformally coupled Galileon is not decoupled from matter on small scales. However, the Vainshtein mechanism is not necessary for this theory. The fifth force is very weak compared to gravity, therefore this theory closely mimics the Einstein theory of gravity at all scales inside the Hubble radius without a screening mechanism.

A disformal coupling is very hard to constrain using cosmology. This is because the disformal coupling is a derivative one, therefore the amplitude for ϕ exchange with a static, non-relativistic source vanishes. The classical force generated between such sources must therefore also vanish. The leading contribution to fifth-forces that result from disformally coupled scalars is generated at one-loop level and is highly suppressed. This statement is consistent with the results that we find in Chapter 4.

Chapter 4

Constraints on a cubic Galileon disformally coupled to Standard Model matter

Abstract

We consider a disformal coupling between Standard Model matter and a cubic Galileon scalar sector, assumed to be a relict of some other physics that solves the cosmological constant problem rather than a solution in its own right. This allows the energy density carried by the Galileon scalar to be sufficiently small that it evades stringent constraints from the integrated Sachs–Wolfe effect, which otherwise rules out the cubic Galileon theory. Although the model with disformal coupling does not exhibit screening, we show there is a 'screening-like' phenomenon in which the energy density carried by the Galileon scalar is suppressed during matter domination when the quadratic and cubic Galileon operators are both relevant and the quadratic sector has a stable kinetic term. We obtain the explicit 3+1 form of Maxwell's equations in the presence of the disformal coupling, and the wave equations that govern electromagnetic waves. The disformal coupling is known to generate a small mass that modifies their velocity of propagation. We use the WKB approximation to study electromagnetic waves in this theory and show that, despite remarkable recent constraints from the LIGO/Virgo observatories that restrict the difference in propagation velocity between electromagnetic and gravitational radiation to roughly 1 part in 10^{15}, the disformal coupling is too weak to be constrained by events such as GW170817 or by the dispersion of electromagnetic radiation at different wavelengths.

4.1 Introduction

For some time, observational constraints on the Hubble rate (both direct and indirect) have yielded strong evidence for a "dark energy" sector causing the expansion rate to accelerate since redshift $z \approx 0.5$. Despite significant effort, the nature of this sector remains largely unknown. It may well imply new forces that effectively produce long-range gravitational repulsion. If so, these forces necessarily couple to Standard Model matter and therefore can be studied using a combination of terrestrial, astrophysical and cosmological measurements. For a recent review, see Ref. Brax et al. (2020).

If one or more new forces *are* present, the minimal possibility is that they are mediated by a scalar field. Even if this is not the case and the intermediate field transforms in a higher-dimensional representation of the Lorentz group, each physical polarization will act like a scalar field—but perhaps with restricted couplings determined by the representation. By constraining the different ways that presently-undetected scalar fields can couple to the Standard Model, we can hope to place indirect constraints on the unknown dark energy sector.

Universal couplings.—To preserve the weak equivalence principle, any new scalar fields should couple to all forms of matter in the same way. What are the possibilities? At linear level, we can write two universal, diffeomorphism-invariant couplings for a scalar field ϕ: first $\phi T/M$, where $T = g^{\mu\nu}T_{\mu\nu}$ is the trace of the energy–momentum tensor $T_{\mu\nu} \equiv (-2/\sqrt{-g})\delta S_{\mathrm{m}}/\delta g^{\mu\nu}$, S_{m} is the matter action, and M is a mass scale characterizing the strength of the force; and second $(\partial_\mu\phi\partial_\nu\phi)T^{\mu\nu}/M^4$, with the same meanings for $T_{\mu\nu}$ and M.

We now wish to promote these interactions into a nonlinear completion. Note that if the constituent species in S_{m} are to obey the weak equivalence principle then they should couple to a single metric $\tilde{g}_{\mu\nu}$, even if this is not the metric $g_{\mu\nu}$ used to construct the gravitational sector. Then the nonlinear interaction must be $S_{\mathrm{m}} = S_{\mathrm{m}}(\tilde{g}^{\mu\nu}, \Psi)$, where Ψ stands schematically for the different species of matter fields and $\tilde{g}_{\mu\nu}$ is related to $g_{\mu\nu}$ by a Bekenstein transformation Bekenstein (1992, 2004),

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + \frac{D(\phi)}{M_D^4}\partial_\mu\phi\partial_\nu\phi. \tag{4.1}$$

We describe $\tilde{g}_{\mu\nu}$ as the "Jordan frame metric". The C-term is a conformal transformation of $g_{\mu\nu}$; in contradistinction, the D-term is called *disformal*. To recover the linear interactions written above we should expand the C- and D-functions in Taylor series, $C = 1 + \phi/M_C + \cdots$ and $D = 1 + \cdots$, and work to lowest order in $1/M_C$ or $1/M_D$, as appropriate. There is no expectation that the mass scales characterizing the conformal and disformal couplings will coincide. Bekenstein showed that this procedure yields the most general interaction between ϕ and matter that respects causality and the weak equivalence principle Bekenstein (1992).

Constraints on M_D.—If M_C and M_D are sufficiently large, so that any new forces are weak, then the linearized interactions will dominate. The linear conformal coupling $\phi T/M_C$ will generate a number of complicated interactions, depending on the vertices already present in S_{m}. Generically, however, any massive species appearing in S_{m} will become endowed with a Yukawa interaction whose coupling constant depends on the particle mass m. It follows that except for very light species, the ϕ-mediated force will be dominated at low momentum transfer by Yukawa exchange, which yields a $1/r^2$ force law with exponential cutoff e^{-mr}. Such Yukawa forces are known to be highly constrained Adelberger et al. (2003). It is now well understood that these unwanted forces can be "screened", but only at the cost of significant complication in the self-interactions of ϕ. In this paper we do not consider the conformal sector any further.

The disformal coupling $(\partial_\mu\phi\partial_\nu\phi)T^{\mu\nu}/M_D^4$ is substantially harder to detect. Because it is derivatively coupled, the amplitude for ϕ exchange with a static, non-relativistic source vanishes. The classical force generated between such sources must therefore also vanish: the leading contribution to such fifth-forces is generated at one-loop level and is highly suppressed Kugo and Yoshioka (2001); Kaloper (2004); Brax and Burrage (2014).

Nevertheless, attempts have been made to constrain M_D. The strongest of these come from collider phenomenology. Kaloper gave the approximate lower bound $M_D \gtrsim 200\,\mathrm{GeV}$ based on unitarity of electron–positron annihilation at LEP Kaloper (2004). It was later shown by Brax & Burrage that because of cancellations the cross-section for scalar-mediated fermion annihilation has an energy dependence that differs from the estimate used in Ref. Kaloper (2004). This yielded a weaker lower bound $M_D \gtrsim 100\,\mathrm{GeV}$ from monophoton searches at LHC Brax and Burrage (2014). Brax, Burrage & Englert went on to consider oblique corrections, Z boson phenomenology, and monophoton, dilepton and monojet events Brax et al. (2015). They concluded that the strongest constraints came from monojet searches by the CMS collaboration during LHC Run 1, which yielded the refined bound $M_D \gtrsim 650\,\mathrm{GeV}$. Currently, the strongest constraint comes from a dedicated ATLAS analysis using $37\,\mathrm{fb}^{-1}$ of LHC data collected in the period 2015–2016 at centre-of-mass energy $\sqrt{s} = 13\,\mathrm{TeV}$. This yields $M_D \gtrsim 1.2\,\mathrm{TeV}$ Aaboud et al. (2019).

Complementary but weaker constraints can be obtained from astrophysics and cosmology. Brax et al. studied spectral distortions in the cosmic microwave background (CMB) due to variations in the speed of light induced by the disformal coupling Brax et al. (2013). Later, Burrage, Cespedes & Davis analysed constraints from the power spectrum of CMB anisotropies in a specific scalar model with "quartic Galileon" self-interactions Burrage et al. (2016). Also, Brax & Davis and Brax, Davis & Kuntz derived constraints from gravitational effects including perhihelion advance, Shapiro time delay, and inspiral of compact objects Brax and Davis (2018); Brax et al. (2019).

Gravitational waves.—The advent of multi-messenger astronomy has changed this picture. It is now possible to test theories of modified gravity using observations of gravitational waves, which have been shown to yield extremely powerful constraints. In particular, in 2017 the LIGO and VIRGO gravitational wave observatories detected radiation emitted from the binary neutron star merger GW170817 Abbott et al. (2017d). Remarkably, this merger event could be associated with an electromagnetic counterpart which was interpreted as a gamma-ray burst. Assuming the gravitational and electromagnetic radiation was emitted at nearly the same time, the observed difference in arrival time $|\Delta t| = (1.74 \pm 0.05)\,\mathrm{s}$ over a path length $\sim 10^{15}\,\mathrm{s}$ implies that the (averaged) propagation speed of gravitational and electromagnetic waves over their common trajectory can differ by no more than roughly 1 part in 10^{15}.

Many authors have noted that disformal couplings modify the speed of propagation of electromagnetic waves relative to gravitational waves. (We will rederive this important result in §4.2.) Therefore, in principle, multi-messenger events such as GW170817 offer an opportunity to place constraints on such couplings, even from a single incident. Indeed, a large number of proposals for dark energy and modified gravity have been strongly disfavoured based on GW170817 alone because they produce a time difference $|\Delta t|$ that is unacceptably large Ezquiaga and Zumalacárregui (2017); Creminelli and Vernizzi (2017b); Baker et al. (2017); Langlois et al. (2018); Sakstein and Jain (2017b).

