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Design and Implementation of Bio-inspired Heterogeneous Particle Swarm Optimisation Algorithms for Unconstrained and Constrained Problems

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I hereby declare that this thesis has not been and will not be, submitted in whole or in part to another University for the award of any other degree.

Signature:

Fevzi Tugrul Varna

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"The man whose sun still moves around the earth is essentially different from the man whose earth is a satellite of the sun. The man who is still dubious about the sum of twice two is different from the thinker for whom nothing is less doubtful than the a priori truths of mathematics."

Western and Eastern Points of View – Psychological Reflections, C.G. Jung

Publications

Earlier drafts of parts of this thesis have appeared elsewhere in peer-reviewed publications:

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F. T. Varna and P. Husbands, "Genetic Algorithm Assisted HIDMS-PSO: A New Hybrid Algorithm for Global Optimisation," 2021 IEEE Congress on Evolutionary Computation (CEC), 2021, pp. 1304-1311, doi: 10.1109/CEC45853.2021.9504852.

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UNIVERSITY OF SUSSEX

Fevzi Tugrul Varna, Doctor of Philosophy

Design and Implementation of Bio-inspired Heterogeneous Particle Swarm Optimisation Algorithms for Unconstrained and Constrained Problems

This thesis presents various novel non-bio and bio-inspired heterogenous PSO algorithms as general-purpose optimisers to handle real-valued unconstrained and constrained real-world problems. The proposed algorithms address various limitations of the canonical PSO, such as rapid loss of population diversity, which is closely associated with the problem of premature convergence to local optima. In general, the proposed algorithms utilise various mechanisms to improve the overall performance of the PSO algorithm by enabling particles to learn from distinct exemplars, maintaining better control of the flow of information between agents, through diversity-aware self-organised subswarms, design and reorganisation of topological structures, manipulation of dynamics of the particle population, and hybridisation.

The algorithms presented in this thesis are global, population-based, single-solution, single-objective, non-derivative algorithms that can cope with real-valued unconstrained and constrained optimisation problems irrespective of features contained in the given problem. The algorithms perform well in both high and relatively low-dimensional search spaces.

The experimental results obtained on three well-recognised CEC benchmark test suites at various dimensions, and 29 non-convex constrained real-world problems, indicate that the algorithms are highly competitive candidates as general-purpose optimisers. Detailed analysis of the results and of the search dynamics are presented.

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Chapter 1 : Introduction

1.1 Motivation

Any metaheuristic is expected to find a satisfactory equilibrium between exploration and exploitation to discover feasible solutions for the given problem. Although we usually cannot calculate in advance what the precise levels of these processes should be, the most prominent stochastic algorithms maintain this balance in different ways. However, it is a widely accepted principle among researchers that the appropriate balance of these two fundamental search activities enhance algorithms' capacities to yield better overall results. But until recently, definitive ways to quantify them were often rather vague. Hence various studies [1]–[5] proposed quantifying metrics that enable us to measure exploration and exploitation through population diversity.

In the majority of metaheuristics, despite utilising different mechanisms, the typical process of balancing exploration and exploitation is to perform a comprehensive search on the problem space during the initial phase of the search, and with the assumption that the promising region(s) is discovered, the exploitation phase is initiated towards the end of the search process to localise the search and discover the best solution within the identified best region. In addition, it is inevitable that the balancing of exploration and exploitation is also imperative for an algorithm to exhibit a reasonable convergence behaviour. Naturally, overstressing the exploration of the search space could result in time-consuming trivial search of the inferior regions and on the contrary, over exploitation of the search space tends to induce rapid loss of population diversity during the early stages of the search which could lead to premature convergence to a local optimum. Hence, it is sensible to recognise that the problems of balancing exploration and exploitation, maintaining population diversity and preventing premature convergence are very much related.

This thesis is concerned with Particle Swarm Optimisation (PSO) and in the particular case of PSO, due to the basic algorithm's search dynamics, it is prone to rapid loss of population diversity and the other aforementioned associated problems. Hence the motivation for this thesis to introduce novel PSO algorithms that can overcome these limitations, using a variety of approaches, resulting in *general-purpose optimisers* capable of tackling both real-valued unconstrained and constrained real-world problems.

1.2 Objectives

Particle swarm optimisation is a one of the most established algorithms in the literature and has been successfully applied to many real-world problems. Since the algorithm has been under investigation for many years, most of the limitations of the canonical PSO algorithm have been noted, as highlighted earlier in this chapter. This thesis aims to propose novel PSO algorithms that are intended to address these various limitations of the PSO algorithm. This will involve a through consideration of relevant existing work with respect to parameter settings, neighbourhood topologies, learning strategies and hybrid methods.

Hence, in this thesis, a number of single-solution, single-objective, global, derivative free, population-based general-purpose optimiser particle swarm optimisation algorithms are to be developed to handle real-valued, single-objective problems, both unconstrained and constrained. The algorithms should perform well irrespective of the characteristics of the problem under consideration, such as dimensionality, convexity, whether or not it is linear, separable, differentiable, unimodal, multimodal, hybrid, compositional, smooth or continuous. The proposed algorithms are to be tested and verified on three distinct benchmark test suites, CEC'13, CEC'14 and

CEC'17 on 10, 30, 50 and 100 dimensions. In addition, the developed optimisers are to be verified on 29 constrained real-world engineering problems. Their performance is to be thoroughly statistically analysed in comparison to many other state-of-the-art algorithms, and their search dynamics is to be investigated.

1.3 Contributions

The major contributions of the thesis are:

- Introduction of the novel HIDMS-PSO algorithm to improve the overall performance of the dynamic multi-swarm PSO (DMS-PSO) algorithm through better exploration and exploitation and prevention of loss of population diversity. The proposed algorithm possessed various mechanisms to overcome the issue of premature convergence and improve the balance of exploration and exploitation. These include: units (small sub-swarms) with master-slave topological structures, heterogeneous particle behaviour with two learning strategies (inward-oriented and outward-oriented) to enable particles to utilise diverse exemplars at each time step and a communication model to control the flow of information between the heterogenous population. The new topological structure enables the algorithm to have more control over the interaction and information exchange between the particles to reduce the loss of diversity and avoid premature convergence. The population is initially divided into two sub-populations, first sub-population is further divided into units that are formed using the introduced master-slave topological structure. The units are comprised of four particles, a single master and three slave particles of distinct types. Instead of the globally best-known position, the particles in the heterogenous population are guided solely using the inward-oriented or outward-oriented learning strategy. In essence, this allows the heterogeneous subpopulation to continue searching the problem space, delay the loss of population diversity to some extent, hence reduce the risk of premature convergence for at least half of the main population. The second sub-population employs the classical PSO search with local and global information to simulate a homogenous behaviour and support exploitation during the search to enable the convergence of the algorithm. Significantly improved search performance was demonstrated.

- Introduction of the novel bio-inspired FFQ-HIDMS-PSO algorithm to further improve the standard HIDMS-PSO algorithm's capacity to maintain better population diversity, . The FFQ-HIDMS-PSO algorithm conceptualises units as *diversity aware* self-organised fission-fusion societies capable of adopting a suitable behaviour to boost the population diversity when population diversity depletes below a certain degree. The proposed algorithm also incorporates a quorum decision mechanism to allow units to individually make decisions to adopt fission or fusion behaviour. Overall, the new bio-inspired variant FFQ-HIDMS-PSO allow units to react to stagnation of particles, and as a result the depletion of population diversity is significantly delayed, resulting in improved search performance.

- Development of the hybrid GA-HIDMS-PSO algorithm as another way to further improve the population diversity maintaining capabilities of the standard HIDMS-PSO algorithm. The new hybrid GA-HIDMS-PSO variant exploits the heterogeneous features of HIDMS-PSO and the evolutionary characteristics of the GA. In the GA-HIDMS-PSO architecture, HIDMS-PSO acts as the primary search engine, and the GA is employed as the secondary method to assist and slow down the loss of diversity for selected proportions of homogeneous and heterogeneous subpopulations of the HIDMS-PSO algorithm. Both methods run

consecutively. As the primary search method, HIDMS-PSO runs for longer periods compared with the GA. The HIDMS-PSO provides the initial solutions for the GA from both homogeneous and heterogeneous subpopulations and final solutions returned from the GA replace prior solutions in the HIDMS-PSO which resumes the search process with potentially more diverse particles to guide the swarm. The new variant significantly boosts the population diversity periodically hence the algorithm can potentially avoid local optima and continue to search the problem space.

- Introduction of the novel A-HIDMS-PSO algorithm to further improve the topological structure of the HIDMS-PSO algorithm. The new algorithm introduces master and slave dominated topologies which significantly change the dynamics of the unit structure by fluctuating the behavioural heterogeneity in individual units via the use of adaptive topologies. The new unit structures are comprised of 8 new topological structures with varying master and slave populations. The master-dominated topologies enforce exploration of the problem space whilst slave-dominated topologies mostly perform exploitation. Hence, unlike the standard HIDMS-PSO, the A-HIDMS-PSO only uses a single heterogeneous population, and by allowing the population to adopt and periodically switch between the two types of topological structures, the algorithm maintains better balance of exploration and exploitation. In addition, several existing components in the standard HIDMS-PSO were trimmed down to simplify the standard HIDMS-PSO algorithm in terms of time complexity. The new A-HIDMS-PSO algorithm also maintains much better population diversity compared to the standard HIDMS-PSO algorithm, resulting in improved search performance.

- To tackle the rapid loss of population diversity and premature convergence issue of the canonical PSO algorithm, the novel altruistic heterogeneous PSO (AHPSO) algorithm was developed. The AHPSO algorithm conceptualises particles as energy-driven agents with bio-inspired altruistic behaviour. In this approach, particles possess a current energy level and an activation threshold and are in one of two possible states (active or inactive) depending on their energy levels. The idea of altruism is used to form lending-borrowing relationships among particles to change an agent's state from inactive to active, and the main search mechanism exploits this idea. Population diversity in the swarm is maintained via agent states and the level of altruistic behaviour particles exhibit. The population diversity is further delayed with an additional mechanism proposed, called the paired particle model (PPM).

- To improve the premature converging tendencies of the canonical PSO algorithm, a bio-inspired variant, biased eavesdropping PSO (BEPSO) was developed. The primary search behaviour of the BEPSO algorithm is inspired by eavesdropping behaviour observed in nature and incorporates a cognitive bias that enables particles to control decisions on cooperation. The algorithm divides the swarm into two distinct particle groups competing to search for a resource. Particles recognise their group members as conspecifics and the other group as heterospecific. When a particle discovers a better position, a contact signal is emitted to attract surrounding conspecifics to the newly discovered position. In nature, multiple species living in the same environment commonly eavesdrop on contact calls originally unintended for them. This can help them to use less energy in searching for resources and potentially increase their fitness. Similarly, in this study, surrounding heterospecifics also eavesdrop on, and exploit, the signal calls originally intended for different conspecific particles. This signaller-recipient interaction experience builds positive

or negative bias among particles over time, and particles' final cognitive bias is used as a decision mechanism to exploit signal information. The BEPSO algorithm yields significantly better convergence rate and maintains better population diversity compared to canonical PSO on different benchmark test suites and dimensions.

1.4 Organisation of the Thesis

This thesis is comprised of 12 chapters.

- **Chapter 1** presents motivations, contribution, objectives, and an outline of the thesis.

- **Chapter 2** presents various optimisation concepts and a literature review of particle swarm optimisation algorithm with emphasis on different parameters, learning strategies neighbourhood topologies, hybrids and real-world applications.

- **Chapter 3** details the experimental setup and the comparison algorithms.

- **Chapter 4** proposes the heterogeneous improved dynamic multi-swarm PSO algorithm (HIDMS-PSO) and the experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites.

- **Chapter 5** proposes the heterogeneous improved dynamic multi-swarm PSO algorithm with bio-inspired fission-fusion behaviour and a quorum decision mechanism (FFQ-HIDMS-PSO). The experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites are also presented.

- **Chapter 6** presents the genetic algorithm assisted heterogeneous improved dynamic multi-swarm PSO algorithm (GA-HIDMS-PSO) and the experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites.

- **Chapter 7** presents the heterogeneous improved dynamic multi-swarm PSO algorithm with an adaptive topological structure (A-HIDMS-PSO) and the experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites.

- **Chapter 8** presents the biased eavesdropping PSO algorithm (BEPSO), and the experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites.

- **Chapter 9** proposes the altruistic heterogeneous PSO algorithm (AHPSO), and presents the experimental results obtained on 10-d, 30-d, 50-d and 100-d CEC'13, CEC'14 and CEC'17 benchmark test suites.

- **Chapter 10** presents a performance comparison and analysis of the six proposed PSO algorithms.

- **Chapter 11** presents the experimental results on various non-convex constrained real-world engineering problems.

- **Chapter 12** presents the performance comparison of the proposed algorithms with additional results published in other studies.

- **Chapter 13** presents final conclusions and future works.

Chapter 2 : Literature Review

2.1 Optimisation

In its simplest terms, a widely accepted definition of mathematical optimisation is the process of finding a solution for a given problem that satisfies a certain criterion. Generally, the type of optimisation problem dictates the method required to solve the problem and typically, the type of optimisation problem is determined based on the type of variables. It is outside the scope of this thesis to discuss these in detail, however, concisely, optimisation problems can be categorised as: continuous (i.e., real-valued), discrete, mixed-discrete, binary (i.e., true/false, on/off etc.) and combinatorial. Taking into account the features (e.g., modality), constraints, objective and other characteristics, optimisation problems can be categorised into broader spectrum such as linear or nonlinear, convex or nonconvex, unimodal or multimodal, differentiable or nondifferentiable, single-objective or multi-objective and unconstrained or constrained optimisation problems.

In this thesis, we solely deal with a particular class of problems, that is: unconstrained and constrained single-objective, real-valued problems. Hence in the following chapters of the thesis, instead of repetitively and explicitly stating the type of problem, they will simply be referred as *optimisation problems*.

2.1.1 Basic Concepts

In this section, we briefly introduce various optimisation concepts that are most relevant to the context of this thesis.

Local and Global Optimisation

Local optimisation can be summarised as the process of finding the optimum for the specified region of the given problem space. Consider the following formal definition:

$$f(x^*) \leq f(x), \forall\, x \in \mathrm{X} \qquad (2.1)$$

where X represent a subset of the problem space.

Majority of the local optimisers are derivative-based and as the name suggests, these methods employ the derivative information to discover the optima [6]. Although gradient-based algorithms are considered mostly efficient with respect to function evaluations and capacity to deal with large number of decision variables, these methods tend to struggle to cope with discrete decision variables, numeric noise and are only able to locate the local optimum. As opposed to the gradient-based methods, direct search algorithms pose as suitable alternatives for local optimisation as they do not require the gradient information, and purely rely on the value of the objective function to guide the search process. Some of the prominent approaches include the simplex method [7], simulated annealing [8] and local variants of evolutionary algorithms. The primary distinction between local and global optimisation lies in the purpose of the search process. Contrary to local optimisation, the objective of global optimisation is to discover the optimal solution located within the entire problem space. Figure 2-1 presents a typical depiction of local and global optima within a simple 1-dimensional problem space.

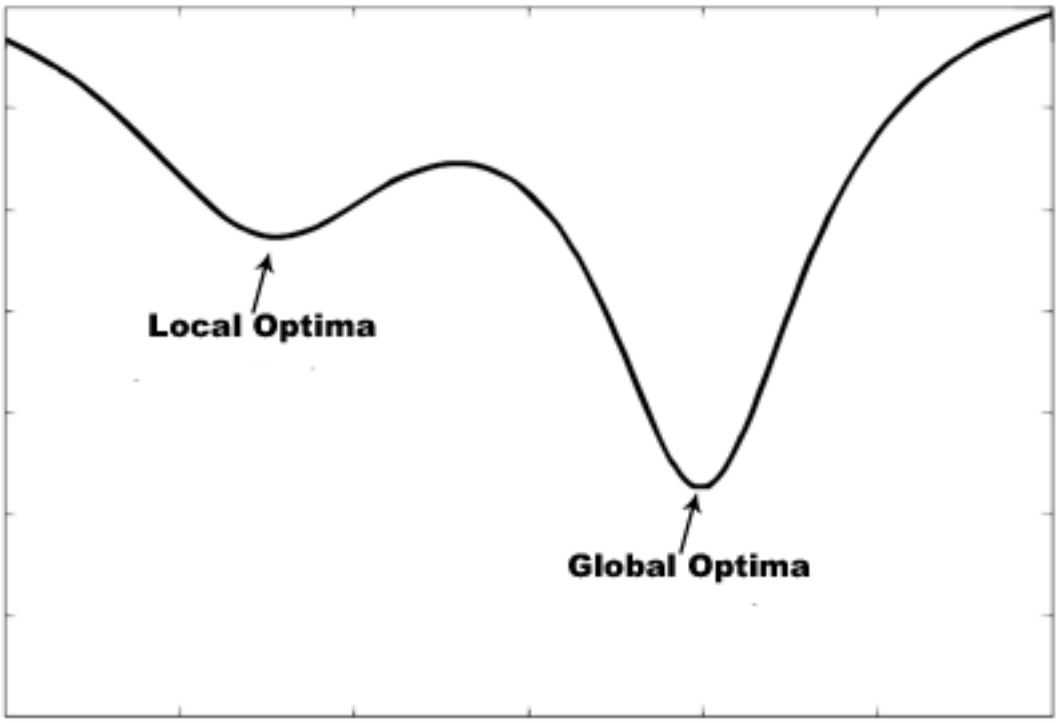


Figure 2-1 Depiction of local and global optima

According to [9], global optimisation approaches can be divided into four classes, namely, *complete*, *asymptotically complete*, *rigorous,* and *incomplete*. The primary concern of this thesis, *the PSO algorithm,* is considered as an *incomplete* method along with the majority of the metaheuristic approaches since these methods generally do not guarantee finding an existing feasible or the optimal solution.

2.1.1.1 No Free Lunch Theorem

The detailed discussion and formal analysis of the no free lunch theorem (NFL) [10] is outside the scope of this thesis. However, in this section we briefly discuss NFL on account of relevance of the first theorem's emphasis, particularly on general-purpose optimisers. Henceforth, for rest of the thesis we will refer to the *first theorem* as the *no free lunch theorem*. In essence, the NFL states that if an algorithm exhibits superior performance on a class of problems compared to another, its performance must be inferior on the remaining set of problem. In other words, the theorem suggests that on average, no algorithm is superior to another. Hence, statements such as "*algorithm A is better than B*" is simply false and the NFL theorem highlights that such statements may only be formed over systematically conducted experiments for the specific class of problems in hand. Moreover, although the theorem is limited to discrete functions, however [11],[12] strongly claim that (at least empirically) NFL holds for unconstrainted and multi-objective optimisation problems.

Objective Function

The objective function is the mathematical representation of the problem under consideration for either minimisation or maximisation. Consider the following mathematical form for an objective function:

$$P = P(\mathbf{Q}); \quad \mathbf{Q} = [Q_1, Q_2, \dots Q_N] \tag{2.2}$$

where P is the function being optimised and $Q_1, Q_2, \ldots Q_N$ are the decision variables. During the optimisation process, the value of the decision variables are manipulated within the permitted range to find the desired solution.

2.1.1.2 Unimodal vs Multimodal Objective Functions

Unimodal and multimodal functions are important categories in optimisation. In essence, unimodality refers to an objective function having a single optimum that is also the global optimum. On the other hand, multimodal objective functions contain one or more local optimums besides the global optimum. Hence, unimodal functions are relatively easier to solve compared to multimodal functions with respect to distinction in modality.

2.1.1.3 Single and Multi-Objective Functions

As previously mentioned, the work presented in this thesis solely targets single-objective optimisation (SOO) problems. However, the importance of multi-objective optimisation (MOO) cannot be overlooked, especially for real-world problems where an optimisation process commonly requires the minimisation or maximisation of a number of objectives simultaneously [13]–[17]. A special case of multi-objective optimisation is sometimes referred to as many-objective optimisation, typically when the objective function contains more than four objectives. In multi-objective optimisation, in contrast to SOO, the idea of the optimal solution is different, and the obtained set of optimal solutions are typically referred to as the Pareto front [18] because Pareto dominance [19] is employed to evaluate the relative quality of the attained solutions. Discussion of the concept of Pareto dominance is outside the scope of this thesis. However, in essence, Pareto dominance enable the comparison of two vectors of objectives in a specific way.

2.1.1.4 Constraints and Constraint Handling

An optimum solution for an objective function can be attained subject to various conditions known as *constraints*. In some cases, the decision variables are not subject to any constraints. However they are only allowed to occupy values within a permitted upper and lower range. On the contrary, a typical constrained optimisation problem usually consists of two types of constraints known as *equality* and *inequality* constraints. Consider the following well-known *process synthesis and design problem:*

Minimise:

$$f(\bar{x}) - x_3 + x_2 + 2x_1$$

Subject to:

$$h_1(\bar{x}) = -2\exp(-x_2) + x_1 = 0, \textit{(Equality constraint)}$$

$$g_1(\bar{x}) = x_2 - x_1 + c_3 \leq 0. \quad \textit{(Inequality constraint)} \qquad (2.3)$$

with bounds:

$$0.5 \leq x_1, x_2 \leq 1.4,$$

$$x_3 \in \{0,1\}.$$

Perhaps the equality constraints are more important since they require constant enforcement. In other words, these types of constraints are generally expected to be satisfied, in some cases, within a specified tolerance. The latter, inequality constraints can perhaps be binding in some cases therefore they tend to permit decision variables to occupy wider spectrum of values.

It should be noted that if all the constraints of a given problem are linear, the problem is considered as a *linearly constrained problem* and on the other hand, in addition to the constraints, when the objective function is also linear, then it becomes a *linear programming problem* [20]. Moreover, a quadratic objective function with linear constraints is referred to as *quadratic*

programming problem. In some cases, as opposed to the real-valued decision variables discussed here, the problem may require decision variables to take integer values, this type of problem is called *integer programming*.

It is worth briefly discussing some of the common methods employed to handle constraints. The simplest and the most common approach is the penalty method [21]. In essence, the idea is to penalise the violated constraints and subsequently the *measure of violation* is added to the value of the objective function. Constraint handling techniques are typically categorised into four divisions, namely, penalty function based methods [21], [22], methods for multi-objective optimisation [23], hybrid methods [24], [25] and approaches based on separation of the objective function and constraints [26]. In addition to the approaches mentioned , other methods [27]–[31] are commonly used in constraint handling.

2.1.2 Deterministic Approaches

In this section, we discuss various deterministic algorithms commonly used for the optimisation of single-objective problems. The detailed mathematical discussion of these algorithms is beyond the scope of the work presented in this thesis; hence the algorithms are briefly discussed.

The gradient decent (sometimes called the steepest decent) algorithm is the simplest gradient-based method. In essence, the algorithm iteratively moves towards the opposing direction of the gradient of the given function to find the minima. On the contrary, to find the local maxima of the function, steps taken in the direction of the gradient (referred to as gradient ascent). Variants using a momentum term [32] and a variable learning rate [33] have been proposed to address the defects of the traditional gradient method. Similarly, the conjugate gradient method [34] was initially introduced for the optimisation of linear equations. Subsequently, Fletcher et. al. [35] and Polak et. al. [36] proposed improvements to solve nonlinear optimisation problems. The CG method employs the general equation used by the gradient descent method; the distinction is the formulation for the direction of descent. Hence, the conjugate method attempts to improve the convergence of the gradient descent method by adopting a *strategy* that determines the direction of descent that assists faster convergence. Different variants of the conjugate methods are proposed in the literature [37],[38] that alter the computation of the standard form for the direction of descent.

As opposed to the first-order gradient descent and the conjugate gradient method, the Newton-Raphson method, named after Isaac Newton and Joseph Raphson, employs the second derivative of the objective function. The N-R approach converges faster compared to other methods mentioned here and it is sometimes called the tangential method as it relies on tangent line of curves to find the approximate range of the root of equations. On the other hand, quasi-Newton methods [39] are computationally faster compared to the Newton-Raphson method however, they exhibit slower convergence. The quasi-Newton approach is ideal when the Jacobian or Hessian is computationally expensive to calculate. In essence, the quasi-Newton algorithm approximates the Hessian based on first-order derivatives. Hence, quasi-Newton methods are computationally fast but poses slower convergence rate compared to the previously mentioned Newton-Raphson method.

Although deterministic methods are capable of solving certain class of problems and guaranteeing convergence to the global optima, they tend to struggle with variety of more complex and real-world problems.

2.1.3 Metaheuristic Approaches

In this section, as appose to the deterministic methods, we briefly overview various approximation methods called metaheuristics, concentrating on those that are relevant to later chapters in the

thesis. This class of algorithms are paramount in state-of-the-art global optimisation methods. The vast literature of metaheuristics is comprised of various algorithms with diverse inspirations, ranging primarily from bio-inspired, nature-inspired to non-metaphorical concepts. Some of the most prominent algorithms in this category are particle swarm optimisation, genetic algorithms, differential evolution, ant colony optimisation and simulated annealing.

The particle swarm optimisation algorithm is inspired by the emergent collective behaviour commonly observed in fish schools and bird flocks. In essence, the particles (agents) fly in the problem space remembering the best position discovered at individual and swarm level, and by updating and exploiting this information, particles determine their trajectories to perform search and discover better solutions. The PSO has been adapted to cope with many different problem types ranging from combinatorial to large-scale [40]–[45], along with extensive applications to various real-world problems [46]–[53].

The genetic algorithm is one of the most widely used evolutionary algorithm, developed in the 1960's by Holland [54]. The algorithm utilises the idea of biological evolution by *natural selection*, inspired by the theory of Charles Darwin. In essence, the genetic algorithm selects and manipulates the solutions (*chromosomes*) using the crossover and mutation operators to generate potentially better solutions. It is worth noting that the crossover and mutation operators are also commonly utilised by many contemporary metaheuristics with the intention of boosting the population diversity. The algorithm has many variants [55]–[57] and has extensively been applied to many real-world problems [58]. The difference between evolutionary algorithms (EAs) and genetic algorithms may be worth briefly mentioning, as these terms are commonly confused or used interchangeably. Typically, evolutionary algorithms are grouped into several categories, including genetic algorithms [59]–[61], genetic programming (GP) [27], evolutionary programming (EP) [62] and evolution strategies (ES) [27]. These techniques share common processes that form the fundamentals of evolution, including reproduction, random variation, competition, and selection. The primary distinction between GA and ES lies in how the population is represented in both algorithms. Traditionally, the solution in GA is represented by binary strings, whereas ES uses the real-valued representation of the solution. In addition, as opposed to the evolution operators utilised by GA, ES solely employs the mutation operator. Another important class of EAs is the estimation of distribution algorithms (EDAs) [63] that has been applied to various problems [64], [65]. Although EDAs are classified as EAs, these algorithms do not utilise crossover and mutation operators. Typically, each solution in EDAs is called *individuals*, forming a *population* that evolves during the search process. In EDAs, the new solutions are generated based on a probabilistic model determined using the best individuals attained from the previous population, and the procedure continues until the specified termination criteria are satisfied. A significant proportion of the studies in the literature focus on designing a better probability distribution model [66]–[69] that can potentially enable EDA to explore more promising regions [70].

The differential evolution method was introduced by Storn and Price [71] in 1997. Although differential evolution possesses similarities to the genetic algorithm in terms of the utilisation of crossover and mutation operators, DE is typically categorised as an evolutionary algorithm. In essence, DE is a vector-based algorithm, and the search mechanism includes generating new position vectors by initially selecting three random vectors and adding the weighted difference of any two selected vectors to the third position vector. Subsequently, the objective value of the new position vector is compared to another randomly selected member from the population. If the objective value of the new position vector is more optimal, the new position vector replaces the selected population member and otherwise it is discarded. The metaheuristics literature contains

extremely capable variants of DE [72], including L-SHADE [73], and the algorithm has been successfully applied to many real-world problems [74]–[78]. Another prominent swarm intelligence algorithm, introduced by Dorigo et al [79] is the ant colony optimisation (ACO) algorithm. As the name suggests, the algorithmic model is inspired by the behaviour of ants observed in nature. Since its introduction, the algorithm has attracted many researchers which led to various extensions of the canonical algorithm such as the elitist ant system [80], ant colony system [81] and rank-based ant system [82]. The ACO algorithm has been applied to a wide spectrum of problems [83]–[86] including path planning [87], protein folding [88] and software design [89]. Simulated annealing (SA) was introduced by Kirkpatrick et. al [90] in 1983. The SA algorithm was initially proposed for combinatorial optimisation problems, but later many extensions were proposed to handle various domains [91]–[93]. Annealing is a technique utilised in metallurgy which includes melting and subsequently slowly cooling a material until solidification. This is the key analogy employed in the SA algorithm.

In the general sense, the metaheuristic approaches have various advantages over deterministic algorithms, perhaps the primary ones being the applicability of these algorithms to a wide spectrum of problems, and the fact that generally, due to their stochastic nature, metaheuristics tend to be *less sensitive* to the characteristics of the problem at hand in comparison to deterministic algorithms. However, despite addressing various shortcomings of the deterministic methods, metaheuristics do not guarantee convergence to global optima, and in some cases implementation and parameter tuning can pose challenges. In this subsection, only a few well-established metaheuristic approaches are briefly mentioned. For a comprehensive review on metaheuristics, interested readers can refer to additional publications [94].

2.1.4 The L-SHADE Algorithm

In the following chapters of this thesis, many PSO variants are discussed, and some are utilised in the experiments. The sole non-PSO algorithm included in the primary experiments is the L-SHADE algorithm [73], an improved variant of DE and a CEC 2014 competition winner. Considering its importance in our work, it will be worth detailing the L-SHADE algorithm and the mechanisms that comprise it.

Although the canonical DE algorithm is competitive compared to many of its peers and applied to various real-world problems [95]–[97]. The DE algorithm utilises several control parameters, N (population size), scaling factor F and crossover rate CR. These control parameters were studied many times [98]–[100], and it was concluded that the optimal values for the parameters are problem dependent. In light of this issue, various extensions of the DE [101], [102] proposed and employed a parameter adaption strategy called the success-history-based adaptive mechanism. In the earlier variants of the success-history-based adaptive mechanism, the F and CR parameters were self-regulated whilst the population size remained constant. However, tuning the population's size is imperative in most metaheuristics, particularly in evolutionary algorithms, as it is well-known that the size of the population contributes to the rate of convergence. However, as with other parameter values, the optimal population size in evolutionary algorithms relies highly on the characteristics of the problem under consideration and the rest of the control parameters.

2.1.4.1 Initialisation Strategy

The initial solution vectors randomly generated using the following equation:

$$\mathbf{x}_{ij}^{0} = LB + rand * \left(UB_j - LB_j\right), j = 1,2,3 \dots, D \tag{2.1.1}$$

where rand is a uniformly distributed random value in the range $[0\ 1]$. LB, UB are the lower and upper bounds, and D is the dimension of the given problem.

The values for the historical memory of CR and F (M_{CR}, M_F) are initialised as 0.5 using the following equation:

$$\begin{cases} M_{CR,k} = 0.5 \\ M_{F,k} = 0.5 \end{cases}, k = 1,2,3 \dots, H \tag{2.1.2}$$

Where H is the size of the historical memories.

2.1.4.2 Mutation Strategy

The L-SHADE algorithm utilises the *current-to-pbest/1* mutation strategy, which uses four vectors from the current population. The mutant vector and the F_i (scaling factor) are calculated using the following two equations:

$$\mathbf{v}_i = \mathbf{x}_i + F_i\left(\mathbf{x}_{pbest} - \mathbf{x}_i\right) + F_i(\mathbf{x}_{r1} - \mathbf{x}_{r2}) \tag{2.1.3}$$

$$F_i = randc\left(M_{F,r3}, 0.1\right) \tag{2.1.4}$$

Where $\mathbf{x}_{r1}$ and $\mathbf{x}_{r2}$ are vectors selected from the current population and the external archive, respectively and $\mathbf{x}_{r1} \neq \mathbf{x}_{r2}$. Similarly, $\mathbf{x}_{pbest}$ is a dominant vector randomly selected from the current population. F_i is a random value in compliance with the Cauchy distribution with mean $M_{F,r3}$ and standard deviation of 0.1.

2.1.4.3 Crossover strategy

After generating the mutant vector, the following equation generates the $\mathbf{u}_{i,j}$ (trial vector) using the $\mathbf{v}_i$ and $\mathbf{x}_i$ vectors by using the binomial crossover strategy:

$$\mathbf{u}_{i,j} \begin{cases} \mathbf{v}_{i,j}, & r < CR_i \quad or\ j = j_{rand} \\ \mathbf{x}_{i,j}, & \text{otherwise} \end{cases} \tag{2.1.5}$$

where j_{rand} is randomly generated value that determines the index of the decision variable and r is a uniformly generated value between 0 and 1. The following equation is used to assign the value of CR_i from the Gaussian distribution:

$$CR_i = \begin{cases} 0, & M_{CR,r3} = \perp \\ randn\left(M_{CR,r3}, 0.1\right), & \text{otherwise} \end{cases} \tag{2.1.6}$$

Where $M_{CR,r3}$ and 0.1 are the mean and the standard deviation values and $\perp$ indicates the terminal value.

2.1.4.4 Selection Strategy

After generating the trial vectors, the selection strategy is adopted to select the survivors to pass on to the next generation:

$$\mathbf{x}_i^{t+1} = \begin{cases} \mathbf{u}_i^t, & f\left(\mathbf{u}_i^t\right) \leq f\left(\mathbf{x}_i^t\right) \\ \mathbf{x}_i^t, & \text{otherwise} \end{cases} \tag{2.1.7}$$

Where f is the objective function of the given problem.

2.1.4.5 External Archive and Historical Memory Update Strategy

The external archive strategy aims to revamp the population diversity in the L-SHADE algorithm and this strategy is expected to minimise the risk of premature convergence. The strategy is based on two simple rules: if the parent vectors $\mathbf{x}_i^t$ is better than the trail vectors $\mathbf{u}_i^t$, then the parent vectors are sustained for the next generation, on the contrary, it is stored in the external archive, denoted

by $\boldsymbol{A}$. The values of F_i and CR_i that generated individuals with better fitness are also stored in S_{CR} and S_F, which are later utilised to update M_{CR} and M_F.

2.1.4.6 Linear Population Size Reduction

The L-SHADE algorithm incorporates the (LPSR) linear population size reduction mechanism. The improved mechanism solely requires a single parameter (initial population) and continuously determines the population size in compliance with a linear function according to the:

$$N^{t+1} = round\left[\left(\frac{N^{min} - N^{init}}{MAX_{NFE}}\right) \cdot NFE + N^{init}\right] \tag{2.1.8}$$

where NFE and MAX_{NFE} are the current and the maximum number of fitness evaluations whilst N^{init} and N^{min} are the initial and minimum population sizes.

The literature comprises many variants of SHADE and L-SHADE algorithms [103], [104]; hence the interested reader is advised to review some of the key [105]–[107] and more comprehensive [108] studies in this area.

2.2 Particle Swarm Optimisation

The word intelligence (from the Latin *intelligere)* can be traced back to the 14th century and even today is still immensely vague and has different definitions in many literatures. Rather *vieux jeu* definitions by various researchers include *the power of good responses from the point of view of truth or facts (Thorndike) — the ability to carry on abstract thinking (Terman) — having learned or ability to learn to adjust oneself to the environment (Colvin) — the capacity for knowledge (Henmon)* [109].However, a more contemporary definition, also relevant to the topic of this thesis is provided by David Fogel in his book, "*intelligence is the capability of a system to adapt its behaviour to meet its goals in a range of environments*" [110].

The term Computational Intelligence (CI) was coined in 1990 by the IEEE Neural Networks Council (now IEEE Computational Intelligence Society) and its explicit definition was provided by Bezdek in 1994. However, a more concise definition of CI [111] reads, "*the study of adaptive mechanisms to enable or facilitate intelligent behaviour in complex and changing environments*". CI thus encapsulates various paradigms including neural networks, evolutionary computation, swarm intelligence, artificial immune systems, and fuzzy systems. However, the scope of this thesis is limited by the swarm intelligence paradigm of CI.

The most prominent, and one of the most cited, swarm intelligence algorithms is particle swarm optimisation (PSO). The PSO algorithm has attracted immense interest from many researchers since its debut in 1995. In the last few decades, approximately ten thousand papers are published every year on PSO which continue to enrich the literature in this area. Over many optimisation algorithms in the literature, the PSO algorithm is preferred due to various advantages such as the simplicity of its mathematical definition, that enables easy implementation of the method, and contrary to many population-based algorithms, the fact it requires the tuning of fewer parameters [112]. Furthermore, PSO can be flexibly integrated with other algorithms, and tends to exhibit minimal sensitivity to stochasticity in the objective functions. Due to these factors, PSO has been extensively studied and successfully applied to numerous problems in diverse fields. The literature of the PSO algorithm and its various applications will now be briefly discussed.

The PSO algorithm [113] was designed by Kennedy and Eberhart in 1995. The particles in the PSO mimic the collective social movements commonly observed in flocks and schools. Each particle

represents a solution in the *D*-dimensional search space, and possesses three attributes: its position, memory and velocity, denoted by x, $pbest$ and v, respectively. Initially, particles' velocity and position are randomly assigned, and subsequently, at each time step, the fitness function is employed to guide particles towards the $pbest$ (particle's memory), and $gbest$, the best position known to the swarm at time *t*. At each time step, the velocity and position of each particle is updated using the following two equations until a termination criterion is met:

$$\mathbf{v}_i^d = w\mathbf{v}_i^d + c_1\mathbf{r}1_i^d\left(\mathbf{pbest}_i^d - \mathbf{x}_i^d\right) + c_2\mathbf{r}_2{}_i^d\left(\mathbf{gbest}^d - \mathbf{x}_i^d\right) \quad (2.4)$$

$$\mathbf{x}_i^d = \mathbf{x}_i^d + \mathbf{v}_i^d \quad (2.5)$$

Where i denotes each particle in the swarm population ($i = 1,2 \dots, N$ population) and d represents the dimension ($d = 1,2 \dots, D$ dimensions). As briefly mentioned before, $\mathbf{v}_i^d$ and $\mathbf{x}_i^d$ represent the velocity and position in the d^{th}dimension of the i^{th} particle whereas $\mathbf{pbest}_i^d$ is the best position found by the i^{th} particle and $gbest$ is the best position discovered in the entire swarm. Moreover, $\mathbf{r}_1$ and $\mathbf{r}_2$ are *d*-dimensional vectors generated in the range of [0 ,1].

In addition, c_1 and c_2 represent the 'cognitive' and 'social' acceleration coefficients. The cognitive coefficient controls the local search whilst the social component controls global exploration by routing the trajectory of particles either towards **pbest** or **gbest**. The control of these two coefficients is immensely important to perform an efficient search as higher values of c_1 result in excessive wandering of the particles and, similarly, a higher value of the c_2 may lead to premature convergence of the particles and prevent the algorithm to discover a feasible solution.

Researchers introduced the inertia weight (w) parameter [114] to improve the balance of exploration and exploitation. In essence, the w parameter reflects the impact of the previous velocity on the calculation of the new velocity. The optimal inertia weight parameter is an imperative factor to achieve the balance of exploration and exploitation and can be assigned a constant value, however, one of the earliest and the most popular setting [114], [115] is the linearly decreasing inertia weight (e.g., $w = 0.9 \rightarrow 0.4$) that enables exploration during the early phase of the search, subsequently, support exploitation towards the end of the search process to accompany the convergence of the algorithm.

Furthermore, [116] introduced a constriction factor as a comparable mechanism to the inertia weight parameter which can be summarised in:

$$\mathbf{v}_i^{t+1} = K[\mathbf{v}_i^t + c_1\mathbf{r}_1(\mathbf{pbest}_i - \mathbf{x}_i) + c_2\mathbf{r}_2(\mathbf{gbest} - \mathbf{x}_i)], \qquad \text{if } \varphi > 4, \quad (2.6)$$

$$K = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|}, \quad (2.7)$$

$$\varphi = c_1 + c_2$$

Eberhart and Shi compared the inertia weight parameter and the constriction factor in equation (2.7) [117] and concluded that the use of the constriction factor, while limiting the maximum velocity to the dynamic range of the variable x_{max} in each dimension, did yield superior results.

Overall, the velocity update equation (2.4) can be broken down into three parts, the first part emulates the impact of the velocity memory or inertia, the second part represents the cognition, or the impact of the best solution discovered by the particle and the final part is the cooperation of particles through **gbest** (best-known solution in the swarm).

2.2.1 Parameter Tuning

Some of the early work on the PSO proposed various ways of controlling the inertia weight, including fixed, linearly increasing/decreasing values[118][115][119]. Study [120] introduced a hybrid model that uses simulated annealing by perturbing the current inertia weight and similarly [121] also employs the idea of simulated annealing to determine the value of the inertia weight. Study [122] introduced a non-linear inertia weight based on the tangent and arc tangent functions. In the study [123], an exponential inertia weight is proposed to enhance the global optimisation efficiency of PSO and [124] introduced a time-varying mechanism for the acceleration coefficients c_1 and c_2 to improve the local search and the convergence of the global optimum solution discovered. Adaptive PSO [125] enhances the search efficiency and convergence speed by automatically controlling the inertia weight and the acceleration coefficients. The SRPSO algorithm uses a self-regulating inertia weight [126] where the best particle is used to enhance the exploration and convergence speed. Study [127] uses half of the value of a random number with Gaussian distribution as the value of the inertia weight and fuzzy Logic (FL) based approach [128] determines the inertia weight, c_1, c_2 and the velocity clamping values independently for each particle.

2.2.2 Neighbourhood Topologies

The neighbourhood topology in the PSO algorithm is critical as it directly impacts how the information flows within the swarm, and information exchange mechanism in the swarm essentially determine the trajectory of particles [129] . One of the main motivations for the introduction of neighbourhood topologies/sociometry is to boost, or leastwise delay the depletion of, population diversity in the swarm to enhance the performance of the algorithm in dealing with various problem complexities such as multimodality [130]. The delayed depletion of population diversity can potentially avoid premature convergence. Hence in theory, the algorithm is generally expected to convergence to a more feasible solution.

The quantity of research published in relation to swarm topology is comprehensive. Hence, for convenience, the discussed work here will be categorised as cellular/diffusion models and distributed/island models. In the following subsections, we discuss the seminal work that falls into each category.

2.2.2.1 Cellular Topologies

In the cellular topology, the particles are generally placed in a 2-d grid arrangement. Typically, the principal idea is to deprive particles from using the $gbest$ and instead, the trajectory of particles is influenced by their respective neighbours. As a consequence, this restriction prevents the entire population from exploiting the same *best-known* solution. Hence, this form of mechanism practically delays the loss of population diversity which in turn can potentially prevent premature convergence and aid particles to continue searching the problem space for extended periods.

Various cellular topologies are proposed for PSO including the cellular PSO algorithm [131] for dynamic environments. The approach segregates particles into groups within cells of cellular automata and each group attempts to discover the optima locally. The researchers claim that the approach yields the discovery of the global optima. The study also verifies that the performance of cellular PSO is superior to mQSO (multi quantum swarm optimization) [132] on various dynamic optimisation problems. Shi et. Al. [133] proposed CPSO (cellular PSO) which integrates the CA model in the particles' velocity update equation to alter the trajectory of particles to avoid local optima problem. The approach employs CPSO-inner and CPSO-outer to improve the searching capability by diverting particles from their original trajectories to explore more potential areas of the search space.

Some of the early work presented by Kennedy and Mendes [129] investigated the impact of circle, wheel, star and random neighbourhood topologies. Both researchers subsequently investigated the impact of various population topologies [134] to discover what comprised a good population structure. Shuntahgan proposed a modified version of PSO [135] with a variable neighbourhood operator where the $gbest$ is replaced with $lbest$ (local best). During search, the neighbourhood of the particle expands from $lbest$ to $gbest$ to progressively include all particles in the neighbourhood. DTT-PSO [136] proposed a dynamic tournament topology. Instead of using the $gbest$ and $lbest$, the approach employs several above-average exemplars from the entire swarm to guide each particle. In addition, the algorithm also incorporates features from fully informed and random topology strategies. The experimental results showed that the approach is efficient compared to various algorithms. Xia et. Al. [137] introduced a multi-swarm PSO with dynamic topology. In the early stages, the population (divided into many sub-swarms) coevolves in parallel and the number of subswarms are gradually decreased to slowly switch the swarm's behaviour from exploration to exploitation. The algorithm also employs a purposeful detecting strategy (PDS), an additional mechanism to help particles escape from local optima. Brits et. Al. proposed a similar approach, named NichePSO [138], which extends the unimodal PSO to obtain multimodal solutions in multimodal problems. In the NichePSO, the population is divided into sub-swarms based on fitness criteria of particles to detect and mark solutions, and a more recent study that employs niching technique is introduced by Huang et. Al. [139] for multimodal optimisation of composite structures.

2.2.2.2 Distributed Topologies

In the distributed approach, the population is divided into simultaneously coevolving subswarms (or islands) that conduct search on potentially distinct regions of the problem space. Typically, during search, direct arbitrary interaction between subswarms does not occur and information flow between swarms take place by exchanging particle(s) according to a specific policy. The information exchange policy may vary to dictate exchange at fixed timesteps, or the exchange may be constrained to transpire between selected subswarms based on a criterion. In essence, the idea of coevolving subswarms reduces the risk of depleting the diversity of the entire population. Instead, the depletion may ensue at specific subswarm(s) whilst the search could still be carried out by the unaffected subswarms. In such topologies, typically the balance of exploration and exploitation is achieved by a *non-interaction* initial search phase to enforce exploration of the problem space. Subsequently, the information exchange policy is initiated to aid exploitation towards the end of the search process.

Some of the most influential work with regards to sub-swarms include dynamic multi-swarm PSO (DMS-PSO) [140]. In the DMS-PSO algorithm, the multi small swarms simultaneously conduct search using its own members and employ a scheduled regrouping strategy to exchange information with other swarms. Zhao et. Al. [141] introduced an extension of DMS-PSO for large scale optimisation with Quasi-Newton local search mechanism. The researchers claim that the proposed neighbourhood structure provides better performance on complex problems. Liang et. Al. [142] proposed another variant of the DMS-PSO algorithm for constrained optimisation problems. They proposed a novel constraint handling technique adaptively assigns subswarms to explore different constraints and guide subswarms to feasible regions.

Ye et.al [143] presented a multi-swarm PSO with a dynamic learning strategy. In this variant, the information exchange between subswarms is assisted with a classification mechanism that distinguishes particles in each subswarm as *ordinary* and *communication* particles. The particles of the former category focus on exploitation whilst the particles in the latter category focus on exploration.

Moreover, it is also common for PSO variants to be classed as static or dynamic with respect to the adopted neighbourhood's type. The first term refers to a *constant* neighbourhood topology, in the latter topology, particle(s) neighbourhoods exhibit variation during the optimisation process. Several static topologies such as hierarchical [144], species-based [145], local-global neighbourhood [146], fully informed PSO [147], Von Neuman, wheel topology and four clusters topology [148] were presented to enhance the performance of the canonical PSO algorithm. Some of the aforementioned static neighbourhood topologies' performances have been investigated by Setayesh et. Al. [149] and applied to image processing.

On the contrary, various dynamic topologies have been introduced. The Clan topology [150] proposed by Carvalho et al. divides the swarm into several clans where each clan is composed of five particles that are fully-connected and the best particle is selected as the *clan leader*. Nasir et al. presented a novel variant of PSO [151] where particles are guided using their own or the historical information attained from the dynamic neighbourhoods. Jie et al. [152] presented a PSO variant with mixed behaviour that initiates the information exchange among subswarms using a cooperative mechanism. The PSO-ITC [153] algorithm introduced a linearly increasing topology connectivity with time and a shuffling mechanism to delay the loss of population diversity.

2.2.3 Learning Strategies

Many learning strategies have been proposed to deal with different drawbacks of the PSO algorithms, including a better convergence toward the global optimum or to maintain the appropriate balance of exploration and exploitation. Some of the seminal work include [154] that presented a new PSO variant called comprehensive particle swarm optimiser (CLPSO) with a novel learning strategy whereby the personal best positions of all particles are employed to update the velocity of a particle to maintain the population diversity for extended periods to minimise the risk of premature convergence. An extension of CLPSO is proposed by Lynn et al. [155] called the heterogeneous CLPSO (HCLPSO) to enhance the exploration and exploitation capabilities of the CLPSO algorithm by dividing the population into two subpopulations where each subpopulation either focuses on exploration and exploitation. Li et al. introduced a self-learning PSO [156] with four learning strategies to cope with different situations in the search space with respect to convergence, exploitation, exploration and escape from local optima. In GOPSO [157], generalized opposition-based learning (GOBL) and Cauchy mutation are presented to address the problem of premature convergence, and the mutation operator is employed to assist particles escape from local optima. Lim et al. [158] proposed ATLPSO-ELS with two strategies, namely the elitist and an orthogonal learning strategy to support particles escape from the local minima. The elitist learning strategy specifically targets evolving the global best exemplar. Xu et al. [159] presented a dimensional learning strategy (DLS) to help discover and integrate the promising information of the populations' best solution according to the personal best experience of each particle, and in social learning PSO (SL-PSO) [160], particles learn from any other particle with lower fitness compared to them.

Some of the recent work on learning strategies for PSO include, Zhang et. Al. [161] with an adaptive learning strategy that addresses the problem of depletion of population diversity. In this approach, the swarm is adaptively divided into subswarms and members of each subswarms are further classified as ordinary and locally best individuals with two distinct learning strategies. Similarly, Wei et Al. [162] also targeted the problem of population diversity and introduced a multiple adaptive strategy. In this approach, particles are adaptively assigned a distinct behaviour in each generation based on their performances. Other recent PSO variants with novel learning strategies include [45], [163]–[168].

2.2.4 The hybridisation of PSO with Other Algorithms

Hybrid algorithms have attracted considerable attention from researchers. In the particular case of PSO, hybridising with other metaheuristics has yielded positive results in enhancing one or many aspects of the canonical PSO algorithm. Hybrid variants of PSO can be categorised as unified purpose hybrids and multiple purpose hybrids. The first category of hybrids involves combining PSO with one or more algorithms where each algorithm is used to solve the same problem. In the latter category, a primary algorithm is utilised to solve the problem, while the sub-algorithm assists the primary algorithm, e.g. by tuning the parameters of the primary algorithm. For the first hybrid category, Sedeh et al. [169] presented a novel GA-PSO hybrid to solve the multi-location facility maintenance scheduling problem. Sedki et al. [170] introduced a PSO-DE hybrid to minimise the cost design of water distribution systems. The hybrid model integrated the differential evolution's operators into PSO to boost the population diversity and assist particles to escape local optima. In PS-ABC [171], ant colony optimisation is hybridised with PSO to solve high-dimensional optimisation problems. In this hybrid variant, the primary role of PSO is to perform local search. In contrast, onlooker and modified scout bee from the ABC algorithm serves as two global search phases. Shankar et al. [172] proposed a new method using harmony search algorithm (HSA) and PSO for energy-efficient cluster head selection. Similarly, [173] proposed a GA-PSO algorithm for the cluster head problem. In this case, the GA is utilised to optimize the cluster head selection and PSO is used to optimise the route for sink mobility. Bonyadi et al. [174] presented a novel hybrid algorithm for constrained optimisation problems where PSO is used to locate the feasible regions at the initial stages of the search; subsequently, the covariance matrix adaptation evolutionary strategy (CMA-ES) algorithm is employed to improve the solutions at the latter stages of the search. Chen et al. [175] introduced PSO-EO which integrates a local heuristic extremal optimisation (EO) with PSO to improve the premature convergence tendency of the PSO algorithm. The hybrid model incorporates the exploration capability of PSO with the exploitation efficiency of extremal optimisation. Similarly, in DMS-PSO-HS [176], the harmony search algorithm is merged into subswarms to benefit from the distributed exploration capability of DMS-PSO and the stochastic exploitation of the harmony search algorithm. The hybrid exhibited superior performance on multimodal and composition functions when individually compared to both algorithms.

Although it is outside the scope of this thesis, an important offshoot of PSO to mention here are the MOPSO (multi-objective PSO) methods [177]–[179] which occupies an important place in the literature and in solving different kinds of multi-objective problems.

2.2.5 Applications

As displayed in Table 2-1, the PSO method have been widely applied to miscellaneous simple to complex real-world problems.

Table 2-1 Application areas of the PSO algorithm

Area	Application	References
Automatic Control System	Cooperative multi-robot target searching in complex unknown environments	[180]
	Load frequency control of power system configurations	[181]
	PID controllers	[182] [183] [184] [185]

	Cascade load force control design for a parallel robot platform	[186]
	Autonomous helicopter stabilisation	[187]
	Biped robot walking in the lateral plane on slope	[188]
Electrical and Electronic Engineering	Electric vehicle charging model	[189]
	Electrical energy transmission system	[190]
	Multistage distribution expansion planning	[191]
	Short-term electrical load forecasting	[192]
	Electroencephalography (EEG) source localization problem	[193]
	State variable estimation of power systems	[194]
Communication Theory	Design of ultrawideband (UWB) pulse waveform	[195]
	The receiver antenna selection problem	[196]
	Determining the required number and locations of transmitting antennas (TXs) to optimize wireless propagation coverage in an indoor UWB communication system	[197]
	Optimisation of printed square monopole antenna	[189]
	Optimal design of optical interference filters	[198]
Operations Research	Task execution time optimisation	[199]
	Optimisation of granulation of the linguistic terms problem	
	Real-time task assignment in heterogeneous multiprocessor	[200]
	Optimisation of multiprocessor task scheduling problem	[201]
Fuel and Energy	Adaptive maximum power point (MPP) tracking approach for PV power generation system	[202]
	Optimisation of power losses and L index in power systems simultaneously	[203]
	Capacity estimation of Li-ion battery based on the charge voltage and current curves	[204]
	Optimal design of hybrid renewable energy systems	[205]
	Scheduling model of thermal power plants	[206]
	Design of MPP tracking in wind power systems	[207]
Engineering (Mechanical, Civil, Chemical, Biological, Medical)	Prediction of protein structures in the 3D hydrophobic polar model	[208]
	Decision-making system for diseases identification	[209]
	Optimisation of TCRO problems in construction project planning	[210]
	Removal of molybdenum using silver nanoparticles from water samples	[211]
	Modelling of mechanical properties of as-cast Mg-Li-Al alloys	

Chapter 3 : Experimental Setup and Benchmark Test Suites

In this thesis, two types of experiments are conducted using unconstrained and constrained problems. The unconstrained problem sets includes the CEC'13 [212], CEC'14 [213] and CEC'17 [214] benchmark test suites at 10, 30, 50, and 100 dimensions. The evaluated unconstrained functions are comprised of single objective benchmark problems of various classes including unimodal, multimodal, hybrid and composition functions.

The CEC'13 benchmark test suites build on and improve the previously published CEC'05 test suite. The major improvements include the addition of new problems and improved composition functions. Similarly, the CEC'14 test suite improved its precedent with various novel problems and features, including the graded level of linkages and rotated trap problems and introducing different shift data for each problem. The entire problem set comprises different problems, including unimodal, multimodal, hybrid and composition problems. The unimodal problems comprise of single optima, which tends to be the global optima. On the contrary, multimodal problems typically pose multiple local optima and are more complex than unimodal problems. The hybrid functions aim to simulate the complicated properties of real-world optimisation problems and are more complex than basic multimodal problems. The hybrid problems comprise subcomponents generated by randomly dividing the variables; subsequently, different subcomponents utilise different basic functions. Hence, the modality of the hybrid problem under consideration may change based on the selected basic function. The composition functions are the most complex problems in the selected benchmark test suites. These problems may utilise hybrid functions as the basic functions; hence problems may have different properties for different subcomponents. In addition, the CEC'14 benchmark test suite consists of a local optimum trap for each composition problem to test algorithms' tendency to converge. Similarly, in the CEC'17 benchmark test suite, the basic functions employed in the composition functions are all shifted and rotated functions.

Overall, a total of 87 unconstrained benchmark functions are used to evaluate the performance of the proposed algorithms, each at four different problem dimensions. These test suites are widely regarded as suitably challenging, enabling thorough evaluation of search algorithms. The evaluation process of each test suite is carried out according to the evaluation criteria set out by the official CEC competitions. Initially, the performance of all employed algorithms (including a large set of state-of-the-art comparator methods) is ranked according to mean error values of the obtained solutions. Subsequently, the pair-wise statistical difference between the proposed algorithm and the comparison algorithms are carried out using the non-parametric Wilcoxon signed-rank test with a significance level of 5%, with appropriate adjustments for multiple comparisons [215]. The symbols $(+)$ is used to denote the algorithms where the proposed algorithm exhibited significantly better performance, $(=)$ indicates no significant differences in the mean performance of the proposed algorithm and a comparison method and finally, $(-)$ represent the comparison algorithms whose performance is significantly better than the proposed algorithm.

After discussing experimental results in relevant chapters, we briefly examine the convergence trend of the algorithms. The convergence behaviour is recorded in compliance with many studies in the literature [216], [217], and the maximum number of iterations is limited to 500 [165]. Moreover, for each problem set, the average (obtained over 20 consecutive runs) convergence characteristics of the proposed algorithm and the four top ranked comparator algorithms on different dimensions. However, as the convergence characteristics are observed to be

relatively similar on different problem dimensions, in the main text we only consider the convergence trend on the 100-dimensional problems for all test suites (*see Appendix 3-5 for convergence rate comparison on various dimensions*). Each unconstrained problem is tested 30 times for $10^4 d$ function evaluations, where d is the problem dimension.

In each chapter, the proposed algorithms are individually compared solely against the 14 comparator algorithms (see Table 3-1). However, in chapter 10, the proposed algorithms are individually compared and analysed against each other on the same benchmark test suites. In addition, the proposed algorithms are verified on 29 non-convex constrained real-world problems [218]. The detailed experimental settings for the constrained real-world problems are provided in chapter 11.

We employ the *default* or *best-known general* parametric settings for all test algorithms and the parameters are not retuned (*including the proposed algorithms*) for any of the experiments conducted. The selection of parameter settings for the proposed algorithms were determined by preliminary experiments conducted for each algorithm on multiple dimensions and test suites (s*ee relevant sections of Chapters 4-9*). Finally, the full list of the new algorithms proposed in this thesis along with the comparison algorithms and their parameter settings are shown in Table 3-1. The selected test algorithms include 13 well-known PSO variants and the CEC competition winner differential evolution variant L-SHADE. For all the comparator algorithms, the original source code provided by the author, or the code used by published studies were employed in the experiments [219], [220]. The entire list of the algorithm source codes (including the proposed algorithms) can be accessed from github.com/FTVarna. During the performance comparison of the algorithms, the obtained mean error value by each algorithm is used as the primary attribute to compare the performance of the algorithms. Hence, for the rest of the thesis, the term "best performance" refers to the algorithms that achieved the lowest mean error value.

Table 3-1 List of algorithms and parametric settings used in the experiments

Key	Algorithm	Parametric Setting
HIDMS-PSO	Heterogeneous Improved Dynamic Multi-Swarm PSO	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $\alpha = 10$
FFQ-HIDMS-PSO	HIDMS-PSO with Bio-inspired Fission-Fusion Behaviour and a Quorum Decision Mechanism	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $FF_{phase} = 100$
GA-HIDMS-PSO	Genetic Algorithm Assisted HIDMS-PSO	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $GA_{phase} = 100$
A-HIDMS-PSO	HIDMS-PSO with an Adaptive Topological Structure	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $phase_{switch} = 100$
BEPSO	Biased Eavesdropping Particles	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $\mathrm{BT} = 20;\ phase = 10$ $SR_{max} = 0.01 * (UB - LB) * d$ $SR_{min} = 0.001 * (UB - LB) * d$
AHPSO	Altruistic heterogeneous PSO	$w = 0.99 \rightarrow 0.29$ $c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $\mathrm{PPM} = \mathrm{on};\ \mathrm{n}_{\mathrm{paired}} = 6$ $\alpha = rand(10 - 18);\ \beta = 0.75$

		$ER = 5;\ LB_{rate} = 10$
L-SHADE[73]	SHADE Using Linear Population Size Reduction	See ref. [73]
BBPSO[221]	Bare bones PSO	See ref. [221]
BreedingPSO[222]	Breeding Swarms: A GA/PSO Hybrid	$w = 0.8 \rightarrow 0.6, c_1, c_2 = 1.49445$ $V_{max} = 0.15 * Range$
HCLPSO[155]	Heterogeneous comprehensive learning PSO	$w = 0.99 \rightarrow 0.29, c_1 = 2.5 \rightarrow 0.5,$ $c_2 = 0.5 \rightarrow 2.5, K: 3 \rightarrow 1.5,$ $V_{max} = 0.5 * Range$
CLPSO[154]	Comprehensive learning PSO	$w = 0.9 \rightarrow 0.2;\ c1, c2 = 1.49445$ $V_{max} = 0.2 * Range$
FIPS[147]	Fully informed PSO	$\chi = 0.729, \sum c_i = 4.1$
FDR-PSO[223]	Fitness-distance ratio PSO	$w = 0.9 \rightarrow 0.4, c_1 = c_2 = 1, c = 2$ $V_{max} = 0.2 * Range$
UPSO[119]	Unified PSO	$\chi = 0.729, c_1, c_2 = 2.05, NR = 1$
EPSO[224]	Ensemble PSO	See ref. [224]
DMS-PSO[140]	Dynamic multi-swarm PSO	$\mathrm{w} = 0.729,\ \mathrm{c}_1 = \mathrm{c}_2 = 1{:}49445$ $\mathrm{m} = 3;\ \mathrm{R} = 15;$ $V_{max} = 0.5 * Range$
HPSO-TVAC[124]	Hierarchical PSO with time-varying acceleration coefficients	$c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$ $V_{max} = 0.5 * Range$
LPSO[225]	Linearly decreasing inertia weight PSO	$w = 09 \rightarrow 0.4;\ c1, c2 = 1.49445$
MaPSO[226]	Macroscopic PSO	$w = 0.9 \rightarrow 0.4, c = 1.49445$
MiPSO[226]	Microscopic PSO	$w = 0.9 \rightarrow 0.4, c = 1.49445$

Chapter 4 : Heterogeneous Improved Dynamic Multi-Swarm Particle Optimisation Algorithm

4.1 Introduction

As discussed in Chapter 2, the interlinked problems of loss of diversity and premature convergence are the two main drawback of the PSO algorithm. This led to numerous variants that tried to address these issues [227].

As explained in chapter 2, the structure of the neighbourhood topology in PSO algorithms, including the use of multiple swarms, can have a significant impact on the algorithm's exploration and exploitation behaviour [228]. This direction is explored in the present chapter which introduces the first of our new PSO algorithms.

In this chapter, we present a novel variant of the PSO, HIDMS-PSO, which builds on and improves the DMS-PSO [140] by introducing a new topological structure for small-subswarms, and novel learning exemplars, to enhance the search behaviour and reduce loss of diversity and premature convergence. In our approach, a portion of the population is assigned master-slave roles at an individual level with a specific topological structure, with further hierarchical ranks assigned to slave particles (as shown in Fig. 4-2) in each unit. The topological structure is supported by the communication model to restrict arbitrary communication among particles to allow hierarchal, one and two-way communication within and among other units, which prevents particles focusing on a single best-solution. In the present study, two movement strategies are introduced, namely, inward-oriented and outward-oriented strategies. The inward-oriented strategy guides particles towards a local solution whilst the latter strategy guides particles away from their own unit. Both strategies combined enables exploitation of local solutions and exploration of solutions discovered by other units. Also, the master-slave roles, the communication model and the heterogeneous behaviour of particles further assist escape from local optima and contribute to maintain the population diversity for extended periods.

4.2 Heterogeneous Improved Dynamic Multi-Swarm Particle Swarm Optimisation Algorithm

The HIDMS-PSO algorithm bases its fundamental idea on the multiswarm mechanism, and the regrouping strategy introduced in the DMS-PSO. The algorithm conceptualises the idea of multi-swarms as "units". The unit is defined as a subswarm-like entity with three features: a fixed population size, dynamic structure and particle roles. These three features are acquired with the unit topological structure, as shown in Figure 4-1. The unit topological structure introduces a hierarchal topological structure for particles with master-slave roles. As displayed in Figure 4-1, a unit comprises a single master, *M,* and three slave particles with distinct types, denoted as $s_1, \dots s_3$. The distinction of slave particles discourages communication among slave particles in the same unit to further delay the depletion of the population diversity of individual units.

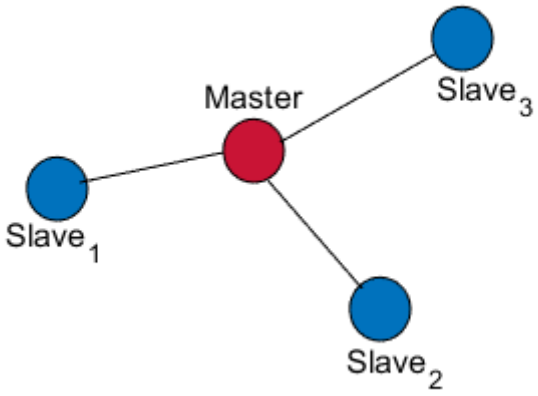


Figure 4-1 Topological structure of a single unit.

The HIDMS-PSO algorithm employs explicit communication rules to avoid arbitrary communication among particles within the same unit or with particles belonging to other units. These rules can be summarised as:

1. Arbitrary particles of the i^{th} unit do not directly communicate with arbitrary particles of the j^{th} unit. Communication is established via the slave particles only.
2. Master particles can only exchange information with one of their slaves.
3. Slave particles can only communicate with slaves of the same type; hence they cannot communicate with the other slaves within their unit.

The rules accommodated by the communication model are also reflected in the exemplar selection in the velocity calculation of particles. In addition, after the positional update, particles communicate with one another using the conditions stated above. In essence, the aforementioned communication rules allow positional information to flow in the swarm in a controlled way to allow particles to convergence relatively fast. Concurrently, the exchange of external information is restricted to only occur between the slave particles of the same type whilst the internal information exchange is permitted solely for master particles to communicate with slave particles in the same unit. Hence, the communication model encourages particles to converge to local best solutions whilst attempting to maintain population diversity to the extent feasible, hence balancing exploitation and exploration.

It should be noted that the particles in the homogeneous subpopulation implicitly communicate across the entire swarm through the use of the **gbest** exemplar.

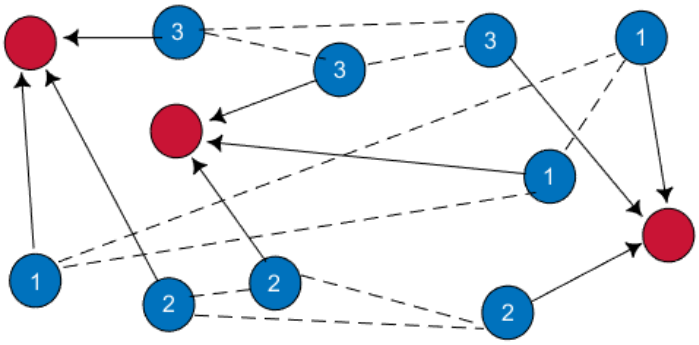


Figure 4-2-The visual depiction of the communication model between 3 units.

4.2.1 The Search Process

The HIDMS-PSO algorithm initiates the search process by dividing the initial population into two equal subpopulations, and these are referred to as the homogeneous subpopulation and the heterogenous subpopulation. The homogeneous subpopulation comprises particles that exhibit uniform behaviour as in the canonical PSO algorithm hence are solely influenced by two exemplars to move in the search space, $pbest$ and $gbest$, respectively, using Eqs. 2.4 and 2.5 (see chapter 2). The particles in the heterogeneous subpopulation are used to form N units, and the master-slave roles are randomly assigned to particles in each unit. The assigned role of the particle determines the learning strategy it adopts, hence allowing the attainment of behavioural heterogeneity in the heterogeneous subpopulation.

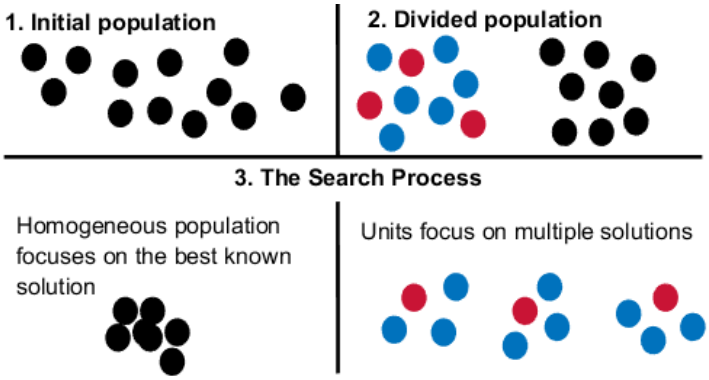


Figure 4-3-Search phases of the HIDMS-PSO algorithm.

The HIDMS-PSO algorithm provides two learning schemes, the inward-oriented and outward-oriented strategy that each group of particles (master or slave) in the heterogeneous subpopulation adopts randomly. In inward-oriented learning, particles are restricted to use guidance information elicited from their unit members, and on the contrary, in the outward-oriented learning strategy, particles' movement is influenced by particles in other units. As briefly mentioned previously, particles' movement is dependent on its assigned role and the randomly selected learning strategy. Hence the velocity update calculation for the master particles with inward-oriented learning can be encapsulated with the following equation:

$$\mathbf{v}_{master}^{t+1} = w^t \mathbf{v}_{master}^t + c_1^t \mathbf{r}_1 (\mathbf{pbest}_{master} - \mathbf{x}_{master}^t) + c_2^t \mathbf{r}_2 (\phi - \mathbf{x}_{master}^t) \quad (4.1)$$

where ϕ is as an exemplar selected randomly as either (1) the position of the most dissimilar (positional dissimilarity) slave particle within the master particle's own unit, (2) the position of the

slave particle with the lowest fitness or (3) the mean of the three slaves' positions in the master's unit. On the contrary, for the outward-oriented learning strategy, one of the velocity update equations from Eq. 4.2-Eq.4.4 is randomly selected:

$$\mathbf{v}_{master}^{t+1} = w^t \mathbf{v}_{master}^t + c_1^t \mathbf{r}_1 (\mathbf{pbest}_{master}^t - \mathbf{x}_{master}^t) + c_2^t \mathbf{r}_2 \left(\mathbf{x}_{master}^{unit} - \mathbf{x}_{master}^t\right) \quad (4.2)$$

$$\mathbf{v}_{master}^{t+1} = w^t \mathbf{v}_{master}^t + c_1^t \mathbf{r}_1 (\mathbf{pbest}_{master}^t - \mathbf{x}_{master}^t) + c_2^t \mathbf{r}_2 \left(\mathbf{x}_{avg}^{unit} - \mathbf{x}_{master}^t\right) \quad (4.3)$$

$$\mathbf{v}_{master}^{t+1} = w^t \mathbf{v}_{master}^t + c_1^t \mathbf{r}_1 \left(\mathbf{x}_{avg}^{master} - \mathbf{x}_{master}^t\right) + c_2^t \mathbf{r}_2 \left(\mathbf{x}_{master}^{unit} - \mathbf{x}_{master}^t\right) \quad (4.4)$$

where $\mathbf{x}_{master}^{unit}$ is the position of a master from another randomly selected unit, $\mathbf{x}_{avg}^{unit}$ is the average position of particles in a randomly selected unit and $\mathbf{x}_{avg}^{master}$ is the average position of particles in the master's unit.

Moreover, the slave particles use the following equation to update velocity:

$$\mathbf{v}_{slave}^{t+1} = w^t \mathbf{v}_{slave}^t + c_1^t \mathbf{r}_1 (\mathbf{pbest}_{slave}^t - \mathbf{x}_{slave}^t) + c_2^t \mathbf{r}_2 (\phi - \mathbf{x}_{slave}^t) \quad (4.5)$$

In this case, for inward-oriented learning, $\vec{\phi}$ is selected as the position of the master particle from the unit the slave particle is a member of, and on the contrary, $\vec{\phi}$ is set to the position of the master particle in another randomly selected unit for the outward-oriented learning strategy. The combination of both strategies enables particles to exploit local information through inward-oriented learning, while particles concurrently gravitate towards particles in other units to sufficiently explore potentially promising regions using outward-oriented learning.

Periodically particles in each unit randomly switch places with a particle of the same type from another unit. This is referred to as "unit reshuffle" (*similar to the regrouping strategy employed in the DMS-PSO*). However, the introduced mechanism does not intend to regroup and exchange information but rather reshape the units while maintaining the master-slave structure. The intention is to further improve, revamp or at least prevent the depletion of the population diversity of the units within the overall swarm by stochastically switching particles of each unit while maintaining their hierarchies.

Algorithm 1: HIDMS-PSO

Input: n as swam size, T_{max} as maximum number of iterations.
Output: **gbest**

c_1=2.5 − (1:T_{max}*2/T_{max}); c_2=0.5 − (1:T_{max}*2/T_{max}); $w_{max} = 0.99; w_{min} = 0.2;$

$$w_1 = \frac{w_{max} + (w_{min} - w_{max})}{1 + exp\left(-5\left(\frac{2t}{T_{max}} - 1\right)\right)}$$

for t=1:T_{max}
  if mod(t,α)==0 **then**
    Reshuffle units
  end
  for i=1:n
    if f($\vec{x}_i$) ≥ f($\vec{x}$) **then**
      w=w_1^t+C; if w>0.99, w=0.99, end
    else
      w=w_1^t+C; if w>0.20, w=0.20, end
    end
    if i^{th} particle is master **then**

```
            Update $\mathbf{v}_i$ using either Eq. 4.1 or 4.2-4.4 and $\mathbf{x_i}$ using 2.5
        elseif $i^{th}$ particle is slave then
            Update $\mathbf{v}_i$ using Eq. 4.5 and $\mathbf{x}_i$ using 2.5
        end
        Mutate $x_i$ using the nonuniform mutation with $p = 0.1$
        Evaluate the fitness of $\mathbf{x}_i$
        Update $\mathbf{pbest_i}$ and $\mathbf{gbest}$
    end
end
```

4.3 Experiments

4.3.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed HIDMS-PSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The results for each experiment are visualised which reflects the total "Number of Functions" (see plots below) best mean performance is obtained by the corresponding algorithms. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

4.3.1.1 Results on 10 dimensional problems

We observe from the results that L-SHADE obtained the lowest mean error values on the entire 10-dimensional unimodal functions (F1~F5) and the majority of the multimodal functions including F8, F10, F11, F12, F13, F14, F16, F17 and F18. For F6 and F9, the best solution is obtained by DMS-PSO whilst for F7 and F15, the lowest mean error value is achieved by the proposed algorithm HIDMS-PSO. The best solution for F19 and F20 is provided by MiPSO and HCLPSO, respectively. On the 10-dimensional composition functions (F21~28), EPSO and L-SHADE solved the same number of functions (2), namely, F22, 27 and F24, F26. For F21, F23, F25 and F28, the best mean performance is exhibited by UPSO, DMS-PSO, MaPSO and LPSO, respectively.

As shown in Figure 4-4, On the 10-dimensional CEC'13 test problems, HIDMS-PSO exhibited the best mean performance solely for 2 functions. We observe a relatively similar performance with other PSO variants employed in this experiment. The best solution for the majority of the 10-dimensional functions is provided by L-SHADE. However, as shown in Figure 4-5, when we compare the performance of HIDMS-PSO with PSO variants only, we can observe that HIDMS-PSO solved more functions than all 13 PSO variants.

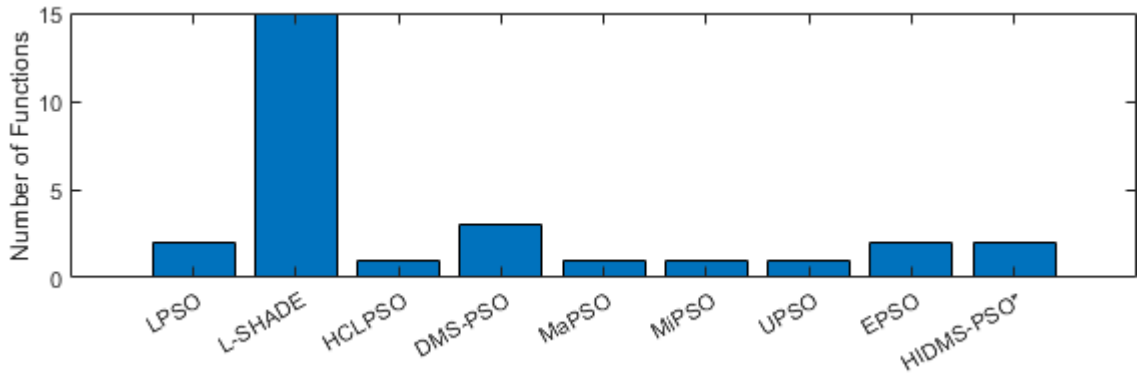


Figure 4-4 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

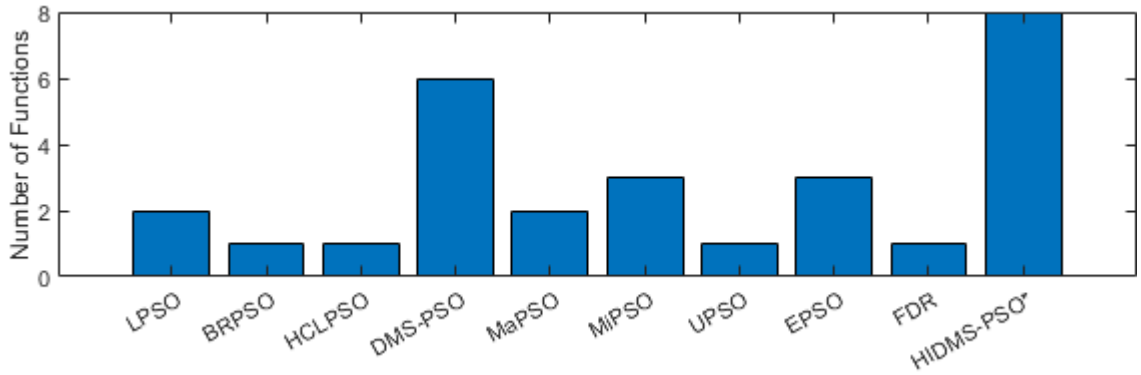


Figure 4-5 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'13 problems

4.3.1.2 Results on 30 dimensional problems

The results on the unimodal functions (F1~F5) indicate that F1 and F5 is solved by DMS-PSO, the best mean performance for F2, F4 and F3 is attained by L-SHADE, EPSO and HIDMS-PSO, respectively. For multimodal functions (F6~F20), L-SHADE solved the majority of the multimodal functions including F6, F8, F10, F11, F13, F14, F16, F17, F18 and F19. For F7 and F9, DMS-PSO obtained the lowest mean error values on F7, F9 and the proposed algorithm HIDMS-PSO exhibited the best mean performance on F15 and F20. The proposed algorithm HIDMS-PSO solved the majority of the composition functions (F23~F28), including F24, F25, F26 and F27. The best solution for F21, F22, F23 and F28 is attained by LPSO, L-SHADE, DMS-PSO and EPSO, respectively.

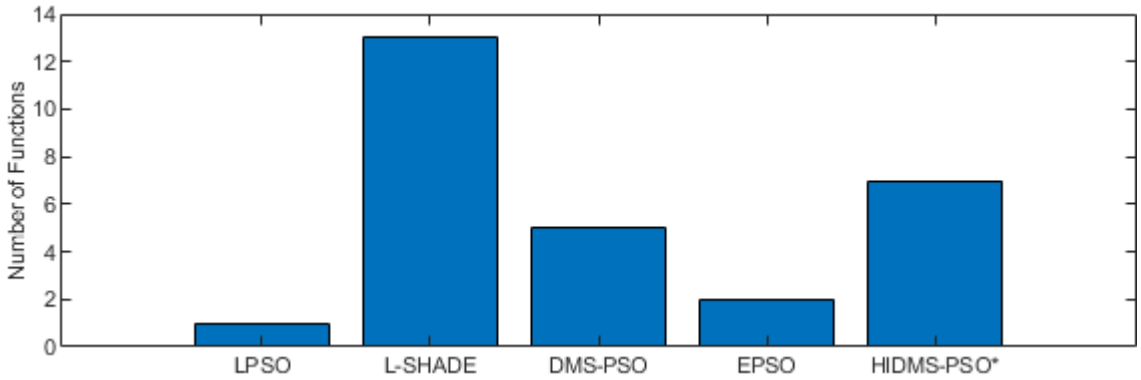


Figure 4-6 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

As shown in Figure 4-6, the best solutions for the 30-dimensional functions are provided by 5 of the 15 algorithms employed in the experiments. The proposed HIDMS-PSO algorithm performed best on 7 of the 28 functions, exhibiting the second-best overall performance and a superior performance compared to all PSO variants.

4.3.1.3 Results on 50 dimensional problems

The results obtained on the unimodal functions (F1~F5) show that for F1 and F5, DMS-PSO performed best whilst L-SHADE obtained the lowest mean error on F2, F3 and EPSO provided the best solution for F4. On the multimodal functions (F6~F20), L-SHADE and the proposed algorithm solved the majority of the multimodal functions where L-SHADE obtained the lowest mean error for F8, F11, F12, F14, F16, F17, F19 and HIDMS-PSO performed best on F7, F9, F10, F15, F18 and F20. The best solutions for F6 and F13 is attained by DMS-PSO and EPSO, respectively. The results on the 8 composition functions (F21~F28), the proposed algorithm HIDMS-PSO solved F24 and F27 whereas L-SHADE obtained the best mean error values on F22 and F23. The best solutions for F21, F25, F26 and F28 is provided by LPSO, DMS-PSO, MiPSO and HCLPSO, respectively.

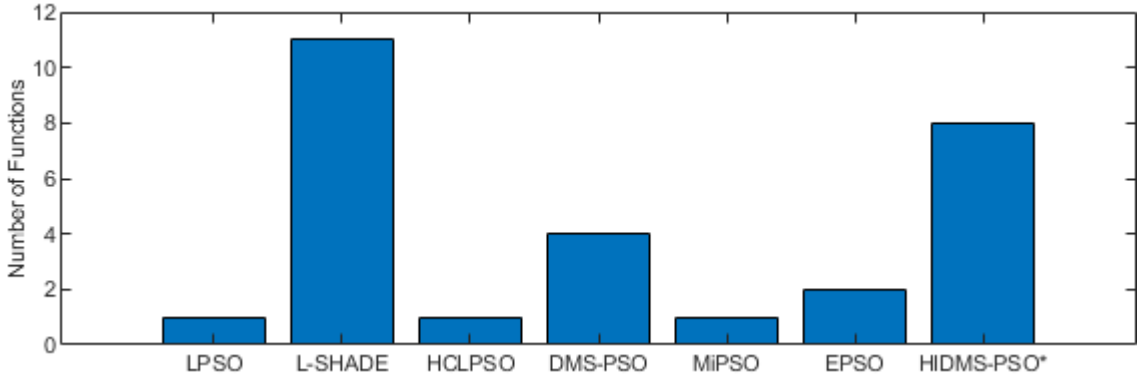


Figure 4-7 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

As shown in Figure 4-7, L-SHADE solved 11 of the 28 functions and the proposed algorithm HIDMS-PSO exhibited the best mean performance on 8 functions in total, the second-best performance attained after L-SHADE. Moreover, HIDMS-PSO solved more problems compared to all PSO variants employed in the experiments on the 50-dimensional functions.

4.3.1.4 Results on 100 dimensional problems

We observe that each unimodal function (F1~F5) is solved by 4 different algorithms. More specifically, the best solutions for F1 and F5 are obtained by DMS-PSO, and L-SAHDE, EPSO, and HIDMS-PSO solved F2, F4 and F3, respectively. On the multimodal functions, the proposed algorithm HIDMS-PSO solved the majority of the multimodal functions including F7, F9, F12, F13, F15, F18 and F20 whereas L-SHADE obtained the lowest mean error values on F8, F10, F14, F16, F17, F19 whereas LPSO and EPSO performed best on F6 and F11, respectively. Finally, on the 100-dimensional composition functions, once again, the proposed algorithm HIDMS-PSO exhibit superior mean performance compared to rest of algorithms and solved 5 of the 8 composition functions including F23, F24, F25, F27 and F28. For F21, F22, and F26, the best solution is provided by UPSO, L-SHADE and EPSO, respectively.

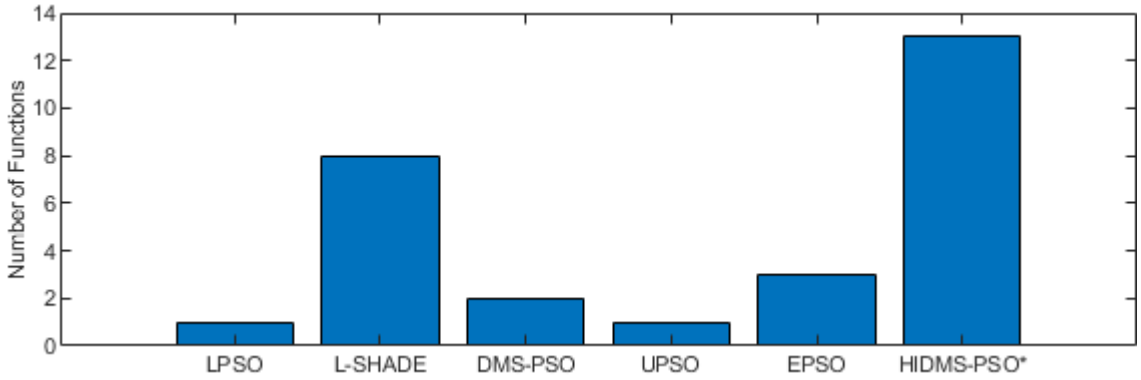


Figure 4-8 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

Overall, as shown in Figure 4-8, the best solutions for 28 100-dimensional CEC'13 functions are provided by only 6 of the 15 algorithms employed in the experiments. We observe a vast difference in the number of functions solved by the proposed algorithm HIDMS-PSO and the comparison algorithms (especially the PSO variants) employed in the experiments, and once again, the closest (second-best) performance to HIDMS-PSO is obtained by L-SHADE. The average and final ranks obtained on the CEC'13 problems are displayed in the final row of Table 4-1:4-4. In summary, the proposed HIDMS-PSO algorithm outranked the majority of the comparison algorithms in all cases where the 4th rank is attained on the 10-dimensional functions outranking 10 algorithms. Moreover, on the 30 and 50-dimensional functions, HIDMS-PSO ranked 2nd and on the 100-dimensional functions, the proposed algorithm HIDMS-PSO obtained the 1st rank, outranking all of the comparison algorithms. Clearly on this test suite the new algorithm's relative performance improves as the problem dimension increases.

In addition, as shown in Table 4-5 the results of the Wilcoxon signed rank test reveals that with the significance level of 5%, the proposed HIDMS-PSO algorithm exhibited significantly superior performance compared to most of the test algorithms and is *not* significantly outperformed by *any* of the other algorithms, including L-SHADE, across all experimented dimensions.

Table 4-1 Wilcoxon Signed Rank Test with significance level of $\alpha = 0.05$ for CEC2013 problems

HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	+	+	=	+	=	=	+	+	+	11/3/0
30	+	+	+	=	=	+	+	=	+	+	=	+	+	+	10/4/0
50	+	+	+	=	=	+	+	=	+	+	=	+	+	+	10/4/0
100	+	+	+	+	=	+	+	=	+	+	=	+	+	+	11/3/0

Figure 4-9 presents the convergence rate comparison of the HIDMS-PSO algorithm with L-SHADE, HCLPSO, EPSO and DMS-PSO on various 100-dimensional CEC'13 problems. The comparison suggests that generally the HIDMS-PSO algorithm exhibits better convergence rate compared to EPSO and HCLPSO. It is worth highlighting that on the multimodal F12, F13, F16 and F19, although the HIDMS-PSO's convergence is not the fastest, however it is not significantly different than L-SHADE and DMS-

PSO. Moreover, on the composition functions (F21, F23, F24, F25, 26 and F28), we clearly observe a more competitive and comparable convergence behaviour between the proposed algorithm HIDMS-PSO, L-SHADE and DMS-PSO.

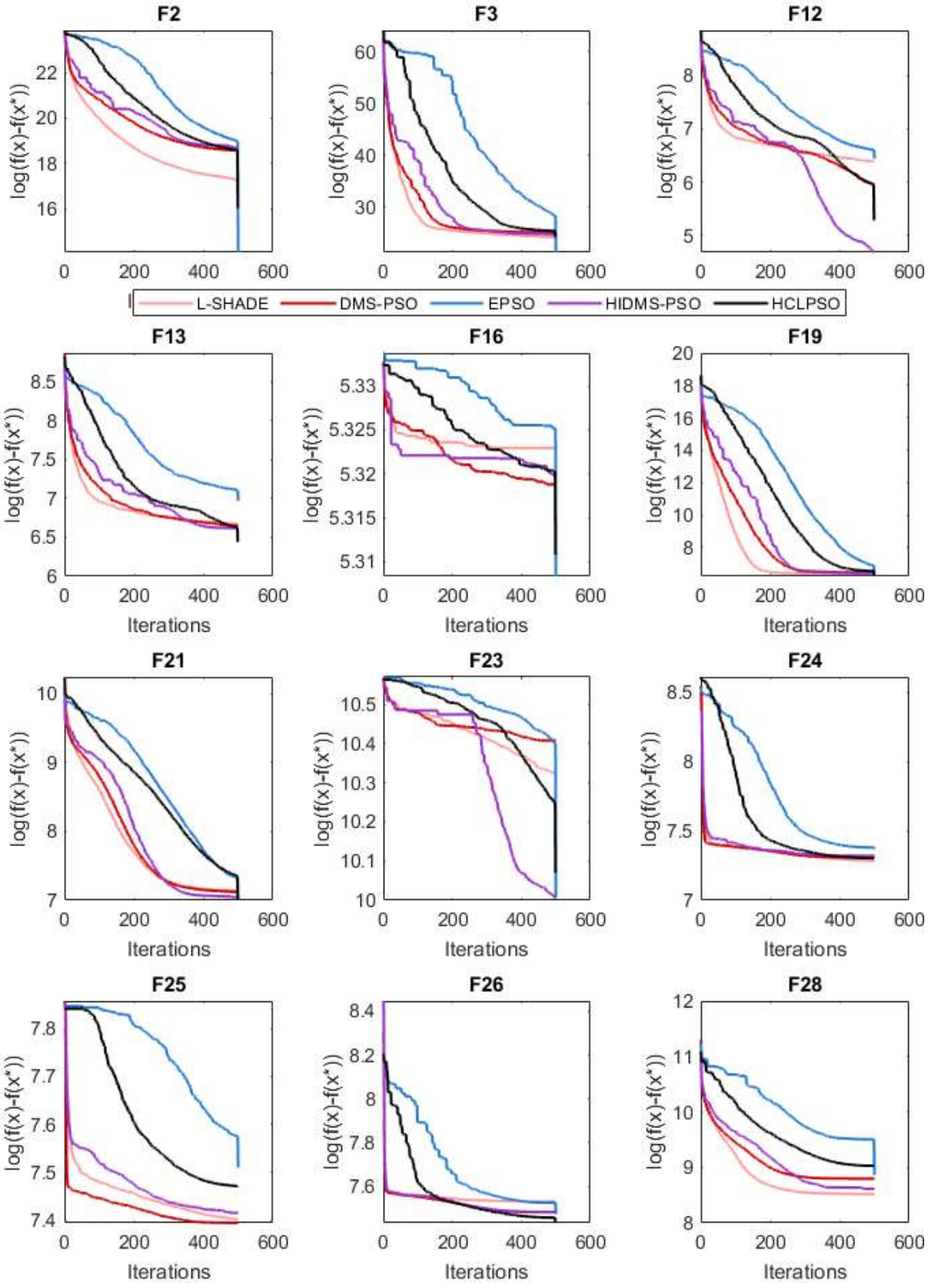


Figure 4-9 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'13 problems

4.3.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed HIDMS-PSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

Results on 10 dimensional problems

The results for the 10-dimensional problems indicate that for unimodal problems (F1~F3), L-SHADE outperformed rest of the algorithms and on the multimodal functions (F4~F15), once again, L-SHADE solved majority (7 of 13) of the multimodal functions including F7, F8, F9, F10, F11, F12 and F15 whereas DMS-PSO exhibited the second-best performance and solved 3 functions including F13, F14 and F16. The best solution for F4 and F5 is provided by EPSO and the proposed algorithm HIDMS-PSO achieved the lowest mean error value on F6. The results on the 10-dimensional hybrid functions (F17~F22) shows that L-SHADE solved all 6 functions. For 8 composition functions (F23~F30), the proposed algorithm HIDMS-PSO obtained the best mean performance on the majority of functions including F24, F27 and F28 whilst rest of the 5 composition functions are solved by individual algorithms. Hence, the best solutions for F23, F25, F26, F29 and F30 are attained by MaPSO, EPSO, DMS-PSO, LPSO and FIPS, respectively.

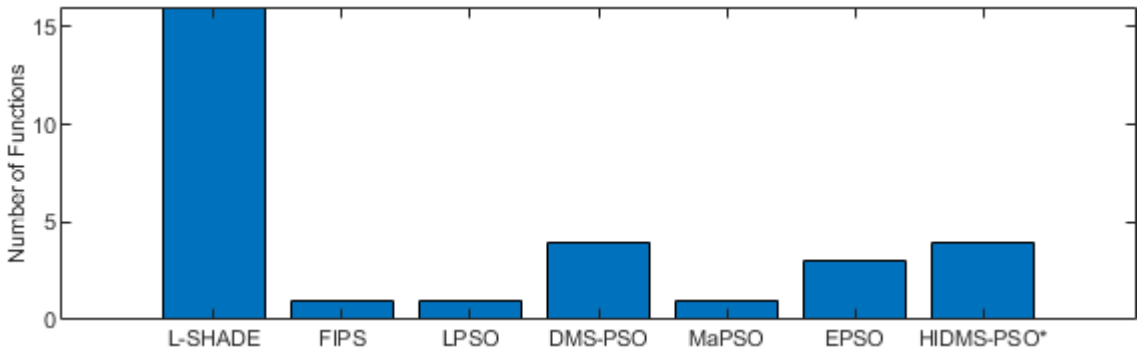


Figure 4-10 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

We can observe from Figure 4-10 that lowest mean error values for the majority of the 10-dimensional CEC'14 functions are obtained by L-SHADE followed by the equal performance of HIDMS-PSO and DMS-PSO.