Leloup et al. applied these constraints to a "full" covariant Galileon model with disformal couplings to matter Leloup et al. (2019). Similar constraints has previously been obtained in the absence of disformal couplings by a number of authors Ezquiaga and Zumalacárregui (2017); Wang et al. (2017); Sakstein and Verner (2015). *Galileons* are scalar field models with possibly higher-derivative kinetic terms that nevertheless yield second-order field equations due to algebraic cancellations Nicolis et al. (2009). The model was generalized to curved spacetime ("covariantized") in Ref. Deffayet et al. (2009). There are five possible Galileon operators that can appear in the Lagrangian, of which $\mathcal{G}_1$ is a linear potential and $\mathcal{G}_2$ is the ordinary kinetic term,

$$\mathcal{G}_1 = \phi, \tag{4.2a}$$

$$\mathcal{G}_2 = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi \equiv X, \tag{4.2b}$$

$$\mathcal{G}_3 = -2X\Box\phi, \tag{4.2c}$$

$$\mathcal{G}_4 = -2X\Big[2(\Box\phi)^2 - 2\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi + XR\Big], \tag{4.2d}$$

$$\begin{aligned}\mathcal{G}_5 = -2X\Big[&(\Box\phi)^3 - 3(\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi)\Box\phi \\ &+ 2\nabla_\mu\nabla^\nu\phi\nabla_\nu\nabla^\rho\phi\nabla_\rho\nabla^\mu\phi - 6G_{\nu\rho}\nabla_\mu\nabla^\mu\nabla^\nu\phi\nabla^\rho\phi\Big].\end{aligned} \tag{4.2e}$$

In these equations, ∇_μ is the covariant derivative constructed from $g_{\mu\nu}$.

Eqs. (4.2a)–(4.2e) are special cases of the operators studied by Horndeski Horndeski (1974), restricted to satisfy a shift symmetry $\phi \to \phi + c$ at the level of the action.[34] Note that $\mathcal{G}_1$ does not spoil this property (at least on a cosmological background) since it transforms as a total derivative. The shift symmetry restricts the operators that can be generated by quantum corrections, making the set $\mathcal{G}_1$ to $\mathcal{G}_5$ radiatively stable among themselves. The "full" model studied by Leloup et al. includes all five operators. This is sometimes described as a *quintic* model. By analogy, if we include all operators except $\mathcal{G}_5$ we have a *quartic* model. If we include all operators except $\mathcal{G}_4$ and $\mathcal{G}_5$ we have a *cubic* model.

A cosmological background spontaneously breaks Lorentz invariance so that time translations $t \to t + c$ are no longer a manifest symmetry. On these backgrounds, $\mathcal{G}_4$ and $\mathcal{G}_5$ modify the speed of propagation of gravitational waves. The conclusion of Refs. Ezquiaga and Zumalacárregui (2017); Wang et al. (2017); Sakstein and Verner (2015) was that in Galileon models where ϕ sources late-time acceleration and is compatible with other cosmological measurements, the time lag $|\Delta t|$ between arrival of gravitational and electromagnetic radiation from GW170817 is much too large. Leloup et al. extended the same conclusion to the quintic Galileon with disformal couplings Leloup et al. (2019).

Outline of this paper.—In this paper we pursue a different but related problem. The conclusions of Refs. Ezquiaga and Zumalacárregui (2017); Wang et al. (2017); Sakstein and Verner (2015); Leloup et al. (2019) were driven by the need to switch on some contribution from $\mathcal{G}_4$ and $\mathcal{G}_5$ in order to evade constraints from the integrated Sachs–Wolfe ("ISW")

[34]The Galileon operators in fact exhibit symmetry under a larger *Galilean* group of transformations $\phi \to \phi + c + b_\mu x^\mu$. This symmetry is softly broken in the covariantized model by terms of order $1/M_{\rm P}$ and is therefore restored in the limit $M_{\rm P} \to \infty$ where gravity decouples. However, this larger symmetry group is not important for the present discussion.

effect Barreira et al. (2014); Brax et al. (2016). (If CMB–galaxy cross correlations measuring the ISW effect are excluded, any self-accelerating Galileon model is typically able to satisfy CMB, BAO and H_0 constraints without tuning the coefficients of the $\mathcal{G}_i$ Renk et al. (2017); Barreira et al. (2014).) In turn, the large ISW signal arises because the Galileon field makes a large contribution to the Hubble rate.

This is not the only scenario in which one can imagine a Galileon scalar sector to arise. For example, it may not happen that the energy density of the Galileon field is itself responsible for sourcing late-time acceleration. Indeed, from one point of view such models are hardly more interesting as a solution of the cosmological constant problem than simply taking $\Lambda^4 \approx (10^{-3}\,\text{eV})^4$ and dispensing with a dynamical component. This is because a self-accelerated Galileon scenario must usually take $\Lambda^4 = 0$ at the outset, which is no more justifiable than choosing $(10^{-3}\,\text{eV})^4$ unless we invoke some unknown symmetry that would make $\Lambda = 0$ a fixed point. While we do not advocate this position dogmatically, it is worth bearing in mind. One might be more willing to tolerate the choice $\Lambda = 0$ required for a dynamical solution if it could naturally explain the scale $10^{-3}\,\text{eV}$, or the redshift associated with the onset of acceleration, but this does not seem to be the case for Galileon scalars.

In this paper we do not assume that Galileon sector is associated with a solution to the cosmological constant problem. It may arise as a vestige of other physics that *is* associated with the solution, for example as the spin-0 polarization of a massive graviton that somehow degravitates the vacuum. Alternatively it may have nothing to do with the cosmological constant at all. In either case, our aim is to keep the Galileon a subdominant contributor to the cosmological energy budget.

The question to be resolved is whether a disformal coupling can be ruled out based on GW170817 alone (or similar measurements), even without modifications to the propagation velocity of gravitational waves from $\mathcal{G}_4$ and $\mathcal{G}_5$. Accordingly we take these operators to be absent. As explained above, the resulting cubic model would be ruled out by measurements of the integrated Sachs–Wolfe effect if its energy density were significant. But provided it is subdominant, the model is cosmologically acceptable.[35]

We do not study the conformal interaction in this paper and therefore set the Bekenstein $C(\phi)$-function to unity. To go further, note that if the ϕ shift symmetry is unbroken

[35] The cubic Galileon model has a well-known instability causing its energy density to grow at late times Chow and Khoury (2009). Ultimately this will set a limit on the period for which our model could be a viable effective description. We will see in §4.3 that there are acceptable models for which the onset of the instability has not yet occurred.

then $D(\phi) = 1$. Making this choice substantially simplifies the analysis, but still yields the leading contribution unless the shift symmetry is strongly broken. (It is also possible that higher-order terms in $\nabla_\mu \phi$ are generated by radiative corrections, but the usual argument of effective field theory shows that these will be subdominant at low energy.)

Summary.—In §4.2 we derive Maxwell's equations with the inclusion of a disformal coupling. Parts of this analysis has already been given in Ref. Brax et al. (2013),[36] but we repeat them here in order to fix notation and make our account self-contained. Because the disformal coupling is wavenumber-dependent, the resulting electrodynamic equations can be regarded as conventional Maxwell theory in a dispersive medium. In §4.3 we discuss cosmological solutions of the disformally coupled scalar field. By solving the Maxwell equations on this background we show that an evolving bundle of light rays propagating over cosmological distances is unstable, in principle, to decay into Galileon particles. A similar "tired light" effect is well-known in theories of axions and axion-like particles, including a dark energy "chameleon" with appropriate coupling Raffelt and Stodolsky (1988); Csaki et al. (2002); Burrage (2008); Brax et al. (2012). Where the loss of photons from the bundle is not catastrophic, there are two key observables. First, the propagation velocity of electromagnetic waves differs from those of gravitational waves, as explained above. Second, because the Maxwell equations on the scalar field background are dispersive, such a bundle of light rays would disperse as it travels over cosmological distances. We discuss both effects and use them to derive constraints on M_D. Finally, we conclude in §4.4.

Notation.—We use metric signature $(-,+,+,+)$ and work in units where $c = \hbar = 1$. The reduced Planck mass is defined by $M_{\mathrm{P}}^{-2} = 8\pi G$ with numerical value $M_{\mathrm{P}} \approx 2.435 \times 10^{18}\,\mathrm{GeV}$. The scalar field action is $\mathscr{L}_{\mathrm{Gal}} \equiv c_2 \mathcal{G}_2 + c_3 \mathcal{G}_3/\Lambda^3$, where c_2 and c_3 are Wilson coefficients that are expected to be of order unity, and Λ is an energy scale that determines when the nonlinear operator $\mathcal{G}_3$ becomes important relative to $\mathcal{G}_2$. We describe $\mathcal{G}_2$ and $\mathcal{G}_3$ as the linear and nonlinear kinetic terms, respectively. In the remainder of this paper we absorb c_3 into Λ without loss of generality. Further, because there is a scaling symmetry in the Galileon sector we must fix c_2 in order to break the redundancy Barreira et al. (2014). The role of c_2 is therefore only to select whether the quadratic kinetic term is *individually* stable or ghostlike, and by making a further scaling transformation we can always arrange that $c_2 = \pm 1$.

[36] The analysis given in this reference also assumes an axion-like coupling to the square of the electromagnetic field strength tensor, $F_{ab}F^{ab}$, which we do not invoke.

With these choices, on a cosmological background, the Lagrangian density specializes to

$$\mathscr{L}_{\mathrm{Gal}}(\phi) = \frac{c_2}{2}\dot{\phi}^2 + \frac{1}{\Lambda^3}\ddot{\phi}\dot{\phi}^2. \tag{4.3}$$

4.2 Maxwell theory

In this section we derive Maxwell's equations in the presence of a disformal coupling. A version of this analysis was previously given using manifestly covariant methods by Brax et al. Brax et al. (2013), who considered electrodynamics in the presence of a disformal coupling together with an additional axion-like interaction with the Maxwell term. (For example, an interaction of this form is known to arise under change of frame; see Ref. Brax et al. (2011).) In comparison with the discussion there, our analysis does not include the axion-like coupling. To make the physical content of the model as transparent as possible we frame our discussion in terms of the Maxwell equations and the physical $\mathbf{E}$ and $\mathbf{B}$ fields. We comment further on the relation between our calculations below.

4.2.1 Disformally-coupled electromagnetism

The action is

$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\mathrm{P}}^2}{2}R - 2\Lambda_{\mathsf{C}} + \mathscr{L}_{\mathrm{Gal}}(\phi)\right] + \int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\mathscr{L}_{\mathrm{SM}}(\tilde{g}_{\mu\nu},\Psi), \tag{4.4}$$

where $\mathscr{L}_{\mathrm{SM}}$ is the Standard Model Lagrangian density, and the remainder of the notation matches that used in §4.1. In particular, $\tilde{g}_{\mu\nu}$ is the Jordan frame metric and Ψ continues to stand schematically for the different species of Standard Model particles. Also, $R = R(g)$ is the Ricci scalar constructed from the 'vanilla' (Einstein frame) metric $g_{\mu\nu}$ and $\Lambda_{\mathsf{C}} \approx (10^{-3}\,\mathrm{eV})^4$ is a cosmological constant that is assumed to drive the observed late-time acceleration of the expansion $a(t)$.

Eq. (4.4) could equivalently be written in the Jordan frame by exchanging $R = R(g)$ for $\tilde{R} = \tilde{R}(\tilde{g})$, the Ricci scalar constructed from the Jordan-frame metric $\tilde{g}$. The disformal coupling between ϕ and matter would then become manifest as a derivative coupling between ϕ and $\tilde{G}_{\mu\nu}$, where $\tilde{G}_{\mu\nu} = \tilde{R}_{\mu\nu} - \tilde{R}\tilde{g}_{\mu\nu}/2$ is the ordinary Einstein tensor. This approach was adopted in Ref. Leloup et al. (2019).

In this paper we focus on the Maxwell term in the Standard Model Lagrangian. In the presence of a source 4-current $J^\mu = (\rho, \mathbf{J})$ this can be written

$$\frac{\mathscr{L}_{\mathrm{EM}}}{\sqrt{-g}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu, \tag{4.5}$$

where, as usual,

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \tag{4.6}$$

The contravariant metric corresponding to $\tilde{g}_{\mu\nu}$ is [cf. Eq. (4.1) with $C = 1$]

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \Lambda_D \partial^\mu \phi \partial^\nu \phi. \tag{4.7}$$

Like D, the scale Λ_D has dimension $[\mathrm{M}^{-4}]$. It is conventionally written in terms of the Einstein-frame kinetic energy for the scalar field $X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$. Then it follows that

$$\Lambda_D = \frac{M_D^{-4}}{1 - 2XM_D^{-4}} = \frac{M_D^{-4}}{1 - \dot{\phi}^2/M_D^4}. \tag{4.8}$$

The last equation applies only in the special case of a cosmological background, where ϕ depends only on coordinate time t. In this equation and subsequently, an overdot denotes a derivative with respect to t.

The integration measures $\sqrt{-g}$ and $\sqrt{-\tilde{g}}$ have a well-known relation Bekenstein (2004); Bettoni and Liberati (2013),

$$\sqrt{-\tilde{g}} = (1 - 2XM_D^{-4})^{1/2}\sqrt{-g} = (1 - \dot{\phi}^2/M_D^4)^{1/2}\sqrt{-g}. \tag{4.9}$$

Moreover the Christoffel symbols are related by Bettoni and Liberati (2013)

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + \Lambda_D \nabla^\rho\phi \nabla_\mu \nabla_\nu \phi. \tag{4.10}$$

These formulae allow us to exchange covariant derivatives ∇_μ compatible with the metric $g_{\mu\nu}$ for derivatives $\tilde{\nabla}_\mu$ compatible with $\tilde{g}_{\mu\nu}$.

Variational principle.—To derive the Maxwell equations it is most convenient to write the action in 'Schwinger' form. This is analogous to the Palatini formulation of Einstein gravity, in which one takes the Ricci scalar $R = R(\Gamma)$ to be constructed from the connection Γ, but without any assumption regarding the relation between Γ and $g_{\mu\nu}$. One then treats Γ and $g_{\mu\nu}$ as independent fields. After variation with respect to Γ and $g_{\mu\nu}$ separately, demanding that bulk contributions vanish yields the Einstein equation whereas demanding that boundary terms vanish requires ∇_μ to be compatible with $g_{\mu\nu}$. Therefore the Palatini variational procedure *also* determines Γ via the fundamental theorem of Riemannian geometry.

A similar approach due to Schwinger can be applied to the Maxwell Lagrangian.[37] To proceed, replace the Maxwell action (4.5) by

$$\frac{\mathscr{L}_{\mathrm{EM}}}{\sqrt{-g}} = -\frac{1}{2}(\nabla_\mu A_\nu - \nabla_\nu A_\mu)F^{\mu\nu} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu. \tag{4.11}$$

[37] See lecture 4 in the notes on quantum field theory by Ludwig Fadeev published in Ref. Deligne et al. (1999).

The field-strength tensor $F_{\mu\nu}$ and the connection A_μ are to be regarded as independent, making the Lagrangian density linear in derivatives as for the Palatini procedure. Variation with respect to $F^{\mu\nu}$ evidently reproduces the expected definition of the Maxwell tensor, Eq. (4.6). Substitution of this result in (4.11) yields (4.5), and therefore we conclude that both variational principles are equivalent.

We now introduce the distinction between $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Following the same procedure that led to (4.11), we find

$$\mathscr{L}_{\mathrm{EM}}[\tilde{g}, A, J] = \sqrt{-\tilde{g}}\left(-\frac{1}{2}(\tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu)\tilde{F}^{\mu\nu} + \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}\right) + \sqrt{-g}\, J^\mu A_\mu, \tag{4.12}$$

where $\tilde{\nabla}$ is the covariant derivative constructed from the connection $\tilde{\Gamma}$ given in Eq. (4.10), and $\tilde{F}_{\mu\nu}$ is the Maxwell tensor built from $\tilde{\nabla}$ and the electromagnetic 4-potential A_μ.

Transformation of the source current.—We are free to express the theory in terms of whatever frame is most convenient. In this section our aim is to obtain the Maxwell equations for the Einstein-frame **E** and **B** fields. This is useful because eventually it is the Einstein-frame metric $g_{\mu\nu}$ that will carry an FRW cosmology. The Hubble rate for this metric will receive contributions from the Einstein-frame **E** and **B** fields, modified by their interactions with the ϕ field.

For this purpose we require the Einstein frame 4-current $J^\mu = (\rho, \mathbf{J})$ appearing in Eq. (4.12), whose time and space components are the charge density ρ and 3-current $\mathbf{J}$ respectively. Although we are working in the Einstein frame, for practical purposes it is convenient to express these in terms of the equivalent *Jordan-frame* quantities, because it is these that would be measured by an experimentalist working in a small freely-falling laboratory in which the influence of gravity and fifth-forces can be neglected. To determine exactly how A_μ couples to these quantities we begin with the action for a Dirac spinor ψ coupled to the Jordan-frame metric $\tilde{g}_{\mu\nu}$,

$$\int \mathrm{d}^4x\, \mathscr{L}_{\mathscr{D}} = \int \mathrm{d}^4x\, \sqrt{-\tilde{g}}\left[-\bar{\psi}\not{\mathscr{D}}\psi + \text{h.c.}\right], \tag{4.13}$$

where $\not{\mathscr{D}} \equiv \gamma^a \tilde{e}_a{}^\mu(\tilde{\nabla}_\mu - \mathrm{i}qA_\mu)$ is the gauge- and diffeomorphism-covariant Dirac operator for a spin-1/2 fermion of charge q, $\tilde{e}_a{}^\mu$ is a vierbein for $\tilde{g}_{\mu\nu}$, and $\bar{\psi} = \psi^\dagger\gamma^0$ is the adjoint spinor to ψ. Greek indices μ, ν, ... label tensor indices transforming under spacetime coordinate diffeomorphisms, whereas Latin indices a, b, ... label indices transforming under the tangent space Lorentz group $SO(1,3)$. In particular, the vierbein satisfies

$$\tilde{e}_a{}^\mu \tilde{e}^{a\nu} = \tilde{g}^{\mu\nu} \qquad \text{and} \qquad \tilde{e}_a{}^\mu \tilde{e}_{b\mu} = \eta_{ab}. \tag{4.14}$$

Finally, the γ^a are Dirac matrices transforming under the tangent space Lorentz group. They satisfy the usual Dirac algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}\mathbf{1}$, where $\mathbf{1}$ denotes the identity matrix in the Dirac spinor representation. The action of the covariant derivative on a spinor can be written

$$\nabla_\mu \psi = \partial_\mu \psi + \frac{1}{8}\omega_\mu^{ab}\gamma_{ab}\psi, \tag{4.15}$$

where $\gamma_{ab} \equiv [\gamma_a, \gamma_b]$ and ω_μ^{ab} is the spin connection.