Results on 30 dimensional problems

The results for 30-dimensional CEC'14 reveal that for unimodal functions (F1~F3), the best solutions for all unimodal functions are provided by L-SHADE and we observe a similar pattern on the multimodal functions (F4~F16) where L-SHADE solved 8 of the 13 multimodal functions including F4,F8, F9,10,11,F12,F15 and F16 whilst the best mean performance on F5, F6, F7, F13 and F14 is attained by individual algorithms namely, HPSO-TVAC, HCLPSO, FIPS, DMS-PSO and the proposed algorithm HIDMS-PSO, respectively. The results obtained on the hybrid functions (F17~F22) show that L-SHADE once again performed best on most of the hybrid functions including F17, F19, F20 and F21 whereas MaPSO and LPSO obtained the lowest mean error values on F18 and F22. Finally, we observe from the results obtained on the 8 composition functions (F23~F30) that DMS-PSO solved 3 of the 8 functions including F23, F25 and F29 whereas the proposed algorithm HIDMS-PSO exhibited

the best mean performance on F24 and F27. L-SHADE, LPSO and MaPSO individually performed best on F26, F28 and F30, respectively.

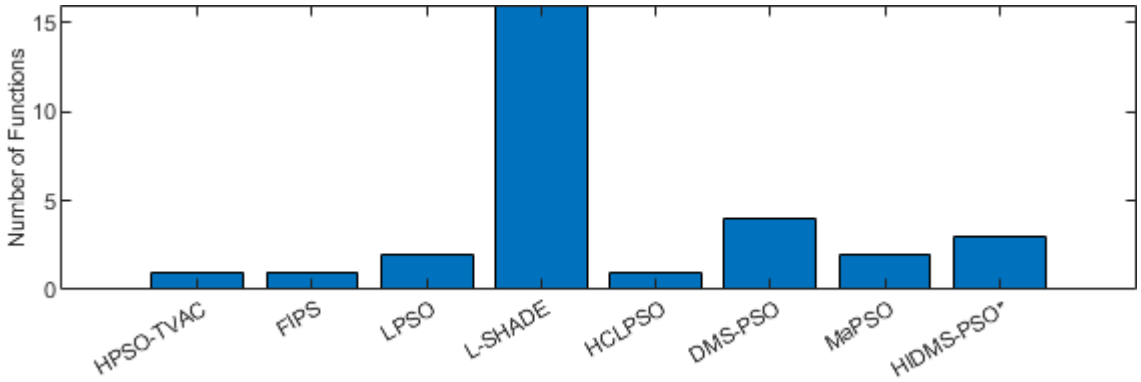


Figure 4-11 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'14 problems

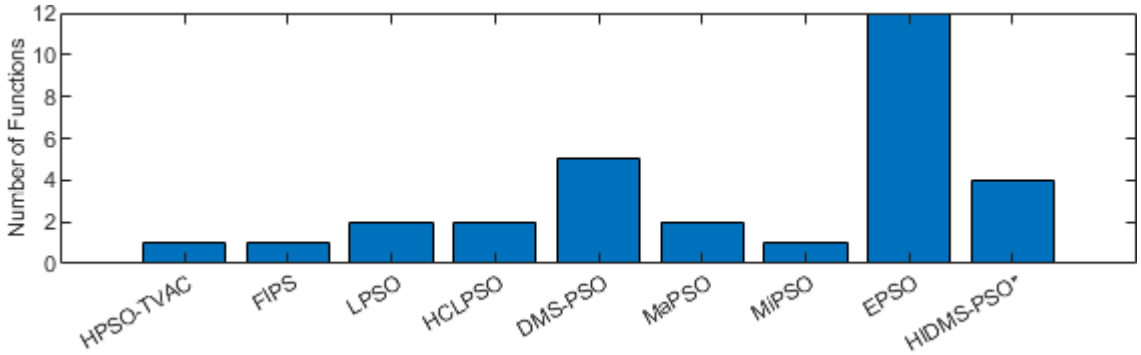


Figure 4-12 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'14 problems

The performance comparison with respect to the number of functions solved is shown in Figure 4-11:4-12 between all test algorithms and PSO variants, respectively. The comparison reveals that in the first case, L-SHADE attained the lowest mean error value for most of the functions at 30 dimensions, and the latter comparison among PSO variants yield that EPSO exhibited better performance for most of the problems compared to other PSO variants. We observe that in both cases, the proposed algorithm HIDMS-PSO is in the third-best place behind DMS-PSO.

Results on 50 dimensional problems

The results for unimodal functions (F1~F3) show that L-SHADE performed best on all unimodal functions and most of the multimodal (F4~F16) functions including F4, F8, F9, F10, F11, F12 and F16 whilst the proposed algorithm HIDMS-PSO solved F6, F14 and F15. For F5, F7 and F13, the best mean solutions are provided by individual algorithms namely, HPSO-TVAC, FIPS and DMS-PSO, respectively. For hybrid functions (F17~F22), once again, L-SHADE performed best on most hybrid functions including F17, F18, F19 and F21 whereas EPSO and the proposed algorithm HIDMS-PSO achieved the best mean performance on F20 and F22, respectively. The experimental results on the 8 composition functions (F23~F30) reveal that DMS-PSO achieved the best mean solution for the majority of the composition functions including F23, F25, F28 and F29 whereas the proposed algorithm HIDMS-PSO solved 2 of the composition functions F24 and F27. The lowest mean error values for F26 and F30 is attained by FIPS and LPSO, respectively.

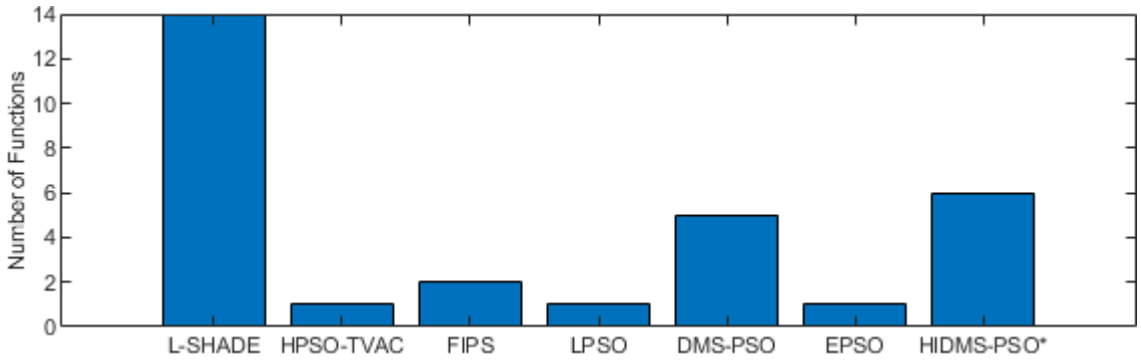


Figure 4-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

We can observe from Figure 4-13 that 50-dimensional CEC'14 functions are solved by only 7 of the 15 algorithms employed in the experiments and the proposed algorithm HIDMS-PSO solved more problems compared to all PSO variants included in the experiment.

Results on 100 dimensional problems

The results obtained on the 100-dimensional CEC'14 functions show that or unimodal functions (F1~F3), L-SHADE achieved the best mean performance on F1 and F2 whilst EPSO performed best on F3. For 13 multimodal functions (F4~F16), the proposed algorithm HIDMS-PSO solved (4) the majority of the functions compared to other algorithms including F6, F11, F14 and F5 whilst L-SHADE performed best on F10, F12 and F16, DMS-PSO attained the lowest mean error values on F9, F13 and the best solutions for F4, F5, F7, F8 is provided individually by LPSO, HCLPSO, UPSO and EPSO, respectively. For hybrid functions (F17~F22), L-SHADE provides the best solution for 2 of the hybrid functions F17 and F21 whilst the remainder 4 functions are solved by individual algorithms, namely, MaPSO (F18), LPSO (F19), EPSO (20) and HIDMS-PSO (F22). On the composition functions (F23~F30), F28, F29 and F30 is solved by LPSO whilst HIDMS-PSO obtained the lowest mean error on F25 and F27. DMS-PSO, HPSO-TVAC and FIPS individually performed best on F23, F24 and F26, respectively.

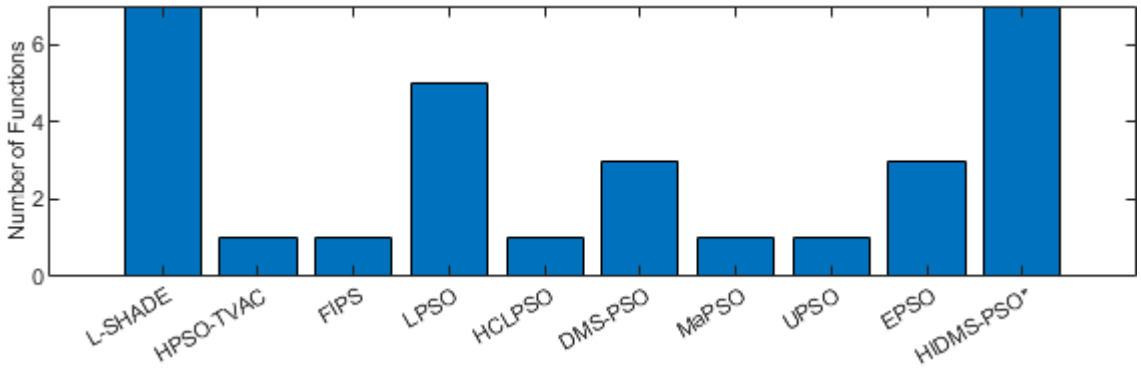


Figure 4-14 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

A shown in Figure 4-14, the best solutions for most of the 100-dimensional CEC'14 test functions are equally provided by the proposed algorithm HIDMS-PSO and L-SHADE. The HIDMS-PSO also solved more functions compared to 13 PSO variants used in the experiments.

The performances of the 15 algorithms on the CEC'14 problems are ranked based on the mean performances and the results yield that the proposed algorithm HIDMS-PSO attained the 7th final rank on 10-dimensional, 5th rank on 30-dimensional, and finally 3rd rank on the 50 and 100-

dimensional CEC'14 problems. Again, its performance improves with the number of problem dimensions. Further, to identify any statistical significance in the mean performance, the Wilcoxon signed rank test is conducted. The detailed results for the Wilcoxon signed rank test are displayed in Table 4-2. By this measure HIDMS-PSO is only outperformed by L-SHADE, and only on the lower dimensional problems. At 100 dimensions it is better than or equal to all other algorithms.

Table 4-2 Wilcoxon Signed Rank Test with significance level of $\alpha = 0.05$ for CEC2014 problems

HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	+	+	=	+	=	-	=	=	+	7/6/1
30	+	+	+	=	=	=	+	=	+	=	-	+	=	+	7/6/1
50	+	+	+	=	=	+	+	=	=	=	-	+	+	+	8/5/1
100	+	+	+	=	=	+	+	=	=	=	=	+	+	+	8/6/0

Furthermore, the convergence rate comparison of the HIDMS-PSO, L-SHADE, EPSO, DMS-PSO and HCLPSO is displayed in Figure 4-15. The comparison suggests that on unimodal, multimodal, hybrid and composition functions, the HIDMS-PSO algorithm exhibit comparable convergence trend to the top ranked algorithms. More specifically, the HIDMS-PSO algorithm generally displayed better convergence rate compared to HCLPSO and EPSO, and in some cases (e.g., multimodal F9), the proposed algorithm HIDMS-PSO demonstrated better mean convergence trend compared to all four algorithms.

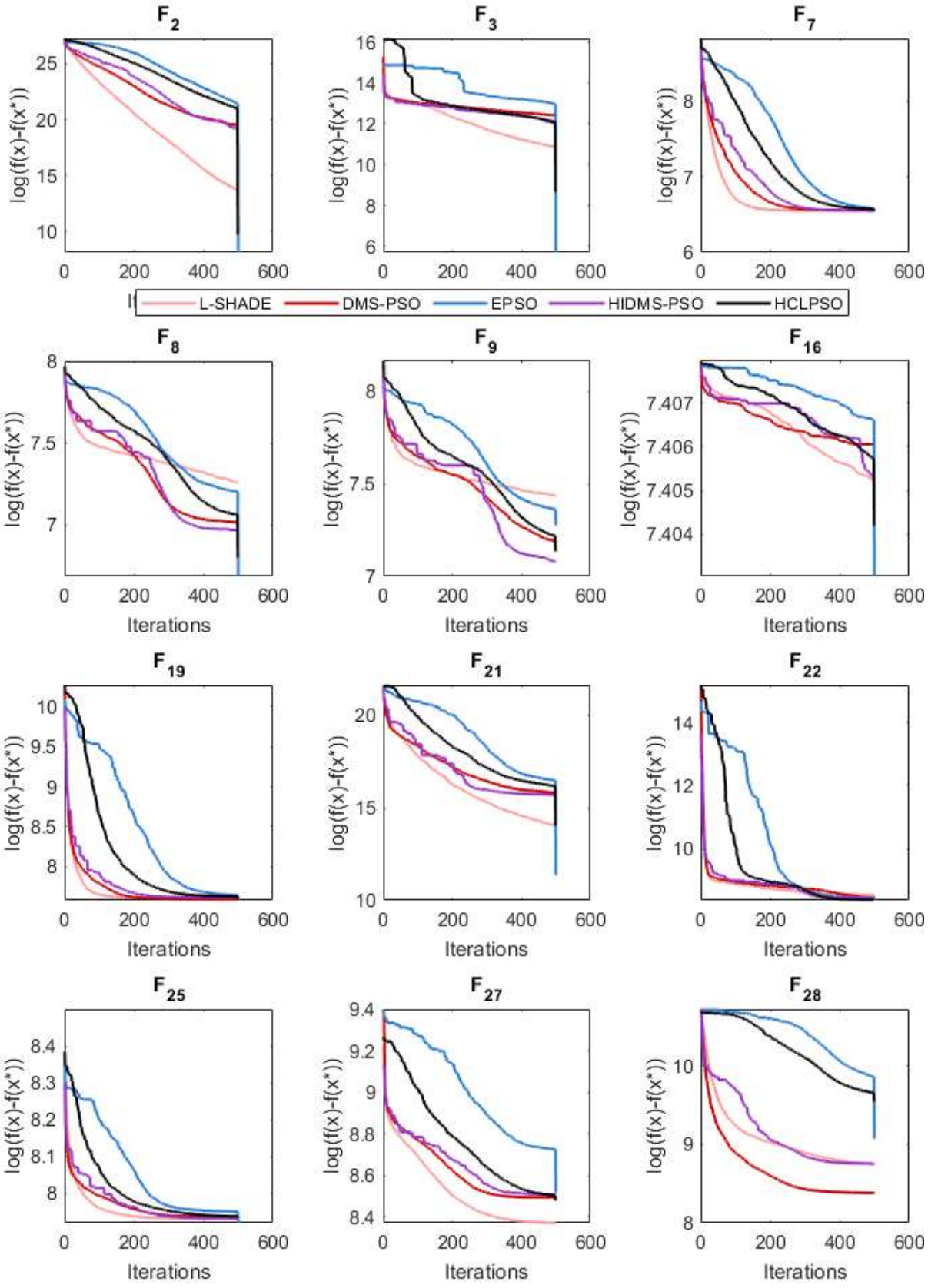


Figure 4-15 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'14 problems

4.3.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed HIDMS-PSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

Results on 10 dimensional problems

The results obtained on the 10-dimensional CEC'17 functions indicate that L-SHADE performed best on the unimodal function F1 and most of the multimodal functions including F3, F4, F5, F7 and F9 whilst EPSO achieved the lowest mean error on F6. On the hybrid functions (F10~F19), we can observe that once again, L-SHADE solved the majority of the hybrid functions including F10~F15, F17~F19. On the other hand, the best performance for F16 is attained by EPSO. On the composition functions (F20~30), MiPSO obtained the best performance on 3 functions, namely F21, F23, F24 whereas MaPSO performed best on F20 and F26. The best performance for F22, F25, F28, F28 and F29 is obtained by individual algorithms, namely EPSO, HCLPSO, LPSO, HIDMS-PSO and L-SHADE, respectively.

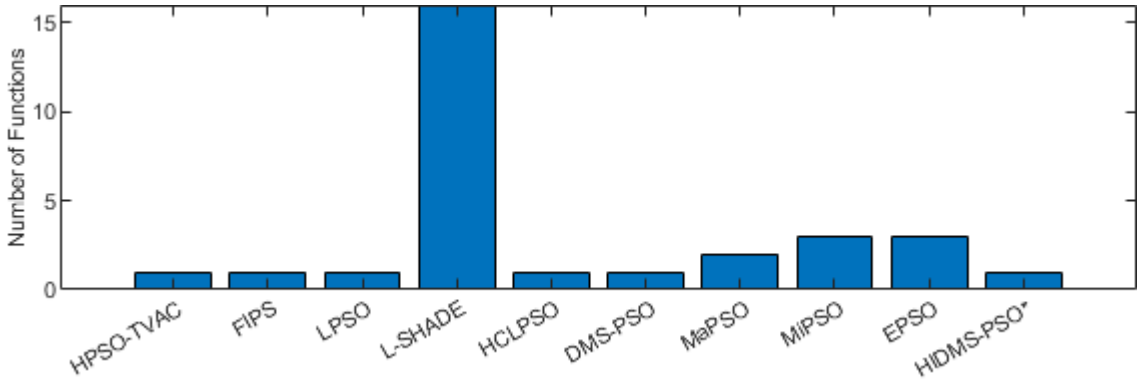


Figure 4-16 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'17 problems

Figure 4-16 displays the total number of functions solved by each algorithm on the 10-dimensional CEC'17 problems. We can observe that most of the 10-diemnsional problems are solved by L-SHADE whilst the PSO variants exhibit approximately comparable performance.

Results on 30 dimensional problems

The mean error values obtained for the 30-dimensional CEC'17 problems reveal that he unimodal F1 is solved by L-SHADE, and for the multimodal functions (F3~F9), the best solution for F5, F7, F8 is attained by L-SHADE whilst EPSO achieved the best mean solution on F3 and F4. The best solution for F6 and F9 is provided by FIPS and MiPSO, respectively. The results on the hybrid functions (F10~F19) show that L-SHADE obtained the best performance for most of the hybrid functions, including, F10, F12~F14, F18 and F19 whilst DMS-PSO performed best on F11 and F16. The best solution for F15 and F17 is obtained by MaPSO and HCLPSO, respectively. Finally, we observe from the results obtained for the composition functions (F20~F30) indicate that the proposed algorithm HIDMS-PSO and L-SHADE solved equal number of functions where HIDMS-PSO exhibited the best performance on F24, F26, F28 and L-SHADE on F20, F21 and F23.

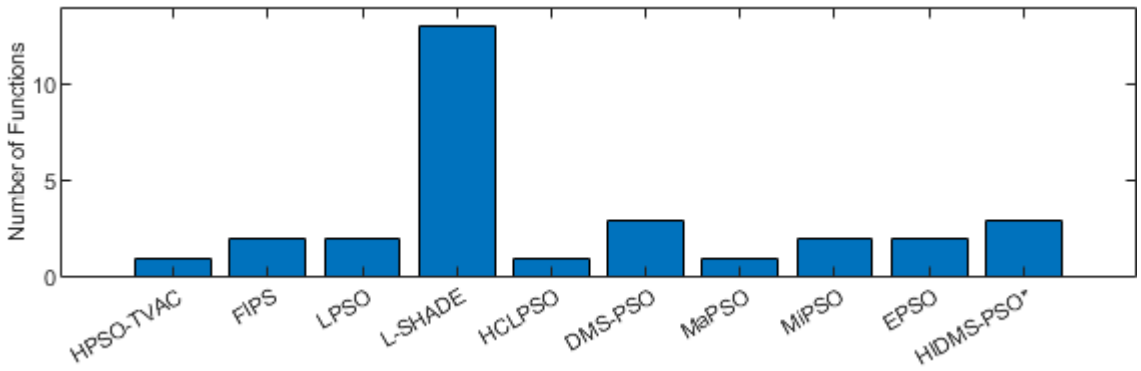


Figure 4-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

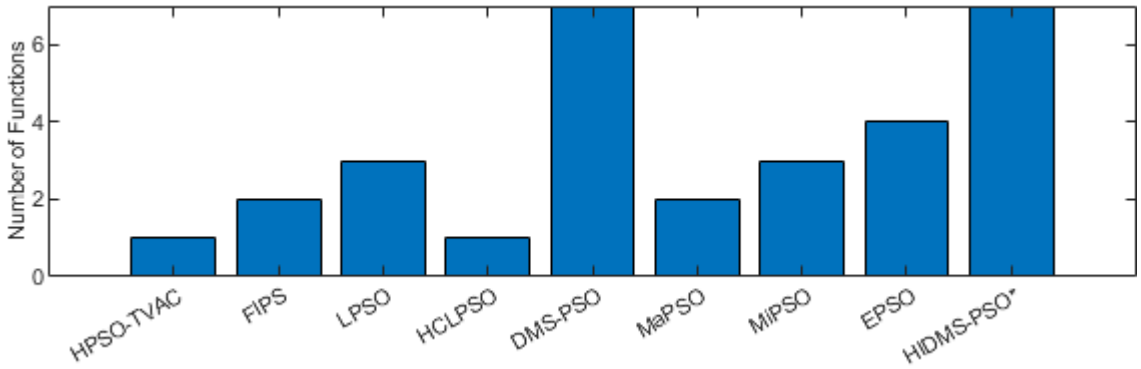


Figure 4-18 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

Overall, the performance of the test algorithms with respect to the best performance exhibited on the total number of functions reveal that, as shown in Figure 4-17, L-SHADE solved more problems compared to all test algorithms whereas HIDMS-PSO exhibited comparable performance to other PSO algorithms. In the second case, shown in Figure 4-18, the comparison among PSO variants suggest that HIDMS-PSO and DMS-PSO demonstrated equal performance (*with respect to number of funcs. solved*) and attained the lowest mean error values most of the 30-dimensional CEC'17 problems.

Results on 50 dimensional problems

The results obtained on the 29 50-dimensional CEC'17 problems indicate that the best solution for unimodal F1 is obtained by L-SHADE and the results on the multimodal functions (F3~F9) show that L-SHADE provides the lowest mean error values for F5, F7, F8 whilst FIPS and EPSO solved 2 functions F6, F9 and F3, F4, respectively. The results on the hybrid functions reveal that L-SHADE performed best on the majority of the hybrid functions including F10, F12, F14, F15, F18, F19 whilst the proposed algorithm HIDMS-PSO performed best on F11, F16 and F17. On the other hand, the best solution for F13 is provided by LPSO. The results on the composition functions (F20~F30) yield that the proposed HIDMS-PSO algorithm and L-SHADE solved equal number of composition functions whereby HIDMS-PSO performed best on F22, F24, F26 and L-SHADE on F21, F23 and F29. Moreover, DMS-PSO attained the lowest mean error value for F20 and F28 whilst remainder composition functions F25, F27 and F30 is solved individually by FIPS, HPSO-TVAC and LPSO, respectively.

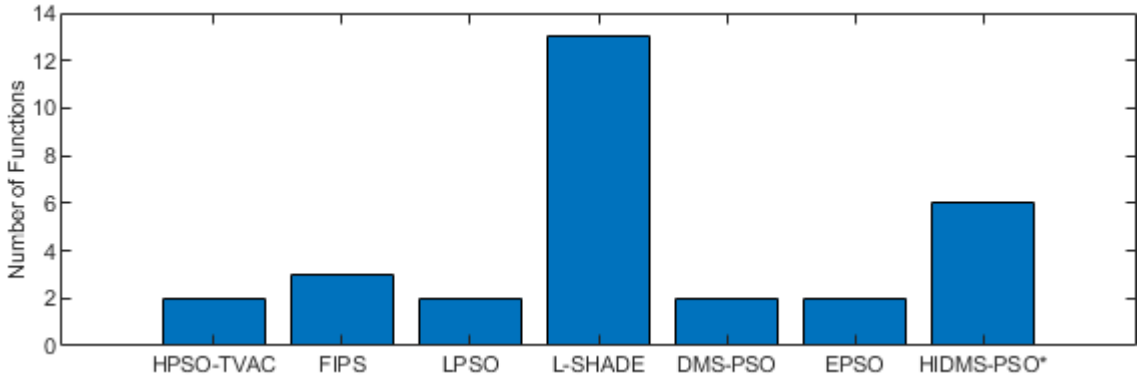


Figure 4-19 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

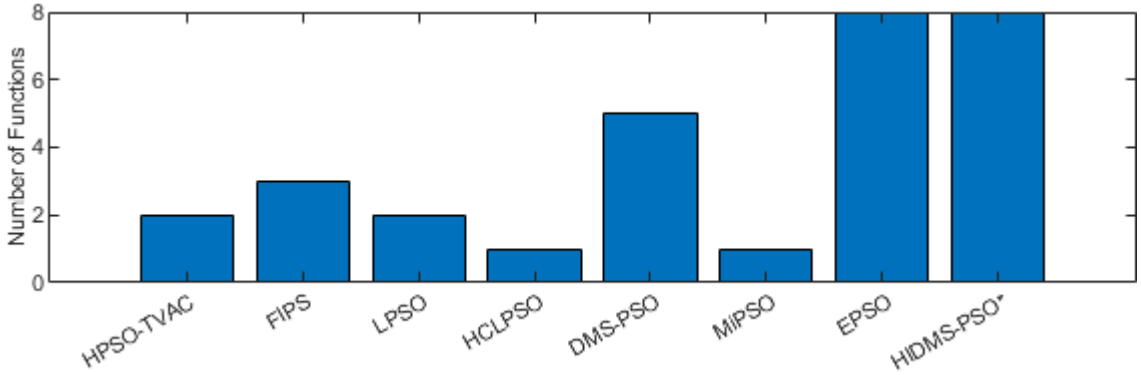


Figure 4-20 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'17 problems

In summary, as shown in Figure 4-19:4-20, for 50-dimensional CEC'17 problems, the best performance for most of the problems are exhibited by L-SHADE followed by the proposed algorithm HIDMS-PSO. In the latter comparison (Fig. 4-20) among PSO variants, we observe that HIDMS-PSO and EPSO solved the majority of the 50-dimensional problems.

Results on 100 dimensional problems

The results yield that L-SHADE is the only algorithm to solve the unimodal F1. On the multimodal functions (F3~F9), DMS-PSO provides the best solution for most multimodal functions including F5, F7 and F8 whilst EPSO solved F3, F6, LPSO and HIDMS-PSO obtained the lowest mean value on F4 and F9, respectively. For the 100-dimensional hybrid functions (F10~F19), L-SHADE exhibited better performance compared to rest of the algorithms and solved 5 of the 11 functions including F10, F12, F14, F15 whilst EPSO solved F13, F18 and F19. On the other hand, the proposed algorithm HIDMS-PSO achieved the best performance on F16, F17 and the best mean solution for F11 is provided by DMS-PSO. The results reveal that the 100-dimensional composition functions (F20~F29) are solved by 3 algorithms only where the proposed algorithm HIDMS-PSO performed best on most of the functions including F22, F23, F24, F25, F26 and F29 whilst DMS-PSO performed best on F20, F21, F27 and LPSO solved 2 of the functions F28 and F30.

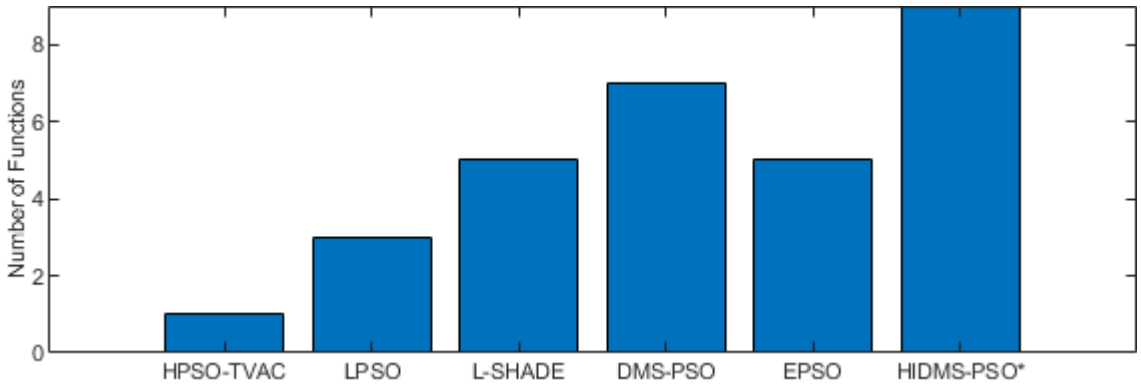


Figure 4-21 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

We can observe from Figure 4-21 that the 100-dimensional CEC'17 functions are solved by 5 algorithms and the proposed algorithm HIDMS-PSO exhibited the best performance for the majority of the functions compared to all algorithms employed in the experiments. Again, on this test suite, its relative performance improved with problem dimensionality.

The proposed HIDMS-PSO algorithm ranked 8th on the 10-dimensional functions. However, with the increase of the problem dimensionality we observe a significant surge in rank. Hence, the proposed algorithm HIDMS-PSO attained the 3rd, 2nd and 1st ranks for the experiments conducted on the 30, 50 and 100-dimensional CEC'17 test problems. Further, the Wilcoxon signed rank test is conducted with the significance level of 5% and the results are shown in Table 4-3. We observe from the results that in general, the performance of the HIDMS-PSO algorithm is statistically significantly better than most algorithms with higher dimensional functions. More specifically, according to the Wilcoxon signed rank test, the HIDMS-PSO algorithm performed better than 6 comparison algorithms on the 10-dimensional functions. Moreover, on the 30, 50 and 100-dimensional functions, HIDMS0-PSO is statistically significantly better than 9, 11 and 10 comparison algorithms, respectively. At 50 and 100 dimensions none of the other algorithms were statistically better than HIDMS-PSO.

Table 4-3 Wilcoxon Signed Rank Test with a Significance Level of $\alpha = 0.05$ for CEC2017 problems

HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	-	+	+	-	+	=	-	=	=	=	6/5/3
30	+	+	+	=	=	+	+	=	+	=	-	+	+	+	9/4/1
50	+	+	+	=	=	+	+	+	+	+	=	+	+	+	11/3/0
100	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0

In addition, the comparison of the convergence characteristics of the HIDMS-PSO algorithm and the 4 top ranked algorithms on the 100-dimensional CEC'17 experiment reveals that (Figure 4-22) the proposed algorithm HIDMS-PSO is capable of converging as fast as the top ranked algorithms in most cases. We also observe from the convergence rates on the multimodal functions (F5, F8)

that the HIDMS-PSO algorithm is better at escaping local optima compared to the top ranked algorithms.

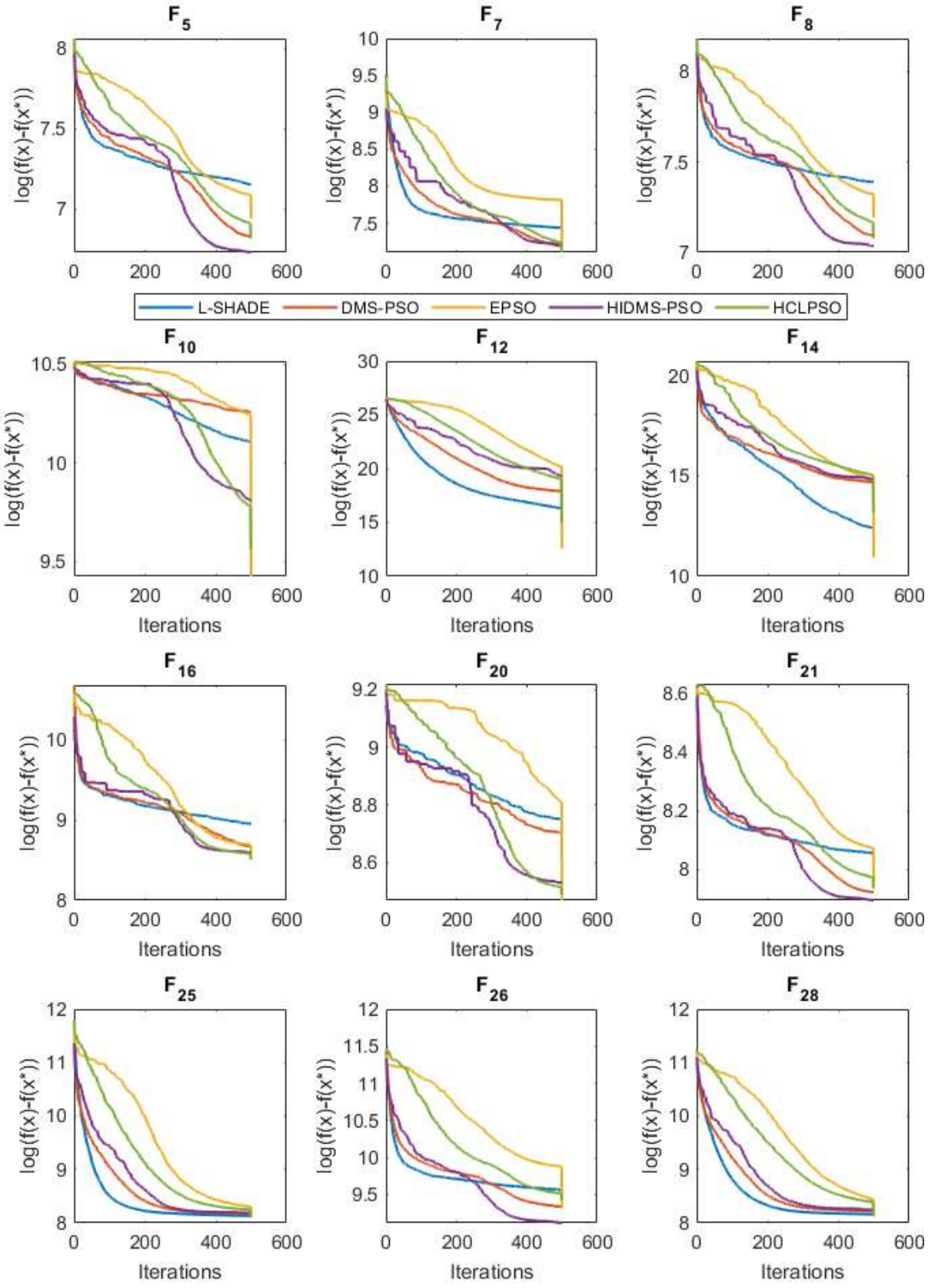


Figure 4-22 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'17 problems

We also individually compared the total number of functions each test algorithm has solved (with regards to the lowest mean error values obtained) against the proposed algorithm HIDMS-PSO on 4 different dimensions of CEC'13, CEC'14 and CEC'17 test suites. We can observe from Figure 4-23:4-25 that on all dimensions of the 3 test suites, the proposed algorithm HIDMS-PSO solved more problems compared to the majority of the algorithms employed. In general, we specifically observe a more distinct gap in the total number of functions solved by HIDMS-PSO and the comparison algorithms, and a rise in the number of algorithms outperformed with increased problem dimensionality.

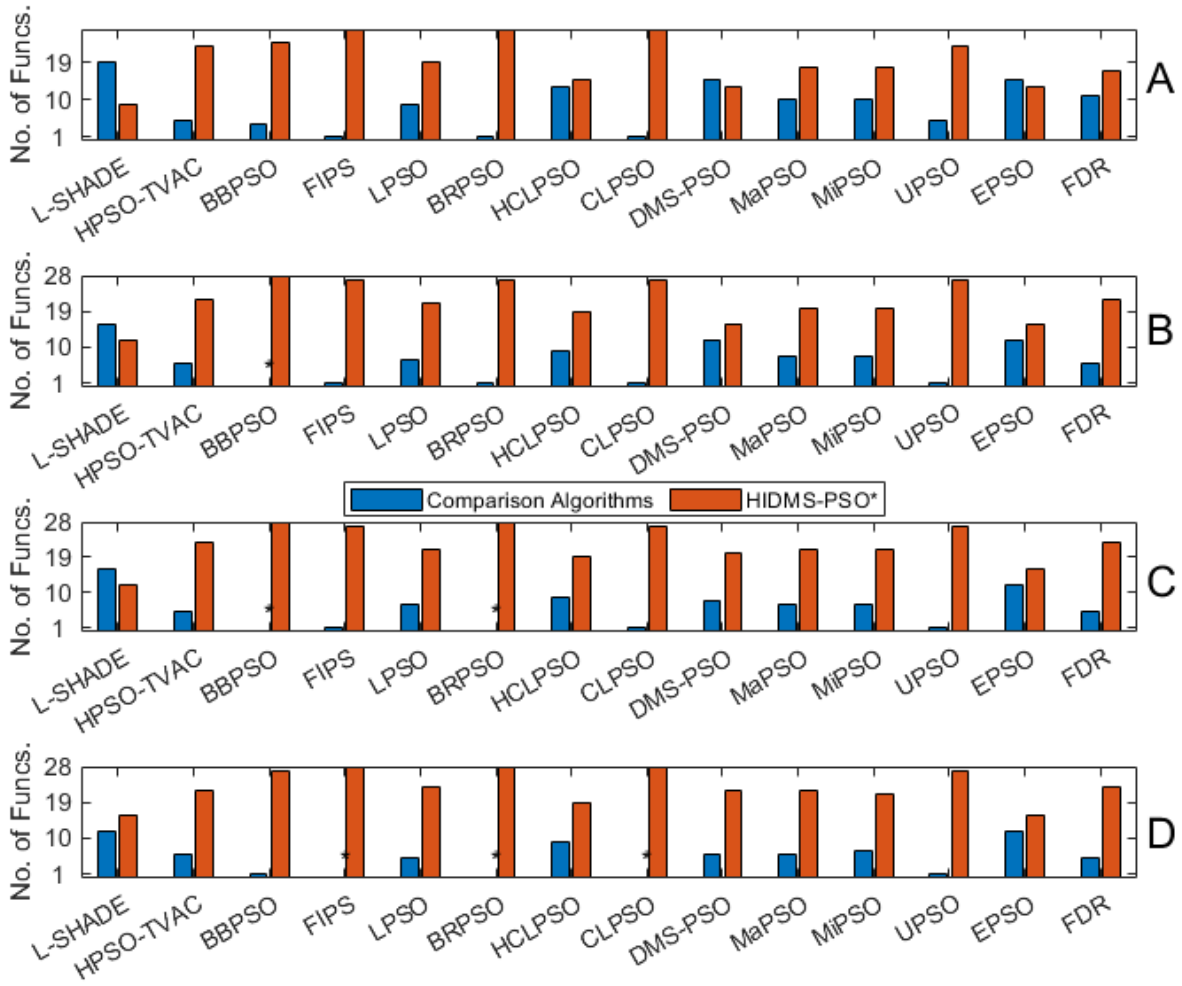


Figure 4-23 Pairwise comparison of HIDMS-PSO and all test algorithms on the sum of problems where the best mean performance is obtained for 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'13 test problems functions

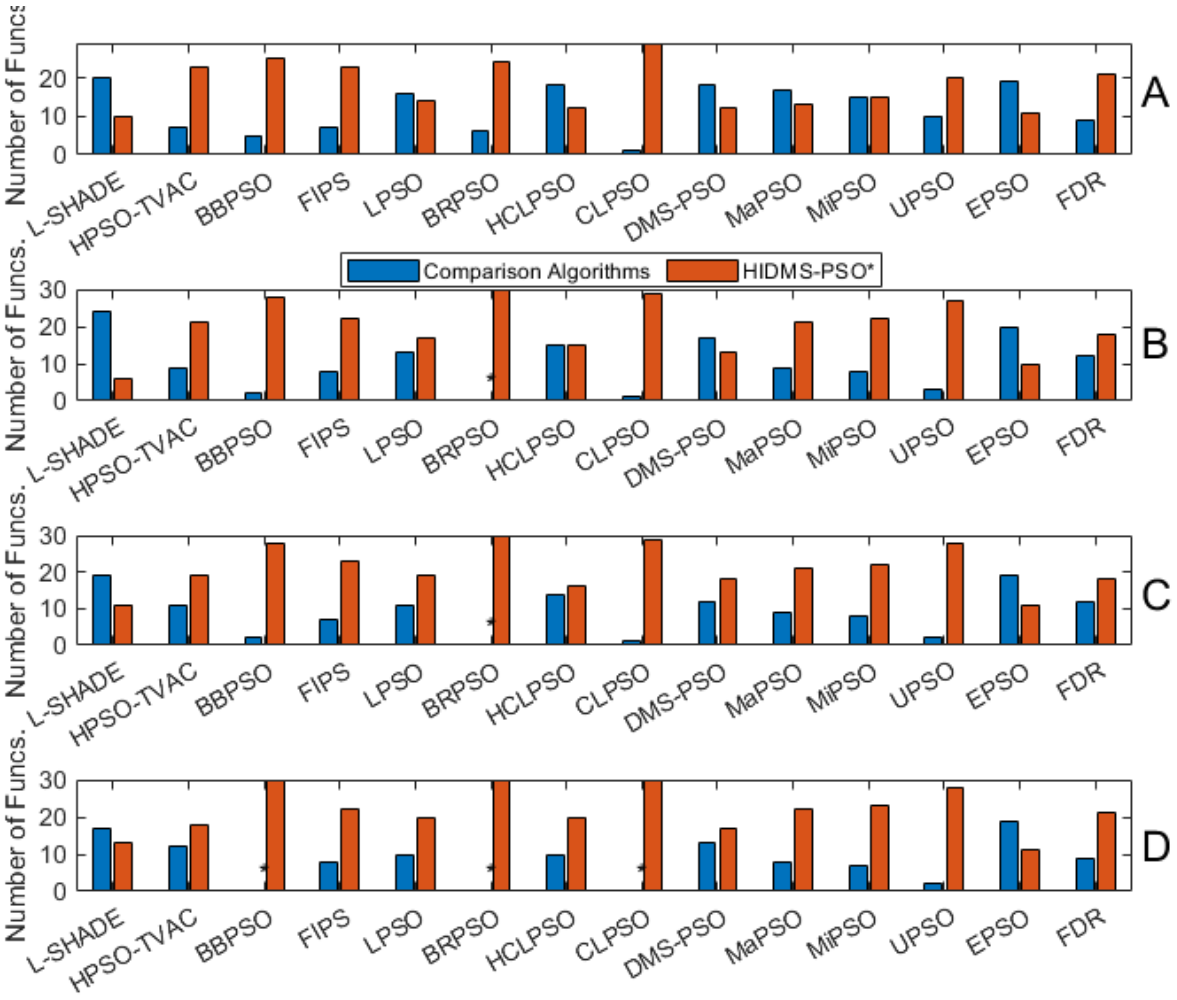


Figure 4-24 Pairwise comparison of HIDMS-PSO and all test algorithms on the sum of problems where the best mean performance is obtained for 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'14 test problems

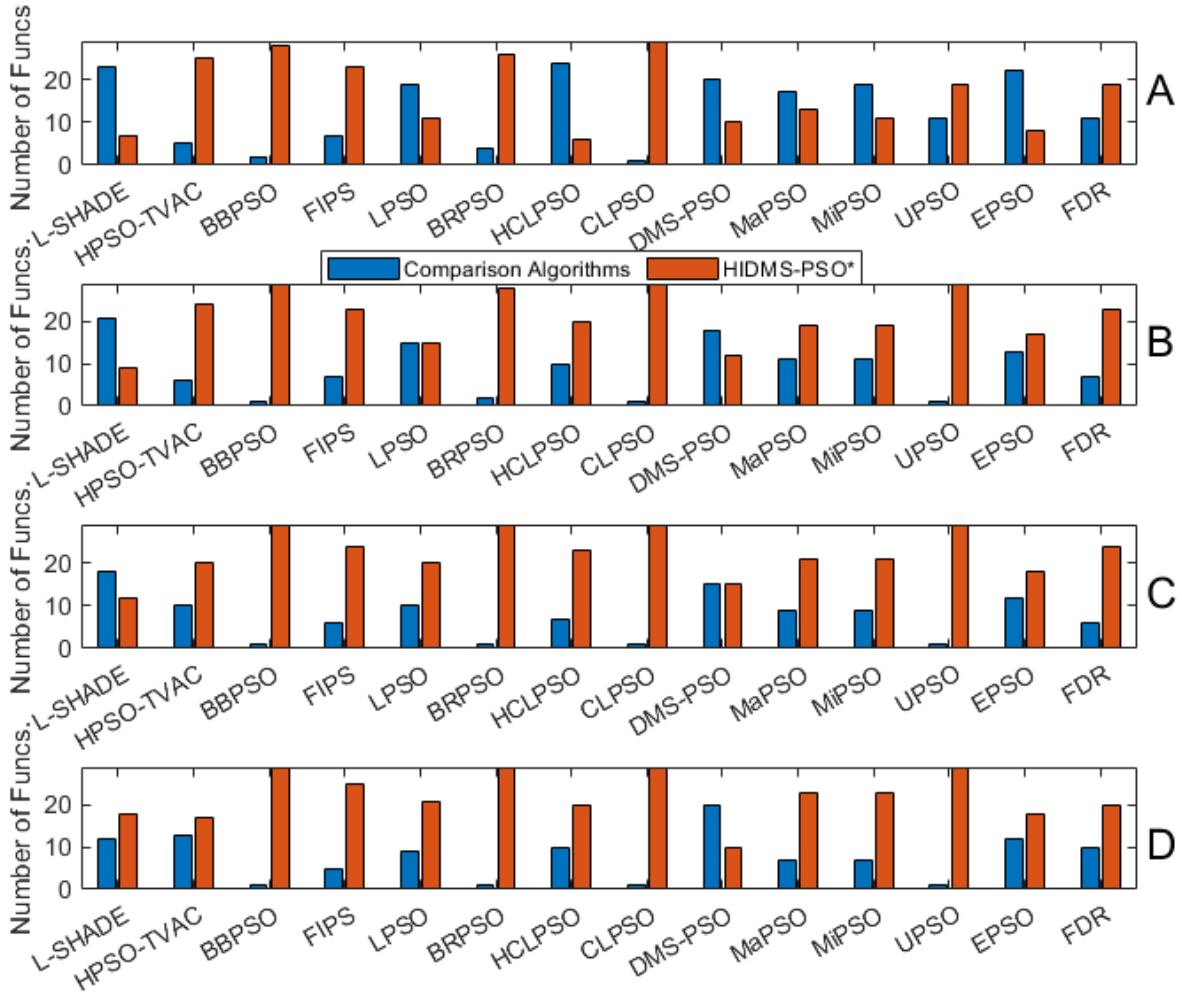


Figure 4-25 Pairwise comparison of HIDMS-PSO and all test algorithms on the sum of problems where the best mean performance is obtained for 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'17 test problems.

4.4. Search Dynamic of HIDMS-PSO

HIDMS-PSO comprises different components, and it is imperative to identify the relative impact of each component and distinguish the core and expandable components of the algorithm. Hence, we have separated the HIDMS-PSO algorithm into different versions that represent a set of components whereby each version incrementally introduces a new component. These versions of the HIDMS-PSO algorithm can be summarised as:

- V1 : HIDMS-PSO with only the homogeneous sub-population.
- V2 : HIDMS-PSO with only the heterogeneous population.
- V3 : HIDMS-PSO with both homogeneous and heterogeneous subpopulations coexisting.
- V4 : V3 + the communication model.
- V5 : V4 + the unit reshuffle mechanism.

4.4.1 The Impact of the Homogeneous and Heterogeneous Sub-populations

Initially, we investigated the behavioural differences of the homogeneous and heterogeneous subpopulations. The impact of the two distinct subpopulations is assessed on the 30 and 50-dimensional CEC'17 benchmark test suites. As stated in previous sections, particles in the homogeneous subpopulation employ the canonical PSO algorithm (with the parameters of HIDMS-

PSO). The heterogeneous subpopulation adopts the distinct velocity update equations depending on the role of the particle and the randomly determined inward-oriented or outward-oriented learning strategy. Hence, particles in the homogeneous subpopulation only use two exemplars $pbest$ and $gbest$ whereas particles in the heterogeneous subpopulation avoid using the $gbest$ exemplar and are guided using multiple exemplars. Hence, better exploitation capability is expected from the homogeneous subpopulation, which is likely to reflect a faster convergence rate. However, the heterogeneous subpopulation may converge to a better final solution at a slower pace due to its better exploration capabilities.

Figures 4-26:4-27 show the result of the initial experiment conducted on the 30 and 50-dimensional CEC'17 functions. As expected, the homogeneous subpopulation exhibits a faster convergence speed in the majority of the problem, usually to a better mean solution. A closer inspection of the convergence rates reveals that the two versions exhibit distinct convergence trends with different function types. As shown in Figure 4-28, the convergence error observed on the composition functions is the lowest, meaning the two versions exhibited the closest convergence behaviour for composition functions followed by the multimodal and hybrid functions. The convergence rate in the 50-dimensional CEC'17 functions does not exhibit a notable change; however, interestingly, at 50 dimensions, V2 demonstrated faster convergence to a significantly better solution for half of the composition functions (F22, F23, F24, F26 and F27). This is most likely the result of composition functions being more complex compared to basic multimodal and hybrid functions; hence the version with only the homogeneous subpopulation cannot sufficiently explore the search space and convergences quickly to a less optimal solution.

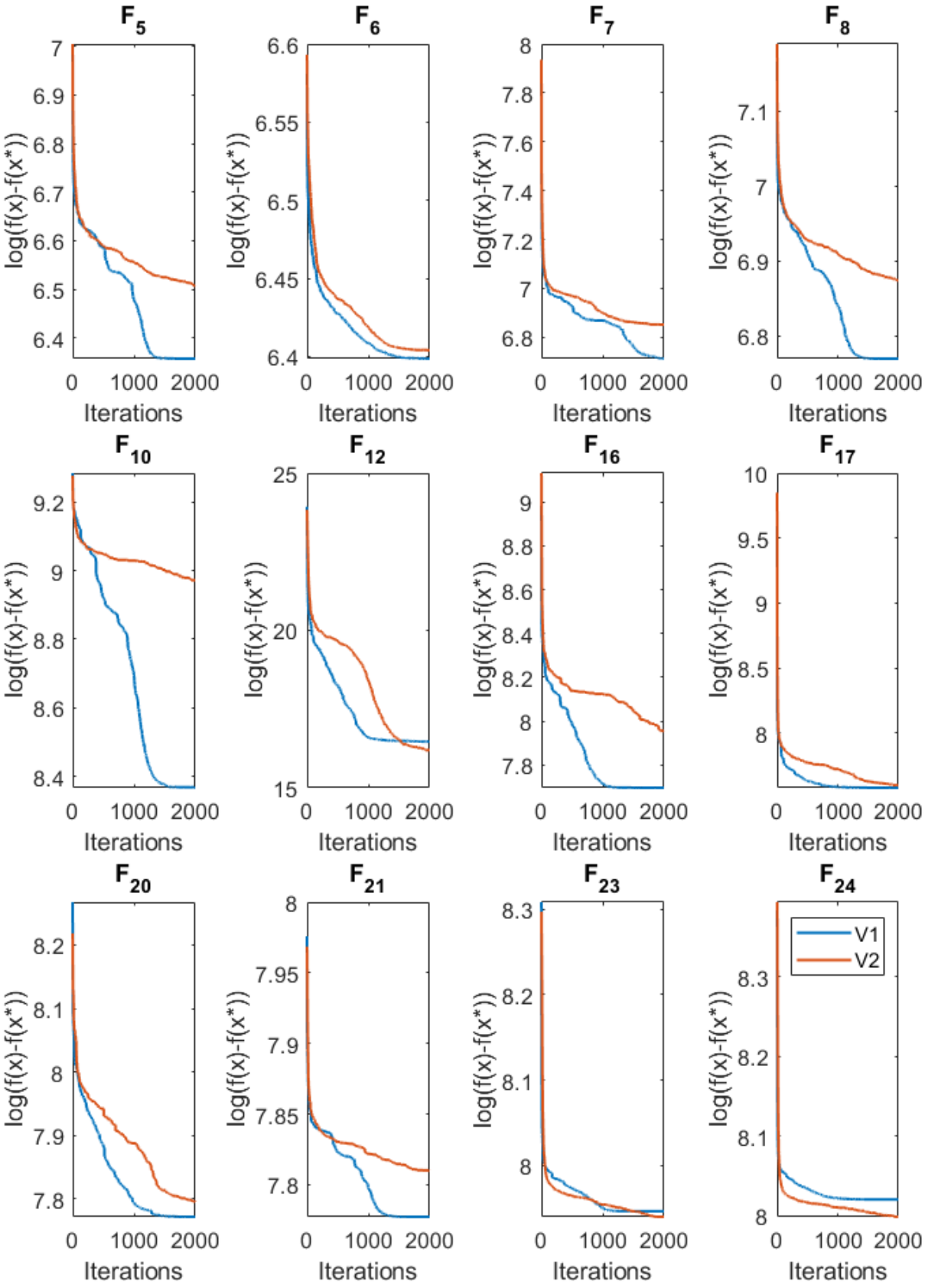


Figure 4-26- Convergence rate comparison of V1 and V2 on the 30-dimensional CEC'17 test functions.

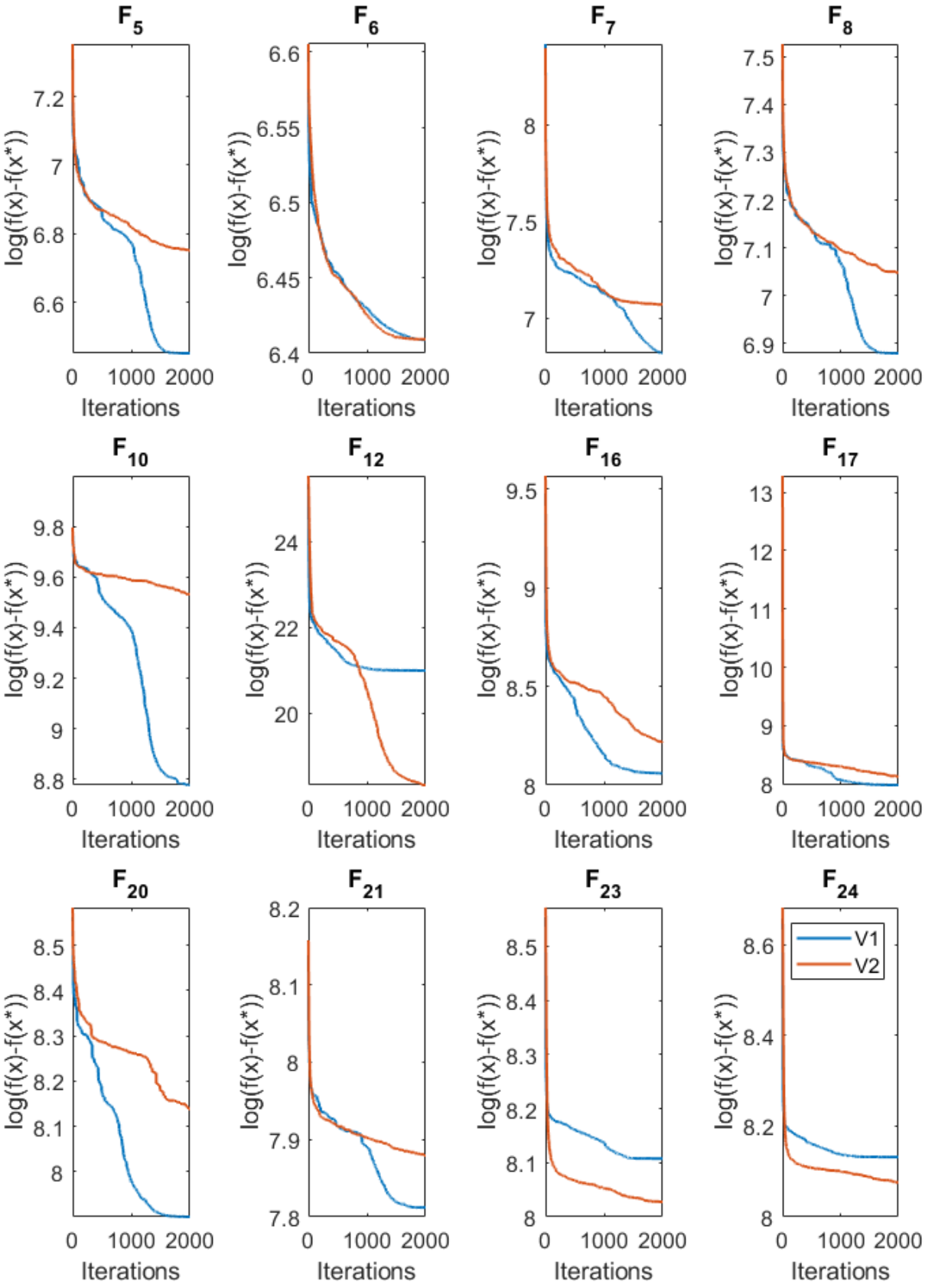


Figure 4-27-Convergence rate comparison of V1 and V2 on the 50-dimensional CEC'17 test functions.

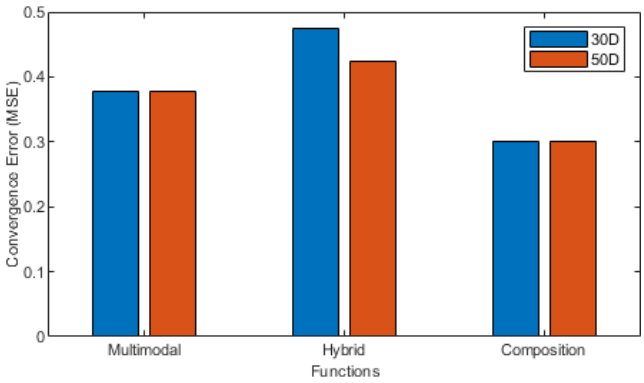


Figure 4-28-Convergence error for V1 and V2 on the 30 and 50-dimensional CEC'17 functions.

4.4.2 The Comparison of Both Sub-populations Coexisting Compared to the Homogeneous and Heterogeneous Sub-populations

The V3 integrates both V1 and V2 (homogeneous and heterogeneous subpopulations), and through the integration of both subpopulations, the balance of exploration and exploitation emerges through the "division of labour" between the two subpopulations.

This section compares the mean performance, convergence rate, and population diversity of V3 with V1 and V2. Figure 4-29 shows the convergence rates for the 30 functions of the CEC'17 test suite for the problem size of 30 dimensions on V1, V2 and V3. Initial experiments to attain the convergence rates indicate that V3 converges to a better solution for most of the functions, usually faster. Initial observations reveal that V3 indiscriminately exhibits equally well convergence behaviour on multimodal, hybrid and composition functions. Moreover, V3 continued to exhibit similar convergence performance when the problem size is increased to 50 by converging to better solutions, usually faster than V1 and V2. However, the increased problem complexity resulted in a noticeable deterioration in the convergence behaviour of V1 for composition problems. Further increasing the problem dimension to 100 resulted in greater deterioration in the convergence behaviour of V1. More specifically, V1 converged to the least feasible solution in 17 of the 30 functions of the CEC'17 test suite. Concurrently, V3 continued to exhibit a faster convergence rate to a better solution while maintaining a higher population diversity than V1. An example of this is shown in Figure 4-30:4-31, where V3 attains faster convergence while maintaining better population diversity rates on functions F13, F14, F15, F16, F18 and F19 at 100 dimensions.

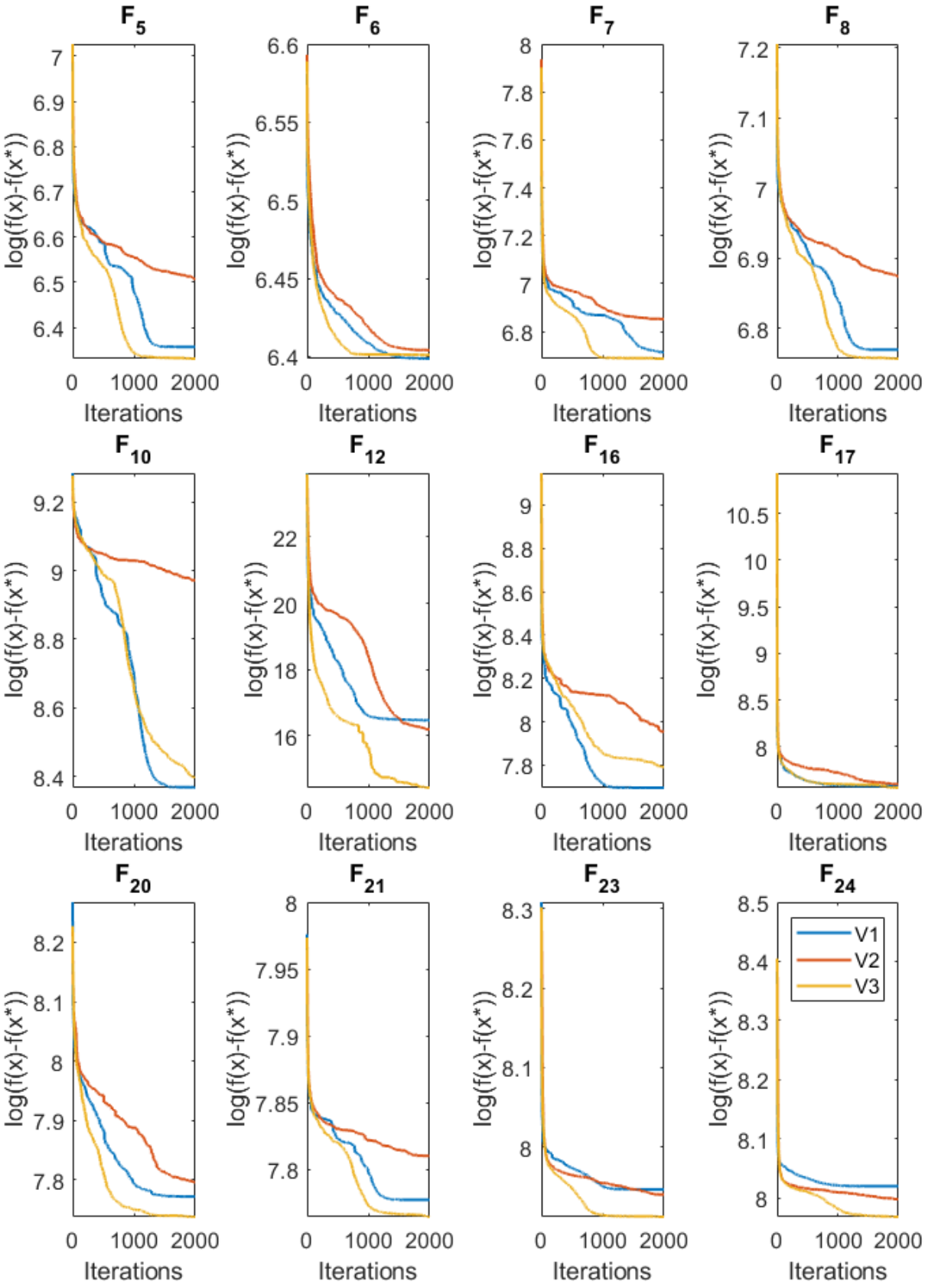


Figure 4-29- Convergence rate comparison of V1, V2 and V3 on the 30-dimensional CEC'17 test functions.

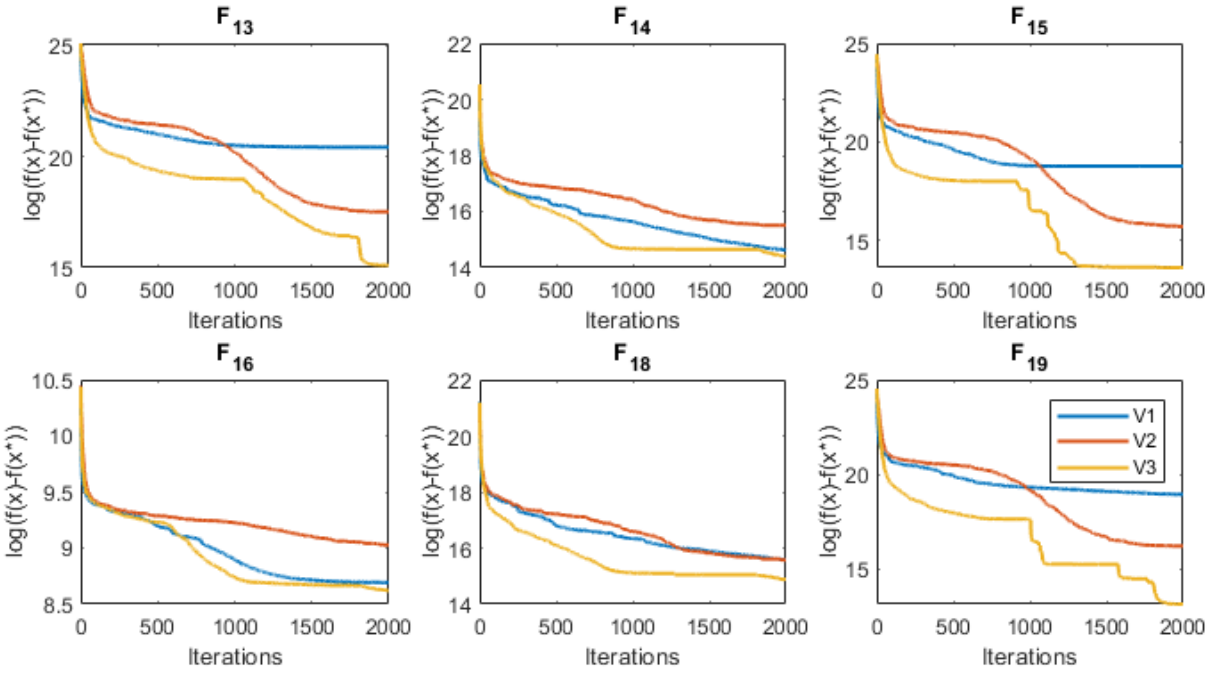


Figure 4-30- Convergence rate comparison of V1, V2 and V3 on the 100-dimensional CEC'17 test functions.

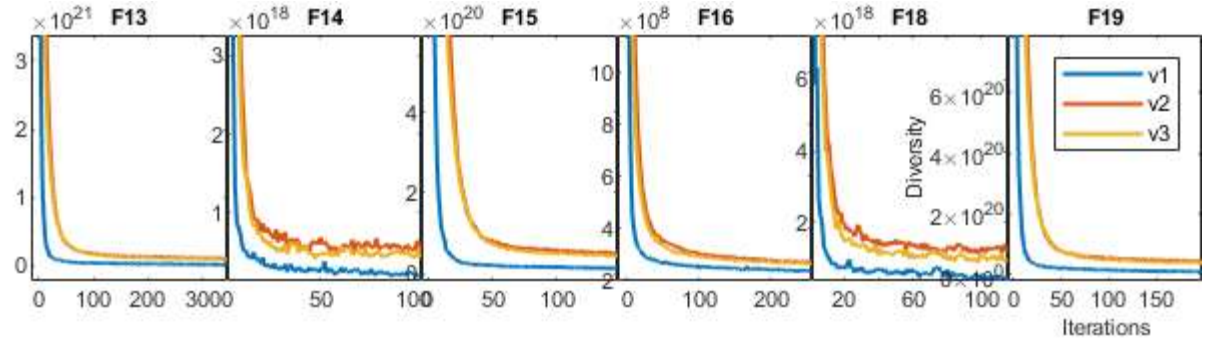


Figure 4-31- Population diversity comparison of V1, V2 and V3 on the 100-dimensional CEC'17 test functions.

In summary, although with 30-dimensional functions, V1 exhibits comparable convergence behaviour, the exploration and exploitation capabilities of V3 are more distinct and reflected on the convergence rate and population diversity when problem dimensionality is increased. This confirms our initial hypothesis that combining both subpopulations enhance the swarm's search capabilities and significantly balances exploration and exploitation through homogeneous and heterogeneous subpopulations.

4.4.3 The Impact of the Communication Model

The previous preliminary analysis on the communication model indicated that it improves the overall performance of the HIDMS-PSO algorithm. However, the precise impact on the heterogeneous subpopulation in functions with varying complexities was unknown. In this section, the impact of the communication model is assessed by observing the mean convergence rates obtained from 30 consecutive runs on the 30, 50 and 100-dimensional CEC'17 functions.

The communication model has a precise role in the HIDMS-PSO algorithm to form an explicit

communication among particles to share positional information in the heterogeneous subpopulation after each positional update in a controlled way. Since particles in the heterogeneous subpopulation do not exploit the $gbest$ exemplar during the search, the communication model is the only form of nonarbitrary explicit communication among particles. Hence, in a way, the communication model substitutes the role of the $gbest$ exemplar in the heterogeneous subpopulation to enable particles in the heterogeneous subpopulation to converge. Figures 4-32:4-34 exhibit the average convergence rates obtained from 30 consecutive runs on the CEC'17 test suite at 30, 50 and 100 dimensions, respectively. The figures show that the convergence rates for with and without the communication model reveal similar convergence patterns. However, a closer inspection reveals that a faster convergence is attained within the initial 100-500 iterations when the communication model is enabled, followed by saturation of both versions of the algorithm in most cases. It is worth noting that the version with the communication model converged to a better final solution for 23 of the 30 functions, at 50 dimensions 20 functions and at 100 dimensions 22 of the 30 functions. That essentially justifies HIDMS-PSO achieving a better overall performance when the communication model is enabled. Hence, despite exhibiting a comparable convergence pattern, the communication model's presence resulted in a faster convergence rate within the initial iterations of the search to a better solution in most functions with different complexities.

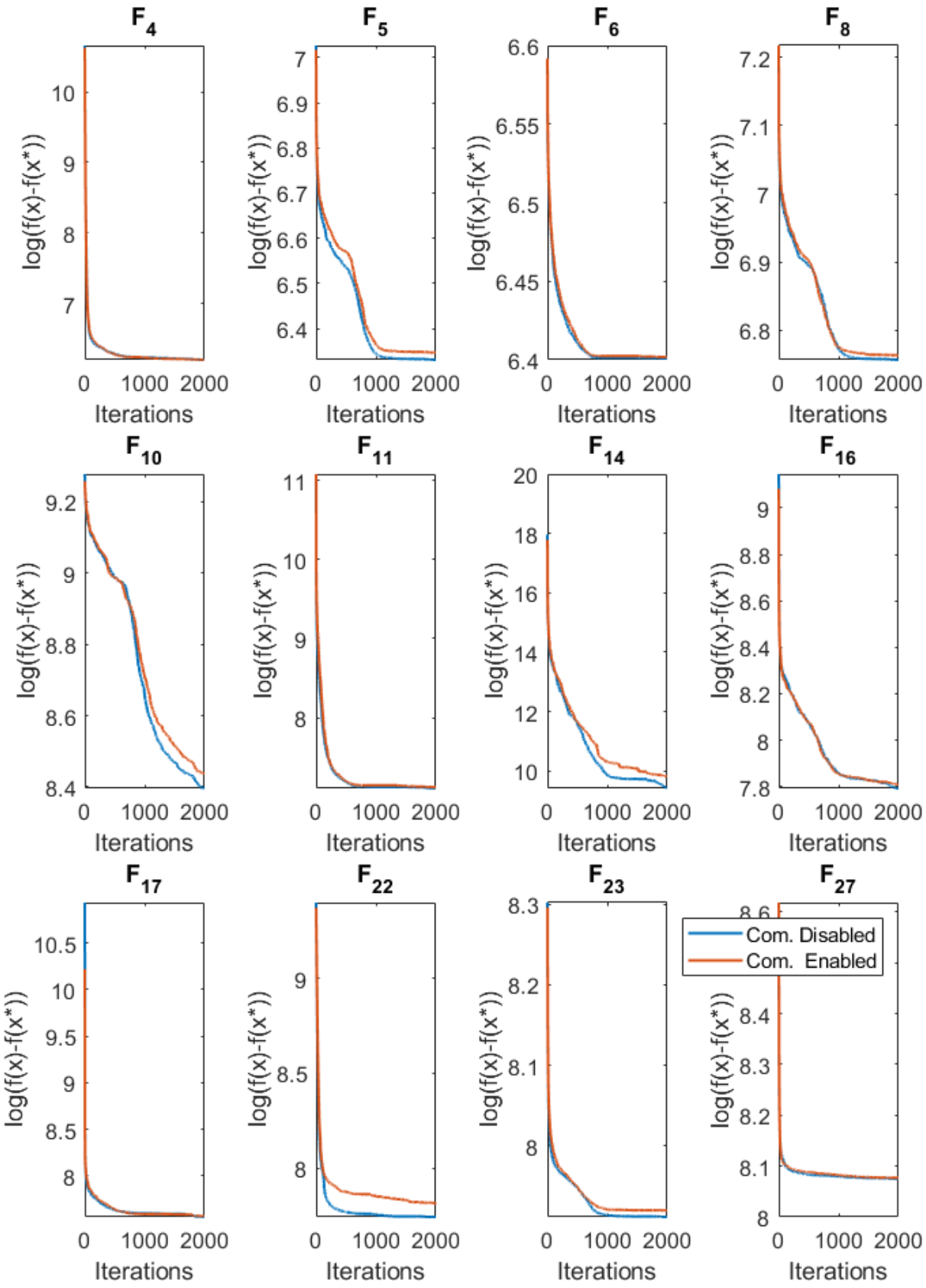


Figure 4-32 Impact of communication model on the convergence for 30-dimensional CEC'17 functions.

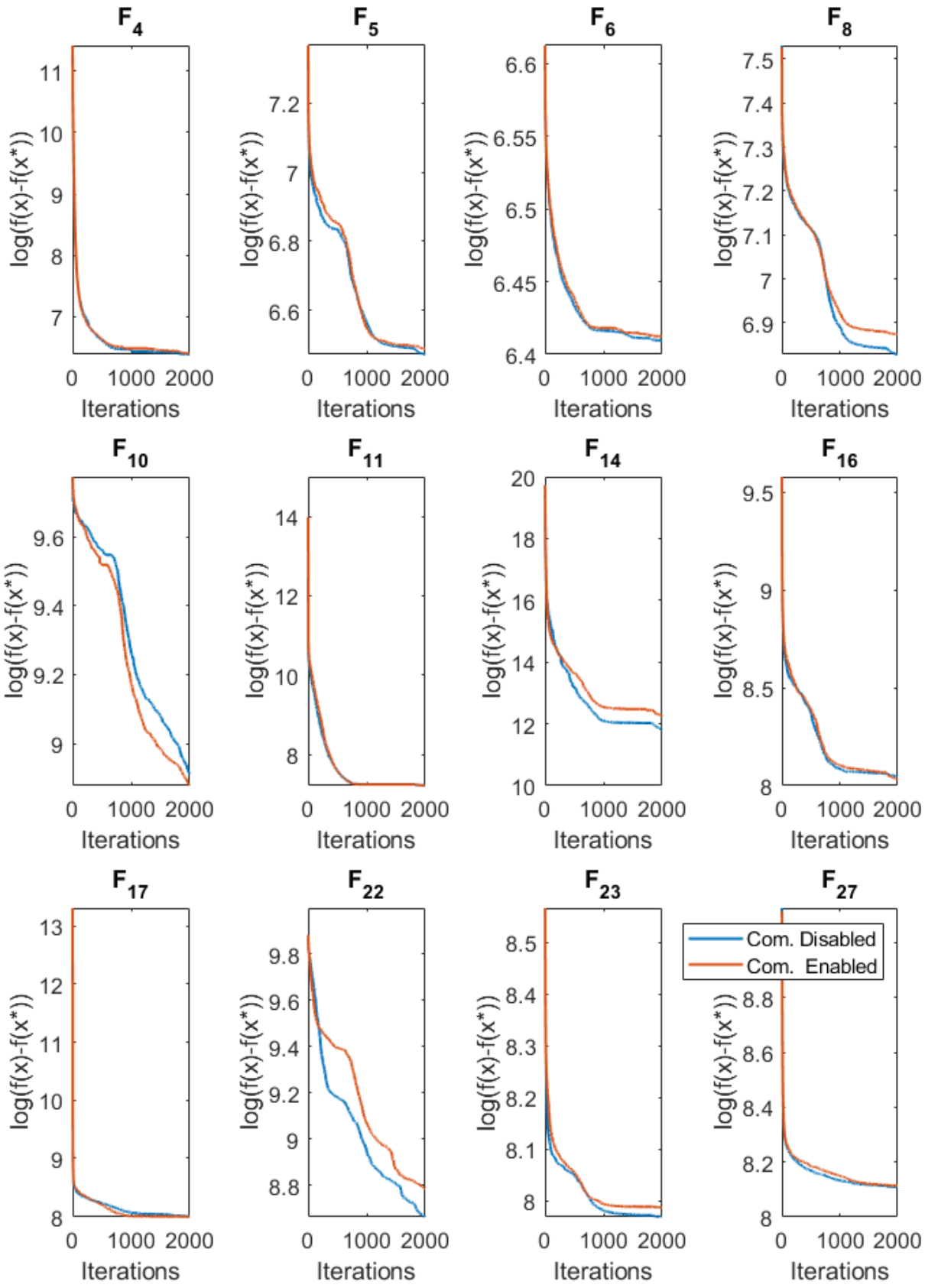


Figure 4-33-Impact of communication model on the convergence for 50-dimensional CEC'17 functions.

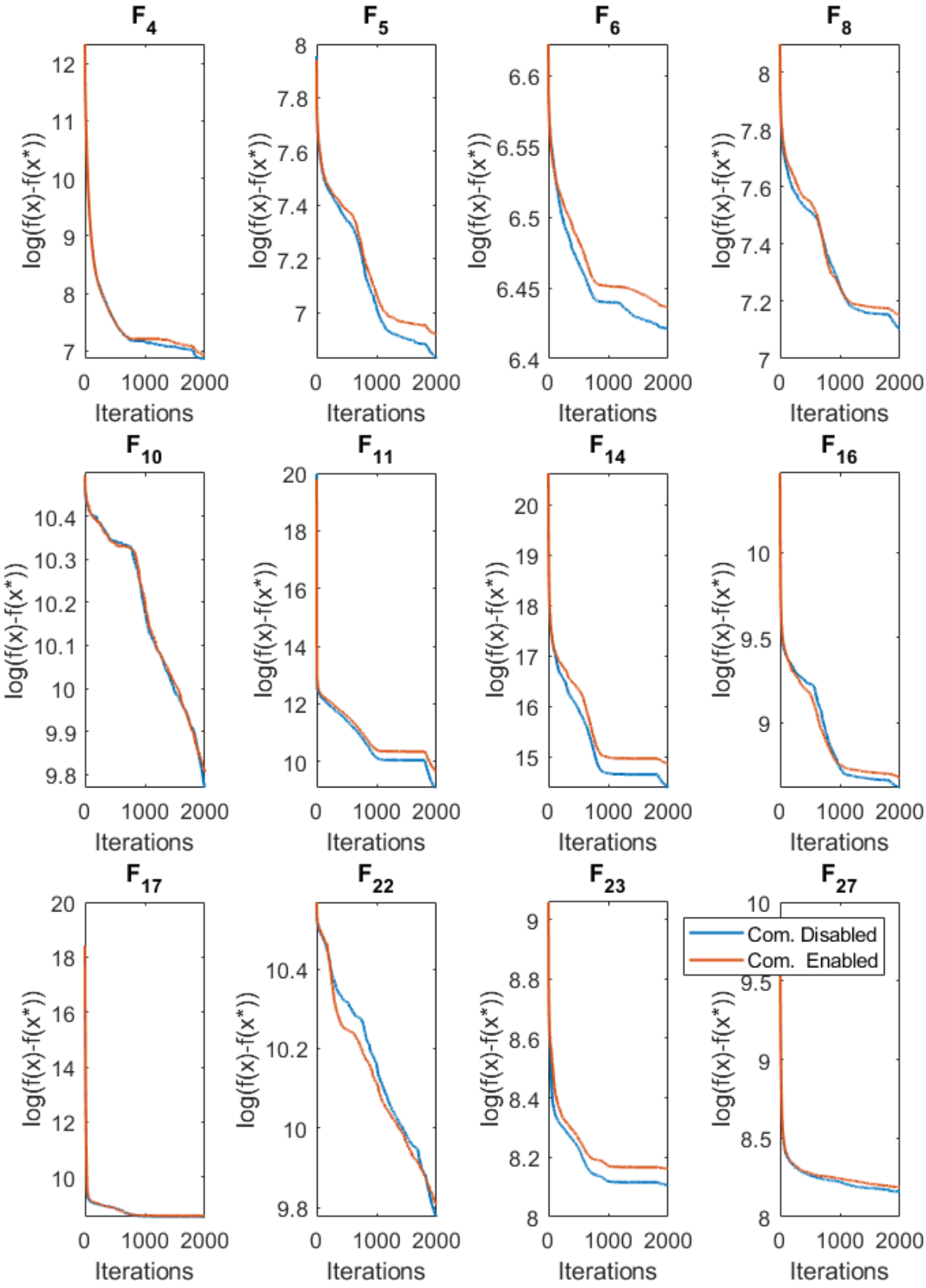


Figure 4-34-Impact of communication model on the convergence for 100-dimensional CEC'17 functions.

4.4.4 The Impact of the Learning Strategies

In this section, we assess the impact of the inward-oriented and outward-oriented learning strategies (see chapter 4.2) on the convergence behaviour of particles. Figures 4-35:4-37 show the convergence rates for the 30, 50 and 100-dimensional CEC'17 functions. For the 30 dimensional functions, the inward-oriented movement strategy generally exhibited a faster convergence within the initial 100-500 iterations and converged to a better solution in 20 of the 30 functions. At 50 dimensions, a different pattern emerges, and we observe a more competitive performance in multimodal functions. The inward-oriented learning strategy exhibits a conspicuously better performance in all hybrid functions by converging to a better solution compared to the outward-oriented learning, usually with a faster initial convergence rate. However, we observe an opposite pattern in the composition functions where the outward-oriented learning exhibits a faster convergence rate in 6 of the 9 functions, to a better mean final solution. Lastly, convergence rates obtained for the 100-dimensional functions reveal a significant deterioration in the average convergence of the inward-oriented learning strategy. On the contrary, the version with the outward-oriented learning strategy exhibits a significantly faster convergence rate to a better final mean solution in 19 of the 30 problems. The increased problem dimensionality reveals a pattern in the type of problems each strategy can handle. For example, we can observe that the outward-oriented learning strategy is capable of converging faster to a much better solution for all multimodal and composition functions at 100 dimensions, whereas for hybrid functions, the inward-oriented learning exhibits a slightly better convergence rate compared to the outward-oriented strategy. Hence, we could conclude that the inward-oriented learning can compete with multimodal, hybrid and composition functions generally in lower-dimensional search space due to the nature of the exemplars in this strategy primarily focusing on the local best solutions; and with lack of appropriate exploration, all units can easily focus on the same solution or different local solutions in close proximity. On the contrary, the outward-oriented learning strategy exhibited inferior performance compared to the inward-oriented strategy in lower-dimensional functions due to lack of satisfactory exploitation capabilities however a superior convergence performance is observed in higher-dimensional search space, especially in multimodal and composition functions, which on the contrary is an indication of good exploration capabilities. Considering the exemplars that influence the movement of agents in either learning strategy, the empirical evidence obtained here justifies the performance of each strategy and combined together, both learning strategies provide agents with the required exploration and exploitation capabilities required for an efficient search.

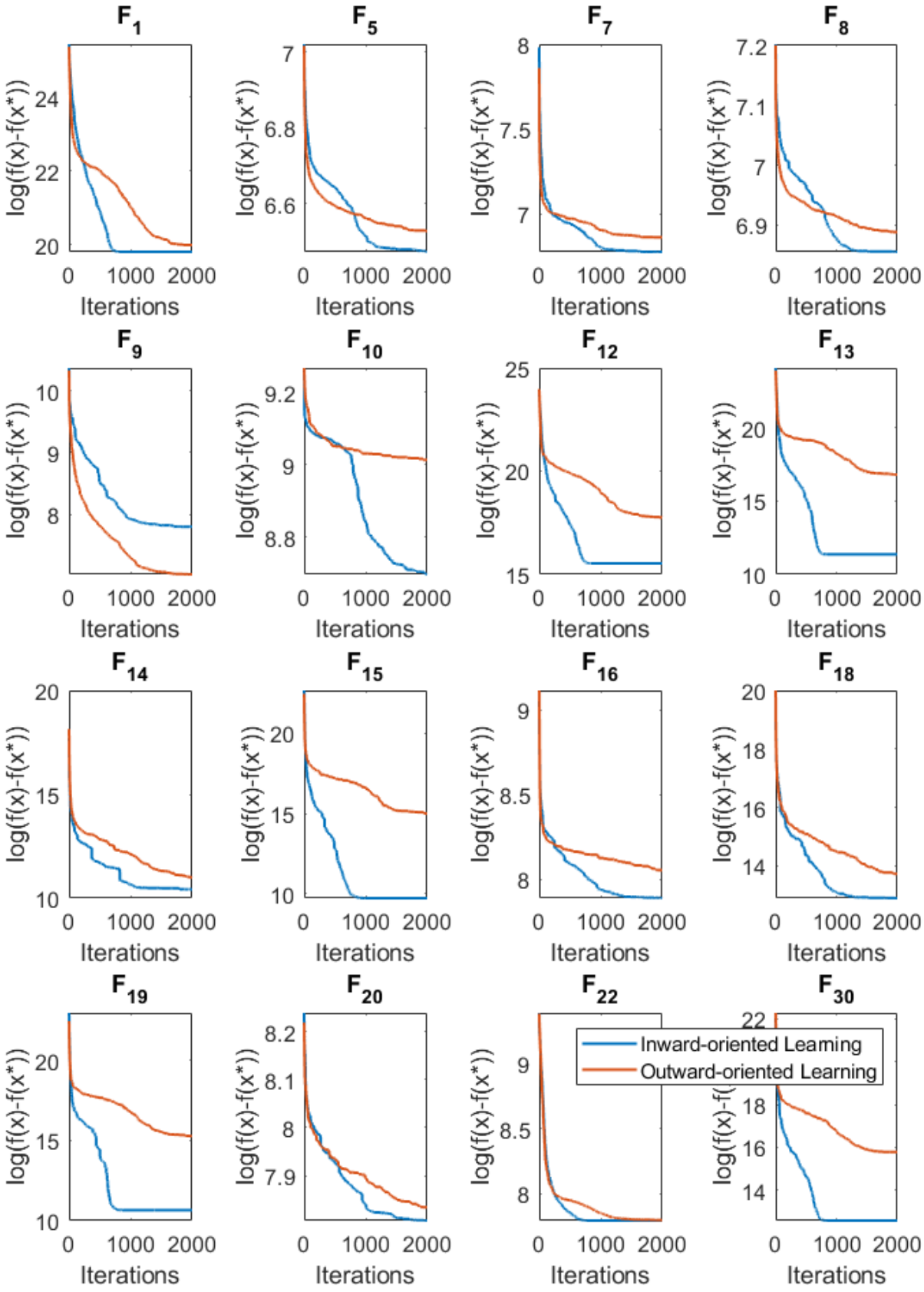


Figure 4-35-Impact of the learning strategies on convergence for 30-dimensional CEC'17 functions.

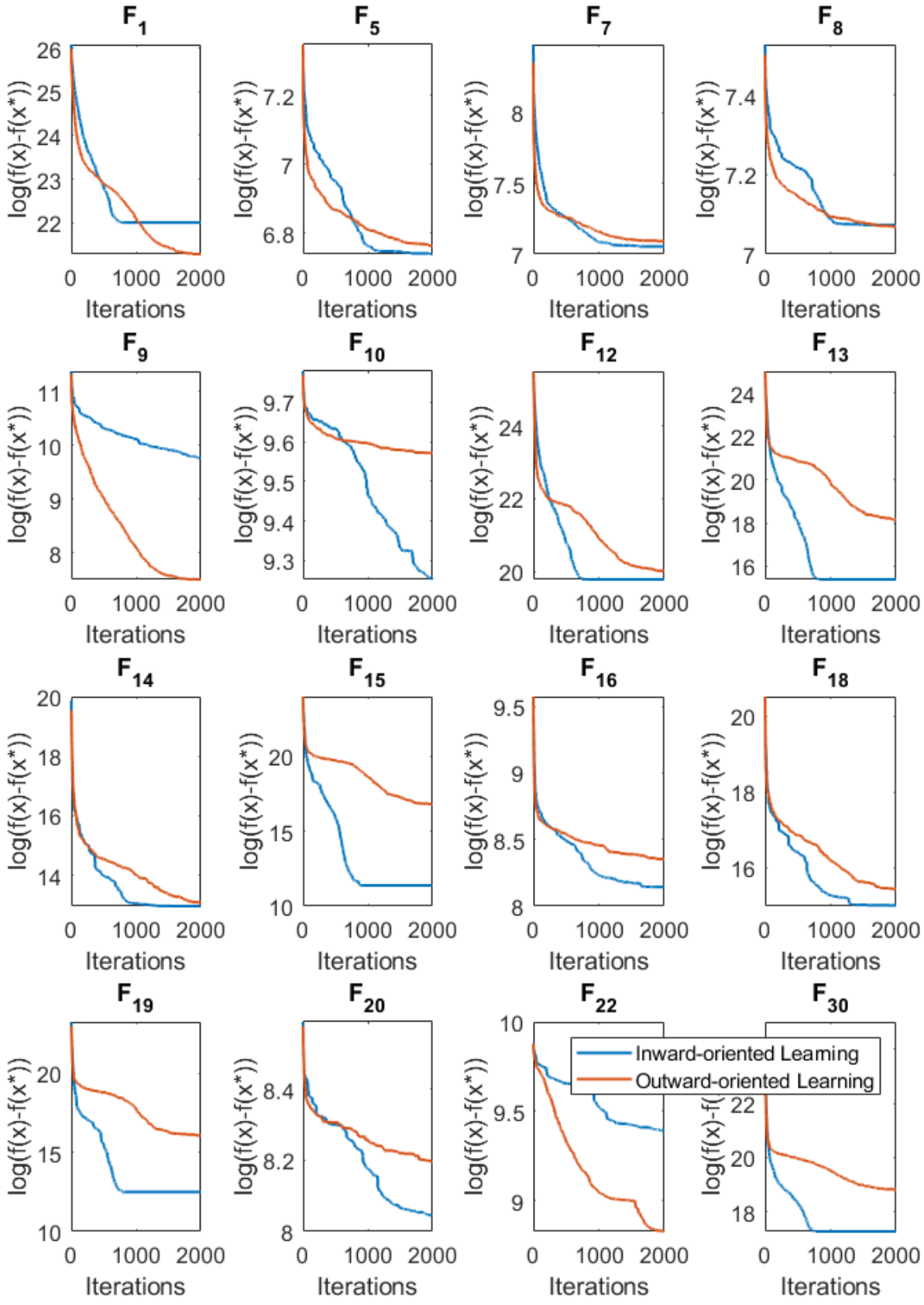


Figure 4-36- Impact of the learning strategies on convergence for 50-dimensional CEC'17 functions.

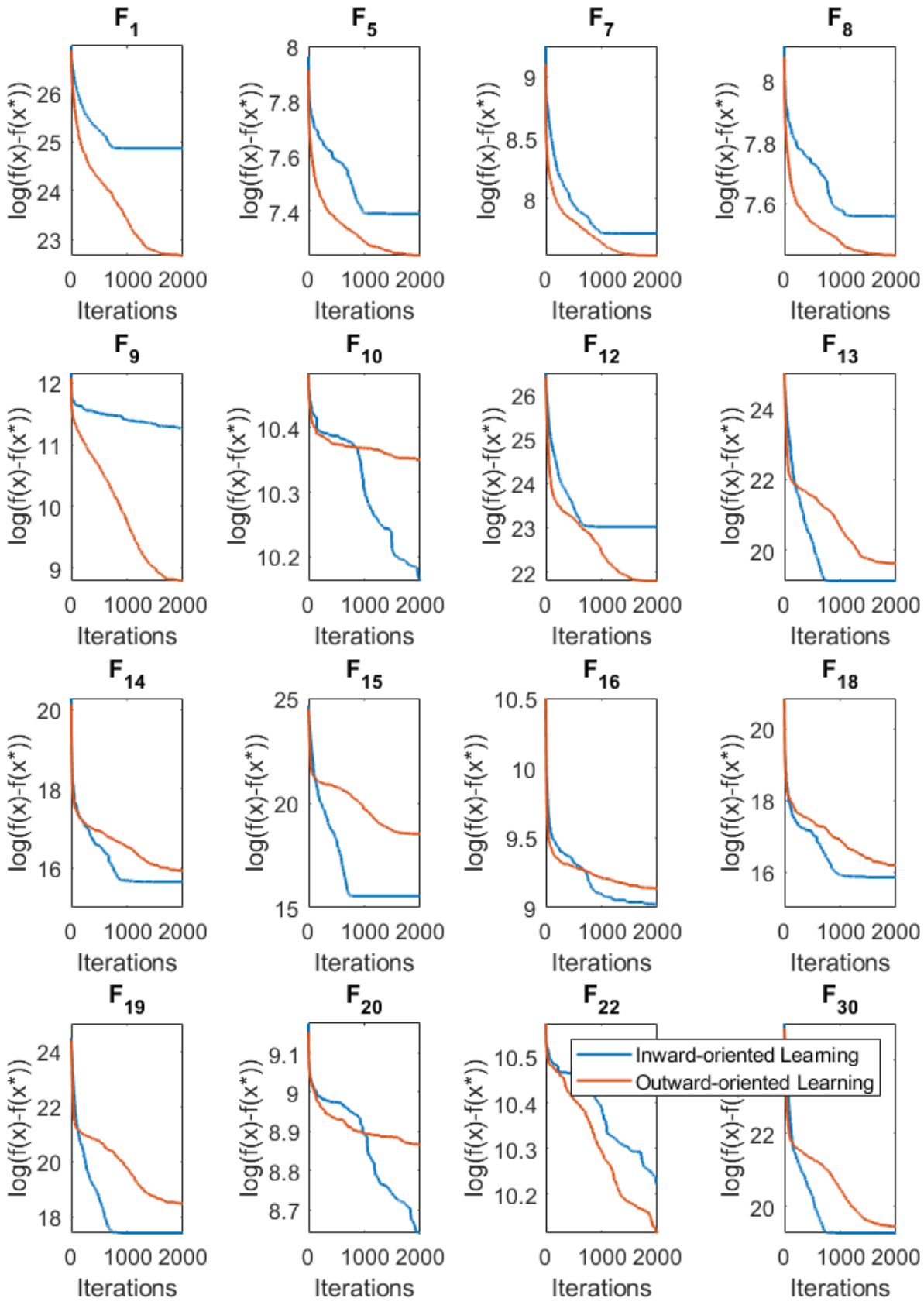


Figure 4-37- Impact of the learning strategies on convergence for 100-dimensional CEC'17 functions.

Moreover, Figure 4-38:4-39 (fitness trend of agents using the inward-oriented and outward-oriented learning for multimodal F5-F9 functions) exhibit the fitness trend for both inward-oriented and

outward-oriented learning strategies. We observe the pattern of a randomly selected particle's **pbest** against the **gbest** over a single run. We can observe from both figures that the fitness trend of an agent employing only inward-oriented learning exhibits minimal exploration and follows a comparable fitness trend to **gbest**. For the same multimodal functions, the outward-oriented learning guided agent exhibits a highly explorative behaviour and fails to converge to **gbest** even at the end of the iterations. Both figures further support the previously attained evidence regarding the exploitation capabilities of the inward-oriented movement and exploration capabilities of the outward-oriented movement strategy.