To identify the charge density ρ and current $\mathbf{J}$ we break Eq. (4.13) into space and time components. Notice that after doing so our expressions appear to mix indices transforming under the $SO(1,3)$ and diffeomorphism groups, although this appearance is fictitious. We find,

$$\begin{aligned}\frac{\mathscr{L}_{\mathscr{D}}}{\sqrt{-g}} =& -\bar{\psi}\gamma^0\tilde{\nabla}_0\psi - \frac{(1-\dot{\phi}^2/M_D^4)^{1/2}}{a}\bar{\psi}\gamma^i\tilde{\nabla}_i\psi + \mathrm{i}q\Big(\bar{\psi}\gamma^0 A_0\psi + \frac{(1-\dot{\phi}^2/M_D^4)^{1/2}}{a}\bar{\psi}\gamma^i A_i\psi\Big) \\ &+ \text{h.c.},\end{aligned} \tag{4.16}$$

where $a = a(t)$ is the scale factor and 'h.c.' denotes the Hermitian conjugate of the entire preceding expression. Spatial indices i, j, ... should be summed using the spatial part of the Einstein-frame FRW metric $g_{ij} = a^2\delta_{ij}$. Identifying $\tilde{\rho} = \mathrm{i}q\bar{\psi}\gamma^0\psi + \text{h.c.}$ as the Jordan-frame charge density and $\tilde{\mathbf{J}} = \mathrm{i}q\bar{\psi}\gamma^i\psi/a + \text{h.c.}$ as the corresponding 3-current, we conclude

$$\rho = \tilde{\rho} \qquad \text{and} \qquad \mathbf{J} = (1-\dot{\phi}^2/M_D^4)^{1/2}\tilde{\mathbf{J}}. \tag{4.17}$$

Returning to Eq. (4.12), expressing all quantities in terms of the Einstein frame, and using (4.17) for J^μ, we obtain

$$\begin{aligned}\frac{\mathscr{L}_{\mathrm{EM}}}{\sqrt{-g}} =& -\epsilon_\phi(\partial_0 A_i - \partial_i A_0)F^{0i} - \frac{1}{2\epsilon_\phi}(\partial_i A_j - \partial_j A_i)F^{ij} + \frac{\epsilon_\phi}{2}F_{0i}F^{0i} + \frac{1}{4\epsilon_\phi}F_{ij}F^{ij} \\ &+ \tilde{\rho}A_0 + \frac{1}{\epsilon_\phi}\tilde{J}^i A_i.\end{aligned} \tag{4.18}$$

We have used (4.8) and defined the quantity ϵ_ϕ to satisfy

$$\epsilon_\phi \equiv \left(1 - \frac{\dot{\phi}^2}{M_D^4}\right)^{-1/2}. \tag{4.19}$$

To proceed we introduce the Einstein-frame $\mathbf{E}$ and $\mathbf{B}$ fields using the conventional definitions[38]

$$aE_i = -F_{0i}, \tag{4.21a}$$

$$aB_i = \frac{1}{2}\epsilon_{ijk}F^{jk}, \tag{4.21b}$$

where ϵ_{ijk} is the covariant Levi-Civita tensor; its components take the values $\pm\sqrt{-g}$. In terms of these fields the action can be rewritten

$$\begin{aligned}\frac{\mathscr{L}_{\mathrm{EM}}}{\sqrt{-g}} = &-\frac{\epsilon_\phi}{a}\mathbf{E}\cdot\frac{\partial}{\partial t}(a^2\mathbf{A}) + \frac{\epsilon_\phi}{a}\phi\nabla\cdot\mathbf{E} - \frac{1}{\epsilon_\phi}\mathbf{A}\cdot\nabla\times\mathbf{B} - \frac{\epsilon_\phi}{2}\mathbf{E}^2 + \frac{1}{2\epsilon_\phi}\mathbf{B}^2 \\ &- \tilde{\rho}\phi + \frac{a^2}{\epsilon_\phi}\tilde{\mathbf{J}}\cdot\mathbf{A},\end{aligned} \tag{4.22}$$

where we have integrated by parts, dropped boundary terms at spatial infinity, and used that $D = 1/M_D^4$ is spatially independent in our model. (If D has spatial dependence then the action has a more complicated formulation.) We have also dropped explicit summation over spatial indices in favour of ordinary dot and cross products in a three-dimensional Euclidean metric. In this 3+1 split the electromagnetic 4-potential satisfies $A^\mu = (\phi, \mathbf{A})$, where ϕ and $\mathbf{A}$ are the three 3-dimensional scalar and vector potential respectively.

4.2.2 Maxwell's equations

Maxwell's equations can be obtained from Eq. (4.22) by variation with respect to ϕ, $\mathbf{A}$, $\mathbf{E}$ and $\mathbf{B}$. Of these, ϕ and $\mathbf{B}$ enter the action in terms that do not involve time derivatives, and therefore produce constraints rather than dynamical equations. The variations of $\mathbf{A}$ and $\mathbf{E}$ produce the evolution equations of the theory.

Constraints.—To see this in detail, first perform the variation with respect to ϕ. This yields Gauss' law,

$$\frac{1}{a}\nabla\cdot\mathbf{E} = \frac{\tilde{\rho}}{\epsilon_\phi}. \tag{4.23a}$$

By analogy with the usual Maxwell equation for a medium with permittivity ϵ, $\nabla\cdot\mathbf{E} = \rho/\epsilon$, we see that the electric field responds to charge density as if it were immersed within a

[38] An observer moving in spacetime with 4-velocity u_μ, normalized (with our metric convention) so that $u_\mu u^\mu = -1$, would observe electric and magnetic fields defined by

$$E_\mu = u^\nu F_{\mu\nu} \tag{4.20a}$$

and

$$B_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}u^\sigma F^{\nu\rho}, \tag{4.20b}$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the four-dimensional Levi–Civita tensor normalized so that $\epsilon_{0123} = \sqrt{-g}$. An observer comoving with the cosmological expansion has $u^\mu = (1, \mathbf{0})$, from which Eqs. (4.21a) and (4.21b) follow. See, e.g., Ref. Tsagas (2005).

medium with electric constant $\epsilon_\phi = (1 - D\dot{\phi}^2)^{-1/2}$. However, we will see that this analogy cannot be extended to all the Maxwell equations. Meanwhile, variation with respect to $\mathbf{B}$ enforces conservation of magnetic flux,

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \quad \frac{1}{a}\nabla \cdot \mathbf{B} = 0. \tag{4.23b}$$

As expected, both Eqs. (4.23a) and (4.23b) are constraints.

Dynamical equations.—The remaining Maxwell equations follow from variation with respect to $\mathbf{A}$ and $\mathbf{E}$. The $\mathbf{A}$ variation yields Ampère's circuital law,

$$\frac{1}{a}\nabla \times \mathbf{B} = a\tilde{\mathbf{J}} + \frac{\epsilon_\phi}{a^2}\frac{\partial}{\partial t}\Big(a^2 \epsilon_\phi \mathbf{E}\Big). \tag{4.23c}$$

By comparison, the form of this law in a medium with fixed electric and magnetic constants ϵ and μ would be $\nabla \times \mathbf{B} = \mu \mathbf{J} + \mu\epsilon\dot{\mathbf{E}}$. Therefore, apparently, there is no assignment of electric and magnetic constants that would maintain the analogy of the scalar condensate as a dielectric medium, as can be done for the gravitational coupling Plebanski (1959).

Finally, variation with respect to $\mathbf{E}$ yields Faraday's law of induction

$$\mathbf{E} = -\frac{1}{a}\nabla\phi - \frac{1}{a}\frac{\partial}{\partial t}(a^2\mathbf{A}) \quad \Rightarrow \quad \frac{1}{a}\nabla \times \mathbf{E} = -\frac{1}{a^2}\frac{\partial}{\partial t}(a^2\mathbf{B}). \tag{4.23d}$$

This is not modified by the disformal coupling.

Relations equivalent to (4.23a)–(4.23d) were given in Ref. Brax et al. (2013), although because this reference worked in terms of a covariant formalism they were not broken out into separate Maxwell equations.

4.2.3 Electromagnetic waves

The wave equation.—Our interest lies in the propagation of electromagnetic radiation from cosmological distances, and for this purpose we require an equation governing electromagnetic waves. After specializing to the vacuum case, a suitable equation from the magnetic field $\mathbf{B}$ can be obtained by taking the curl of Ampère's law, Eq. (4.23c), and substituting the time derivative of Faraday's law (4.23d) to eliminate $\nabla \times \dot{\mathbf{E}}$. The wave equation resulting from this procedure is (cf. Refs. Turner and Widrow (1988); Tsagas (2005))

$$\ddot{\mathbf{B}} + \left(5H + \frac{\dot{\epsilon}_\phi}{\epsilon_\phi}\right)\dot{\mathbf{B}} + \left(2(3H^2 + \dot{H}) + 2H\frac{\dot{\epsilon}_\phi}{\epsilon_\phi}\right)\mathbf{B} - \frac{1}{a^2\epsilon_\phi^2}\nabla^2\mathbf{B} = 0. \tag{4.24a}$$

The coupling to gravity has been well-studied. Both gravitational effects and the disformal coupling generate a soft mass term that does not spoil gauge invariance. In particular note that this wave equation describes coupled electric and magnetic oscillation of a fixed frequency, rather than a free magnetic field. Despite the large friction term $5H$ appearing in (4.24a), the energy density $\sim \mathbf{B}^2$ of a free magnetic field still redshifts at the rate $1/a^4$ expected for radiation. The same applies for a free electric field. See, eg. Barrow and Tsagas (2011). Once a solution for $\mathbf{B}$ is known, it can be used to generate a solution for $\mathbf{E}$ via Eq. (4.23d). Alternatively, $\mathbf{E}$ can be solved directly using

$$\ddot{\mathbf{E}} + \left(5H + 3\frac{\dot{\epsilon}_\phi}{\epsilon_\phi}\right)\dot{\mathbf{E}} + \left(2(3H^2 + \dot{H}) + 7H\frac{\dot{\epsilon}_\phi}{\epsilon_\phi} + \frac{\dot{\epsilon}_\phi^2}{\epsilon_\phi^2} + \frac{\ddot{\epsilon}_\phi}{\epsilon_\phi}\right)\mathbf{E} - \frac{1}{a^2\epsilon_\phi^2}\nabla^2\mathbf{E} = 0. \tag{4.24b}$$

Electromagnetic waves governed by Eqs. (4.24a) or (4.24b) do not propagate with velocity $c = 1$. Neglecting the effect of the mass term, their phase velocity is

$$c_{\mathrm{EM}}^2 = \frac{1}{\epsilon_\phi^2} = 1 - \frac{\dot{\phi}^2}{M_D^4}. \tag{4.25}$$

This result is accurate to leading order in $1/M_D$. The same formula was previously given in Ref. Brax et al. (2013).