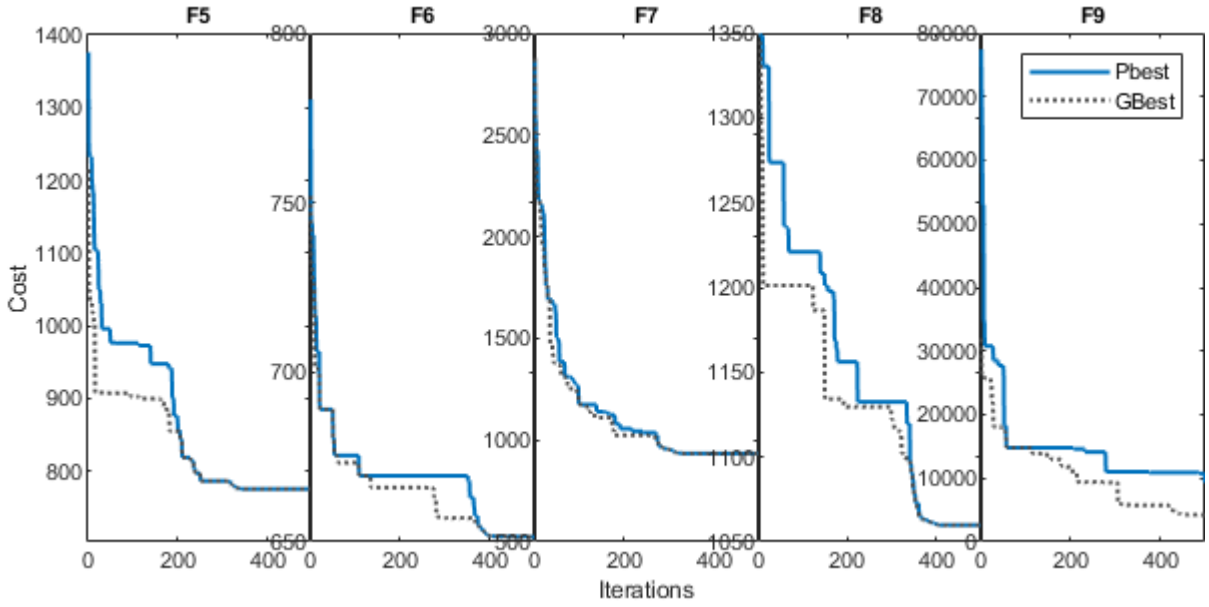


Figure 4-38- $pbest$ Fitness trend of a random particle using only inward-oriented learning on the multimodal functions.

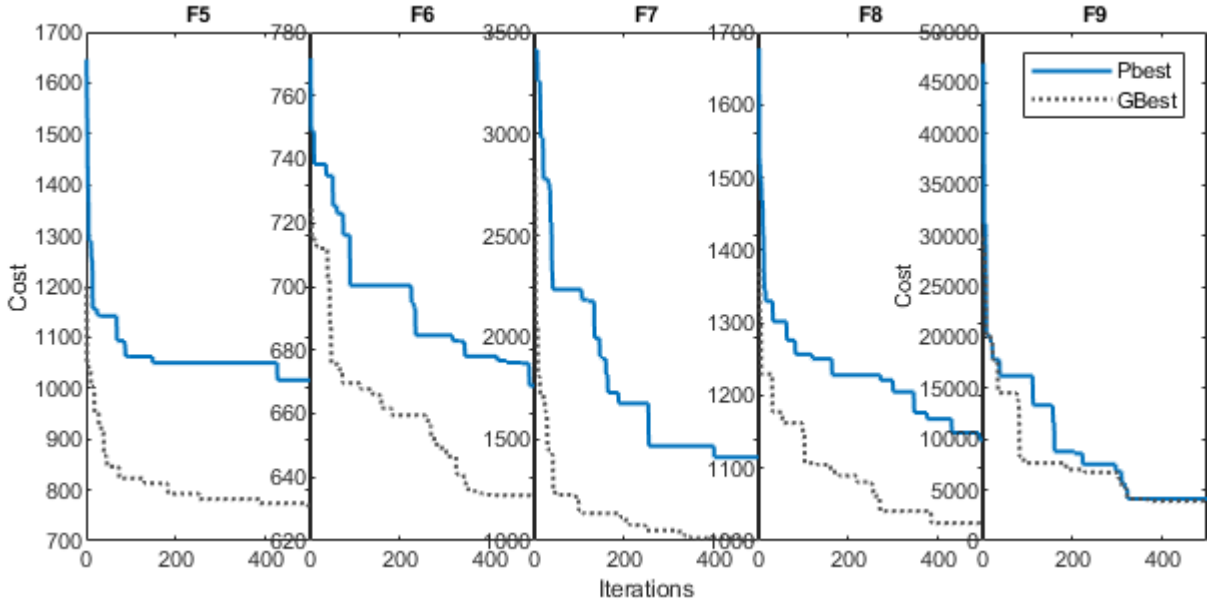


Figure 4-39- $pbest$ fitness trend of a random particle using only outward-oriented learning on the multimodal functions.

4.4.5 The Impact of the Unit Reshuffling Mechanism

During the search, particles in each unit periodically switch places with another particle of the same type. This is referred to as “unit reshuffle” and it could have a significant impact on units’ best known solution, population diversity, hence, the algorithm's convergence. In this section, we assess the impact of the reshuffle mechanism on the convergence behaviour of the HIDMS-PSO algorithm using various fixed periods (10,50,100,250,500,1000 iterations) as the interval to trigger the reshuffling mechanism. In addition, we test the HIDMS-PSO algorithm without the reshuffle mechanism and with a linearly decreasing (1000→10) reshuffling period.

Figure 4-40 shows the convergence rates for 30-dimensional CEC’17 functions. No conclusive evidence was attained from the convergence behaviour to determine a single best value for the reshuffle period. However, for multimodal functions, α value between 100-500, for hybrid functions α value between 10-50 and for composition functions α value between 10-250 yields a better convergence rate on the CEC’17 test functions. Since we are unable to distinguish a single value for α through convergence rates, as shown in Figure 4-41, we observe the quality of the final solution, which yields $\alpha = 10$ leads to convergence to a better final solution compared to the other tested parameter values on the 30, 50 and 100-dimensional CEC’17 functions. Hence in general, $\alpha = 10$ can be used; however, when faster convergence is a priority instead of the final quality of the solution, particular values stated above can be experimented to achieve the desired results.

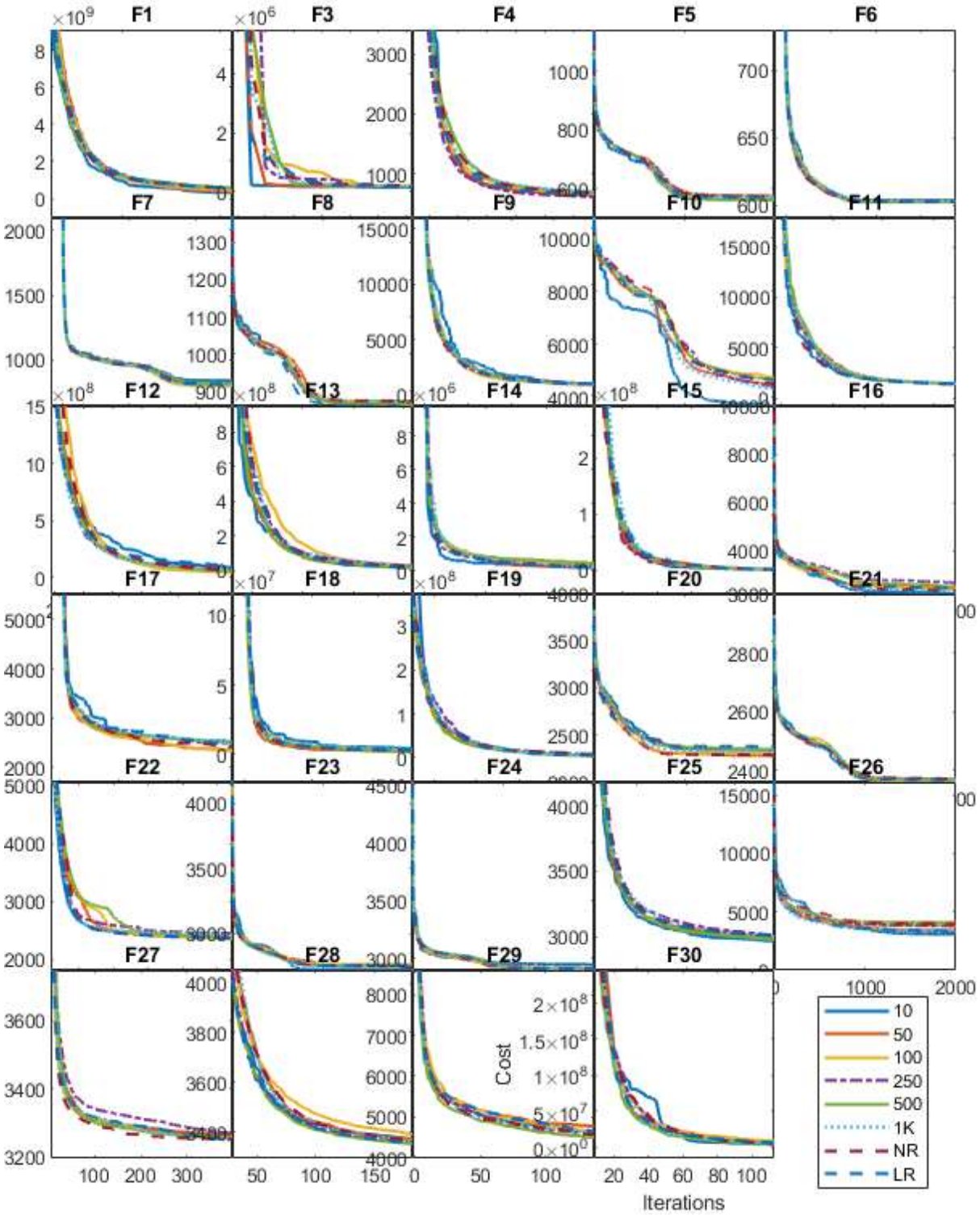


Figure 4-40-Impact of the reshuffle mechanism on the convergence of the HIDMS-PSO algorithm for the 30-dimensional CEC'17 functions (NR=no reshuffle and LR=linearly decreasing reshuffle period).

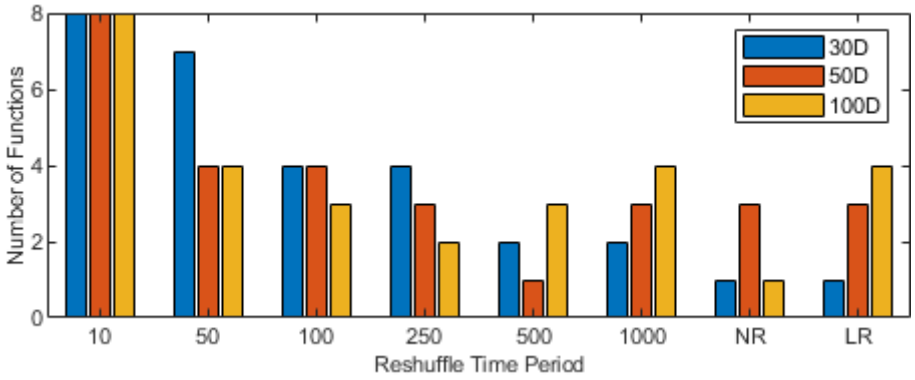


Figure 4-41- Number of best mean final converged solutions at 30,50, and 100 dimensions using various reshuffle periods (NR=no reshuffle and LR=linearly decreasing reshuffle period).

4.5 Discussions

The performance of the HIDMS-PSO algorithm is verified over three test suites (see chapter 3 for details of the test suites), a total of 87 functions each at 4 different dimensions. The 100-dimensional versions of these functions comprise the most complex instances of the problems. Hence, we observe from the experimental results that on the 100-dimensional 87 functions, the proposed algorithm HIDMS-PSO provided the best mean solution for 30 of them among the 14 test algorithms. On the other hand, HIDMS-PSO exhibited the second-best mean performance on 11 functions. Moreover, the HIDMS-PSO algorithm solved more problems on 30 and 50 dimensional CEC'13 functions compared to all PSO variants employed and on 100 dimensions, HIDMS-PSO performed best on more functions than all algorithms including L-SHADE. Similarly, on 50 dimensional CEC'14 and CEC'17 test functions, HIDMS-PSO provided the best mean solutions for more functions compared to all PSO variants employed in the experiments and on the 100-dimensional functions of both test suites, once again, the proposed algorithm solved the majority of the problems compared to all 14 test algorithms.

As mentioned in previous sections, L-SHADE is the CEC'14 competition winner and performed very efficiently on the experiments conducted in this work hence it is a particularly challenging algorithm to compare against the HIDMS-PSO algorithm. The numerical comparison with L-SHADE reveals that L-SHADE performed best on 8, 7 and 6 problems on the 100-dimensional CEC'13, CEC'14 and CEC'17 test suites whereas in the same experiments, the proposed algorithm HIDMS-PSO solved 13, 7 and 10 problems, respectively. Further comparison with L-SHADE on different classes of functions experimented on different test suites show that L-SHADE solved 4 and HIDMS-PSO one unimodal functions. The comparison on the multimodal functions reveals that HIDMS-PSO solved 12 whilst L-SHADE performed best on 9 functions. The proposed HIDMS-PSO algorithm performed best on 3 hybrid functions whereas L-SHADE solved 6 of them and finally on the composition functions, HIDMS-PSO provided the best solution for 13 of them whereas L-SHADE solved a single composition function. As apparent from the comparison, although the proposed algorithm is unable to outperform L-SHADE on unimodal and hybrid functions, HIDMS-PSO can compete with L-SHADE on functions with diverse characteristics which exhibits the capacity of the proposed algorithm to handle wide range of problems. Further, the comparison with L-SHADE on multimodal and composition functions further highlight the capacity of HIDMS-PSO in handling multimodality and complexity.

In addition, the Wilcoxon signed ranked test verified that HIDMS-PSO's performance is statistically significantly better than the majority of the comparison algorithms, especially on the 50

and 100-dimensional functions, and, importantly, there are very few instances where any of the other algorithms are statistically better than the proposed algorithm. The convergence rate comparison further verified the convergence speed and efficiency of the discovered solution compared well to the top ranked algorithms.

4.6 Conclusion

This chapter presented the heterogeneous improved dynamic multi-swarm PSO algorithm. The HIDMS-PSO algorithm divides the population into two subpopulations, one homogeneous and one heterogenous. The homogeneous subpopulation employs the classical PSO velocity update equation whilst the heterogeneous subpopulation is used to form units with a novel master-slave topological structure. Further, particles in the heterogeneous subpopulation use two learning strategies, inward-oriented and outward-oriented learning, to determine their trajectories and the communication model constricts the flow of information among particles. These elements work together to improve performance by maintaining population diversity and reducing premature convergence. The performance of the HIDMS-PO was tested on 87 functions from three distinct test suites CEC'13, CEC'14 and CEC'17 on all available dimensions (10-d, 30-d, 50-d, 100-d).

Comprehensive comparative performance evaluation on multiple test suites shows that the proposed algorithm HIDMS-PSO outperformed the majority of the comparator algorithms in most cases. More specifically, the modality and dimensionality significantly increase the complexity of the problem under consideration, and it is common for algorithms with similar agent behaviour to lose population diversity in the search space and suffer premature convergence, especially when dealing with multimodal landscapes. Therefore, the better performance achieved by the HIDMS-PSO algorithm, particularly on more complex (e.g., multimodal and composition) and higher dimensional problems may be justified by the heterogeneous agent behaviour and the additional mechanisms incorporated to delay the depletion of diversity.

Hence, this verifies the effectiveness of the proposed algorithm and the corresponding components that comprise the HIDMS-PSO algorithm. In addition, individual components (*subpopulations, learning strategies, communication model and reshuffling mechanism*) of the HIDMS-PSO algorithm were assessed and the results revealed that, in summary, as well as the overall mean performance, both subpopulations significantly impacted the algorithm's convergence trend. The observations indicated that the homogeneous subpopulation converged faster on lower dimensional problems, especially when experimented with specific function types. However, this pattern changed with increased dimensions, as in this case, the heterogeneous subpopulation exhibits better mean performance and convergence rate. Hence, it is evident that dimensionality and different function types directly and distinctly impact the performance of both subpopulations. Similarly, the assessment of the two learning strategies adopted by particles revealed that both learning strategies could cope better with different situations. Although the difference in coping with various landscapes was expected for both strategies, the impact of the landscape had a mild effect on the convergence rate, and the primary weight appeared as the dimensionality of the problem under consideration. In summary, the inward-oriented learning strategy exhibited better overall mean performance and convergence rate on most 30-dimensional problems compared to outward-oriented learning. However, when the problem set is experimented with 50-D and 100-D problems, the outward-oriented learning strategy exhibited superior performance for the majority of the test problems. Besides the two major components (*subpopulations* and *learning strategies)* that significantly alter the HIDMS-PSO algorithm's performance, our findings on the communication model and the reshuffling mechanism indicate that these two components have less impact on the overall performance of the algorithm. More precisely, the presence of the communication model

resulted in faster convergence within the initial stages of the search and a better mean performance for the majority of the test problems at different dimensions. Similarly, for the reshuffling mechanisms, $\alpha = 10$ is suggested as the default value; however, different optimal values were identified for multimodal, hybrid and composition functions (see chapter 4.4.5 for details).

Chapter 5 : HIDMS-PSO with Bio-inspired Fission-Fusion Behaviour and a Quorum Decision Mechanism

5.1 Introduction

The vast majority of PSO variants proposed in the literature address the problem of premature convergence to improve the performance of the algorithm. The HIDMS-PSO algorithm, presented in Chapter 4, performs search using two fixed subpopulations, one homogeneous and one heterogeneous, and an explicit communication model to slow down the loss of population diversity. The intention of this chapter is to present a new variant of the HIDMS-PSO algorithm to further improve the depletion of population diversity by redesigning the unit structures in the standard HIDMS-PSO algorithm as self-organising social groups. Self-organisation is widespread in nature [229]–[231], occurring in species from bacteria colonies to humans. Eusocial organisms exhibit self-organising behaviour to resolve various issues in order to survive [232][233]. In this chapter, we introduce a mechanism composed of a bio-inspired fission-fusion behaviour and a quorum decision mechanism to form self-organised units capable of reacting to improve their diversity.

Fission-fusion behaviour involves social groups changing their formation over time through either splitting into smaller groups (fission) or merging with other groups (fusion). It is observed in many organisms, including social insects, birds, fish and even humans, as a form of fitness beneficiary mechanism in a social group or colony, used to maximise survival or reproduction, or to minimise the chances of becoming prey. One of the most sophisticated examples of the fission-fusion system is observed in African elephant societies, where a decision is made to fission into small groups of one-two individuals or fuse to form aggregations of tens or hundreds where group composition become a dynamic property. Splitting may occur in certain environments to minimise the risk from predators, or merging to exhibit defensive/deterrent behaviour towards or to protect a resource using the density of the population.

In our algorithmic model, fission-fusion behaviour is employed as a reactive mechanism through the creation of “diversity aware” units that exhibit fission-fusion behaviour when a unit's diversity exhibits a downward trend. Many colonies and social group based species, such as ants and honeybees, make group decisions, and in our behavioural model units make a group decision on when to adopt fission-fusion behaviour, based on a quorum response. As a result, these two incorporated mechanisms provide units with the ability to self-organise by reaching a decision and adopting a behaviour through consensus, which in turn significantly improves population diversity and the overall performance of the algorithm.

5.2 HIDMS-PSO with Bio-inspired Fission-Fusion Behaviour and a Quorum Decision Mechanism

The FFQ-HIDMS-PSO extends the HIDMS-PSO algorithm to address the aforementioned problem of premature convergence by redesigning the unit structures as bio-inspired self-organising social groups. Self-organisation is a paramount feature of all living systems, and the standard definition reads as “*a process in which pattern at the global level of a system emerges solely from numerous interactions among lower-level components of the system*” [234]. In nature, animals prevalently exhibit self-organising behaviour to survive, and a common example of this is observed in fission-fusion societies.

The FFQ-HIDMS-PSO algorithm incorporates a bio-inspired fission-fusion system and a quorum decision mechanism to form self-organised units capable of managing their population diversity. These two mechanisms allow units to be "diversity aware" and to split up into smaller segments or merge with other units to form a single large unit when their population diversity exhibits a downward trend to minimise the loss of diversity in the overall population. Each unit individually assesses their diversity and decide to adopt fission or fusion behaviour using the quorum decision mechanism.

The search process of the FFQ-HIDMS-PSO algorithm initiates with a single population of n units, where the units are as in the HIDMS-PSO algorithm. The diversity of each unit, denoted by δ_k, is calculated at every $\frac{FF_{period}}{10}$ iterations, where FF_{period} is 1-10% of the total number of iterations, and it is the maximum period for which a unit can adopt the fission or fusion behaviour. After the expiration of the FF_{period}, fissioned and fused particles randomly reform as standard units to resume the search. The δ_k^t is calculated using:

$$\delta_k^t = \sum_{j=1}^{N} MSE(\mathbf{x}_m, \mathbf{x}_j) \tag{5.1}$$

Where MSE is the mean squared error, $\mathbf{x}_m$ is the position of the master particle and $\mathbf{x}_j$ is the jth slave particle's position in unit k, and N is the number of slave particles.

The fission-fusion behaviour is initiated when the δ_k^t (delta of the k^{th} unit at time t) fall below the average δ across all units. The group decision to determine whether to adopt the fission or fusion behaviour is based on the quorum response of each member of the unit, denoted by QR_j, calculated as:

$$QR_j = \frac{MSE(\mathbf{x}_m, \mathbf{x}_j)}{1 + \left(\frac{\alpha}{\beta}\right)^{\frac{\gamma}{10}}} \tag{5.2}$$

Where QR_j is the quorum response of the jth slave member of the unit, $\mathbf{x}_m$, $\mathbf{x}_j$ are positions of the master and the j^{th} slave's positions, α is the number of fissioned/fused conspecific of the j^{th} slave particle, β is the total number of particles adopting the fission/fusion behaviour at time t and γ is an integer between 1 and 4, assigned as the rank of the j^{th} slave particle based on its fitness (4th rank given to the fittest slave particle). Eq. 5.2 is a modified version of the original equation introduced in [235] as a model for collective decision making in cockroaches. In summary, the calculation of QR_j enables each member of a unit to individually gather information. For example, the similarity between the master and the jth slave particle reduces the quorum response of the jth slave resulting in the particle facilitating minimal contribution to the final decision. Similarly, a higher value of γ and a greater number of particles that have already adopted the fission/fusion behaviour at time t increases the quorum response of the particle. This encourages particles to adopt the fission-fusion behaviour more strongly when there are a sufficient number of particles already employing the fission or fusion behaviour. In essence, the calculated quorum response is used to mimic how animal groups without a leader form a consensus to make a decision.

After the quorum response is calculated, the decision is finalised by counting the number of unit members with $QR_j > \overline{QR}$, where $\overline{QR}$ is the average quorum response for the unit and the following rules are employed to determine the final quorum decision:

- If more than 2 unit members have greater QR values than $\overline{QR}$, fission or fusion behaviour is adopted.
- If more than 2 unit members have lower QR values than $\overline{QR}$, fission or fusion is not adopted.
- If the number of unit members with QR values greater than and less than $\overline{QR}$ are the same, or if all QR values of unit members are the same, we use the QR value of the unit master in place of $\overline{QR}$ and proceed according to the first two rules.

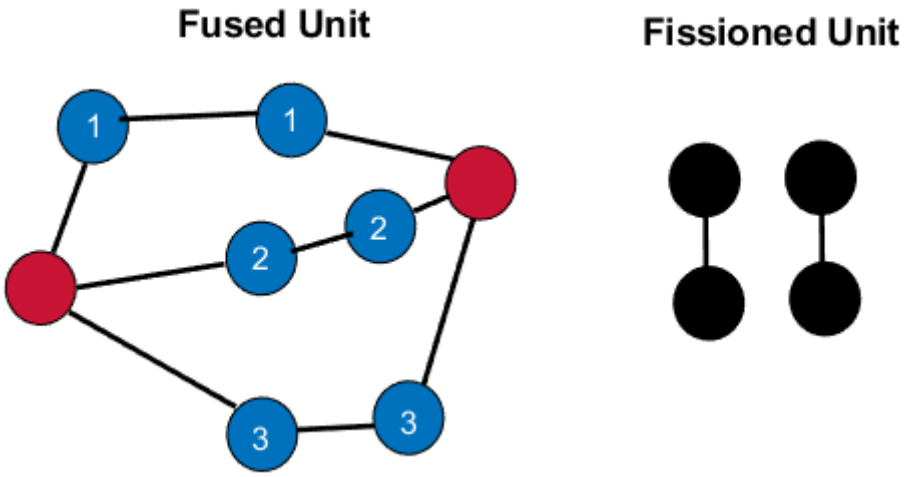


Figure 5-1-Visual depictions of fused units and fissioned units.

As shown in Figure 5-1, units that adopt fission behaviour are split into two sub-units, and as opposed to the unit structure shown in Figure 4-1, these particles are divested of their master-slave roles. On the contrary, in fused units, particles maintain their assigned roles, and the fusion behaviour enables particles to influence each other at an individual level, enabling a revamped population diversity to emerge at a higher level. The fissioned particles employ the following equation to update their velocities:

$$\mathbf{v}_i^{t+1} = w^t \mathbf{v}_i^t + c_1^t \mathbf{r}_1 \left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t \mathbf{r}_2 \left(\phi - \mathbf{x}_i^t\right) \quad (5.3)$$

Where ϕ is the position of a particle randomly selected from: the most diverse fissioned sub-unit (diversity of a sub-unit can be calculated using the MSE function), a randomly selected fissioned sub-unit, the first fissioned, or the last fissioned sub-units. These randomly selected four exemplars broaden the movement capabilities of fissioned particles to regain their population diversity within a short period of time, reforming into more diverse units later to continue the search.

On the contrary, the particles that adopt the fusion behaviour employ the following velocity update equation:

$$\mathbf{v}_i^{t+1} = w^t \mathbf{v}_i^t + c_1^t \mathbf{r}_1 \left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t \mathbf{r}_2 \left(\phi - \mathbf{x}_i^t\right) \quad (5.4)$$

Where the exemplar ϕ is randomly chosen as either the position of a conspecific particle randomly selected from across all fused units, or the most dissimilar particle relative to any

randomly selected master within the fused unit the particle belongs to.

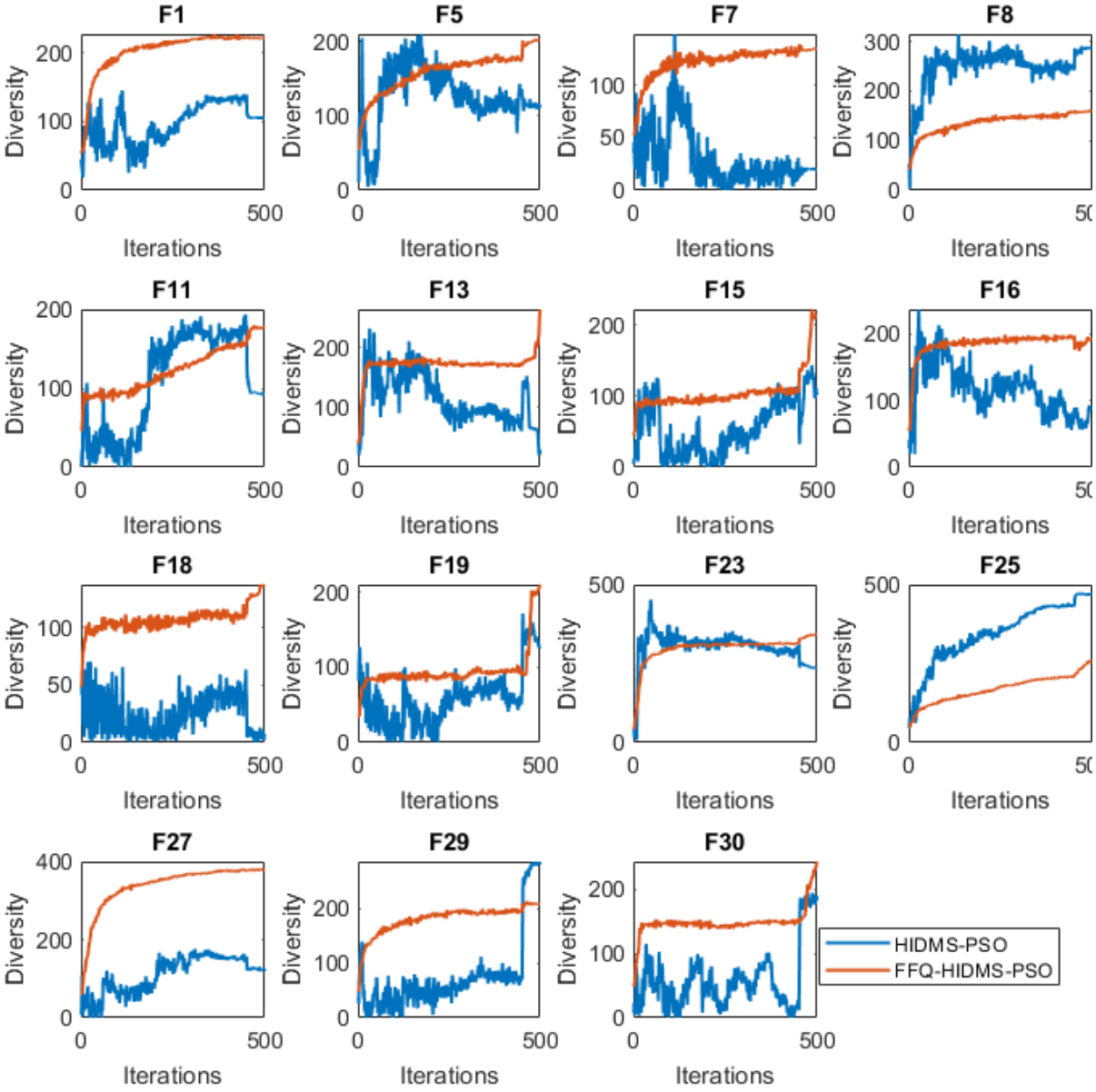


Figure 5-2 Comparison of population diversity between the HIDMS-PSO and FFQ-HIDMS-PSO algorithms on representative 30-dimensional CEC'17 problems

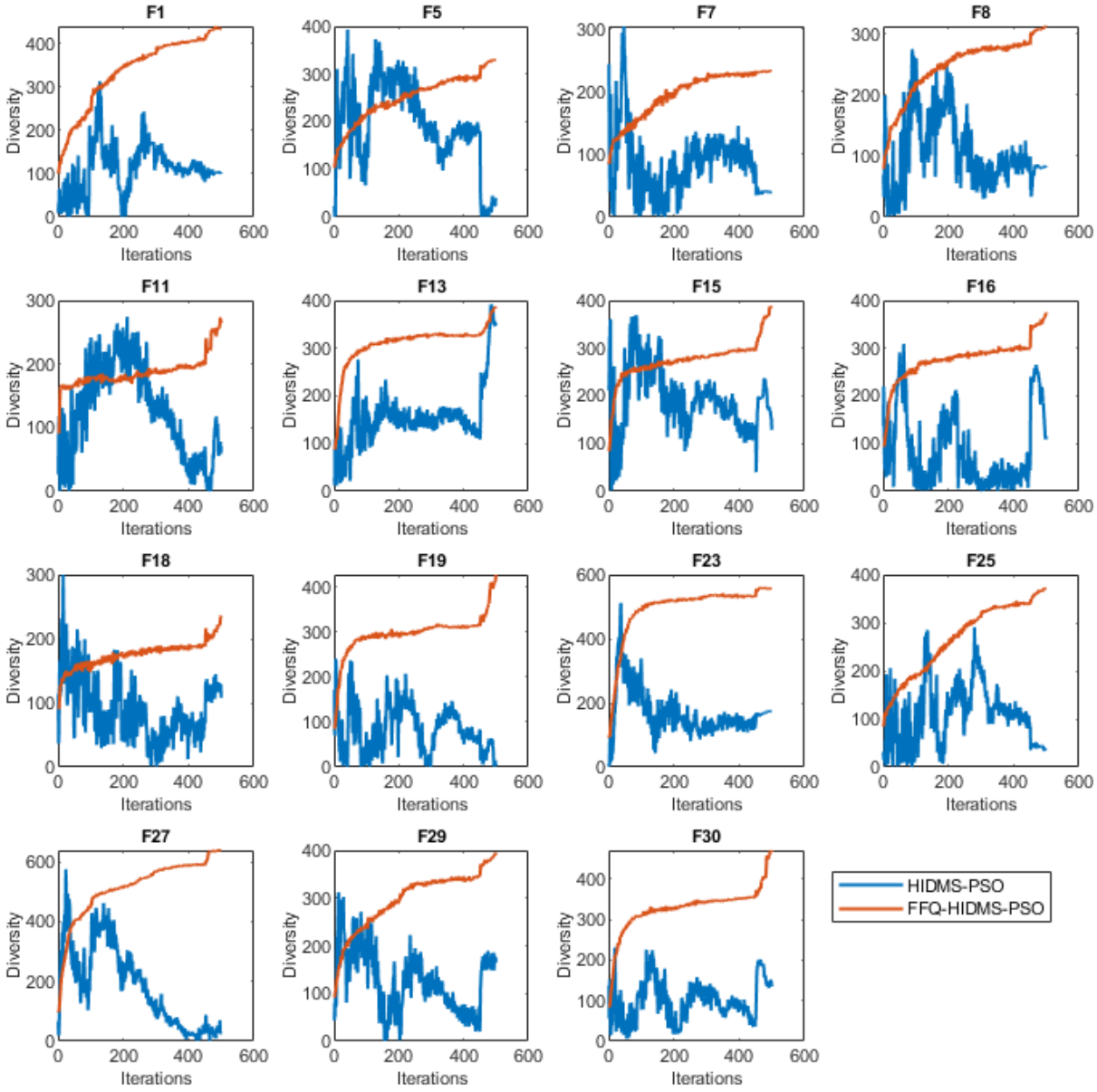


Figure 5-3 Comparison of population diversity between the HIDMS-PSO and FFQ-HIDMS-PSO algorithms on representative 100-dimensional CEC'17 problems

As apparent from Figure 5-2:5-3, by combining fission and fusion behaviour and a quorum decision mechanism allows units to monitor their own population diversity and react accordingly through consensus. As a result, when these particles reform units to re-join the heterogeneous subpopulation to continue the search, the overall population diversity is boosted or at least the loss of population diversity in the population is systematically slowed down to enable particles to continue moving in the search space to avoid particles becoming stuck in local optima.

Algorithm 2: FFQ-HIDMS-PSO

Input: n as swam size, T_{max} as maximum number of iterations

Output: $gbest$

1 c_1=2.5 − (1:T_{max}*2/T_{max}); c_2=0.5 − (1:T_{max}*2/T_{max}); $w_{max} = 0.99; w_{min} = 0.2;$

2 $$w_1 = \frac{w_{max} + (w_{min} - w_{max})}{1 + exp\left(-5\left(\frac{2t}{T_{max}} - 1\right)\right)}$$

```
for t=1:T_max
    if time to reshuffle then
        Reshuffle units
    end
    if mod(t, FF_period) == 0 then
        Randomly reform new units from fissioned and fused particles
    end
    if mod(t, round(FF_period / 10)) == 0 then
        Calculate δ for all units using Eq. 5.1
        if any unit's δ < δ̄ then
            Randomly select fission or fusion behaviour
            Calculate QR for each unit member using Eq. 5.2
            Use the rules in Chapter 5.1 to determine the final decision
        end
    end
    for i=1:n
        if f(x_i) ≥ f(x̄) then
            w=w_1^t+C; if w>0.99, w=0.99, end
        else
            w=w_1^t+C; if w>0.20, w=0.20, end
        end
        if i^th particle belongs to a unit that is not fissioned/fused then
            if i^th particle is master then
                Update v_i using either Eq. 4.1 or 4.2-4.4 and x_i using Eq. 2.5
            elseif i^th particle is slave then
                Update v_i using Eq. 4.5 and x_i using Eq. 2.5
            end
        elseif particle belongs to a fissioned sub − pop then
            Update v_i using Eq. 5.3 and x_i using Eq. 2.5
        elseif particle belongs to a fused sub − pop then
            Update v_i using Eq. 5.4 and x_i using Eq. 2.5
        end
        Evaluate the fitness of x_i
        Update pbest_i and gbest
    end
end
```

Table 5-1 display the mean and final ranks obtained on 10-d, 30-d, 50-d and 100-d CC'17 problems using different FF_{period} values. The results indicate that $FF_{period} = 100$ exhibited better mean overall performance compared other tested parameter values. It is worth noting that the values employed in this experiment were narrowed down to the most feasible values based on the previous experiments conducted on related research. Considering the results, $FF_{period} = 100$ is used in the main experiments presented in the following sections of this thesis. For thorough assessment of all parameters utilised by the standard HIDMS-PSO algorithm (which are also used in FFQ-HIDMS-PSO), refer to section 4.4.

Table 5-1 Mean and final ranks obtained on various dimensions of the CEC'17 problems using different FF_{period} parameter values

FF_{period}	10-d		30-D		50-D		100-D	
$FF_{period} = 50$	3.40	4	3.40	4	2.86	2	2.93	2
$FF_{period} = 100$	3.10	1	2.76	1	2.66	1	2.76	1
$FF_{period} = 200$	3.86	6	3.16	2	3.56	3	3.56	3
$FF_{period} = 300$	3.33	2	3.66	5	3.63	4	3.83	6
$FF_{period} = 400$	3.36	3	3.33	3	4.03	6	3.76	5
$FF_{period} = 500$	3.43	5	4.16	6	3.73	5	3.63	4

5.3 Experiments

5.3.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed FFQ-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

5.3.1.1 Results on 10 dimensional problems

The results obtained on the 10-dimensional CEC'13 problems indicate that L-SHADE tends to perform much more efficiently compared to all test algorithms on the low dimensional functions and the results show that the five 10-dimensional unimodal functions (F1~F5) are solved by L-SHADE in addition to the majority of the multimodal functions including F8, F10,F11,F12,F13,F14,F16,F17 and F18. DMS-PSO performed best on F6, F9, F15 and the best solution for F7, 19 and F20 is attained by FFQ-HIDMS-PSO, MiPSO and HCLPSO, respectively. The results on the composition functions (F21~F28) indicate that L-SHADE achieved the best performance on F22 and F27 whilst rest of the composition functions are solved individually by UPSO(F21), DMS-PSO (F23), EPSO (F24), MaPSO (F25), FFQ-HIDMS-PSO (F26), and LPSO (F28).

As shown in Figure 5-4, a significant portion of the 10-dimensional problems are solved by L-SHADE while the proposed algorithm FFQ-HIDMS-PSO exhibited the best performance on 2 functions. However, when the proposed algorithm's performance is compared solely with the PSO variants, as displayed in Figure 5-5, it can be observed that the proposed algorithm FFQ-HIDMS-PSO solved more problems compared to all PSO variants with the exception of DMS-PSO.

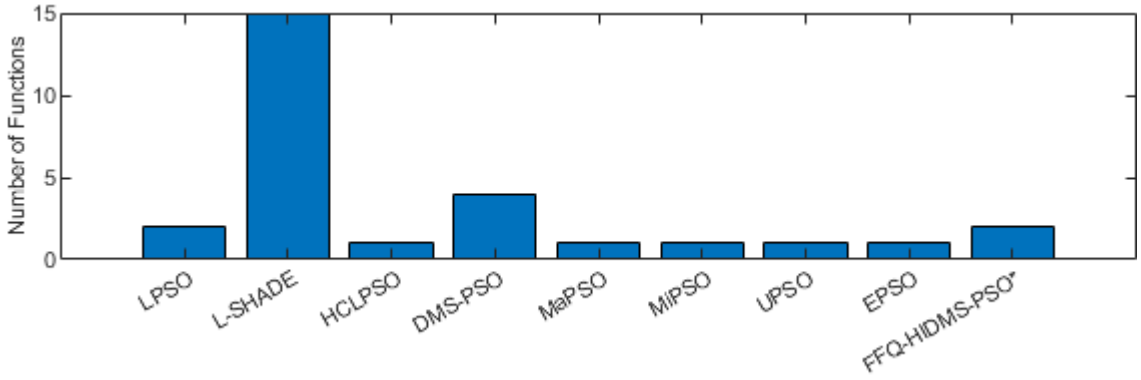


Figure 5-4 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

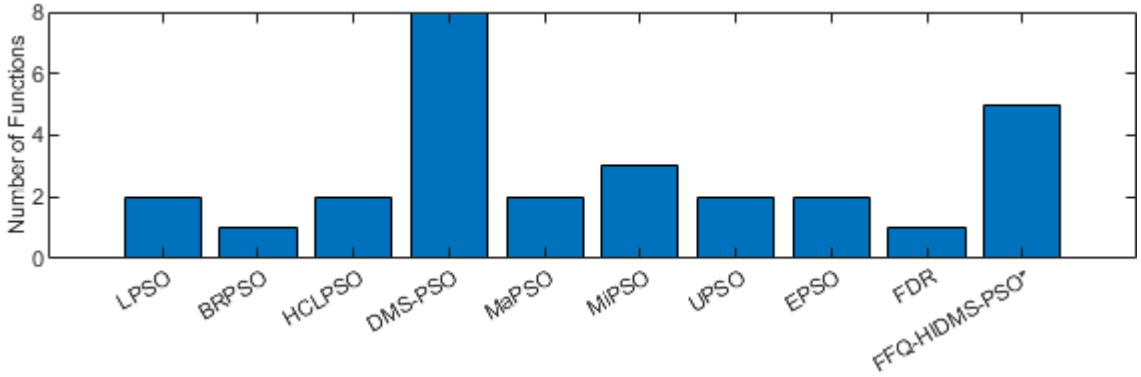


Figure 5-5 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'13 problems

5.3.1.2 Results on 30 dimensional problems

The experimental results obtained on the 30-dimensional CEC'13 problems indicate that on the unimodal functions (F1~F5) indicate that the best solutions for the 5 unimodal functions are achieved by DMS-PSO, L-SHADE and EPSO. The DMS-PSO algorithm solved F1, F5, L-SHADE performed best on F2, F3 and EPSO achieved the lowest mean error value on F4. For multimodal functions (F6~F20), the results indicate that 12 of the 15 multimodal functions are solved by L-SHADE including F6, F8, F10, F11, F12, F13, F14, F15, F16, F17, F18, F19 whilst FFQ-HIDMS-PSO performed best on F7, F20 and DMS-PSO provided the best solution for F9. For the 8 30-dimensional composition functions (F21~F28), the proposed algorithm FFQ-HIDMS-PSO and DMS-PSO solved 2 functions individually where FFQ-HIDMS-PSO performed best on F24, F27 and DMS-PSO on F23, F25. The best performance for F21, F22, F26 and F28 is achieved by LPSO, L-SHADE, MiPSO and EPSO, respectively.

We can observe from Figure 5-6 that the 28 30-dimensional CEC'13 functions are solved by 6 of the 15 algorithms employed in the experiments. In this experiment, the proposed algorithm FFQ-HIDMS-PSO exhibited better performance on functions compared to 12 of the 13 PSO variants, except DMS-PSO. However, as shown in Figure 5-7, the sole comparison of PSO variants (excluding L-SHADE) reveal that the proposed algorithm FFQ-HIDMS-PSO exhibited an equal performance to EPSO and DMS-PSO in the number of functions solved.

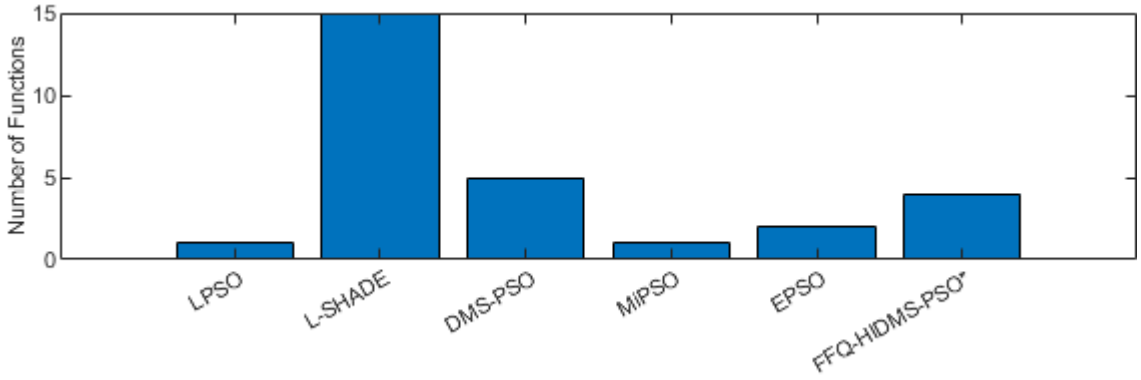


Figure 5-6 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

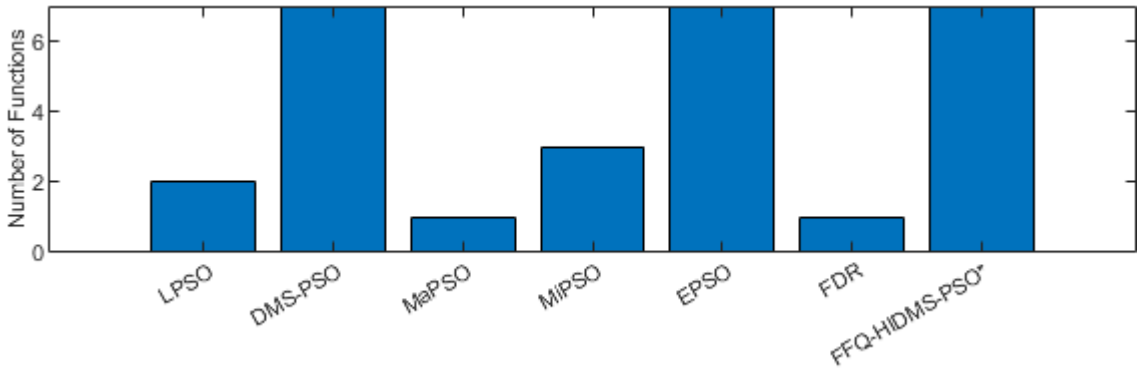


Figure 5-7 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'13 problems

5.3.1.3 Results on 50 dimensional problems

The experimental results obtained on the 50-dimensional CEC'13 problems reveal that the 5 unimodal functions (F1~F5) are solved by 4 different algorithms. The DMS-PSO algorithm performed best on F1 and F5, the rest of the functions including F2, F3 and F4 are solved individually by L-SHADE, FFQ-HIDMS-PSO and EPSO, respectively. For the 15 multimodal functions (F6~F20), we observe that L-SHADE and the proposed algorithm FFQ-HIDMS-PSO solved 14 of the 15 multimodal functions. The L-SHADE algorithm exhibited the best performance on 8 functions namely, F8, F10, F11, F14~F17 and F19 whereas the proposed algorithm FFQ-HIDMS-PSO achieved the best performance on 6 functions including F7, F19, F12, F13, F18 and F20 whilst the best solution for F6 is attained by EPSO. For 50-dimensional composition functions (F21~F28), the proposed algorithm FFQ-HIDMS-PSO exhibited the best performance on 4 of 8 composition function including F23, F24, F26 and F27 whereas the best solutions for F21, F22, F25 and F28 is attained individually by LPSO, L-SHADE, DMS-PSO and HCL-PSO, respectively.

The performances obtained on the 50-dimensional functions indicate a deterioration in the performance of most algorithms compared to 10 and 30-dimensional functions due to increased dimensionality which is primarily reflected on the number of functions solved by the test algorithms. However, we observe an opposite trend in the performance of the proposed algorithm FFQ-HDIMS-PSO. Figure 5-8 displays the only algorithms that achieved the best performance on 50-dimensional CEC'13 functions and the number of problems solved by each algorithm. Further, in Figure 5-9, we display the same comparison exclusively among PSO variants. We can observe from Figure 5-8:5-9 that the proposed algorithm FFQ-HIDMS-PSO solved more problems in both comparisons.

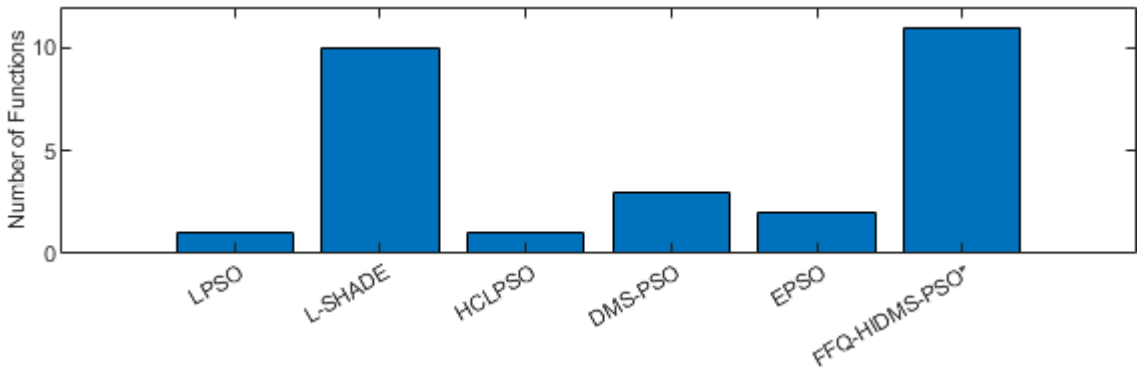


Figure 5-8 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

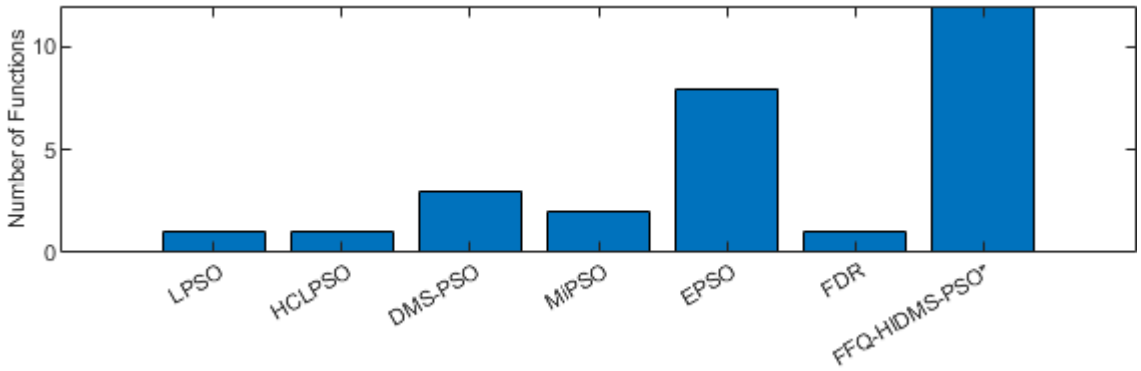


Figure 5-9 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'13 problems

5.3.1.4 Results on 100 dimensional problems

The results obtained on the 100-dimensional CEC'13 test problems indicate that for unimodal functions (F1~F5), the best solutions for F1 and F5 is attained by DMS-PSO whilst for F2, F3 and F4, the lowest mean errors are achieved individually by L-SHADE, FFQ-HIDMS-PSO and EPSO, respectively. The majority of the multimodal functions (F6~F20) are solved by L-SHADE and the proposed algorithm FFQ-HIDMS-PSO where L-SHADE performed best on F8, F10, F14, F15, F16, F17, F19 and FFQ-HIDMS-PSO provided the best mean solution for F7, F9, F12, F13, F18 and F20. On the other hand, the lowest mean error values for F6 and F11 are obtained by LPSO and EPSO, respectively. The results for the composition functions (F21~F28) indicate that the proposed algorithm FFQ-HIDMS-PSO solved 5 of 8 of the composition functions including F23, F24, F25, F27 and F28 whereas the best solutions for F21, F22 and F26 are attained individually by UPSO, L-SHADE and EPSO.

As shown in Figure 5-10, the overall performance of algorithms on the 100-dimensional CEC'13 problems indicate that the proposed algorithm FFQ-HIDMS-PSO provided the best solutions for the majority (12 of 28) of the problems whereas L-SHADE, in second place solved the most problems (9 of 28) in this experiment. Further comparison shown in Figure 5-11 exhibit the comparison of FFQ-HIDMS-PSO with PSO variants which indicates that the proposed algorithm successfully solved significantly more problems compared to all 13 PSO variants employed in the experiments. Clearly, by this measure, the relative performance of the algorithm significantly improves with dimensionality.

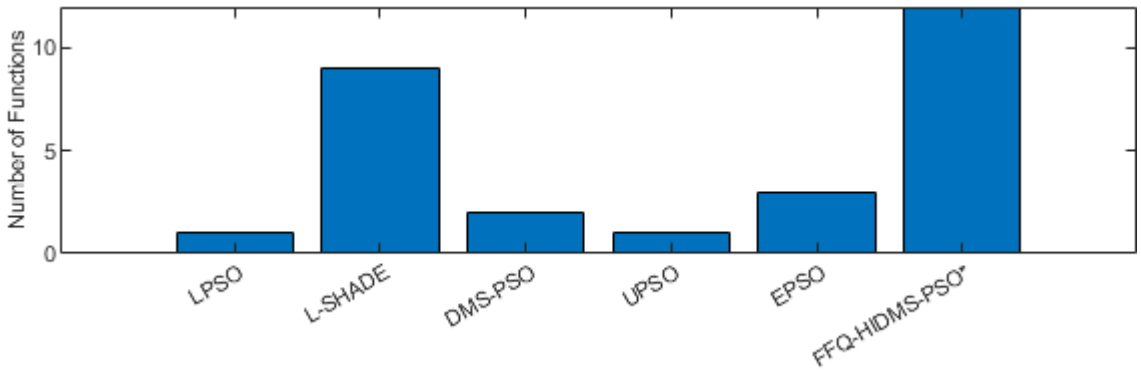


Figure 5-10 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

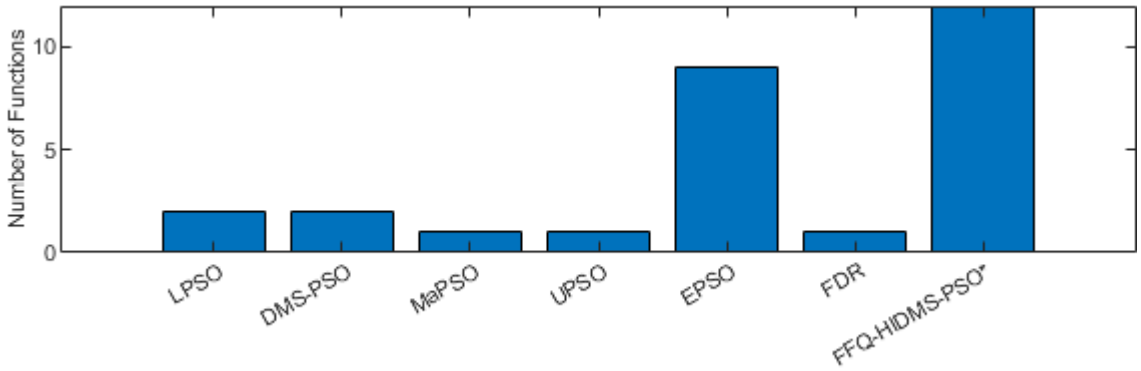


Figure 5-11 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'13 problems

The proposed algorithm FFQ-HIDMS-PSO obtained comparable ranks across different dimensions which can be seen as an indication of consistency in performance. On the 10-dimensional functions, FFQ-HIDMS-PSO ranked 6th, for the experiment conducted on the 30 and 50-dimensional functions, FFQ-HIDMS-PSO attained the 4th rank, and finally for 100-dimensional function the proposed algorithm ranked 5th among 14 test algorithms. However, despite not ranking in the top 3 algorithms, when we observe the individual ranks obtained for functions, it can be seen that (*see Appendix 2 for details*) the proposed algorithm FFQ-HIDMS-PSO attained the 1st rank for the majority of problems however it does not perform on a par for several functions (see F1, F5, F6, F14, F16, F21) which skews the overall average rank for the proposed algorithm. Further, the calculation of average and final ranks without considering the mentioned functions (F1, F5, F6, F14, F16, F21) yield that the proposed algorithm FFQ-HIDMS-PSO ranked 1st with the average rank of 2.90 whereas L-SHADE ranked 2nd with the average rank of 4.1.

Table 5-2 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2013 problems

FFQ-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−

10	+	+	+	-	=	=	+	=	+	=	-	=	=	+	6/6/2
30	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
50	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
100	+	+	+	=	=	=	+	=	=	+	=	+	+	+	8/6/0

Moreover, the Wilcoxon signed rank test results are shown in Table 5-2. The results indicate that the proposed algorithm FFQ-HIDMS-PSO performed significantly better than the majority of the comparison algorithms on the 30, 50 and 100-dimensional functions, and indeed at these dimensions no other algorithm performed significantly better than it. Further, Figure 5-12 displays the convergence rate comparison of FFQ-HIJDMS-PSO with the 4 top ranked algorithms on the 100-dimensional problems. The comparison of the convergence characteristics reveals that the proposed algorithm consistently exhibits faster convergence compared to EPSO and HCLPSO in unimodal, multimodal and composition functions. We also observe that on various multimodal functions (e.g., F12), FFQ-HIDMS-PSO exhibit comparable convergence characteristics to L-SHADE but converges to a better overall solution, also the general convergence trend of the proposed algorithm on the rest of the multimodal functions indicates that FFQ-HIDMS-PSO is capable of escaping local optima to converge to a similar or better solution compared to the top ranked algorithms on the CEC'13 experiment.

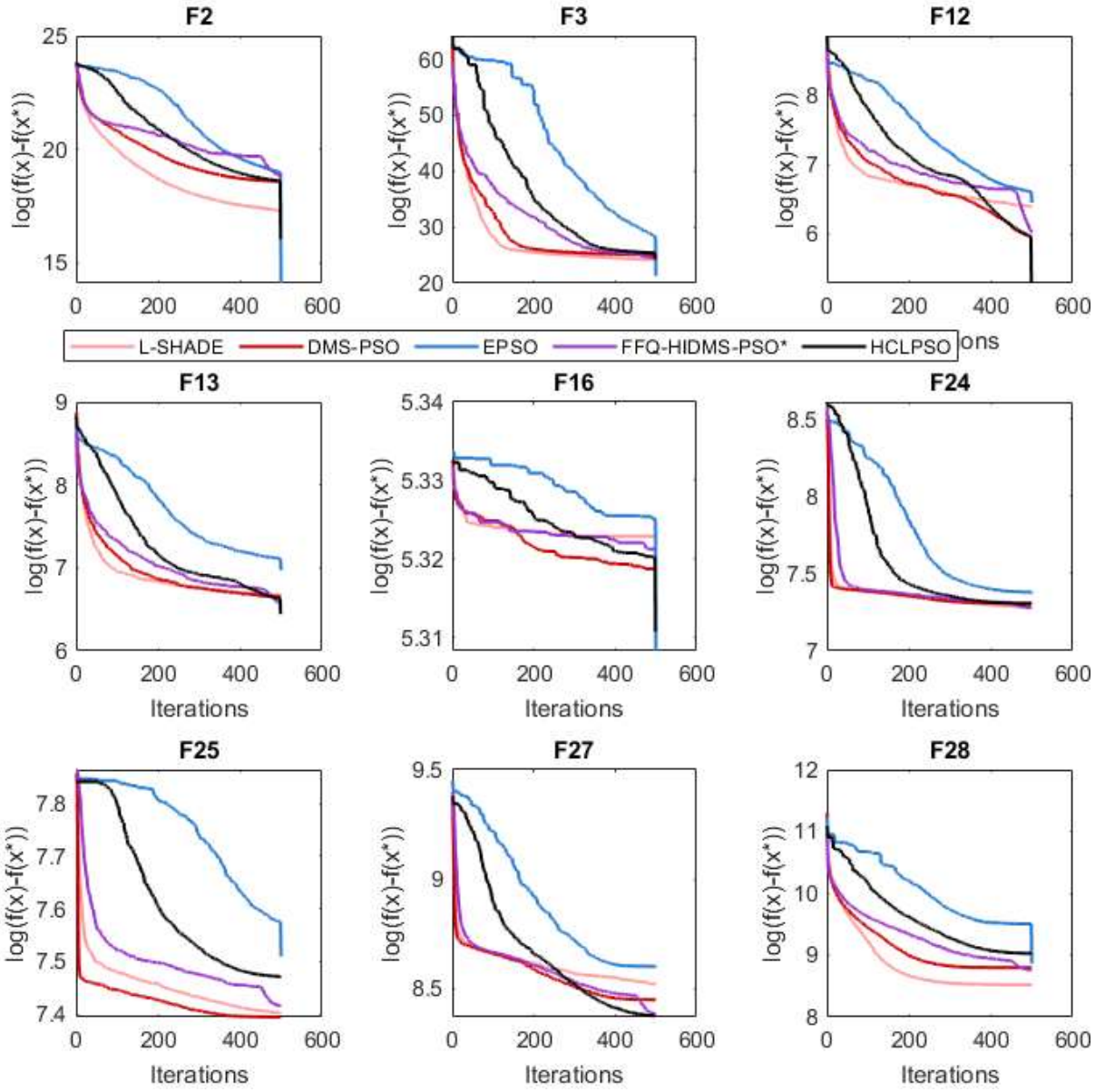


Figure 5-12 Average convergence rate comparison of FFQ-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'13 problems

5.3.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed FFQ-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

5.3.2.1 Results on 10 dimensional problems

For 10-dimensional CEC'14 problems, the results reveal that L-SHADE exhibited the best performance for 3 10-dimensional unimodal functions (F1~F3) and similar on the multimodal functions (F4~F16), L-SHADE solved F7, F8, F9, F10, F11, F12, F15 whereas DMS-PSO attained the best solutions for F6, F13, F14 and F16. On the other hand, the lowest mean error values for F4 and F5 are obtained by EPSO. The results for the 6 hybrid functions (F17~F22) indicates that L-SHADE

provided the best solutions for all of them. The results for the 10-dimensional composition functions reveal that in this case FFQ-HIDMS-PSO performed best on F24, F25, MaPSO on F23, F27 and LPSO on F28, F29. The best mean solutions for F26 and F30 are provided by FMS-PSO and FIPS, respectively.

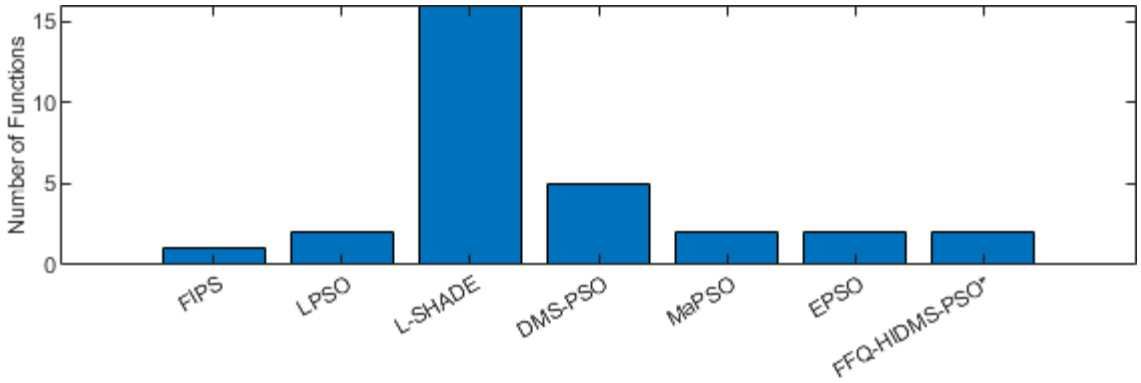


Figure 5-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

Overall, as shown in Figure 5-13, majority of the low dimensional functions are solved by L-SHADE. On the other hand, the performance of the proposed FFQ-HIDMS-PSO algorithm appears approximately in line with the other PSO variants on the 10-dimensional functions.

5.3.2.2 Results on 30 dimensional problems

The results for 30-dimensional CEC'14 problems indicate that L-SHADE obtained the best performance for all unimodal functions (F1~F3) and most of the multimodal functions (F4~F16) including F4,F8,F9,F10,F11,F12,F15 and F16 whereas the lowest mean error values for F5, F6, F7, F13 and F14 are obtained individually by HPSO-TVAC, FIPS, FFQ-HIDMS-PSO, DMS-PSO and FDR, respectively. Similarly, on the 30-dimensional hybrid functions (F17~F22), L-SHADE solved F27, F19, F20, F21 whereas MaPSO and LPSO provided the best solution for F18 and F22, respectively. The performances on the composition functions indicate that DMS-PSO solved 3 of the 8 functions including F23, F25, F29 whilst the proposed algorithm FFQ-HIDMS-PSO achieved the best performance on F24 and F27. The lowest mean error values for F26, F28 and F30 are obtained by L-SHADE, LPSO and MaPSO, respectively.

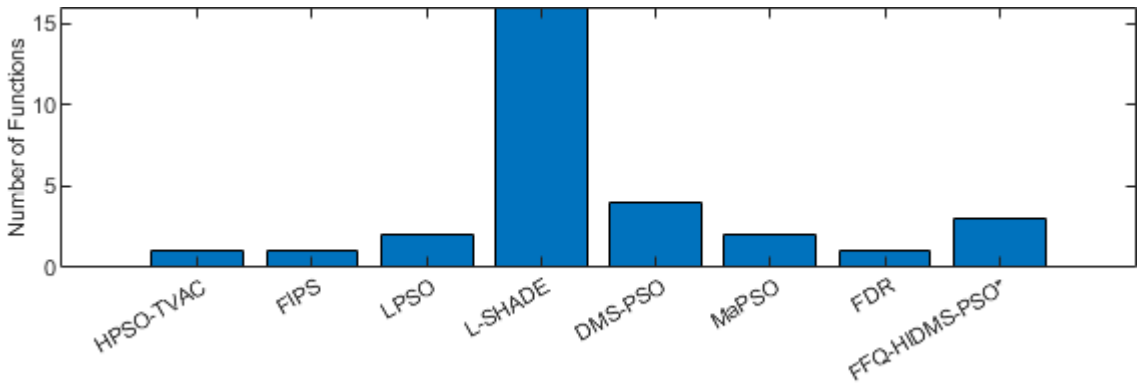


Figure 5-14 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'14 problems

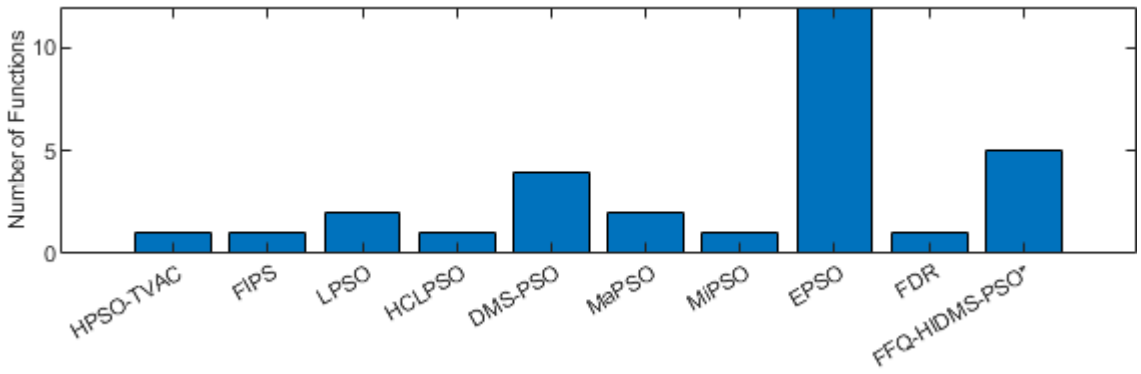


Figure 5-15 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'14 problems

From Figure 5-14, we observe that 8 (half of the algorithms employed) of the 15 algorithms were able to solve at least one of the 29 CEC'14 functions. The majority of the functions are solved by L-SHADE and the performances of the PSO variants exhibit approximately similar trend. In Figure 5-15, the comparison among PSO variants indicate that EPSO solved most of the problems whereas the proposed algorithm FFQ-HIDMS-PSO exhibit the second-best overall performance among the 13 PSO variants on the 30-dimensional CEC'14 functions.

5.3.2.3 Results on 50 dimensional problems

The results obtained on the 50-dimensional CEC'14 functions illustrate that L-SHADE solved all 3 unimodal functions and 6 of the multimodal functions including F4, F8, F10, F11, F12 and F16 whereas the proposed algorithm FFQ-HIDMS-PSO solved F6, F9 and F15. The best solutions for F5, F7, F13 and F14 is provided by HPSO-TVAC, FIPS, DMS-PSO and LPSO, respectively. The results on the 50-dimensional hybrid functions show that L-SHADE, EPSO and the proposed algorithm FFQ-HIDMS-PSO are the sole algorithms to solve the hybrid functions where L-SHADE achieved the best performance on F17, F18, F19, F21 and EPSO on F20 whilst FFQ-HIDMS-PSO performed best on F22. For the composition functions (F23~F30), we observe that DMS-PSO solved 4 of the 8 functions including F23, F25, F28, F29 whilst the best solutions for F24, F26, F27, F30 are provided individually by MiPSO, FIPS, FFQ-HIDMS-PSO and LPSO, respectively.

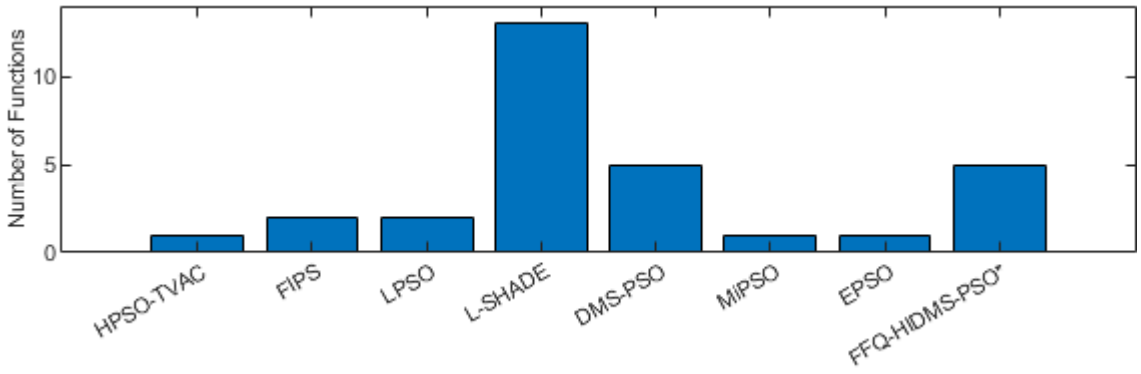


Figure 5-16 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

We observe from Figure 5-16 that the 30 CEC'14 problems at 50 dimensions are solved by 8 of the 15 algorithms employed. The majority of the functions are solved by L-SHADE and the FFQ-HIDMS-PSO algorithm provided the best solution for the second most problems along with DMS-PSO, solving more problems compared to rest of the PSO variants.

5.3.2.4 Results on 100 dimensional problems

The results for the 100-dimensional CEC'14 problems indicate that for unimodal functions (F1~F3), L-SHADE solved F1 and F2 whilst the best performance for F3 is achieved by EPSO. On the multimodal functions (F4~F16), the lowest mean error values for majority of the functions are obtained by LSHADE and the proposed algorithm FFQ-HIDMS-PSO where L-SHADE solved F10, F11, F12, F16 and FFQ-HIDMS-PSO performed best on F6, F9, F15 whilst HCLPSO exhibit the best performance on F5 and F14. The best mean solutions for F4, F7, F8, and F13 are attained individually by LPSO, UPSO, EPSO and DMS-PSO, respectively. On the hybrid functions (F17~F22), L-SHADED solved 2 functions including F17 and F21. For F18, F19, F20 and F22, best solutions are obtained by MaPSO, LPSO, EPSO and DMS-PSO. The results for the composition functions (F23~F30), LPSO solved 4 composition functions including F25, F28, F29, F30 whilst the best solutions for F23, F24, F26 and F27 are attained by DMS-PSO, HPSO-TVAC, FIPS and FFQ-HIDMS-PSO.

We observe from Figure 5-17 that overall, L-SHADE solved 8 functions whilst LPSO exhibited the best performance on 6 across 30 functions. Among the PSO variants employed in the experiments, the proposed algorithm FFQ-HIDMS-PSO solved more problems compared to 12 PSO variants except LPSO.s

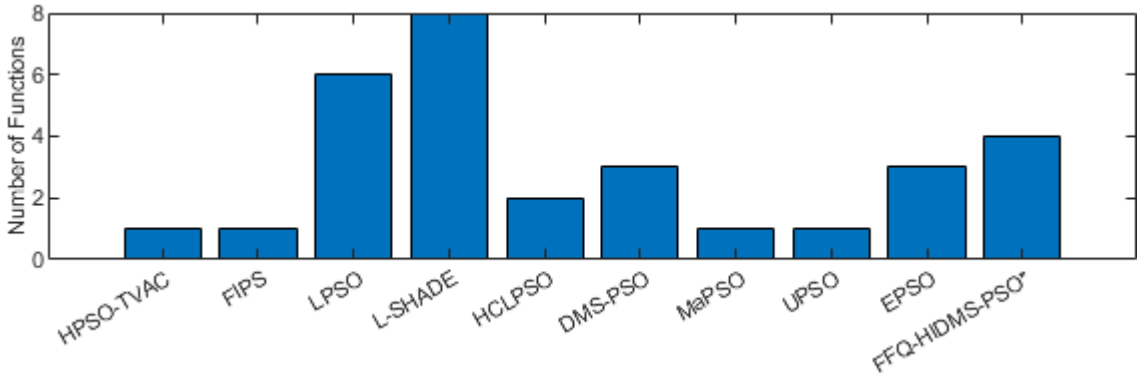


Figure 5-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

The average and final ranks obtained across 4 dimensions on the CEC'14 test problems reveal that the proposed algorithm FFQ-HIDMS-PSO ranked 7th on the 10-dimensional, 6th on the 50-dimensional and 5th on the 30 and 100-dimensional problems. Further, Table 5-3 display the results for the Wilcoxon signed rank test with significance level of 5%. In summary, the results indicate that the proposed algorithm FFQ-HIDMS-PSO is significantly better than 7 algorithms on the 10 and 100-dimensional functions. Similarly, on the 30 and 50-dimensional problems, the FFQ-HIDMS-PSO algorithm's performance is statistically significant compared to 5 algorithms. At dimension 10, 2 algorithms are statistically better than FFQ-HIDMS-PSO, at the other higher dimensions one algorithm is statistically better than FFQ-HIDMS-PSO.

Table 5-3 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2014 problems

FFQ-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−

10	+	+	+	=	-	+	+	=	+	=	-	=	=	+	7/5/2
30	+	+	+	=	=	=	+	=	=	=	-	=	=	+	5/8/1
50	+	+	+	=	=	=	+	=	=	=	-	=	=	+	5/8/1
100	+	+	+	=	-	=	+	=	=	=	=	+	+	+	7/6/1

The convergence rate comparison of FFQ-HIDMS-PSO, L-SHADE, DMS-PSO, EPSO and HCLPSO is shown in Figure 5-18. The convergence comparison suggests that also FFQ-HIDMS-PSO is outranked by EPSO and HCLPSO, the proposed algorithm consistently exhibits faster convergence rate compared to these two algorithms on unimodal, multimodal, hybrid and composition functions.

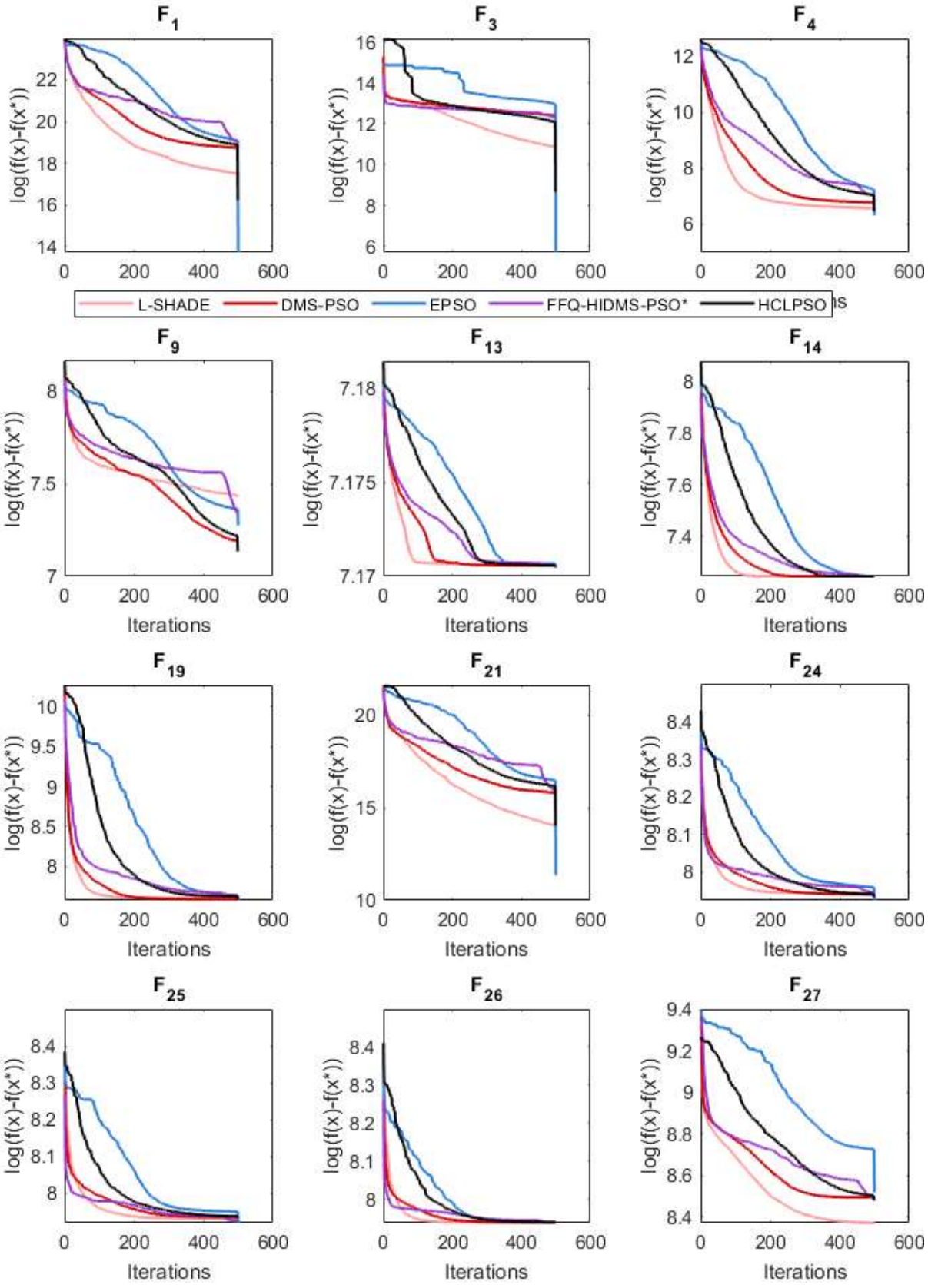


Figure 5-18 Average convergence rate comparison of FFQ-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'14 problems

5.3.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed FFQ-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

5.3.3.1 Results on 10 dimensional problems

The mean errors obtained on the 10-dimensional CEC'17 problems indicate that for unimodal F1, L-SHADE performed best and as well as the majority of the multimodal functions (F3~F9) including F3, F4, F5, F7, F8 and F9 whilst EPSO provided the best solution for F6. On the hybrid functions (), once again, we observe that all hybrid functions are solved by L-SHADE and EPSO where L-SHADE performed best on F10, F11, F12, F13, F14, F15, F17, F18 and F19 whilst EPSO solved F16. For the unimodal, multimodal and hybrid functions we observe that two algorithms (L-SHADE and EPSO) exhibit dominating performance however the results obtained on the composition functions (F20~F30) we observe a more distributed performance among 7 algorithms where MiPSO solved F21, F23, F24, MaPSO performed best on F20, F26 and HCLPSO provided the best solution on F25 and F28. The best performance for F22, F27, F29 and F30 is attained by EPSO, LPSO, L-SHADE and DMS-PSO, respectively.

Overall, as shown in Figure 5-19, we observe that on 10-dimensionL CEC'17 problems, the proposed algorithm FFQ-HIDMS-PSO is unable to exhibit the best performance for any problems. On the other hand, L-SHADE appear to significantly outperform rest of the algorithms in this experiment with regards to number of functions solved.

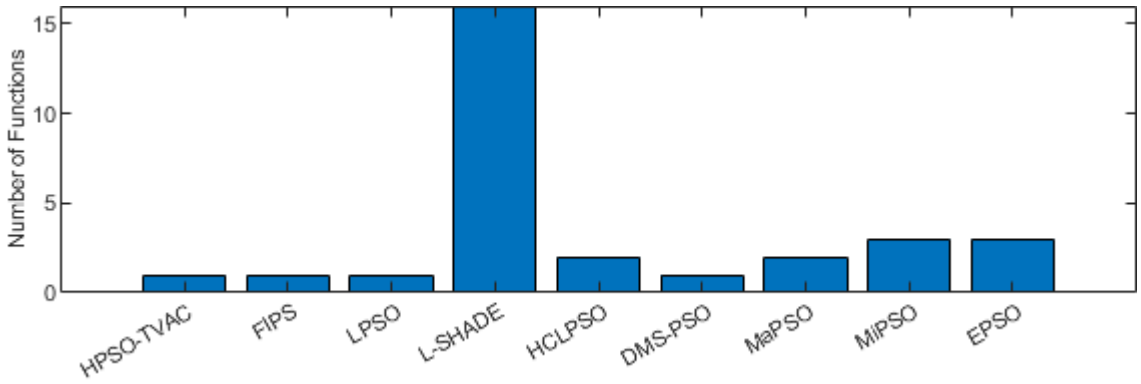


Figure 5-19 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'17 problems

5.3.3.2 Results on 30 dimensional problems

The results obtained on the 30-dimensional CEC'14 problems indicate that L-SHADE achieved the best performance on unimodal F1 and on the multimodal functions (F3~F9), the best performance for F5, F7 is attained by L-SHADE whereas EPSO solved F3 and F4. The lowest mean error values for F6, F8 and F9 is achieved by MiPSO, FFQ-HIDMS-PSO and FIPS, respectively. For hybrid functions (F10~F19), the majority of the hybrid functions including F10, F12, F13, F14, F18 and F19 is solved by L-SHADE whereas the best solution for F11, F15, F16 and F17 is provided by FFQ-HIDMS-PSO, MaPSO, DMS-PSO, and HCLPSO, respectively. For composition functions (F20~F30), L-SHADE performed best on F20, F21, F23, F24 whilst LPSO obtained the best solution on F22 and F30. For F25, F26, F27, F28 and F29, the best solutions are attained individually by FIPS, FFQ-HIDMS-PSO, MiPSO, EPSO and DMS-PSO, respectively.

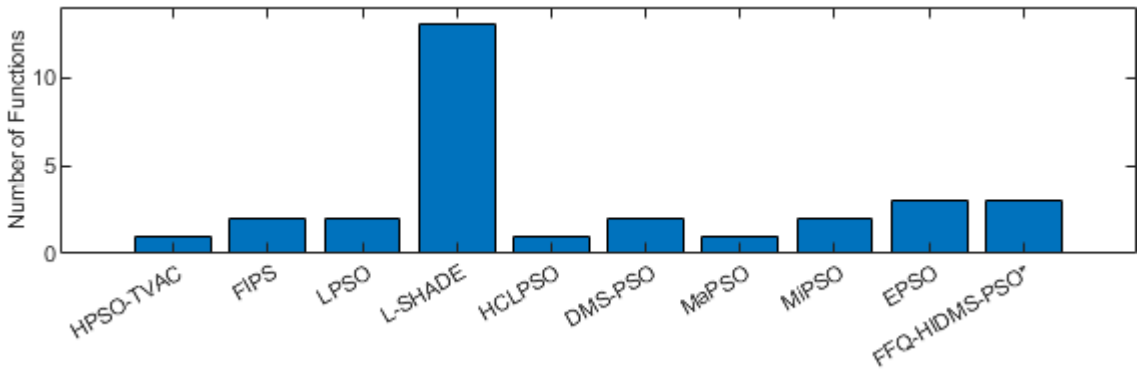


Figure 5-20 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

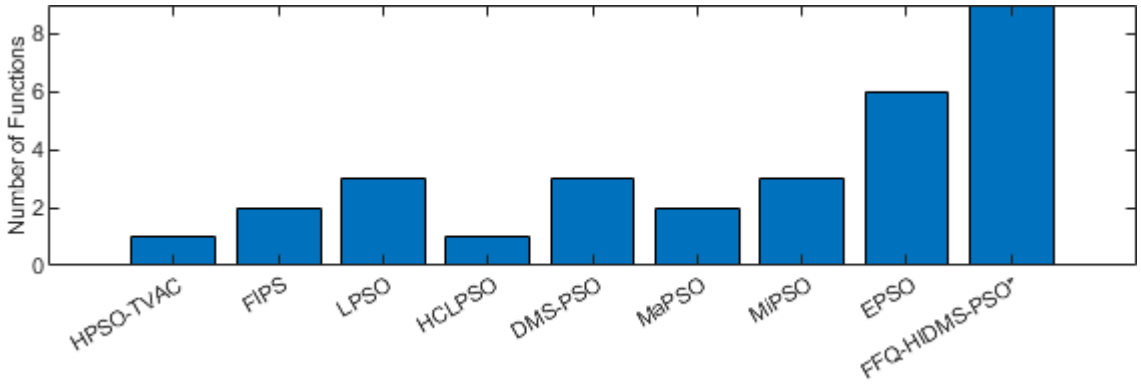


Figure 5-21 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

We observe from Figure 5-20 that overall, on the 30-dimensional functions, L-SHADE solved most of the functions in this experiment however when compared with the PSO variants (Fig. 5-21), the proposed algorithm FFQ-HIDMS-PSO exhibit superior performance by solving more functions in comparison to the employed PSO variants. In addition, the proposed algorithm FFQ-HIDMS-PSO provided the second-best solution for eight 30-dimensional functions.

5.3.3.3 Results on 50 dimensional problems

The results obtained on the 50-dimensional CEC'17 benchmark problems reveal that for most of the multimodal functions at 50 dimensions, the proposed algorithm FFQ-HIDMS-PSO achieved the best performance including F5, F7 and F8 whereas FIPS and EPSO solved two functions individually namely, F6,F9 and F3,F4, respectively. The 50-dimensional hybrid functions are solved by 3 of the 14 algorithms employed in the experiments where L-SHADE performed best on F10,F12,F14,F15,F18 and F19. The proposed algorithm FFQ-HIDMS-PSO solved F11, F16, F17 and EPSO provided the best solution for F13. We observe a deterioration in the performance of most of the algorithms on composition functions however an opposite pattern in performance of the proposed algorithm FFQ-HIDMS-PSO which solved the majority of the functions including F21, F22, F23, F26 and F29 whereas DMS-PSO performed best on F20 and F28. The lowest mean error values for F24, F25, F27 and F30 is attained individually by L-SHADE, FIPS, HPSO-TVAC and LPSO, respectively.

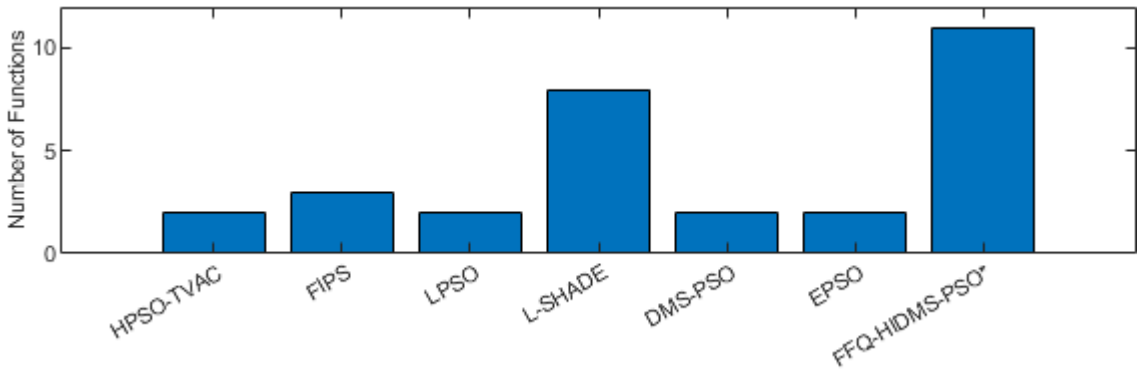


Figure 5-22 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

Figure 5-22 shows the algorithms and the corresponding number of functions solved by them. We observe that the proposed FFQ-HIDMS-PSO algorithm successfully solved more functions compared to all test algorithm employed in the experiment on the 50-dimensional CEC'17 problems.

5.3.3.4 Results on 100 dimensional problems

The results obtained on the 100-dimensional CEC'17 problems suggest that the multimodal functions are solved by 3 algorithms where the proposed algorithm FFQ-HIDMS-PSO solved the majority of the multimodal functions including F5, F7, F8 and F9 whereas EPSO attained the lowest mean error for F3, F6 and LPSO performed best on F4. On 100-dimensional hybrid functions (F10~F19), the best performance for F10, F12, F14, F15 is obtained by L-SHADE and EPSO performed best on F13, F18 and F19. The DMS-PSO algorithm provides the best solution for F11 and the proposed algorithm FFQ-HIDMS-PSO solved F16 and F17. We can observe from the results on the 100-dimensional CEC'17 test problems that the only 3 of the 15 algorithms were able to solve the composition functions. The significant portion of the composition functions are solved by the proposed FFQ-HIDMS-PSO algorithm including F21, F22, F23, F24, F26, F29 whereas DMS-PSO performed best on F20, F25, F27 and the best solutions for F28, F30 is attained by LPSO.

Figure 5-23 displays the algorithms and the total number of functions each algorithm solved at 100 dimensions. We observe from Figure 5-23 that FFQ-HIDMS-PSO performed best on significantly more functions compared to the 14 test algorithms employed in the experiment.

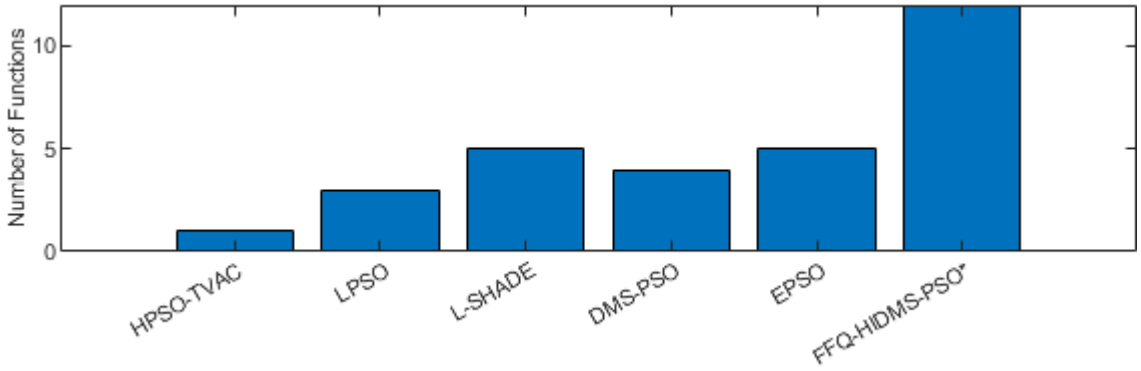


Figure 5-23 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

The average and final ranks obtained on different dimensions of CEC'17 problems suggest that the proposed algorithm FFQ-HIDMS-PSO obtained the 8th rank at 10 dimensions whilst on 30-dimensional problems, the FFQ-HIDMS-PSO ranked 3rd, and on the 50 and 100-dimensinal problems, FFQ-HIDMS-PSO ranked 2nd. Hence, on the 30, 50 and 100-dimensional experiments, the proposed algorithm outranked the majority of the comparison algorithms. Further, the Wilcoxon Signed rank test results (displayed in Table 5-4), indicate that the proposed algorithm FFQ-HIDMS-PSO exhibited statistically significantly better performance than 9, 11 and 10 algorithms at 30, 50 and 100 dimensions, respectively. At the higher dimensions no other algorithm was statistically better than FFQ-HIDMS-PSO. Taking into account the various measures described above, it is clear that FFQ-HIDMS-PSO's performance on this test suite was exceptional.

Table 5-4 Wilcoxon Signed Rank Test Results with a Significance Level of $\alpha = 0.05$ for CEC2017 problems

FFQ-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	-	+	+	-	+	=	-	=	=	=	6/5/3
30	+	+	+	=	=	+	+	=	+	=	-	+	+	+	9/4/1
50	+	+	+	=	=	+	+	+	+	+	=	+	+	+	11/3/0
100	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0

In addition, as shown in Figure 5-24, the convergence rate comparison on the 100-dimensional CEC'17 functions with the 4 top ranked algorithms L-SHADE, EPSO, DMS-PSO and HCLPSO reveal that FFQ-HIDMS-PSO mostly exhibit comparable convergence characteristics to DMS-PSO and L-SHADE whilst consistently converging faster than EPSO and HCLPSO.

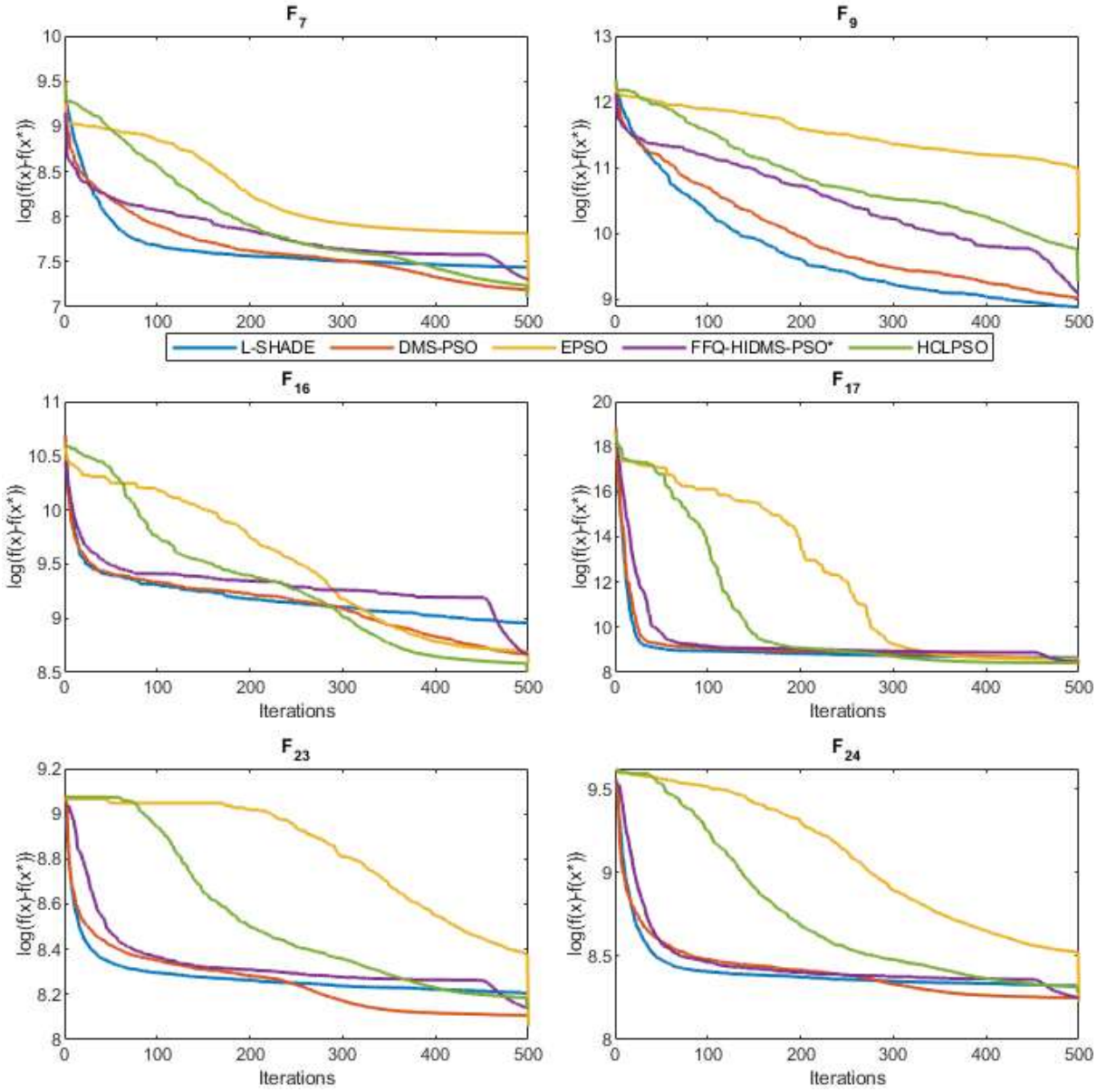


Figure 5-24 Average convergence rate comparison of FFQ-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'17 problems

Moreover, we individually compared the total number of functions solved by the proposed algorithm and each comparison algorithm on all dimensions of CEC'13, CEC'14 and CEC'17 test suites. The comparison is shown in Figure 5-25:5-27, and we observe that the proposed algorithm FFQ-HDMS-PSO solved more problems compared to most of the other algorithms. We consistently observe a very efficient performance on high dimensional (50 and 100-dimensional) functions.

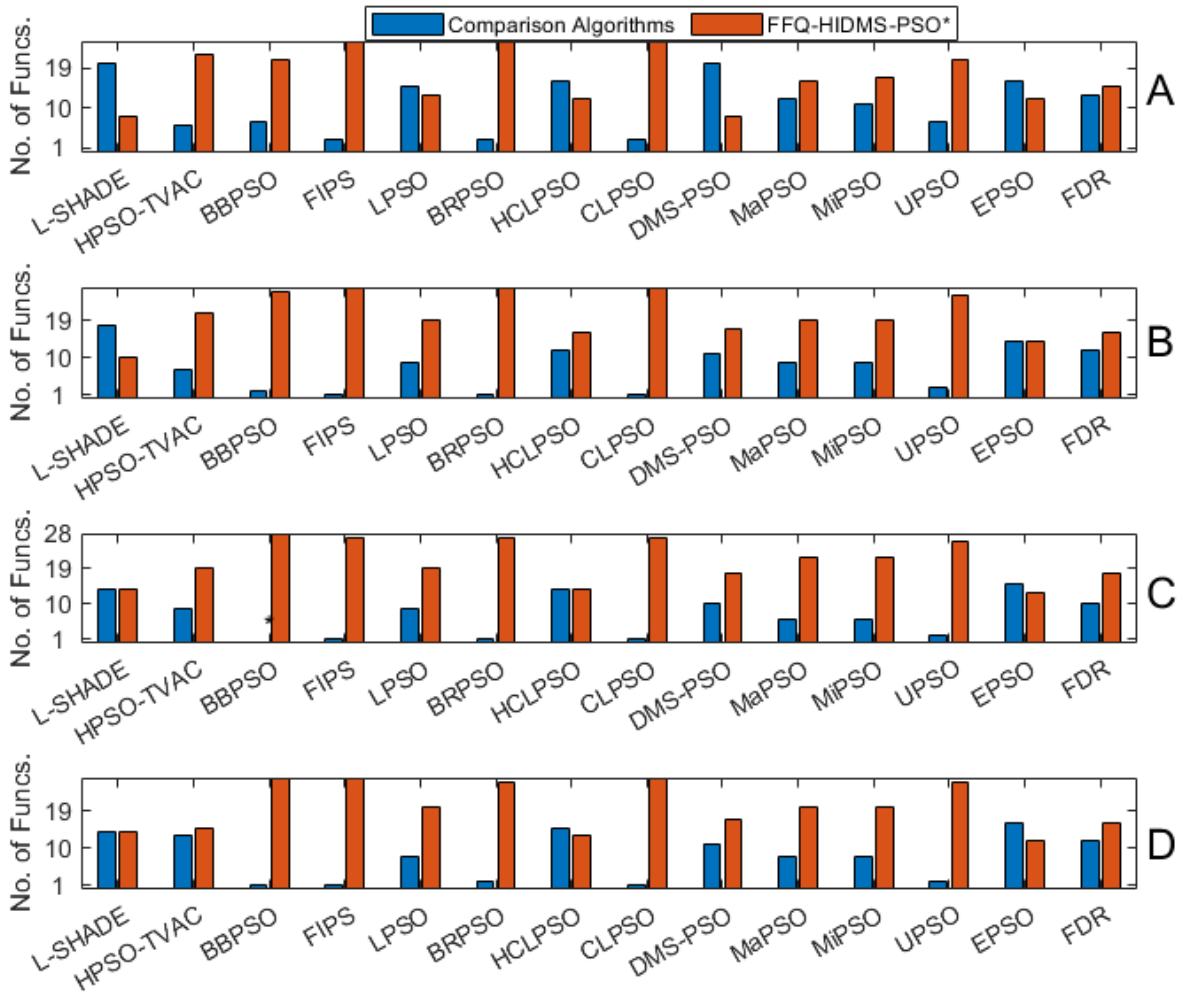


Figure 5-25 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'13 test functions

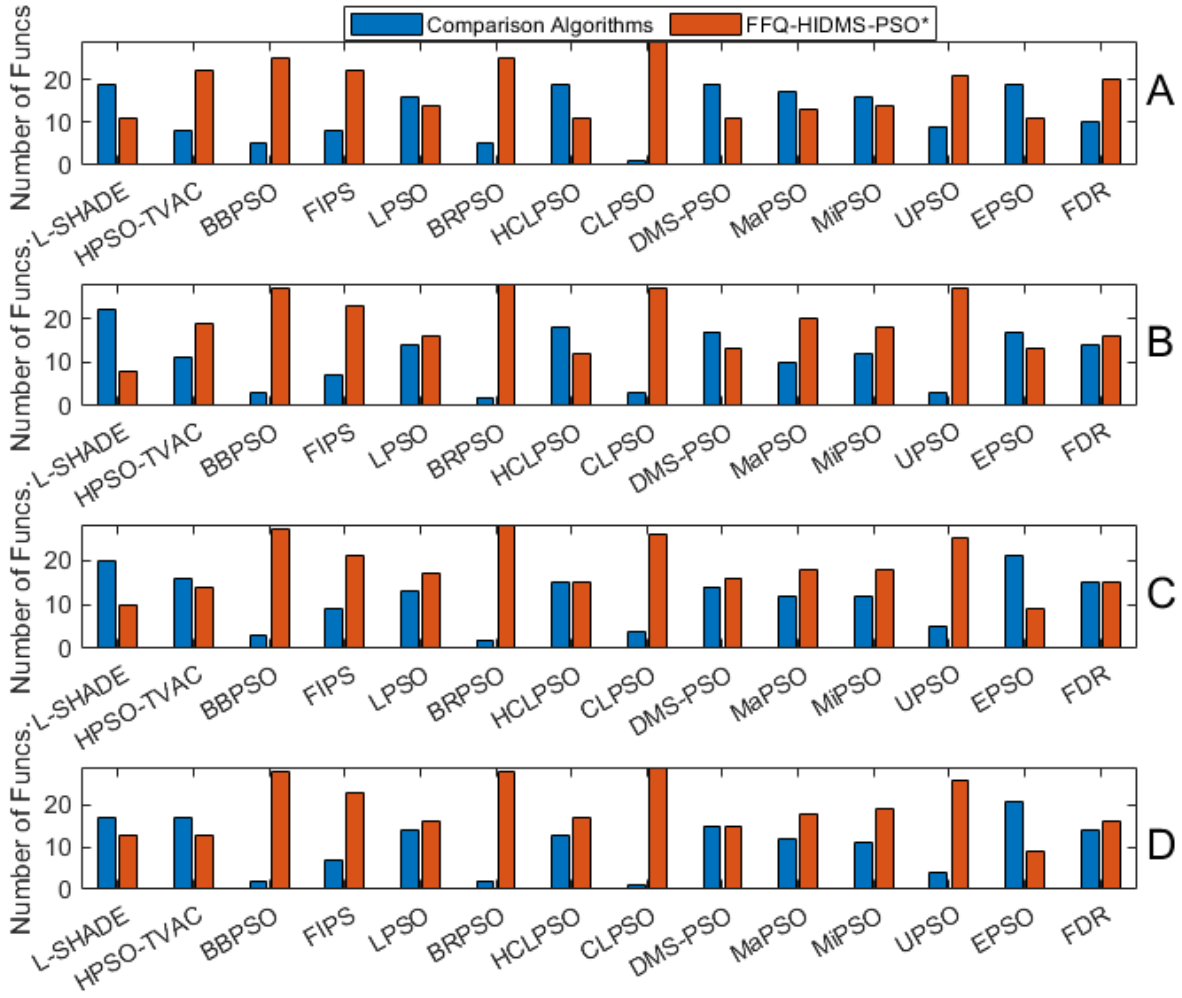


Figure 5-26 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'14 test functions

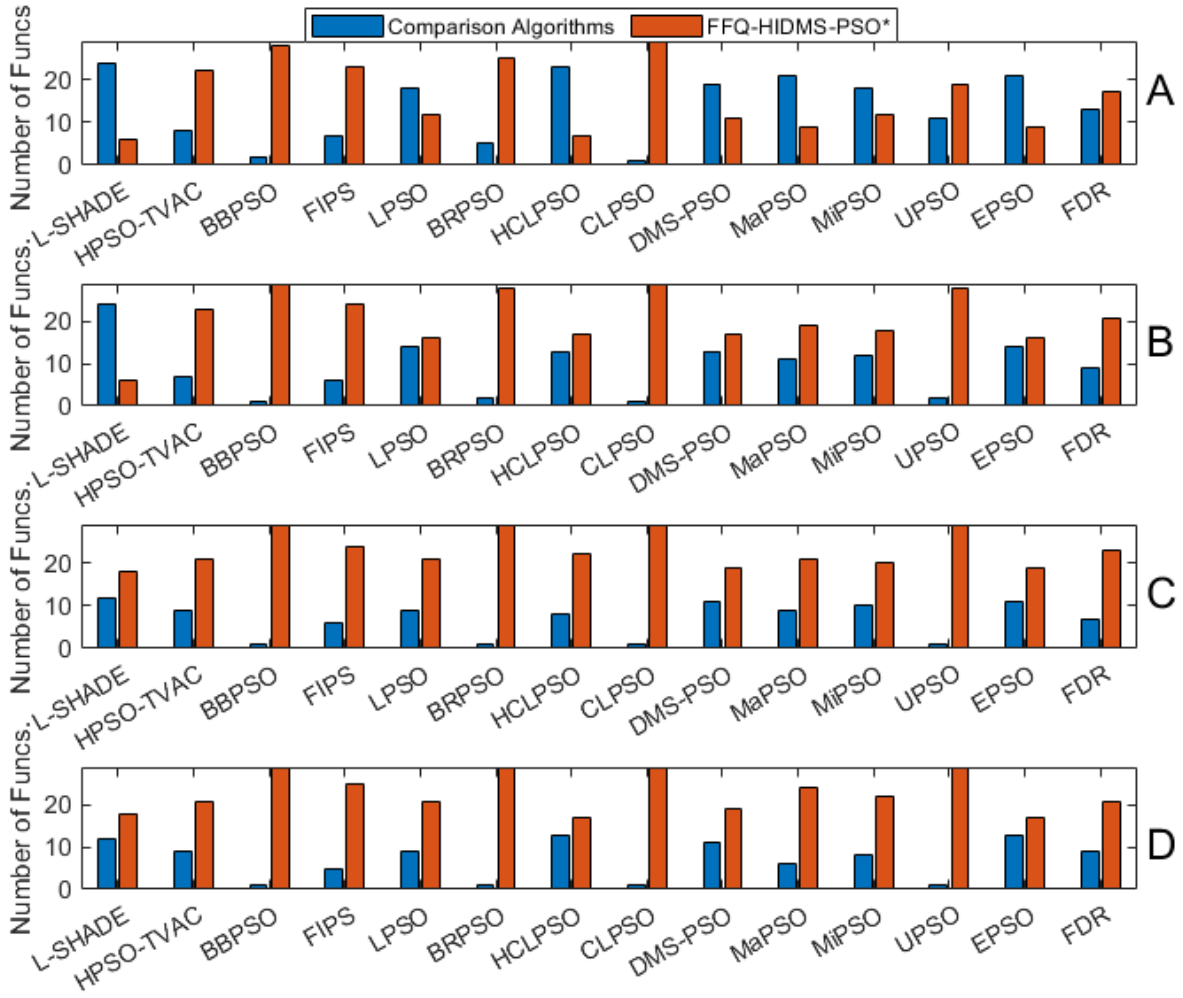


Figure 5-27 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'17 test functions

5.4 Discussions

The performance of the proposed FFQ-HIDMS-PSO algorithm is tested on a set of 87 problems on 10, 30, 50 and 100 dimensions comprised from CEC'13, CEC'14 and CEC'17 test suites. The problem set includes 9 unimodal, 35 multimodal, 16 hybrid and 26 composition functions.

Overall, the FFQ-HIDMS-PSO algorithm attained the best performance for 4 problems on 10-dimensional, 10 problems on 30-dimensional, 27 problems on 50-dimensional and 28 problems on the 100-dimensional problems. These numbers signify that performance of the proposed algorithm improves with increased problem dimensions. On the 87 100-dimensional problems, the comparison of the FFQ-HIDMS-PSO, L-SHADE and EPSO in terms of the number of functions solved reveal that L-SHADE achieved the lowest mean error values for 22 100-dimensional problems, on the other hand EPSO solved 11 100-diemnsional problems in total whereas the proposed algorithm FFQ-HIDMS-PSO exhibited the best performance for 28 problems. Figure 5-28 exhibits the same comparison on different types of problems, and we observe that FFQ-HIDMS-PSO solved more multimodal and composition functions compared to L-SHADE.

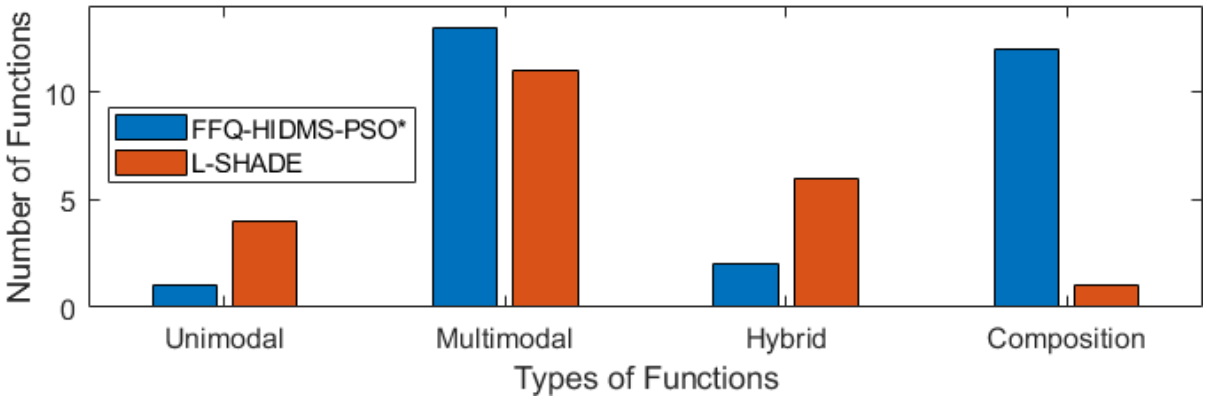


Figure 5-28 – The comparison of FFQ-HIDMS-PSO and L-SHADE on different types of functions solved

Besides the comparison on the number of problems solved, the Wilcoxon signed rank test indicated that the proposed algorithm's performance is significantly better than most test algorithm on each experiment conducted. Further, the comparison of the convergence revealed that FFQ-HIDMS-PSO exhibited comparable convergence characteristics to the top ranked algorithms, and generally exhibited convergence trend faster than EPSO and HCLPSO.

5.5 Conclusion

This chapter introduced a bio-inspired variant of the HIDMS-PSO algorithm named the FFQ-HIDMS-PSO algorithm. The FFQ-HIDMS-PSO algorithm incorporates fission-fusion behaviour and a quorum decision mechanism to primarily improve the population diversity maintaining capabilities of the standard HIDMS-PSO and the quorum decision mechanism is utilised to allow units to simulate self-organised organisms. The performance evaluation of the proposed algorithm is conducted on all available dimensions of the CEC'13, CEC'14 and CEC'17 benchmark test suites. The experimental results and comparisons with emphasis to overall mean performance, performance on different dimensions, different types of problems, and the number of problems solved signifies that the FQ-HIDMS-PSO algorithm exhibits better performance compared to majority of the test algorithms in most cases. Case in point, on the 50 and 100-dimensional CEC'17 problems, the FFQ-HIDMS-PSO ranked 2nd and exhibited statistically significant performance compared to 11 (50-d) and 10 (100-d) of the 14 test algorithms employed. Hence, the accumulating evidence presented throughout this chapter reaffirms the efficacy of the proposed algorithms on variety of problems and complexities.

Chapter 6 : Genetic Algorithm Assisted HIDMS-PSO Algorithm

6.1 Introduction

In the last decade, researchers have increasingly turned towards a highly effective class of algorithms, namely the hybridisation of metaheuristics. The hybridisation of evolutionary algorithms (EAs) with other types of algorithms is popular due to its practicality and competence in handling uncertainty and noise.

The problem of premature convergence is a core issue in the metaheuristic literature [236], [237], and it mainly occurs due to lack of diversity. In a typical search process, initially, diversity is high, and depletion of diversity ensues as the population moves closer to the best-known optimum. Although in theory, high population diversity may help to guarantee finding the optimal solution, it may also result in slow convergence, meaning that an algorithm is in theory capable of finding the optimal solution but may never converge or meet the termination criteria in a reasonable timeframe. In contrast, in a search process with a low population diversity, fast convergence is usually observed with poor solution accuracy (convergence to local optima). The study [238] refers to the ideal balance between convergence and accuracy as the trade-off point. It is apparent that convergence is not guaranteed, even with sufficient diversity, but maintaining the balance of exploration and exploitation may boost an algorithm to perform at maximum capacity. To tackle this issue, hybridisation has become a widely accepted method to promote diversity during the search for the global optimum. In this chapter we explore hybridisation as a possible way to further improving the HIDMS-PSO algorithm.

As previously mentioned, the HIDMS-PSO algorithm introduced in Chapter 4 possesses heterogeneous features and adopts several strategies to delay the loss of diversity in the population to tackle the problem mentioned above. In light of this, we aim to exploit the heterogeneous qualities of HIDMS-PSO, while extending its diversity-handling capabilities further, by hybridising it with a GA in a collaborative architecture, thus boosting particles' abilities to escape local optima. In a more recent study [238], hybrid algorithms are grouped into two main categories as collaborative hybrid and integrative hybrid approaches. The former methodology refers to combining two or more algorithms running in either a parallel or sequential manner with several frameworks including multistage, sequential and parallel. In this approach, the contributing weight of each algorithm can be assumed to be equal (50/50). The latter hybrid method refers to integrating one of the algorithms into the main/master algorithm as a subordinate. This model offers two approaches, namely, full manipulation and partial manipulation. In this case, the contributing weight of the second algorithm is around 10 to 20%.

To maintain the aforementioned ideal *trade-off point* between convergence rate and accuracy, in our hybrid model, we combine the approaches of the sequential collaborative and the partial manipulative integrative hybrid frameworks to efficiently exploit the heterogeneous features of HIDMS-PSO. In our model, the GA is employed for short periods (50 iterations) to assist HIDMS-PSO (which runs consecutively for 100 iterations) by evolving a proportion of both the homogeneous and heterogeneous subpopulations of the HIDMS-PSO. The sole purpose of this collaborative hybrid interaction is to prevent depletion of diversity within the population of HIDMS-PSO by periodically feeding subpopulations of HIDMS-PSO with the evolved solutions from the GA. The evolved solutions returned from the GA replace the positions (not $pbests$) of the same particles from both subpopulations of the HIDMS-PSO. As a result, this causes fluctuations in the diversity of randomly selected proportions of both subpopulations as particles are deviated from their intended trajectories. In addition, the replaced positions result particles to continue search from new positions discovered by GA, further assisting particles to discover potentially better regions to revamp the search. By only exchanging a proportion of both subpopulations between the two algorithms, we retain a significant fraction of agents unchanged in the HIDMS-PSO algorithm. This

strategy allows us to avoid the slow convergence issue while retaining diversity during the overall search, enabling convergence within a reasonable time to an accurate solution.

6.2 Genetic Algorithm Assisted Heterogeneous Dynamic Multi-Swarm Particle Swarm Optimisation Algorithm

Hybridisation is an effective approach to compensate for the drawbacks of one or more algorithms by eliciting the desirable features of the chosen methods. Many PSO-GA hybrids have been successfully applied to various problems [239] [240]. In terms of GA-PSO hybrids, typical approaches include the integration of genetic operators into PSO, for example, integrating crossover or mutation operators to revamp the population diversity in the PSO or determination of the initial solutions of the GA algorithm by PSO (or vice versa) or tuning the parameters of the GA using PSO.

Instead of some of the conventional approaches mentioned, the hybrid model of GA-HIDMS-PSO employs a combination of features from the collaborative and the integrative hybrid frameworks. Instead of integrating specific components of each algorithm, both algorithms are employed intact with specific roles to compose an overall efficient search process. As the primary algorithm, the role of the HIDMS-PSO is to initiate, conduct and control the main search and at specific intervals, a randomly selected half of the solutions from the homogenous and heterogeneous subpopulations of HIDMS-PSO is passed to the genetic algorithm as initial solutions to allow the GA to resume the search for the specified period. Subsequently, the returned final solutions from the GA replace the same solutions in both subpopulations. In this cycle, the role of the genetic algorithm is to evolve a proportion of both homogeneous and heterogeneous subpopulation of the HIDMS-PSO algorithm to revamp the population diversity in the main population.

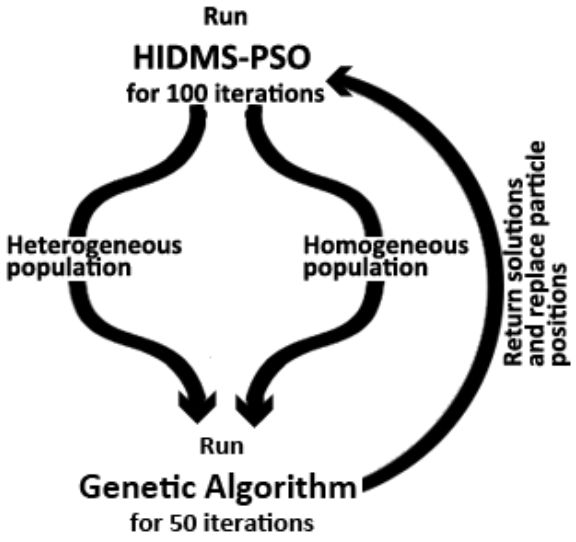


Figure 6-1-Cycle of the GA-HIDMS-PSO.

The fundamental idea of the hybrid model used in the GA-HIDMS-PSO extends several existing hybrid frameworks. The collaborative exchange between the two algorithms and the consecutive executions is derived from the collaborative framework's sequential structure. On the other hand, the integrative hybrid framework's partial manipulation approach inspires the genetic algorithm's role to evolve a proportion of solutions from both subpopulations. The hybrid model of GA-HIDMS-PSO relies on two main factors to exhibit efficient results: the execution ratio/interval of both algorithms and the size and selection of the solutions from the HIDMS-PSO subpopulations. The

optimal values for the two parameters mentioned above were determined by conducting a preliminary experiment to observe the convergence rate and population diversity with different values. Figure 6-2 exhibits the convergence rate obtained over 20 consecutive runs with different contribution weights (intervals) of both HIDMS-PSO and the genetic algorithm. We observe a consistent fast convergence when HIDMS-PSO and the genetic algorithm are executed in short cycles, and on the contrary, slower convergence to a less optimal solution is observed when HIDMS-PSO is executed for longer periods before the initiation of the genetic algorithm. This is most likely caused by the significant and irreversible depletion of population diversity in the HIDMS-PSO algorithm. Hence, the initiation of the genetic algorithm after running HIDMS-PSO for longer periods causes minimal fluctuation in the population diversity of the HIDMS-PSO algorithm as by the time the genetic algorithm is initiated, the population diversity of the HIDMS-PSO algorithm already exhibits major depletion. This essentially results in the genetic algorithm starting the search process with a similar set of solutions; hence, the GA cannot "diversify" the given solutions. As a result, this reflects a minor to no contribution to the population diversity of HIDMS-PSO with almost no practical impact on the search process. On the contrary, running shorter periods of HIDMS-PSO enables the genetic algorithm to revamp the *slightly* depleted population diversity prior to any significant loss of diversity. Hence, we initially settled on 100 iterations of the HIDMS-PSO followed by 50 iterations of the GA as shown in Figure 6-1. However, as shown in Table 6-1, the assessment of other *execution durations* for GA and HIDMS-PSO algorithms reveal that 50 iterations of both algorithms consistently render better overall results. Hence, in the main experiments the said setting will be adopted.

Table 6-1 Mean and final ranks obtained on the CEC'17 problems by running HIDMS-PSO and GA for different durations

$duration$	10-d		30-D		50-D		100-D	
$50-25$	3.30	3	3.33	3	4.07	4	4.07	4
$50-50$	2.43	1	3.13	1	2.37	1	3.03	1
$100-50$	3.13	2	3.23	2	3.40	2	3.10	2
$100-100$	4.60	4	4.63	4	3.77	3	3.60	3
$200-50$	5.07	6	4.87	6	5.33	6	5.20	6
$300-100$	5.10	7	4.70	5	4.67	5	4.67	5
$400-100$	5.03	5	5.57	7	5.83	8	5.50	7
$500-250$	5.33	8	5.60	8	5.63	7	5.90	8

A real coded genetic algorithm is employed in the hybrid model presented here, and, in the experiments, we used tournament selection (size of 3), uniform crossover and a Gaussian mutation operator with probability of 0.1 per chromosome. Our preliminary experiments had shown that using various other genetic operators with the genetic algorithm did not yield significantly different performance as the role of optimising the given problem is primarily undertaken by the HIDMS-PSO algorithm.

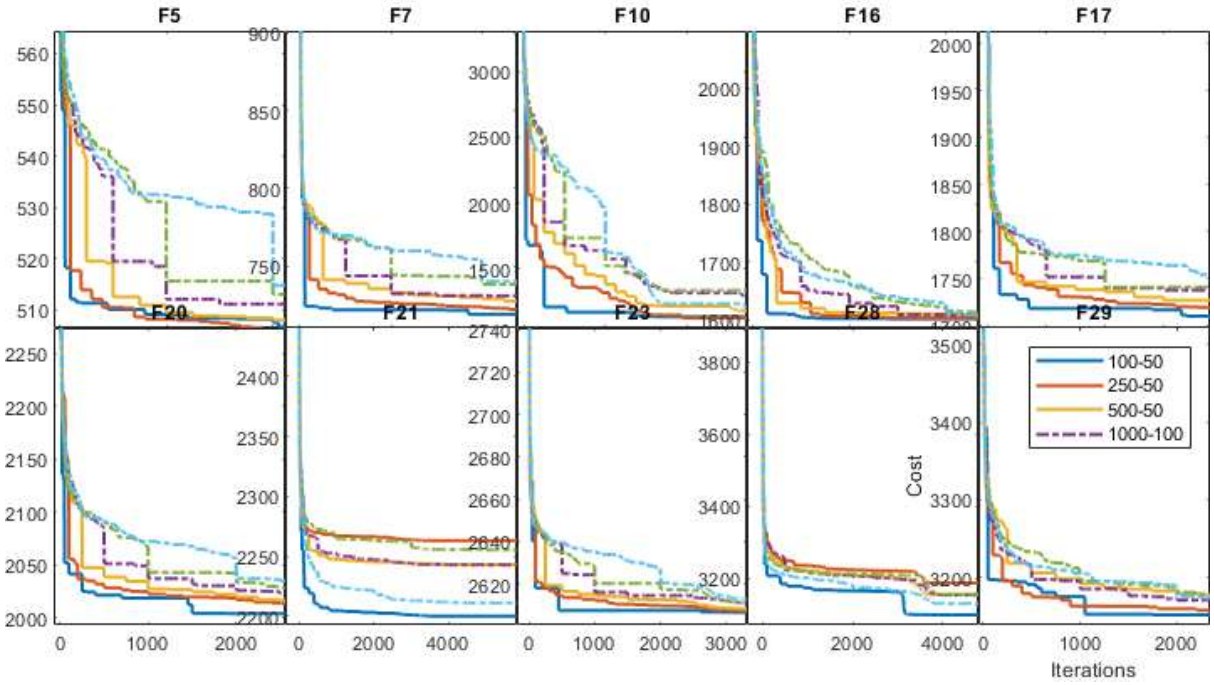


Figure 6-2-The impact of different length of executions of HIDMS-PSO and GA in the GA-HIDMS-PSO hybrid model on the convergence

In Figure 6-3:6-4, we exhibit the average convergence rate and population diversity comparisons of the standard HIDMS-PSO and GA-HIDMS-PSO algorithm obtained over 20 consecutive runs on 30-dimensional CEC'17 functions. The convergence rates clearly show that GA-HIDMS-PSO converges faster to a better solution compared to HIDMS-PSO in almost all cases while maintaining a higher population diversity, verifying the effectiveness of the hybrid algorithm.

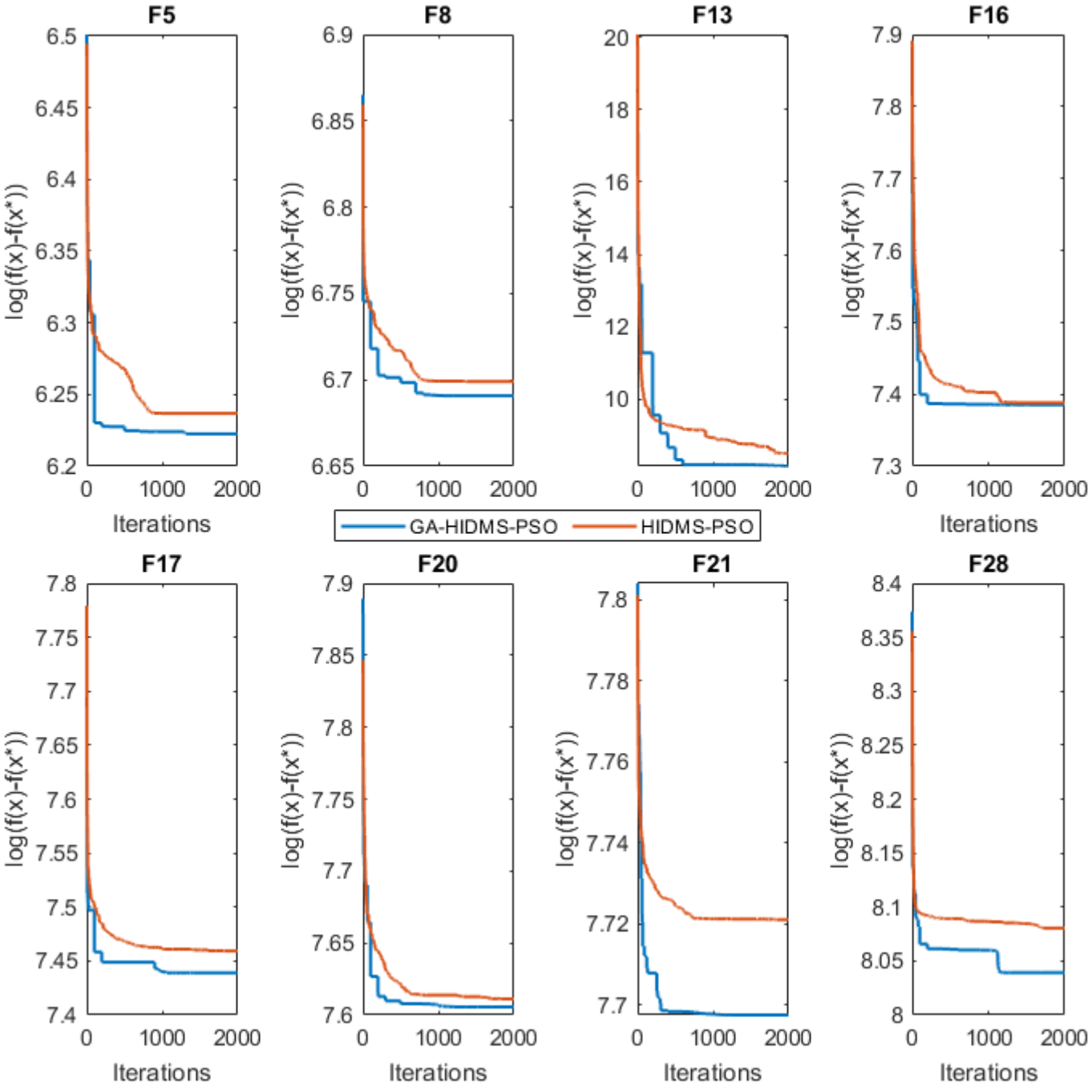


Figure 6-3- Convergence rate comparison of GA-HIDMS-PSO and HIDMS-PSO on representative 30-dimensional CEC'17 functions

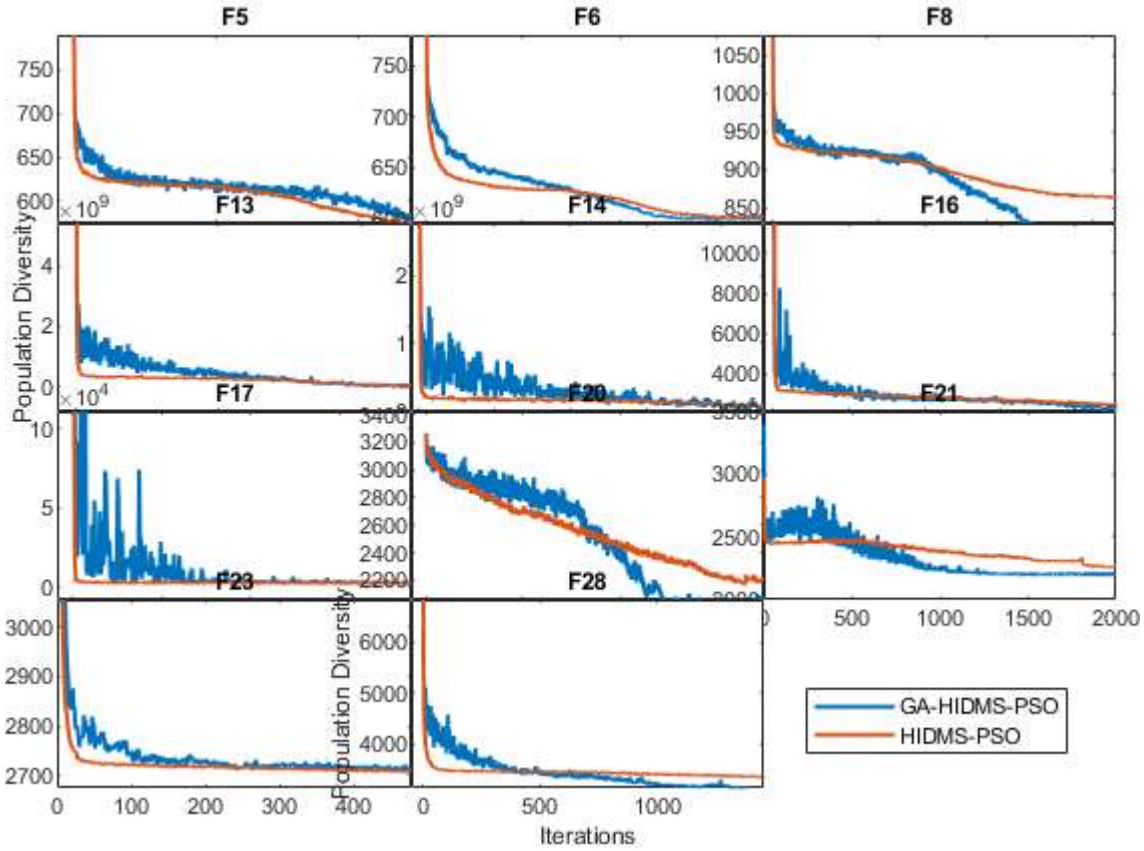


Figure 6-4- Population diversity comparison of GA-HIDMS-PSO and HIDMS-PSO on the 30-dimensional CEC'17 functions

Algorithm 3: GA-HIDMS-PSO

Input: n as swam size, T_{max} as maximum number of iterations

Output: $gbest$

c_1=2.5 − (1:T_{max}*2/T_{max}); c_2=0.5 − (1:T_{max}*2/T_{max}); $w_{max} = 0.99; w_{min} = 0.2;$

$$w_1 = \frac{w_{max} + (w_{min} - w_{max})}{1 + exp\left(-5\left(\frac{2t}{T_{max}} - 1\right)\right)}$$

for t=1:T_{max}
  if time to reshuffle **then**
    Reshuffle units
  end
  if mod(t,100)==0 **then**
    GA_{phase}=true;
  end
  if GA_{phase}==true **then**
    Select $GA_{initial}$ as the initial solutions from both subpopulation of HIDMS-PSO
    $GA_{finalSols}$=GeneticAlgorithm($GA_{initial}$);
    GA_{phase}=false;
  else
    for i=1:n
      if f($\vec{x}_i$) ≥ f($\vec{x}$) **then**
        w=w_1^t+C; if w>0.99, w=0.99, end

```
            else
                w=w_1^t+C; if w>0.20, w=0.20, end
            end
            if i^th particle is master then
                Update v_i using either Eq. 4.1 or 4.2-4.4 and x_i using 2.5
            elseif i^th particle is slave then
                Update v_i using Eq. 4.5 and x_i using 2.5
            end
            Mutate x_i using the nonuniform mutation operator with the p = 0.1
            Evaluate the fitness of x_i
            Update pbest_i and gbest
        end
    end
```

6.3 Experiments

6.3.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed GA-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

6.3.1.1 Results on 10 dimensional problems

The experimental results for the 10-dimensional CEC'13 problems suggest that for 10-dimensional unimodal functions, L-SHADE attained the lowest mean error values, and for multimodal functions (F6~F20), once again, L-SHADE performed best on F8, F10~F14, F16~F18 whilst DMS-PSO solved F6, F7, F9 and F15. Moreover, the best performances for F19 and F20 are observed by MiPSO and HCLPSO, respectively. On the composition functions (F21~F28), the results indicate that LSHADE performed best on F22 and F27 whilst UPSO, DMS-PSO, EPSO, MaPSO, GA-HIDMS-PSO, LPSO exhibited the best performance on F21, F23, F24, F25, F26 and F28, respectively. It is worth noting that the proposed algorithm GA-HIDMS-PSO obtained the second-best solutions on unimodal F2, F3, multimodal F7 and F18.

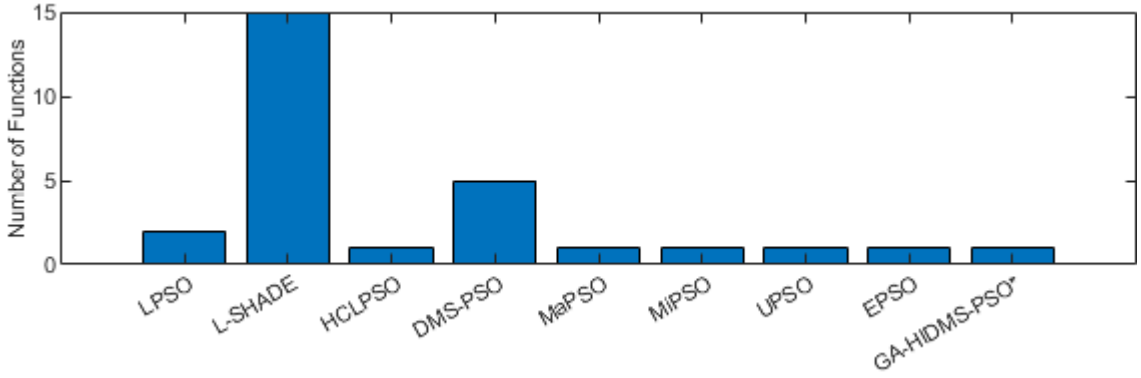


Figure 6-5 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

Overall, as shown in Figure 6-5, L-SHADE exhibited superior performance for the majority of the 10-diemnsional CEC'13 problems followed by the performance of DMS-PSO. The GA-HIDMS-PSO algorithm exhibit similar performance to EPSO, UPSO, HCLPSO, MaPSO and MiPSO in terms of the total number of functions solved with the best mean error.

6.3.1.2 Results on 30 dimensional problems

The experimental results on the 28 30-dimensional CEC'13 problems reveal that the results yield that for unimodal functions (F1~F5), DMS-PSO performed best on F1, F5 whilst L-SHADE, GA-HIDMS-PSO and EPSO attained the best performance on F2, F3 and F4, respectively. The best mean solutions for majority of the multimodal functions (F6~F20) are provided by L-SHADE including F6, F8, F10~19 whilst the proposed algorithm GA-HIDMS-PSO performed best on F7, 20 and DMS-PSO on F9. Moreover, the performances on the composition functions (F1~F28) indicate that the proposed algorithm GA-HIDMS-PSO and DMS-PSO performed best on F24, F27 and F23, F25, respectively. The best solutions for F21, F22, F26 and F28 are achieved individually by LPSO, L-SHADE, MiPSO and EPSO, respectively.

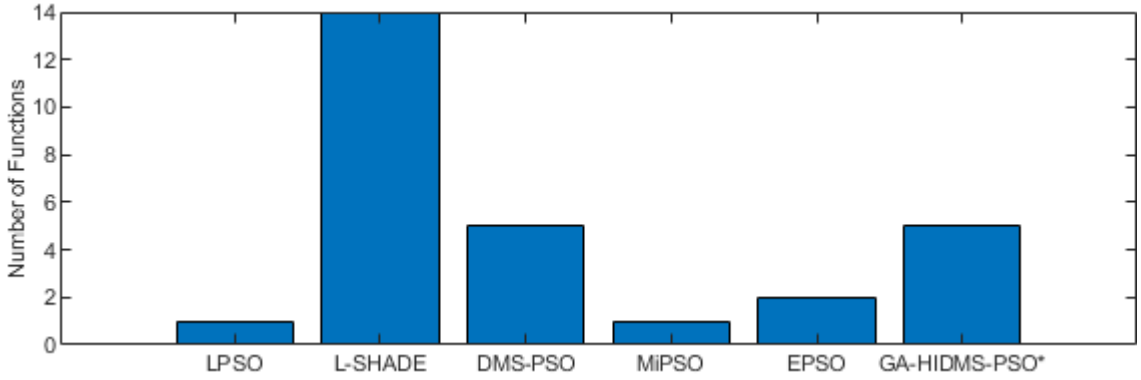


Figure 6-6 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

As shown in Figure 6-6, the best performances for the 30-dimensional CEC'13 problems are attained by 6 of the 14 algorithms experimented. In this case, the GA-HIDMS-PSO solved the second-most functions along with DMS-PSO after L-SHADE.

6.3.1.3 Results on 50 dimensional problems

The mean errors achieved for 28 50-dimensional CEC'13 reveal that on the unimodal functions (F1~F5), DMS-PSO performed best on F1 and F5 whilst L-SHADE, EPSO and the proposed algorithm GA-HIDSMS-PSO individually exhibited the best performance on F2, F3 and F4, respectively. The results on the multimodal functions (F6~F20) indicate that the best solutions for all problems are achieved solely by L-SHADE and the proposed algorithm GA-HIDMS-PSO where L-SHADE performed best on F8, F10, F11, F14, F16~F19 and GA-HIDMS-PSO exhibited the best performance on F7, F9, F12, F13, F15 and F20. On the 50-dimensional composition functions (F21~F28), the proposed algorithm GA-HIDMS-PSO solved more problems compared to rest of the algorithms and obtained the best performance on F23, F24 and F27 whereas the best mean solutions for F21, F22, F25 and F26 are achieved individually by LPSO, L-SHADE, DMS-PSO, MiPSO and HCLPSO, respectively.

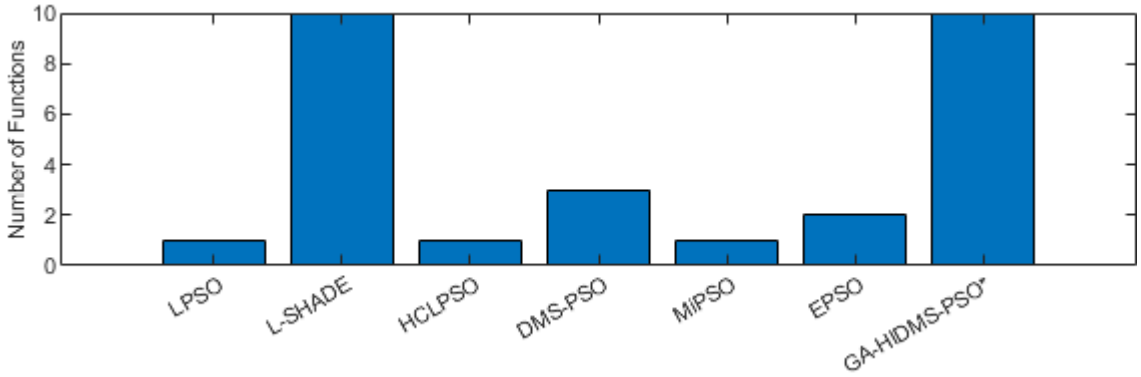


Figure 6-7 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

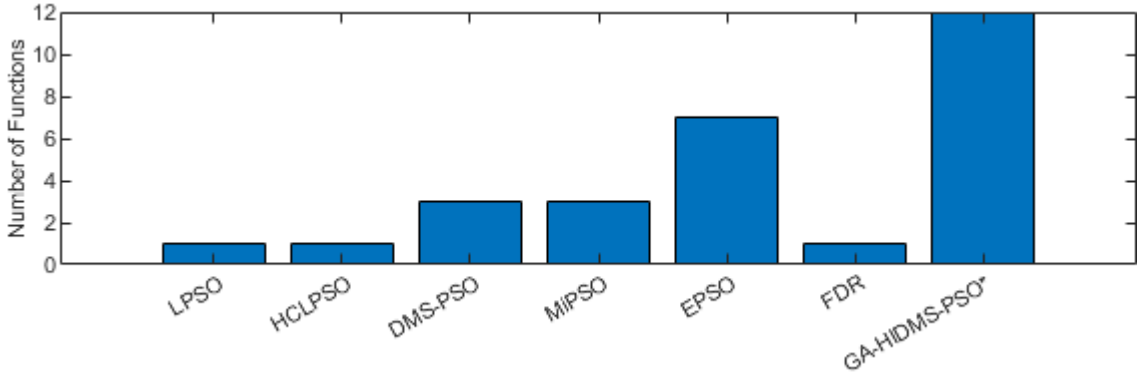


Figure 6-8 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'13 problems

In Figure 6-7, the total number of 50-dimensional CEC'13 functions solved by each algorithm is presented, we can observe that the proposed algorithm GA-HIDMS-PSO and L-SHADE exhibited equal performance on the number of functions solved and both algorithms provided the best solutions for significantly more problems compared to the rest of the algorithms. In Figure 6-8, the same comparison among the PSO variants also yield that, once again, the GA-HIDMS-PSO algorithm provided better mean solutions for significantly more functions.

6.3.1.4 Results on 100 dimensional problems

The results obtained on the 28 100-dimensional CEC'13 problems indicate that for 100-dimensional unimodal functions (F1~F5), DMS-PSO attained the lowest mean error values for F1, F5 whilst L-SHADE, GA-HIDMS-PSO and EPSO performed best on F2, F3 and F4, respectively. The best solutions for majority of the multimodal functions (F6~F20) are provided by the proposed algorithm GA-HIDMS-PSO including F7, F9, F12, F13, F15, F18 and F20 whereas L-SHADED obtained the lowest mean error values for F8, F10, F14, F16, F17 and F10. On the other hand, LPSO and EPSO exhibited the best performance on F6 and F11, respectively. The results on the composition functions (F21~F28) reveal that the proposed algorithm GA-HIDMS-PSO exhibited better performance on more problems compared to other algorithms and performed best on F23, F24, F27 whereas the best mean solutions for F21, F22, F25 and F26 are attained by UPSO, L-SHADE, DMS-PSO, EPSO and HCLPSO, respectively.

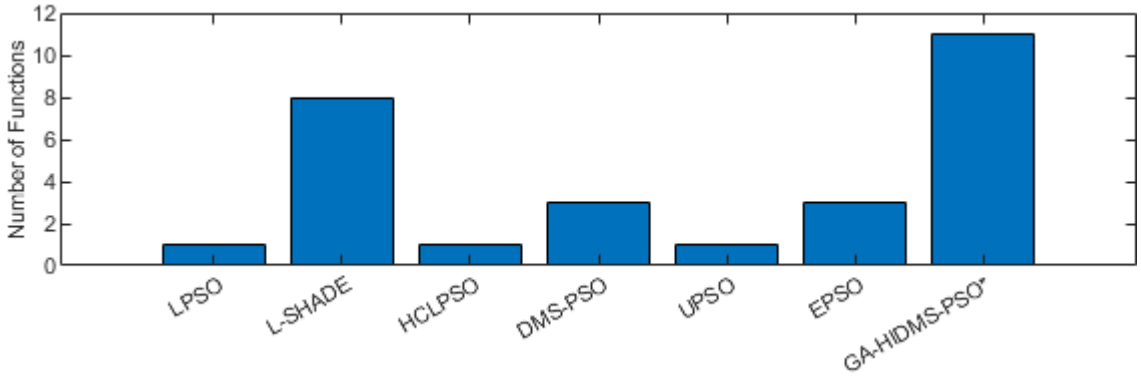


Figure 6-9 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

As shown in Figure 6-9, overall, for the 100-dimensional CEC'13 problems, the proposed algorithm GA-HIDMS-PSO exhibited superior performance on more problems compared to all test algorithms. In addition, the proposed algorithm also provided the second-best solution for 4 problems.

The obtained ranks on four different dimensions reveal that the proposed algorithm GA-HIDMS-PSO is capable of outranking most of the test algorithms at both low and high dimensional problems. More specifically, at 30 dimensions, the GA-HIDMS-PSO ranked 3rd, at 30 and 50 dimensions, GA-HIDMS-PSO achieved the 2nd rank and finally, for the experiment conducted on 100-dimensional problems, the proposed algorithm outranked all comparison algorithms and attained the 1st rank. Further, the Wilcoxon Signed-rank test results are shown in Table 6-2. The results yield that at the significance level of 5%, the proposed algorithm GA-HIDMS-PSO exhibited statistically superior performance than 11 of the 14 test algorithms on 30, 50 and 100-dimensional CEC'13 problems, and at the higher dimensions no other algorithm was statistically better than it.

Table 6-2 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2013 problems

GA-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	+	+	=	+	=	-	+	+	+	9/4/1
30	+	+	+	+	=	+	+	=	+	+	=	+	+	+	11/3/0
50	+	+	+	+	=	+	+	=	+	+	=	+	+	+	11/3/0
100	+	+	+	+	=	+	+	=	+	+	=	+	+	+	11/3/0

Figure 6-10 displays the convergence rate comparison of the GA-HIDMS-PSO against L-SHADE, EPSO, DMS-PSO and HCLPSO on selected 100-dimensional unimodal, multimodal and composition CEC'13 problems. As we can observe from Figure 6-10, the proposed algorithm GA-HIDMS-PSO exhibits comparable convergence trend to L-SHADE and DMS-PSO in most cases. It is also worth highlighting that in case of multimodal F12 and composition F21, the proposed algorithm converged faster than all considered algorithms.

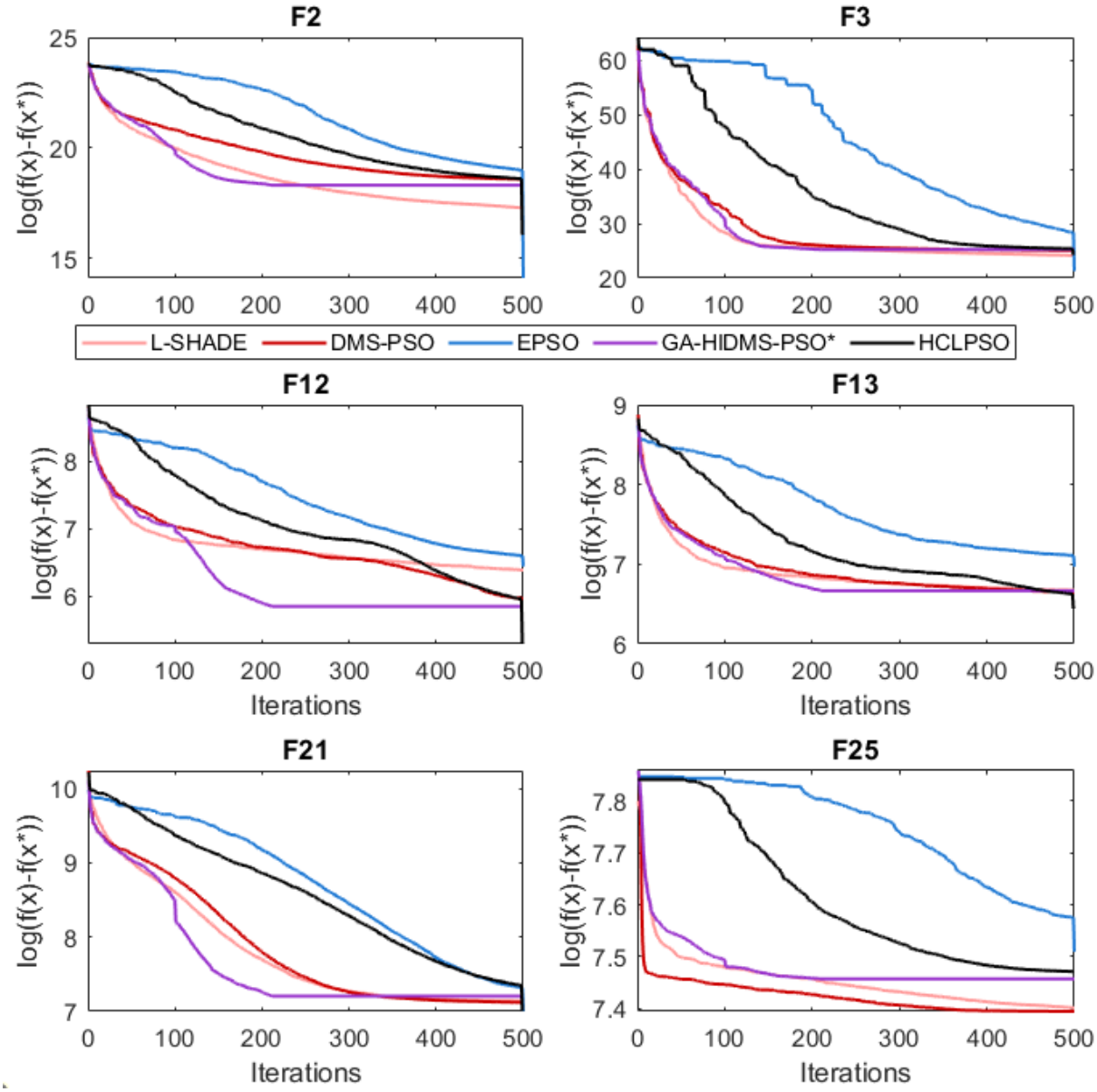


Figure 6-10 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'13 problems

6.3.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed GA-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

6.3.2.1 Results on 10 dimensional problems

The experimental results achieved on the 10-dimensional CEC'14 problems suggest that he best solutions for the entire unimodal subset (F1~F3) of functions are attained by L-SHADE and on the multimodal functions (F4~F15), L-SHADE solved F7~F12, F15 whilst DMS-PSO discovered the best solutions for F6, F13, F14 and F16. On the other hand, the best mean solutions for F4 and F5 are achieved by EPSO and GA-HIDMS-PSO, respectively. The results on the 10-dimensional hybrid

functions show that L-SHADE performed best on all hybrid functions (F17~F22) and on the composition functions, LPSO exhibited the best performance on F28, F29 whilst MaPSO performed best on F23 and F27. The best solutions for F24, F25 and F30 are individually obtained by L-SHADE, EPSO and FIPS, respectively.

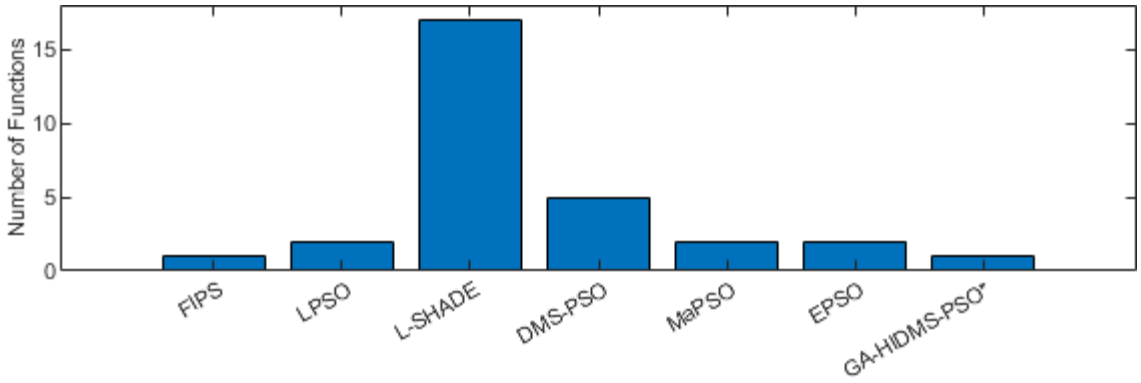


Figure 6-11 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

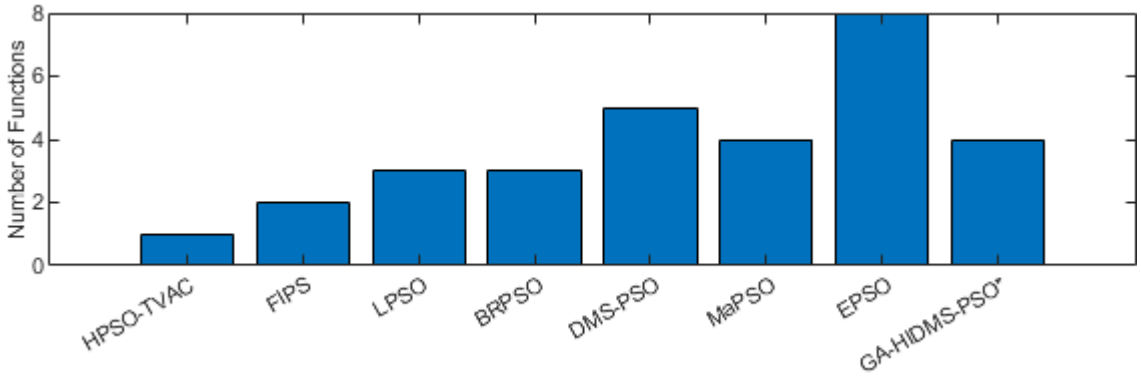


Figure 6-12 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'14 problems

Figure 6-11 displays the total number of problems each algorithm provided the best mean solution and in Figure 6-12, same comparison is display only for PSO variants (without L-SHADE). We can observe that in the first case, L-SHADE provided the best solutions for more functions compared to rest of the algorithms and the latter case, GA-HIDMS-PSO exhibited comparable efficiency to MaPSO and DMS-PSO.

6.3.2.2 Results on 30 dimensional problems

The experimental results for 30-dimensional CEC'14 problems indicate that the best performance for 3 unimodal functions (F1~F3) are obtained by L-SHADE and for multimodal functions (F4~F15), once again, L-SHADE performed best on F4, F8~F12, F15 and F16. On the other hand, the lowest mean error values for F5, F6, F7, F13 and F14 are attained individually by HPSO-TVAC, GA-HIDMS-PSO, FIPS, DMS-PSO and FDR, respectively. The results on the hybrid functions (F17~F22) yield that the best solutions for F17, F19, F20 and F21 are provided by L-SHADE whereas the best performance for F18 and F20 are obtained by LPSO and MaPSO, respectively. The results on the composition functions (F23~F30) indicate that DMS-PSO solved F23, F25 and F29 whilst the lowest mean error values for F24, F26, F27, F28 and F30 are individually provided by FIPS, L-SHADE, GA-HIDMS-PSO, LPSO and MaPSO, respectively.

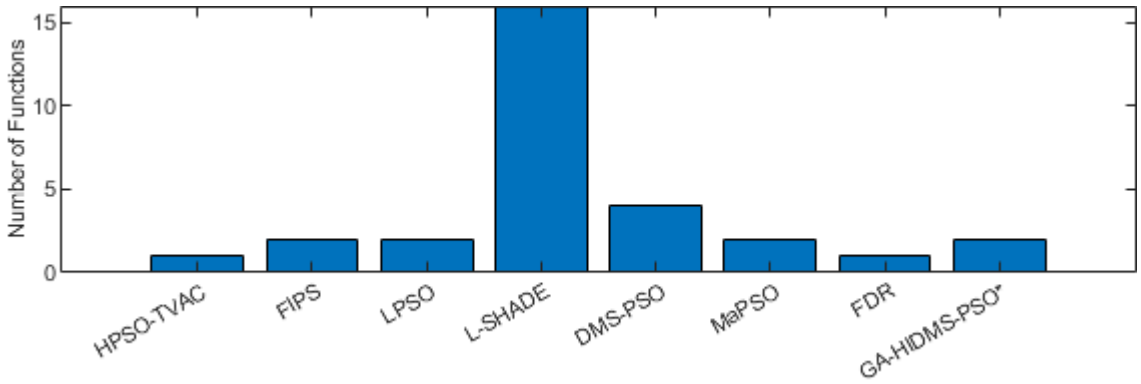


Figure 6-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'14 problems

Figure 6-13 presents the total number of functions solved with the lowest mean error values by each algorithm. As indicated, the proposed algorithm GA-HIDMS-PSO exhibited comparable performance to most of the algorithms with respect to the number of functions solved.

6.3.2.3 Results on 50 dimensional problems

The results obtained on the 50-dimensional CEC'14 problems suggest that L-SHADE exhibited the best performance for the unimodal functions (F1~F3). Moreover, on the multimodal functions (F4~F16), once again, L-SHADE performed best no most of the multimodal functions including F4, F8~F12, F15, F16 whilst the lowest mean error values for F5, F6, F7, F13 and F14 are provided by HPSO-TVAC, GA-HIDMS-PSO, FIPS, DMS-PSO and LPSO, respectively. The results on the hybrid functions (F17~F22) yield that L-HADE performed best on F17~F19, F21 whilst GA-HIDMS-PSO and DMS-PSO exhibited the best performance on F20 and F22, respectively. For composition functions (F23~F30), we observe that DMS-PSO obtained the performance on F23, F25, F28, F29 whilst the best mean solutions for F24, F26, F27 and F30 are attained by MiPSO, FIPS, GA-HIDMS-PSO and LPSO, respectively.

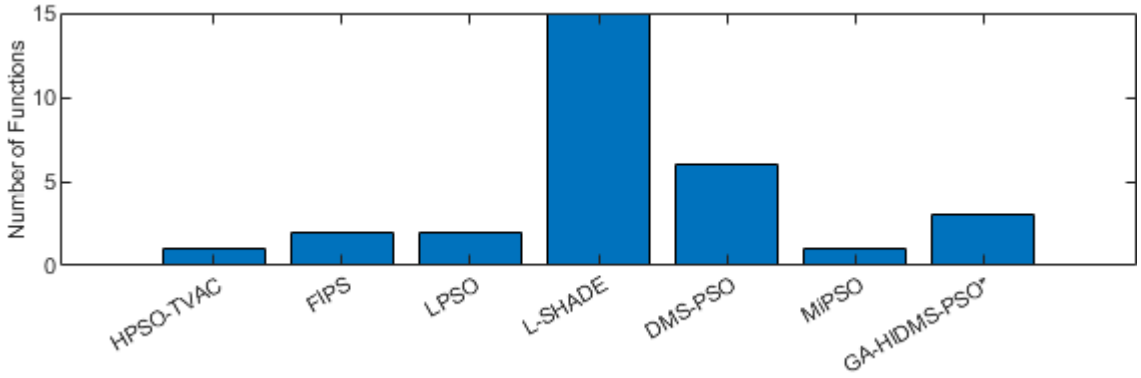


Figure 6-14 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

Overall, as shown in Figure 6-14, the lowest mean error values for 50-dimensional CEC'14 problems are obtained by 7 of the 14 algorithms employed in the experiments. The results suggests that in this case, L-SHADE performed best on most problems whilst GA-HIDMS-PSO exhibited the third-best performance in terms of the number of functions provided with the best mean error.

6.3.2.4 Results on 100 dimensional problems

The results on the 100-dimensional unimodal functions reveal that L-SHADE performed best on F1, F2 whilst EPSO solved F3. We observe from the results obtained for multimodal functions (F4~F16) that L-SHADE exhibited the best performance on F10~F12, F16 whilst GA-HIDMS-PSO solved F6, F9, F15 and HCLPSO performed best on F5 and F14. The lowest mean error values for F4, F7, F8 and F13 are attained by LPSO, UPSO, EPSO and DMS-PSO, respectively. For the 100-dimensional hybrid functions (F17~F22), the performance of L-SHADE yields the best mean error values on F17 and F21 whilst MaPSO, LPSO, EPSO and DMS-PSO each performed best on a single function namely, F18, F19, F20 and F22, respectively. The performances on the composition functions (F23~F30) yield that LPSO obtained the best performance for most of the composition functions including F25, F28, F29 and F30 whilst the lowest mean error values for F23, F24, F26 and F27 are provided individually by DMS-PSO, HPSO-TVAC, FIPS and GA-HIDMS-PSO, respectively.

We can observe from Figure 6-15 that on the 100-dimensional CEC'13 problems, L-SHADE and LPSO obtained the lowest mean error values for majority of the problems. For the same problem set, the proposed algorithm GA-HIDMS-PSO placed third and exhibited the best mean performance for 4 problems after L-SHADE and LPSO.

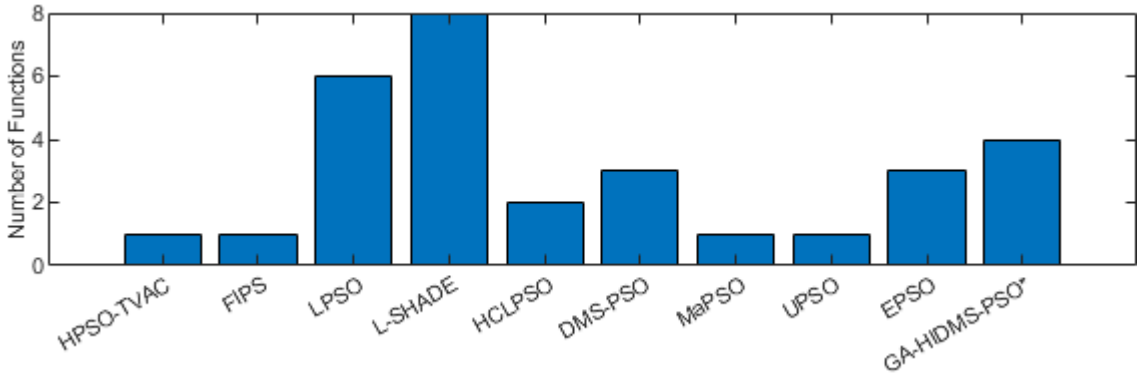


Figure 6-15 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

The performance of all algorithms is ranked based on the mean error values obtained. Details of the results and ranks can be seen in Appendix 2. The mean and final ranks render that for the 10, 30 and 100-dimensional CEC'14 test problems, the proposed algorithm ranked 4th and on the 50-dimensional problems, the GA-HIDMS-PSO algorithm obtained the 5th rank. Moreover, Table 6-4 display the Wilcoxon signed rank test that is conducted to determine the statistical significance between GA-HIDMS-PSO and other algorithms. The results suggest that at 10 dimensions, the GA-HIDMS-PSO is significantly better than 7 algorithms. On the other hand, for 30, 50 and 100-dimensional cases, the GA-HIDMS-PSO algorithm exhibited superior mean performance against 8 test algorithms, the only algorithm that is statistically significantly better than it is L-SHADE (at 10,30 and 50 dimensions but not at 100).

Table 6-3 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2014 problems

GA-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	+	+	=	+	=	-	=	=	+	7/6/1
30	+	+	+	=	=	+	+	=	+	=	-	+	=	+	8/5/1
50	+	+	+	=	=	+	+	=	=	=	-	+	+	+	8/5/1
100	+	+	+	=	=	+	+	=	=	=	=	+	+	+	8/6/0

Furthermore, in Figure 6-16, the convergence characteristics of the proposed GA-HIDMS-PSO is compared with L-SHADE, EPSO, HCLPSO and DMS-PSO on the 100-dimensional unimodal, multimodal, hybrid and composition functions from the CEC'14 benchmark test suite. The comparison does not provide conclusive evidence to point out a single best converging algorithm. However, in most cases, the proposed algorithm exhibited better convergence trend than HCLPSO and EPSO, and in various test cases (e.g., multimodal F7, F9, F14, hybrid F19 and composition F23, F25), the proposed algorithm GA-HIDMS-PSO exhibited better convergence characteristics in terms of convergence speed and the solution quality compared to most algorithms.

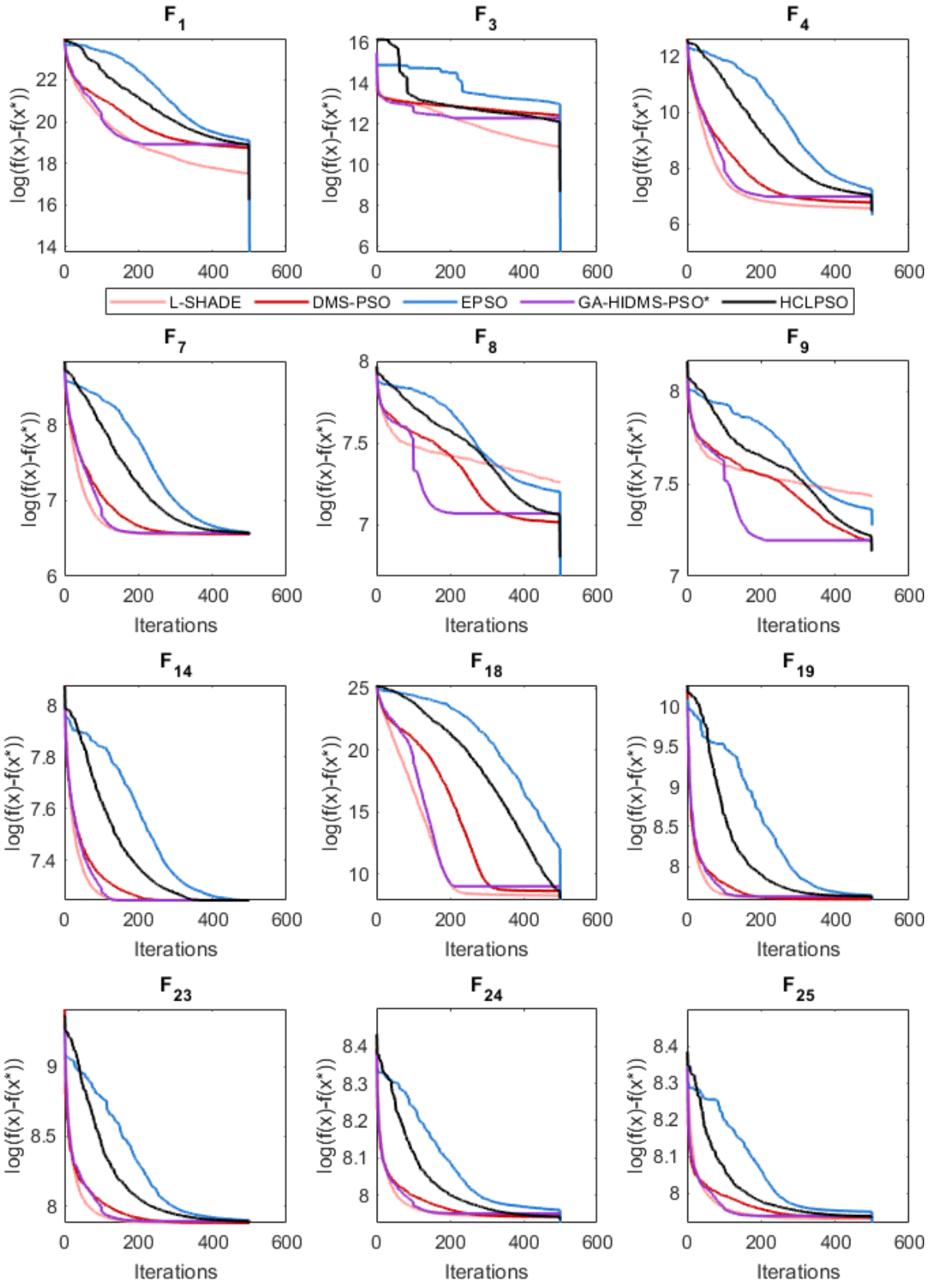


Figure 6-16 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'14 problems

6.3.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed GA-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

6.3.3.1 Results on 10 dimensional problems

The mean error values obtained on the 10-dimensional CEC'17 problems suggest that for the multimodal functions (F3~F9), L-SHADE exhibited the best performance on F3, F4, F5, F7 and F8. On the other hand, EPSO and FIPS obtained the lowest mean errors for F6 and F9, respectively. Similarly, on the hybrid functions (F10~ F19), L-SHADE solved most 9 of the 10 hybrid functions including F10~F15, F17~F19 whilst for F16, the best mean performance is exhibited by EPSO. On the composition functions (F20~F30), MiPSO (F21,F24), MaPSO (F20, F26) and HCLPSO (F25, F28) solved 2 functions individually whilst EPSO, GA-HIDMS-PSO, LPSO, L-SHADE and DMS-PSO individually provided the best mean performance on F22, F23, F27, F29 and F30, respectively.

In Figure 6-17, we present the performance of each algorithm in terms of the total number of functions solved with the best mean error. We can observe that L-SHADE provided better mean performance for most of the CEC'17 problems at dimensions, whilst the GA-HIDMS-PSO exhibited comparable performance to most of the PSO variants employed in the same experiment.

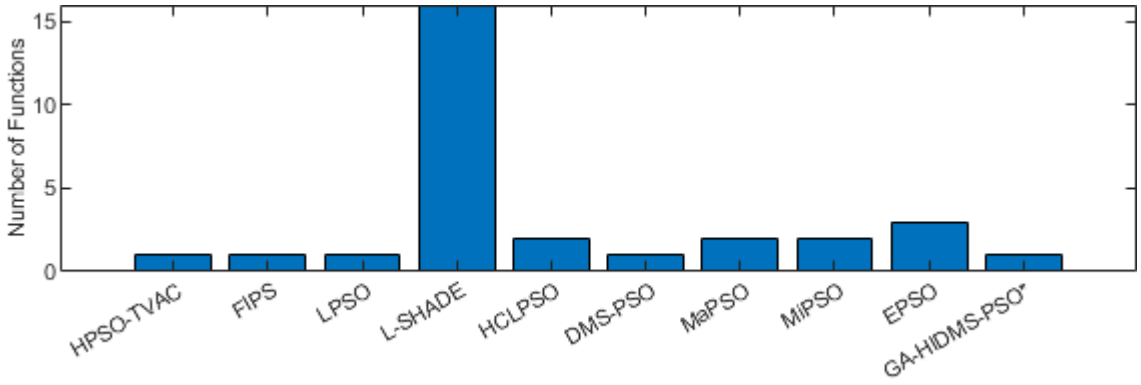


Figure 6-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'17 problems

6.3.3.2 Results on 30 dimensional problems

The experimental results on the 30-dimensional CEC'17 problems indicate that on the multimodal functions (F3~F9), the best mean performance for F5, F7 and F8 is provided by L-SHADE whilst EPSO achieved the best performance on F3 and F4. On the other hand, MiPSO and FIPS attained the lowest mean errors for F6 and F9, respectively. For hybrid functions (F10~F19), once again, L-SHADE obtained the best performance on F10, F12, F13, F14, F18 and F19 whilst DMS-PSO solved F11, F16 and the lowest mean error values for F15 and F17 are achieved by MaPSO and HCLPSO, respectively. The results obtained on the 30-dimensional composition functions suggest that GA-HIDMS-PSO and L-SHADE solved 3 functions individually whilst LPSO performed best on F22 and F30. Moreover, the best performances on F25, F27 and F29 are individually obtained by FIPS, MiPSO and DMS-PSO, respectively.

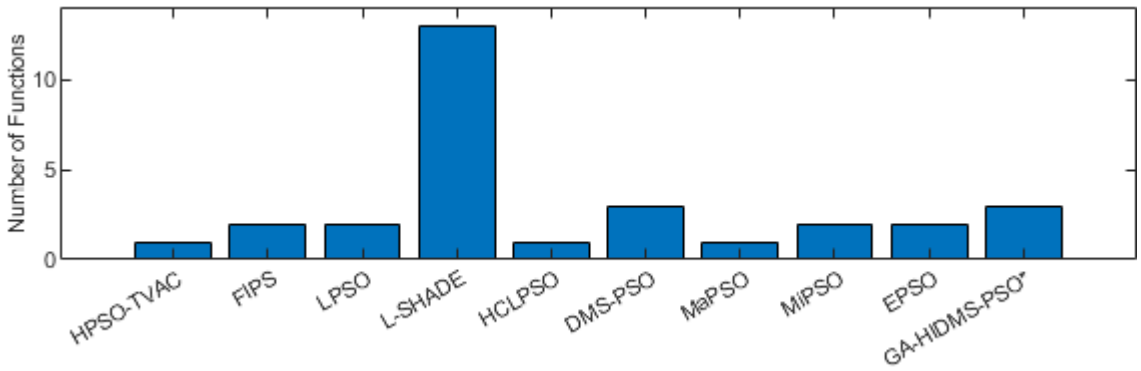


Figure 6-18 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

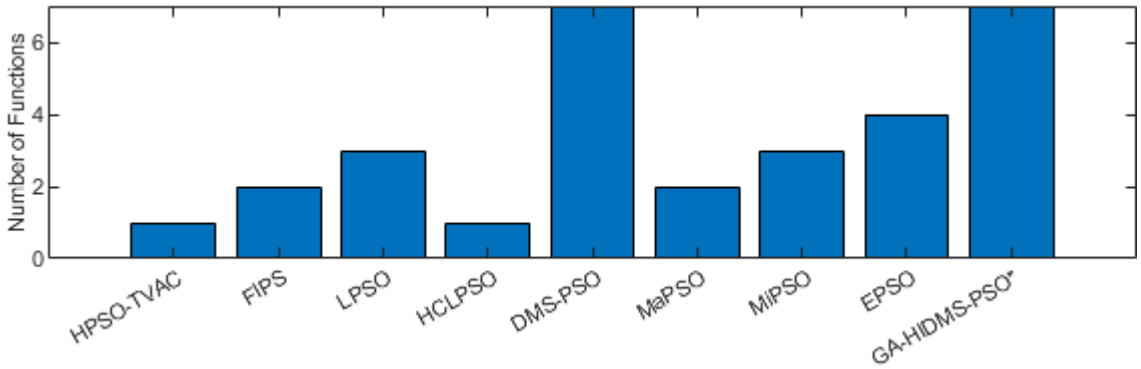


Figure 6-19 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

Figure 6-18:6-19 display the total number of test problems each algorithm outperformed rest of the algorithms with regards to mean performance. The first comparison (Figure 6-18) yields that on the 30-dimensional CEC'17 problems, L-SHADE obtained better solutions for most of the problems. However, in the second case (Figure 6-19), the comparison among PSO variants reveal that the proposed algorithm GA-HIDMS-PSO and DMS-PSO outperform other PSO algorithms.

6.3.3.3 Results on 50 dimensional problems

The results obtained on the 50-dimensional CEC'17 problems suggest that for multimodal functions (F3~F9), EPSO (F3, F4), FIPS (F6, F9) and L-SHADE (F5, F8) individually solved 2 problems whilst GA-HIDMS-PSO provided the lowest mean error value for F7. For hybrid functions (F10~F19), the results indicate that L-SHADE performed best on F10, F12, F14, F15, F18 and F19 whilst the proposed algorithm GA-HIDMS-PSO performed best on F11, F16 and F17. Furthermore, the results on the 50-dimensional composition functions reveal that the proposed algorithm GA-HIDMS-PSO exhibited the best performance on majority of the composition functions including F20, F22, F23, F26 and F29 whilst L-SHADE solved F21 and F24. On the other hand, the lowest mean error values for F25, F27, F28 and F30 are individually obtained by FIPS, HPSO-TVAC, DMS-PSO and LPSO, respectively.

Overall, the comparison of the number of test problems each algorithm solved is shown in Figure 6-20. The comparison suggests that L-SHADE and the proposed algorithm GA-HIDMS-PSO obtained the best mean solutions for majority of the 50-dimensional CEC'17 problems. In Figure 6-21, the same comparison is displayed for solely PSO variants which yields that in this case, the proposed GA-HIDMS-PSO algorithm exhibited better performance for most of the problems compared to all PSO variants.

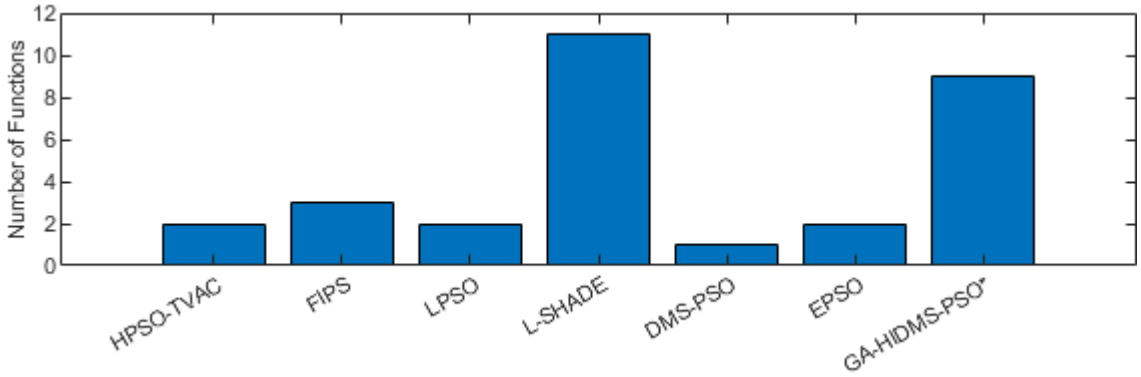


Figure 6-20 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

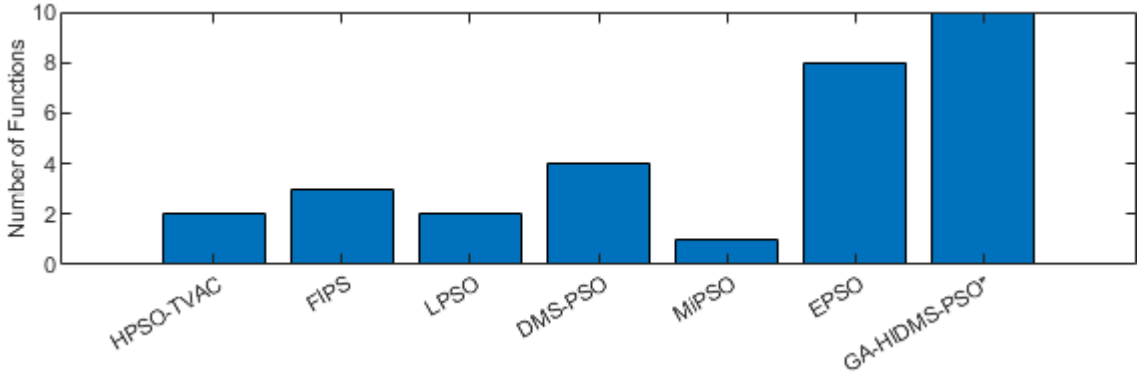


Figure 6-21 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'17 problems

6.3.3.4 Results on 100 dimensional problems

The mean errors obtained for each 100-dimensional CEC'17 problem indicate that for the multimodal functions (F3~F9), the proposed algorithm GA-HDIMS-PSO (F7, F9), EPSO (F3, F6) and DMS-PSO (F5, F8) solved 2 functions individually whilst LPSO achieved the best mean performance for F4. For hybrid functions (F10~F19), the mean performance for F10, F12, F14 and F15 is achieved by L-SHADE whilst EPSO performed best on F13, F18 and F19. Further, the proposed algorithm GA-HIDMS-PSO attained the lowest mean error values on F16, F17 and DMS-PSO achieved the best mean performance on F11. The results obtained on the 100-dimensional composition functions (F20~30) yield that the proposed algorithm GA-HIDMS-PSO provided the best performance for most of the composition functions including F20, F24, F25, F26 and F29 whilst L-SHADE (F22, F23), LPSO (F28, F30) and DMS-PSO (F21, F27) performed best on 2 functions individually.

On the 100-dimensional CEC'17 problems, the lowest mean error values achieved for total number of test problems are shown in Figure 6-22. The comparison reveals that the proposed algorithm GA-HIDMS-PSO obtained better mean solutions for more problems compared to all test algorithms. Similarly in Figure 6-23, the same comparison among only PSO variants also suggest that the GA-HIDMS-PSO algorithm once again exhibited better mean performance for more problems compared to all PSO variants. The relative performance of GA-HIDMS-PSO again significantly improved with problem dimension.

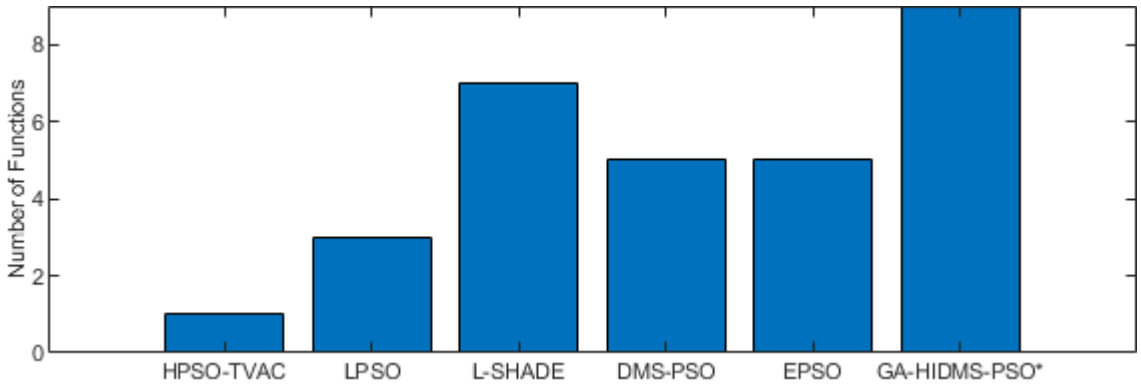


Figure 6-22 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

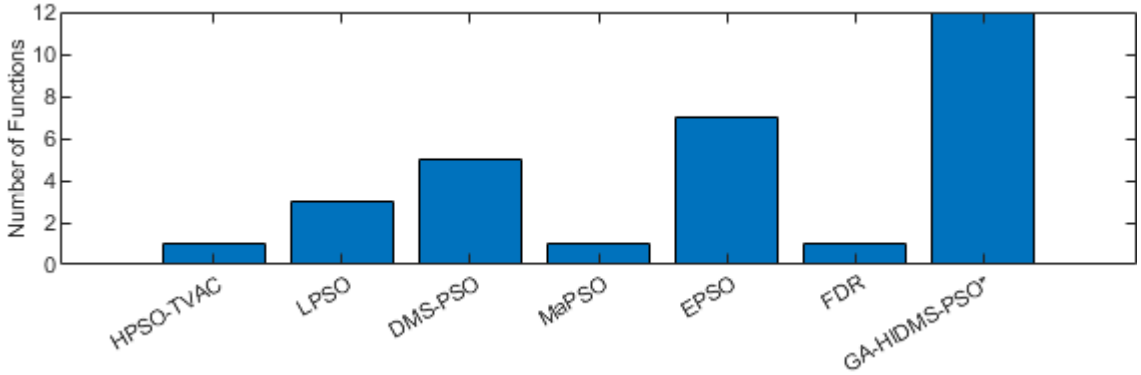


Figure 6-23 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'17 problems

The mean and final ranks calculated for each algorithm based on the mean error values obtained on the 29 problems at four different dimensions suggest that the proposed algorithm GA-HIDMS-PSO obtained the 7th, 3rd, 2nd and 1st final ranks for 10, 30, 50 and 100-dimensional CEC'17 test problems, respectively. Moreover, the Wilcoxon signed rank test is conducted to determine if the proposed algorithm GA-HIDMS-PSO's performance is significantly different to the performance of any of the test algorithms employed in the experiments. The details of the Wilcoxon signed rank test is presented in Table 6-6 and the results yield that the mean performance of GA-HIDMS-PSO is significantly superior compared to most of the algorithms on all dimensions experimented. It is also worth mentioning that the Wilcoxon signed rank test results also suggest that none of the test algorithms exhibited statistically significantly better performance against GA-HIDMS-PSO at 30, 50 and 100 dimensions.

Table 6-4 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2017 problems

GA-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	-	+	+	-	+	=	-	=	-	=	6/4/4
30	+	+	+	=	=	+	+	=	+	=	=	+	+	+	9/5/0

50	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0
100	+	+	+	+	+	+	+	+	+	+	=	+	+	+	13/1/0

Furthermore, the convergence rate comparison of the GA-HIDMS-PSO against the 4 top ranked algorithms L-SHADE, EPSO, DMS-PSO and HCLPSO is shown in Figure 6-24. The comparison suggests that the proposed algorithm GA-HIDMS-PSO generally exhibits better convergence trend against EPSO and HCLPSO, and comparable convergence behaviour against L-SHADE in most cases. It is worth noting that the GA-HIDMS-PSO algorithm specifically exhibits better convergence compared to other algorithms on the composition functions (F23, F24, F29).

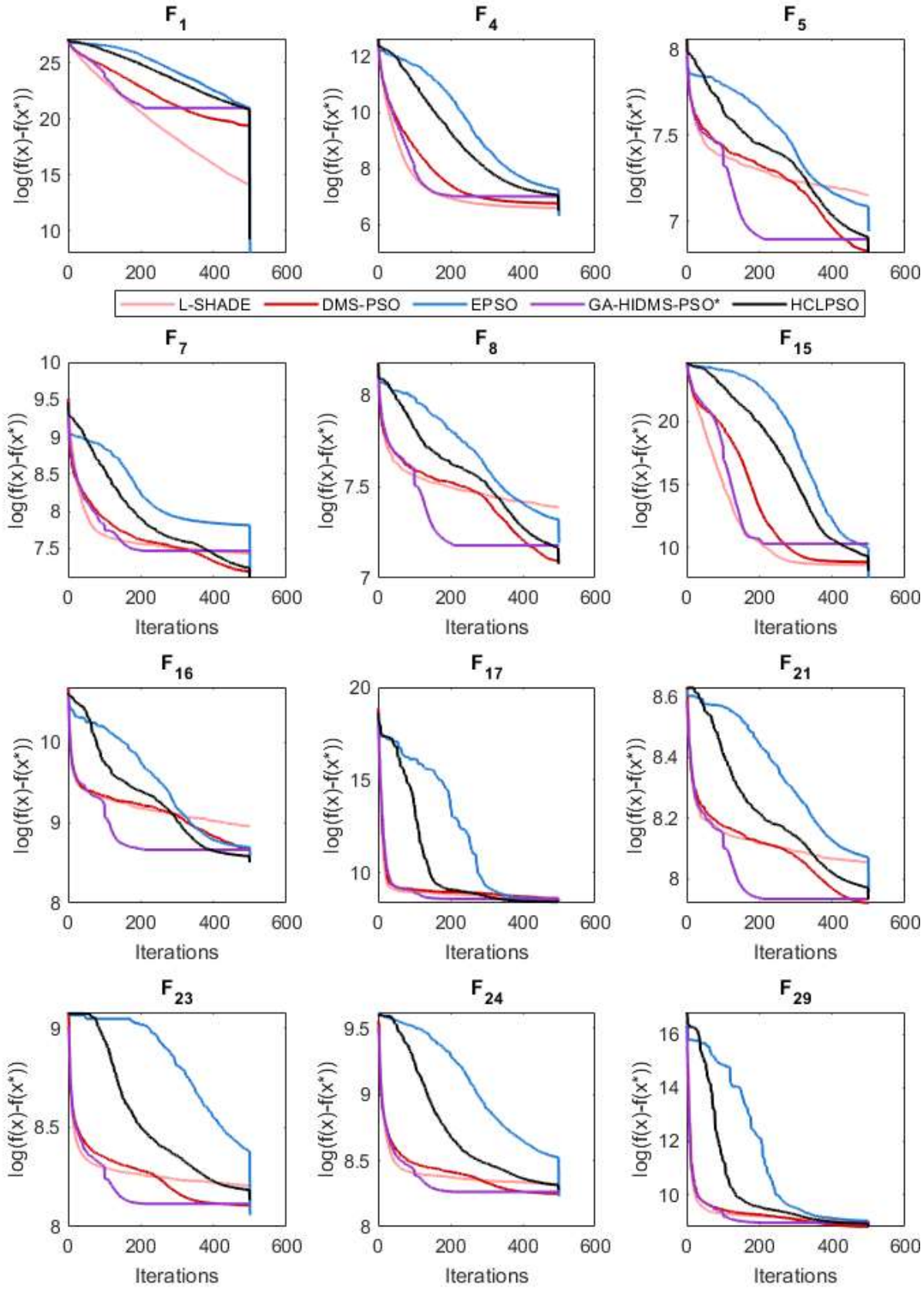


Figure 6-24 Average convergence rate comparison of HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'17 problems

In addition, the total number of problems solved by the proposed algorithm GA-HIDMS-PSO is compared with all test algorithms on all dimensions of the three test suites CEC'13, CEC'14 and CEC'17. Figure 6-25-6-27 exhibit the pairwise comparison and in summary, the results suggest that on all three test suites and at each dimension, the proposed algorithm GA-HIDMS-PSO obtained better mean error values for significantly more problems compared to the majority of the test algorithms, especially on higher dimensional problems.