Notice that Eq. (4.25) is always subluminal provided $\dot{\phi}^2/M_D^4 \ll 1$. In this calculation we have worked to leading order in $1/M_D$, so this limitation is effectively a consistency condition. When $c_{\mathrm{EM}} \ll 1$ higher terms in $1/M_D$ must become important and we cannot use our calculation to make a clear statement about the phenomenology. We describe this as the "nonlinear region". Whether a given model falls in this region depends explicitly on M_D and the scalar field profile, and as we will see below this depends in turn on Λ and the expansion history $H(t)$ of the model. Loop corrections will almost certainly renormalize the numerical coefficients at each order, making this region very difficult to study.

WKB solution.—Eqs. (4.24a) and (4.24b) can be reduced to a common form by making a field redefinition to remove the friction term (that is, the term linear in $\dot{\mathbf{E}}$ or $\dot{\mathbf{B}}$). Making the transformations

$$\mathbf{E}(t,\mathbf{x}) = \frac{\mathbf{G}(t,\mathbf{x})}{a^{5/2}\epsilon_\phi^{3/2}}, \qquad \text{and} \qquad \mathbf{B}(t,\mathbf{x}) = \frac{\mathbf{G}(t,\mathbf{x})}{a^{5/2}\epsilon_\phi^{1/2}} \tag{4.26}$$

it can be checked that both fields can be built from solutions to the equation

$$\ddot{\mathbf{G}} + m_{\mathrm{eff}}^2(t)\mathbf{G} - \frac{1}{a^2\epsilon_\phi^2}\nabla^2\mathbf{G} = 0, \tag{4.27a}$$

where the time-dependent mass $m_{\mathrm{eff}}^2(t)$ is defined by

$$m_{\mathrm{eff}}^2(t) \equiv -\frac{H^2}{4} - \frac{\dot{H}}{2} - \frac{H}{2}\frac{\dot{\epsilon}_\phi}{\epsilon_\phi} + \frac{1}{4}\frac{\dot{\epsilon}_\phi^2}{\epsilon_\phi^2} - \frac{1}{2}\frac{\ddot{\epsilon}_\phi}{\epsilon_\phi}. \tag{4.27b}$$

Clearly, this merely reflects the fact that both $\mathbf{E}$ and $\mathbf{B}$ are derived from the same underlying gauge field.

The terms involving H^2 and $\dot{H}$ are generated by mixing with the metric Breitenlohner and Freedman (1982). They are both roughly of order H^2. The terms involving derivatives of ϵ_ϕ will depend on the detailed profile of the scalar field $\phi(t)$. In Ref. Brax et al. (2013) it was suggested that these terms would also typically be of order H^2 unless the field is undergoing a sudden transition. In §4.3 below we will see that this expectation is borne out for the scalar field profile generated when the nonlinear kinetic term $\mathcal{G}_3$ is relevant.

In this situation we can expect $|\dot{m}_{\text{eff}}/m_{\text{eff}}^2|$ to be of order $|\dot{H}/H^2| \sim 1$, which implies that the mass changes significantly on timescales of order the Hubble time. For electromagnetic waves whose wavelength is much smaller than the horizon we can regard m_{eff}^2 as roughly fixed over many cycles of the wavetrain. It follows that we can obtain an approximate description of its evolution using the WKB procedure. We expand $\mathbf{G}$ in a suitable basis of polarization matrices with Fourier mode functions $\psi(t, \mathbf{k})$. Then the WKB approximation consists in writing

$$\psi(t, \mathbf{x}) = \alpha(t) \exp\Big(\mathrm{i}\mathbf{k}\cdot\mathbf{x} - \mathrm{i}\theta(t)\Big), \tag{4.28}$$

where the phase $\theta(t)$ varies rapidly on the timescale of $\alpha(t)$.

Substitution of (4.28) in (4.27a) yields

$$\frac{\ddot{\alpha}}{\alpha} - \dot{\theta}^2 + m_{\text{eff}}^2 + \frac{k^2}{a^2\epsilon_\phi^2} + \mathrm{i}\left(2\frac{\dot{\alpha}}{\alpha}\dot{\theta} + \ddot{\theta}\right) = 0. \tag{4.29}$$

Demanding that the real and imaginary parts cancel separately shows that the amplitude $\alpha(t)$ varies like $\alpha(t) = \alpha_0/\dot{\theta}(t)^{1/2}$. Meanwhile, the phase function $\theta(t)$ satisfies

$$-\frac{1}{2}\frac{\dddot{\theta}}{\dot{\theta}} + \frac{3}{4}\frac{\ddot{\theta}}{\dot{\theta}}\frac{\ddot{\theta}}{\dot{\theta}} - \dot{\theta}^2 + m_{\text{eff}}^2 + \frac{k^2}{a^2\epsilon_\phi^2} = 0. \tag{4.30}$$

We write

$$\theta = \theta_{\text{fast}} + \theta_{\text{slow}}, \tag{4.31}$$

where θ_{fast} is designed to absorb the 'fast' variation due to integration of the source term,

$$\frac{\mathrm{d}\theta_{\text{fast}}}{\mathrm{d}t} = \left(m_{\text{eff}}^2 + \frac{k^2}{a^2\epsilon_\phi^2}\right)^{1/2} \equiv \omega_{\text{eff}}(t), \tag{4.32}$$

where the last equality defines the effective frequency ω_{eff}. The solution is $\theta_{\text{fast}} \approx \int^t \omega_{\text{eff}}(t')\,\mathrm{d}t'$. In particular, $\theta_{\text{fast}}(t_2) - \theta_{\text{fast}}(t_1) \approx \omega_{\text{eff}}(\bar{t})\Delta t$ if t_1 and t_2 are not too widely separated, where $\bar{t} = (t_1 + t_2)/2$ and $\Delta t = t_2 - t_1$. It follows that the second derivative $\dot{\theta}_{\text{fast}}$ varies much more slowly than θ_{fast}, making $\ddot{\theta}_{\text{fast}}$ a slowly-varying function that determines θ_{slow}. If desired we could solve (4.30) perturbatively for θ_{slow}, although for the purposes of this paper we do not need such precision. It follows that a typical mode function of the field $\mathbf{G}$ approximately satisfies

$$\psi(t, \mathbf{k}) = \frac{\alpha_0}{\sqrt{\omega_{\text{eff}}(t)}} \exp\left(\mathrm{i}\mathbf{k}\cdot\mathbf{x} - \mathrm{i}\int^t \omega_{\text{eff}}(t')\,\mathrm{d}t'\right). \tag{4.33}$$

This solution was previously given in Ref. Brax et al. (2013), neglecting the mass term m_{eff}^2. It was described there as the "eikonal approximation". In this case the WKB approximation reduces to the same procedure.

Time-of-flight formula.—The phase velocity of the wavetrain at time t is roughly $\omega_{\text{eff}}(t)/k$. If we regard an electromagnetic wave as a coherent superposition of very many collimated photons, it is clear that the phase velocity of the wave must equal the propagation velocity of the photons because of the peculiar properties of massless particles in special relativity. For *massive* particles travelling at less than the speed of light this relationship is no longer so clear. In certain circumstances (such as water waves) it can happen that the phase velocity is unrelated to the velocity of individual particles that participate in the wavetrain.

Recalling (4.25) it follows that the phase velocity for (4.33) can be written

$$c_p = \frac{\omega_{\text{eff}}}{k_{\text{phys}}} = \frac{c_{\text{EM}}}{k_{\text{phys}}}\left(k_{\text{phys}}^2 + \frac{m_{\text{eff}}^2}{c_{\text{EM}}^2}\right)^{1/2} = c_{\text{EM}}\left(1 + \frac{m_{\text{eff}}^2}{c_{\text{EM}}^2 k_{\text{phys}}^2}\right)^{1/2} \tag{4.34}$$

where $k_{\text{phys}} = k/a$ is the physical wavenumber corresponding to the comoving wavenumber k. Note that c_{EM} is only equal to the phase velocity if $m_{\text{eff}}^2 = 0$. However, it is clearly unsatisfactory to regard the phase velocity c_p as an estimate for the characteristic particle velocity in the beam. First, c_p can easily become superluminal even if m_{eff}^2 is positive. Second, there is an unwanted divergence at small k_{phys}, and at large k_{phys} (where energies are ultrarelativistic) c_p approaches c_{EM} rather than unity. These properties are entirely characteristic of phase velocities and have nothing to do with the disformal coupling or the fact that (4.33) is propagating on a curved background.

Instead, we proceed as follows. We are still considering (4.33) to describe a coherent superposition of very many collimated particles that share an approximate common momentum 4-vector. Therefore, consider a small box of spacetime that lies along the particles' trajectory. According to the equivalence principle we can regard (4.33) as the wavefunction for an on-shell particle with 4-momentum k_μ in the interior of the patch. For small displacements $\delta x^\mu = (\delta t, \delta \mathbf{x})$ within the patch this yields

$$\psi \approx \exp\Big(\mathrm{i}\mathbf{k}\cdot\delta\mathbf{x} - \mathrm{i}\omega_{\text{eff}}\delta t\Big) \approx \mathrm{e}^{\mathrm{i}k_\mu \delta x^\mu} \tag{4.35}$$

which implies that we should regard $\omega_{\text{eff}}(t)$ as the characteristic energy for particles in the beam at time t. This is related to their propagation velocity via the usual special relativistic formula $E = \gamma m$, and hence

$$v = \sqrt{1 - \frac{m_{\text{eff}}^2}{\omega_{\text{eff}}^2}} = c_{\text{EM}}\left(\frac{1}{c_{\text{EM}}^2 + m_{\text{eff}}^2/k_{\text{phys}}^2}\right)^{1/2}. \tag{4.36}$$

Clearly v has more satisfactory properties than c_p. As $k_{\text{phys}} \to 0$ the propagation velocity approaches zero. In the ultrarelativistic limit $k_{\text{phys}} \to \infty$ we have $v \uparrow 1$. Further, v is always subluminal if m_{eff}^2 is positive.

Using v to estimate particle velocities in the beam, the time of flight between two locations A and B is

$$T(A \to B) = \int_A^B \frac{\mathrm{d}r}{v}, \tag{4.37}$$

where $\mathrm{d}r$ is an element of length along the trajectory and we have assumed that A and B are sufficiently local that the effects of curvature can be ignored. This is typically the case for LIGO sources. For local sources it will also be a reasonable approximation to take v as time independent, in which case the travel time of electromagnetic radiation T_{EM} compared to the travel time of gravitational radiation T_{grav} will be

$$T_{\text{EM}} \approx \frac{1}{v} T_{\text{grav}}. \tag{4.38}$$

To obtain a quantitative estimate requires information about the scalar field profile. We discuss this in §4.3 before applying (4.37) to GW170817 in §4.4.

4.3 The dynamics of the scalar field

Our task is now to solve for the evolution $\phi(t)$ of the Galileon scalar. To leading order in $1/M_D$ the action can be obtained by linearizing Eq. (4.4) in the disformal coupling. Explicitly, this is

$$\int \mathrm{d}^4x \sqrt{-g} \left[\frac{M_{\text{P}}^2}{2}(R - 2\Lambda_{\mathsf{C}}) + \frac{c_2}{2}\dot{\phi}^2 + \frac{1}{\Lambda^3}\ddot{\phi}\dot{\phi}^2 + \left(1 - \frac{\dot{\phi}^2}{2M_D^4}\right)\rho \right], \tag{4.39}$$

where ρ is the density of baryonic and cold dark matter. Eq. (4.39) applies at any epoch, but we mostly use it during matter domination where $\rho = \rho_m$. As explained in §4.1, we fix c_2 in order to break a scaling symmetry in the Galileon sector; its role is to make the quadratic kinetic term stable if $c_2 = +1$ and ghostlike if $c_2 = -1$. We will allow the cubic self-interaction scale Λ to vary over a suitable parameter range. It can be positive or negative.

Remarkably, Eq. (4.39) admits an exact solution in which the linear and nonlinear ϕ kinetic terms combine to support a nontrivial field profile. Applying the Euler–Lagrange equation to (4.39) in terms of $\dot{\phi}$ requires $\partial\mathcal{L}/\partial\dot{\phi} = \mathrm{d}/\mathrm{d}t(\partial\mathcal{L}/\partial\ddot{\phi})$. This leads immediately to the *algebraic* solution

$$\dot{\phi}(t) = -\left(c_2 + \frac{\rho_m(t)}{M_D^4}\right)\frac{\Lambda^3}{3H(t)}. \tag{4.40}$$

In the absence of the nonlinear term the only solution available is $\dot{\phi} = c$ for constant c. This is substantially less interesting and does not lead to interesting time-dependent effects from the disformal coupling in Eqs. (4.24a)–(4.24b) or (4.27a). In the late universe ρ_m is negligible in comparison with M_D once we impose the ATLAS constraint $M_D \gtrsim 1.2\,\text{TeV}$ Aaboud et al. (2019). Therefore we see that the disformal coupling cannot play an important role in the evolution of ϕ except during the very early universe, before the time of the electroweak phase transition.

Eq. (4.40) is valid for all cosmological backgrounds and parameter values, provided that $\dot{\phi}^2 \ll M_D^4$. In the late universe its time variation is set by $H(t)$, as indicated in §4.2.3. Notice that this solution bears a strong resemblance to the "tracker" solutions described by Barreira et al. Barreira et al. (2013); Renk et al. (2017), which are defined so that

$$\frac{\dot{\phi} H}{M_\text{P} H_0^2} \equiv \xi = \text{constant}. \tag{4.41}$$

These solutions yield $\dot{\phi} \sim 1/H$, as does (4.40) when the disformal coupling can be neglected.

Parameter constraints.—Recall that to evade stringent constraints from the ISW effect we do not allow the Galileon to contribute significantly to the energy budget of the Universe. The energy density contributed by the Galileon sector is

$$\rho_\text{Gal} = \rho_{\mathcal{G}_2} + \rho_{\mathcal{G}_3} = \frac{1}{2} c_2 \dot{\phi}^2 + \frac{1}{\Lambda^3} \ddot{\phi} \dot{\phi}^2, \tag{4.42}$$

where $\rho_{\mathcal{G}_2}$ and $\rho_{\mathcal{G}_3}$ measure the energy density contributed by the quadratic and cubic Galileon operators $\mathcal{G}_2$ and $\mathcal{G}_3$, respectively. We constrain the model parameters Λ and M_D so that ρ_Gal evaluated at the present day is smaller than the current background energy density $\sim \text{meV}^4$. In terms of the solution (4.40) we can evaluate $\rho_{\mathcal{G}_2}$ and $\rho_{\mathcal{G}_3}$ individually,

$$\rho_{\mathcal{G}_2} \equiv \frac{1}{2} c_2 \dot{\phi}^2 \approx \frac{1}{2} c_2 \frac{\Lambda^6}{9H^2}, \tag{4.43}$$

$$\rho_{\mathcal{G}_3} \equiv \frac{1}{\Lambda^3} \ddot{\phi} \dot{\phi}^2 = \frac{2}{3} \frac{\dot{H}}{H^2} \left(c_2 + \frac{\rho_m}{M_D^4} - 6 \frac{H^2}{M_D^4} \right) c_2 \rho_{\mathcal{G}_2} \approx \frac{c_2}{3} \frac{\dot{H}}{H^2} \frac{\Lambda^6}{9H^2}. \tag{4.44}$$

In the final expressions we have assumed $\rho_m \ll M_D^4$.

4.4 Conclusion

We are now in a position to apply the time-of-flight formula (4.37) to GW170817. Assuming (4.40) for the scalar field profile, we find that the difference in travel time is negligible, despite the tightness of the constraint. We assume the lower limit for M_D allowed by collider measurements Aaboud et al. (2019), and take $\Lambda = 2.4 \times 10^{-13}\,\mathrm{eV}$, which is the largest value permitted by the constraints on ρ_{Gal} for this value of M_D. These choices maximize the time-of-flight difference. Unfortunately, for any physically reasonable k_{phys}, it can be checked that T_{EM} and T_{grav} are indistinguishable to more than 15 significant figures for any low-redshift event such as $GW170817$. Therefore we conclude that the disformal coupling *alone* is so weak it cannot be constrained even by precise time-of-flight observations. A similar conclusion applies to the dispersion of light implied by the k-dependence of (4.33) and (4.36).

In Fig. 4.1 we show constraints in the parameter space for M_D and Λ. Notice that there is a curious competition between $\rho_{\mathcal{G}_2}$ and $\rho_{\mathcal{G}_3}$ during matter domination. Because the linear kinetic energy density is proportional to $\dot{\phi}^2$, the constraint is independent of the sign of Λ. It is also almost independent of the sign of c_2.[39] Prior to dark energy domination the linear and non-linear terms $\rho_{\mathcal{G}_2}$ and $\rho_{\mathcal{G}_3}$ in Eq. (4.42) have almost exactly the same amplitude; their ratio is $c_2 2\dot{H}/(3H^2) \simeq c_2$ during matter domination, up to corrections of order ρ_m/M_D^4. Hence if $c_2 = -1$ the total scalar kinetic energy is always suppressed during matter domination and until dark energy dominates at $z \lesssim 0.5$. This 'screening-like' effect is curious and is worthy of further attention. Today, for a flat ΛCDM cosmology with $\Omega_m \simeq 0.29$ the linear energy dominates by a factor of 3.5 over the non-linear term.