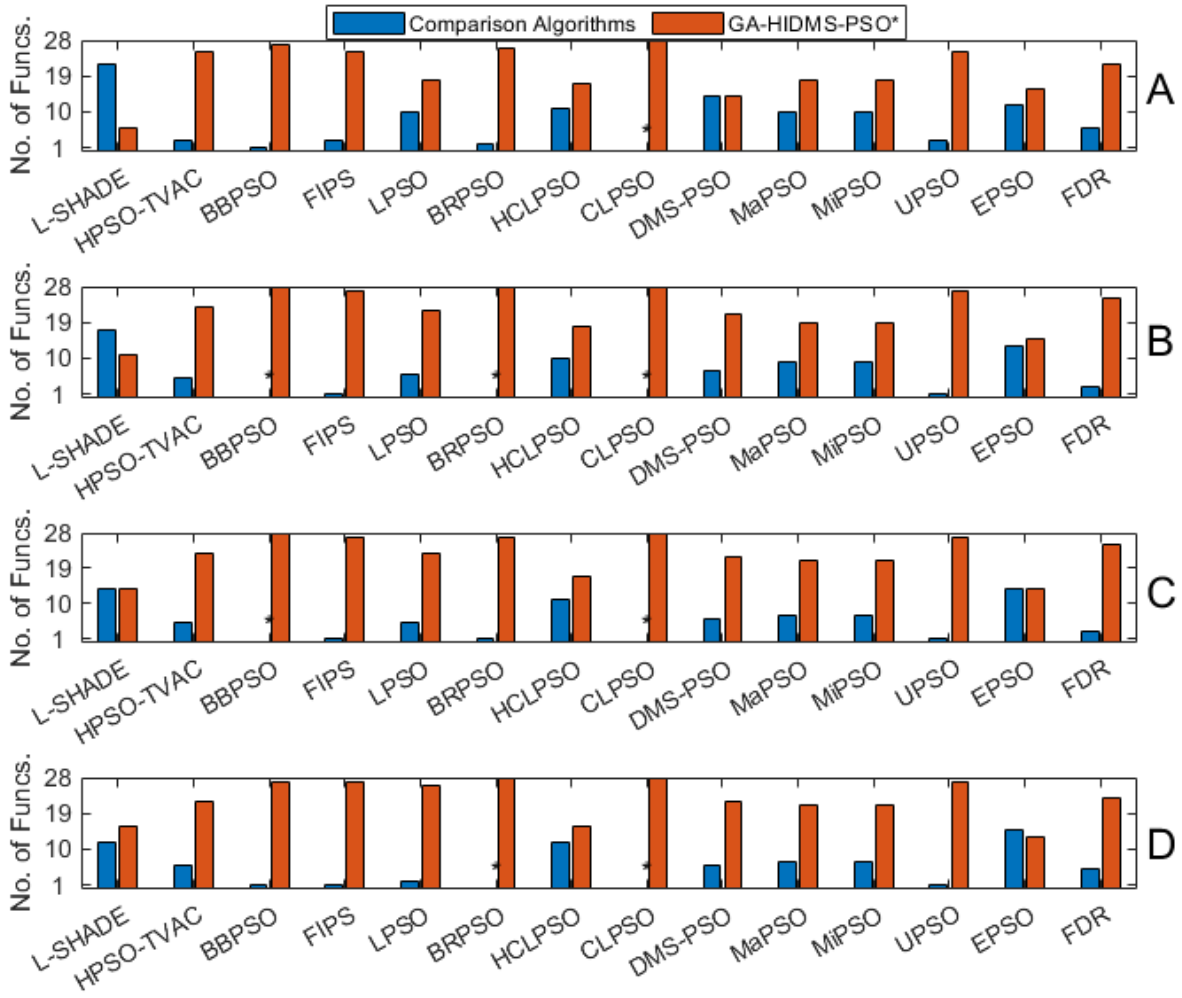


Figure 6-25 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'13 test functions

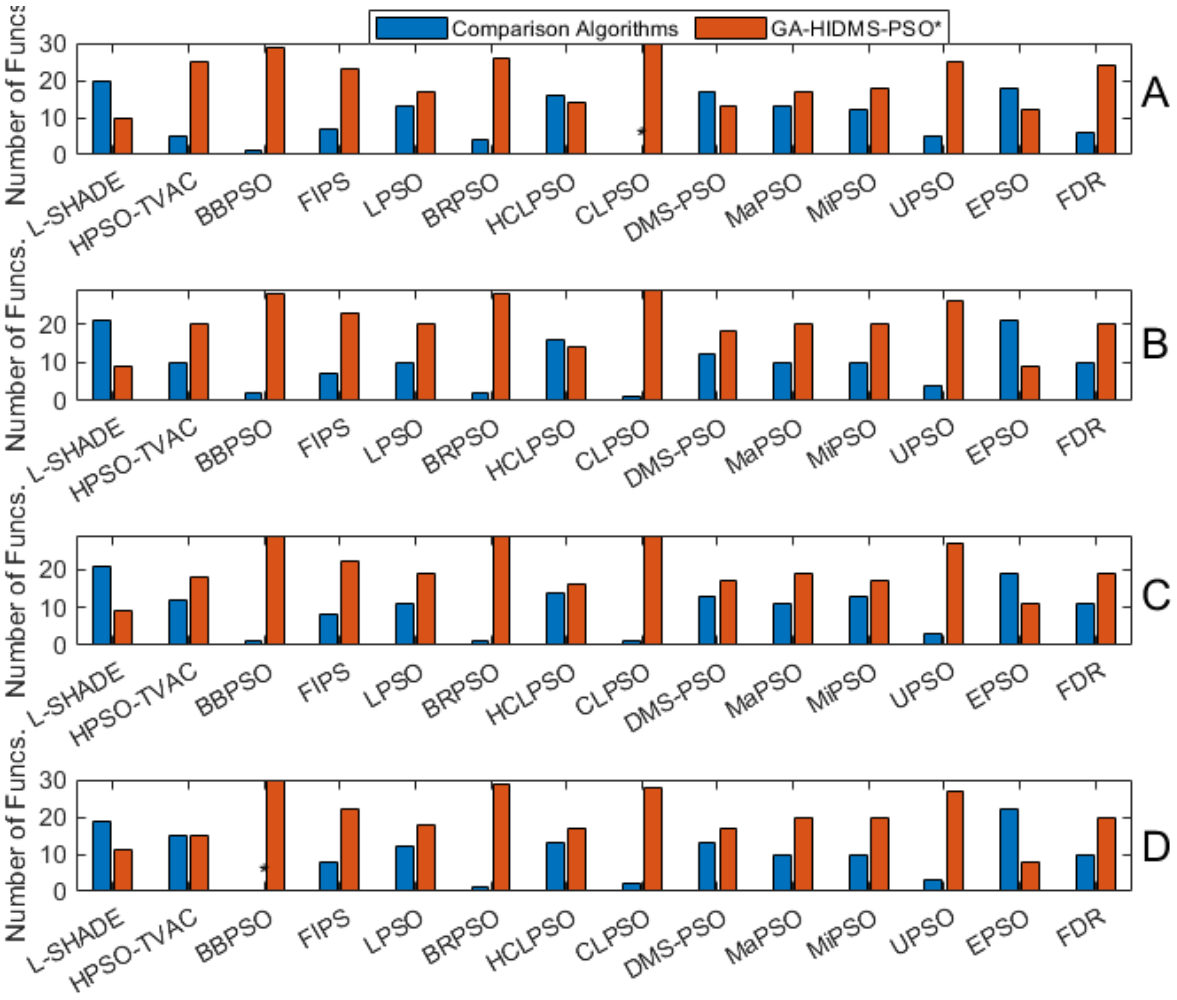


Figure 6-26 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'14 test functions

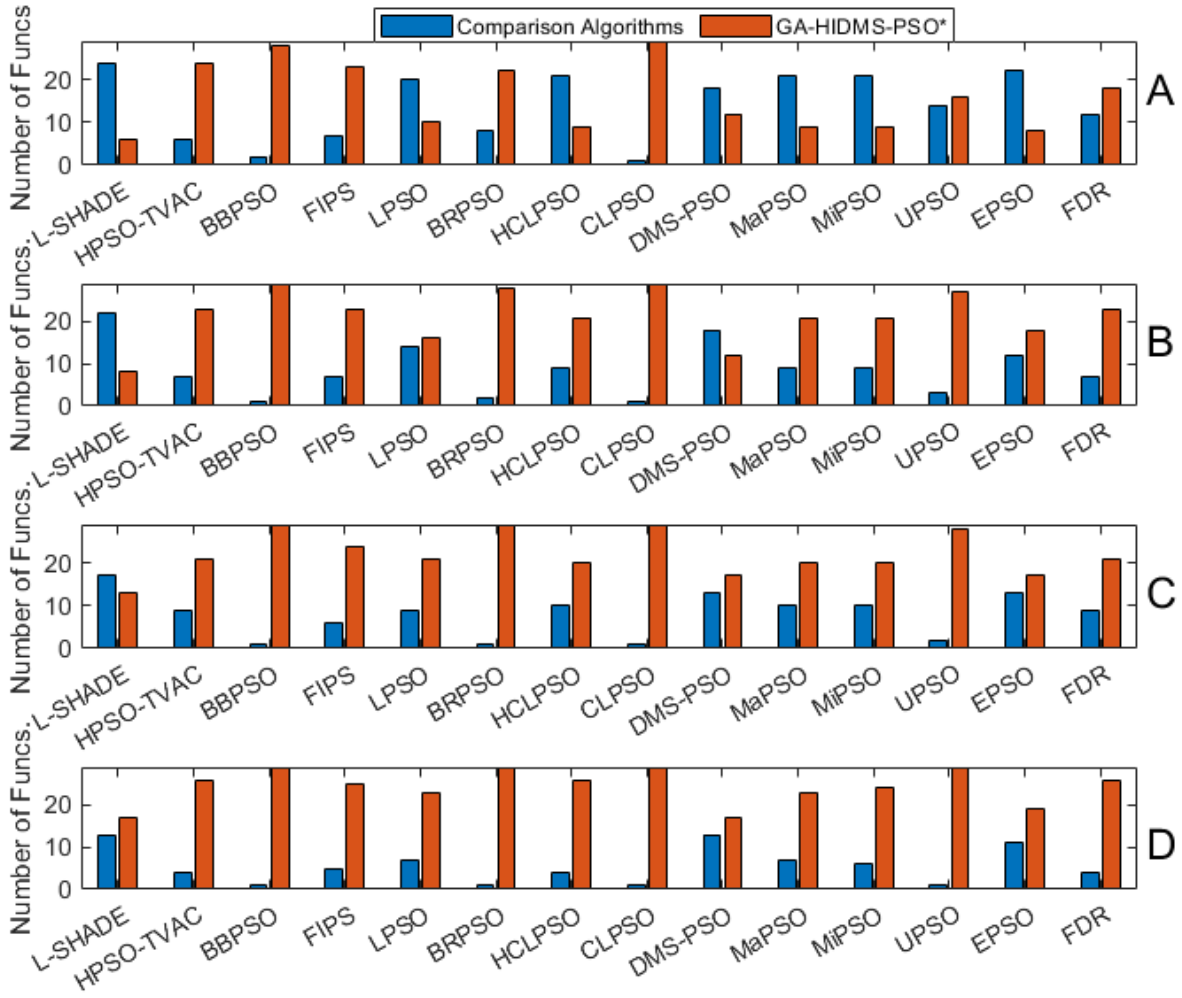


Figure 6-27 Pairwise mean performance comparison of HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'17 test functions

6.4 Discussions

The efficacy of the genetic algorithm assisted HIDMS-PSO was tested on all available dimensions of the three distinct test suites of CEC'13, CEC'14 and CEC'17. The entire problem set includes 8 unimodal, 35 multimodal, 16 hybrid and 26 composition functions and the performance was compared with 13 well-known PSO variants and the CEC competition winner L-SHADE.

A brief glance to the number of functions solved by the proposed algorithm yields that at dimensions 10,30,50 and 100 dimensions, the GA-HIDMS-PSO algorithm attained the lowest mean error values for 3, 10, 22 and 24 (out of 86) problems, respectively. For the same experiments, EPSO solved 6 problems at 10 dimensions, 4 problems at 30 and 50 dimensions and finally, EPSO exhibited the best performance on 11 problems for 100-dimensional problems. A more detailed comparison is shown in Figure6-28 that highlights the number of unimodal, multimodal, hybrid and composition functions solved by GA-HIDMS-PSO, EPSO and L-SHADE at 50 and 100 dimensions across CEC'13, CEC'14 and CEC'17 test suites. We can observe from Figure 6-28 that the proposed algorithm GA-HIDMS-PSO solved more multimodal and composition functions compared to top ranked EPSO and L-SHADE. On the other hand, for unimodal and hybrid functions, GA-HIDMS-PSO did exhibit acceptable performance compared to rest of the test algorithms by solving a reasonable number of

unimodal and hybrid functions, however the performance is inferior compared to EPSO and L-SHADE.

However, a broader consideration of the mean performance based on the obtained ranks on 50 and 100-dimensional CEC'3 and CEC'17 benchmark problems reveal that the proposed algorithm obtained the 2nd rank on the 50-dimensional CEC'13 and CEC'17 and the 1st rank on the 100-dimensional CEC'13 and CEC'17 benchmark test suites. Similarly, the Wilcoxon signed rank test reaffirmed the superiority of the GA-DHISMS-PSO against most of the test algorithms, specifically on 50 and 100-dimensional problems.

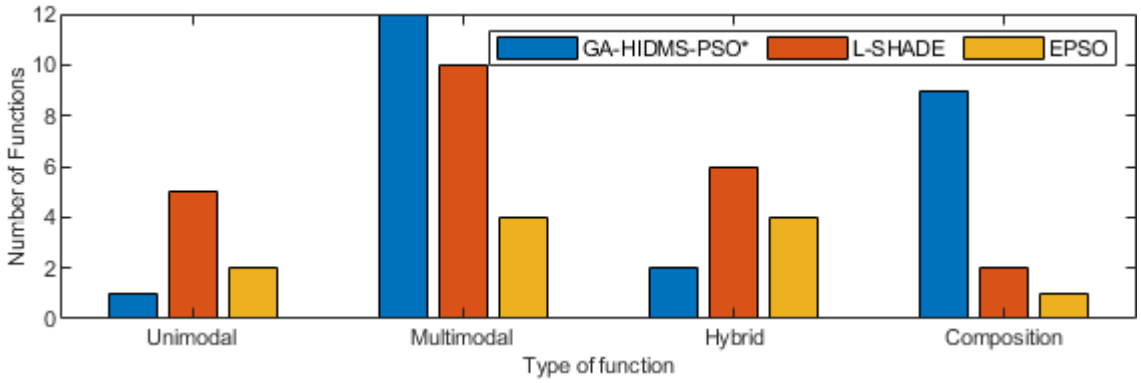


Figure 6-28 Comparison of different function types solved at 100 dimensions across all test suites by EPSO, L-SHADE and GA-HIDMS-PSO

6.5 Conclusion

This chapter presented the genetic algorithm assisted HIDMS-PSO. The GA-HIDMS-PSO utilises the sequential collaborative and partial manipulative integrative hybrid frameworks to efficiently exploit the heterogeneous features of HIDMS-PSO. The proposed extension employs HIDMS-PSO as the primary method whereas the role of GA is limited to solely boost the population diversity of the heterogeneous subpopulation in the HIDMS-PSO. Both algorithms are executed consecutively which in return yields a periodic revamping of the main population possessed by HIDMS-PSO. The evidence produced in this is obtained by conducting experiments on 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 benchmark test suites using 14 test algorithms. The experimental results, various comparisons on different aspect of performances and statistical significance comparison suggest that the GA-HIDMS-PSO is capable of outperforming most, and in some cases all, of the test algorithms employed in the experiments.

Chapter 7 : Heterogeneous Improved Dynamic Multi-Swarm Particle Optimisation Algorithm with Adaptive Topologies

7.1 Introduction

The efficacy of the canonical PSO algorithm is particularly limited when dealing with multimodal problems due to premature convergence to a local solution triggered by particles' rapid loss of population diversity [241]. In the last few decades, PSO has been extensively applied in diverse real-world problems [242] and with many variants emerging to solve problems that would be very challenging to the canonical version. The HIDMS-PSO algorithm introduces subswarm-like entities, called units, with a master-slave inspired topological structure. The unit structure introduced in HIDMS-PSO (Fig 4-1) is an essential component for efficient performance, and any alteration to the structure of the unit topology could significantly change the dynamics of the algorithm's search behaviour. In the standard HIDMS-PSO algorithm, units are designed as static entities. The algorithm does not possess a mechanism to monitor and prevent a loss of individual units' diversity; instead, units are employed as the primary mechanism to maintain population-wide diversity during the search process, to prevent the problem of premature convergence to local optima. Hence, considering the importance of the unit structure and the irrefutable impact of topology [228] on PSO's general performance, in this chapter, we present a new variant of the HIDMS-PSO algorithm with dynamically changing multiple topologies for the unit structure introduced in the standard HIDMS-PSO, in order to explore the potential of dynamic topologies. The eight newly introduced topological structures are categorised as slave dominated, or master dominated, based on the density of the dominating particle type. The search behaviour of the proposed algorithm maintains a balanced master-slave population by forming either slave or master dominated units in the heterogeneous subpopulation. Periodic switches between the two classes of topological structures allow units to adapt their behaviour at individual and swarm levels. The aim of these mechanisms is to maintain better population diversity and prevent too-rapid convergence throughout the search process, leading to a more efficient search.

7.2 Heterogeneous Improved Dynamic Multi-Swarm Particle Optimisation Algorithm with Adaptive Topologies

The variant A-HIDMS-PSO exploits the static form of the unit topological structure introduced in the standard HIDMS-PSO algorithm by introducing various new topologies for the unit structure to enable units to maintain a better balance of exploration and exploitation. As shown in Figure 7-1, the new topologies consist of four slave-dominated and four master-dominated topologies. The connection lines in Figure 7-1 show the influence of the higher-ranked particle (e.g. master) on the lower-ranked particle (e.g. slave). The population of slave-dominated topologies is comprised predominantly by particles with slave roles. On the contrary, the population is formed mostly by the particles with master roles. In the new variant, units maintain their behavioural balance of exploration and exploitation by switching units' topological structures between the master-dominated and slave-dominated topologies. In contrast to the standard HIDMS-PSO, the master particles in the A-HIDMS-PSO solely use the outward-oriented learning (*see Equation 4.1 and 4.2*) strategy whilst the slave particles randomly choose either inward or outward-oriented learning to guide their movements in the search space.

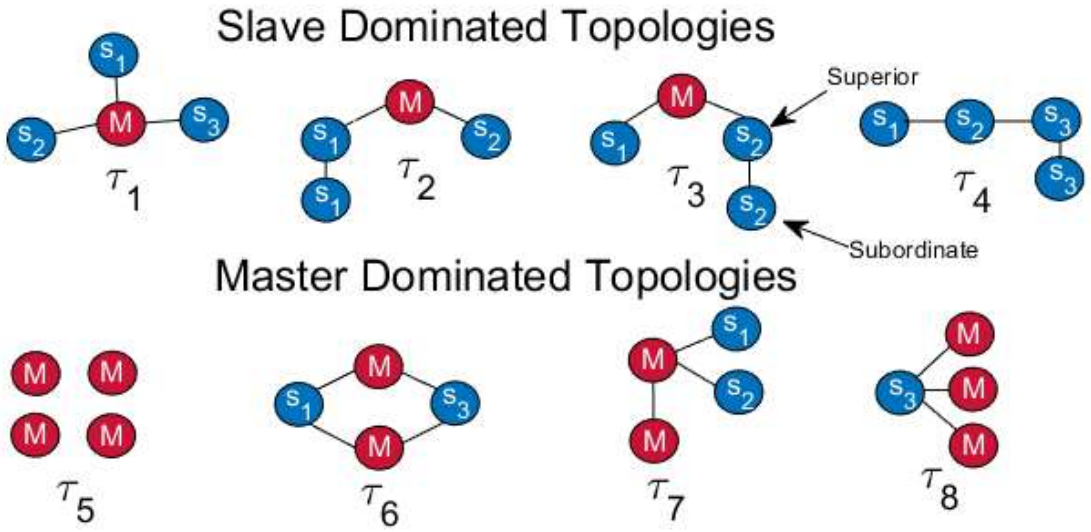


Figure 7-1-Topology depiction of master-dominated and slave-dominated units.

The outward-oriented strategy has a higher capacity to support exploration (discussed in the next section) whilst the inward-oriented learning allows particles to perform the appropriate level of exploitation to enable faster convergence. Hence, by limiting the behavioural scope of the master particles to a single learning strategy, the A-HIDMS-PSO algorithm aims to allow master particles to only explore, while the slave particles randomly adopt either inward or outward-oriented learning to maintain the balance of exploration and exploitation at the lower level. Generally, the population of master particles in the swarm is expected to have a lower density compared to slave particles, hence the role of the slave particles has significant importance in maintaining the overall balance between exploration and exploitation. The search process of the A-HIDMS-PSO algorithm is initiated similarly to the standard HIDMS-PSO. The swarm is randomly divided into two equal subpopulations, one homogeneous and one heterogeneous; subsequently, the heterogeneous subpopulation is used to form N units with either master-dominated or slave-dominated topologies *(all units adopt topologies from the same randomly chosen class)* from the corresponding topologies (from Figure 7-1) randomly assigned to each unit.

As units adopt different topologies during the search, in some cases (e.g., τ_4 or τ_7), a unit can possess two slave particles of the same type concurrently. Similarly, in τ_4 and τ_5, units are composed of only slave or only master particles. In these two situations, particles do not adopt inward or outward-oriented learning. Instead, in the first case, one of the slave particles become subordinate to its conspecific (as shown in Figure 7-1). The search dynamics of a unit is significantly affected by the adopted topology, and in some cases, e.g., in τ_7, the unit is guided by two master particles (explorers) whilst the *superior* slave particle either employs inward or outward-oriented learning to move in the search space using the positions of either of the master particles or by the position of another slave particle located in other units (Eq. 4.5). On the contrary, the sole role of the subordinate slave particle is to exploit the solutions discovered by the *superior* slave particle (Eq. 7.1). In a way, the superior slave particle acts as an interface between the explorer particles (masters) and the subordinate slave particle by implicitly transferring positional information from both masters and slave particles in other units to the subordinate, to be exploited. This implicit cooperation within different types of particles essentially provides division of labour within units, which further contributes to the efficient exploration and exploitation of the search space. In the latter case, as shown in τ_4 and τ_5, units are composed of homogenous particles of either master or

slave type. As opposed to the exploitative behaviour of the subordinate particle, in this case, particles use their cognitive exemplar $pbest$ and the mean position of particles in their unit. This allows particles to coherently perform a search on the local information as opposed to the movement of other particles. Hence, the velocity of slave and master particles in units with topologies τ_1, τ_2, τ_3, τ_6, τ_7 and τ_8 are updated using the standard HIDMS-PSO algorithm's velocity update Eqs. 4.1 and 4.2, respectively. In addition, the velocity of the subordinate particle is updated using:

$$\mathbf{v}_{sub}^{t+1} = w^t \mathbf{v}_{sub}^t + c_1^t \mathbf{r}_1 (\mathbf{pbest}_{sub} - \mathbf{x}_{sub}^t) + c_2^t \mathbf{r}_2 \left(\mathbf{x}_{slave}^{superior} - \mathbf{x}_{sub}^t\right) \quad (7.1)$$

Where $\mathbf{v}_{sub}^t$ and $\mathbf{x}_{sub}^t$ are the velocity and position of the subordinate particle in units with topologies τ_2 and τ_3 and $\mathbf{x}_{slave}^{superior}$ is the position of the superior particle to the subordinate. The velocity of the particles in units with topologies τ_4 and τ_5 are updated using:

$$\mathbf{v}_i^{t+1} = w^t \mathbf{v}_i^t + c_1^t \mathbf{r}_1 \left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t \mathbf{r}_2 \left(\mathbf{x}_{unit}^{avg} - \mathbf{x}_i^t\right) \quad (7.2)$$

Where $\mathbf{v}_i^t$, $\mathbf{x}_i^t$ and $\mathbf{pbest}_i$ are the velocity, position and personal best position of a particle in units with topologies τ_4 and τ_5, and $\mathbf{x}_{unit}^{avg}$ is the average position of the three particles in the unit.

As briefly mentioned in the beginning, during the search, the population is comprised of either master-dominated or slave-dominated units. Hence this requires switching between the two types of topologies at specific intervals. Table 7-1 displays the experiments conducted on various dimensions of the CEC'17 problems using different durations utilised for the adoption of both types of topologies. The results indicate that switching between the topologies every 100 iterations render better mean overall performance, hence, this setting will be employed in the main experiments.

Table 7-1 Mean and final ranks obtained on various dimensions of the CE'17 problems using different intervals

$duration$	10-D		30-D		50-D		100-D	
50	4.27	3	4.27	3	5.07	8	4.33	4
100	3.57	1	3.50	1	3.87	1	3.63	1
200	4.67	6	4.17	2	4.13	3	4.17	3
250	4.63	5	4.47	6	4.40	6	3.73	2
300	3.97	2	4.33	5	4.23	4	5.00	7
400	4.50	4	4.83	7	4.33	5	4.70	5
500	4.27	3	5.20	8	5.03	7	4.70	5
1000	5.20	7	4.30	4	4.00	2	4.80	6

Algorithm 4: A-HIDMS-PSO

Input: n as swam size, T_{max} as the maximum number of iterations
Output: $gbest$

1 c_1=$2.5-(1{:}T_{max}*2/T_{max})$; c_2=$0.5-(1{:}T_{max}*2/T_{max})$; $w_{max}=0.99; w_{min}=0.2;$

2 $$w_1 = \frac{w_{max} + (w_{min} - w_{max})}{1 + exp\left(-5\left(\frac{2t}{T_{max}} - 1\right)\right)}$$

3 **for** t=$1{:}T_{max}$

```
    if mod(t,100)==0 then
      Switch to master-dominated/slave-dominated units
    end
    for i=1:n
        if f(x̄_i) ≥ f(x̄) then
          w=w_1^t+C; if w>0.99, w=0.99, end
        else
          w=w_1^t+C; if w>0.20, w=0.20, end
        end
        if i^th particle is in the heterogeneous sub-pop then
            if i^th particle's unit topology is τ1,τ2,τ3,τ4,τ6,τ7 or τ8 then
                if i^th particle is a master then
                    Update v_i using Eqs. 4.2-4.4 and x_i using 2.5
                elseif i^th particle is slave then
                    Update v_i using 4.5 and x_i using 2.5
                end
            elseif unit's topology is τ4 or τ5 then
                if i^th particle is subordinate then
                    Update v_i using Eq. 7.1
                else
                    Update v_i using Eq. 7.2
                end
            end
        else
            Update v_i using Eqs. 2.4 and 2.5
        end

        Mutate x_i using the nonuniform mutation
        Evaluate the fitness of x_i
        Update pbest_i and gbest
    end
end
```

7.3 Experiments

7.3.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed A-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

7.3.1.1 Results on 10 dimensional problems

The experimental results for the 10-dimensional CEC'13 problems indicate that L-SHADE performed best on all unimodal functions (F1~F5), also on multimodal functions (F5~F20), we observe that L-SHADE solved F8, F10, F11~F14,F6~F18 whereas DMS-PSO achieved the best mean solutions for F6, F7, F9, and F15. On the other hand, MiPSO and HCLPSO obtained the lowest mean error values on F19 and F20, respectively. The results on the 8 composition functions (F21~F28) show that EPSO and

L-SHADE individually solved 2 functions including F24, F26 and F22, F27, respectively. The best solutions for F21, F23, F25 and F28 are solved by UPSO, DMS-PSO, MaPSO and LPSO, respectively.

The results indicate that for 10-dimensional CEC'13 problems, the proposed algorithm A-HIDMS-PSO was unable to attain the best mean performance for any functions, however, overall, the A-HIDMS-PSO algorithm exhibited the second-best performance on 3 functions. A shown in Figure 7-2, L-SHADE solved most of the 10-dimensional functions, whereas among PSO variants, DMS-PSO appears to perform best for more problems compared to rest of the PSO variants in this experiment.

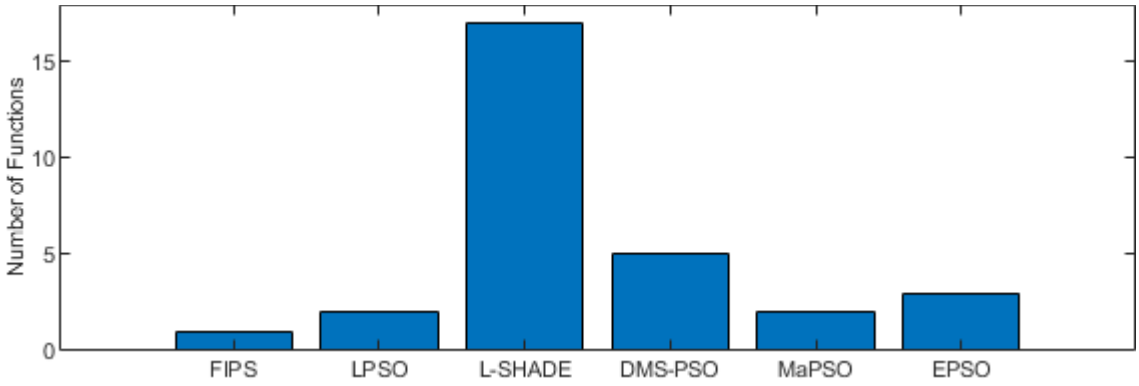


Figure 7-2 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

7.3.1.2 Results on 30 dimensional problems

The experimental results for the 30-dimensional CEC'13 problems indicate that for unimodal functions (F1~F5), L-SHADE and DMS-PSO individually solved 2 functions where L-SHADE performed best on F2, F3 and DMS-PSO on F1, F5. On the other hand, EPSO achieved the best performance on F4. The results on the multimodal functions (F6~F20) indicate that most of the multimodal functions are solved by L-SHADE including F6, F8, F10, F11~F19 whereas DMS-PSO obtained the best solution for F7 and F9. Finally, we observe that the lowest mean error value for F20 is attained by A-HIDMS-PSO. The results for the 8 composition functions (F21~F28) reveal that the proposed A-HIDMS-PSO algorithm exhibited the best performance on F24, F26, F27 whilst DMS-PSO obtained the best solutions for F23 and F25. The lowest mean error values for F21, F22 and F28 is individually attained by LPSO, L-SHADE and EPSO, respectively.

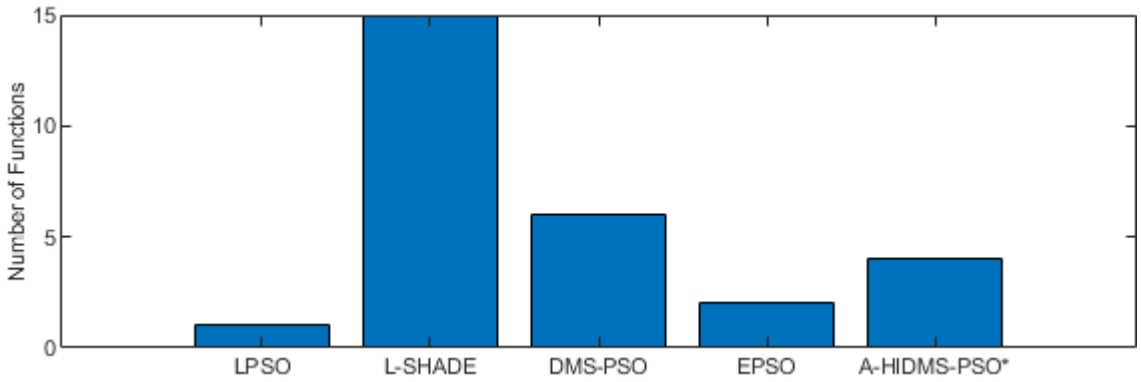


Figure 7-3 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

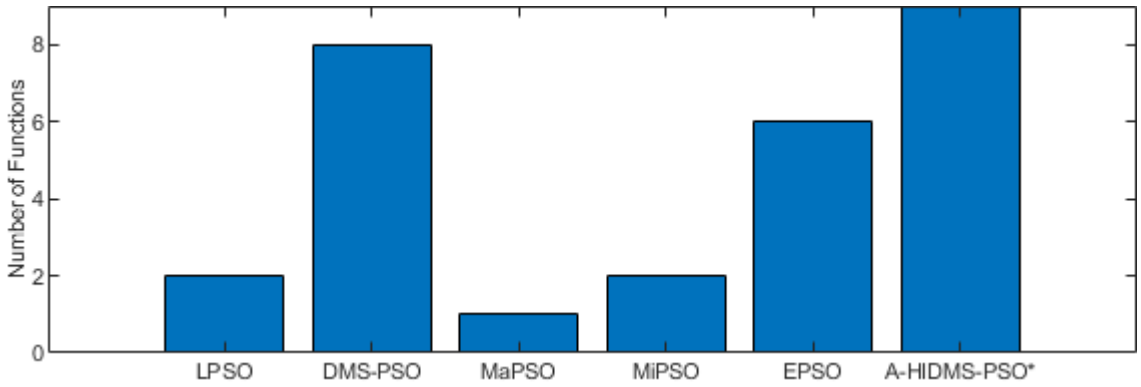


Figure 7-4 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'13 problems

Overall, as shown in Figure 7-3, the 28 30-dimensional CEC'13 functions are solved by only 5 algorithms. It can be observed that L-SHADE exhibited the best performance for majority of the problems. Figure 7-4 displays the number of functions solved among PSO variants; we observe that the proposed algorithm A-HIDMS-PSO successfully solved more problems compared to all PSO variants.

7.3.1.3 Results on 50 dimensional problems

The mean errors obtained on the 50-dimensional CEC'13 problems suggest that for the 5 unimodal functions (F1~F5) are obtained by L-SHADE (F2, F3), DMS-PSO (F1, F5) and EPSO (F4). On the multimodal functions, we observe that L-SHADE exhibit the best performance for 10 functions including F8, F10~F12, F14~F19, DMS-PSO and EPSO performed best on F13 and F6. On the other hand, the proposed algorithm A-HIDMS-PSO performed best for more 50-dimensional multimodal functions compared to all PSO variants including F7, F9 and F20. The results on the 8 composition functions (F21~F28) reveal that the proposed algorithm A-HIDMS-PSO solved more composition functions compared to all comparison algorithms, namely F23, F24 and F27. The lowest mean error values for F21, F22, F25, F26 and F28 is attained individually by LPSO, L-SHADE, DMS-PSO, MiPSO and HCLPSO, respectively.

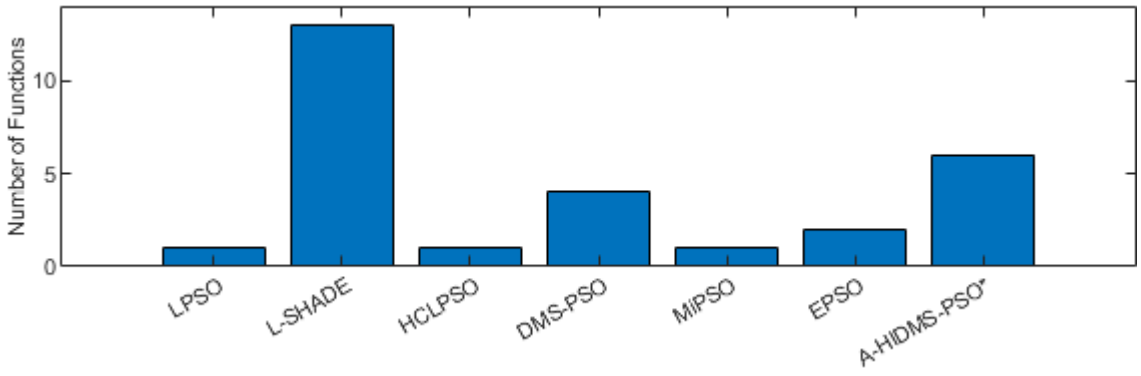


Figure 7-5 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

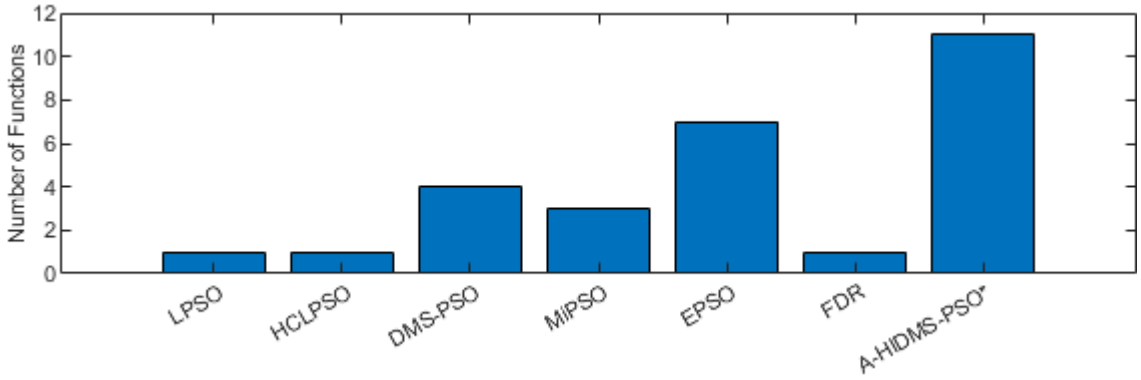


Figure 7-6 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'13 problems

From Figure 7-5, we can observe that only 7 of the 15 comparison algorithms succeeded in finding the best solution for 50-dimensional CEC'13 functions. We also observe that L-SHADE solved most of the functions and the proposed algorithm exhibited the second best overall performance and managed to solve more functions compared to all PSO variants employed in the experiment. Further, in Figure 7-6, we exhibit the same comparison among PSO variants (without L-SHADE), in this case, the proposed algorithm once again exhibited better performance and discovered the best solutions for more problems compared to all PSO variants.

7.3.1.4 Results on 100 dimensional problems

The mean errors obtained on the 100-dimensional CEC'13 indicate that for 100-dimensional unimodal functions, DMS-PSO performed best on F1,F5 whilst F2, F3, F4 is solved by L-SHADE, A-HIDMS-PSO and EPSO, respectively. The results on the multimodal functions (F6~F20) indicate that the proposed algorithm solved majority of the multimodal functions including F7, F9, F12, F12, F15, F18 and F20 whilst L-SHADE performed best on F8, F10, F14, F16, F17 and F19. On the other hand, the lowest mean error values for F6 and F11 are obtained by LPSO and EPSO, respectively. We observe from the performance on the composition functions that the proposed algorithm A-HIDMS-PSO solved more functions compared to the rest of the algorithms employed in the experiment. The A-HIDMS-PSO algorithm solved F23, F24, F27 and F28 whereas the best solutions for F21, F22, F25 and F26 are discovered by UPSO, L-SHADE, DMS-PSO and EPSO, respectively.

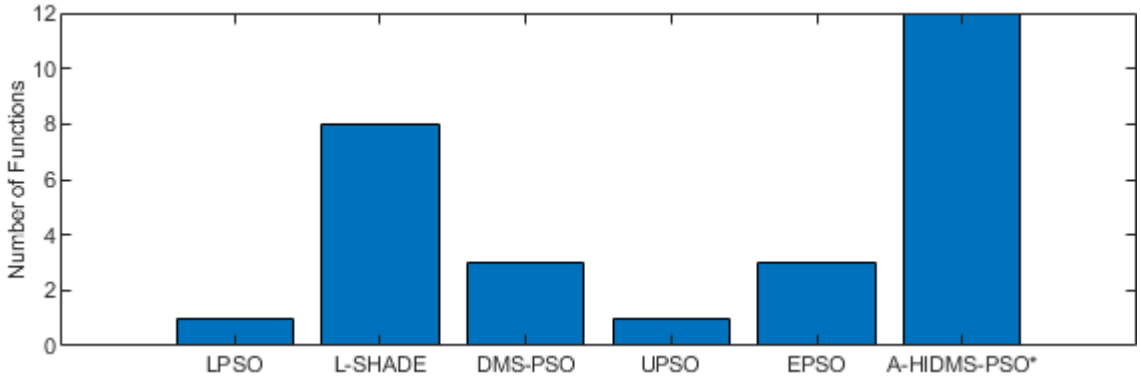


Figure 7-7 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

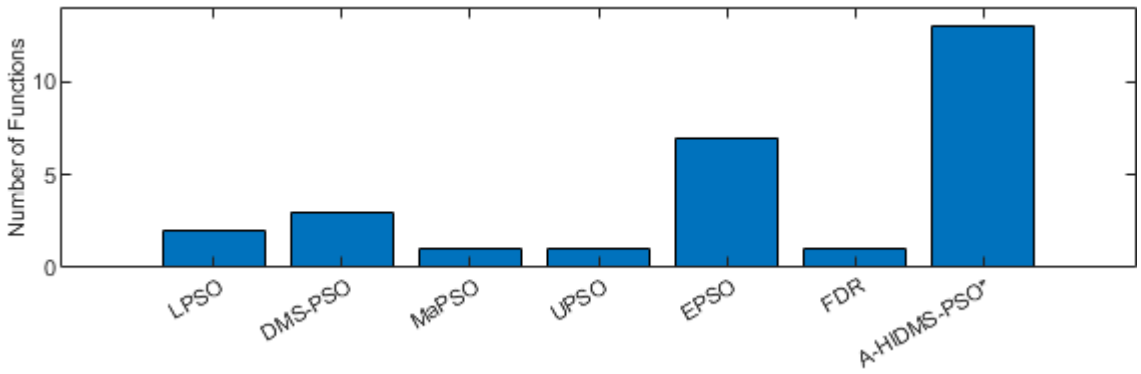


Figure 7-8 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'13 problems

We can observe from Figure 7-7, that compared to 10, 30 and 50-dmensional functions, the proposed algorithm performed significantly better on the 100-dimensional functions and successfully solved more problems compared to all test algorithms. Moreover, as shown in Figure 7-8, the comparison of the total number of functions solved among PSO variants yield that the proposed algorithm once again solved significantly more functions.

The algorithms are ranked based on the mean error values obtained on the 10, 30, 50 and 100-dimensional CEC'13 problems. The details of the mean and final ranks are given in Appendix 2. In summary, the A-HIDMS-PSO algorithm ranked 5th on the 10 dimensional problems, however, we observe continues improvement in the average performance of A-HIDMS-PSO. Hence, the final ranks suggest that on the 30-dimensional CEC'13 problems A-HIDMS-PSO ranked 3rd, and on the 50 and 100-dimensional problems, the proposed algorithm obtained the 2nd rank.

Table 7-2 Wilcoxon Signed Rank Test Results with a Significance Level of $\alpha = 0.05$ for CEC2013 problems

A-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	+	+	=	+	=	-	=	=	+	7/6/1
30	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
50	+	+	+	+	=	+	+	=	+	+	=	+	+	+	11/3/0
100	+	+	+	=	=	+	+	=	+	+	=	+	+	+	10/4/0

Table 7-2 presents the Wilcoxon signed rank test results of the A-HIDMS-PSO algorithm against other algorithms employed in the experiments on four different dimensions. The number of (+/=/-) are shown in the last column of the Table 7-2 which suggests that on all dimensions, the proposed A-HIDMS-PSO algorithm performed significantly better than most of the test algorithms. It is also worth mentioning that at 30, 50 and 100 dimensions, the Wilcoxon signed rank test reveals that none of the comparison algorithms exhibited significantly superior performance against the A-HIDMS-PSO algorithm. Further, Figure 7-9 exhibits the convergence characteristic comparison of the proposed algorithm A-HIDMS-PSO with the 4 top ranked algorithms on the CEC'13 problems. The comparison suggests that in most cases, the A-HIDMS-PSO algorithm exhibits comparable

convergence characteristics in terms of speed and the converged solution quality on various 100-dimensional unimodal, multimodal and composition functions in comparison to the algorithms under consideration.

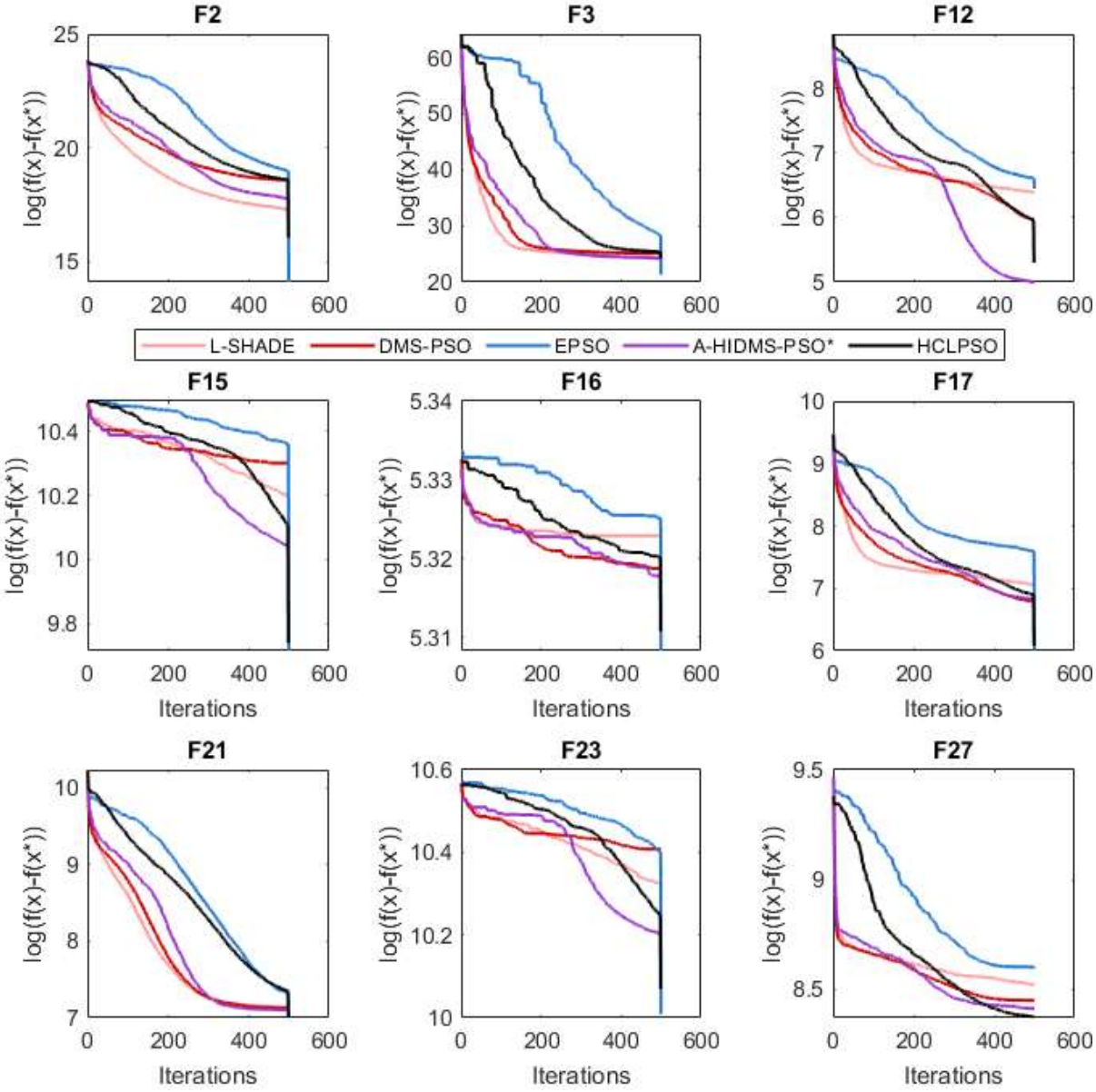


Figure 7-9 Average convergence rate comparison of A-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'13 problems

7.3.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed A-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

7.3.2.1 Results on 10 dimensional problems

The results achieved on the 10-dimensional CEC'14 problems reveal that on the unimodal functions (F1~F3) show that L-SHADE solved all 3 functions, similarly, on the multimodal functions L-SHADE performed best on 7 functions including F7~F12, F15 whilst DMS-PSO solved F6, F12, F14 and F16. On the other hand, the best mean solutions for F4 and F5 are obtained EPSO. The results on the hybrid functions (F17~F22) indicate that L-SHADE achieved the best performance on all hybrid functions. Finally, on the composition functions (F23~F30), LPSO and MaPSO solved F28, F29 and F23, F27, respectively. The best mean solutions for F24, F25, F26 and F30 are obtained individually by L-SHADE, EPSO, DMS-PSO and FIPS, respectively.

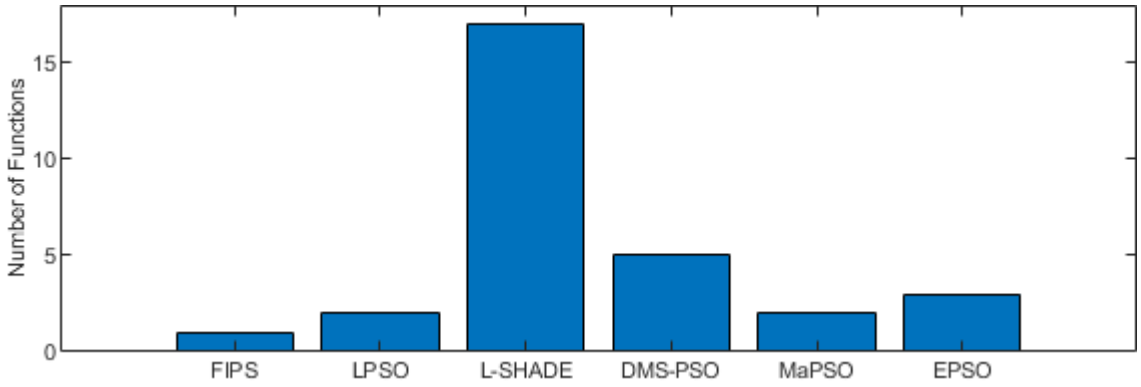


Figure 7-10 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

Figure 7-10 displays the 6 algorithms that attained the lowest mean error values on the 10-dimensonal functions, and the total number of functions solved by them. We observe that at 10 dimensions, L-SHADE solved majority (17) of the 30 problems. The proposed algorithm A-HDIMS-PSO did not exhibit the best performance for any problems at 10 dimensions however, A-HIDMS-PSO algorithm obtained the second lowest mean error values for 4 functions.

7.3.2.2 Results on 30 dimensional problems

The experimental results obtained on the 30-dimensional CEC'14 problems suggest that best solutions for all unimodal (F1~F3) and 8 multimodal functions are attained by L-SHADE including F4, F8~F12, F15, F16 whilst for F5, F6, F7, F13 and F14, the lowest mean error values are obtained by HPSO-TVAC, A-HIDMS-PSO, FIPS, DMS-PSO and FDR, respectively. On the hybrid functions (F17~F22), the results reveal that L-SHADE achieved the best performance on F17, F19, F20, F21 whilst MaPSO and LPSO performed best on F18 and F22, respectively. Finally, the experimental results for the composition functions (F23~F30) show that DMS-PSO solved F23, F25, F29 whilst the proposed algorithm A-HIDMS-PSO performed best on F24 and F27. On the other hand, the lowest mean error values for F26, F28 and F30 are attained individually by L-SHADE, LPSO and MaPSO, respectively.

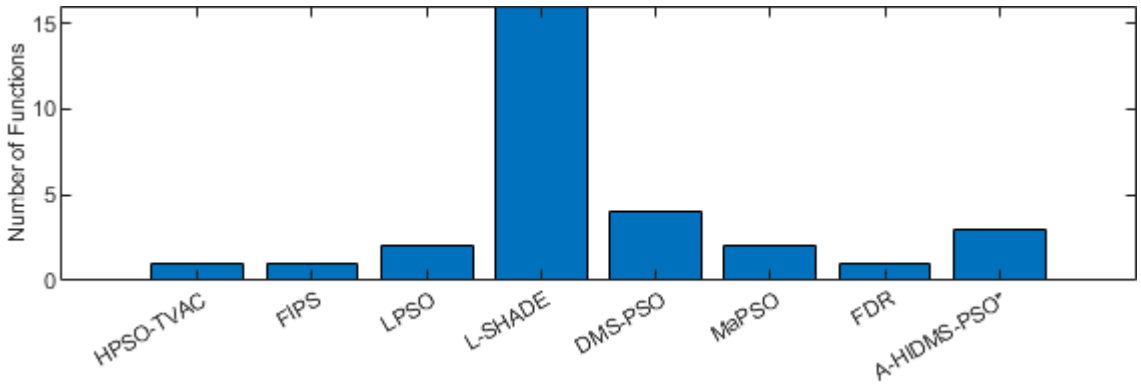


Figure 7-11 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'14 problems

We observe from Figure 7-11 that L-SHADE performed best on most of the 30-dimensional on the CEC'14 test suite. The proposed algorithm A-HIDMS-PSO exhibited the 3rd best overall performance in this case. In addition, the proposed algorithm A-HIDMS-PSO attained the second-best mean solution for four 30-dimensional CEC'14 problems.

7.3.2.3 Results on 50 dimensional problems

The mean error obtained on the thirty 50-dimensional CEC'14 test problems suggest that for the unimodal functions (F1~F3) indicate the L-SHADE exhibited the best performance on the entire subset of functions and on the multimodal functions (F4~F16), L-SHADE performed best on 7 functions including F4, F8, F10~F12, F16 whilst the proposed algorithm A-HIDMS-PSO solved F6 and F15. The lowest mean error values for F5, F17, F13 and F14 are attained by HPSO-TVAC, FIPS, DMS-PSO and LPSO, respectively. Moreover, the best solutions for the six 50-dimensional hybrid functions (F17~F22) are achieved by L-SHADE (F17, F18, F19, F21), DMS-PSO (F22) and EPSO (F20) whilst the results on the composition functions (F23~F30) yield that DMS-PSO and A-HIDMS-PSO exhibited the best performance on F23, F25, F28, F29 and F24, F27, respectively. The lowest mean error values on F26 and F30 are obtained individually by FIPS and LPSO, respectively.

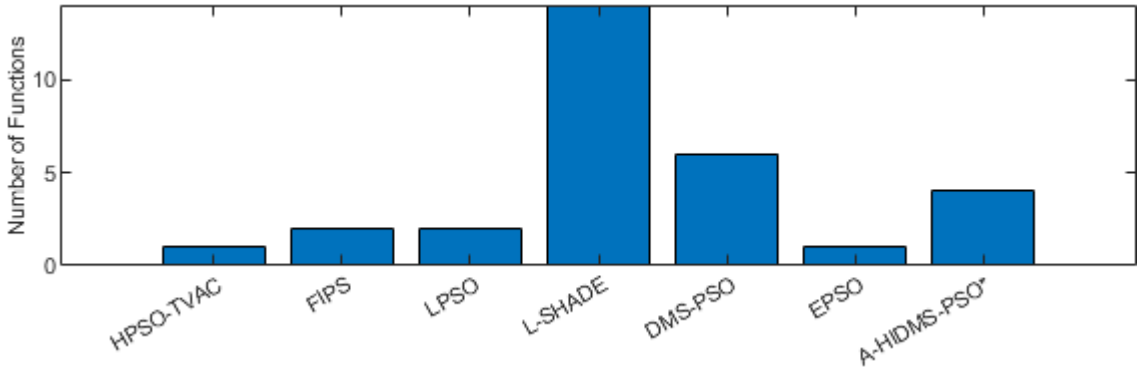


Figure 7-12 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

From Figure 7-12, we observe that L-SHADE solved most of the (14) problems on the 50-dimensional CEC'14 test problems whilst the second and third-best overall performances are attained by DMS-PSO and the proposed algorithm A-HIDMS-PSO.

7.3.2.4 Results on 100 dimensional problems

The results obtained on the 100-dimensional CEC'14 problems indicate that L-SHADE (F1, F2) and EPSO (F3) exhibited the best performance on the unimodal functions. The results on the multimodal functions (F4~F16) show that L-SHADE solved 4 of the multimodal functions including F10, F11, F12, F16 whilst the proposed algorithm A-HIDMS-PSO performed best on F6, F14, F15 whereas DMS-PSO achieved the best solutions for F9 and F13. The best mean solutions for F4, F5, F7 and F8 are provided individually by LPSO, HCLPSO, UPSO and EPSO, respectively. On the other hand, the results for hybrid functions (F17~F22) indicate that L-SHADE solved F17, F21 whilst the lowest mean error for F18, F19, F20 and F22 are obtained by MaPSO, LPSO, EPSO and DMS-PSO, respectively. For composition functions (F23~F30), LPSO solved 3 of the 8 functions whereas the proposed algorithm A-HIDMS-PSO provided the best solution for F25 and F27. Moreover, the lowest mean error values for F23, F24 and F26 are provided by DMS-PSO, HPSO-TVAC and FIPS, respectively.

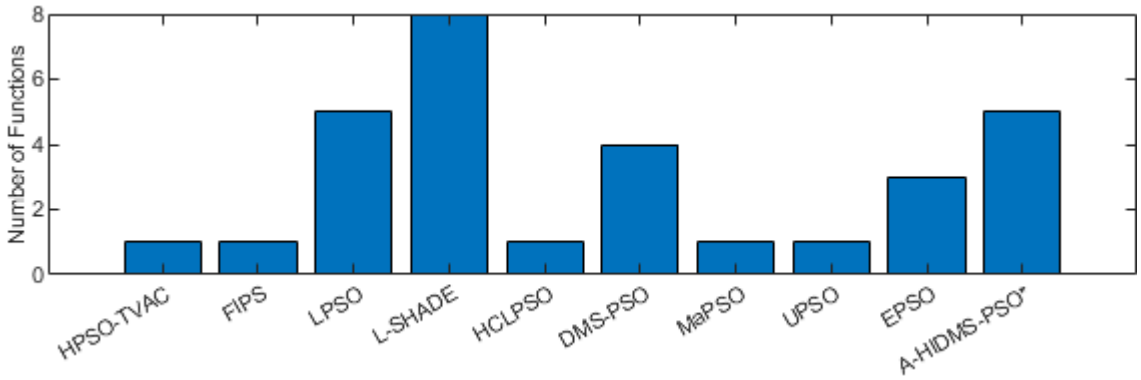


Figure 7-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

We can observe from Figure 7-13 that, overall, L-SHADE achieved the best performance on 8 of 30 100-dimensional CEC'14 problems whereas the proposed algorithm A-HIDMS-PSO and DMS-PSO exhibited the best performance on 5 problems.

Further, the details of the mean and final ranks obtained on four different dimensions of the CEC'14 test problems are shown in Appendix 2. The obtained ranks yield that the proposed algorithm A-HIDMS-PSO's final rank improved with increased problem dimensions. In summary, on the 10-dimensional CEC'14 problems, A-HIDMS-PSO ranked 6th, subsequently at 30 and 50 dimensions, A-HIDMS-PSO ranked 4th and at 100 dimensions, the proposed algorithm attained the 3rd final rank.

Table 7-3 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2014 problems

A-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	-	-	+	+	=	+	=	-	=	=	+	7/4/3
30	+	+	+	=	=	=	+	=	+	=	-	+	=	+	7/6/1
50	+	+	+	=	=	+	+	=	=	=	-	+	+	+	8/5/1
100	+	+	+	=	=	+	+	=	=	=	=	+	+	+	8/6/0

Table 7-4 presents the details of the Wilcoxon signed rank test conducted on the experimental results for 10, 30, 50 and 100-dimensional CEC'14 problems. In summary, in each case, the performance of the A-HIDMS-PSO algorithm is observed to be superior compared to more than half of the algorithms employed. Considering the Wilcoxon signed rank test results for 30, 50 and 100 dimensions, we also observe that only L-SHADE exhibited statistically superior performance against the A-HIDMS-PSO algorithm on the 30 and 50-dimensional problems, but no algorithms are statistically superior at 100 dimensions. In addition, we have compared the convergence rates of L-SHADE, EPSO, DMS-PSO and HCLPSO against the proposed algorithm A-HIDMS-PSO on various 100-dimensional unimodal, multimodal, hybrid and composition functions selected from the CEC'14 benchmark problems. The comparison is shown in Figure 7-14, which shows that, in most cases the proposed algorithm A-HIDMS-PSO has faster convergence compared to EPSO and HCLPSO. The overall convergence characteristics of all 5 algorithms appear comparable and in particular, we do not observe any of the algorithms' convergence behaviour to be drastically different or worse on a particular class of function.

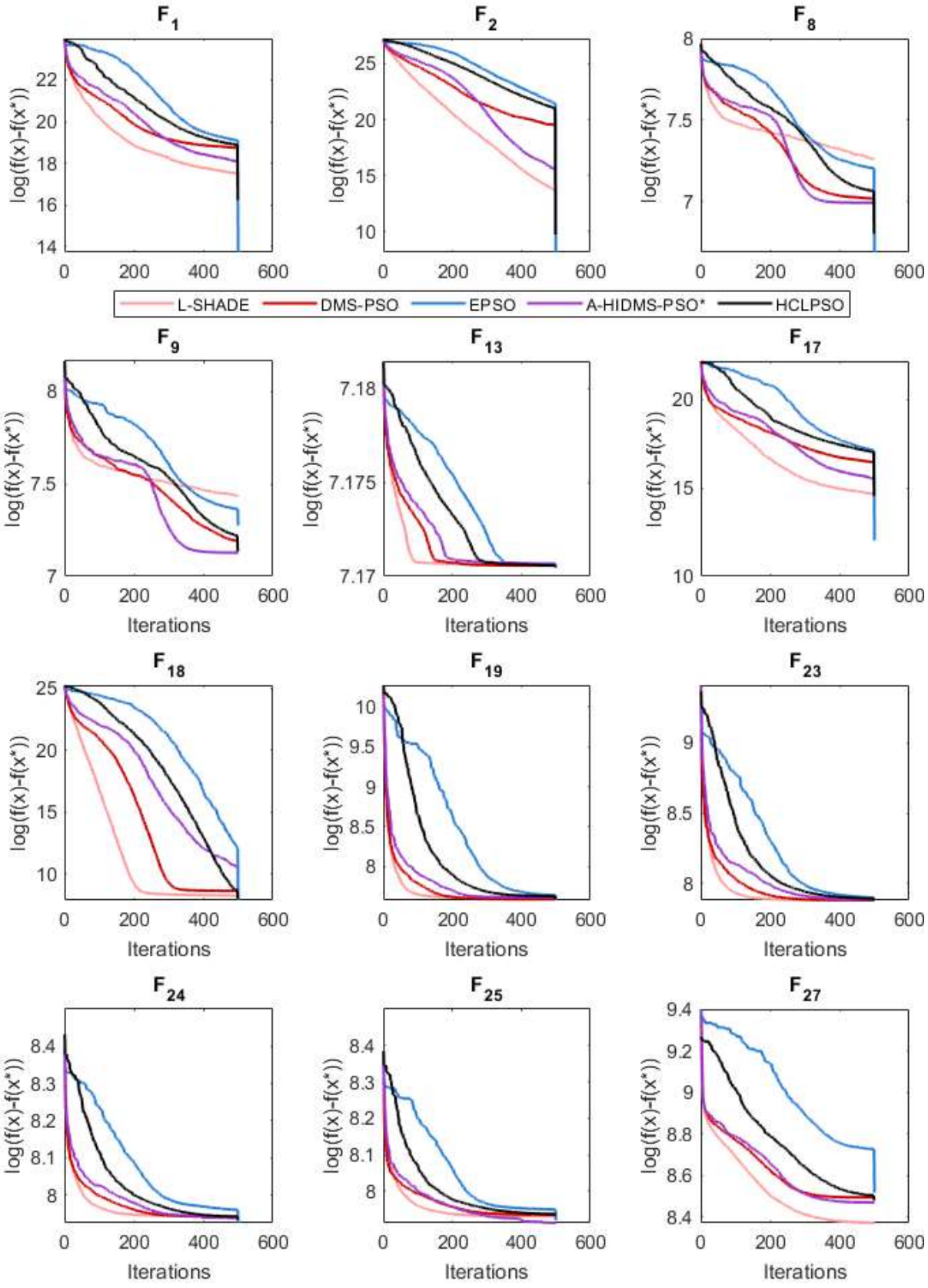


Figure 7-14 Average convergence rate comparison of A-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'14 problems

7.3.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed A-HIDMS-PSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

7.3.3.1 Results on 10 dimensional problems

The results obtained on the 10-dimensional CEC'17 test problems indicate that the multimodal functions, L-SHADE solved F3, F4, F5, F7, F8 and F9 and on the hybrid functions, once again, L-SHADE obtained the best performance on F10~F15, F7~F9. On the other hand, EPSO provided the best solution for multimodal F6 and hybrid F6. The results on the composition functions indicate that MiPSO (F23, F24), MaPSO (F20, F26) and HCLPSO (F25, F28) individually solved 2 functions. We observe that the best solutions for composition F21, F22, F27, F29 and F30 is attained by A-HIDMS-PSO, EPSO, LPSO, L-SHADE and DMSPSO, respectively.

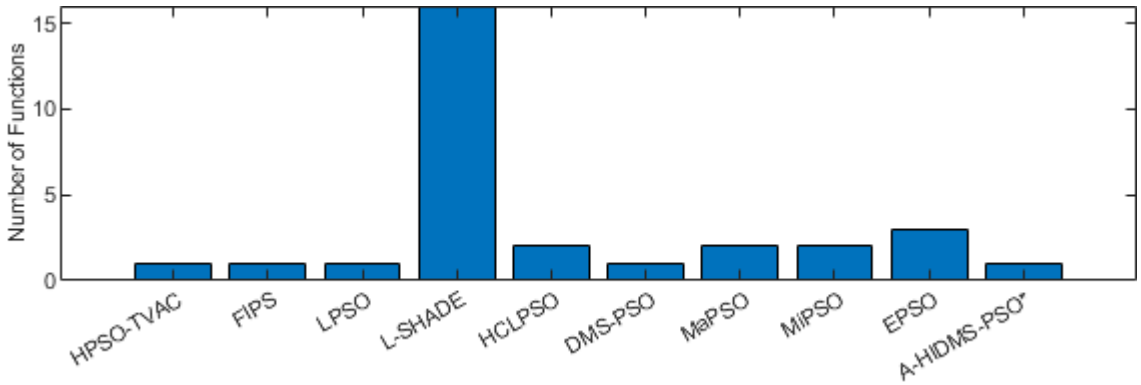


Figure 7-15 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'17 problems

As shown in Figure 7-15, for 10-dimensional CEC'17 problems, the best mean error values for majority of the problems are attained by L-SHADE. In this test case, the proposed algorithm A-HIDMS-PSO exhibited comparable performance to LPSO, FIPS AND HPSO-TVAC with respect to the number of functions solved.

7.3.3.2 Results on 30 dimensional problems

The results obtained on the 30-dimensional CEC'17 test problems suggest that on the multimodal functions, L-SHADE performed best on F5, F7, F8 and on the hybrid functions, once again L-SHADE solved F10, F12, F13, F14, F18 and F19. Moreover, EPSO performed best on multimodal F3, F4, MiPSO on F6 and FIPS on F9. On the hybrid functions, DMS-PSO solved F11, F16, MaPSO performed best on F15 and the proposed algorithm A-HIDMS-PSO achieved the best performance on F17. The results on the composition functions reveal that L-SHADE attained the best performance on F20, F21, F23, F24 whilst LPSO solved F22 and F30. The best solutions for F25, F26, F27, F28 and F29 are individually achieved by FIPS, EPSO, MiPSO, A-HIDMS-PSO and DMS-PSO, respectively.

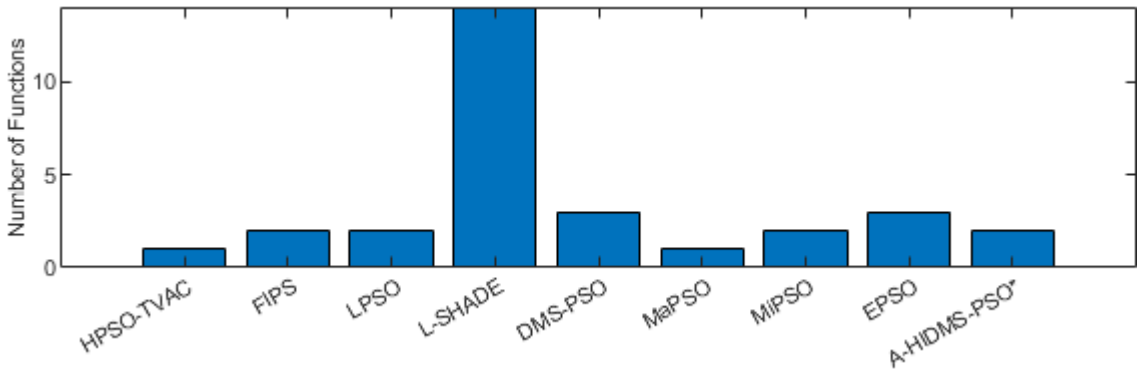


Figure 7-16 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

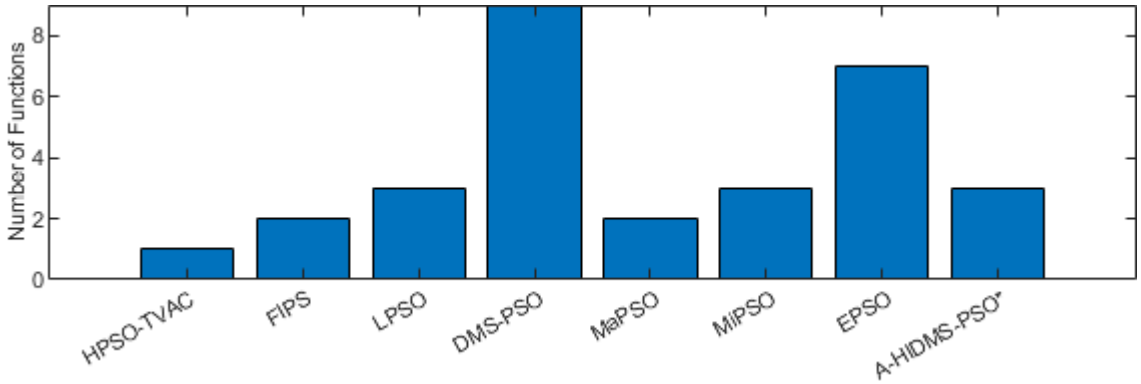


Figure 7-17 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

Overall, as shown in Figure 7-16, most of the 30-dimensional functions are solved by L-SHADE and the PSO variants exhibit relatively similar performances in finding the best solution for the 30-diemsnional 29 CEC'17 problems. Further comparison solely among PSO variants (Figure 7-17) indicate that DMS-PSO and EPSO solved majority of the problems at 30 dimensions. Although the proposed algorithm only attained the best solutions for 3 functions, it achieved the second-best solutions for 8 30-dimensional problems in this experiment.

7.3.3.3 Results on 50 dimensional problems

The mean errors obtained on the 50-dimensional CEC'17 problems indicate that L-SHADE, FIPS and EPSO solved the 7 50-dimensional multimodal functions where L-SHADE obtained the lowest mean errors on F5, F7, F8, FIPS on F6, F9 and EPSO on F3, F4. On the hybrid function, the results show that L-SHADE solved F10, F12, F14, F15, F18, F19 whilst DMS-PSO performed best on F11, F16, F17 and LPSO achieved the best solution for F13. On the 50-dimensional composition functions, L-SHADE exhibited the best performance for F21, F23, F24, F29 whilst DMS-PSO performed best on F20, F26 and F28. The best solutions for F22, F25, F27 and F30 are provided individually by A-HIDMS-PSO, FIPS, HPSO-TVAC and LPSO, respectively.

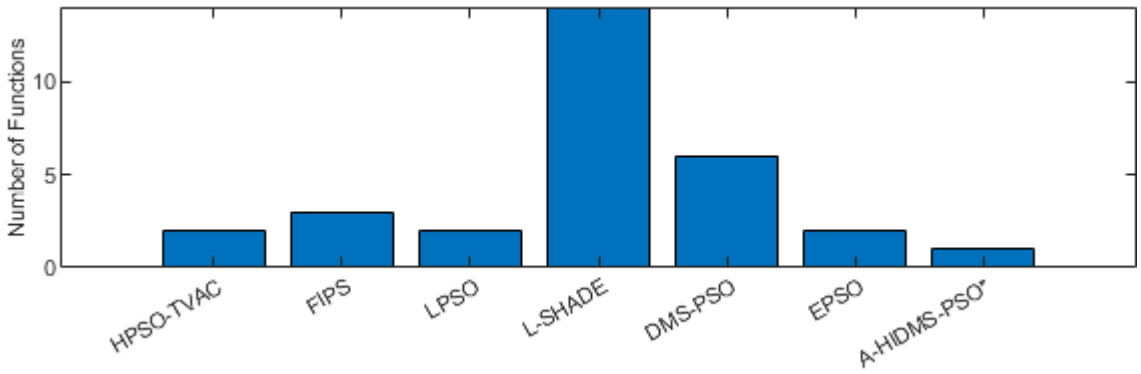


Figure 7-18 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

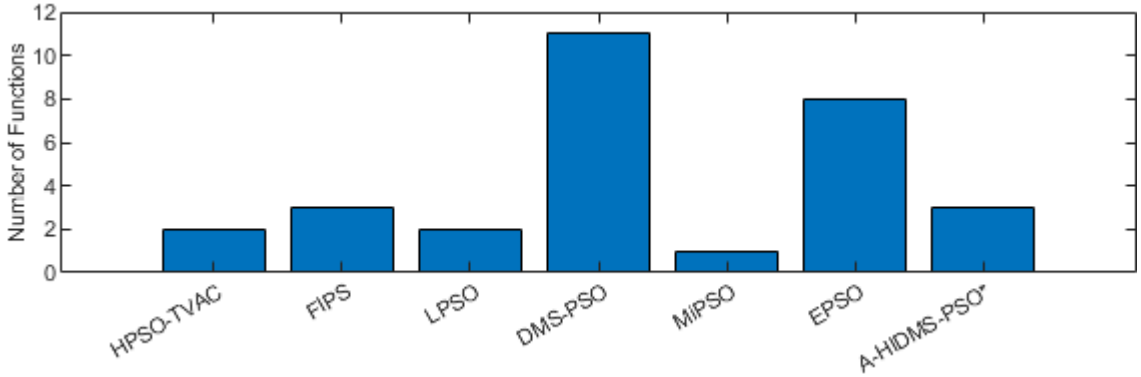


Figure 7-19 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'17 problems

Figure 7-18:7-19 display the comparison with respect to the number of functions solved by each algorithm. In Figure 7-18, the comparison exhibits all comparison algorithms and in Figure 7-19 exhibit comparison among PSO variants solely. In the first case, L-SHADE demonstrated the best performance for more functions compared to all test algorithms and in the latter case, DMS-PSO performed better on more functions compared to other PSO variants.

7.3.3.4 Results on 100 dimensional problems

The experimental results obtained on the 100-dimensional CEC'17 test problems reveal that for multimodal functions, we observe that DMS-PSO solved 4 of the 7 functions inducing F5, F7, F8, F9 whilst EPSO performed best on F3, F6 and LPSO attained the lowest meane error on F4. We observe that the best solutions for 10 hybrid functions are attained by L-SHADE (F10, F12, F14, F15), EPSO (F13, F18, F19), A-HIDMS-PSO (F16, F17) and DMS-PSO (F11). Moreover, the results on the composition functions reveal that DMS-PSO and the proposed algorithm solved F20, F21, F24, F27 and F25, F26, F29, respectively. On the other hand, L-SHADE performed best on F22, F24 and LPSO solved F28, F30.

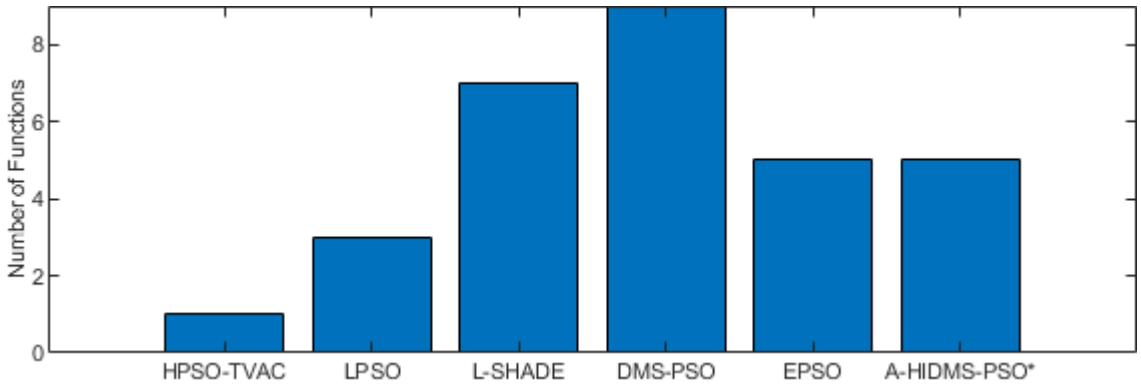


Figure 7-20 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

Figure 7-20 displays the 6 algorithms that provided the best solution for 29 100-dimensional CEC'17 problems. We observe that DMS-PSO solved the most problems followed by the performance of L-SHADE. On other hand, the proposed algorithm A-HIDMS-PSO exhibited best performance on 5 problems along with EPSO.

The average and final ranks are shown in Appendix 2. The final ranks obtained on the 10, 30, 50 and 100-dimensional CEC'17 problems indicate that the proposed algorithm A-HIDMS-PSO ranked 8th on the 10-dimensional functions. However, at 30 and 50 dimensions, the A-HIDMS-PSO algorithm achieved the 4th rank with comparable average rank (4.7) to EPSO (EPSO ranked 2nd with avg. rank of 4.3). Finally, on the 100-dimensional CEC'17 problems, the proposed algorithm A-HIDMS-PSO outranked 11 of the 14 test algorithms and achieved the 3rd rank. Once again, we observe that the average rank of A-HIDMS-PSO is 4.06 for 100-dimensional problems whereas DMS-PSO (2nd rank) obtained a very close average rank of 3.9 and L-SHADE (1st rank) exhibit the average rank of 3.4. Moreover, Table 7-6 displays the results for the Wilcoxon signed rank test conducted with the significance level of 5%. In summary, the Wilcoxon signed rank test shows that at every dimension, the proposed algorithm A-HIDMS-PSO exhibited significantly better performance against most of the test algorithms under consideration. Moreover, it is also worth noting that at 30, 50 and 100 dimensions, the A-HIDMS-PSO algorithm is only outperformed by L-SHADE (at 30-d and 50-d), however for 100-dimensional CEC'17 problems, the mean performance comparison of the proposed algorithm A-HIDMS-PSO and L-SHADE is statistically non-significant.

Table 7-4 Wilcoxon Signed Rank Test Results with a Significance Level of $\alpha = 0.05$ for CEC2017 problems

A-HIDMS-PSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	-	+	+	-	+	=	-	=	=	=	6/5/4
30	+	+	+	=	=	+	+	=	+	=	-	+	=	+	8/5/1
50	+	+	+	=	=	+	+	=	+	=	-	+	+	+	9/4/1
100	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0

Further, the comparison of the convergence characteristics (Figure 7-21) of the A-HIDMS-PSO algorithm with the 4 top ranked algorithms L-SHADE, DMS-PSO, EPSO and HCLPSO at 100 dimensions reveal that in general, the proposed algorithm converges faster than EPSO and HCLPSO, moreover we observe that A-HIDMS-PSO exhibit comparable convergence trend to DMS-PSO and L-SHADE. On the multimodal functions (see F5, F8 in Figure 7-21), the proposed algorithm initiates converging at similar pace to DMS-PSO and L-SHADE however in comparison, it converges to a significantly better solution which may be seen as the A-HIDMS-PSO algorithm's capability to escape local optima.

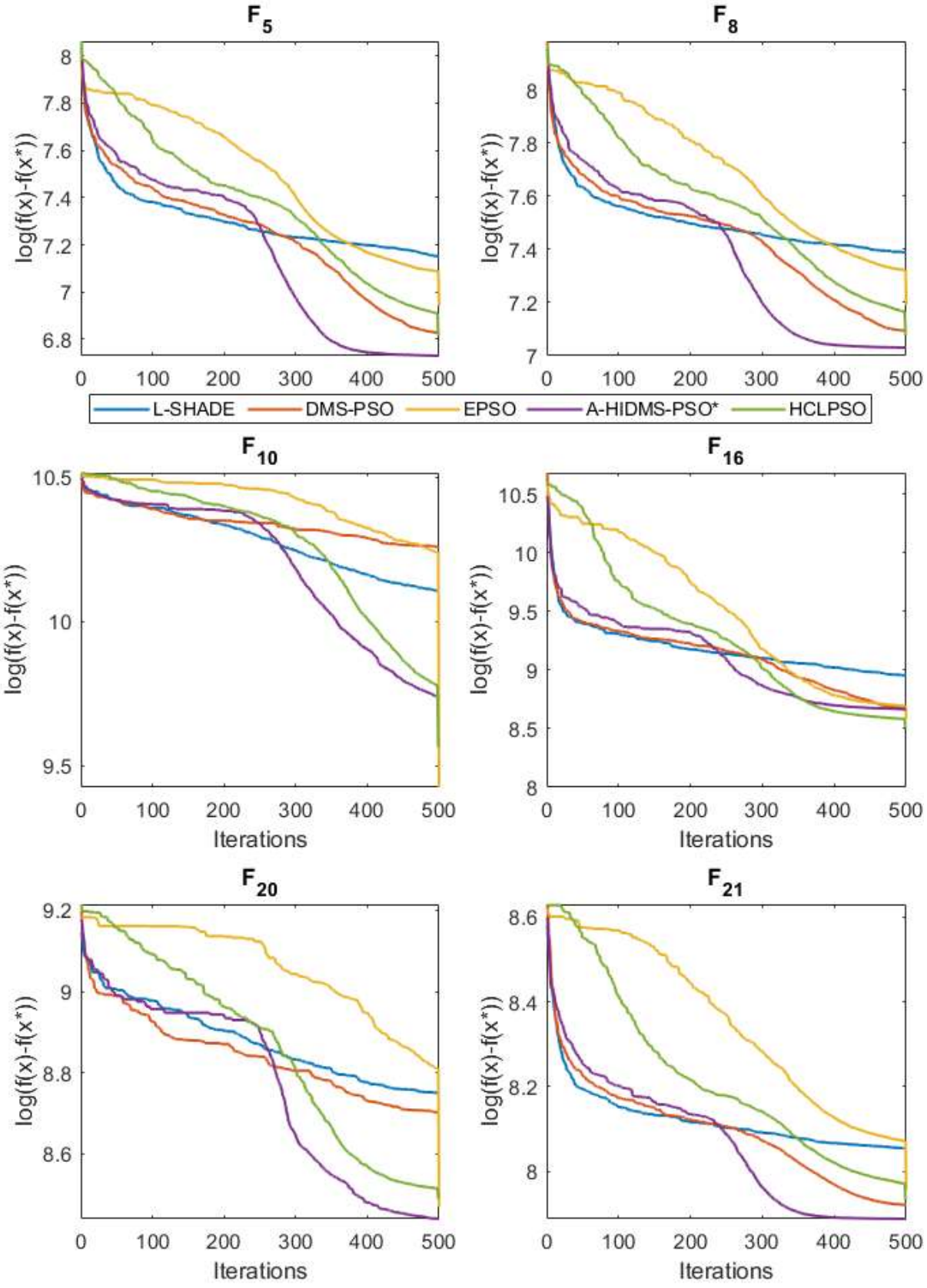


Figure 7-21 Average convergence rate comparison of A-HIDMS-PSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'17 problems

Moreover, Figure 7-22:7-24 display the individual comparison of the total number of functions where each algorithm attained the best performance against the proposed algorithm A-HIDMS-PSO at different dimensions of CEC'13, CEC'14 and CEC'17 test problems. We can observe from Figure 7-22:7-24 that the proposed algorithm A-HIDMS-PSO exhibited better mean performance for more functions compared to majority of the algorithms, and in some cases the proposed algorithm A-HIDMS-PSO is observed to be superior compared to all test algorithms.

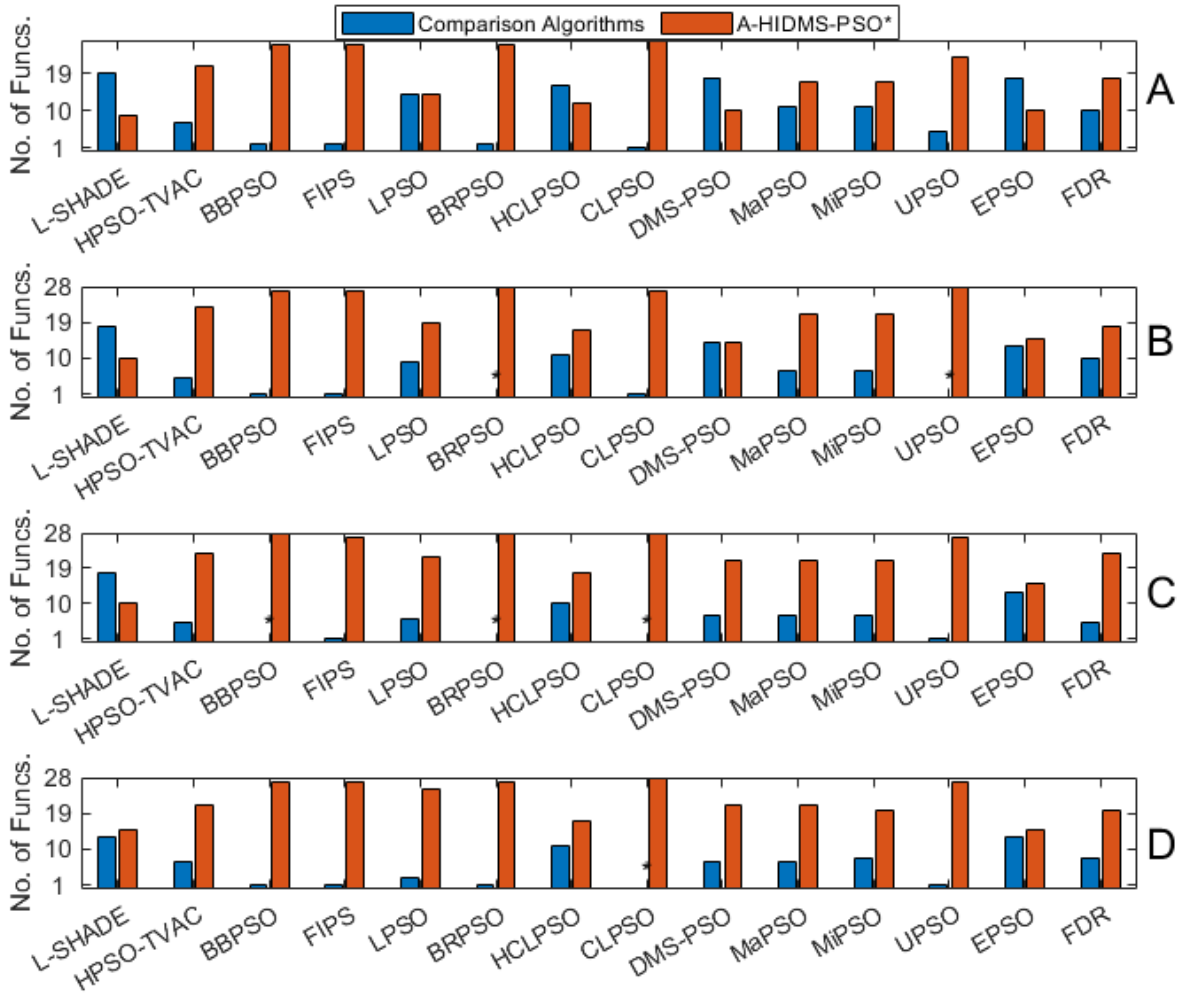


Figure 7-22 Pairwise mean performance comparison of A-HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'13 test functions

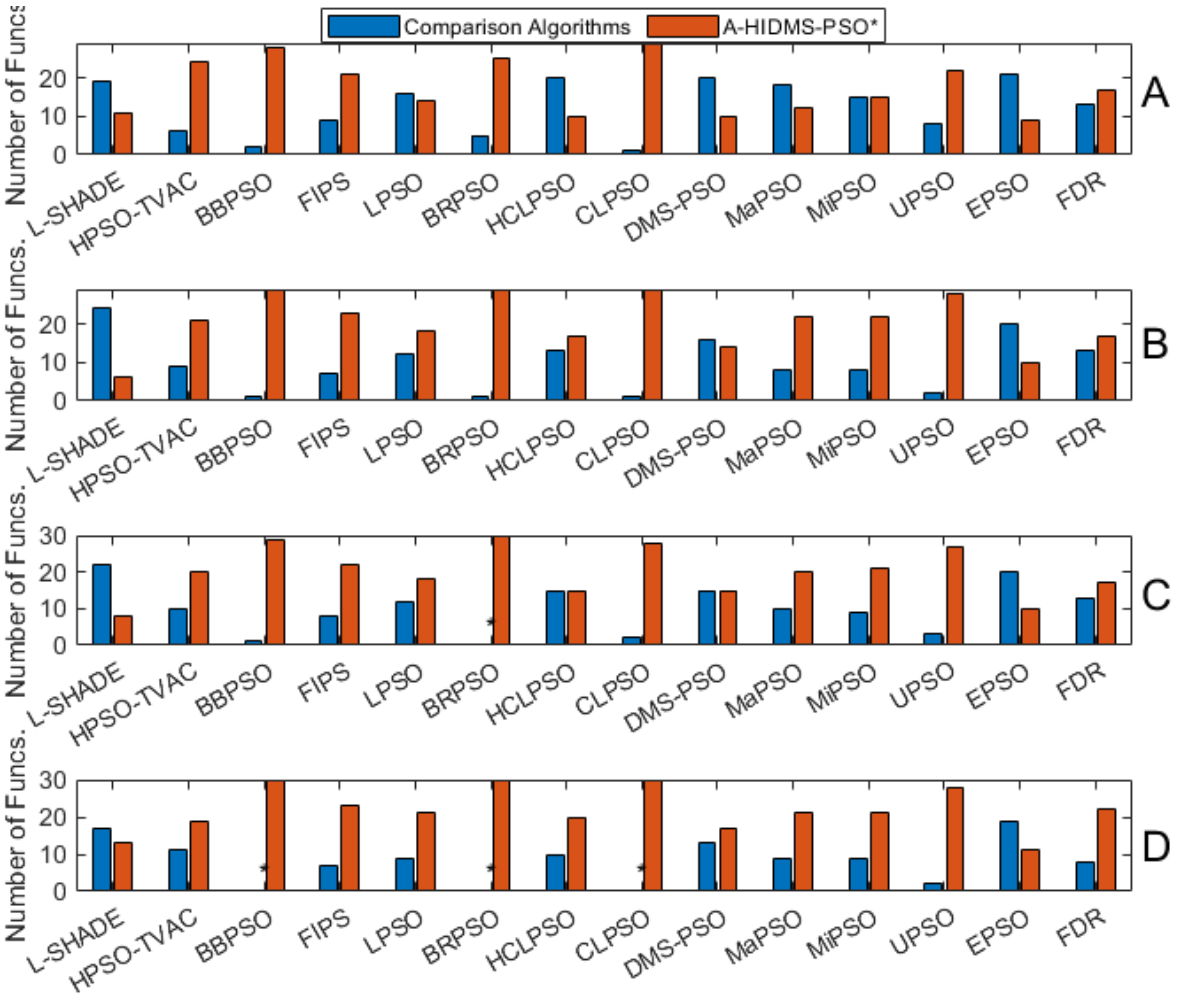


Figure 7-23 Pairwise mean performance comparison of A-HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'14 test functions

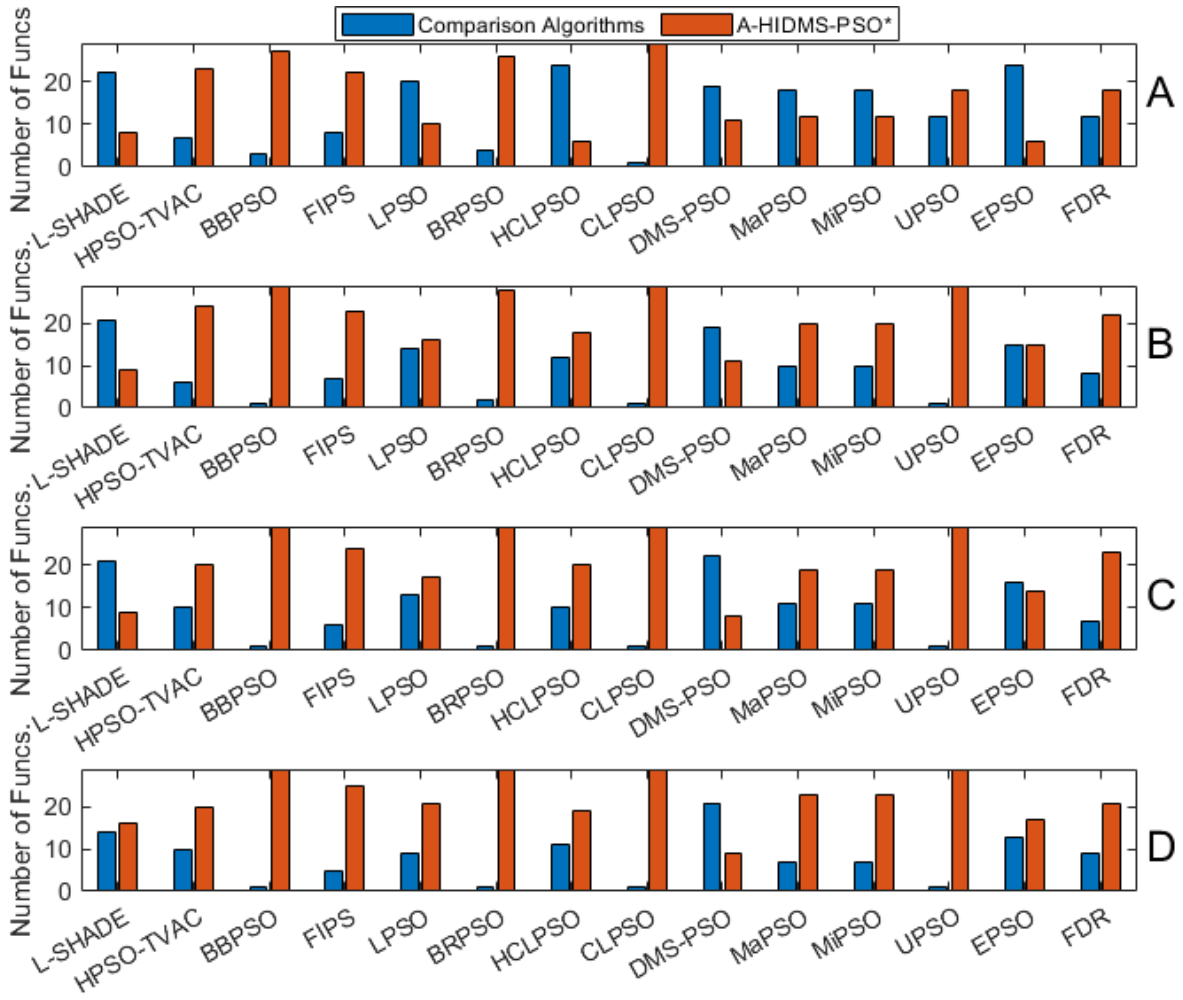


Figure 7-24 Pairwise mean performance comparison of A-HIDMS-PSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'17 test functions

7.4 Discussions

The proposed algorithm's performance was evaluated on a total of 87 problem derived from the CEC'13, CEC'14 and CEC'17 across 10, 30, 50 and 100 dimensions. The problem set encapsulates 9 unimodal, 35 multimodal, 16 hybrid and 26 composition functions. The performance of the proposed algorithm was compared with 13 PSO variants and L-SHADE.