Our analysis supports earlier impressions that a disformal coupling is difficult to constrain using cosmology alone. If so, then collider physics will remain the best prospect for determining constraints on the disformal coupling scale M_D, but conversely we cannot expect the current ATLAS bound $M_D \gtrsim 1.2\,\mathrm{TeV}$ to be dramatically superseded in the near- or medium-term future. While previous studies including a Galileon sector and a disformal coupling have reported that best-fit models typically yield a time-lag Δt too large to be compatible with LIGO/VIRGO constraints, the effect in these analyses is driven by the internal structure of the Galileon sector and not by the disformal coupling.

[39] A discussion of the frame dependence is considered in Faraoni et al. (1999); Flanagan (2004).

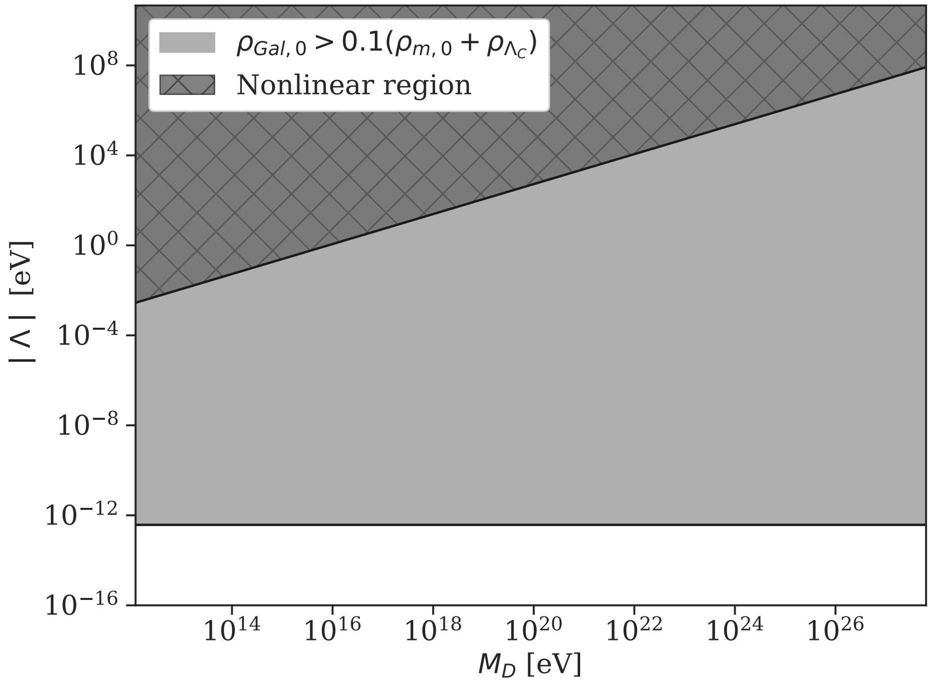

Figure 4.1: The plot shows the parameter space for the cubic Galileon model disformally coupled to matter, in the cubic Galileon parameter Λ and the disformal scale M_D. For definiteness we show the constraint with $c_2 = +1$, but there is only an order unity shift when $c_2 = -1$. The x-axis runs from 1.2×10^{12}eV, the minimum value for M_D given by collider constraints up to M_{P}. The light grey region is ruled out due to observations of the ISW effect. The dark grey region contains models which cannot be accurately described by the calculation performed here. The allowed parameter space is shown in white.

Chapter 5

Conclusion

Here I will briefly review the most exciting things to happen in the cosmology community over the last five years, the time it took me to complete my PhD. I will then describe the most compelling expectations of the field of cosmology over the next 15 years. Finally, I will summarise the conclusions of the specific work contained in this thesis before giving my expectations for the sequester theory and for disformal couplings in the future.

A few days before the start of my PhD, the Laser Interferometer Gravitational-Wave Observatory (LIGO) made the first direct detection of gravitational waves on 14 September 2015. It was inferred that the signal, dubbed GW150914, originated from the merger of two black holes with masses 36^{+5}_{-4} $M_\odot$ and 29 ± 4 $M_\odot$, resulting in a 62 ± 4 $M_\odot$ black hole Abbott et al. (2016a). The announcement of the discovery was not made until early the next year. The phenomena had been predicted 100 years earlier by Einstein as a consequence of his general theory of relativity Einstein (1916, 1917). This observation and those that followed resulted in the 2017 Nobel Prize in Physics being awarded for the detection of gravitational waves Nobel Media AB (2017).

LIGO is made up of two interferometers, one in Hanford, Washington state, and one in Livingston, Louisiana. Virgo is a similar experiment based near Pisa, Italy. In August 2017, LIGO and Virgo observed the first gravitational waves coming from a binary neutron star inspiral (GW170817) Abbott et al. (2017b). Because this event was detected at all three sites at slightly different times, this allowed for an improvement of the localization of the source by a factor of ten. The localisation of the source facilitated the electromagnetic follow-up of the event; neutron star mergers were expected to be accompanied by an electromagnetic counterpart. Multiple observatories detected the electromagnetic counterpart, a kilonova in the galaxy NGC 4993, 40 Mpc away, emitting a short gamma ray burst (GRB 170817A) 1.7 seconds after the merger Goldstein et al. (2017); Abbott et al. (2017a); Savchenko et al. (2017); Abbott et al. (2017c); Pan et al. (2017).

These observations were very important for the modified gravity and dark energy community. More details are given in Subsection 3.1. This is not the only major result to affect the cosmology community in the last five years.

The Planck collaboration released their final results in 2018 Aghanim et al. (2018). The data taken by the Planck satellite gave the highest resolution detections of both the intensity and polarization of the CMB anisotropies. It also catalogued galaxy clusters through observations of the the Sunyaev-Zel'dovich effect and observations of gravitational lensing of the CMB. The satellite also took measurements of the integrated Sachs-Wolfe effect. This data gave us very precise measurements of several cosmological parameters, including the inferred Hubble constant.

The final release of the Planck results, along with more recent local measurements of H_0 (for examples see Riess et al. (2019); Birrer et al. (2019)), has resulted in the tension that these measurements exhibit (as outlined in Section 1.2) becoming a trending topic of discussion in the cosmology community. Independent estimations of the errors of the local measurements have been made specifically with regard to: errors in redshift measurements of supernovae Davis et al. (2019), errors in measurements of the local Hubble flow Wojtak et al. (2014); Odderskov et al. (2017); Wu and Huterer (2017), and Cepheid calibration Rose et al. (2019); Jones et al. (2018). More exotic solutions to the tension have also been proposed including alternative theories of dark energy Peirone et al. (2019); Martinelli et al. (2019).

The first direct image of a black hole was taken in 2017 and released in 2019 Akiyama et al. (2019a). The black hole that was pictured was at the center of the galaxy M87 and has an approximate mass of $10^9 \mathrm{M}_\odot$ Akiyama et al. (2019b). The image was made with radio-wave observations and provides a new tool to test gravity at its most extreme limits and on a mass scale that was not accessible until now. As such, the article has already been cited over 500 times, with many of those articles describing tests of general relativity and alternative theories using observations like this, for some recent examples see Zeng and Zhang (2020); Banerjee et al. (2020).

One could say that the last five years have been a good time to be a cosmologist. In the next section we discuss the work done in this thesis in light of these exciting events. First, we explain why the next 15 years will be just as exciting.

The European Space Agency (ESA) test mission, Laser Interferometer Space Antenna (LISA) Pathfinder, was launched in 2015 to test the technology necessary to put a test mass in (almost) perfect free fall conditions McNamara et al. (2008). The full LISA mission will provide a new gravitational wave observatory with an arm length of 2.5 million km (and frequency range of 8.3 seconds or 0.12 Hz) Amaro-Seoane et al. (2017); Stroeer and Vecchio (2006); Berti et al. (2019); Amaro-Seoane et al. (2012). This will enable further tests of gravity Belgacem et al. (2019).

A future gravitational wave observatory is not he only new observatory cosmologists are looking forward to. MeerKAT is a radio telescope consisting of 64 antennas and is the first phase of the Square Kilometer Array (SKA). When finished SKA will have a total collecting area of approximately one square kilometre. It will operate over a wide range of frequencies and its size would make it 50 times more sensitive than any other radio instrument built to date. The amount of data coming from this telescope will be approximately an exabyte (10^{18} bytes) a day, giving the community plenty to work with to further test gravity and cosmology Bacon et al. (2020); Schwarz et al. (2015); Raccanelli et al. (2015); Maartens et al. (2015); Weltman et al. (2020).

The final experiments we will describe in this far from exhaustive list of future observatories are those which will take large galaxy surveys. This includes the following[40]: the Dark Energy Survey (DES) Abbott et al. (2005), the Dark Energy Spectroscopic Instrument (DESI) Levi et al. (2019), Euclid Laureijs et al. (2011), the Vera C. Rubin Observatory, also known as the Large Synoptic Survey Telescope (LSST) Ivezić et al. (2019), and 4MOST de Jong et al. (2019). Although the first three are satellites and the last two are terrestrial; they share some common goals. Galaxy surveys measure the distribution of large scale structure including galaxies and galaxy clusters. These galaxies can be used as bias tracers of the underlying dark matter field in the Universe.

5.1 Summary of work done here

We have discussed the biggest advancements in cosmology and gravity in the last five years and hopefully the reader is convinced that the next 15 will be just as insightful. This thesis has touched on two different theories which each address different problems but both have consequences for our understanding of cosmology.

[40]The SKA could fit into this list but because nothing like it on the scale planned had ever been made before, it warranted its own paragraph

5.1.1 The sequester

The sequester proposal was built to solve the problem that we have called in this thesis, the radiative stability problem (see section 1.3.2 for a definition). Our work and the work of others El-Menoufi et al. (2019) has shown that the most recent realisation of the sequester Kaloper and Padilla (2017) seems to solve the problem of radiative stability.