Overall, in terms of the number of problems solved, the proposed algorithm A-HIDMS-PSO performed best on 1, 9, 11 and 22 problems on 10, 30, 50 and 100-dimensional cases, respectively. Further, the A-HIDMS-PSO algorithm attained the second-best performance on 8, 14, 12 and 14 problems for 10, 30, 50 and 100 dimensional problems, respectively. When compared with L-SHADE and EPSO, at 100 dimensions, L-SHADE performed best on 23 and EPSO solved 11 of the 86 problems. Figure 7-25 exhibits the comparison of A-HIDMS-PSO and EPSO on the number of unimodal, multimodal, hybrid and composition functions solved. In summary, the proposed algorithm exhibited superior performance and solved more multimodal and composition functions compared to EPSO.

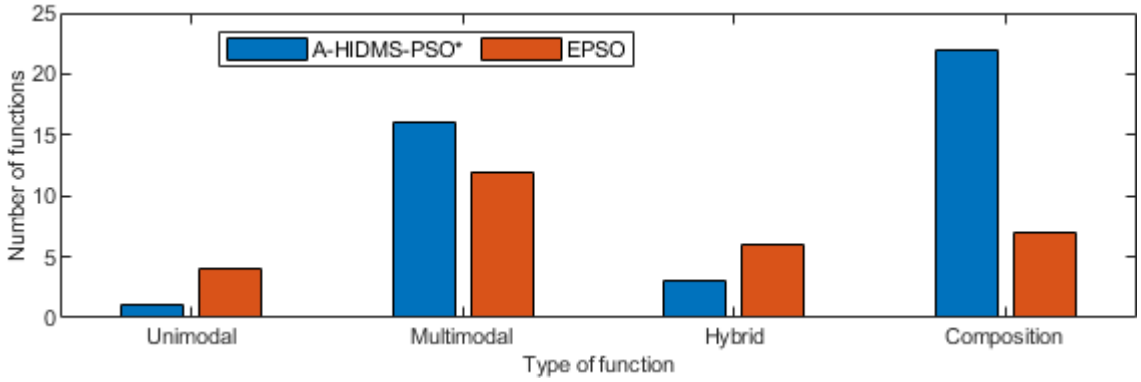


Figure 7-25 Comparison of unimodal, multimodal, hybrid and composition functions solved by A-HIDM-PSO and EPSO.

Moreover, the final ranks indicated that the A-HIDMS-PSO algorithm exhibited better overall mean performance compared to most of the algorithm employed in the experiments, especially in high dimensional problems. Case in point, for 100-dimensional CEC'13 problems, A-HIDMS-PSO ranked 2nd and for CEC'14 and CEC'17 problems, the proposed algorithm attained the 3rd rank. Similarly, the Wilcoxon signed rank test also indicated that the A-HIDMS-PSO algorithm exhibited significantly superior performance compared to most of the algorithms on high dimensional problems. Furthermore, the convergence rate comparison of the A-HIDMS-PSO algorithm revealed in most cases, A-HIDMS-PSO exhibited better convergence rate compared to EPSO and HCLPSO, and in some cases exhibited comparable convergence trend to L-SHADE.

Overall, the general performance of the A-HIDMS-PSO algorithm on high dimensional problem appear superior in comparison to its performance on low dimensional problems. This may be related to low dimensional problems that typically pose lesser complexity, hence resulting in more algorithms to exhibit better performance which yields a more competitive comparison.

7.5 Conclusion

This chapter presented the A-HIDMS-PSO algorithm that extends the standard HIDMS-PSO algorithm. The new variant proposed 8 new topological structures for the unit structure and divides them as slave-dominated and master-dominated. As opposed to the standard HIDMS-PSO, the A-HIDMS-PSO algorithm possesses a single population of particles that adopts either slave dominated or master-dominated topologies and during the search process, unit topological structures witch between the two support exploration and exploitation to yield a better balance between the two concepts. The performance of the A-HIDMS-PSO algorithm is verified over three benchmark test suites namely, CEC'13, CEC'14 and CEC'17 on 10, 30, 50 and 100 dimensions using 14 comparison algorithms (13 PSO variants and L-SHADE). The evidence attained on the results, convergence rate comparison and statistical significance comparison suggests that the A-HIDMS-PSO exhibited acceptable performance on low dimensional problems and is capable of outperforming the majority of the test algorithms employed on high dimensional problems.

Chapter 8 : Biased Eavesdropping Particle Swarm Optimisation Algorithm

8.1 Introduction

Emerging complex problems significantly challenge the current computational tools; hence in the last decade or so, bio-inspired approaches have become the dominant inspiration for the majority of meta-heuristic algorithms developed to handle new, complex problems in different domains. Although PSO's fundamental idea is based on biological inspiration, its model of particle movement is relatively simple compared to natural flocking behaviour.

In most cases, the homogeneous nature of particles' behaviour to move towards a common goal using the two exemplars, cognitive and social influences, tends to trigger rapid loss of diversity, leading to premature convergence. We have seen various approaches to addressing this problem in the previous chapters. Another contemporary way to potentially minimise this issue is to design an efficient heterogeneous agent behaviour to avoid the stagnation of particles and improve the algorithm's overall performance by avoiding premature convergence. Hence, in this chapter, we present a new heterogeneous PSO variant, BEPSO, inspired by the alert-signalling behaviour of animals to attract conspecifics to a discovered resource location for potential exploitation, and the way in which surrounding heterospecific eavesdropper animals try to exploit that information to improve their own fitness. In addition, our particle model incorporates a cognitive bias component to allow particles to build a form of evolving perception about each other through social experience, and this is used to allow particles to individually decide which behaviour to adopt to maintain heterogeneity in the swarm.

8.1.1 Eavesdropping Behaviour in Animals

Eavesdropping plays a significant role in animal communication and the evolution of communication among animals [243]. In short, eavesdropping occurs as a result of animals accessing communication signals, transmitted by heterospecifics (of a different group or species), that were not intended for them. In nature, it is more common for signal interceptors to be intraspecific (of the same species) in order to perceive the call and extract the required information accurately; however, it is not uncommon to observe interspecific animals (competitors of a different species) intercept signals and use them as an advantage to increase their own fitness. Interspecific eavesdropping is particularly interesting as different species may be proficient in distinct areas within the same habitat and capable of recognising different threats through their distinct sensory capabilities. A concrete example of interspecies eavesdropping is illustrated by the relationship between red squirrels and Eurasian jays [244]. In this case truly astonishing evolutionary dynamics have resulted in communication between a mammal and a bird, which have become positively biased towards one another and are able to warn and guard each other within the same habitat.

8.2 Biased Eavesdropping Particle Swarm Optimisation Algorithm

In the proposed algorithm, BEPSO, the particle's bio-inspired behaviour model comprises three components: recognition, communication, and bias. The recognition component refers to the particle's ability to distinguish between conspecific (of the same type) and heterospecific particles. The communication component refers to the implicit signal-based communication particles perform when they discover a new and better position. The bias component enables particles to build a form of perception towards each other that evolves through social experiences. This perception is used to adopt different behaviours during the search process.

The idea of eavesdropping in nature inspires the proposed algorithm's main search behaviour. In summary, initially, particles are divided into two groups. The particles of the same group recognise each other as conspecifics, and on the contrary, both groups' members recognise those from the other as heterospecific. All particles are assigned an initial random bias towards the rest of the particles in the swarm in such a fashion that any two conspecific particles may be either negatively biased or unbiased towards each other, while any two heterospecifics may be positively biased towards each other. Particles' search behaviour initiates by updating velocity and position using the canonical PSO algorithm's update equations (Eqs. 2.4 and 2.5). After the positional update, if particles discover a better position, in an attempt to guide conspecifics to a potentially better location, the particle implicitly communicates with the surrounding conspecific particles by transmitting a signal. The signaller particle intends the communication signal recipients to be solely conspecifics, but surrounding heterospecific particles eavesdrop and exploit the information the communication signal provides (detailed later). The signal recipients (from both conspecific and heterospecific groups) either accept or refuse to use the information provided by the signal considering several factors, including their bias towards the signaller particle. Before transmitting the signal, the particle determines the transmission point $\vec{\tau}$ for the intended communication signal. $\vec{\tau}$ is a position that lies between the previous and the current position of the signaller particle and $\vec{\tau}$ is calculated as:

$$\vec{\tau}_i^{t+1} = \mathbf{x}_{signaller}^{t} + rand * \left(\mathbf{x}_{signaller}^{t+1} - \mathbf{x}_{signaller}^{t}\right) \tag{8.1}$$

Where $\mathbf{x}_{signaller}^{t}$ and $\mathbf{x}_{signaller}^{t+1}$ are the previous and the current (newly discovered) position of the signaller particle, and $rand$ is a single uniformly distributed random number in the range (0,1). The communication signal has a radius defined by the SR parameter with minimum and maximum bounds. The $SR_{max} = 0.01 * (UB - LB) * d$ and $SR_{min} = 0.001 * (UB - LB) * d$ where UB, LB are upper and lower bounds of the given problem and d is the dimension of the problem. The radius of the communication signal is determined individually for each signal based on the particle's fitness compared to the average fitness in the swarm. This simulates the signal's loudness; hence, the range of the signal extends or shrinks based on the quality of the discovered position. This behaviour aims to mimic the confidence of the particle in the quality of the discovered location to attract more conspecifics; hence, the signaller particle transmits a signal with a wider influence range, and on the contrary, particles with less confidence in the quality of the discovered position use lower SR with the intention of transmitting a signal to the least number of conspecifics to minimise potential loss of fitness in the conspecific population. The SR parameter is calculated using the following equation:

$$SR(f(\mathbf{x}), \delta, SR_{min}, SR_{max}) = \begin{cases} rand(SR_{min}, \dfrac{(SR_{min} + SR_{max})}{2}), & f(\mathbf{x}) < \delta \\ rand(\dfrac{(SR_{min} + SR_{max})}{2}, SR_{max}), & else \end{cases} \tag{8.2}$$

Where $f(\mathbf{x})$ is the fitness of the signaller particle, and δ is the average fitness in the swarm at time t. The communication signal consists of *k* signal layers to mimic the environmental noise and distortion of the signal as it travels further to the boundary of the signal range. Hence, recipient particles located in different locations of the signal "hear" a distorted variant of the original signal. Figure 8-1 shows the visual depiction of the communication signal with intended conspecific recipients and eavesdroppers. Particles (both conspecific and eavesdroppers) can accept or ignore the information provided by the communication signal depending on their bias and the signaller

particle's confidence in the newly discovered position (detailed later). The recipient particles closest to the transmission point receive the least distorted signal. On the contrary, recipients located furthest from the transmission point receive the most distorted signal. This mechanism essentially minimises multiple particles clustering in the same region and avoids potential stagnation within recipient particles.

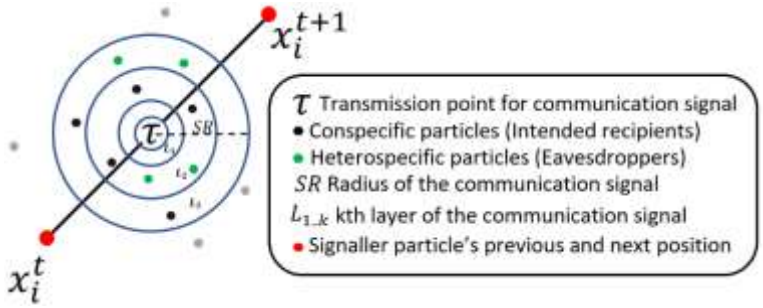


Figure 8-1-Visual representation of the communication signal sent by the signaller particle.

Among animals, survival and cooperation mainly depend on the bias, either through genetic influence, e.g., cooperating with a conspecific for the first time regardless of any lack of previous experience, or through social learning where positive association play a role [245], [246]. As briefly mentioned earlier, in our algorithmic model, particles are either positively biased, negatively biased or neutral (unbiased) towards each other, and particles use their bias to decide whether to accept or reject the information the signal provides as a guide. It might be worth highlighting that the negative bias refers to the particle's defence mechanism built over time to avert potential negative social guidance to minimise loss of fitness due to inconsequential communication signals transmitted by signaller particles. The latter, positive bias, on the contrary, enables particles to form an implicit cooperative relationship and accept guidance through signal calls from established "social partners" formed over time in an attempt to improve particles' fitness. Both mechanisms accent each other and allow particles to form a decentralised communication that evolves based on particles' individual social experiences. Particles' biases form over time as a result of the accumulation of consecutive positive or negative experiences attained from the usage of the signal information. The experience (λ) is positive when the signal improves the fitness of the recipient particles, and on the contrary case, the particle's experience of the guidance is negative. The accumulator decision model in the free-response paradigm is employed to enable particles to accumulate evidence through a signaller-recipient relationship and determine a final bias when sufficient evidence accumulates over time through this relationship until either of the accumulated variables reaches a threshold to trigger a decision response. The following equation is used to build the bias "evidence" between a recipient and a signaller particle:

$$\vec{\theta}(\lambda)_{ij}^{t} = \begin{cases} [\alpha^t + \lambda, \beta^t], & f(\mathbf{x})^{t+1} < f(\mathbf{x})^t \\ [\alpha^t - \lambda, \beta^t], & f(\mathbf{x})^{t+1} \geq f(\mathbf{x})^t \end{cases} \tag{8.3}$$

Where $\vec{\theta}(\lambda)_{ij}^{t}$ is the vector-valued output of the response variables the j^{th} particle (recipient) collects for the i^{th} (signaller) particle, α^t is the positive and $\beta^t$is the negative bias response variable at time t, $\lambda = MSE(f(\mathbf{x})^{t+1}, f(\mathbf{x})^t)0.01$ is the accumulating factor that contributes towards the negative or positive bias response variables. Figure 8-2 shows the visual depiction of Eq. 8.2. Each time evidence is collected, subsequently, Eq. 8.3 is used to determine if either of the response variables reached the specified bias threshold values.

$$Bias_{ij}^{t}\left(\vec{\theta}(\lambda)_{ij}^{t},\eta\right)=\begin{cases}1, & \vec{\theta}(\lambda)_{ij}^{t}\geq\eta\\ -1, & \vec{\theta}(\lambda)_{ij}^{t}\leq-\eta\\ 0, & \eta>\vec{\theta}(\lambda)_{ij}^{t}>-\eta\end{cases} \tag{8.4}$$

Where $Bias_{ij}^{t}$ is the j^{th} particle's bias towards the i^{th} particle, η is an integer in the range [10, 100]. The value of η controls the pace for particles bias; hence the value of η can have a direct impact on the behaviour of particles. Particles tend to be rapidly biased when η is set to approximate to the lower range. On the contrary, particles can remain unbiased towards most particles in the swarm for extended periods when η is in the higher range.

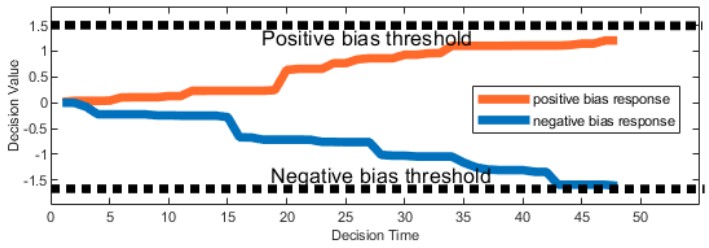


Figure 8-2 – The accumulator decision model exhibits i^{th} particle's bias towards the j^{th}particle at time t.

At the beginning of the search process, all particles are given random biases; positive, negative or neutral represented by corresponding values $1, -1$ and 0, respectively, to allow heterogeneity from the start of the search process. Since there would be no transmission of signals at $t = 1$, all particles update their velocity and position using the following update equation:

$$\mathbf{v}_i^{t+1} = w^t\mathbf{v}_i^t + c_1^t\mathbf{r}_1\left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t\mathbf{r}_2\left(\mathbf{gbest} - \mathbf{x}_i^t\right) \tag{8.5}$$

After the positional update, if the particle discovers a better position, it must transmit a communication signal to attract conspecifics to a potentially better location; hence the particle is required to determine a transmission point $\vec{\tau}$ using Eq. 8.6. As mentioned previously, to approximately simulate signal distortion, we mutate the signal vector k times (for k signal layers) starting with the smaller mutations ($p = 0.1$), and as k increases, the mutation probability increases to trigger larger mutations. This enables particles closest to the signal transmission point to receive the least distorted signal while particles located further away receive the most distorted signal call. In our algorithmic model, we employ the non-uniform mutation operator; however, an alternative mutation operator or addition of white noise to the signal vector can be used for the same purpose in a situation where time efficiency might be a priority. The particles within the signal range are identified by calculating the Euclidian distance between all particles and the signal transmission point τ. If the distance between the i^{th} particle and $\vec{\tau}$ is less than SR, then i^{th} particle is a recipient of the signal (see the pseudocode). Relying on information from another individual could be costly to recipients regardless of the signaller being a conspecific or not. Hence, when particles receive a contact signal, both the conspecifics of the signaller particle and surrounding eavesdropping heterospecific particles decide to exploit or ignore the signal information by a simple risk versus reward assessment. The particles' decisions to exploit or ignore the signal information allow particles to adopt two distinct behaviours, namely *signal-based* and *non-signal based* guidance, and following rules that define the criteria both conspecifics and eavesdroppers use to exploit or ignore the signal information:

1. The conspecific recipient particles accept to exploit signal information only if the recipient particle is positively biased or unbiased towards the signaller and the signaller particle's confidence in the newly discovered position is high. (The signaller particle's confidence is determined as high if $SR \geq \frac{(SR_{min}+SR_{max})}{2}$ and is low if $SR < \frac{(SR_{min}+SR_{max})}{2}$).
2. The eavesdropper particles accept to exploit signal information if the eavesdropper is positively or negatively biased, but the signaller's confidence in the newly discovered position is high.

In nature, animals adopt various strategies to deter eavesdroppers or makes their signals less desirable or noticeable to heterospecifics [247]. In this study, the signaller particle aims to evade eavesdroppers by occasionally deliberately using smaller SR values to attract fewer particles. This behaviour enables signaller to appear less confident in the quality of the discovered position to make the signal less conspicuous for eavesdroppers. This evasion strategy mostly affects eavesdroppers, even whilst negatively biased towards the signaller particle. The eavesdroppers consider the confidence of the signaller as a criterion to decide whether to exploit the signal information. However, this evasion strategy comes at a cost to conspecifics of the signaller as smaller SR simply narrows the signal range, hence targeting fewer conspecifics. Recipients of both conspecific and eavesdroppers that adopt the signal-based guidance use the following equation to update their velocity:

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1^t\mathbf{r}_1\left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t\mathbf{r}_2\left(\mathbf{L}_k - \mathbf{x}_i^t\right) \tag{8.6}$$

Where $\mathbf{v}_i^t$, $\mathbf{pbest}_i$ and $\mathbf{x}_i^t$ are velocity, personal best and current position of either conspecific or eavesdropper particle at time t and $\mathbf{L}_k$ is the signal vector for the k^{th} layer of the signal.

The non-signal based behaviour comprises of two *repellent* and a *collaboration* exemplar. The two repellent exemplars $\mathbf{x}_{furthest}^t$ and $\mathbf{x}_{OutOfRange}^t$ are the furthest located particle to the signaller and a non-recipient particle that is outside of the signal range. The two repellent exemplars are selected from the recipient's conspecific group. The *collaboration* exemplar $\mathbf{x}_{collaborative}^t$ requires randomly selecting four recipients from the corresponding group, and the $\mathbf{x}_{collaborative}^t$ exemplar is calculated as the mean of two vectors $\mathbf{x}_1^{rand}$ and $\mathbf{x}_2^{rand}$ where $\mathbf{x}_1^{rand}$ is a vector that lies between the positions of the first and second selected recipient particles and similarly $\mathbf{x}_2^{rand}$ lies between the third and fourth selected recipients. Figure 8-3 visualises the selection of the collaboration exemplar, and the following equations are used to calculate $\mathbf{x}_1^{rand}$, $\mathbf{x}_2^{rand}$ and the $\mathbf{x}_{collaborative}^t$ exemplar.

$$\begin{aligned} \mathbf{x}_1^{rand} &= \mathbf{x}_1^{ev} + rand * (\mathbf{x}_2^{ev} - \mathbf{x}_1^{ev}) \\ \mathbf{x}_2^{rand} &= \mathbf{x}_3^{ev} + rand * (\mathbf{x}_4^{ED} - \mathbf{x}_3^{ev}) \\ \mathbf{x}_{col}^t &= \overrightarrow{R} = \sum_{i=1}^{2} \frac{\mathbf{x}_i^{rand}}{2} \end{aligned} \tag{8.7}$$

Where $\mathbf{x}_1^{ev}$ to $\mathbf{x}_4^{ev}$ are the randomly selected four eavesdropper particles within the signal range. The particles that adopt the non-signal based guidance update their velocities using the following equation:

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1^t\mathbf{r}_1\left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t\mathbf{r}_2\left(\mathbf{S} - \mathbf{x}_i^t\right) \tag{8.8}$$

Where $\mathbf{S}$ is randomly selected exemplar as either $\mathbf{x}^t_{furthest}$, $\mathbf{x}^t_{OutOfRange}$ or $\mathbf{x}^t_{col}$.

Imitation is one of the most common social learning behaviours animals adopt. In our algorithmic model, the unbiased recipients imitate the dominating behaviour of their conspecifics. Hence, if p particles of the conspecifics or eavesdroppers adopt the signal-based guidance while q of them adopt the non-signal based guidance and $p > q$, then the unbiased conspecific/eavesdroppers imitate either of the behaviour dominantly adopted by their conspecifics. When $p = q$ or unbiased particles dominate one or both groups, signal-based or non-signal based behaviour is randomly adopted by the unbiased recipient particles.

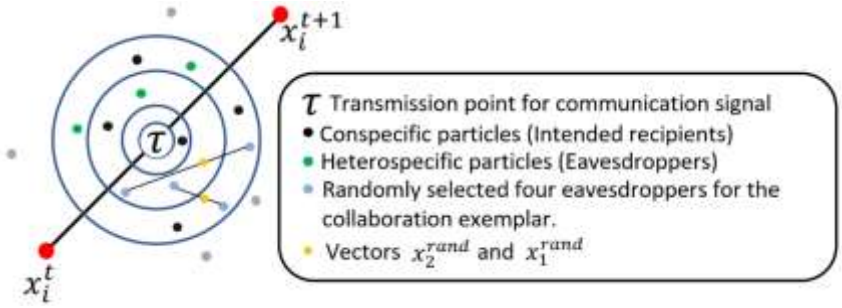


Figure 8-3- Visual depiction of the particles selected for collaboration exemplar.

The heterogeneity in the swarm is formed by signal-based and non-signal based behaviour adopted by recipient particles, and the particles' biases formed over time maintain the balance of particles adopting either behaviour. The entitlement of particles as recipients depends on several factors, including the previous and the discovered position by the signaller, SR value, and the calculated transmission point. Hence, a small fluctuation to one of these factors significantly alters the list of potential recipients of both conspecific and eavesdroppers at time t. Consequently, which set of particles become recipients is highly imponderable for each transmitted signal. In return, this unpredictable yet self-organising behaviour further supports particles to avoid monotonous behaviour. As a result, the continuous boost in the population diversity minimises the risk of particles being stuck at local optima. Concerning the sufficient exploitation of existing potential solutions, the bio-inspired model described so far is supported by incorporating multi-swarms introduced in the study [140]. In our algorithm, multi-swarms are periodically used as reinforcements to sufficiently exploit the potential local solutions discovered by the bio-inspired model. Preliminary experiments conducted to find the optimum time required for the multi-swarm search phase revealed that the best expected performance is achieved with an approximately 10:1 execution ratio. The initiation of the multi-swarm mechanism requires the division of the swarm into *N* subswarms. Instead of randomly splitting the swarm into N equal subswarms, in our study, three subswarms are formed based on particles' dominating biases to enable each subswarm to possess an asymmetrical and self-regulating population. Hence, a particle is a member of $subswarm_1$, if it is predominantly positively biased towards most particles in the swarm. Similarly, if a particle is mostly negatively biased or unbiased, then the particle belongs to the corresponding groups $subswarm_2$ and $subswarm_3$. Each member of the subswarm uses the following equation to update their velocity:

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1^t\mathbf{r}_1\left(\mathbf{pbest}_i - \mathbf{x}_i^t\right) + c_2^t\mathbf{r}_2\left(\mathbf{lbest}_k - \mathbf{x}_i^t\right) \tag{8.9}$$

Where $\mathbf{lbest}_k$ is the position of the fittest particle in the k^{th} subswarm.

In summary, particles' bias built over time ensures behavioural heterogeneity by enforcing particles to dynamically adopt signal-based and non-signal based guidance. This primary search strategy is supported by the periodically activated subswarm searches to efficiently exploit the existing solutions. The asymmetrical populations in the subswarms exploit different local solutions by varying densities of particles at different search phases, resulting in an efficient search behaviour with a good balance of exploration and exploitation.

Algorithm 5: BEPSO

Input: n as swam size, T_{max} as maximum number of iterations.
Output: $gbest$

$c_1=2.5-(1{:}T_{max}*2/T_{max})$; $c_2=0.5-(1{:}T_{max}*2/T_{max})$; $w_{max}=0.99; w_{min}=0.2$; C=0.15;

$$w_1=\frac{w_{max}+(w_{min}-w_{max})}{1+exp\left(-5\left(\frac{2t}{T_{max}}-1\right)\right)}$$

for t=1:T_{max}
  for i=1:n
    if $f(\bar{\mathbf{x}}_i) \geq f(\bar{\mathbf{x}})$ **then**
      w=w_1^t+C; if w>0.99, w=0.99, end
    else
      w=w_1^t+C; if w>0.20, w=0.20, end
    end
    if $phase_{subswarm} == true$ **then**
      Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 8.9 and 2.5
    else
      if $status_i == 0$ **then**
        Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 8.5 and 2.5
      else
        Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 8.5 and 2.5 (use guidance instead of the gbest exemplar)
        Evaluate the fitness of $\mathbf{x}_i$
        Calculate θ using Eq. 8.3
        Update ith particle's bias using Eq. 8.4
      end
      if $f(\mathbf{x}_i^{t+1}) < f(\mathbf{x}_i)$ **then**
        Calculate τ using Eq. 8.1
        if $randi([0\ 1]) == 0$ **then**
          $SR = U\left(SR_{min}, \frac{SR_{min}+SR_{max}}{2}\right)$
        else
          Calculate SR using Eq. 8.2
        end
        for $j = 1{:}\,n$ **do**
          if $norm(\tau - \mathbf{x}_j) < SR$ **then**
            if $j^{th} particle\ is\ a\ conspecific$ **then**
              if $j^{th} particle\ qualifies\ for\ signal-based\ guidance$ **then**
                Calculate $guidance_j$ using Eq. 8.6 and 2.5
              else
                Calculate $guidance_j$ using Eq. 8.8 and 2.5
              end
              $status_j = 1$
            elseif $j^{th} particle\ is\ a\ heterospecific$ **then**
              if $j^{th} particle\ qualifies\ for\ signal-based\ guidance$ **then**
                Calculate $guidance_j$ using Eq. 8.6 and 2.5
              else
                Calculate $guidance_j$ using Eq. 8.8 and 2.5
              end

```
                    status_j = 1
                end
            end
        end
    end
end
```

8.3 Parameter Analysis of BEPSO

Because the new algorithm has various parameters that can significantly impact performance, in this section we assess these parameters to find the best general set of values. The impact of the BEPSO parameters are evaluated on different dimensions of the CEC'14 and CEC'17 problem sets.

8.3.1 The impact of Population Size

The impact of the population size on the BEPSO algorithm was tested on the 10,30,50 and 100-dimensional CEC'14 and CEC'17 test functions. Tables 8-1:8-2 display the mean and final ranks for different population sizes and the best and second-best mean performance for 10,30 and 100-dimensional CEC'14 test functions was attained using the population size of 30 and 40, on the other hand for 50-dimensional functions, the $pop = 20$ exhibited the best and $pop = 30$ exhibited the second-best performance. The results on the CEC'17 test functions yield similar results and $pop = 20$ consistently attained the best mean performance for 30,50 and 100-dimensional functions. On the contrary, for 10-dimensional CEC'17 functions, we observe that pop=100, a higher population value, produced superior performance compared to rest of the population sizes. It is also worth noting that as shown in Figure 8-4, although pop=100 exhibited better mean performance for 10-dimensional CEC'17 functions, we observe that when $pop = 30$, the BEPSO algorithm attained lower mean values for significantly more functions and considering the marginal difference in the mean ranks (4.53 and 4.40), pop=30 may also serve as an effective population setting for 10-dimensional CEC'17 functions. Overall, we conclude that the population size of 20-40 generally perform well in most cases hence can be employed for most problems. In the comparative experiments presented later in the thesis, population size of 40 is employed for the BEPSO algorithm.

Table 8-1- Mean and final ranks of obtained using different population sizes on the 10, 30, 50 and 100 dimensional CEC'14 problems

Population	10D		30D		50D		100D	
20	4.00	3	4.67	5	2.93	1	3.73	3
30	3.73	2	3.20	1	3.43	2	3.27	2
40	3.47	1	3.63	2	3.90	3	2.83	1
50	5.03	5	3.90	3	3.43	2	4.57	5
60	4.40	4	4.50	4	4.50	4	4.47	4
70	5.33	7	4.77	6	5.33	6	5.27	6
80	5.23	6	5.57	7	5.27	5	5.70	7
100	6.13	8	6.17	8	7.07	7	6.33	8
200	6.47	9	7.40	9	7.93	8	7.63	9

Table 8-2- Mean and final ranks of obtained using different population sizes on the 10, 30, 50 and 100 dimensional CEC'17 problems

Population	10D		30D		50D		100D	
20	4.97	7	3.30	1	2.73	1	2.30	1
30	4.53	4	3.83	2	3.23	2	3.23	2
40	4.50	3	4.07	3	3.57	3	3.57	3
50	4.97	7	4.20	4	4.27	4	4.17	4
60	4.73	5	4.93	6	5.00	5	4.47	5
70	4.80	6	4.87	5	5.57	7	5.33	6
80	4.43	2	5.53	7	5.43	6	5.93	7
100	4.40	1	5.87	8	6.17	8	6.70	8
200	6.47	8	7.20	9	7.83	9	8.10	9

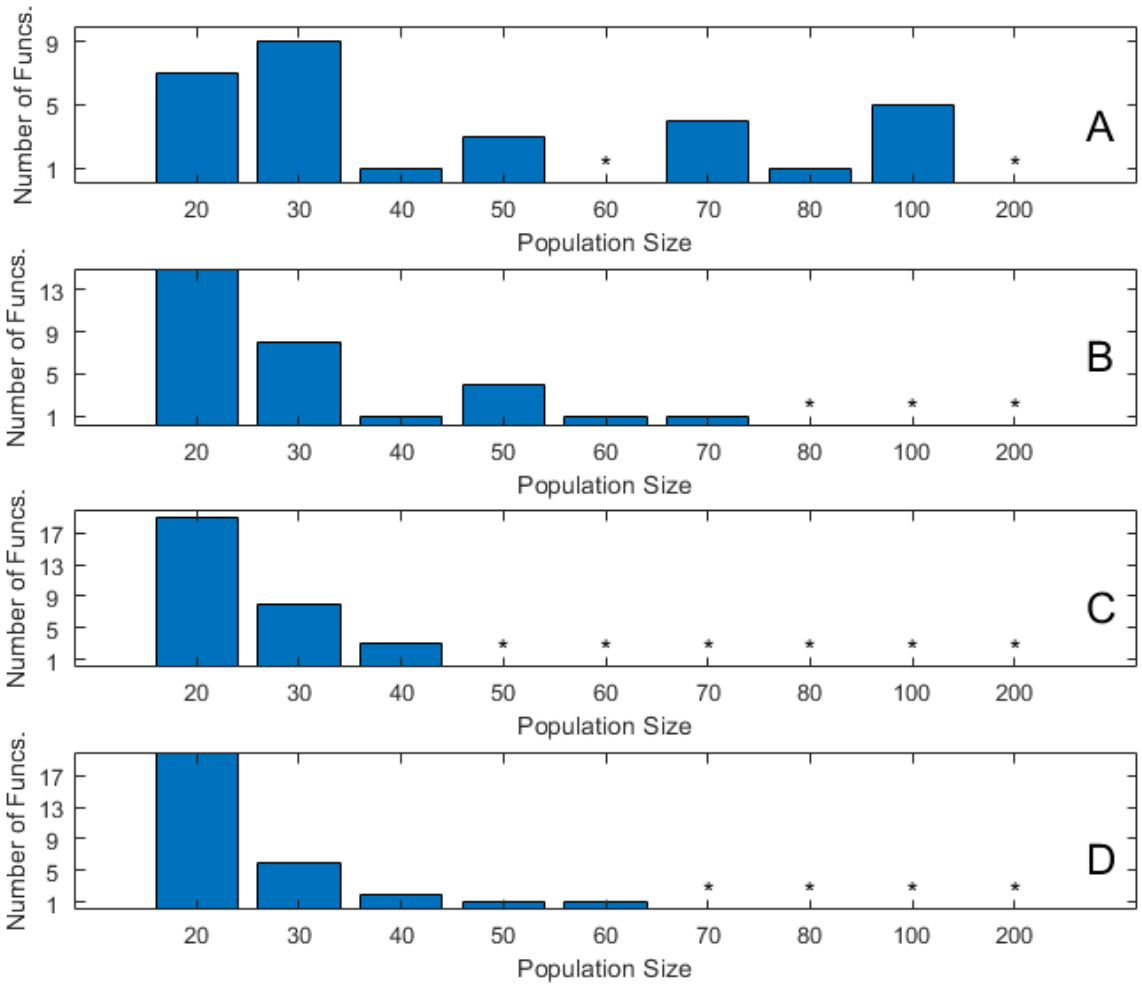


Figure 8-4 The total number of best performances achieved using different population sizes with respect to mean error values on the 10 (A), 30(B), 50(D), 100 (D) dimensional CEC'17 problems

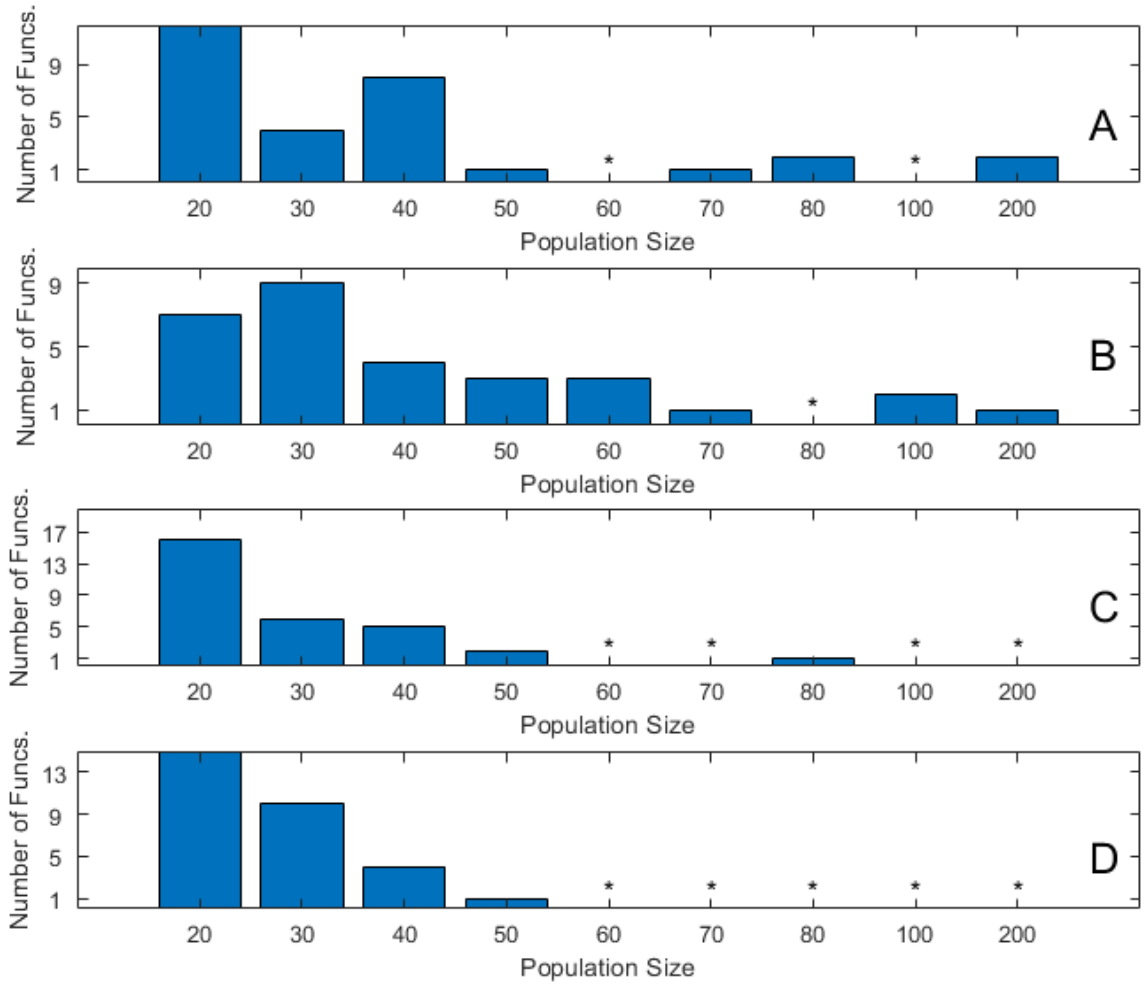


Figure 8-5 The total number of best performances achieved using different population sizes with respect to mean error values on the 10 (A), 30(B), 50(D), 100 (D) dimensional CEC'14 problems

8.3.2 The Impact of Search Phase Duration

In this section, we assess the impact of the duration of multi-swarm and non-multi-swarm search phases (the *phase* parameter). Initially, we had identified that executing both search phases equally for relatively short intervals (approximately 100 iterations) yield sufficiently good results. However, here a more extensive assessment is conducted on multiple test suites across various dimensions. We have tested 12 parameter values as intervals, the shortest interval being 10 and the longest 1000 iterations. The results obtained on the CEC'14 and CEC'17 test functions are ranked, mean and final ranks are displayed in Tables 8-3:8-4. The ranks indicate that the value of 10 attained the best performance for the entire CEC'14 test suite, and for 30 and 100-dimensional CEC'17 test functions. For 10 and 50-dimensional CEC'17 functions, phase=25 achieved the lowest ranks however $phase = 10$ also attained the second-best performance in these two cases. Hence, shorter intervals for the $phase$ parameter yield better overall mean performance on both test suites and four different dimensions. In addition, Figure 8-6 shows the lowest mean, standard deviation and minimum values each parameter value attained by for the number functions. We also observe that phase=10 not only achieved the lowest mean value for the majority of functions, but also attained the lowest standard deviation and found the lowest minimum value for the greater number of functions. Moreover, the convergence rate for 100-dimensional CEC'17 test functions is shown in Figure 8-7. The convergence rates reveal that phase=10 consistently convergences faster to a better final solution in 19 of the 30

functions. Hence, $phase = 10$ was used for both non-multi-swarm and multi-swarm search phases for all the main comparative experiments conducted in the following chapters.

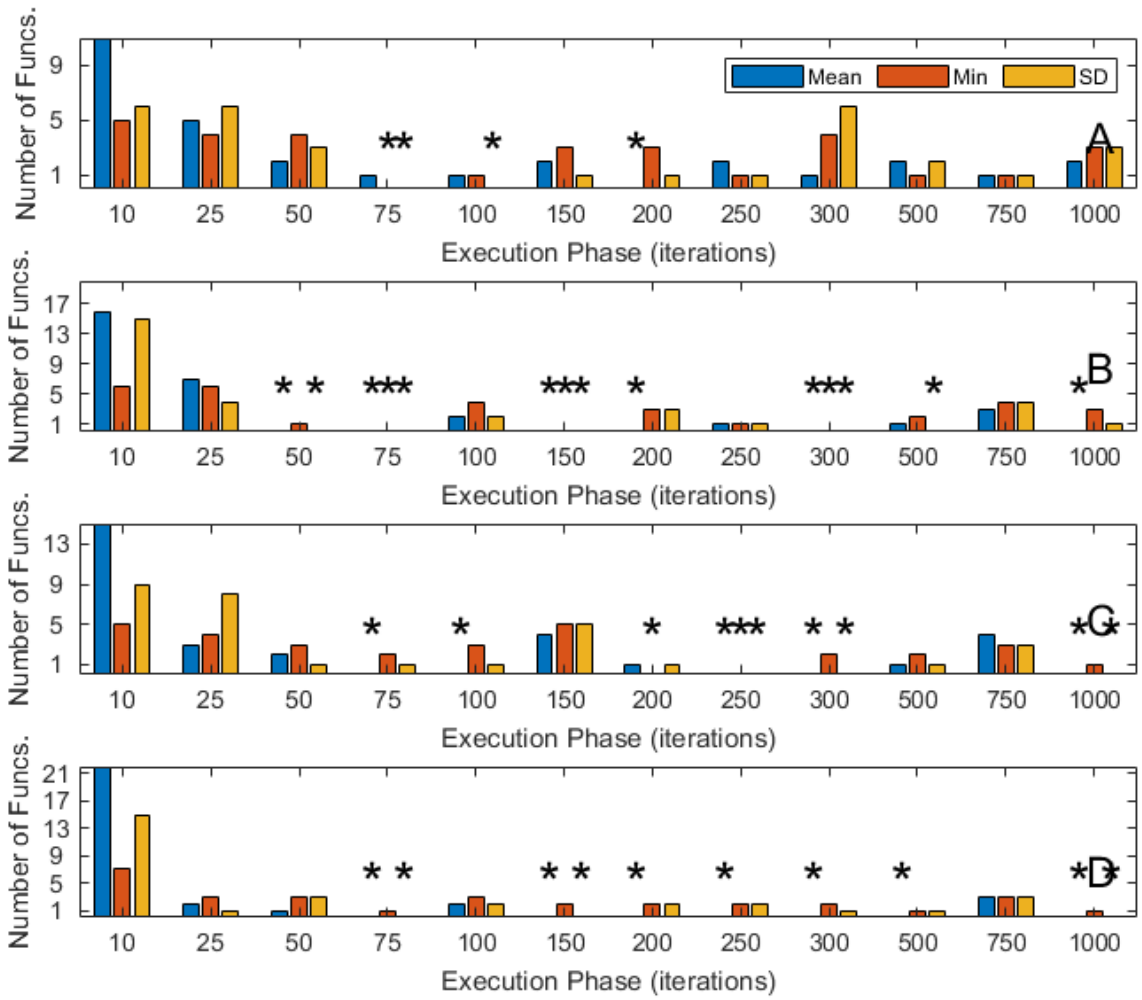


Figure 8-6 The impact of execution phases on obtaining the lowest mean error, min, and standard deviation values on 10 (A), 30 (B), 50 (D) and 100 (D) dimensional CEC'17 problems

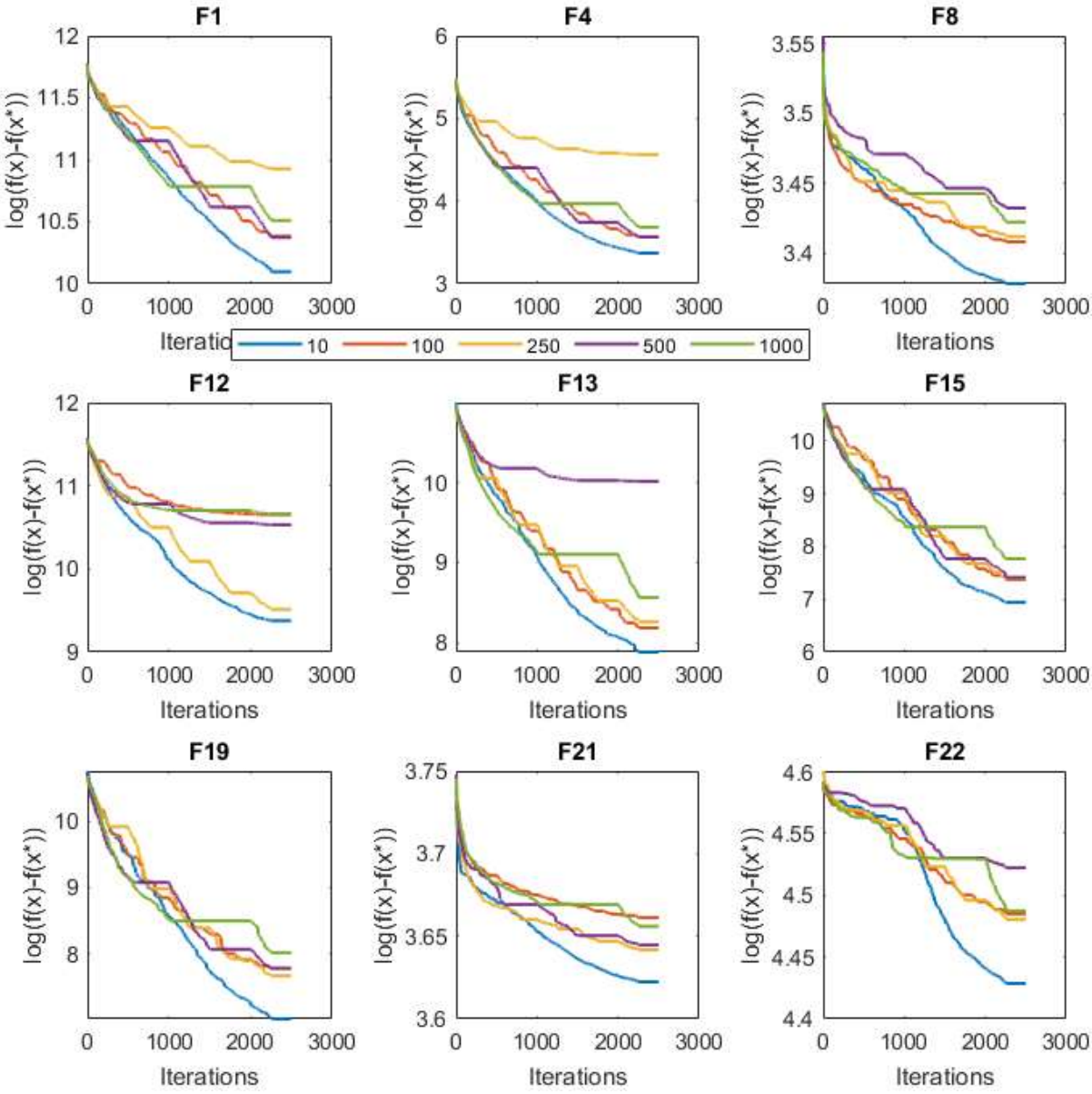


Figure 8-7 Impact of different execution phases on convergence rate for 100-dimensional CEC'17 functions

Table 8-3 Mean and final ranks obtained on the CEC'14 problems using different execution rates

Execution Phase	10D		30D		50D		100D	
10	4.67	1	3.87	1	4.60	1	3.03	1
25	5.73	3	4.93	2	5.03	2	5.00	3
50	5.47	2	5.37	3	5.47	3	5.87	5
75	6.73	8	7.13	10	6.87	10	6.47	6
100	7.60	12	6.67	8	6.83	9	6.73	7
150	6.80	9	6.63	7	6.87	10	6.97	8
200	6.00	5	7.10	9	6.73	8	7.63	11
250	6.70	7	5.60	4	5.90	4	5.67	4
300	6.23	6	7.23	11	6.37	6	7.33	9
500	7.20	11	6.43	5	6.30	5	7.37	10

750	6.87	10	6.60	6	6.40	7	4.60	2
1000	5.80	4	8.23	12	8.43	11	9.13	12

Table 8-4 Mean and final ranks obtained on the 10, 30, 50 and 100 dimensional CEC'17 problems using different execution rates

Execution Phase	10D		30D		50D		100D	
10	5.13	2	2.87	1	4.27	2	2.33	1
25	4.97	1	3.27	2	4.00	1	4.83	2
50	6.67	8	6.87	6	6.17	7	5.57	4
75	6.37	6	6.90	7	7.27	9	7.80	10
100	7.17	10	7.53	11	6.00	5	6.63	6
150	7.63	11	7.37	10	5.83	4	7.00	8
200	6.73	9	7.33	9	6.03	6	7.20	9
250	6.33	5	6.73	5	6.67	8	6.50	5
300	6.33	5	7.13	8	8.00	10	7.20	9
500	6.57	7	6.47	4	7.27	9	6.77	7
750	5.77	3	5.27	3	5.77	3	5.47	3
1000	6.13	4	8.07	12	8.53	11	8.50	11

8.3.3 The Impact of the Signal Range

The SR parameter range (SR_{min} ,SR_{max}ontrols the scope of influence the signaller particle has on the recipients. Hence, the value of SR could significantly alter the dynamics of the BEPSO algorithm. The mean and final ranks for the calibration of the SR parameter are shown in Table 8-5:8-6. In general, lower values of for the SR parameter tend to yield better results. For 10 and 50-dimensional CEC'14 functions, $SR_{max} = 25; SR_{min} = 25$ and for 30 and 100-dimensional CEC'14 functions $SR_{max} = 30; SR_{min} = 5$ exhibited the best performance. For the CEC'17 test functions, at 10 dimensions $SR_{max} = 600; SR_{min} = 60$, at 30 dimensions $SR_{max} = 100; SR_{min} = 100$, at 50 dimensions $SR_{max} = 100; SR_{min} = 10$ and at 100 dimensions $SR_{max} = 30; SR_{min} = 5$ attained the best performance. In addition, as shown in Figure 8-8, the results on the CEC'14 test functions reveal that $SR_{max} = 3; SR_{min} = 0.6$ consistently attained the lowest mean values for more functions compared to other SR values, and we particularly observe that the impact is more evident with increased dimensionality.

Table 8-5 Mean and final ranks obtained on the 10, 30, 50 and 100 dimensional CEC'14 problems using different signal range values

SR values	10D		30D		50D		100D	
$SR_{max} = 0.01; SR_{min} = 0.001$	4.23	6	3.63	2	3.30	1	3.30	1
$SR_{max} = 0.02; SR_{min} = 0.01$	4.23	6	3.83	4	3.97	4	4.33	6
$SR_{max} = 0.05; SR_{min} = 0.01$	3.53	2	4.00	5	3.83	3	3.67	2
$SR_{max} = 0.1; SR_{min} = 0.05$	3.23	1	4.00	5	4.37	7	3.77	3
$SR_{max} = 0.2; SR_{min} = 0.1$	4.03	4	3.77	3	4.03	5	4.00	5
$SR_{max} = 0.3; SR_{min} = 0.2$	4.07	5	4.63	6	4.10	6	3.90	4
$SR_{max} = 0.4; SR_{min} = 0.3$	3.97	3	3.43	1	3.70	2	4.33	6
$SR_{max} = 0.01; SR_{min} = 0.001$	4.23	6	3.63	2	3.30	1	3.30	1
$SR_{max} = 0.02; SR_{min} = 0.01$	4.23	6	3.83	4	3.97	4	4.33	6
$SR_{max} = 0.05; SR_{min} = 0.01$	3.53	2	4.00	5	3.83	3	3.67	2

Table 8-6 Mean and final ranks obtained on the 10, 30, 50 and 100 dimensional CEC'17 problems using different signal range values

SR values	10D		30D		50D		100D	
$SR_{max} = 0.01; SR_{min} = 0.001$	4.10	5	4.37	7	3.63	2	3.90	5
$SR_{max} = 0.02; SR_{min} = 0.01$	4.20	6	3.93	4	3.90	4	4.50	6
$SR_{max} = 0.05; SR_{min} = 0.01$	3.70	2	4.17	6	4.20	6	3.43	2
$SR_{max} = 0.1; SR_{min} = 0.05$	4.03	3	3.43	1	4.10	5	4.63	7
$SR_{max} = 0.2; SR_{min} = 0.1$	4.07	4	3.87	3	4.10	5	3.20	1
$SR_{max} = 0.3; SR_{min} = 0.2$	3.60	1	3.53	2	3.50	1	3.80	3
$SR_{max} = 0.4; SR_{min} = 0.3$	3.60	1	4.00	5	3.87	3	3.83	4
$SR_{max} = 0.01; SR_{min} = 0.001$	4.10	5	4.37	7	3.63	2	3.90	5
$SR_{max} = 0.02; SR_{min} = 0.01$	4.20	6	3.93	4	3.90	4	4.50	6
$SR_{max} = 0.05; SR_{min} = 0.01$	3.70	2	4.17	6	4.20	6	3.43	2

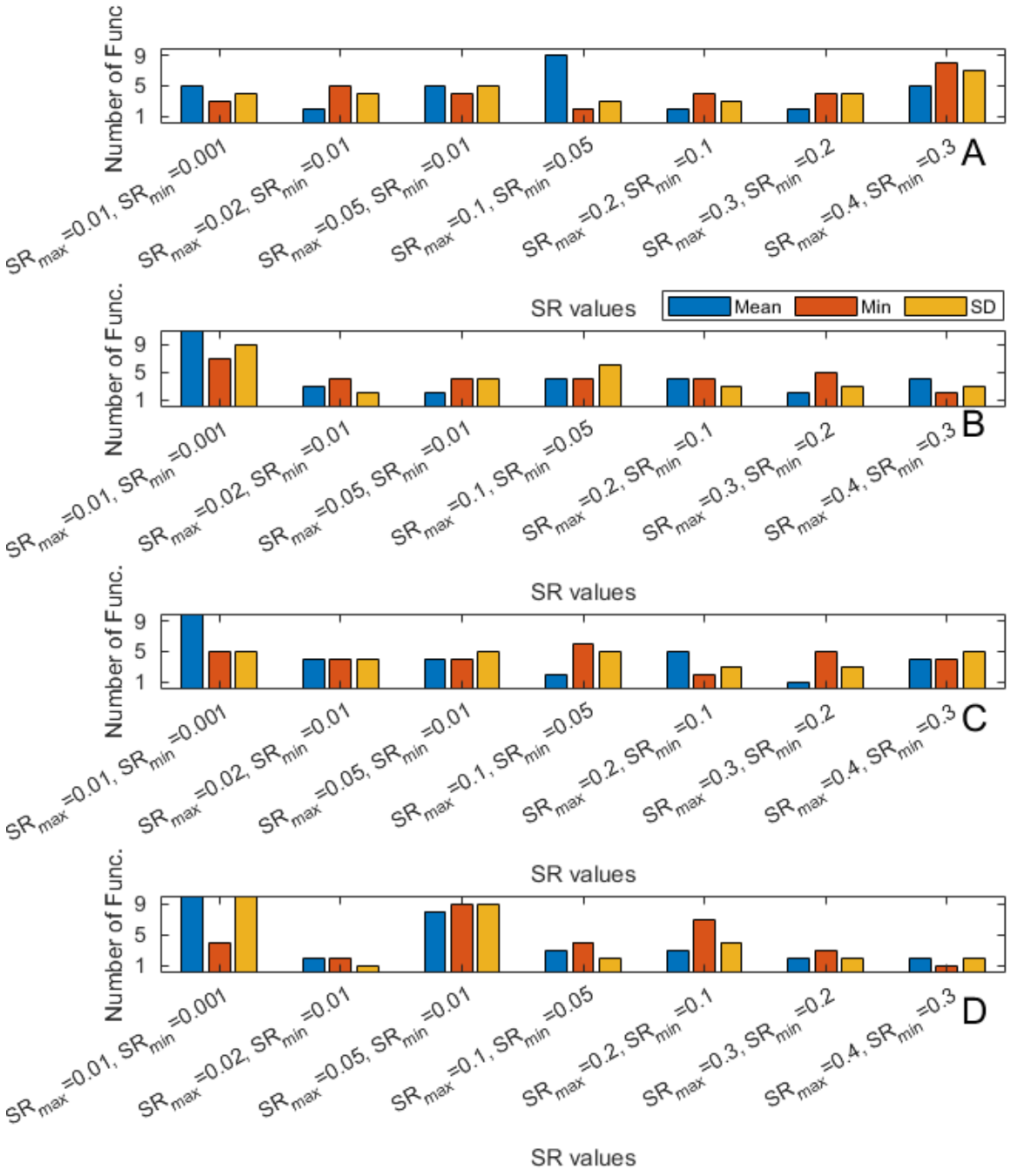


Figure 8-8 The impact of signal range on obtaining the lowest mean error, min, and standard deviation values on 10 (A), 30 (B), 50 (D) and 100 (D) dimensional CEC'14 problems

A single optimal SR value was not observed in our experiments however when we examine the results closely, we find that $SR_{max} = 0.01, SR_{min} = 0.001$ consistently exhibit the best performance for multimodal functions. For hybrid and composition functions, we observe three values $SR_{max} = 0.05, SR_{min} = 0.01$, $SR_{max} = 0.1, SR_{min} = 0.05$ and $SR_{max} = 0.2, SR_{min} = 0.1$ that dominates rest of the tested parameter values. Based on the outcome shown in Figure 8-8, $SR_{max} = 0.01, SR_{min} = 0.001$ will be used the main experiments.

8.3.4 The Impact of the Bias Threshold

In this subsection, the impact of the bias threshold parameter (BT) is assessed on the mean performance and convergence of the BEPSO algorithm. The bias threshold controls particles' negative or positive *"perception"* towards other particles which in return influences the guidance of the particles. Hence, in essence, the higher values set for the bias threshold upper and lower limit tend to require more *evidence* for particles to be become positively or negatively biased, and on the contrary, the lower values force particles to rapidly attain a bias with minimal evidence.

Tables 8-9:8-10 display the mean and final ranks obtained on 10, 30, 50 and 100-dimensional CEC'14 and CEC'17 benchmark problems for various bias threshold values. The ranks obtained suggest that in general, lower BT values (rapid bias) consistently yield better overall mean result across all dimensions of the CEC'14 and CEC'17 test suites. More specifically, for CEC'14 benchmark problems, $BT = 10 - 20$ and for CEC'17 problems, $BT = 20 - 25$ values exhibit the best mean performance. Hence, in the comparative experiments, $BT = 20$ is used for all test suites.

Table 8-9 Mean and final ranks obtained on the 10, 30, 50 and 100-dimensional CEC'14 problems using different (BT) bias threshold values

BT	10D		30D		50D		100D	
5	7.07	6	7.00	5	7.60	10	7.57	8
10	7.10	7	6.13	1	6.70	4	6.17	2
15	7.27	9	6.63	2	5.93	1	8.00	9
20	5.77	1	7.17	8	6.83	6	5.90	1
25	6.80	4	7.03	6	6.77	5	6.30	3
30	6.57	3	6.77	3	8.80	13	6.90	6
40	7.00	5	7.30	9	6.53	3	8.47	13
50	7.77	11	8.40	14	7.57	9	6.50	4
60	7.83	12	6.87	4	6.90	8	7.13	7
70	7.20	8	7.07	7	9.00	14	8.10	10
80	8.37	13	8.13	12	6.87	7	8.20	11
90	9.37	14	8.17	13	7.97	11	6.87	5
100	7.47	10	7.83	11	8.20	12	7.57	8
200	6.40	2	7.47	10	6.30	2	8.30	12

Table 8-10 Mean and final ranks obtained on the 10, 30, 50-and 100-dimensional CEC'17 problems using different (BT) bias threshold values

BT	10D		30D		50D		100D	
5	7.47	8	6.97	5	7.00	6	7.57	10
10	7.30	7	6.57	2	8.13	10	6.63	3
15	8.60	14	7.57	8	8.17	11	8.10	12
20	5.80	1	8.37	13	5.50	1	6.10	1
25	7.60	9	6.07	1	5.87	2	6.37	2
30	6.23	2	6.57	2	6.53	4	7.27	6
40	7.87	12	6.83	3	8.23	12	7.70	11
50	6.60	3	6.87	4	8.57	13	7.37	7
60	6.83	4	7.10	6	8.63	14	6.77	4
70	6.93	5	7.87	10	7.87	9	9.73	13

80	7.67	10	7.37	7	6.80	5	7.03	5
90	7.83	11	8.03	12	6.10	3	6.37	2
100	8.20	13	8.00	11	7.13	7	7.43	8
200	7.03	6	7.80	9	7.43	8	7.53	9

8.4 Experiments

8.4.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed BEPSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

8.4.1.1 Results on 10 dimensional problems

The mean errors obtained on the 10-dimensional CEC'13 benchmark problems suggest that for the 5 unimodal functions (F1~F5), the best mean solution for F2, F3, F4 and F5 is provided by L-SHADE whilst LPSO exhibited the best performance on F1. The results on the multimodal functions (F6~F20), L-SHADE solved F8, F10~F14, F16~F18 whilst DMS-PSO obtained the lowest mean error values on F6, F7, F9 and F15. Moreover, the best mean performance for F19 and F20 is individually exhibited by MiPSO and HCLPSO, respectively. On the composition functions (F21~F28), the best solutions are obtained by L-SHADE, LPSO, DMS-PSO, MaPSO, UPSO, EPSO and BEPSO where L-SHADE exhibited the best performance on F22 and F27. UPSO and DMS-PSO achieved the best results on F21 and F23, respectively. The lowest mean error for F24 and F25 are obtained by EPSO and MaPSO, respectively. Similarly, the proposed algorithm BEPSO and LPSO provided the best mean solution for a single function, namely F26 and F28.

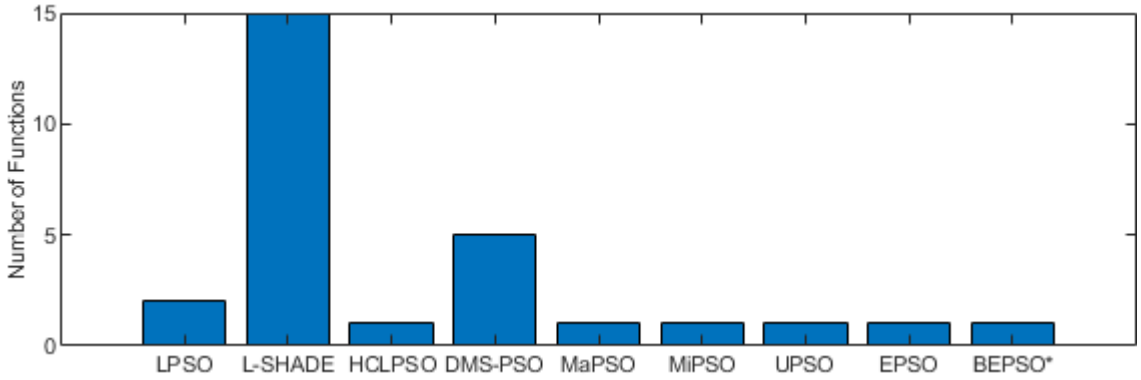


Figure 8-9 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

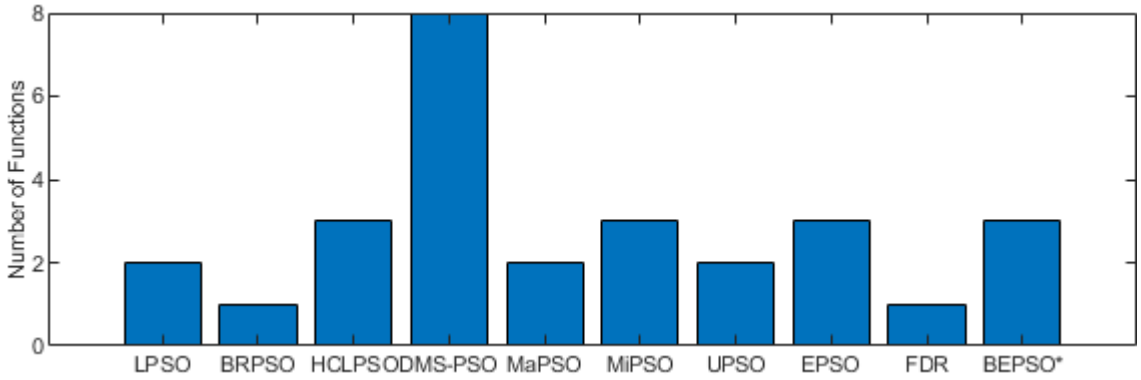


Figure 8-10 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'13 problems

Overall, in Figure 8-9:8-10, the total number of functions solved by each algorithm is displayed. In Figure 8-9, the comparison includes L-SHADE and in Figure 8-10, the comparison only includes the 13 PSO variants. As we can observe from both figures, in the first case the proposed algorithm BEPSO solved comparable number of problems to most PSO variants. In the latter case, the BEPSO algorithm attained lowest mean error values for equal number of functions to EPSO, MiPSO and HCLPSO.

8.4.1.2 Results on 30 dimensional problems

The results obtained on the 30-dimensional problems are indicate that for unimodal functions (F1~F5), DMS-PSO performed best on F1, F5, L0-SHADE exhibited the best performance on F2, F3 and finally, EPSO obtained the lowest mean error value for F4. On the multimodal functions (F6~F20), L-SHADE performed best on F6, F8, F10, F11, F14, F16, F17, F18 and F19. The proposed algorithm BEPSO provided the best mean solution on F7, F12, F13, F15, F20 and EPSO achieved the best mean solution for F9. The results on the 30-dimensional composition functions (F21~F28) show that the proposed algorithm BEPSO solved the majority of the composition functions and obtained the lowest mean error mean on F23, F24, F26 and F27. The best solution for F21, F22, F25 and F28 are obtained by (in the same order) LPSO, L-SHADE, DMS-PSO and EPSO, respectively.

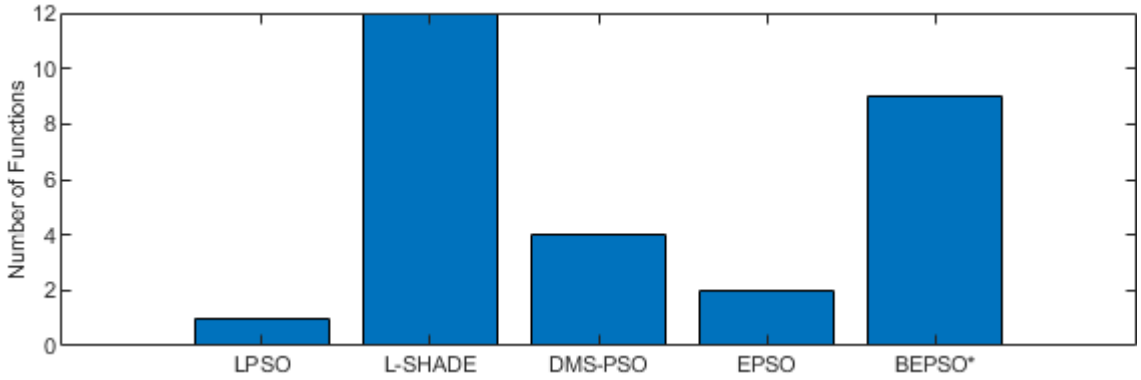


Figure 8-11 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

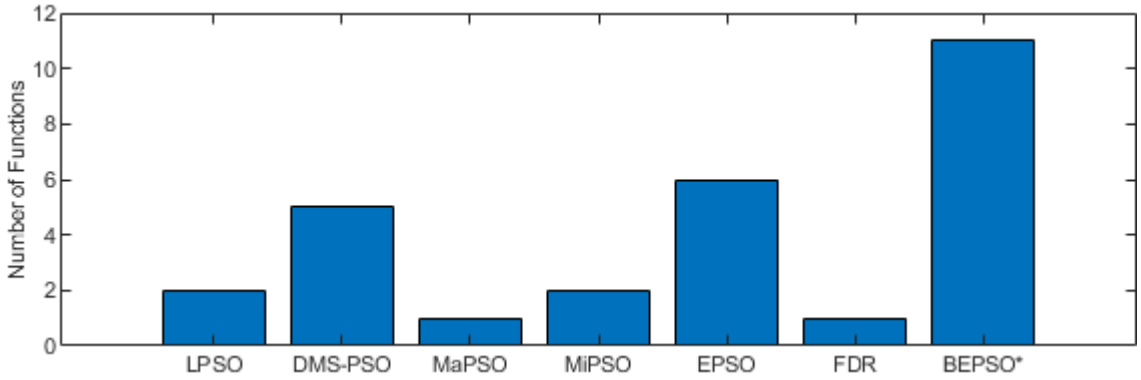


Figure 8-12 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'13 problems

Overall, as shown in Figure 8-11, the best solutions for the 28 30-dimensional functions are obtained only by L-SHADE, BEPSO, DMS-PSO, EPSO and LPSO where L-SHADE performed best on 12 of the 28 functions whilst the proposed algorithm BEPSO exhibited close performance and provided the best solution on 9 of the 28 problems. In Figure 8-12, the comparison among PSO variants reveal that the proposed algorithm BEPSO solved more problems compared to all PSO variants employed in the experiment.

8.4.1.3 Results on 50 dimensional problems

The mean errors values obtained on the 50-dimensional CEC'13 benchmark problems indicate that on the unimodal functions (F1~F5), DMS-PSO obtained the best mean errors for F1, F5 whilst L-SHADE, EPSO and the proposed algorithm BEPSO performed best on F2, F4 and F3, respectively. We observe that the best solutions are obtained by the same 4 algorithms on the multimodal functions. L-SHADE performed best on F8, F10, F11, F14, F15, F16, F17 and F19 whereas BEPSO achieved the best performance on F7, F12, F13, F18 and F20. The lowest mean error values for F6 and F9 are achieved by EPSO and DMS-PSO, respectively. On the composition functions (F21~F28), the proposed algorithm BEPSO (F23, F24) and DMS-PSO (F25, F27) achieved the best mean performance best on 2 problems. LPSO, L-SHADE, MiPSO and HCLPSO obtained the lowest mean error values for a single function, namely, F21, F22, F26 and F28, respectively.

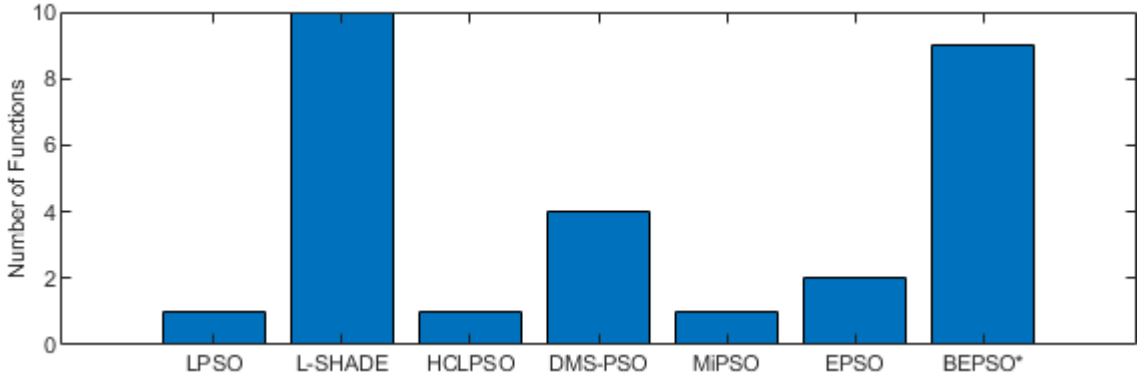


Figure 8-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

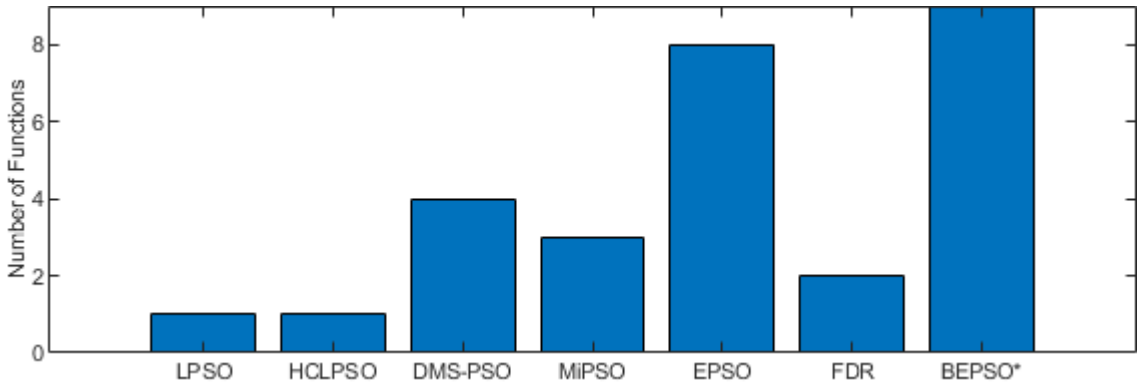


Figure 8-14 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'13 problems

A shown in Figure 8-13, the overall performance of the test algorithms indicates that L-SHADE and BEPSO provided best solutions for majority of the 50-dimensional problems where L-SHADE performed best on 10 of the 28 problems and BEPSO on 8 of the 28 50-dimensional problems. The closest performance to these two algorithms is attained by DMS-PSO which found the best mean solutions for 5 functions. Similarly, in Figure 8-14, the same comparison among PSO variants suggest that the BEPSO algorithm exhibited better mean performance on most of the 50-dimensional CEC'13 problems compared to all PSO variants.

8.4.1.4 Results on 100 dimensional problems

The mean error values obtained on the 100-dimensional CEC'13 problems indicate that on the unimodal functions (F1~F5), the best mean solution for each unimodal function is attained by 5 different algorithms namely, DMS-PSO (F1), L-SHADE (F2), BEPSO (F3), EPSO (F4) and MiPSO (F5), respectively. On the multimodal functions (F6~F20), L-SHADE and the proposed algorithm BEPSO solve majority of the multimodal functions where L-SHADE obtained the lowest meane error values on F8, F10, F14, F15, F16, F17, F19 and BEPSO performed best on F7, F12, F13 and F18. For functions F9 and F20, DMS-PSO exhibit the best performance whilst LPSO and EPSO provide the best mean solution for F6 and F11, respectively. The results on the composition functions (F21~F28) show that the BEPSO algorithm solve most of the composition functions including F23, F24 F27 and F28. The best mean solutions for problems F21, F22, F25 and F26 are attained by UPSO, L-SHADE, DMS-PSO and EPSO, respectively.

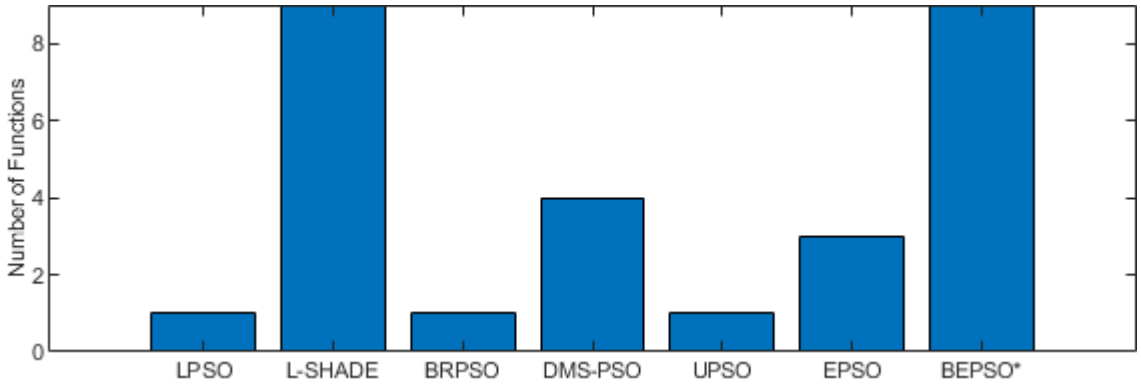


Figure 8-15 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

As shown in Figure 8-15, the top overall mean performance on the 100-dimensional CEC'13 problems are observed by BEPSO and L-SHADE. Both algorithms exhibit better performance in

solving the majority of the 100-dimensional problems and with vast difference compared to rest of the algorithms. Besides the overall problems, when observed, the performance of BEPSO on 100-dimensional unimodal is comparable to other algorithms. For the multimodal functions, BEPSO attained better performance on more problems compared to all PSO variants and finally, on the composition functions, the proposed algorithm BEPSO solved more problems compared to all comparison algorithms.

The final ranks indicate that the proposed algorithm BEPSO ranked 5th on the 10-dimensional and 3rd on the 30-dimensional problems, outranking 12 of the 14 comparison algorithms. Further, the final ranks obtained on other dimensions show that BEPSO ranked 4th and 5th on 50 and 100-dimensional problems. Overall, in the worst case, the proposed algorithm BEPSO outranked 10 of the 14 comparison algorithms and exhibit particularly efficient mean performance on the 30-diemnsional CEC'13 problems where it outranked 12 of the 14 test algorithms. The details of the mean and final ranks are given in Appendix 2.

Table 8-11 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2013 problems

BEPSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	=	=	+	=	+	=	-	=	=	+	6/7/1
30	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
50	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
100	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0

In addition, Table 8-12 displays the results of the Wilcoxon signed rank test for the CEC'13 problems. The performance of BEPSO is significantly better than 9 algorithms for 30, 50 and 100-dimensional CEC'13 problems and for the same dimensions, no significance is found in the performance of 5 algorithms and moreover, none of the comparison algorithms' performance was statistically significant compared to BEPSO's performance. For the entire experiment conducted on the CEC'13 test suite across multiple dimensions, only 2 algorithms (on the 10-dimensional probs.) yield significantly better performance than BEPSO.

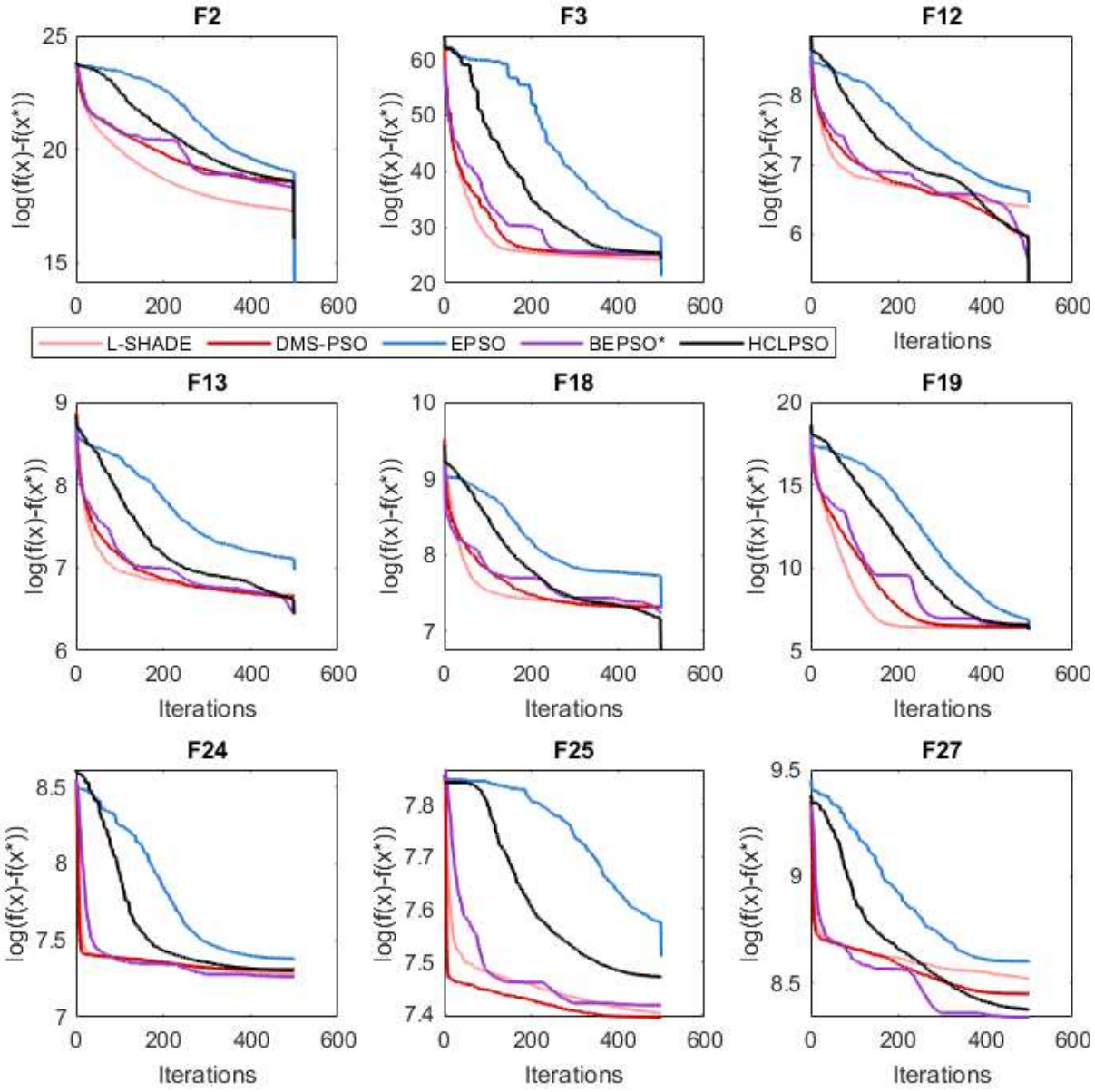


Figure 8-16 Average convergence rate comparison of BEPSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'13 problems

In Figure 8-16, the convergence characteristics of the BEPSO algorithm is compared with L-SHADE and three PSO variants EPSO, DMS-PSO and HCLPSO. The comparison suggests that the proposed algorithm can convergence as efficient as the compared algorithms in most test cases.

8.4.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed BEPSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

8.4.2.1 Results on 10 dimensional problems

The results on the 10-dimensional CEC'14 problems suggest that for unimodal problems F1, F2 and F3, L-SHADE exhibited the best mean performance. For multimodal problems (F4~F16), once again L-

SHADE obtained the best performance for F8, F10, F11, F12 and F15 whilst DMS-PSO performed best on F6, F12, F14 and F16. Moreover, EPSO (F4, F5) and the proposed algorithm BEPSO (F7, F9) attained the lowest mean error values for two functions. The results on the hybrid functions (F17~F22) reveal that L-SHADE provided the best mean performance for all hybrid functions at 10 dimensions and the performances for the composition functions (F22~F30), MaPSO (F23, F27) and LPSO (F28, F29) individually solved two problems whilst BEPSO, EPSO, DMS-PSO and FIPS individually provided the best mean performances for F24, F25, F26 and F30, respectively.

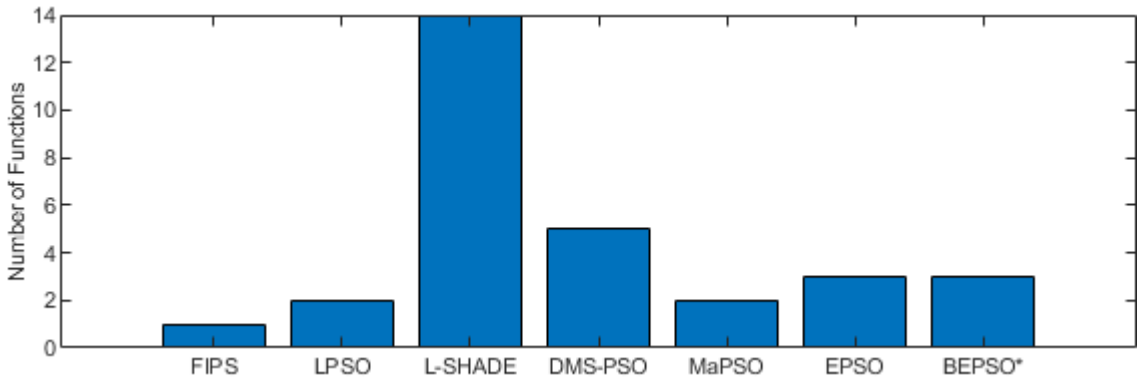


Figure 8-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

As shown in Figure 8-17, the overall mean performance on the 10-dimensional CEC'14 problems yield that L-SHADE obtained the best performance for majority of problems. Among the PSO variants, DMS-PSO solved more functions compared to other PSO variants.

8.4.2.2 Results on 30 dimensional problems

The experimental results obtained on the 30-dimensional CEC'14 problems suggest that for the unimodal problems (F1~F3) indicate that L-SHADE solved all unimodal problems. Similarly, for multimodal functions (F4~F16), once again L-SHADE exhibited the best performance for most of the multimodal functions including F4, F8, F10, F11, F12, F15 and F16 whilst HPSO-TVAC, FIPS, DMS-PSO and FDR individually obtained the lowest mean error values on F5, F7, F13 and F14, respectively. Moreover, the proposed algorithm BEPSO exhibited the best performance on F6 and F9. For hybrid functions (F17~F22), the lowest mean errors for F17, F19, F20 and F21 are achieved by L-SHADE whilst MaPSO and LPSO provided the best mean solutions for F18 and F22, respectively. The results on the composition functions (F23~F30) reveal that DMS-PSO solved F23, F25 and F29 whilst FIPS, L-SHADE, BEPSO, LPSO and MaPSO individually obtained the lowest mean errors for F24, F26, F27, F28 and F30, respectively.

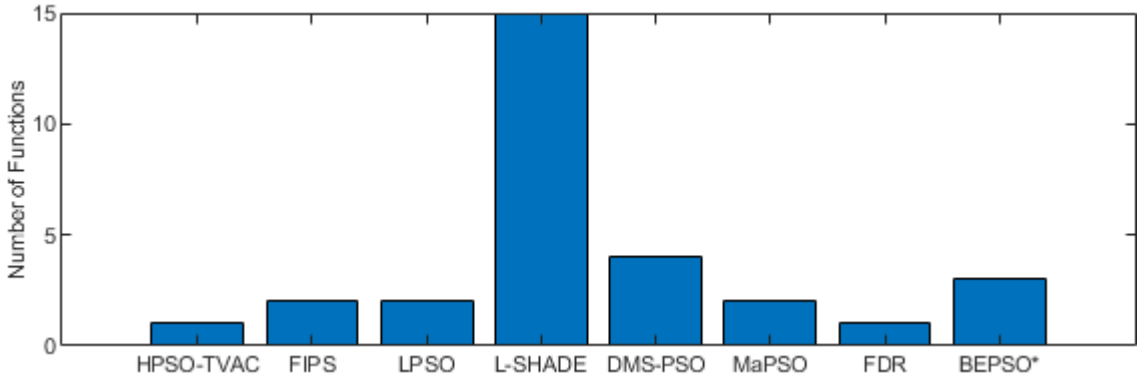


Figure 8-18 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'14 problems

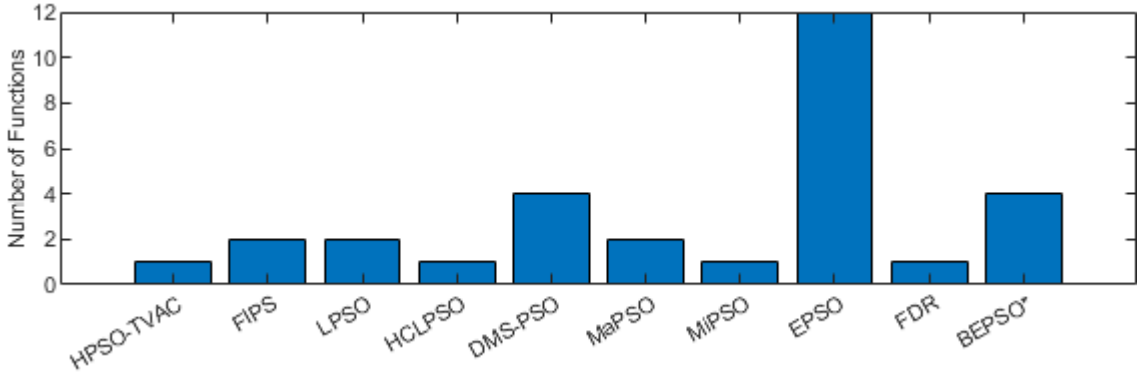


Figure 8-19 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'14 problems

Figure 8-18:8-19 demonstrate the total number of problems each algorithm performed best on for the 30-dimensional CEC'14 problems. The first (Fig. 8-18) exhibits the comparison among all test algorithms and the latter (Fig. 8-19) reveals the comparison among PSO variants. In the first case, BEPSO solved more problems compared to 12 test algorithms (except L-SHADE and DMS-PSO) and in the second comparison case, EPSO exhibited the best performance for most of the 30-dimensional problems where the proposed algorithm BEPSO solved equal number of problems as DMS-PSO.

8.4.2.3 Results on 50 dimensional problems

The experimental results on the 50-dimensional CEC'14 problems reveal that on the 3 unimodal problems indicate that L-SHADE exhibited the best performance for F1, F2 and F3. For multimodal problems (F4~F16), once again L-SHADE solved majority of the problems including F4, F8, F10, F11, F12 and F16 whilst the proposed algorithm BEPSO exhibited the best mean performance on F6, F9 and F15. Moreover, HPSO-TVAC, FIPS, DMS-PSO and LPSO individually obtained the lowest mean error values on F5, F7, F13 and F14, respectively. Furthermore, the results on the hybrid functions (F17~F22) reveal that for F17, F18, F19 and F21, L-SHADE achieved the best mean performance whilst for F20 and F22, EPSO and the proposed algorithm BEPSO exhibited the best performance. Finally, the results on the composition functions show that DMS-PSO exhibited better performance on most of the composition functions including F23, F25, F28 and F29 whilst MiPSO, FIPS, BEPSO and LPSO individually attained the lowest mean error values on F24, F26, F27 and F30, respectively.

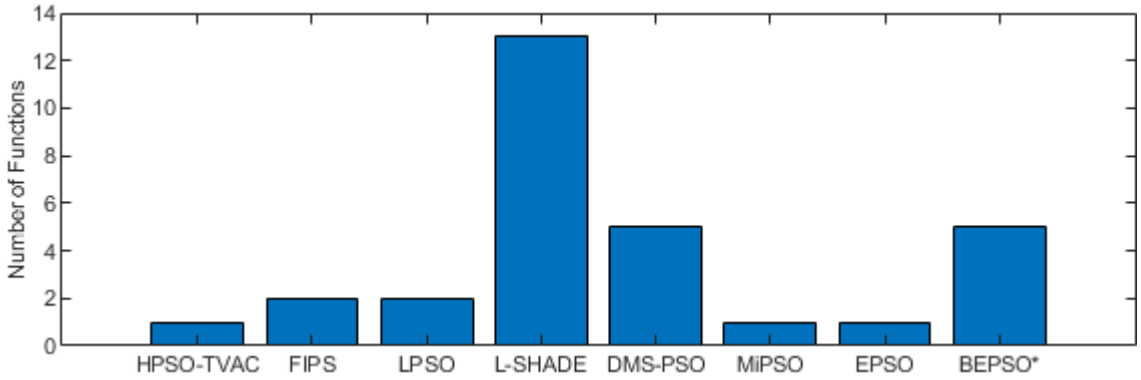


Figure 8-20 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

The overall comparison of the best mean performances for the 50-dimensional CEC'14 problems suggest that, as shown in Figure 8-20, apart from L-SHADE's superior performance, the proposed algorithm BEPSO and DMS-PSO solved equal, and more problems compared to all PSO variants.

8.4.2.4 Results on 100 dimensional problems

The experimental results obtained on the 30 100-dimensional CEC'14 problems reveal that for F1 and F2, the best mean performance is attained by L-SHADE whilst EPSO achieved the best performance on F3. The results on the multimodal functions yield that the proposed algorithm BEPSO (F6, F9, F13, F15) and L-SHADE (F10, F11, F12, F16) each solved 4 multimodal functions. Moreover, HCLPSO performed best on F5, F14, and LPSO, UPSO and EPSO solved F4, F7 and F8, respectively. For hybrid functions (F17~F22), the lowest mean error values for F17 and F21 are obtained by L-SHADE whilst for F18, F19, F20 and F22, the best performances are individually achieved by MaPSO, LPSO, EPSO and BEPSO, respectively. Further, on the 8 composition functions (F24~F30), the results yield that LPSO solved most of the composition functions including F25, F28, F29 and F30 whilst DMS-PSO, HPSO-TVAC, FIPS and BEPSO attained the lowest mean error values on F23, F24, F26 and F27, respectively.

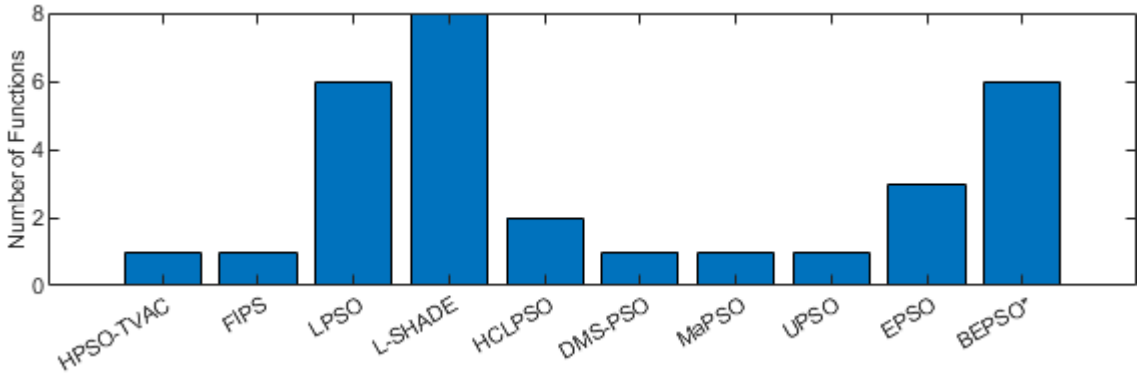


Figure 8-21 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

As shown in Figure 8-21, the overall best mean performances on the 100-dimensional CEC'14 problems suggest that L-SHADE performed best on 8 problems whereas LPSO and the proposed algorithm BEPSO exhibited superior performance compared to other PSO variants with respect to the number of functions solved, and both attained the lowest mean error values for 6 problems.

The 15 algorithms are ranked according to the mean error values obtained on the 10, 30, 50 and 100-dimensional CEC'14 problems (*see Appendix 2*). The final ranks indicate that the proposed algorithm BEPSO obtained the 7th rank on the 10 and 100-dimensional, and 6th rank on the 30 and 50-dimensional problems. Although initially it appears that the proposed algorithm did not exhibit comparably well performance to its previous performance on the CEC'13 problems. However, when we observe the average ranks of L-SHADE and BEPSO (*L-SHADE=4.36 and BEPSO=6.5*), the observed difference in the average ranks of both algorithms is not far substantial. Further, the Wilcoxon signed rank test is conducted to discover if the performance of any of the test algorithm is significantly different to the proposed algorithm BEPSO. The results of the Wilcoxon signed rank test is shown in Table 8-14, and in summary, the results yield that at 10 dimensions, the proposed algorithm exhibited statistically significantly better performance against 7 algorithms. Similarly, at 30, 50 and 100 dimensions, BEPSO exhibited superior performance than 5 comparison algorithms.

Table 8-12 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2014 problems

BEPSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	-	-	+	+	=	+	=	-	=	=	+	7/4/3
30	+	+	+	=	-	=	+	-	=	=	-	=	=	+	5/6/3
50	+	+	+	=	=	=	+	=	=	=	-	=	=	+	5/8/1
100	+	+	+	=	-	=	+	=	=	=	-	=	=	+	5/7/2

The convergence trend of the BEPSO algorithm is compared against the 4 top ranked algorithms on the 100-dimensional CEC'14 problems including L-SHADE and 3 PSO variants HCLPSO, EPSO and DMS-PSO. Although the proposed algorithm obtained the 7th rank on experiment conducted on the 100-dimensional CEC'14 problem set. As shown in Figure 8-22, the comparison reveals that the convergence trend of the proposed algorithm BEPSO is generally better than HCLPSO and EPSO, and in some test cases comparable to L-SHADE across unimodal, multimodal, hybrid and composition functions.

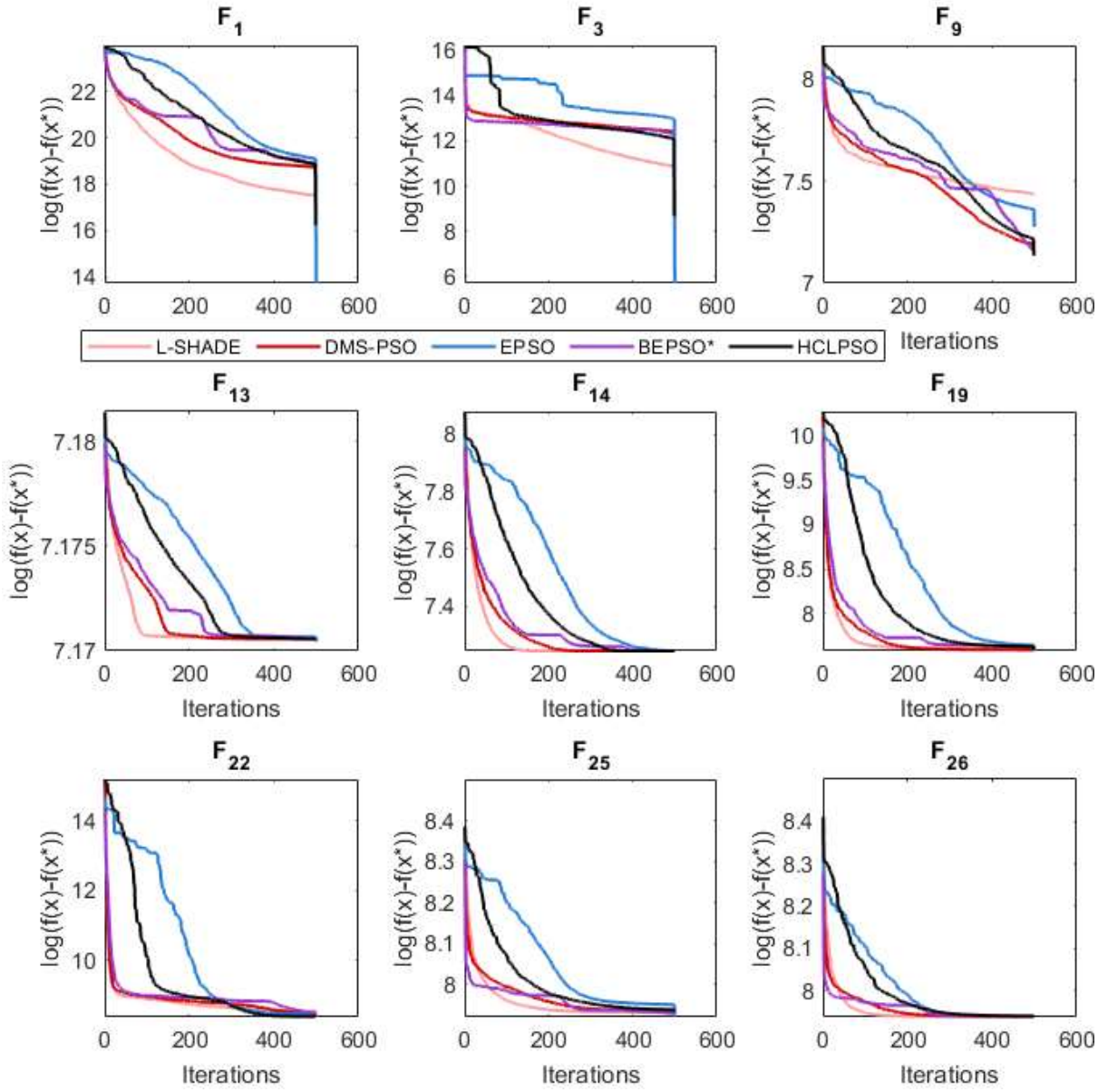


Figure 8-22 Average convergence rate comparison of BEPSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'14 problems

8.4.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed BEPSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

8.4.3.1 Results on 10 dimensional problems

The mean error values obtained on the 10-dimensional CEC'17 problems suggest that L-SHADE solved majority of the multimodal functions including F3~F5, F7 and F9. The lowest mean errors for F6 and F8 are obtained by EPSO and BEPSO, respectively. On the hybrid functions, once again L-SHADE performed best on 9 of the 10 functions including F10, F11, F12, F13, F14, F15, F17, F18 and F19 whilst EPSO obtained the best mean error for F16. Unlike for rest of the functions, the best

results obtained on the composition functions are dispersed among several algorithms where MiPSO performed best on F21, F23, F24, HCLPSO and LPSO on F25, F28 and F27, F30.

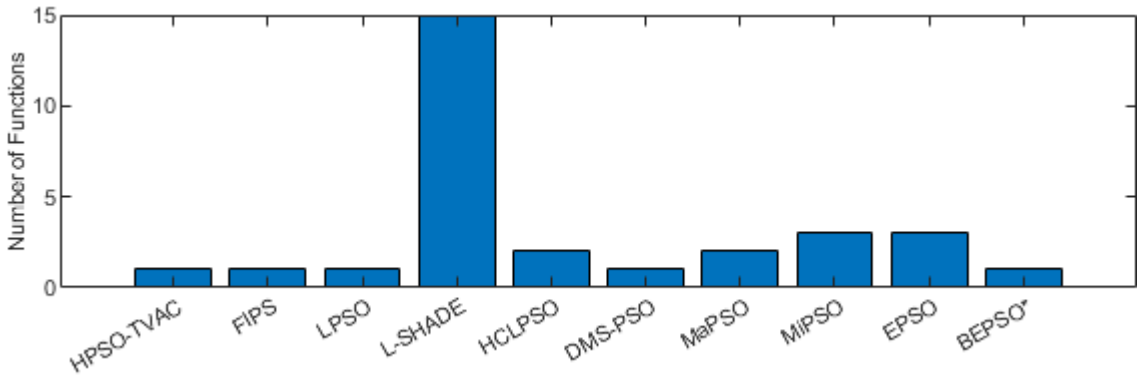


Figure 8-23 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'17 problems

As shown in Figure 8-23, we observe that the overall L-SHADE exhibited superior performance on 17 of the 29 functions. On the other hand, rest of the algorithms yield approximately similar performances on the 10-dimensional functions.

8.4.3.2 Results on 30 dimensional problems

The results on the 30-dimensional CEC'17 problems indicate that for the unimodal function F1, L-SHADE exhibit the best performance and on the multimodal functions BEPSO (F5, F8) and EPSO (F3, F4) solved more functions compared to rest of the algorithms. MiPSO, L-SHADE and FIPS obtained the lowest mean error on a single function F6, F7 and F9, respectively. For the 10 hybrid functions (F20~F29), L-SHADE provides the best solution for 6 functions including F10~F14, F18, F19 and DMS-PSO performed best on F11, F16 whilst MaPSO and BEPSO attained the best solution for F15 and F17, respectively. The results on the 11 composition functions (F20~F30) indicate that DMS-PSO performed best on 3 functions (F20, F23, F29) whilst LPSO (F22, F30), EPSO (F26, F28) and BEPSO (F21, F24) obtained the lowest mean error values on 2 functions and for F25 and F27, FIPS and MiPSO obtained the best mean performance.

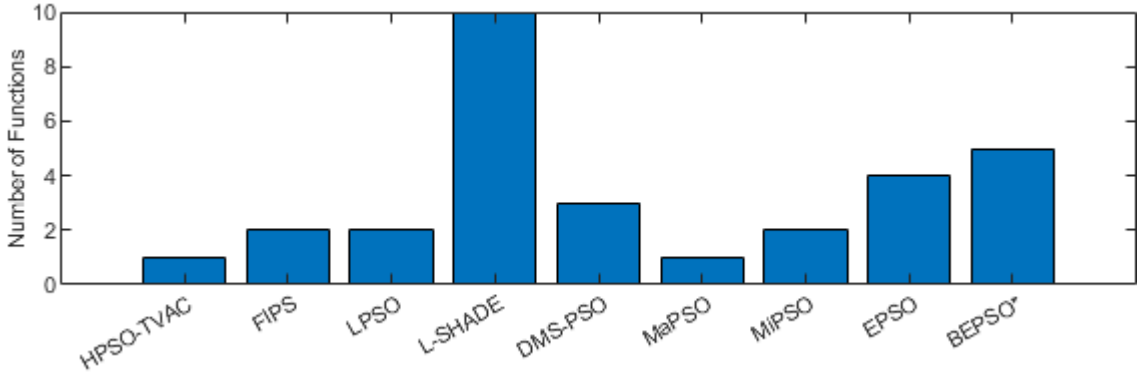


Figure 8-24 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

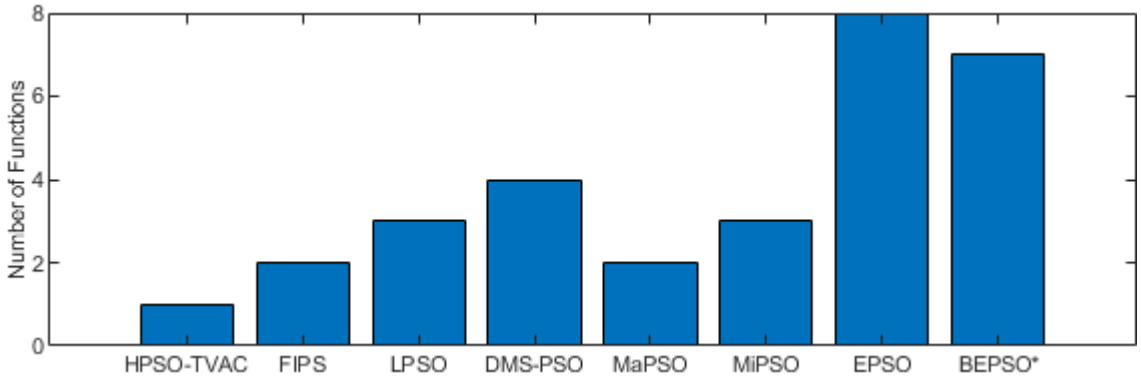


Figure 8-25 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

Figure 8-24:8-25 display the number of 30-dimensional CEC'17 problems solved by each algorithm. In Figure 8-24, the comparison yields that L-SHADE performed best on most 30-dimensional problems, followed by the performance of the proposed algorithm BEPSO. In Figure 8-25, the comparison among PSO variants suggests that EPSO exhibited the best performance for majority (8 of 29) of the problems, and on the other hand, the proposed algorithm BEPSO solved 7 of the 29 problems in this case.

8.4.3.3 Results on 50 dimensional problems

The results achieved on the 50-dimensional CEC'17 problems suggest that on the multimodal functions (F3~F9), the proposed algorithm BEPSO obtained the best mean solutions for F5, F7, F8 whilst EPSO performed best on F3, F4 and FIPS achieved the lowest mean error on F6 and F9. For the hybrid functions (F10~F19), L-SHADE performed best on 6 of the 11 functions including F10, F12, F14, F15, F18 and F19. The second top performance on the hybrid functions are attained by the proposed algorithms BEPSO where it provided the best mean solution on F11, F16 and F17. Finally, the best mean solution for F13 is achieved by LPSO. The results on the composition functions (F20~F30), the proposed algorithm BEPSO performed best on majority of the composition functions at 50 dimensions including F21, F22, F23, F24, F26 and F29. On the other hand, DMS-PSO obtained the best mean solution on F20 and F28. Moreover, FIPS (F25), HPSO-TVAC (F27) and LPSO (F30) achieved the lowest mean error values on a single function.

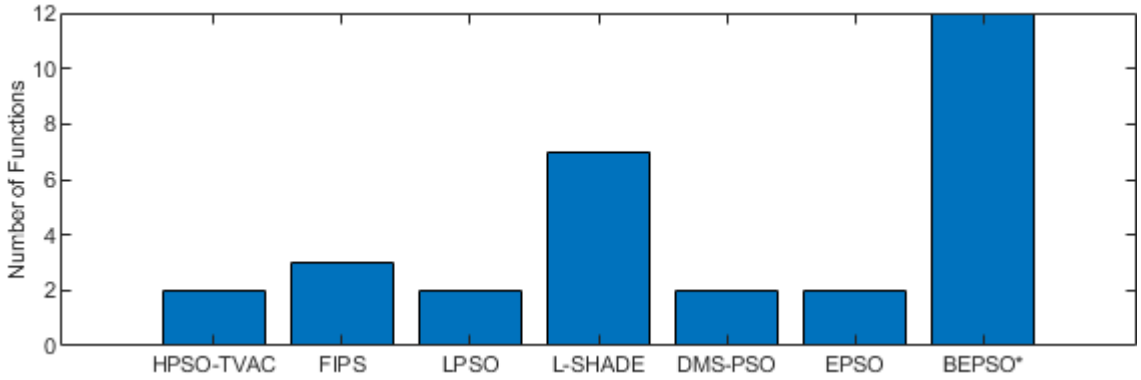


Figure 8-26 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

As we can observe from Figure 8-26, the proposed algorithm BEPSO exhibited the best performance for most (12 of 30 funcs.) of the CEC'17 test problems at 50 dimensions. The second-

best overall mean performance is attained L-SHADE, where the algorithm obtained the lowest mean error values on 8 of the 30 functions.

8.4.3.4 Results on 100 dimensional problems

The results on the 100-dimensional CEC'17 test problems indicate that on the unimodal F1, L-SHADE obtained the best performance. On the multimodal functions (F3~F9), the proposed algorithm BEPSO exhibited the best performance for majority of the multimodal functions including F5, F7, F8, and F9 whereas EPSO performed best on F3, F6 and LPSO on F4. The results on the hybrid functions reveal that L-SHADE obtained the lowest mean error value for 4 functions including F10, F12, F14 and F15 followed by the performance of EPSO on 3 functions (F13, F18, F19). The DMS-PSO algorithm found the best solutions for F11 and F16 whereas the proposed algorithm BEPSO performed best on F17. Finally, the results on the 11 composition functions (F20~F30) reveal that the proposed algorithm BEPSO solved majority (6 of 10) of composition functions including F20, F21, F22, F23, F24 and F26 whilst DMS0-PSO performed best on F25, F27, F29 and LPSO on F28 and F30.

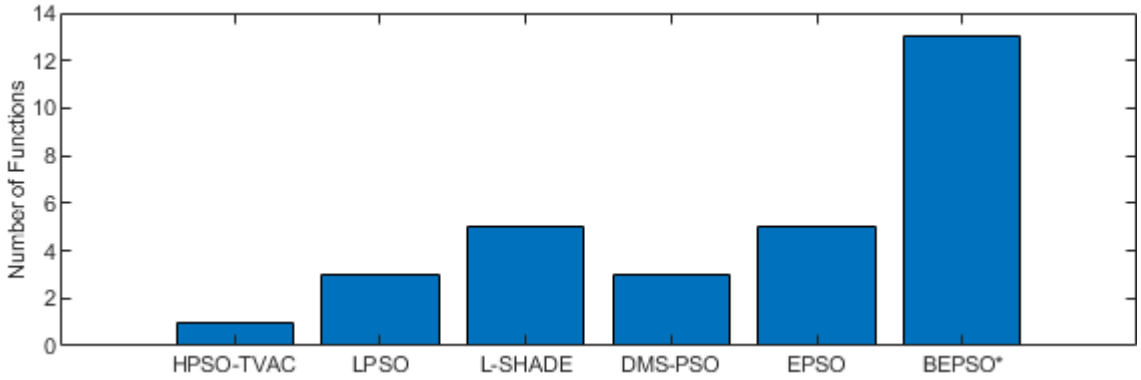


Figure 8-27 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

Figure 8-27 illustrates the total number of functions each algorithm provided the best solution on the 100-dimensional CEC'17 functions. As we can observe, the proposed algorithm BEPSO exhibited superior overall mean performance on majority of functions and attained the lowest mean error value for 11 of 30 problems.

The performance of each algorithm is ranked according to the obtained mean error value and the average/final ranks for 10-d,30-d,50-d and 100-d CEC'17 problems are shown in Appendix 2. According to the final ranks across different dimensions, the proposed algorithm BEPSO ranked 8th on the 10-dimensional functions. However, with the increased problem dimensionality, we observe a sharp improvement, hence, on the 30, 50 and 100-dimensional functions, BEPSO ranked 4th, 2nd, and 3rd, respectively.

Table 8-13 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2017 problems

BEPSO versus															
Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	=	-	+	+	=	+	=	-	=	=	=	6/6/2/

30	+	+	+	=	=	+	+	=	+	=	-	+	+	+	9/4/1
50	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0
100	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0

Moreover, the results of the Wilcoxon signed ranked is displayed in Table 8-16. In summary, the results indicate that the proposed algorithm BEPSO exhibited statistically superior performance compared to 10 of the 14 algorithms on 50 and 100-dimensional CEC'17 problems whilst the performance of 4 algorithms is statistically insignificant compared to BEPSO, no algorithms are statistically significantly better than BEPSO at these higher dimensions. Further, as shown in Figure 8-28, we have compared the convergence rate characteristics of BEPSO with the 4 top ranked algorithms on the CEC'17 test problems.

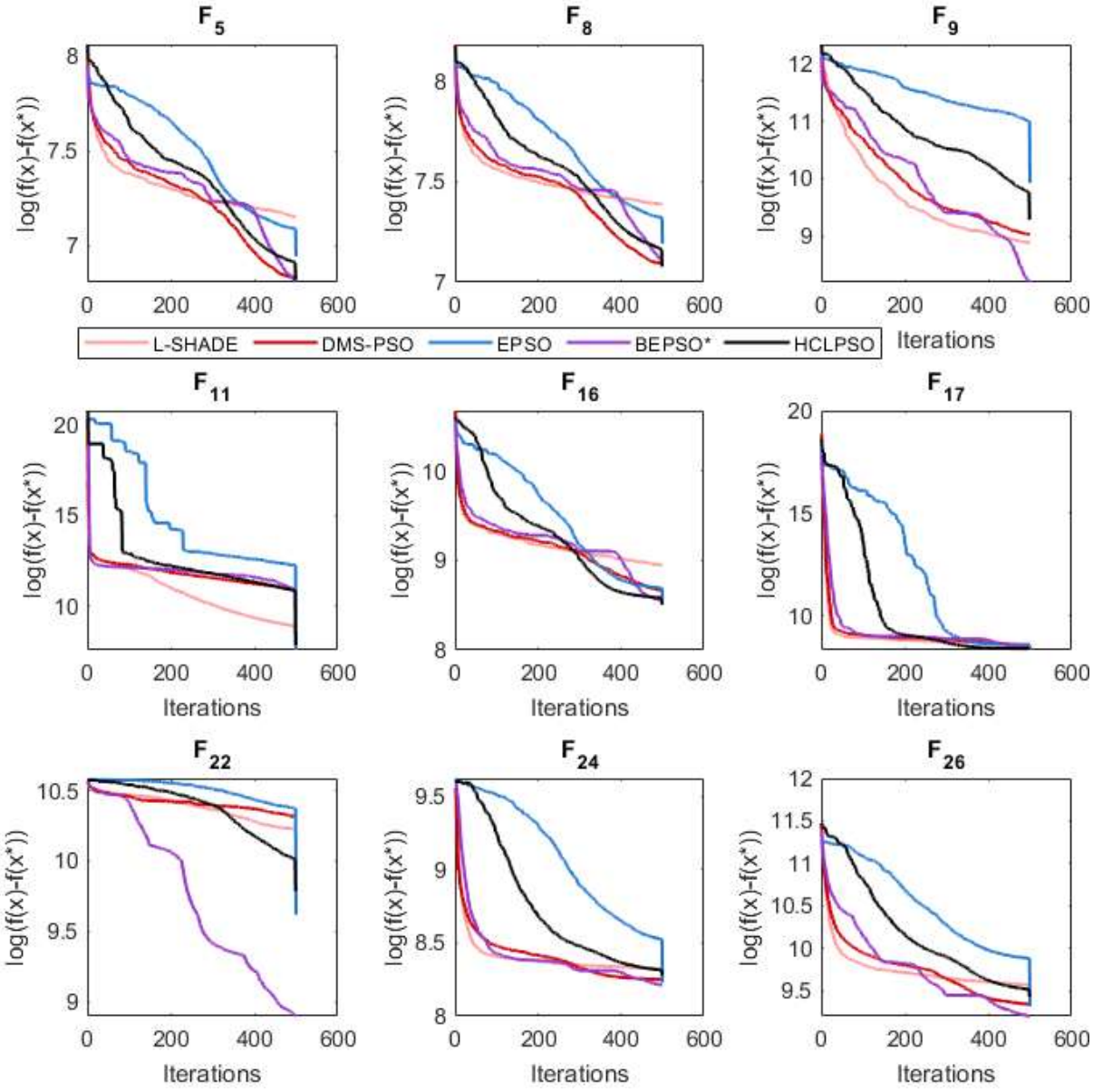


Figure 8-28 Average convergence rate comparison of BEPSO with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 100-dimensional CEC'17 problems

The convergence rate comparison reveals that the proposed algorithm BEPSO mostly exhibit distinctly faster convergence trend within significant portion of the search process compared to EPSO and HCLPSO. In addition, we generally observe DMS-PSO and L-SHADE illustrate faster initial convergence (e.g. multimodal F8, F9), however, in some cases we observe that towards the end of the search process, BEPSO exhibit better convergence rate. Since F8 and F9 are multimodal functions, this also indicates that in some cases BEPSO exhibits better capability to escape from local optima.

In addition, we assess the performance of the proposed algorithm BEPSO with the 14 test algorithm based on the total number of functions solved on 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test suites. As shown in Figure 8-29:8-31, the comparisons yield that the BEPSO algorithm exhibited superior mean performance for more problems compared to all 14 test algorithms on 50-dimensional CEC'13, 50 and 100-dimensional CEC'17 problems. For rest of the comparisons, in most cases, the BEPSO algorithm solved more problems compared to majority of the algorithms included in the experiments.

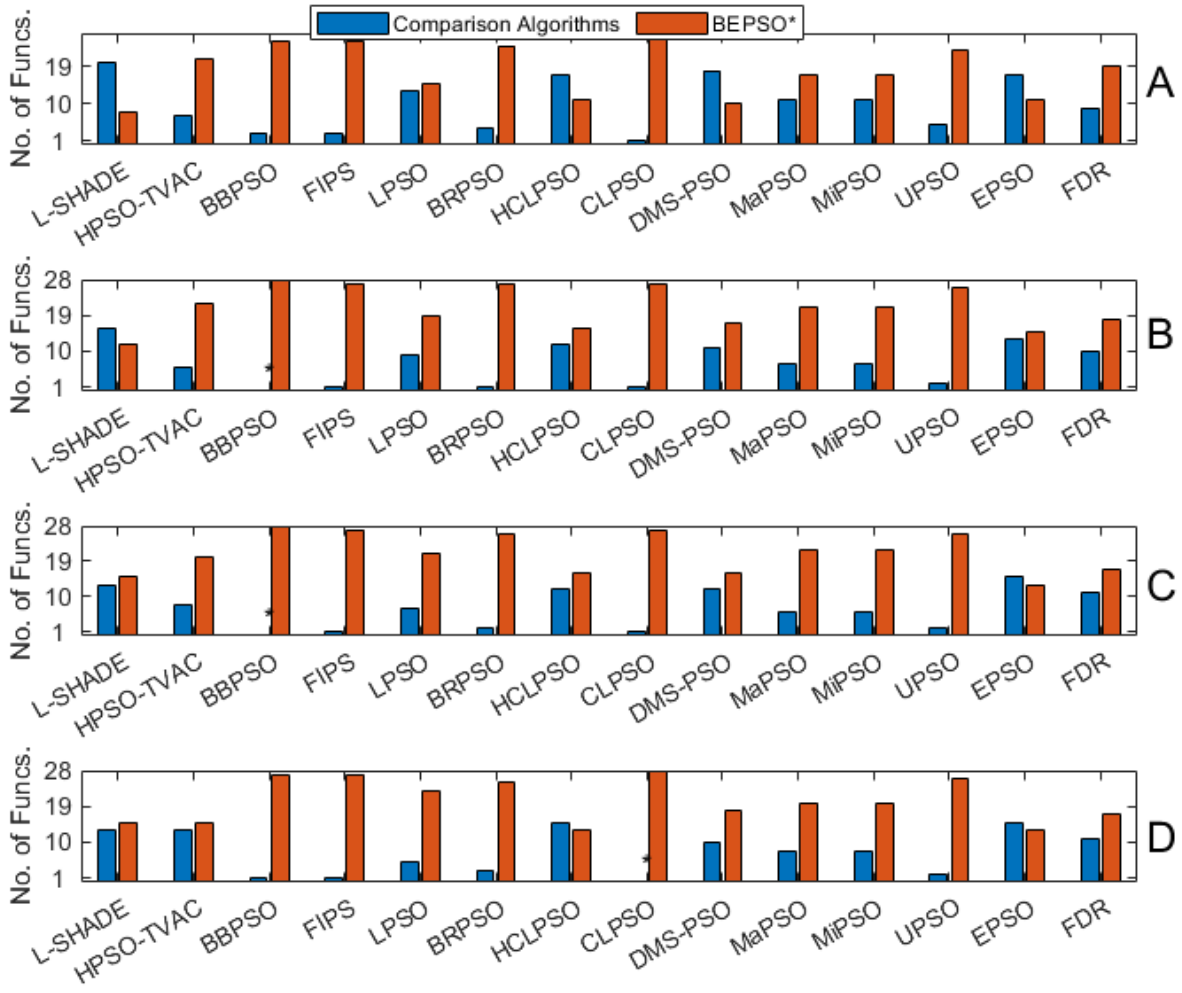


Figure 8-29 Pairwise mean performance comparison of BEPSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'13 test functions

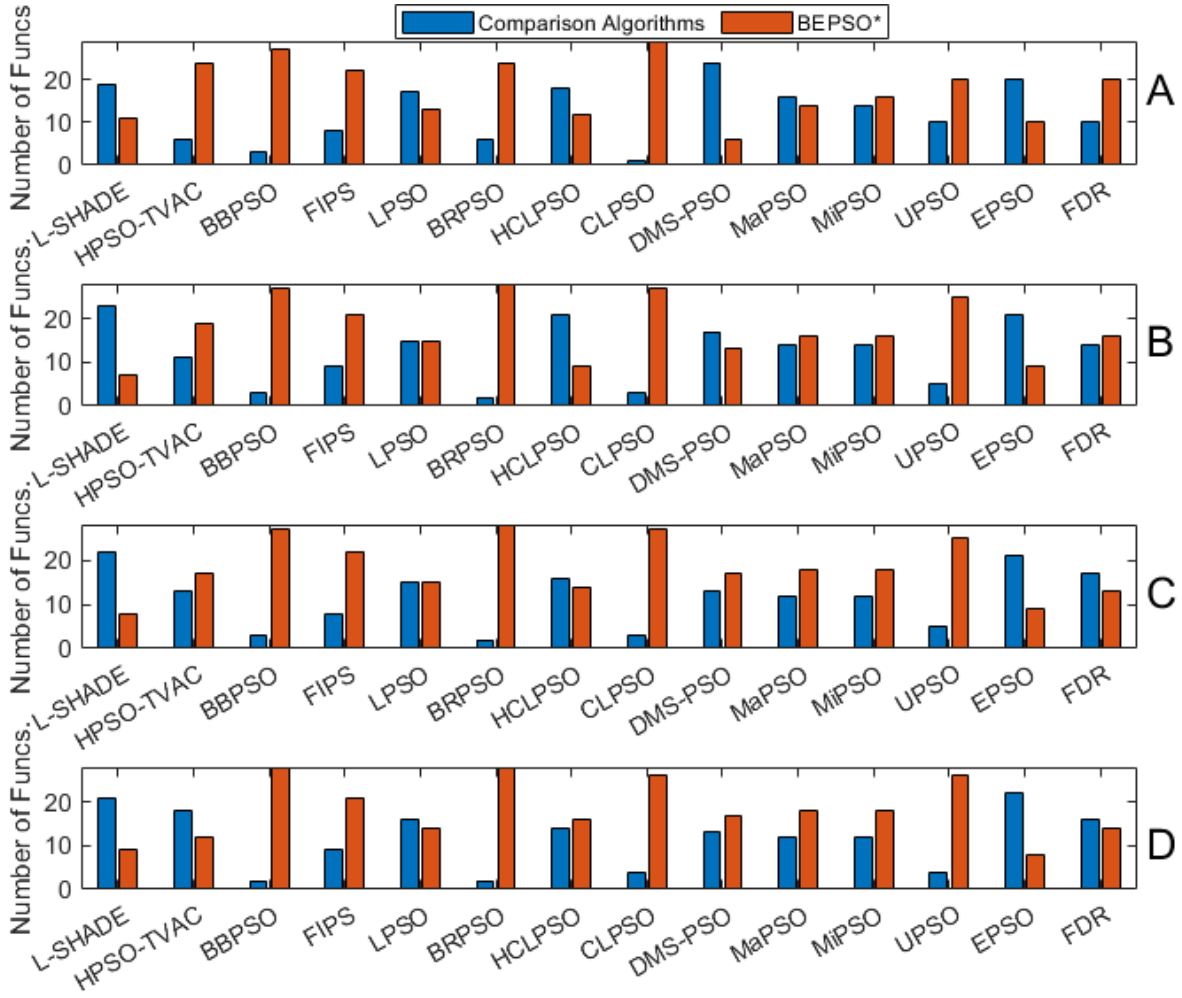


Figure 8-30 Pairwise mean performance comparison of BEPSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'14 test functions

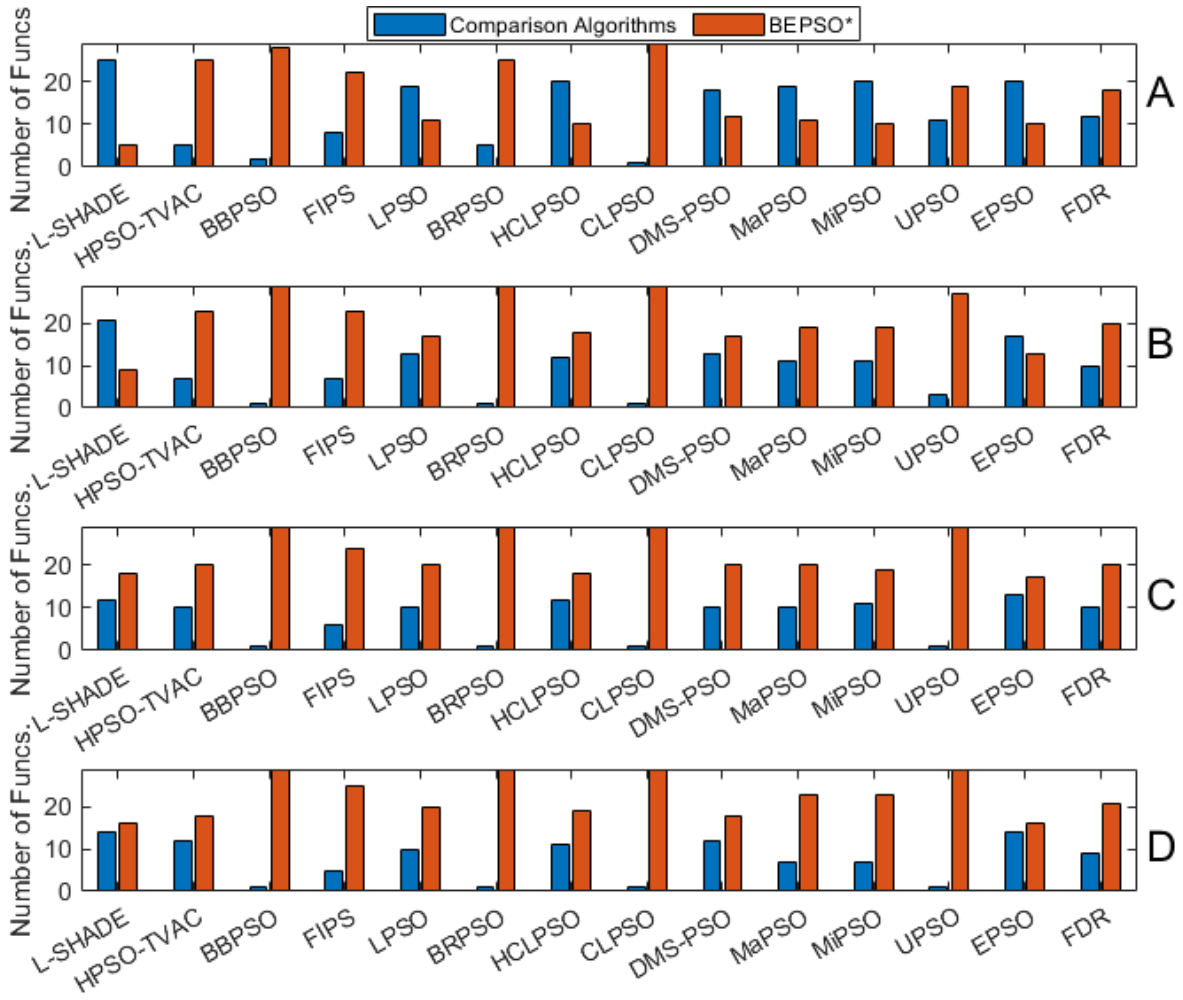


Figure 8-31 Pairwise mean performance comparison of BEPSO and all test algorithms on 10-dimensional(A), 30-dimensional(B), 50-dimensional(C) and 100-dimensional(D) CEC'17 test functions

8.5 Discussions

The performance evaluation of the proposed BEPSO algorithm is conducted on the CEC'13, CEC'14 and CEC'17 benchmark test suites across 10, 30, 50 and 100 dimensions. The test suites consist of 87 total functions with a variety of different classes including unimodal, multimodal, hybrid and composition.

Overall, the proposed BEPSO algorithm exhibited better performance on higher dimensional problems compared to low (10-d and 30-d) problems. More specifically, on the entire 10-dimensional problems, the proposed algorithm BEPSO provided the best solution 7 functions and the second-best solution for 9 functions. However, with the increased problem dimensionality we clearly observe a sudden increase in the number of solved problems. For the 30-dimensional problems, across three test suites, the proposed algorithm BEPSO performed best on 17 and exhibited the second-best performance on 9 problems. Similarly, for 50 and 100-dimensional problems, BEPSO exhibited the best performance on 25 and 27 problems, respectively. Once again, the performance of the proposed algorithm is further verified using the Wilcoxon signed ranked test and the convergence rates which confirmed that overall, the performance of the BEPSO algorithm is statistically significantly better compared to the majority of the test algorithms, especially on higher dimensional functions. The convergence rate comparison also revealed that BEPSO exhibit faster

convergence rate compared to some of the top ranked algorithms. A closer look into the performance of BEPSO on different classes of functions (100-d) indicate that on unimodal functions, BEPSO is capable of performing as good as other PSO variants employed in the experiments. In addition, based on the data obtained from three test suites, BEPSO produced better solutions compared to all PSO variants (13 algorithms) on all three test suites. Moreover, on the CEC'14 test problems, BEPSO and L-SHADE solved equal number of multimodal functions at 100 dimensions. We observe a similar pattern on the composition functions too, once again, BEPSO provided better solutions for more functions compared to all 14 algorithms on the composition functions of CEC'13 and CEC'17 test suites at 100 dimensions. In the case of hybrid functions, our observations on the 100-dimensional CEC'14 and CEC'17 hybrid functions indicate that BEPSO does not pose a superior performance however it exhibits a comparable performance to other PSO variants employed in the experiments. Moreover, the convergence rate plots for the hybrid functions reveal that BEPSO illustrates faster convergence rate compared to two of the four top ranked algorithms on the hybrid functions.

The performance comparison of the BEPSO algorithm with the CEC2014 competition winner L-SHADE, we observe that BEPSO exhibited better performance on the 10,30,50 and 100-dimensional composition functions of the CEC'13 benchmark test suite. Similarly, on the 10-dimensional CEC'14 test functions, once again, BEPSO outperformed L-SHADE by solving majority of the composition functions. Moreover, BEPSO also performed better than L-SHADE for 50 and 100-dimensional multimodal and composition functions on the CEC'17 test suite. As the comparison with L-SHADE yield, BEPSO is capable of outperforming L-SHADE on different classes of functions and on various dimensions which signifies the effectiveness of the algorithms. Especially, the performance of BEPSO against L-SHADE on the high dimensional multimodal functions indicates scope of the proposed mechanisms' capacity to efficiently avoid premature convergence to a local optimum.

8.6 Conclusion

This chapter introduced a bio-inspired PSO variant named BEPSO. The algorithm BEPSO algorithm incorporates the idea of eavesdropping observed in many animals in nature, cognitive bias and the accumulator decision mechanism. In the BEPSO algorithm, particles are divided into two groups where the same-group members recognise each other as conspecific, and on the contrary, the other group members are recognised as heterospecific. Each particle sends out a signal, which contains a guidance information, and intended solely for conspecifics to guide them to a potentially better location. The signal is intercepted and exploited by heterospecifics as well as conspecifics based on the cognitive bias and the group particle belongs to. In addition, the accumulator decision model is utilised to enable each particle to build evidence based on the guidance received and determine the final cognitive bias.

The performance of the proposed algorithm was verified using the CEC'13, CEC'14 and CEC'17 benchmark test suites on all available dimensions against 14 test algorithms (13 PSO variants and L-SHADE). The evidence suggests that the proposed algorithm's mean performance is superior to most algorithms employed in the experiments. For the case in point, on the 30 and 50-dimensional CEC'13 problem set BEPSO was ranked 3rd, on the 30-dimensional CEC'17 problem set it was ranked 3rd, and on the 50 and 100-dimensional CEC'17 problem set it was ranked 2nd. For the remainder of the experiments, the proposed algorithm demonstrated good performance in most cases, including the individual performance on unimodal, multimodal, hybrid and composition functions.

Chapter 9 : Altruistic Heterogeneous Particle Swarm Optimisation

9.1 Introduction

As we have seen, the inability of the canonical PSO to tackle more complex problems is usually associated with premature convergence triggered by a loss of diversity in the population. As we have explored in the previous chapters, researchers have proposed many different mechanisms to handle this issue to maintain population diversity and enable an efficient search process. With many variants expanding in diverse directions, including various nature/bio-inspired behaviours, topological structures, multi-swarms and heterogeneous particle behaviours, the PSO algorithm now possesses its own vast literature in computational intelligence. In this chapter we add to that literature by introducing a novel PSO variant inspired by a certain kind of altruistic behaviour found in nature, a direction that has not been explored before.

The role of altruism in evolutionary dynamics was first analysed in mathematical detail by W.D. Hamilton in 1964 [248]. He showed how altruism could arise and be maintained within the Darwinian framework, conferring overall benefit to the group if not to the individual. Traditionally, researchers tended to assess the benefit of altruism to an organism by examining the average number of offspring, with contributors exhibiting less reproductive success in comparison to beneficiaries. However, several different types of altruistic behaviours have been discovered in various specifies. Some exemplary altruistic behaviours are observed in social insect colonies such as ants, wasps, bees and termites. In these colonies, the sterile workers are devoted to the queen by protecting the nest and foraging food. By doing so, sterile workers have no reproduction fitness and hence do not reproduce, but they contribute to the queen's reproductive efforts. A prevalent example of a more complex organism exhibiting altruistic behaviour is the blood regurgitating vampire bats that feed undernourished bats to avoid starvation in their group [249]. Velvet monkeys exhibit similar behaviour to their groups by giving alarm calls to warn of the presence of predators, in doing so putting themself at risk [250]. The type, level, and results of altruistic behaviour vary widely between organisms, as do the relationships evolved between helper and beneficiary actors.

The proposed AHPSO algorithm incorporates nature-inspired conditional altruistic behaviour and a heterogeneous particle model that delays the loss of population diversity and prevents premature convergence, resulting in a highly effective search algorithm. In our approach, particles are conceptualised as energy-driven entities with two possible states, active and inactive. Particles have a current energy level and activation threshold and are inclined to be active by maintaining their current energy level above the activation threshold. The distinction in a particle's state is used to create a heterogeneous population, and particles' tendency to become active culminates in lending-borrowing relationships among them. Hence conditional altruistic behaviour is exhibited by particles which lend energy to inactive particles to allow them to change their state. In doing so, helper/lender particles risk downgrading their own state from active to inactive. To minimise the risk of reducing their own fitness, a group of lenders assess the situation of the beneficiary/borrower particle, based on the level of altruistic behaviour it has exhibited, to determine the final decision. In our behavioural model, heterogeneity is attained through altruism and better population diversity through heterogeneity. Hence, these two concepts depend on, feed and maintain each other.

In the following subsections, two behaviour models are introduced. The first is the altruistic particle model, which constitutes the particle's primary behavioural model. The latter, the paired particle model, extends the former to boost population diversity further.

9.2 AHPSO: Altruistic Particle Model (APM)

In the proposed algorithm, as briefly mentioned above, particles are conceptualised as energy-driven entities that are either active or inactive. The activation status of particles is dependent on two factors, current energy level and activation threshold, represented with the notations $E_{current}$ and $E_{activation}$ respectively. Both values are randomly assigned initially. The current energy level refers to the level of energy acquired at time t, and the activation threshold refers to the level of energy required to trigger activation at time t. Hence, when the particles' current energy levels are lower than its activation threshold, its state is determined to be *inactive*, and *active* in the contrary situation. The concept of activation is employed to determine the type of movement strategy for particles at the individual level and as a result, controls the behavioural heterogeneity in the swarm.

In the proposed behaviour model, particles have a tendency to be active; hence particles in the inactive state are expected to borrow energy from other particles when their $E_{current} < E_{activation}$. As a result, particles attempt to form altruistic lending and borrowing relationships among each other to upgrade their state from inactive to active. The main factor influencing and maintaining the swarm heterogeneity is the particles' altruistic behaviour. A particle that behaves altruistically by making significant energy contributions to other swarm members is highly unlikely to be rejected when in need of energy itself, and on the contrary, particles that exhibit lower altruistic behaviour are inclined to be rejected.

The idea behind the lending-borrowing ecology is that the overall circulating energy in the swarm is assumed to be a resource for active particles to make use of within the swarm. However, if a particle has limited energy resources, this leads to it seeking and borrowing resources from other particles; persistent borrowing behaviour in a particle over prolonged periods results in a highly unstable lending-borrowing ratio and reduces the altruism value, A_i, of the particle (as the particle consumes a lot more resources than it contributes to the swarm). As the name suggests, the altruism value, A_i, of a particle quantifies the level of altruistic behaviour that particle exhibits and is calculated as:

$$A_i = \frac{L_i^{(t)}}{B_i^{(t)}} \tag{9.1}$$

where $L_i^{(t)}$ and $B_i^{(t)}$ are the number of times the ith particles lent and borrowed energy, respectively, to and from other particles up to time t. When a particle is unable to activate, it attempts to borrow energy from randomly selected potential lenders, and in order to lend energy, *potential lender* particles expect energy requesting particle to meet an altruism criterion defined by φ (Eq. 9.3). This criterion is based on the independent probability of two events P_1 and P_2. For P_1 , the altruism value of the borrower particle A_i is used, and P_2 is calculated as

$$P_2 = \frac{\delta}{N} \tag{9.2}$$

Where δ is the number of particles in the swarm with active status at time t, and N is the population size.

$$\phi = P_1 \times P_2 \tag{9.3}$$

P_2 gives a rough measure of the probability that a lender particle will return the lent energy from the swarm. In addition to enforcing altruistic behaviour, the criterion ϕ provides a form of altruistic assessment of the lender particles' probability of returning lent energy.

The potential lender particles use the γ value described by Eqn. 9.4 as the final decision to either lend energy or reject the request of the borrower particle.

$$\gamma = \begin{cases} false, & \phi < \beta \\ true, & \phi \geq \beta \end{cases} \tag{9.4}$$

where β is the average altruism value in the swarm.

If the decision γ is in favour of the energy requesting particle (i.e. true), an equal amount of energy is borrowed from each lender to compensate the required energy of the borrower particle. This is calculated as

$$E_{required} = \frac{(E_{activation} - E_{current})}{\alpha} \tag{9.5}$$

Where $E_{required}$ is the amount of required energy from each lender and α is the number of selected lenders. The movement strategy adopted by particles in the altruistic behaviour model is based on the altruistic traits of particles. Particles that are active use the canonical update equation shown in Eqs. 2.4-2.5, whereas inactive particles who do not meet the criterion φ, and therefore cannot borrow, use Eq. 9.6 to update their velocity (and position via Eq. 2.5).

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\mathbf{pbest}_i^t - \mathbf{x}_i^{\mathrm{t}}\right) + c_2\mathbf{r}_2\left(\mathbf{pbest}_{\min A}^t - \mathbf{x}_i^{\mathrm{t}}\right) \tag{9.6}$$

Where $\mathbf{pbest}_{\min\gamma}^t$ is the personal best position of the least altruistic particle at time t. In the AHPSO framework, particles who do not meet the criterion φ are less altruistic at time t hence behave together with *similar*ly less altruist particles. Considering the evolutionary characteristics of particles in the altruist model, the least and most altruist particles fluctuate. Hence, in this case, guidance towards the least altruistic particle partially enables cooperation through altruism and supports heterogeneity. Energy sharing takes place between the lender particles and the borrower who meets the criterion ϕ. Post-energy borrowing, the borrower particle can be in two states, in the first case, the borrower particle is presumed to have borrowed sufficient energy to be active. However, this is not always the case as lenders are randomly selected without any criteria, hence there is a distinct possibility of some lenders not having excess energy to lend. Therefore, after borrowing energy, the borrower particle may still lack sufficient energy to activate. In this case, an exemplar for the particle is generated by the mean position of half of the lender particles and their velocity is updated as follows

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1\mathbf{r}_1\left(\mathbf{pbest}_i^t - \mathbf{x}_i^t\right) + c_2\mathbf{r}_2\left(\mathbf{x}_{mean} - \mathbf{x}_i^{\mathrm{t}}\right) \tag{9.7}$$

Where $\mathbf{x}_{mean}$ is the mean position of the randomly selected $\frac{\alpha}{2}$ lender particles.

As commonly seen in certain PSO variants, in our behavioural model, particles do not explicitly exchange positional information, hence, by using the mean position of a proportion of lender particles, we aim to form an implicit communication between lender-borrower particles.

If, however, the borrower particle succeeds in borrowing sufficient energy to activate, the particle's velocity is calculated using:

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\mathbf{pbest}_i^t - \mathbf{x}_i^{\mathrm{t}}\right) + c_2\mathbf{r}_2\left(\mathbf{pbest}_{\max A}^t - \mathbf{x}_i^t\right) \tag{9.8}$$

Where $\mathbf{pbest}_{\max A}^{(t)}$ is the personal best position of the most altruistic particle at time t. Similar to the Eq. 9.7, here, ith particle is guided towards the most altruist particle to form partial cooperation at altruistic basis and maintain heterogeneity. It is worth noting that, $E_{current}$ and $E_{activation}$ are randomly reinitialised at specific intervals for the entire swarm. The idea behind this is to fluctuate the altruism value of particles and allow less altruistic particles at time t to cooperate, contribute and evolve as an altruistic particle. In contrast, an altruistic particle could "devolve" and exhibit selfish behaviour. As a result, this model allows altruistic and selfish particles to adopt distinct movement strategies that change and adapt depending on the level of a particle's 'evolution', leading to an adaptive and heterogeneous particle population.

9.3 AHPSO: Paired Particle Model (PPM)

The paired particle model proposed in this section is an extension to the altruistic behaviour model described in the previous section. The purpose of the paired particle model is to further boost the heterogeneity properties of the algorithm, leading to increased population diversity. A proportion of the population is used for the paired particle model, this is set at least as half of the main population. This model employs two movement strategies for the selected particles, namely, a coupling-based strategy and an opposition-based strategy. The paired particle model enables particles to randomly form and maintain pair-style bonds similar to the mechanism employed in study [30] and an altruistic particle may abandon its pair if the pair is less altruist than the swarm's average.

9.3.1 Coupling-based Strategy

As briefly mentioned, the paired particle model provides two movement strategies for particles to *influence* and *oppose* paired particles. The first strategy distinguishes pairs as *tightly* and *loosely* coupled or *neutral* particles to determine the type of movement strategy. The following rules govern the type of coupling relationship paired particles adopt:

1. A pair is tightly coupled, if both particles are active at time t.
2. A pair is loosely coupled, if both particles are inactive at time t.
3. A pair is neutral, if one particle is active and the other is inactive at time t.

The tightly coupled paired particles tend to have more influence on each other whereas loosely coupled pairs are less affected by each other. The tightly and loosely coupled particles update their velocities using Eq. 9.9 and Eq. 9.10, respectively.

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\left(\mathbf{x}_{pair}^i \times E_{current}^i\right) - \mathbf{x}_i^t\right) + c_2\mathbf{r}_2\left(\left(\mathbf{pbest}_{pair}^i \times E_{current}^i\right) - \mathbf{x}_i^t\right) \tag{9.9}$$

Where $\mathbf{x}_{pair}^i$ and $\mathbf{pbest}_{pair}^i$ are the ith particle's pair's position, and personal best position and $E_{current}^i$ is the current energy level of the ith particle.

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\left(\mathbf{pbest}_i^t \times E_{current}^i\right) - \mathbf{x}_i^t\right) + c_2\mathbf{r}_2\left(\left(\mathbf{x}_{pair}^i \times E_{current}^i\right) - \mathbf{x}_i^{\mathrm{t}}\right) \tag{9.10}$$

$E_{current}^i$ is used as a damping factor to restrict particles from rapid oscillations and as a result particle perform secondary movements in partial steps. In essence, the coupling-based strategy empowers particles within the paired behaviour model to influence each other regardless of any distance constraints within pairs. Thereby, clustered particles take small steps towards their pair, depending on the type of coupling relationship formed, causing perturbations in the current position without explicit impact on $\mathbf{pbest}$. However, these fluctuations in particle position subsequently influences the next position of the particle, helping to escape local optima.

9.3.2 Opposition-based Strategy

The opposition-based movement guides paired particles towards exemplars with opposite features. By guiding both particles of a pair in potentially distinct directions, the strategy aims to maintain diversity within all pairs, and hence within a proportion of the population. In a way, this movement strategy partly compensates for the previous coupling-based strategy where pairs influence each other. The opposition-based strategy aims to slow down the learning between pairs, without destroying it, and delays loss of diversity between them by guiding both in the direction of distinct exemplars. The altruism value of the paired particles is used as the determining factor to distinguish the type of movement a particle performs. Exemplar selection for members of paired particles can be summarised as follows; If the i^{th} particle is more altruist than its couple, it randomly selects to move towards either $\mathbf{pbest}$ or the position of the most altruistic individual of the most altruistic pair

at time t. On the contrary, if the i^{th} particle is less altruist than its pair, it randomly selects to move towards either **pbest** or the position of the least altruistic individual of the least altruistic pair at time t. The velocity for the former and the latter particles are updated using

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\mathbf{pbest}_i^t - \mathbf{x}_i^{\mathrm{t}}\right) + c_2\mathbf{r}_2\left(\mathbf{x}_{maxA}^{\mathrm{pair}} - \mathbf{x}_i^t\right) \tag{9.11}$$

Where $\mathbf{x}_{maxA}^{\mathrm{pair}}$ is either the **pbest** or the position of the most altruistic individual of the most altruistic pair at time t.

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^{\mathrm{t}} + c_1\mathbf{r}_1\left(\mathbf{pbest}_i^t - \mathbf{x}_i^t\right) + c_2\mathbf{r}_2\left(\mathbf{x}_{minA}^{\mathrm{pair}} - \mathbf{x}_i^t\right) \tag{9.12}$$

Where $\mathbf{x}_{minA}^{\mathrm{pair}}$ is either the **pbest** or the position of the least altruistic individual of the least altruistic pair at time t.

Since both movement strategies in the paired particle model updates the position of particles consequently, this acts as a stabiliser mechanism that enables particles to partially escape from local optima and continue the search process. It's also worth noting that in both cases of coupling-based and opposition-based learning, the fitness of the exemplar particles is deliberately not considered to minimise particles clustering around local solutions, but instead, these strategies aim to maintain diversity and hence guard against premature convergence.

Algorithm 6: AHPSO

Input: n as swam size, T_{max} as maximum number of iterations.
Output: **gbest**

c_1=2.5 − (1:T_{max}*2/T_{max}); c_2=0.5 − (1:T_{max}*2/T_{max}); $w_{max} = 0.99; w_{min} = 0.2;$ C=0.15;

$$w_1 = \frac{w_{max} + (w_{min} - w_{max})}{1 + exp\left(-5\left(\frac{2t}{T_{max}} - 1\right)\right)}$$

for t=1:T_{max}
  β = average A value in swarm at t
  δ = number of active particles at t
  for i=1:n
    if f($\bar{\mathbf{x}}_i$) ≥ f($\bar{\mathbf{x}}$) **then**
      w=w_1^{t}+C; if w>0.99, w=0.99, end
    else
      w=w_1^{t}+C; if w>0.20, w=0.20, end
    end
    if $E_{current} \geq E_{activation}$ **then**
      Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 2.4 and 2.5
    else
      Calculate P_1, P_2 and ϕ using Eqs. 9.2 and 9.3
      if $\phi < \beta$ **then**
        Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 9.6 and 2.5
      else
        Randomly select α potential lenders
        Calculate $E_{required}$ using Eq. 9.5
        Deduct $E_{required}$ from $E_{current}$ of each lender
        Update $E_{current}$ of the borrower
        Increment L by one for all lenders
        Increment B by one for the borrower
        if $E_{current} \geq E_{activation}$ **then**
          Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 9.8 and 2.5
        else
          Update $\mathbf{v}_i$ and $\mathbf{x}_i$ using Eq. 9.7 and 2.5
        end
      end
    if i^{th} particle is paired **then**
      if randi([0 1]) == 0 **then**

```
                    if pair is tightly coupled then
                        Update v_i and x_i using Eq. 9.9 and 2.5
                    else if pair loosely coupled then
                        Update v_i and x_i using Eq. 9.10 and 2.5
                    end
                else
                    if A_i > A_couple then
                        Update v_i and x_i using Eq. 9.11 and 2.5
                    else
                        Update v_i and x_i using Eq. 9.12 and 2.5
                    end
                end
            end
            Evaluate the fitness of x_i
            Update pbest_i and gbest
        end
    Reinitialise E_current^{1..n} and E_activation^{1..n}
    end
```

9.4 The Search Process of AHPSO

Considering all the concepts described in the previous sections, we can divide the search process into two behaviour models, namely altruistic and paired particle models. Both models complement each other and are run consecutively at each iteration, and the search process is initiated with the altruistic particle model by which particles attempt to change their state from inactive to active. Those with sufficient energy levels are active and use equation Eq. 2.4 to update their velocity. Behaviourally, active particles tend to be more focused on exploitation. On the contrary, inactive particles attempt to borrow energy to compensate. However, as mentioned in the previous sections, some particles may not meet the criterion φ and do not qualify for altruism. These particles update velocity using Eq. 9.6. In contrast, in the post-borrowing phase, if a particle managed to borrow sufficient energy and was able to change its state to active, the velocity is updated using equation Eq. 9.8, while those still incapable of activating after borrowing energy use Eq. 9.7 to update their velocity. Eqs. 9.6, 9.7 and 9.8 focus on exploring the search space and are mainly influenced by the most and least altruistic particles at time t. The level of altruistic behaviour exhibited by particles varies highly and particles evolve frequently from more to less or less to more altruistic in the swarm. Hence the aforementioned three types of particles are guided by a highly diverse and fast evolving exemplars enabling a efficient search behaviour while maintaining population diversity.

This constitutes the part of the search process handled by the altruistic behaviour model. Next the second, paired particle, behaviour model is initiated. Unlike the main altruistic particle model, this is used for only a selected proportion of the population. The main purpose of the paired particle model is to further increase heterogeneity and prevent stagnation of particles for the selected proportion of the population. The paired particle model provides two movement strategies for the paired population: coupling and opposition-based movement. Particles randomly select either strategy and for the former strategy the coupling relationship between the pair is determined based on their activation status as tightly, loosely or neutral and this is used as a criterion to select the type of movement particle undertakes. Tightly coupled particles tend to be more influenced by each other and on the contrary loosely coupled particles are less influenced by one another. The latter strategy, opposition-based movement provides guidance based on the altruism value of paired particles and the type of movement depends on the altruistic trait of paired particles. Particle that are more altruist than their pair are guided towards the most altruist particle of the most altruist pair. On the other hand, if particle is less altruistic then its pair, it's guided towards the least altruist particle of the least altruist pair. It's worth noting that, the change in the energy levels and the A_i

values shape particles' behavioural patterns and stochastically switch between different types of movements, leading to diverse behaviour and evolution of strategy.

9.5 Parameter Tuning and Analysis

AHPSO has a number of parameters, hence in this section, we assess the impact they have on the performance of the algorithm, testing on various dimensions and multiple test suites.

9.5.1 The impact of the alpha parameter

As briefly mentioned previously, the α parameter determines the size of the randomly selected number of potential lenders when a particle demands energy from other particles. Higher alpha value leads borrowing menial energy from each lender which precludes the expected imbalance in the current energy acquired by particles, which is required to create the behavioural heterogeneity (by triggering different behaviours) in the swarm. On the other hand, too low alpha value leads to large scale of energy borrowing from fewer particles. Once again, this procrastinates the required imbalance in the current energy levels which enforces the interaction between particles. Hence, in this section we aim to determine the optimal value for the alpha parameter that will enable the algorithm to maintain the aforementioned balance whilst maximising the overall performance. Initially, we conducted an experiment on the 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test functions and ranked the results based on the mean error values as shown in Table 9-1:9-3.

According to the final ranks, we have identified five distinct values that consistently exhibited better performance compared to other values used for the α parameter. From Table 9-1:9-3, we observe that $\alpha = 14$ attained the best performance for 10-dimensional and 100-dimensional functions for CEC'13 and CEC'17 test suites, $\alpha = 12$ exhibited the top rank for 30-dimensional CEC'13 and 100-dimensional CEC'14 test suites and $\alpha = 16$ achieved the first rank on the 50-dimensional CEC'13 and 10-dimensional CEC'14 whilst $\alpha = 10$ exhibited superior mean performance for 50-dimensional CEC'14 and CEC'14 functions and finally, $\alpha = 18$ ranked first for 30-dimensional CEC'17 functions. The calibration of the α parameter reveals that the optimal values attained over three different dimensions and test suites spread across 10-18 hence, the optimal value is determined as $10 \leq \alpha \leq 18$. Hence, in the main experiments, the value of α is randomly determined between $10 - 18$ each time the parameter is used.

Table 9-1- Mean and final ranks of different α values on the 10, 30, 50 and 100 dimensional CEC'13 problems

d	10-d		30-d		50-d		100-d	
$\alpha = 4$	4.96	3	5.75	5	5.46	7	5.68	7
$\alpha = 6$	5.89	8	6.93	10	5.07	3	5.25	5
$\alpha = 8$	5.18	4	6.04	6	5.21	5	5.18	4
$\alpha = 10$	5.64	6	5.32	3	5.18	4	4.61	2
$\alpha = 12$	5.86	7	4.39	1	4.61	2	5.11	3
$\alpha = 14$	4.46	**1**	5.04	2	5.32	6	4.29	1
$\alpha = 16$	5.64	6	6.32	8	4.21	1	5.61	6
$\alpha = 18$	4.68	2	5.61	4	6.14	8	6.36	8
$\alpha = 20$	6.25	9	6.07	7	6.86	9	6.61	9
$\alpha = 25$	5.46	5	6.39	9	8.32	10	8.11	10
$\alpha = 30$	6.29	10	7.82	11	9.61	11	9.11	11

Table 9-2- Mean and final ranks of different α values on the 10, 30, 50 and 100 dimensional CEC'14 problems

d	10-d		30-d		50-d		100-d	
$\alpha = 6$	6.20	9	7.13	9	5.70	6	7.30	11
$\alpha = 8$	5.63	4	5.83	6	6.77	10	5.07	3
$\alpha = 10$	5.90	7	5.57	4	5.57	4	6.33	8
$\alpha = 12$	5.77	5	5.70	5	4.63	1	5.40	5
$\alpha = 14$	5.37	3	4.93	2	5.67	5	4.83	1
$\alpha = 16$	5.10	2	5.00	3	4.97	3	4.87	2
$\alpha = 18$	4.57	1	4.47	1	4.77	2	5.57	6
$\alpha = 20$	5.63	4	5.83	6	5.83	7	5.33	4
$\alpha = 25$	5.80	6	5.90	7	5.87	8	5.80	7
$\alpha = 30$	6.33	10	6.47	8	6.67	9	6.47	9
$\alpha = 4$	6.13	8	7.33	10	7.73	11	7.20	10

Table 9-3- Mean and final ranks of different α values on the 10, 30, 50 and 100 dimensional CEC'17 problems

d	10-d		30-d		50-d		100-d	
$\alpha = 4$	6.40	10	6.70	10	5.97	8	6.93	9
$\alpha = 6$	5.10	5	6.57	8	5.27	3	5.83	8
$\alpha = 8$	5.43	6	6.00	7	5.73	5	4.93	3
$\alpha = 10$	5.63	7	5.03	3	4.77	1	4.97	4
$\alpha = 12$	5.03	4	5.53	6	4.87	2	5.57	6
$\alpha = 14$	4.50	1	4.87	2	5.80	6	4.33	1
$\alpha = 16$	5.93	8	5.20	4	5.43	4	5.03	5
$\alpha = 18$	4.90	3	4.73	1	5.73	5	4.67	2
$\alpha = 20$	4.53	2	5.40	5	5.90	7	5.77	7
$\alpha = 25$	6.23	9	6.67	9	7.53	10	7.37	10
$\alpha = 30$	6.70	11	7.47	11	7.17	9	8.77	11

9.5.2 The impact of the energy redistribution rate

The impact and calibration of the ER parameter was assessed on the 10, 30, 50 and 100-dimensinoal CEC'13, CEC'14 and CEC'17 test functions. The performance of each parameter value is ranked based on the mean performance and ranks are shown in Tables 9-4:9-6. The final ranks indicate that lower values consistently exhibit superior performance. ER=1 consistently exhibited the best mean performance for 10, 50 and 100-dimensional CEC'14 functions, in addition, $ER = 1$ provides the best performance for 10 and 100-dimensional CEC'17 test functions. The second-best value was observed as $ER = 5$ where superior performance is attained on the 30-dimensional CEC'14 and CEC'17 functions and for 50-dimensional CEC'17 functions. Besides $ER = 1$ and $ER = 5$, $ER = 10$ is the only value that achieved the best performance for 50-dimensional CEC'13 functions. Hence, $ER = 5$ and $ER = 1$ dominated and generally exhibited superior performance for the experiments conducted on 10, 30, 50 and 100-dimensions on CEC'13, CEC'14 and CEC'17 test suites.

Table 9-4 - Mean and final ranks of different ER values on the 10, 30, 50 and 100 dimensional CEC'13 problems

d	10-d	30-d	50-d	100-d

$ER = 1$	6.89	5	5.04	2	5.54	2	3.64	2
$ER = 5$	4.86	1	4.07	1	5.86	3	3.25	1
$ER = 10$	5.61	2	5.32	3	3.68	1	4.93	4
$ER = 15$	8.57	14	5.64	4	6.93	5	4.54	3
$ER = 20$	8.14	10	7.46	5	6.39	4	6.29	5
$ER = 25$	6.96	6	7.75	6	7.29	6	6.89	6
$ER = 30$	6.71	3	8.21	7	8.57	9	8.64	8
$ER = 40$	7.93	8	9.00	9	8.21	8	9.39	9
$ER = 50$	6.82	4	9.36	12	7.86	7	8.50	7
$ER = 75$	7.43	7	8.89	8	10.00	14	10.54	12
$ER = 100$	8.68	15	9.18	11	9.21	10	9.43	10
$ER = 150$	8.07	9	10.68	14	11.32	15	10.32	11
$ER = 200$	8.50	13	10.68	14	9.68	11	10.61	14
$ER = 250$	8.43	12	9.57	13	9.75	13	10.57	13
$ER = 500$	8.29	11	9.14	10	9.71	12	10.86	15

Table 9-5- Mean and final ranks of different ER values on the 10, 30, 50 and 100 dimensional CEC'14 problems

d	10-d		30-d		50-d		100-d	
$ER = 1$	5.97	1	4.87	2	3.60	1	3.03	1
$ER = 5$	6.53	2	4.67	1	4.30	2	3.30	2
$ER = 10$	6.53	2	7.23	5	6.23	4	4.77	3
$ER = 15$	6.90	4	5.80	3	6.63	5	5.33	4
$ER = 20$	6.87	3	7.77	7	5.90	3	6.50	5
$ER = 25$	7.77	7	7.80	8	6.87	6	6.53	6
$ER = 30$	7.43	5	7.27	6	7.40	8	6.97	7
$ER = 40$	8.33	11	6.93	4	7.30	7	8.53	8
$ER = 50$	7.73	6	9.73	14	8.00	9	8.63	9
$ER = 75$	8.00	9	9.10	10	9.50	11	9.40	10
$ER = 100$	7.80	8	8.53	9	10.47	14	9.77	11
$ER = 150$	8.80	13	9.47	12	11.13	15	10.47	12
$ER = 200$	8.17	10	9.20	11	9.93	12	11.40	15
$ER = 250$	9.07	14	9.60	13	9.03	10	10.80	13
$ER = 500$	8.70	12	8.53	9	10.20	13	11.07	14

Table 9-6- Mean and final ranks of different ER values on the 10, 30, 50 and 100 dimensional CEC'17 problems

d	10-d		30-d		50-d		100-d	
$ER = 1$	5.67	1	4.50	2	4.53	2	3.07	1
$ER = 5$	6.93	5	3.97	1	4.10	1	3.47	2
$ER = 10$	6.10	2	6.13	3	5.30	3	3.60	3
$ER = 15$	8.40	13	6.87	5	5.87	4	5.10	4
$ER = 20$	8.20	11	6.50	4	6.23	5	7.03	6
$ER = 25$	7.63	6	8.13	7	7.50	6	6.80	5
$ER = 30$	8.47	14	8.33	8	8.27	9	7.30	7
$ER = 40$	6.73	4	8.93	10	7.73	7	8.33	8
$ER = 50$	8.27	12	9.07	12	8.17	8	8.77	9

$ER = 75$	8.07	9	7.60	6	9.10	10	10.67	13
$ER = 100$	8.63	15	9.63	14	10.70	15	10.73	14
$ER = 150$	6.47	3	9.63	14	9.77	13	10.37	12
$ER = 200$	7.87	8	9.00	11	10.07	14	11.63	15
$ER = 250$	7.83	7	8.83	9	9.47	11	10.13	11
$ER = 500$	8.17	10	9.37	13	9.70	12	9.50	10

Further assessment indicates that, as shown in Figure 9-1:9-3, among the sixteen different values, $ER = 1$ and $ER = 5$ yield significantly better performance in attaining the lowest mean, minimum and standard deviation values for majority of functions across different dimensions of multiple test suites. In addition, we observe that $ER = 1$ and $ER = 5$ exhibit better performance compared to other ER values tested for the majority of the unimodal, multimodal, hybrid and composition functions. Hence, considering the results presented in Figure 9-1, for the main experiments $ER = 1$ is used.

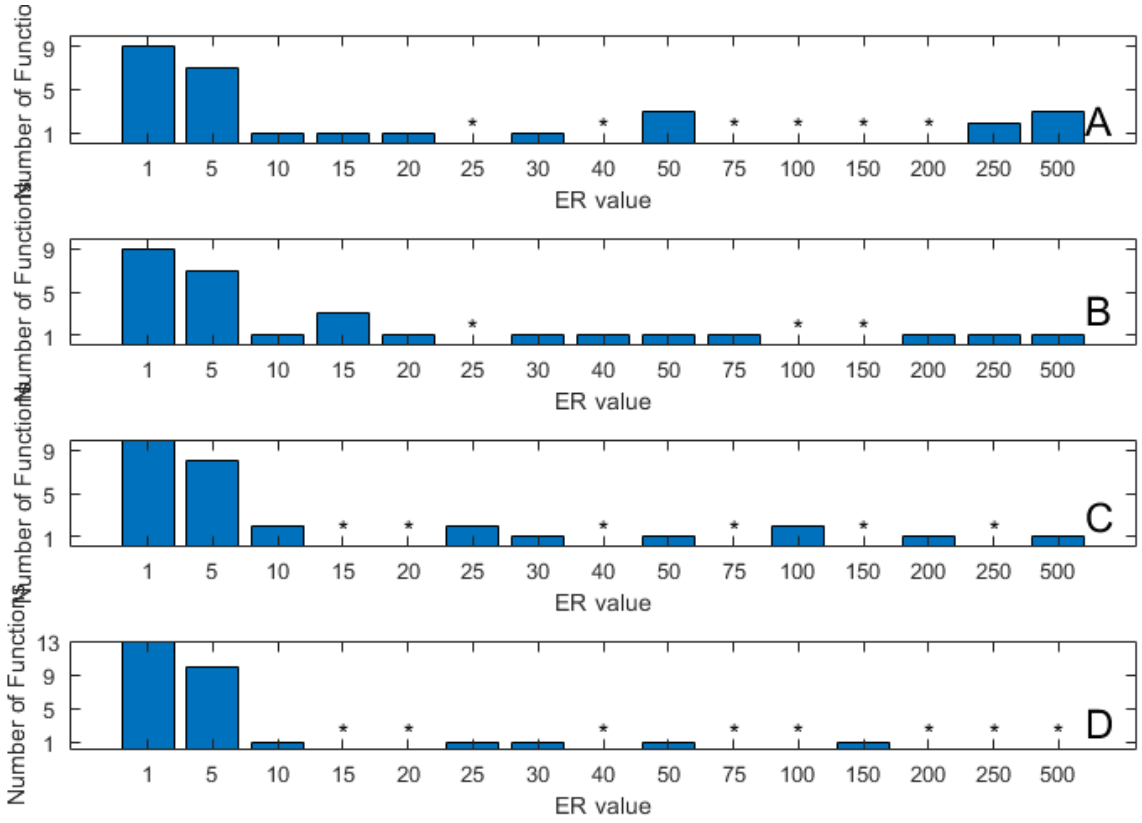


Figure 9-1-Total number of problems alpha values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'13 problems

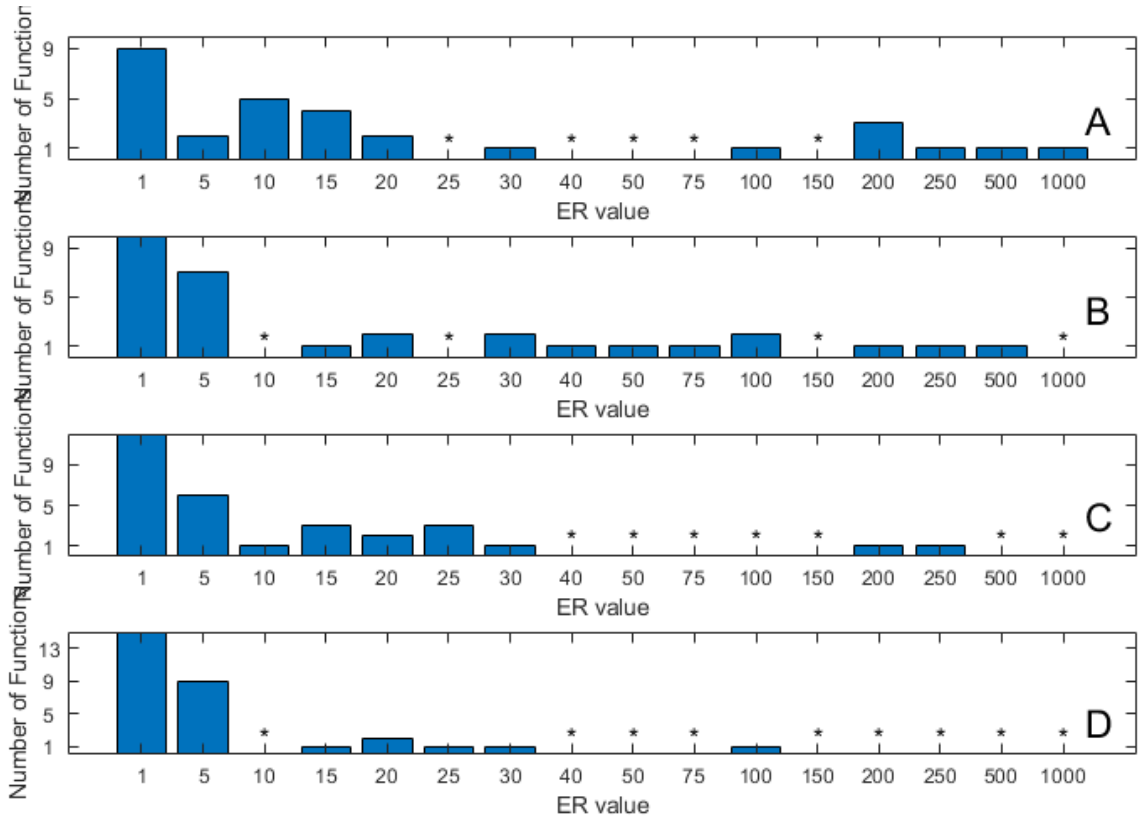


Figure 9-2-Total number of problems alpha values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'14 problems

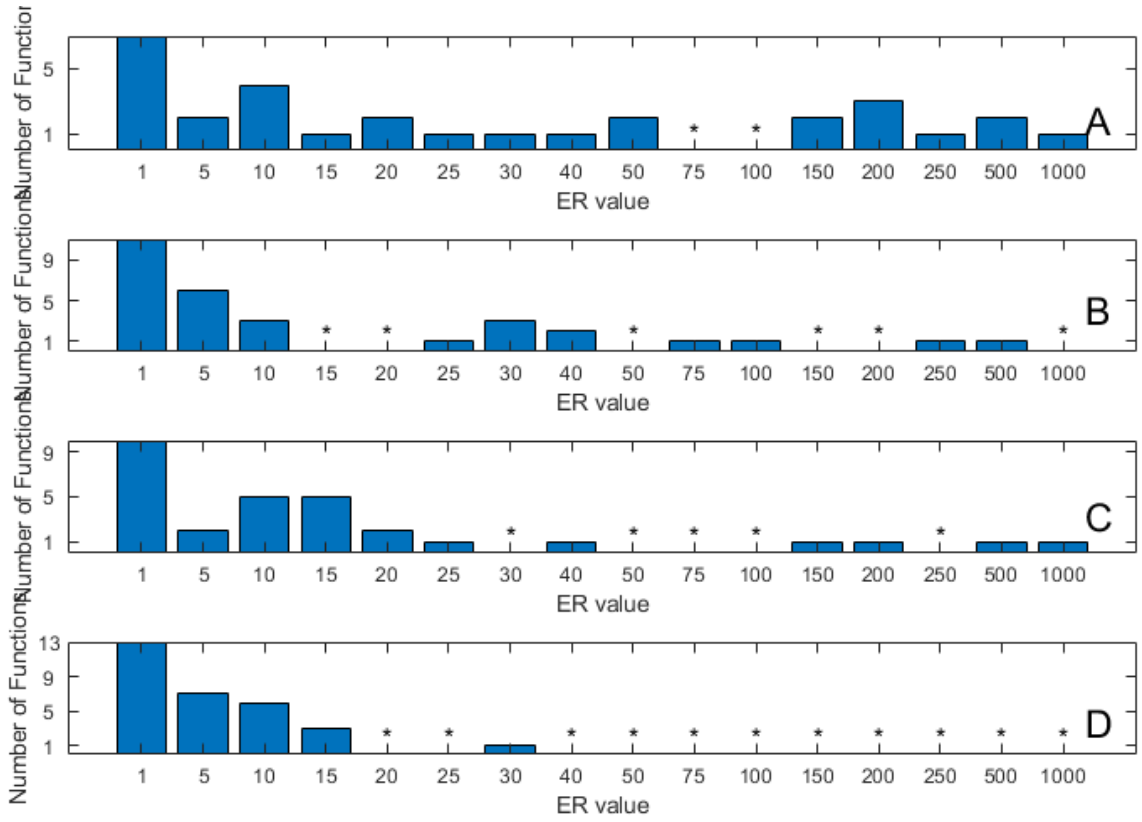


Figure 9-3-Total number of problems alpha values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'17 problems

9.5.3 The Impact of the lending-borrowing reset rate

The lending-borrowing profile builds over time as particles move in the search space and interact with other particles. After a certain number of iterations, the gap between L and B can be irreversible which at higher level directly affects the value of β, a parameter that is used by the swarm and at an individual level. The mentioned gap causes repetitive rejection or approval (depending on which is higher) for energy borrowing requests of particles. Hence, the aforementioned issue is prevented by periodically resetting the values of L and B to maintain an approximate balance between them. The optimal interval to reset L and B is expected to be short but at the same time, it should provide sufficient time to allow particles to build a lending-borrowing profile. Hence to determine the optimal value, we conducted experiments on the 10, 30 and 50-dimensional CEC'13, CEC'14 and CEC'17 test suites using 17 different values as periods to reset the values of L and B. The obtained results are ranked according to their mean error values and are shown in Table 9-7:9-9. The final ranks indicate that for three test suites across four dimensions only $LB_{rate} = 5$ and $LB_{rate} = 10$ exhibit the best performances compared to rest of the values employed for the LB_{rate} parameter. $LB_{rate} = 5$ achieved the best performance for 50 and 100-dimensional CEC'13, 30, 50 and 100-dimensional CEC'14 and for all dimensions on the CEC'17 test suite. Moreover, $LB_{rate} = 10$ attained the 1st rank on the 10-dimensional CEC'13, CEC'14, and 30-dimensional CEC'13 test functions. Hence, we conclude that $LB_{rate} = 5$ is the optimal value on the three test suites employed and the second-best setting is $LB_{rate} = 10$.

Table 9-7- Mean and final ranks of different LB_{rate} values on the 10, 30, 50 and 100 dimensional CEC'13 problems

LB_{rate}	10-d		30-d		50-d		100-d	
5	5.36	2	5.07	2	3.25	1	2.82	1
10	4.89	1	5.04	1	4.18	2	4.32	2
15	7.86	6	6.07	3	6.32	3	5.25	3
20	6.43	3	6.25	4	7.18	5	6.75	6
25	9.14	12	6.86	5	6.39	4	6.25	4
30	8.93	11	8.25	7	8.07	6	6.50	5
35	6.79	4	8.36	8	8.21	7	9.29	8
40	8.86	10	7.86	6	9.00	9	8.14	7
45	7.50	5	8.82	9	10.36	12	9.54	9
50	9.18	13	9.68	10	11.61	14	10.18	11
60	7.89	7	10.89	12	10.29	11	10.11	10
70	8.39	8	10.61	11	8.61	8	10.43	12
80	8.50	9	11.36	13	11.14	13	11.54	13
100	10.57	15	11.96	16	9.57	10	12.07	15
150	11.25	17	11.79	14	12.00	15	11.93	14
200	11.21	16	12.25	17	13.21	16	13.89	17
250	10.39	14	11.89	15	13.61	17	13.79	16

Table 9-8-CEC'14- Mean and final ranks of different LB_{rate} values on the 10, 30, 50 and 100 dimensional CEC'14 problems

LB_{rate}	10-d		30-d		50-d		100-d	
5	5.37	2	4.73	1	2.93	1	2.93	1
10	4.00	1	5.00	2	4.30	2	3.93	2
15	7.80	3	5.67	3	6.27	3	4.87	3
20	8.07	5	6.97	4	6.27	3	6.17	4
25	8.90	9	8.27	6	7.40	5	7.47	5
30	7.90	4	8.43	8	6.73	4	8.07	7
35	8.63	7	9.03	9	9.17	7	7.57	6
40	9.93	12	9.43	11	10.17	10	8.10	8
45	8.43	6	9.23	10	9.33	8	9.57	10
50	9.43	10	8.33	7	10.07	9	8.83	9
60	10.17	13	7.97	5	9.10	6	9.80	11
70	9.93	12	10.50	15	10.70	13	10.90	13
80	8.87	8	9.90	13	10.20	11	10.37	12
100	9.60	11	9.63	12	12.00	16	11.03	14
150	10.53	14	10.47	14	10.43	12	12.73	15
200	9.60	11	12.80	17	11.50	14	13.13	17
250	10.53	14	12.10	16	11.90	15	13.00	16

Table 9-9-CEC'17- Mean and final ranks of different LB_{rate} values on the 10, 30, 50 and 100 dimensional CEC'17 problems

LB_{rate}	10-d		30-d		50-d		100-d	
5	4.93	1	4.03	1	3.10	1	2.30	1
10	6.10	3	4.17	2	4.43	2	2.83	2
15	6.80	5	4.90	3	5.80	3	4.37	3
20	8.33	8	6.50	4	6.40	4	4.43	4
25	8.03	6	7.43	7	6.87	6	6.27	5
30	6.50	4	7.10	6	6.57	5	8.07	7
35	10.50	13	8.97	8	7.70	7	8.43	8
40	8.30	7	6.80	5	8.97	8	8.67	9
45	9.67	11	9.23	9	9.80	11	7.93	6
50	5.60	2	10.43	11	9.20	9	10.03	10
60	9.87	12	10.47	12	9.77	10	10.87	12
70	8.73	9	9.83	10	9.80	11	10.23	11
80	9.63	10	11.37	15	10.67	12	11.80	13
100	10.87	14	11.33	14	11.83	14	12.00	14
150	11.00	15	11.20	13	12.80	15	12.87	15
200	9.87	12	11.87	16	11.40	13	13.60	16
250	11.63	16	12.83	17	13.37	16	13.77	17

We further assess the performance of the LB_{rate} parameter on the mean, minimum and standard deviation values. Figure 9-4:9-6 exhibits the parameter values tested for the LB_{rate} and the comparison of the total number of functions each value attained the lowest f_{mean}, $f_{\min}$ and the standard deviation on different dimensions for CEC'13, CEC'14 and CEC'17 test functions. As shown in Figure 4-6, we also observe improvements in the performance of AHPSO with increased dimensionality when $LB_{rate} = 5$, which indicates that the LB_{rate} parameter is more influential and contributes effectively to the search process in higher dimensional problems. Hence, for the main experiments conducted, $LB_{rate} = 5$ is used.

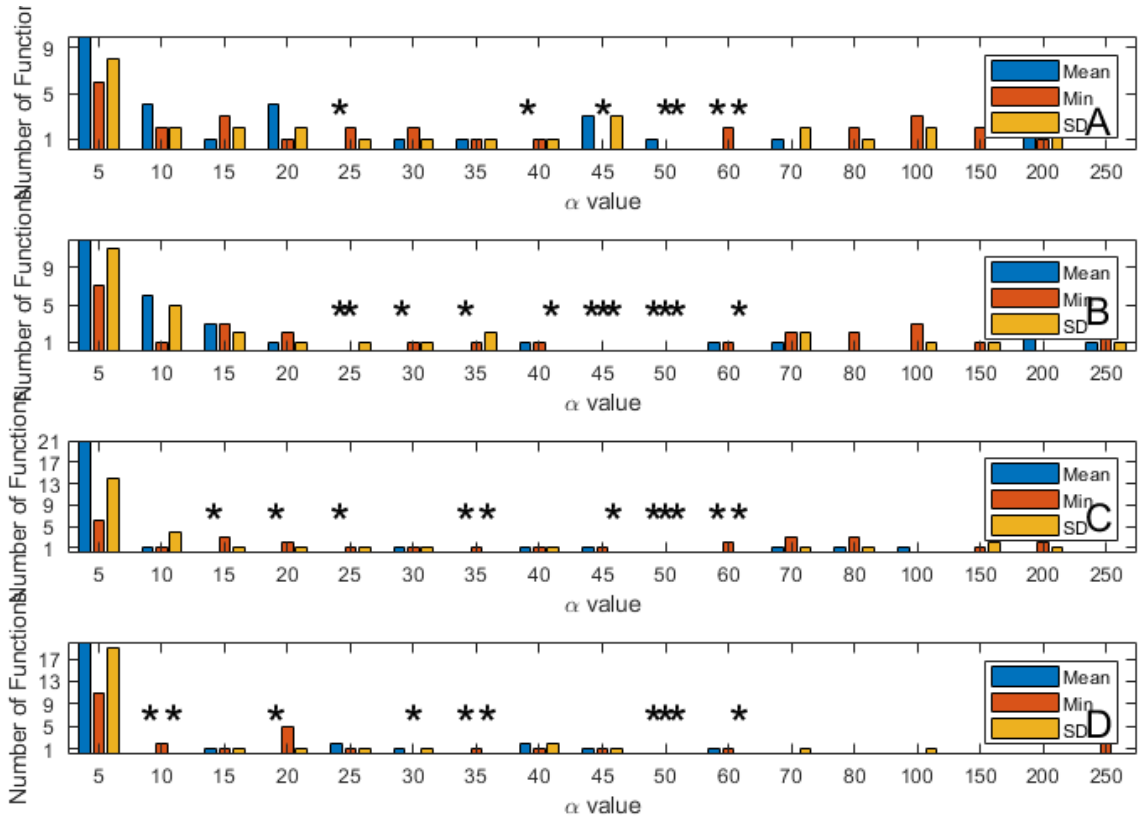


Figure 9-4-Total number of problems LB_{rate} values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'13 problems

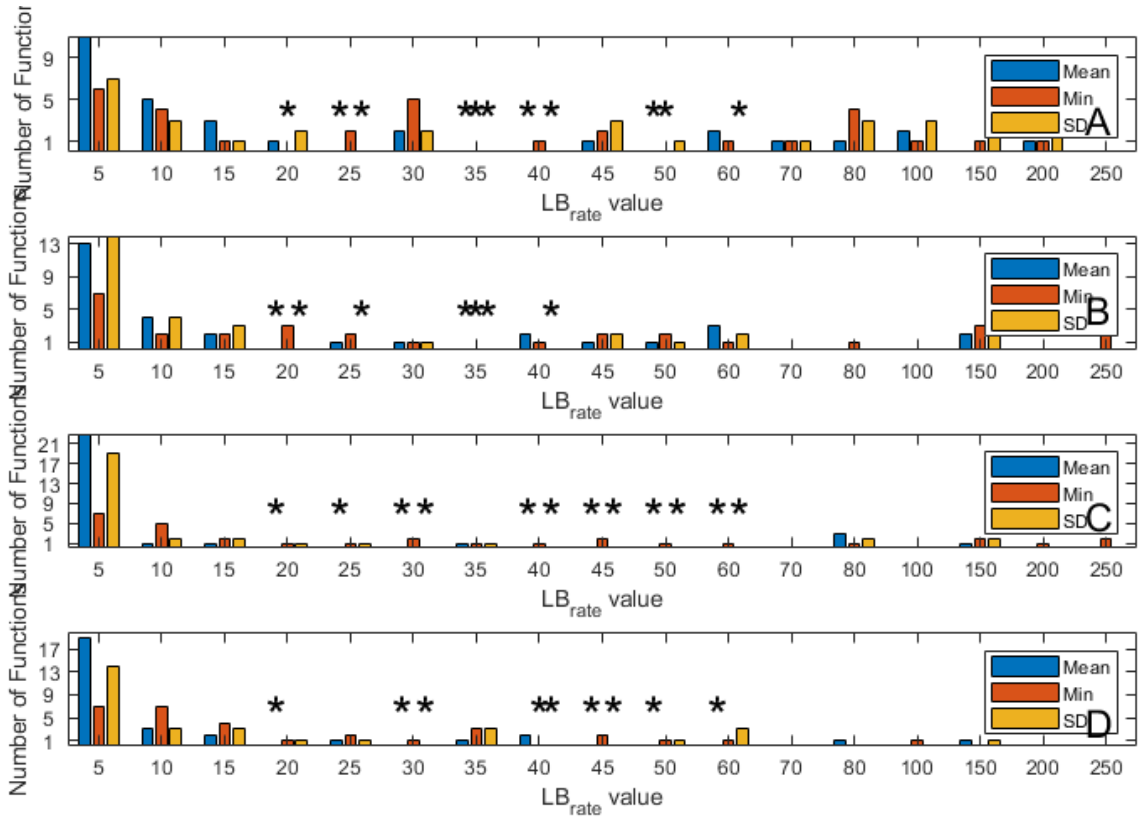


Figure 9-5- Total number of problems LB_{rate} values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'14 problems

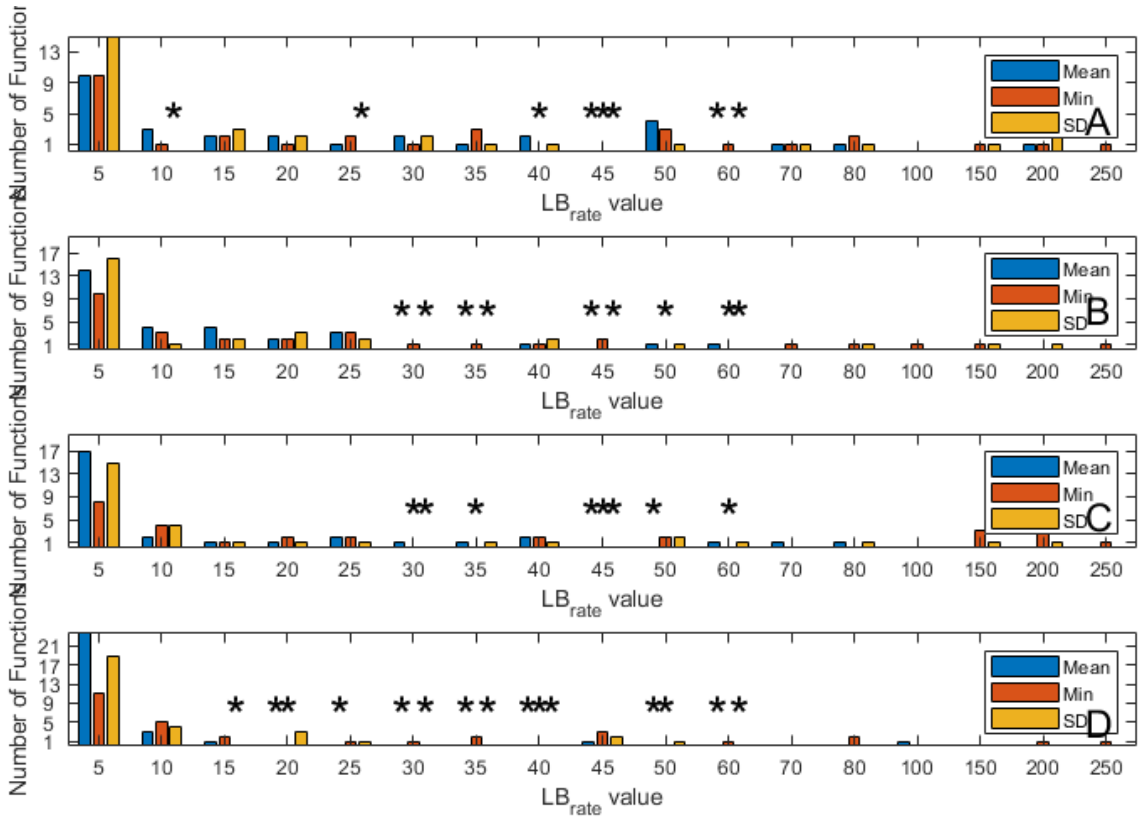


Figure 9-6- Total number of problems LB_{rate} values obtained the lowest mean error value on 10(A), 30(B), 50(C) and 100(D) dimensional CEC'17 problems

9.5.4 The Impact of the population size

To assess the impact of population size on the performance of AHPSO, we have conducted an experiment on the 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test functions using various population sizes and observed the impact on the overall mean performance. The obtained mean error results across three test suites are ranked and are shown in Table 9-10:9-11. Our findings indicate that for 30, 50 and 100-dimensional functions on the three test suites, the population size of 60 (n=60) consistently exhibit better performance compared to the rest of the values tested for particle population. On the other hand, for 10-dimensional functions, we generally observe that denser population (e.g., n=80, n=100, n=200) exhibit better performance on the CEC'13, CEC'14 and CEC'17 test functions. However, since n=60 exhibited better mean performance on all test suites and across most dimensions, hence in the main experiments, $n = 60$ will be used.