A cornerstone of sequestering the vacuum energy is that additional terms, σ and $\hat{\sigma}$ are added to the standard gravity and matter action, at the most global scale,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{P}}^2}{2}R + \theta R_{\mathrm{GB}} - \Lambda^4 - \mathcal{L}_{\mathrm{m}}\right) + \sigma\left(\frac{\Lambda^4}{\mu^4}\right) + \hat{\sigma}(\theta). \tag{5.1}$$

Because σ and $\hat{\sigma}$ do not couple to the metric, they do not source gravitational fields. They are constant, they do not change even with changes in the spacetime volume. The beneficial effects of these terms are not diluted as spacetime expands. These terms seem unusual, however they do have a microscopic interpretation, integrals of a four-form flux F_4 over spacetime Kaloper et al. (2016b). If one accepts that it is possible for these kind of terms to exist in the action[41], then the large low-energy loop contributions can be stored in these non-gravitating sectors designed to do exactly that and nothing more.

On the sequester solving the cosmological constant problem there are more questions here. Within effective field theory, renormalized couplings of operators cannot be predicted but must be measured, Kaloper & Padilla argue this must be the case for the cosmological constant (arguments against this explanation can be found in 1.3.2.). To Kaloper & Padilla solving the radiative stability problem amounts to solving the cosmological constant problem itself.

If we were to disagree with the argument above, then the sequester solution does not explain the current value of the cosmological constant energy density. It is proposed in Kaloper and Padilla (2014b) that a quintessence like field would be an appropriate explanation. The sequester has no comment on the size of the bare cosmological constant, which there is no reason to expect not to exist. This could also explain the observed value of the cosmological constant energy density.

5.1.2 Disformal couplings

We stated in section 3.3 that a disformal and conformal coupling are both parts of the most general line element that can be written down which obeys the weak equivalence

[41] The four-form fluxes are an effective description which ultimately will have to be described using a UV complete theory, this is discussed more in the next part of the conclusions, Subsection 5.2.1.

principle. In this thesis we explored the possibility of a disformal coupling between a Galileon scalar field and matter.

We attacked the theory of a cubic Galileon disformally coupled to matter with an arsenal consisting of: measurements of the speed of gravitational waves compared to light with GW170817, null observations of missing energy at colliders, observations of the ISW effect, null observations of the difference in speeds of different frequencies of light, and considerations of phase velocity of electromagnetic waves. Ultimately we found that the disformal coupling is very resilient to cosmological tests, with the best constraint on the disformal scale M_D coming from collider constraints. The probes we tested here also did not constrain the cubic Galileon itself any further than has already been done Renk et al. (2017).

5.2 Future work

We have already outlined what we can broadly expect from cosmology over the next few years, here we address the future of the two focuses of this thesis.

5.2.1 The sequester

The cosmological constant problem still remains the biggest problem in theoretical physics. Its solution would surely warrant a Nobel prize. The idea of sequestering the vacuum loops was not *new* in the sense that similar work had come before it, for example Coleman's wormholes Coleman (1988).

There is nothing more to be said about observational consequences of the sequester. It is formulated to solve a single problem and nothing more, leaving all other physics untouched. One might be concerned that the sequester may affect inflation. In slow-roll inflation the inflaton source behaves like a constant vacuum energy to zeroth order. We see a de Sitter phase in inflation and today. Kaloper, Padilla and their collaborators found that the sequester does not interfere with inflation Kaloper and Padilla (2014a); Coltman et al. (2019).

Future work on the sequester theory will likely involve the development of a UV complete theory, or rather finding a UV complete theory that permits the sequester in its low energy effective description. The sequester theory alone states that there is some form of matter, for which all the vacuum energy flows to, that does not gravitate. There is more to be understood about what this matter *is* exactly.

Indeed, work in this direction has already begun. In Kaloper (2019) it is stated that the terms that do not couple to the metric in Equation 5.1 resemble terms which typically appear in string compactifications.

5.2.2 Disformal couplings

The future study of disformal couplings is much more open than that of the sequester theory. It has been repeatedly shown in the literature and in Chapter 4 that it is very hard to constrain the disformal coupling with cosmological probes. The best constraints we have on the coupling so far come from looking for missing energy at colliders, and this will likely remain.

One may ask if Horndeski theories, disformally coupled or otherwise, play any role in the growth or distribution of structure in the Universe. There may be another way to test the disformal coupling through the observation of nonlinear structure. N-body simulations have been made for a disformally coupled quintessence model Llinares et al. (2020)[42]. Interestingly, in Llinares et al. (2020) "excellent" agreement was found between exact and approximated solutions, which opens the way for running disformal gravity cosmological simulations using Newtonian solvers.

We propose a test of a Horndeski theory, specifically the cubic Galileon coupled conformally and/or disformally to Standard Model matter using data from a weak lensing survey and redshift space distortions. Weak lensing is a good probe of modified gravity models because it is sensitive to the growth of large scale structure and the relationship between the gravitational potentials and matter. Although the slip in Galileon gravity is the same as that in GR i.e. the Bardeen potentials are equal; the geodesic equation in Galileon gravity is different due to the "fifth force" effects.

If one were to go down this road of investigation, one may use of a developed Boltzman solver code e.g. Hi-CLASS Bellini et al. (2020), to output the matter power spectrum. In predicting the matter power spectrum for such a universe it would be beneficial to also take into account screening (if it exists in the theory). One could also build a pipeline to compute the redshift-space multipoles from the Kaiser formula for the redshift-space over density using the previously computed power spectrum. An example of a similar pipeline was used in Burrage et al. (2017).

These predictions could then be assembled into a single likelihood code that compares the shear power spectrum to lensing observations from e.g. KiDS Kuijken et al. (2019), and the redshift space distortions power spectra to a galaxy survey e.g. 2dFLens Blake

[42]For other work that has been done on testing a disformally coupled quintessence field acting as dark energy to matter see e.g. Ref Teixeira et al. (2020)

et al. (2016). The final step will be to embed this likelihood code in an MCMC code e.g. MontePython Brinckmann and Lesgourgues (2019).

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Appendix A

Sequester calculation details

Here, we give some details of the computation of the left hand diagram in Figure 2.1. The explicit calculation of the diagram was made to check that no miraculous cancellation of contribution occurred. The amplitude is,

$$I_{\rm tot} = i\frac{8^2}{M_p^4}\int\frac{{\rm d}^4k}{(2\pi)^D}\int\frac{{\rm d}^4p}{(2\pi)^D}\frac{1}{(p^2+m^2)^a}\frac{p^2(p-k)^2}{[(p-k)^2+m^2]^b}, \tag{A.1}$$

where p is the momentum in the scalar loop and k the momentum of the gravition. We can write,

$$I = \int\frac{{\rm d}^4k}{(2\pi)^D}\int\frac{{\rm d}^4p}{(2\pi)^D}\frac{1}{(p^2+m^2)^a}\frac{p^2(p-k)^2}{[(p-k)^2+m^2]^b}, \tag{A.2}$$

where we have changed the powers in the denominator to a and b. First, using the Feynman parameterisation to combine denominators and then shifting the integration variable $p' \to p-(1-x)k$,

$$I = \int\frac{{\rm d}^4k}{(2\pi)^D}\int\frac{{\rm d}^4p'}{(2\pi)^D}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1{\rm d}x\; x^{a-1}(1-x)^{b-1}\frac{[p'+(1-x)k]^2[p'-xk]^2}{[(p')^2+m^2+x(1-x)k^2]^{a+b}}. \tag{A.3}$$

We are integrating over a symmetric integral and therefore after multiplying out the numerator odd terms will vanish. Breaking up the integral and computing each one. Dropping the $'$ from p',

$$I_1 = \int\frac{{\rm d}^4k}{(2\pi)^D}\int\frac{{\rm d}^4p}{(2\pi)^D}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1{\rm d}x\; x^{a-1}(1-x)^{b-1}\frac{p^4}{[p^2+M^2]^{a+b}}, \tag{A.4}$$

where $M = m^2 + x(1-x)k^2$. Rotating to spherical coordinates,

$$I_1 = \frac{2\pi^{D/2}}{\Gamma(D/2)}\int\frac{{\rm d}^4k}{(2\pi)^D}\int_0^\infty\frac{{\rm d}p}{(2\pi)^D}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1{\rm d}x\; x^{a-1}(1-x)^{b-1}\frac{p^{D-1}p^4}{[p^2+M^2]^{a+b}}. \tag{A.5}$$

Using the standard integral,

$$\int_0^\infty{\rm d}\kappa\frac{\kappa^{l-1}}{(\kappa^2+\nu^2)^m} = \nu^{l-2m}\frac{\Gamma(l/2)\Gamma(m-l/2)}{2\Gamma(m)}. \tag{A.6}$$

The result is,

$$I_1 = \frac{2\pi^{D/2}}{\Gamma(D/2)} \frac{\Gamma(D/2+2)\Gamma(a+b-D/2-2)}{(2\pi)^D} \int \frac{\mathrm{d}^4 k}{(2\pi)^D} \frac{1}{\Gamma(a)\Gamma(b)} \int_0^1 \mathrm{d}x \, x^{a-1}(1-x)^{b-1} M^{D-2a-2b+4}. \tag{A.7}$$

We take the number of dimensions to be $D+\epsilon$ and then perform the x integral up to order ϵ, we then compute the integral numerically, the result is,

$$\Lambda \frac{m^4}{M_p^4} \mathcal{O}(1). \tag{A.8}$$

Computing all other contributions to I as expected, does not yield cancellation of the contribution.