Table 9-10-Calibration of population size on the 10, 30, 50 and 100 dimensional CECE'13 problems

n	10-d		30-d		50-d		100-d	
10	11.21	12	12.61	13	12.32	12	12.68	13
15	9.96	11	10.86	12	10.86	11	10.89	12
20	8.86	10	9.61	11	9.54	10	9.93	11
30	6.39	9	7.68	10	7.32	9	8.32	10
35	5.75	8	5.68	5	6.18	7	6.50	9
40	5.07	2	4.79	2	5.71	5	5.89	8
45	5.43	5	5.89	6	5.36	3	5.50	5
50	5.07	2	5.39	3	4.96	2	5.61	6
60	5.21	3	4.68	1	4.71	1	4.50	1

70	5.71	7	5.93	7	5.79	6	4.71	2
80	5.29	4	6.32	9	5.64	4	5.25	4
100	4.75	1	5.54	4	5.79	6	5.11	3
200	5.54	6	6.04	8	6.82	8	5.75	7

Table 9-11- Calibration of population size on the 10, 30, 50 and 100 dimensional CECE'14 problems

n	10-d		30-d		50-d		100-d	
10	11.27	13	11.83	12	12.13	12	12.47	12
15	9.73	12	10.97	11	10.17	11	10.83	11
20	9.40	11	9.23	10	8.20	10	9.67	10
30	6.70	9	6.67	9	6.50	8	8.07	9
35	6.73	10	6.27	8	6.10	6	6.97	8
40	6.20	8	5.63	5	6.00	5	5.70	7
45	5.83	6	5.67	6	5.70	4	5.27	6
50	5.87	7	5.67	6	4.77	2	5.23	5
60	4.73	2	4.67	1	4.73	1	4.60	1
70	5.20	4	4.93	2	5.03	3	4.63	2
80	4.63	1	5.13	3	6.40	7	4.73	3
100	4.93	3	5.57	4	6.00	5	5.00	4
200	5.27	5	6.17	7	6.67	9	5.23	5

Table 9-12- Calibration of population size on the 10, 30, 50 and 100 dimensional CECE'17 problems

n	10-d		30-d		50-d		100-d	
10	10.87	13	12.17	13	12.27	12	12.33	13
15	10.30	12	10.23	12	10.63	11	11.10	12
20	7.80	11	9.13	11	9.17	10	9.67	11
30	6.73	10	7.17	10	6.20	8	7.20	10
35	6.67	9	6.80	9	5.83	5	6.23	8
40	6.23	8	6.30	7	6.20	8	4.50	3
45	5.70	7	5.23	4	5.60	4	5.17	5
50	5.47	6	5.97	6	5.00	3	5.00	4
60	5.07	3	3.37	1	4.03	1	4.23	1
70	5.20	5	4.77	2	4.97	2	4.47	2
80	4.57	2	4.87	3	5.93	6	5.80	6
100	5.17	4	5.83	5	6.13	7	5.90	7
200	4.37	1	6.57	8	6.43	9	6.80	9

9.5.5 The impact of the PPM and paired population size

The paired particle model proposed in the AHPSO algorithm aimed to delay the loss of population diversity to assist the AHPSO algorithm in prevention of premature convergence. In this section, we assess the impact of the PPM on the population diversity and the overall mean performance on the 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test suites with the PPM mechanism enabled and disabled. Table 9-13 display the average and final ranks for the mean performances of AHPSO algorithm with and without the PPM mechanism. The mean and final ranks reveal that AHPSO with the PPM mechanism achieved better ranks compared to the version without the PPM on the 30,50

and 100-dimensional CEC'13, CEC'14 and CEC'17 test suites. We also observe from Table 9-13 that the PPM mechanism is more effective on the higher dimensional functions in all three test suites.

Table 9-13- Mean ranks with the PPM mechanism enabled and disabled on the 10, 30, 50 and 100 dimensional CEC'13, CEC'14 and CEC'17 problems

	CEC'13		CEC'14		CEC'17	
	PPM on	PPM off	PPM on	PPM off	PPM on	PPM off
10D	1.60	1.39	1.53	1.43	1.70	1.26
30D	1.42	1.72	1.33	1.63	1.36	1.60
50D	1.31	1.87	1.30	1.76	1.33	1.73
100D	1.26	1.79	1.23	1.73	1.26	1.80

In addition, as shown in Figure 9-7, the population diversity comparison reveals that the AHPSO algorithm with PPM successfully maintains significantly better population diversity for 30, 50 and 100-dimensional functions. Moreover, Figure 9-8 shows the convergence rates for the same problem set and it is worth noting that AHPSO with PPM converged to a better or similar mean solution in most cases whilst maintaining a more diverse population. Moreover, Figure 9-9:9-11 displays the total number of functions where each version of AHPSO (with and without PPM) attained the best mean error values over 30 consecutive runs, once again, AHPSO with PPM obtains better mean error values for higher dimensional (50-d and 100-d) functions. The results shown in Table 9-13 and Figure 9-11 corroborate the impact of the PPM mechanism and we clearly observe the true impact of the PPM mechanism with increased dimensionality on the overall performance of the AHPSO algorithm.

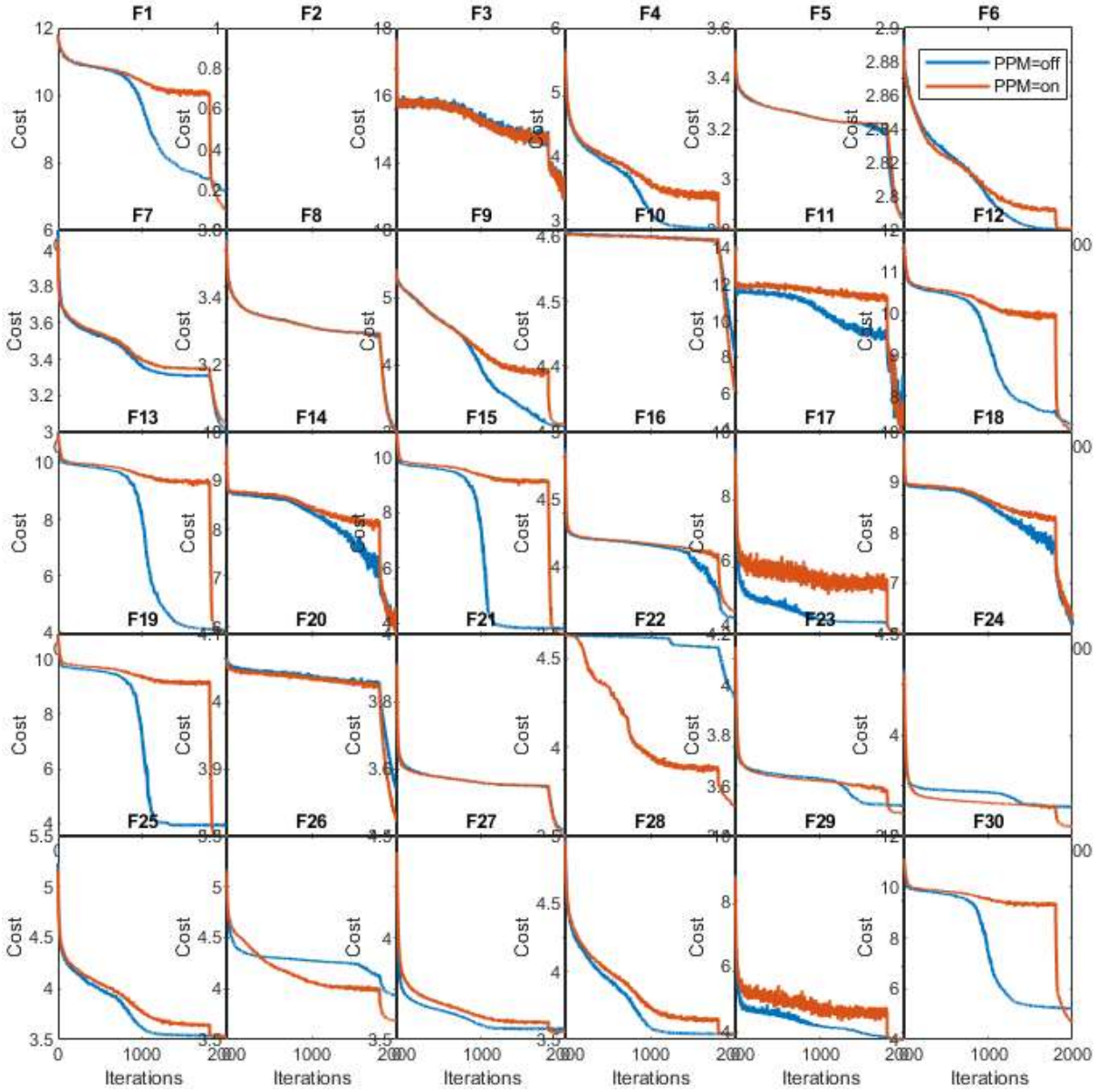


Figure 9-7- Population diversity comparison of AHPSO with PPM enabled and disabled on the 100 dimensional CEC'17 problems

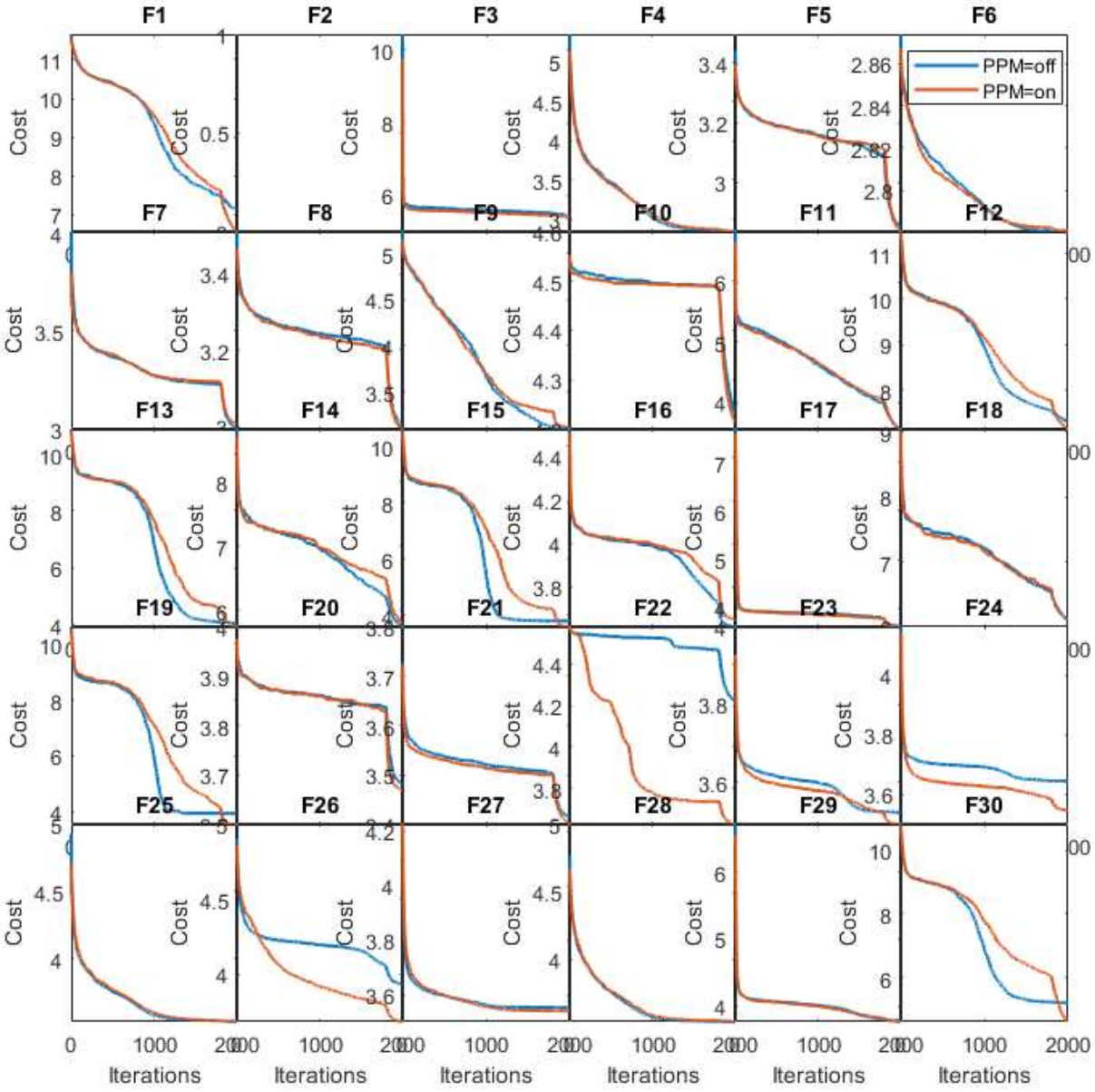


Figure 9-8- Convergence rate comparison of AHPSO with PPM enabled and disabled on the 100 dimensional CEC'17 problems

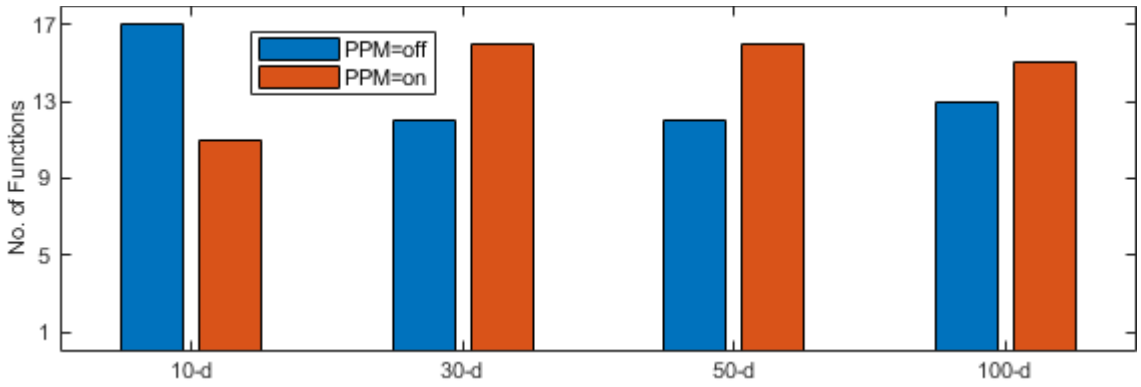


Figure 9-9- The total number of problems where AHPSO with and without the PPM exhibited the best performance on the 10, 30, 50 and 100-dimensional CEC'13 problems.

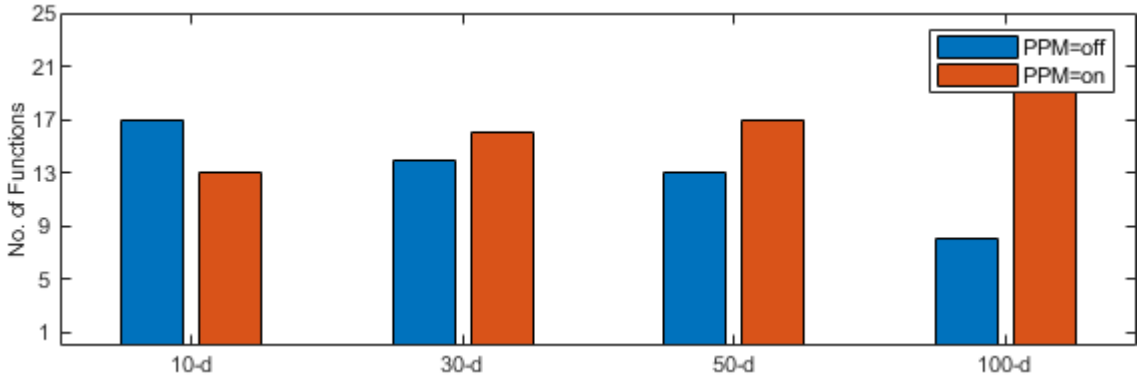


Figure 9-10- The total number of problems where AHPSO with and without the PPM exhibited the best performance on the 10, 30, 50 and 100-dimensional CEC'14 problems.

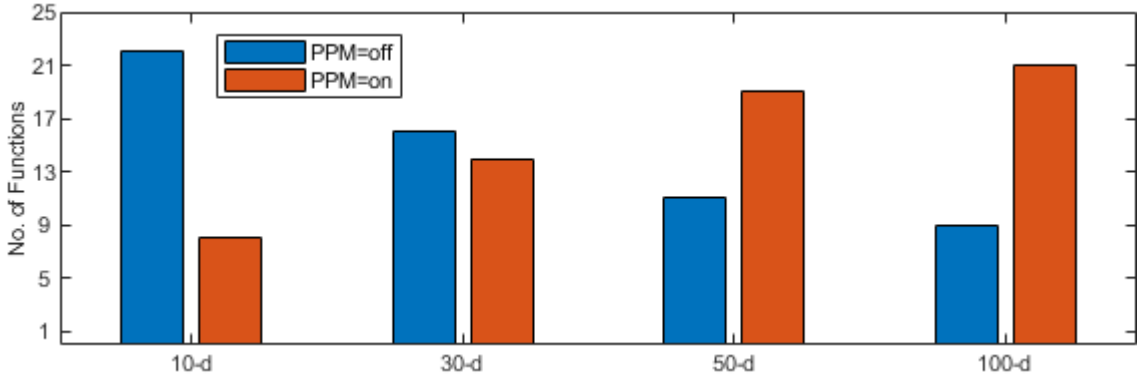


Figure 9-11- The total number of problems where AHPSO with and without the PPM exhibited the best performance on the 10, 30, 50 and 100-dimensional CEC'17 problems

After verifying the general impact of the PPM mechanism, we conducted a series of experiments to determine the optimal paired particle population that can reduce or increase the impact of the PPM mechanism. Hence, using 11 different paired particle population sizes, we have conducted experiments on the 10, 30, 50 and 100-dimensional CEC'17 test functions. Table 9-14 displays the mean and final ranks on the obtained mean error values for different paired particle population sizes for 10, 30, 50 and 100-dimensional CEC'17 functions. The results indicate that for 10-dimensional functions $n_{paired} = 16$ performed best, subsequently, for 30, 50 and 100-dimensional functions, $n_{paired} = 10$, $n_{paired} = 30$ and $n_{paired} = 20$ exhibited superior performance compared to the rest of the values tested. The second-best value was observed to be $n_{paired} = 6$ for 10 and 30-dimensional functions. $n_{paired} = 6$ was also observed to exhibit fair performance on the 50 and 100-dimensional functions.

Figure 9-12 exhibits the total number of functions where each n_{paired} value attained the lowest mean error. Although in Figure 9-12, we observe that $n_{paired} = 6$ consistently enabled AHPSO to attain lower mean errors for more functions compared to rest of the n_{paired} values employed, however as shown in Table 9-14, the ranks portray slightly different results. The convergence rates shown in Figure 9-13 indicate that the lower the n_{paired} value, the faster convergence is attained as PPM aims to delay the population diversity, hence in some cases, the denser paired particle population tend to cause slightly slower convergence however the final converged solution is hardly any different. Although the best mean performance is attained by different n_{paired} values at different dimensions (with slight mean difference). We observe compelling evidence that lower n_{paired} values favour convergence and appear as a more suitable fixed *general setting*. Hence,

considering the results exhibited in Figure 9-12 and Figure 9-13, in the main experiments $n_{paired} = 6$ will be used.

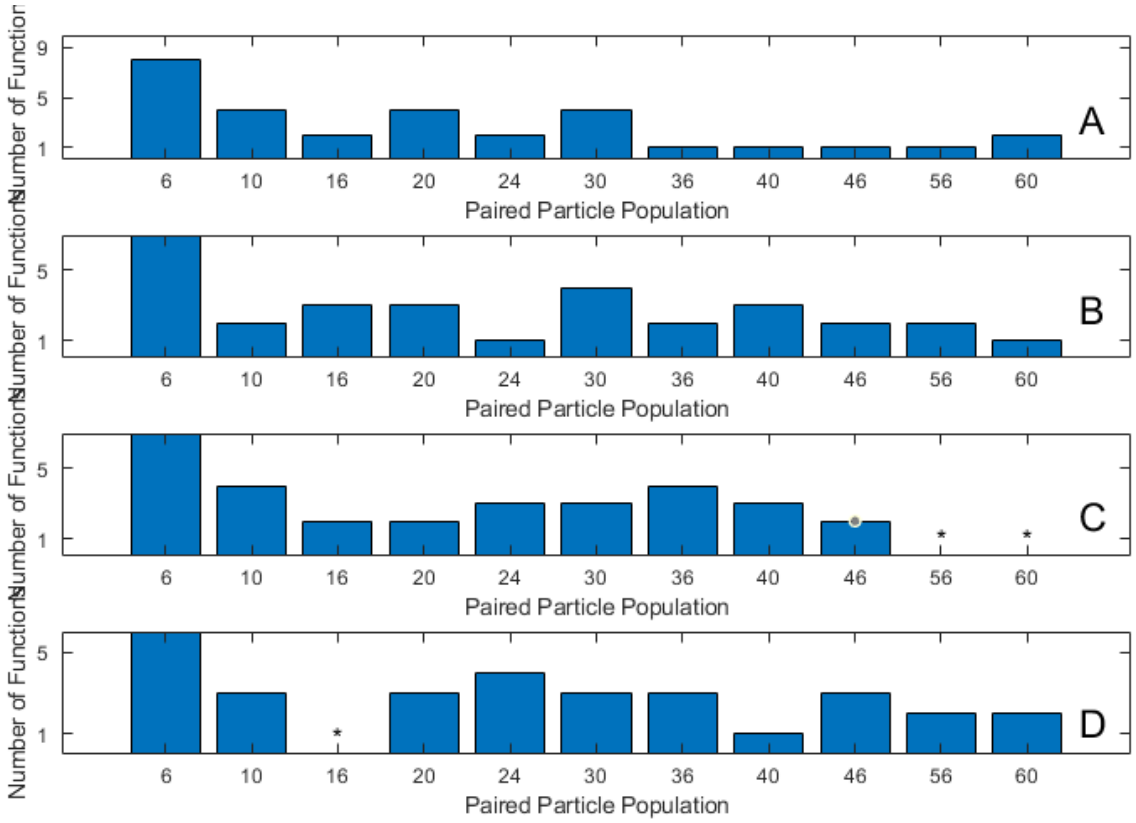


Figure 9-12- The impact of different paired population sizes on the number of best performances obtained by AHPSO for 10(A), 30(B), 50(C) and 100(D) dimensional CEC'17 problems

Table 9-14-Mean and final ranks obtained using different paired particle population sizes on the 10, 30, 50 and 100 dimensional CEC'17 problems

n_{paired}	10D		30D		50D		100D	
6	4.57	2	4.90	2	5.57	4	5.37	3
10	5.00	4	4.73	1	5.53	3	5.37	3
16	4.53	1	5.27	3	5.63	5	5.57	6
20	5.27	5	6.17	8	4.97	2	4.50	1
24	5.37	6	5.40	5	5.97	7	5.50	4
30	4.97	3	5.37	4	4.90	1	5.30	2
36	5.87	7	6.60	9	5.57	4	5.53	5
40	6.00	8	5.60	6	6.03	8	6.10	7
46	6.23	9	6.10	7	5.83	6	6.57	8
56	7.67	10	7.07	11	7.03	9	7.30	10
60	7.90	11	6.97	10	7.13	10	7.07	9

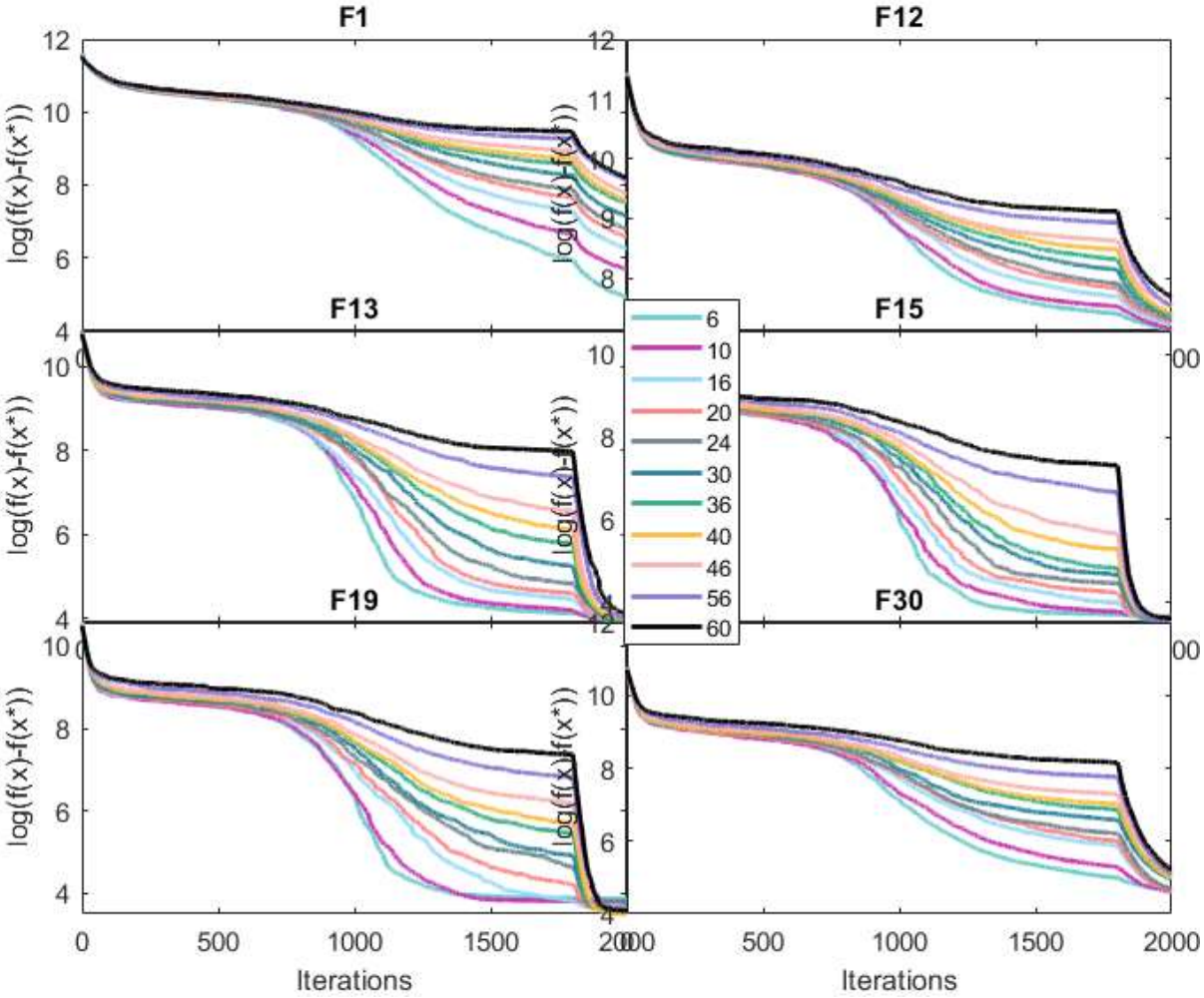


Figure 9-13-The convergence rate comparison of AHPSO with different population sizes on various 100 dimensional (CEC'17) unimodal, multimodal, hybrid and composition functions.

9.5.6 The Impact of the Beta parameter

During search, the beta parameter is self-regulated, and its value is always equal to the average altruism value of particles. Beta is employed as a threshold value/criterion to initiate the energy sharing process hence has significant influence on the type of velocity update equation particles adopt. In summary, the beta parameter directly impacts the heterogeneity properties of the AHPSO algorithm. Although the AHPSO algorithm exhibits sufficient performance with the self-regulated version of the beta parameter, in this section we assess the precise impact of beta on the overall performance. In addition, we observe the impact of the balance of the heterogeneity in the swarm during search and the rate of convergence. We have used 12 different values for the beta parameter on the 10,30,50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test functions. The results are ranked according to mean error values and are shown in Tables 9-15:9-17. Overall, the top ranks obtained on the CEC'13 test functions is spread across different beta values however we observe a more stable pattern on the CEC'14 and CEC'17 test suites, as $\beta = 0.7 - 0.8$ exhibit the best performance.

Table 9-15-- Mean and final ranks obtained using different β values on the 10, 30, 50 and 100 dimensional CEC'13 problems

β	10-d	30-d	50-d	100-d

$\beta = 0.05$	6.75	11	7.18	8	6.25	2	6.57	8
$\beta = 0.1$	6.04	8	6.36	5	6.32	4	6.50	7
$\beta = 0.15$	4.89	1	6.75	7	6.89	7	6.50	7
$\beta = 0.2$	5.71	5	6.21	3	6.32	4	5.68	1
$\beta = 0.25$	6.11	9	6.04	2	6.21	1	6.18	4
$\beta = 0.3$	5.86	6	6.36	5	6.46	5	7.00	10
$\beta = 0.4$	6.79	12	7.25	9	6.32	4	6.75	9
$\beta = 0.5$	5.04	2	6.46	6	6.29	3	6.46	6
$\beta = 0.6$	6.00	7	7.25	9	6.71	6	5.93	2
$\beta = 0.7$	6.54	10	6.32	4	7.00	9	6.14	3
$\beta = 0.8$	5.36	4	7.18	8	6.96	8	7.32	11
$\beta = 0.9$	5.25	3	4.64	1	6.25	2	6.43	5

Table 9-16- Mean and final ranks obtained using different β values on the 10, 30, 50 and 100 dimensional CEC'14 problems

β	10-d		30-d		50-d		100-d	
$\beta = 0.05$	6.97	9	7.23	8	8.47	10	9.00	12
$\beta = 0.1$	7.00	10	7.97	10	8.57	11	8.37	11
$\beta = 0.15$	6.87	8	7.97	10	8.30	9	7.53	10
$\beta = 0.2$	6.30	6	7.80	9	7.37	8	7.03	8
$\beta = 0.25$	6.63	7	7.97	10	7.37	8	7.40	9
$\beta = 0.3$	7.33	11	6.87	7	7.13	7	6.60	7
$\beta = 0.4$	7.53	12	6.43	6	5.37	5	5.97	6
$\beta = 0.5$	5.43	5	4.97	5	5.73	6	5.83	5
$\beta = 0.6$	5.27	3	4.73	3	4.60	3	4.43	3
$\beta = 0.7$	4.67	1	4.47	1	4.23	2	4.13	1
$\beta = 0.8$	5.00	2	4.63	2	3.97	1	4.30	2
$\beta = 0.9$	5.40	4	4.77	4	4.70	4	5.20	4

Table 9-17- Mean and final ranks obtained using different β values on the 10, 30, 50 and 100 dimensional CEC'17 problems

β	10-d		30-d		50-d		100-d	
$\beta = 0.05$	7.73	10	8.10	9	7.73	9	9.07	11
$\beta = 0.1$	7.50	9	8.00	8	8.20	11	8.97	9
$\beta = 0.15$	8.00	11	8.40	10	8.10	10	9.40	12
$\beta = 0.2$	7.00	7	8.47	11	8.40	12	9.03	10
$\beta = 0.25$	7.00	7	6.90	6	7.23	7	7.30	7
$\beta = 0.3$	7.37	8	7.07	7	7.47	8	7.53	8
$\beta = 0.4$	5.20	5	5.00	4	6.00	5	6.30	6
$\beta = 0.5$	5.27	6	6.03	5	6.10	6	4.57	5
$\beta = 0.6$	4.83	4	4.40	2	4.53	3	3.97	4
$\beta = 0.7$	4.20	2	4.53	3	4.37	2	3.30	2
$\beta = 0.8$	4.10	1	4.37	1	3.10	1	3.33	3
$\beta = 0.9$	4.67	3	4.53	3	4.57	4	3.03	1

The heterogeneity in the AHPSO algorithm is attained by the four velocity update equations denoted as $v_1 \dots v_4$ which allow particles to have distinct influences whilst moving in the search space. As previously mentioned, the beta parameter directly impacts the balance of the velocity update equation adopted by particles. The minimum percentage of adoption for each velocity update equation is expected to be 20% and the maximum is 35%, approximately during the entire search process. The stated minimum and maximum values allow each velocity update equation to be employed sufficiently to enable the required heterogeneity and maximise the performance of the algorithm. Figure 9-14 exhibits how the heterogeneity of the AHPSO algorithm is affected by different beta values. We can clearly observe from Figure 9-14 that the values between 0.05 and 0.6 does not yield the expected balance between the velocity update equations and in all cases, v_4 dominates the search process by 37% to 53% and as the value of the beta increases, the usage of v_2 increases in parallel, whilst the usage of v_4 decreases. Subsequently v_2 initiates dominating the search process. We consistently observe the expected heterogeneity on the 100-dimensional functions when $\beta = 0.7 - 0.75$, where each velocity update equation is exploited within the tolerated range. We have conducted further experiments using $\beta = 0.7 \text{ and } 0.75$ on different dimensions (10-d, 30-d 50-d) and results exhibited nominal difference in the heterogeneity during the search process across various dimensions hence, a stable heterogeneity is maintained using $\beta = 0.7 \text{ and } 0.75$ in most cases. Hence, $\beta = 0.75$ is used for the main experiments detailed in the following sections.

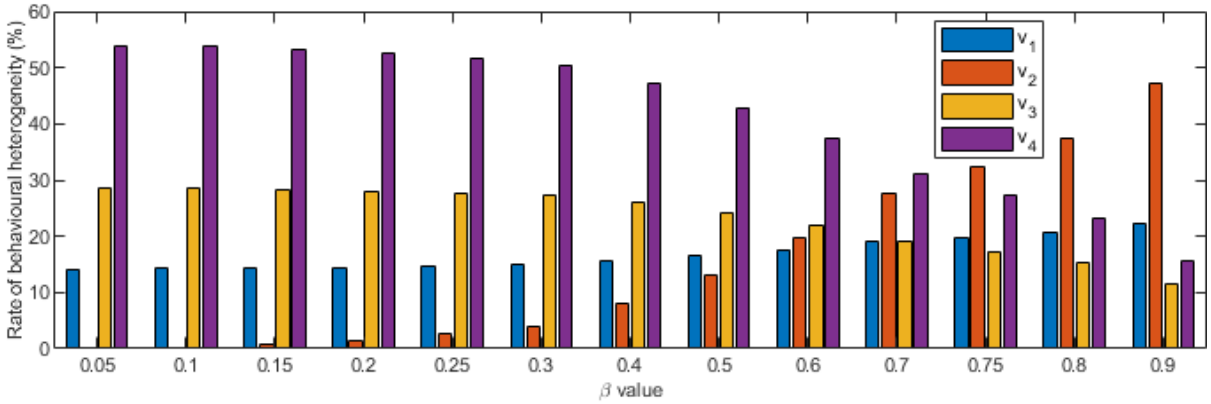


Figure 9-14-The impact of the β on the behavioural heterogeneity.

9.6 Experiments

9.6.1 Comparative Results on the CEC'13 Benchmark Problems

In this section, we present the results obtained on the CEC'13 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed AHPSO algorithm is compared with 14 algorithms on a total of 28 functions including 5 unimodal, 15 multimodal, and 8 composition functions. The full details of the problem definitions for CEC'13 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

9.6.1.1 Results for 10 Dimensional Functions

The experimental results on the 10-dimensional CEC'13 problems suggest that for the 10-dimensional unimodal functions, only L-SHADE performed best on all unimodal functions (f1-f5). On the multimodal functions, L-SHADE outperforms other algorithms on function f8, f10, f11, f12, f13, f14, f16, f17 and f18. DMS-PSO exhibited the second-best overall performance and provides the best mean solution on f6, f7, f9 and f15. Finally, HCLPSO and MiPSO attained the best mean solution on a

single function f19 and f20, respectively. The performances on the composition functions indicate that L-SHADE and EPSO provides the best mean solution on two functions f22, f27 and f24, f26, respectively. Furthermore, UPSO, DMS-PSO, MaPSO and LPSO, each exhibited the best solution on a single function f21, f23, f25 and f28, respectively.

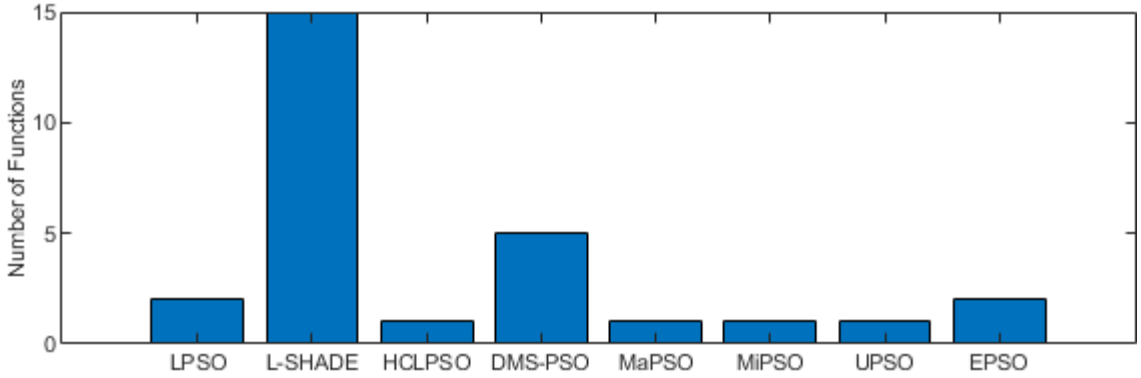


Figure 9-15 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'13 problems

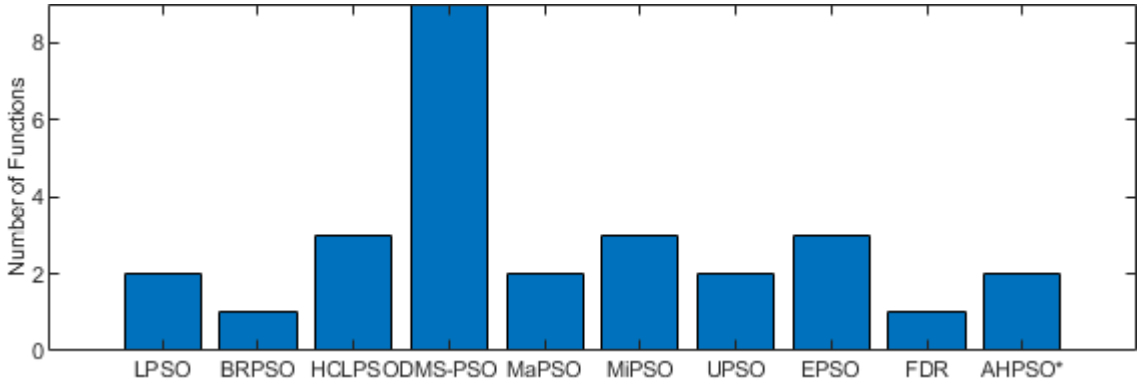


Figure 9-16 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'13 problems

Overall, L-SHADE performed best on 16 of the 28 10-dimensional CEC'13 problems and ranked first over other algorithms. On the other hand, the proposed algorithm AHPSO attained the second-best performance on 6 functions. As shown in the last row of Table 9-18, L-SHADE ranks first on the 10-dimensional problems whereas the proposed algorithm obtained the 6th rank in this case.

9.6.1.2 Results for 30 Dimensional Functions

The mean error values obtained on the 30-dimensional CEC'13 problems suggest that for the 5 unimodal functions, L-SHADE and DMS-PSO obtained the best performance on two functions f2, f3 and f1, f5, respectively. For f4, the best performance is attained by EPSO. On the multimodal functions, once again, L-SHADE exhibited the best performance for 12 of the 15 functions, namely, f6 f8, f10, f11, f12, f13, f14, f15, f16, f17, f18 and f19. For the remainder 3 functions, DMS-PSO attained the best solution for f7,f9 and AHPSO achieved the best mean performance for f20. The performance of algorithms on the 8 composition functions indicates that only AHPSO and HCLPSO obtained the best performance for 2 function f23, f28 and f24, f27, respectively. Whilst rest of the algorithms L-SHADE, LPSO, DMS-PSO and MiPSO attained the best performance for a single function f22, f21, f25 and f26, respectively.

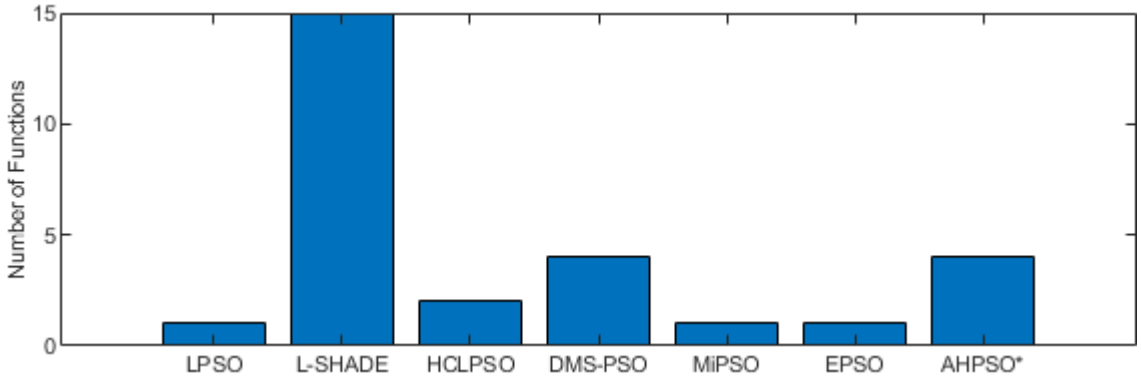


Figure 9-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'13 problems

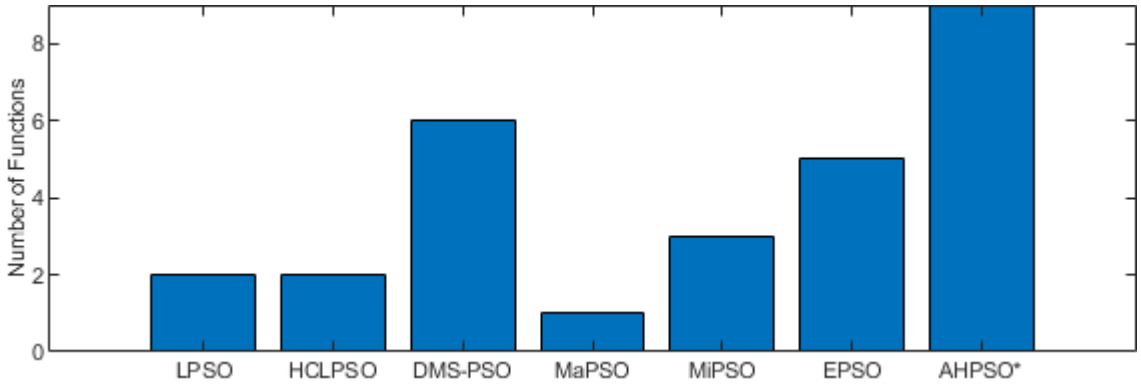


Figure 9-18 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'13 problems

9.6.1.3 Results for 50 Dimensional Functions

Th experimental results attained on the 50-dimensional CEC'13 problems suggest that L-SHADE attained the best performance on function f8, f10, f11, f14, f15, f16, f17 and f19. AHPSO exhibited the second-best overall mean performance and obtained the best solution for 6 functions namely, f7, f9, f12, f13, f18 and f20. Furthermore, on function f6, EPSO obtained the best mean solution. For composition functions, only AHPSO and DMS-PSO attained the best performance for two functions, namely f23, f24 and f25, f27, respectively. For rest of the composition functions f21, f22, f26 and f28, the performance is obtained by LPSO, L-SHADE, MiPSO and HCLPSO, respectively.

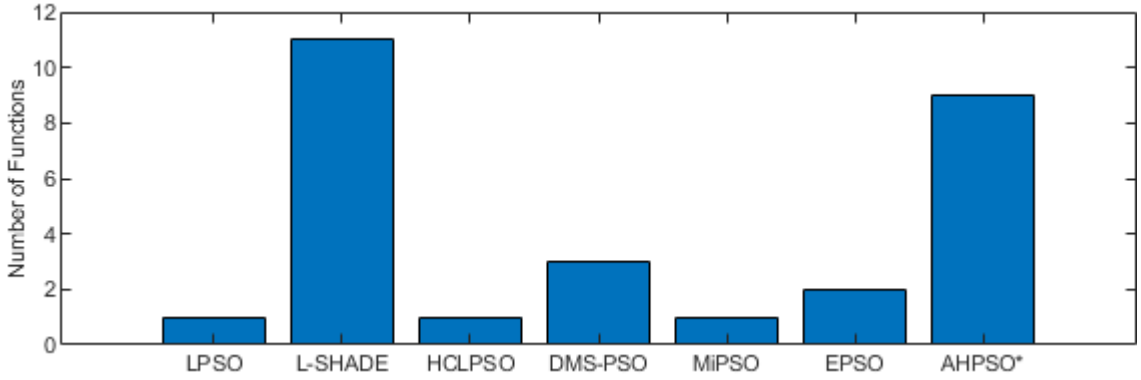


Figure 9-19 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'13 problems

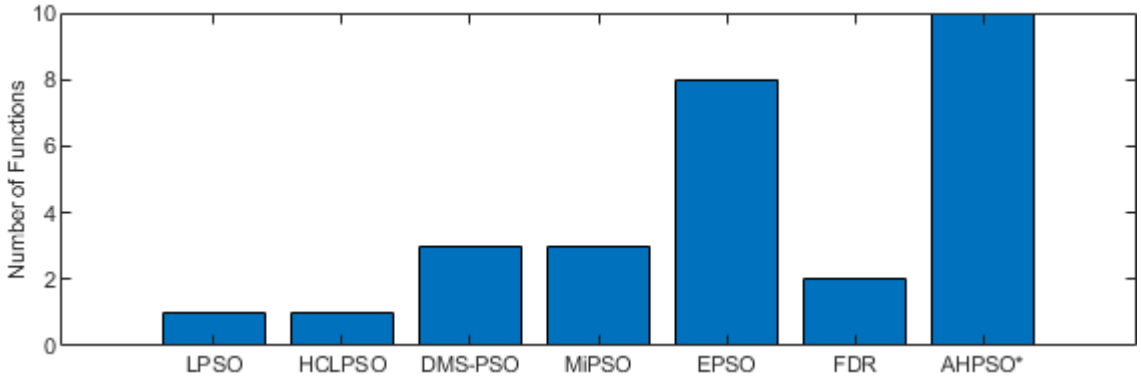


Figure 9-20 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'13 problems

In summary, L-SHADE ranked first on the 50-dimensional functions whereas the proposed algorithm AHPSO obtained the 5th rank, however as it can be observed from Table 21, no substantial difference is observed in the average rank of AHPSO and L-SHADE. In addition, L-SHADE provides the best performance for 10 of the 28 problems and despite attaining the 5th rank, AHPSO exhibit similar performance and performed best on 8 of the 28 functions. The closest performance to AHPSO and L-SHADE is observed in the performance of DMS-PSO where best mean error values are obtained on 4 of the 28 functions.

9.6.1.4 Results for 100 Dimensional Functions

The results for the 100-dimensional problems reveal that for unimodal functions, DMS-PSO attained the best performance for most unimodal functions, namely F1 and F3. The best performance on the F2, F4 and F5 are exhibited by L-SHADE, EPSO and MiPSO, respectively. On the multimodal functions, L-SHADE provides the best performance on F8, F10, F14, F15, F16, F17 and F19. The proposed AHPSO algorithm exhibited superior mean performance on multimodal F7, F9, F12, F13, F18 and F20. Furthermore, the best performance for F6 and F11 is obtained by LPSO and EPSO.

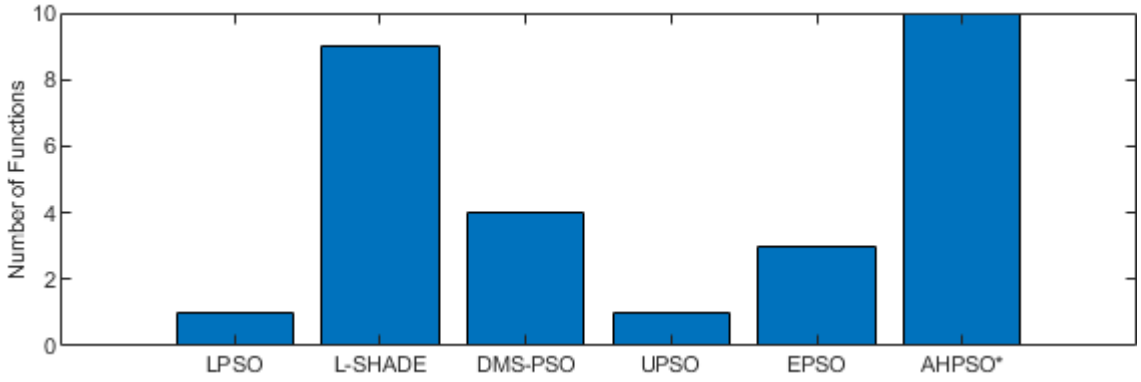


Figure 9-21 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'13 problems

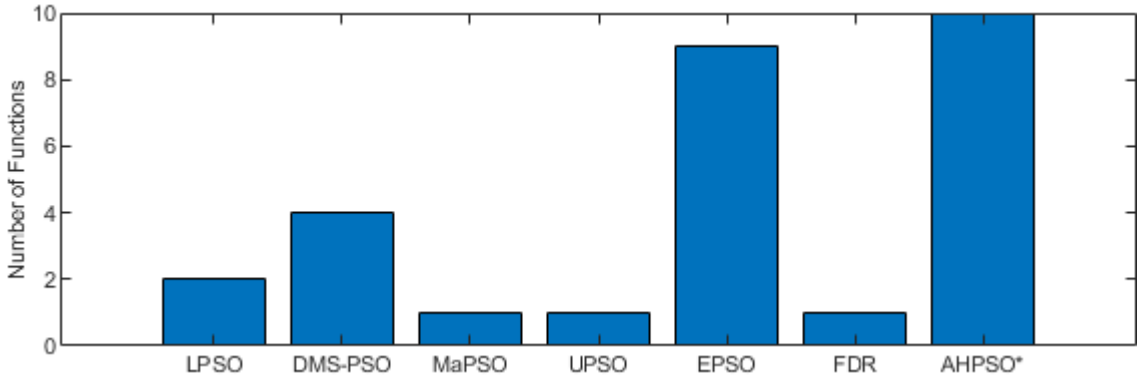


Figure 9-22 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'13 problems

Overall, the proposed algorithm AHPSO and L-SHADE performed best on the majority of the 100-dimensional problems and attained the lowest mean error values for equal number of functions (9 of 28). The second performance closest to AHPSO and L-SHADE is observed in the performance of DMS-PSO where it performed best on 4 of 28 functions.

The average and final ranks for 10, 30, 50 and 100-dimensional CEC'13 test functions is shown in Appendix 2. In summary, on the 10-dimensional functions, the proposed algorithm is ranked 6th and on the 30, 50 and 100-dimensional CEC'13 functions, AHPSO obtains the 5th rank. However, for the 100-dimensional functions, the average rank of AHPSO is not substantially different compared to the average rank EPSO (2nd rank). The Wilcoxon signed rank test results of AHPSO against comparison algorithms are shown in Table 9-18. As it can be observed in the last column, the proposed algorithm AHPSO exhibited significantly better performance than the majority (9 of 14) of the test algorithms whereas none of the algorithms exhibit statistically significantly better performance against AHPSO on the 30, 50 and 100-dimensional CEC'13 functions.

Table 9-18- Wilcoxon Signed Rank Test Results of Single-Problem Analysis with a Significance Level of $\alpha = 0.05$ for CEC2013 problems

Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	−	−	=	+	=	+	+	−	=	=	+	6/5/3
30	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
50	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0
100	+	+	+	=	=	=	+	=	+	+	=	+	+	+	9/5/0

Moreover, Figure 9-23 displays the convergence rate comparison of the proposed algorithm AHPSO with the 4 top ranked algorithms on the CEC'13 test suite. In summary, the proposed algorithm AHPSO exhibit better convergence rate compared to EPSO and HCLPSO in all cases. Mostly, L-SHADE and DMS-PSO exhibit slightly better convergence compared to AHPSO however, AHPSO also exhibit comparable convergence characteristics to L-SHADE and DMS-PSO on various functions.

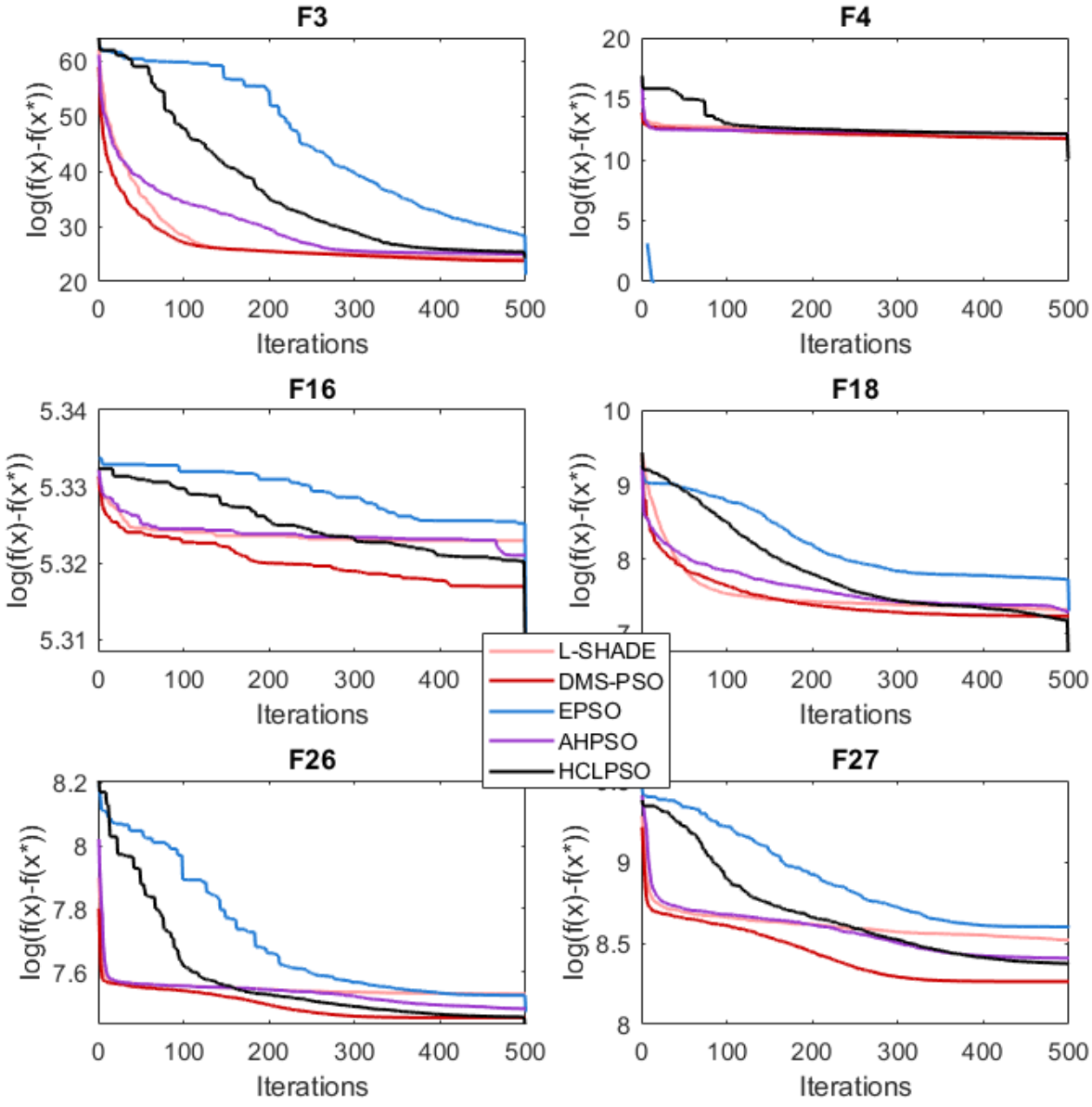


Figure 9-23 Average convergence rate comparison of AHPSO with the 4 top ranked algorithms on the 100-dimensional unimodal(F3,F4), multimodal(F16,F18) and composition(F26,F27) functions of the CEC'13 test suite.

9.6.2 Comparative Results on the CEC'14 Benchmark Problems

In this section, we present the results obtained on the CEC'14 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed AHPSO algorithm is compared with 14 algorithms on a total of 30 functions including 3 unimodal, 13 multimodal, 6 hybrid and 8 composition functions. The full details of the problem definitions for CEC'14 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

9.6.2.1 Results for 10 Dimensional Functions

The results on the 10-dimensional problems indicate that for unimodal functions, L-SHADE obtained the best mean solution for the unimodal function F1, F2 and F3. On the multimodal functions, L-SHADE and DMS-PSO exhibited the best mean performance for the same number of functions. More specifically, L-SHADE exhibited superior mean performance on F7, F8, F9, F10, F12 and F15 whereas DMS-PSO on F5, F6, F11, F13, F14 and F16. Moreover, the best solution for multimodal F4 is

provided by EPSO. Once again, on the hybrid functions, L-SHADE achieved the best mean solution for all 10-dimensional hybrid functions F17, F18, F19, F20, F21 and F22. On the composition functions, the proposed algorithm AHPSO discovered the best solutions for more functions (F24, F25 and F28) compared to rest of the algorithms. The second-best overall performance on the composition functions is attained by MaPSO where best mean solution is attained on F23 and F27. Moreover, DMS-PSO, LPSO and FIPS algorithms exhibited superior performance on a single composition function F26, F27 and F30, respectively.

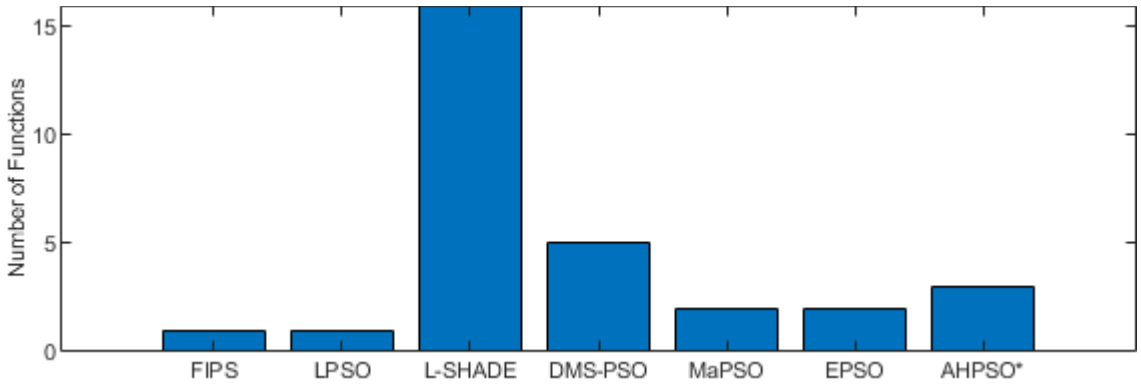


Figure 9-24 The total number of best performances achieved by each algorithm with respect to mean error values on the 10-dimensional CEC'14 problems

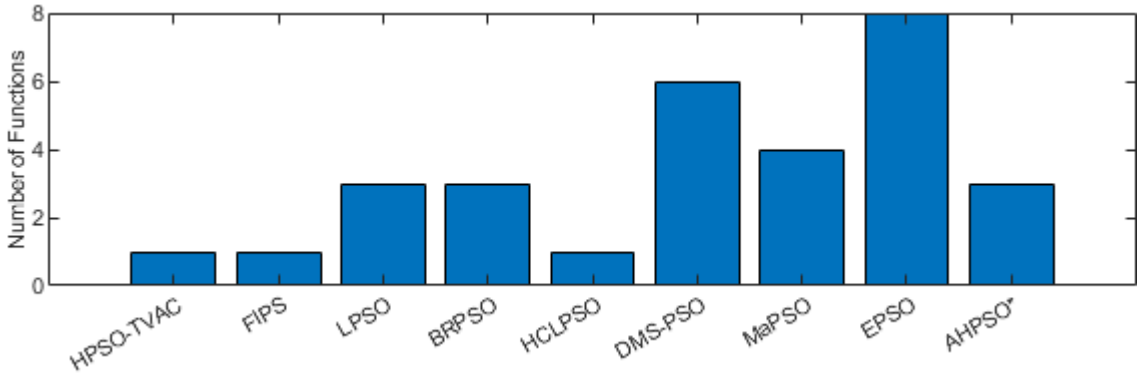


Figure 9-25 The total number of best performances achieved by PSO variants with respect to mean error values on the 10-dimensional CEC'14 problems

9.6.2.2 Results for 30 Dimensional Functions

The results obtained on the 30-dimensional CEC'14 problems suggest that on the unimodal functions indicate that L-SHADE performed best on all 3 unimodal functions F1, F2 and F3. L-SHADE also performed best on majority of the multimodal functions F4, F8, F9, F10, F11, F12, F15 and F16 while HPSO-TVAC, DMS-PSO, FIPS and the proposed algorithm AHPSO exhibited the best performance on a single function, F5, F6, F7 and F14, respectively. The results on the 10-dimensional hybrid CEC'14 test functions display that L-SHADE provides the best performance on F17, F19, F20 and F21 while DMS-PSO and MaPSO attained the best mean solution for F22 and F18, respectively. Finally, on the composition functions, the proposed algorithm AHPSO and DMS-PSO exhibited superior performance compared to rest of the test algorithms and achieved the best performance for F24, F25 and F23, F29, respectively. On the other hand, L-SHADE, LPSO, HCLPSO, and MaPSO performed best on a single composition function F26, F27, F28 and F30, respectively.

9.6.2.3 Results for 50 Dimensional Functions

The results on the 50-dimensional CEC'14 problems indicate that on the unimodal functions, L-SHADE performed best on all unimodal functions F1-F3. On the other hand, the results on the multimodal functions, once again L-SHADE attained the lowest mean value for majority of multimodal functions including F4, F5, F8, F10, F11, F12 and F16 while DMS-PSO yield the best results on F6, F12, F15 and the proposed AHPSO algorithm obtained the best performance on F9 and F14. On the other hand, HPSO-TVAC and FIPS performed best on multimodal functions F5 and F7, respectively. On the 50-dimensional hybrid functions, the performance of L-SHADE yields the best performance on F17, F18, F19 and F21 whilst DMS-PSO and EPSO obtained the best solution on F22 and F20, respectively. On the 8 composition functions, DMS-PSO performed best on F23, F27, F28 and F29 whilst the proposed algorithm AHPSO yield the best result on F24 and F25.

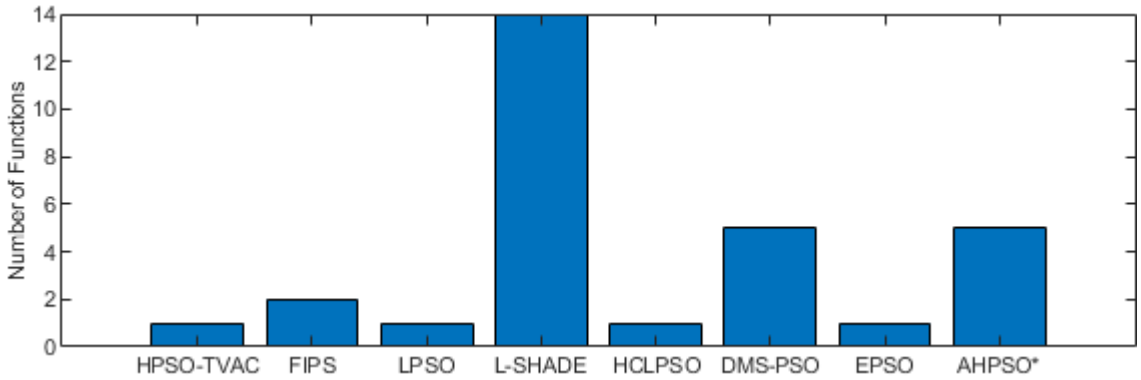


Figure 9-26 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'14 problems

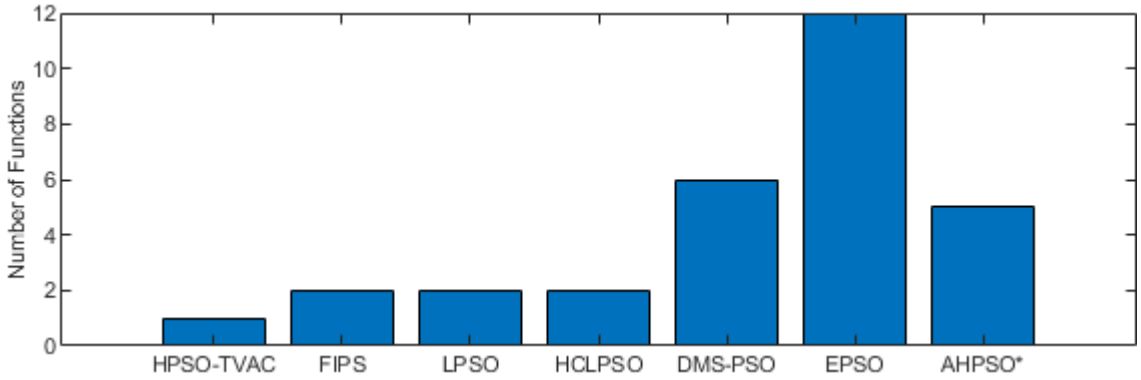


Figure 9-27 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'14 problems

9.6.2.4 Results for 100 Dimensional Functions

The mean error values obtained on the 100-dimensional CEC'14 problems indicate that on the unimodal functions, L-SHADE attained the best performance on function F1 and F2 while EPSO obtained the best mean solution on F3. On the multimodal functions, the proposed AHPSO algorithm performed best for majority of the multimodal functions, namely on F6, F9, F11, F13 and F14 while L-SHADE performed best on F10, F12 and F16. Moreover, LPSO, HCLPSO, UPSO, EPSO and DMS-PSO obtained the best mean solution for a single function F4, F5, F7, F8 and F15. The results on the hybrid function indicate that L-SHADE performed best on F17, F21 while MaPSO, DMS-PSO, EPSO and the proposed algorithm AHPSO obtained the best mean performance on a single function, namely F18, F19, F20 and F22, respectively. The best performance for the 8 composition functions

were attained by the proposed algorithm AHPSO, DMS-PSO and FIPS where AHPSO performed best on F24, F25 and F27. DMS-PSO obtained the best means solution on F23, F28, F29 and F30 while FIPS exhibit the best performance on F26.

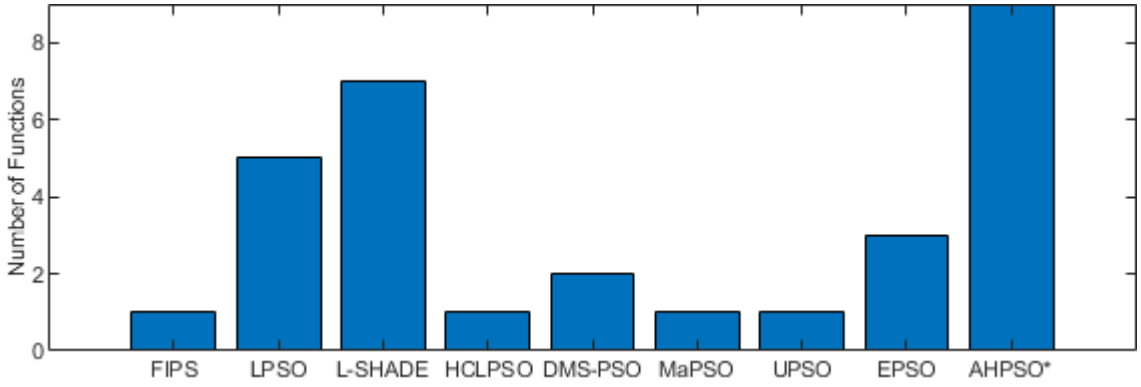


Figure 9-28 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'14 problems

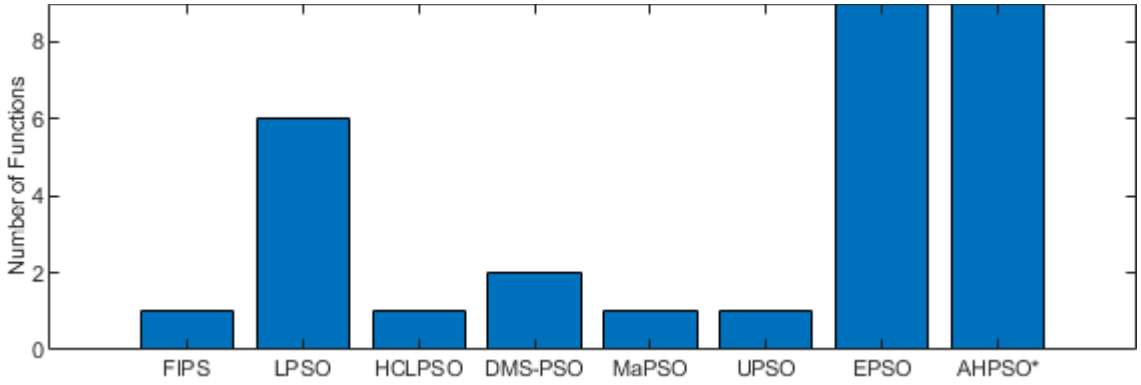


Figure 9-29 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'14 problems

Overall, the proposed algorithm AHPSO obtained the lowest mean error values for majority of the 100-dimensional problems (9 of 30) followed by L-SHADE which performed best on 7 of 30 problems. Although L-SHADE ranked superior to AHPSO on the 100-dimensinonal CEC'14 problems, AHPSO exhibit better performance on more functions compared to all 14 algorithms.

The average and final ranks obtained on the 10, 30, 50 and 100-dimensional CEC'14 test functions are shown in Appendix 2. The proposed AHPSO algorithm ranked significantly better on the 100-dimensional functions compared to low dimensional (10-d) functions. On the 10-dimensional functions, AHPSO ranked 8th, on the 30 and 50-dimensional functions, AHPSO ranked 6th and on the 100-dimensional functions, AHPSO attained the 4th rank. Once again, on the 100-dimensional functions, we observe that the average rank of AHPSO (5.6) is not vastly different to the average rank of L-SHADE (4.5). The results for the Wilcoxon signed rank test is shown in Table 9-19. We observe that the proposed algorithm AHPSO is statistically significantly better than 22 algorithms in total across 10, 30, 50 and 100 dimensions. Moreover, 8 algorithms exhibit significantly better performance compared to AHPSO and according to the Wilcoxon signed rank test, the difference in performance of 26 algorithms is insignificant compared to the AHPSO algorithm.

Table 9-19- Wilcoxon Signed Rank Test with a Significance Level of α = 0.05 for CEC2014 problems

Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	−	−	=	+	=	+	−	−	=	=	=	5/5/4
30	+	+	+	=	−	=	+	−	=	=	−	=	=	+	5/6/3
50	+	+	+	=	=	=	+	=	=	=	−	=	=	+	5/8/1
100	+	+	+	=	=	=	+	=	=	=	=	+	+	+	7/7/0

Figure 9-30 displays the convergence rate of mean solution values obtained from 30 consecutive runs for multimodal, hybrid and composition functions of 100-dimensional CEC'14 and benchmark functions. It can be observed that the proposed algorithm AHPSO consistently exhibits better convergence progress compared to EPSO and HCLPSO and in some cases, comparable and better convergence rate than DMS-PSO and L-SHADE, especially on the composition functions.

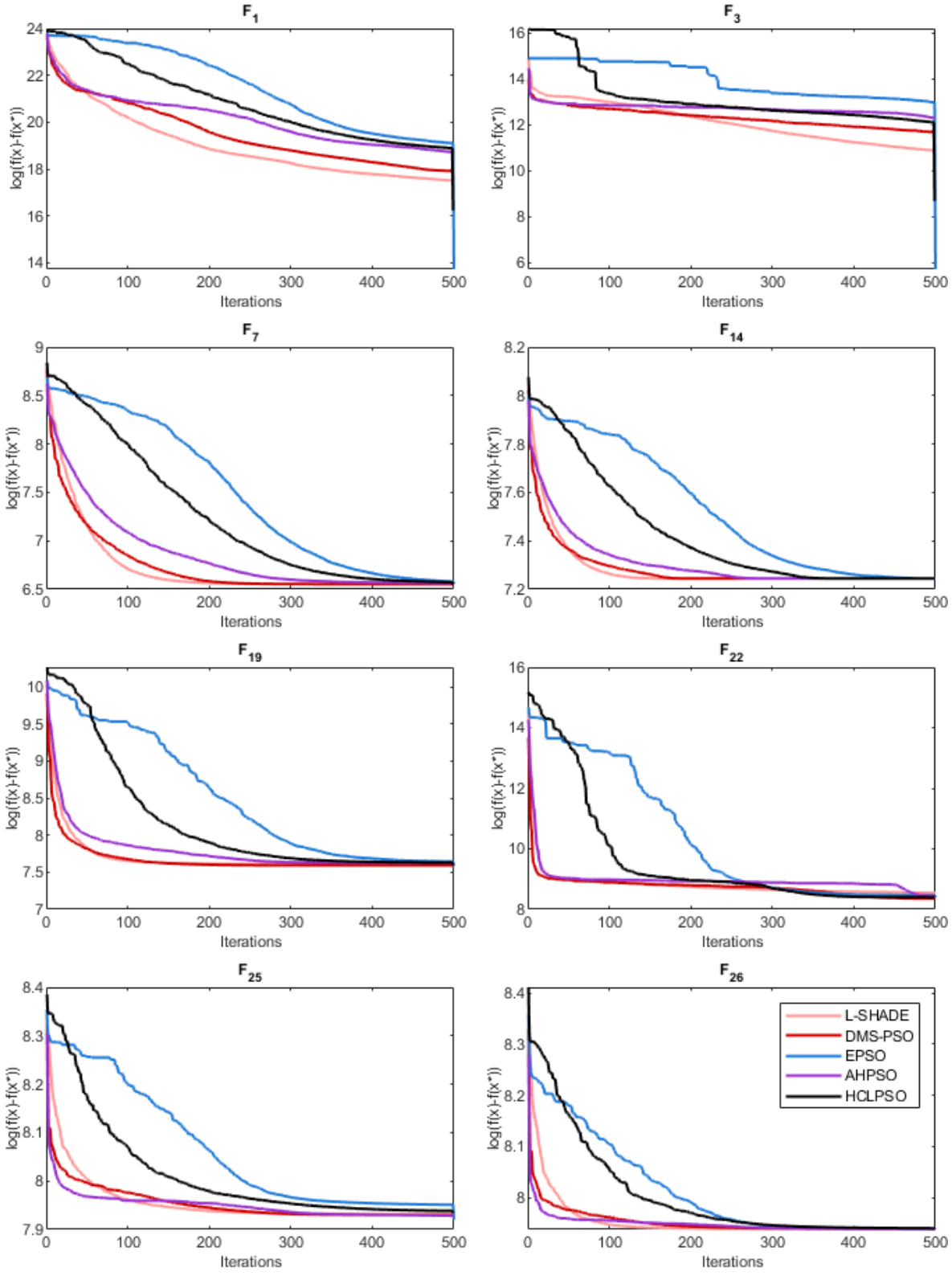


Figure 9-30 Average convergence rate comparison of AHPSO with the 4 top ranked algorithms on the 100-dimensional unimodal(F1,F3), multimodal(F7,F14), hybrid(F19,F22) and composition(F25,F26)functions of the CEC'14 test suite

9.6.3 Comparative Results on the CEC'17 Benchmark Problems

In this section, we present the results obtained on the CEC'17 benchmark test suite across 10, 30, 50 and 100-dimensions. The performance of the proposed AHPSO algorithm is compared with 14 algorithms on a total of 29 functions including 1 unimodal, 7 multimodal, 10 hybrid and 11 composition functions. The full details of the problem definitions for CEC'17 benchmark test suite is given in Appendix 1 and the experimental results are included in Appendix 2.

9.6.3.1 Results for 10 Dimensional Functions

The experimental results obtained on the 10-dimensional CEC'17 problems reveal that on the multimodal functions, L-SHADE performed best on 6 of the 7 functions including F3, F4, F5, F7, F8 and F9 whilst the lowest mean error value for F6 is obtained by EPSO. Similarly, for hybrid functions, L-SHADE attained the best performance for 9 of the 10 hybrid functions namely, F10, F11, F12, F13, F14, F15, F17, F18 and F19. On the other hand, EPSO performed best on F16. Finally, on the 10-dimensional composition functions, MiPSO exhibit the best performance for majority of composition functions including F21, F23 and F24. MaPSO (F20, F26), HCLPSO (F25, F28) and LPSO (F27, F30) performed best on two functions whilst L-SHADE provides the best mean solution on F29. Overall, L-SHADE exhibit superior performance on the 10-dimensional unimodal, multimodal and hybrid functions however the performance of L-SHADE deteriorated over composition functions hence, in this case, we observe a more diffused performance.

9.6.3.2 Results for 30 Dimensional Functions

The experimental results obtained on the 30-dimensional problems suggest that for multimodal functions, L-SHADE performed best on F5, F7 and F8 while EPSO attained the best mean performance on F3 and F4. Moreover, MiPSO and FIPS obtained the best mean solution on F6 and F9. On the hybrid functions, L-SHADE dominates rest of the algorithms and attains the lowest mean value for majority of hybrid functions including F10, F12, F13, F14, F18 and F19. The second-best overall performance on the hybrid functions is attained by DMS-PSO on F11 and F16 while MaPSO and HCLPSO obtained the best performance on a single function, namely F15 and F17, respectively. On the composition functions, DMS-PSO exhibits an overall superior performance by achieving the best mean solution for more functions compared to rest of the algorithms namely, F20, F23 and F29 while L-SHADE and LPSO obtained the best performance on F21, F24 and F22, F30, respectively. The best performance on F25, F26, F27 and F28 is achieved by FIPS, AHPSO, MiPSO and EPSO, respectively.

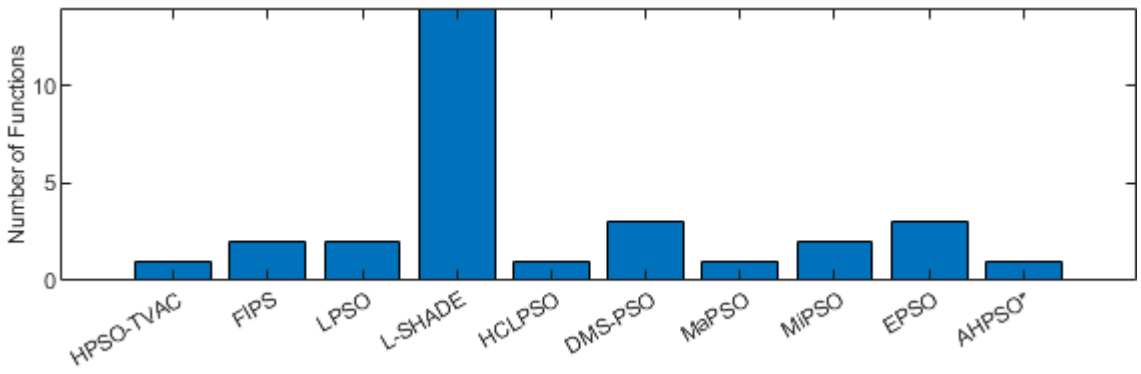


Figure 9-31 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

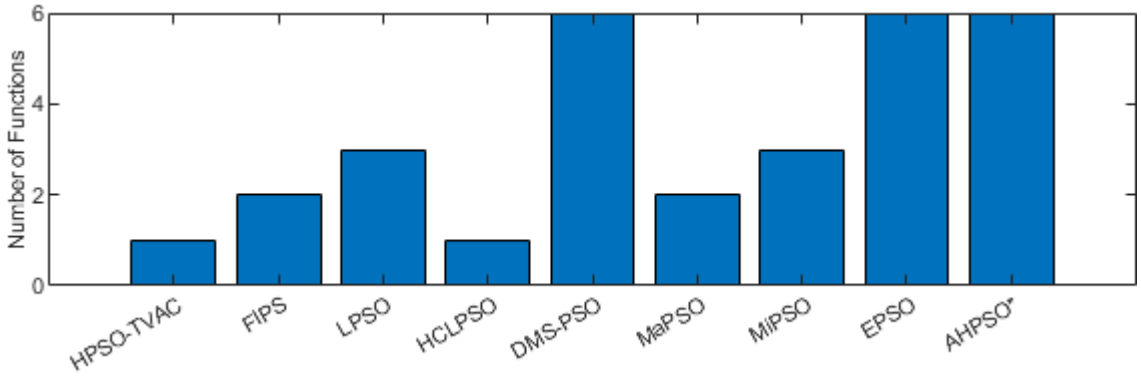


Figure 9-32 The total number of best performances achieved by PSO variants with respect to mean error values on the 30-dimensional CEC'17 problems

9.6.3.3 Results for 50 Dimensional Functions

The mean error values achieved for 50-dimensional CEC'17 problems indicate that on the multimodal functions, the proposed algorithm AHPSO, EPSO and FIPS attain the best performance on two functions where AHPSO exhibited the top performance on F5,F8, EPSO on F3,4 and FIPS on F6 and F9 while L-SHADE obtained the best solution on F7. For the hybrid functions L-SHADE attained best performance for most functions including F10, F12, F14, F15, F18 and F19 while the proposed algorithm AHPSO performed best on F16 and F17. Moreover, DMS-PSO and LPSO gained the best mean performance on F11 and F13, respectively. On the composition functions, L-SHADE and DMS-PSO exhibit comparable performance and attain the obtain the best mean solution for 3 functions, F21, F23, F24 and F20, F28, F29, respectively. The proposed algorithm AHPSO performed best on F22 and F26 while FIPS, LPSO and HPSO-TVAC attained the best mean solution on F25, F27 and F30, respectively.

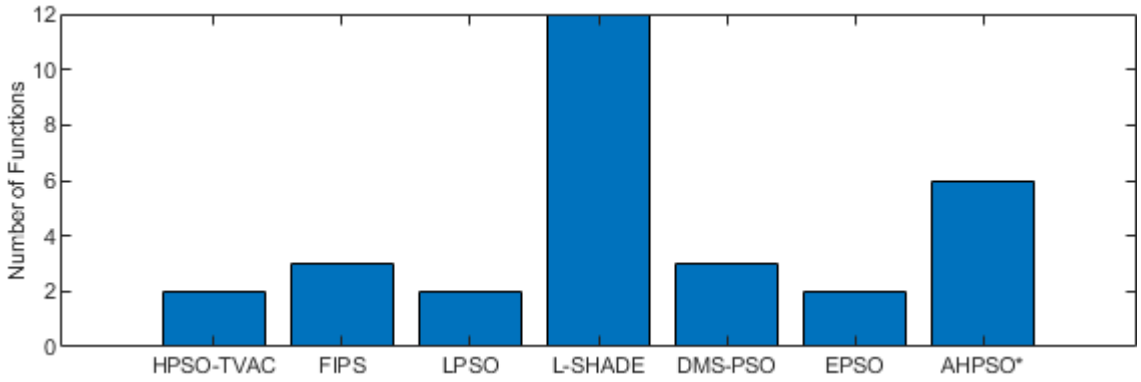


Figure 9-33 The total number of best performances achieved by each algorithm with respect to mean error values on the 50-dimensional CEC'17 problems

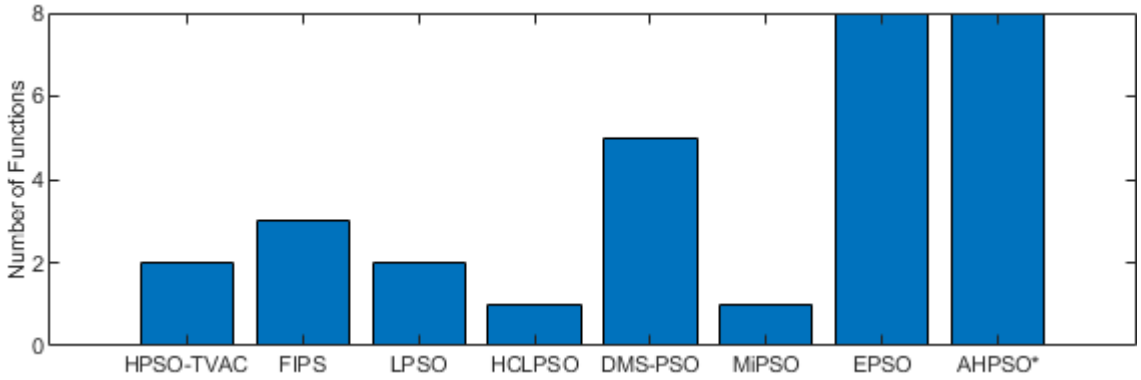


Figure 9-34 The total number of best performances achieved by PSO variants with respect to mean error values on the 50-dimensional CEC'17 problems

The top mean overall performance on majority of 50-dimensional problems is obtained by L-SHADE (12 of 29 functions) followed by the second-best overall performance of AHPSO which obtained the lowest mean error values on 6 of the 29 50-dimensional problems.

9.6.3.4 Results for 100 Dimensional Functions

The results on the 100-dimensional CEC'17 problems suggest that on the multimodal functions, the proposed algorithm attained the best performance for the majority of multimodal functions including F5, F7, F8 and F9 while EPSO performed best on F3 and F6. Moreover, the best mean solution on F4 is achieved by LPSO. The results on the hybrid functions indicate that L-SHADE outperformed rest of the algorithms for majority of the hybrid functions performed best on F10, F12, F14 and F15 while EPSO achieved the best performance on F13, F18 and F19. The proposed algorithm AHPSO outperformed rest of the test algorithms and achieved the best solution for F16 and F17 while the best performance for F11 is obtained by DMS-PSO. The results on the 100-dimensional composition functions indicate that the proposed algorithm AHPSO and DMS-PSO exhibit comparable performance and outperformed comparison algorithms by achieving the best solution for 4 functions, F20, F21, F22, F26 and F24, F25, F27, F29, respectively.

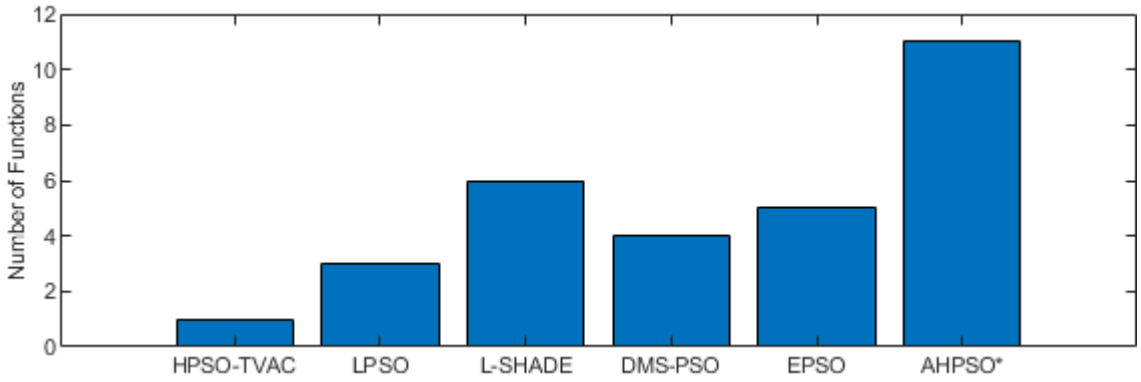


Figure 9-35 The total number of best performances achieved by each algorithm with respect to mean error values on the 100-dimensional CEC'17 problems

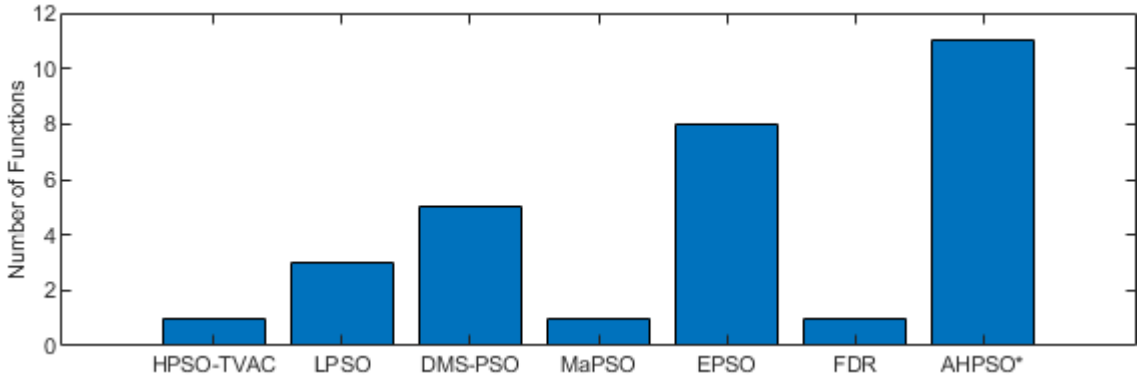


Figure 9-36 The total number of best performances achieved by PSO variants with respect to mean error values on the 100-dimensional CEC'17 problems

Overall, the proposed algorithm AHPSO exhibit the best performance on majority of 100-dimensional problems (10 of 29 functions). Once again, although L-SHADE ranked first and AHPSO ranked third in this case, AHPSO exhibit the best performance on more functions compared to all comparison algorithms (including L-SHADE).

Based on the mean and final ranks obtained on all three test suites, we observe that the rank of the proposed algorithm AHPSO consistently improves with increased problem dimensionality on all test suites. The ranks on the CEC'17 test suite indicate that the AHPSO algorithm ranks 3rd on the 50 and 100-dimensional CEC'17 functions, outranking majority of the comparison algorithms. Once again, the proposed algorithm AHPSO attains relatively close average rank compared to the 1st ranked L-SHADE (average rank of AHPSO=4.10, L-SHADE=3.50). The results of the Wilcoxon signed rank test on the 10, 30, 50 and 100-dimensional CEC'17 functions are shown in Table 9-20. The AHPSO algorithm performed statistically significantly better against 10 of the 14 comparison algorithms at 50 and 100 dimensions. On the same problem set, the difference in performance of 4 algorithms (including L-SHADE) are statistically insignificant compared to AHPSO. Further, the convergence rate comparison of AHPSO against the 4 top ranked algorithms is shown in Figure 9-37. The comparison reveals that on the 100-dimensional CEC'17 test functions, the AHPSO algorithm exhibits faster convergence compared to EPSO and HCLPSO in most cases.

Table 9-20 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for CEC2017 Problems

Dimension	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO	+/=/−
10	+	+	+	−	−	=	=	−	+	−	−	−	−	=	4/3/7
30	+	+	+	=	=	+	+	=	+	=	−	+	=	+	8/5/1
50	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0
100	+	+	+	=	=	+	+	+	+	=	=	+	+	+	10/4/0

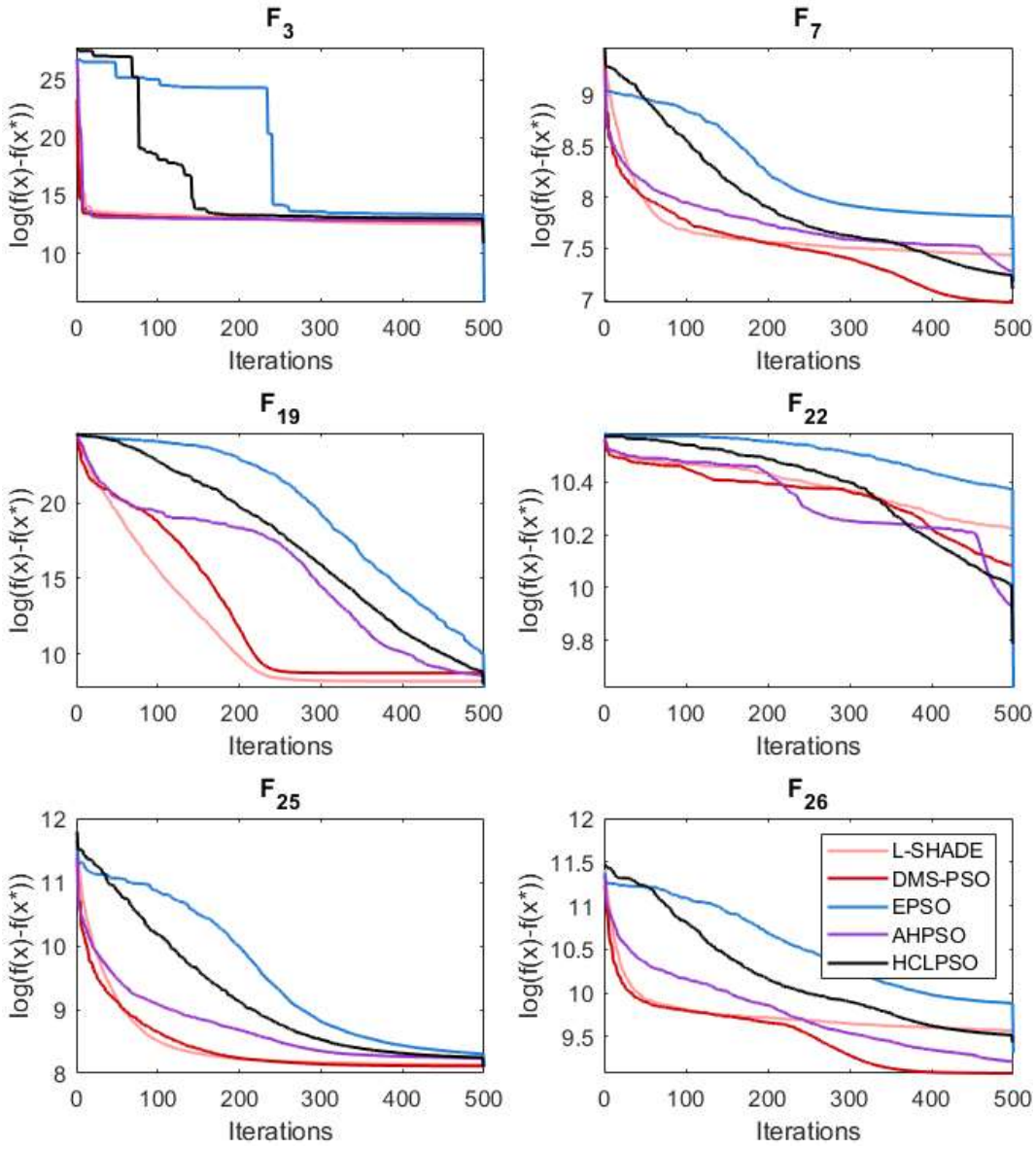


Figure 9-37 Average convergence rate comparison of AHPSO with the 4 top ranked algorithms on the 100-dimensional multimodal(F3,F7), hybrid(F19,F22) and composition(F25,F26)functions of the CEC'17 test suite

In addition, we individually compare the total number of functions each comparison algorithm obtained against the proposed algorithm AHPSO on the mean error values. The pairwise performance comparison on the obtained mean error values indicate that the proposed AHPSO algorithm outperformed majority of the comparison algorithms on the 10,30,50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test functions. As shown in Figure 9-38:9-40, we observe significantly better performance with higher dimensional functions, a clear example of this can be observed on the 100-dimensional CEC'17 functions where the AHPSO algorithm outperformed all comparison algorithms in attaining the best mean solutions for majority of the functions.

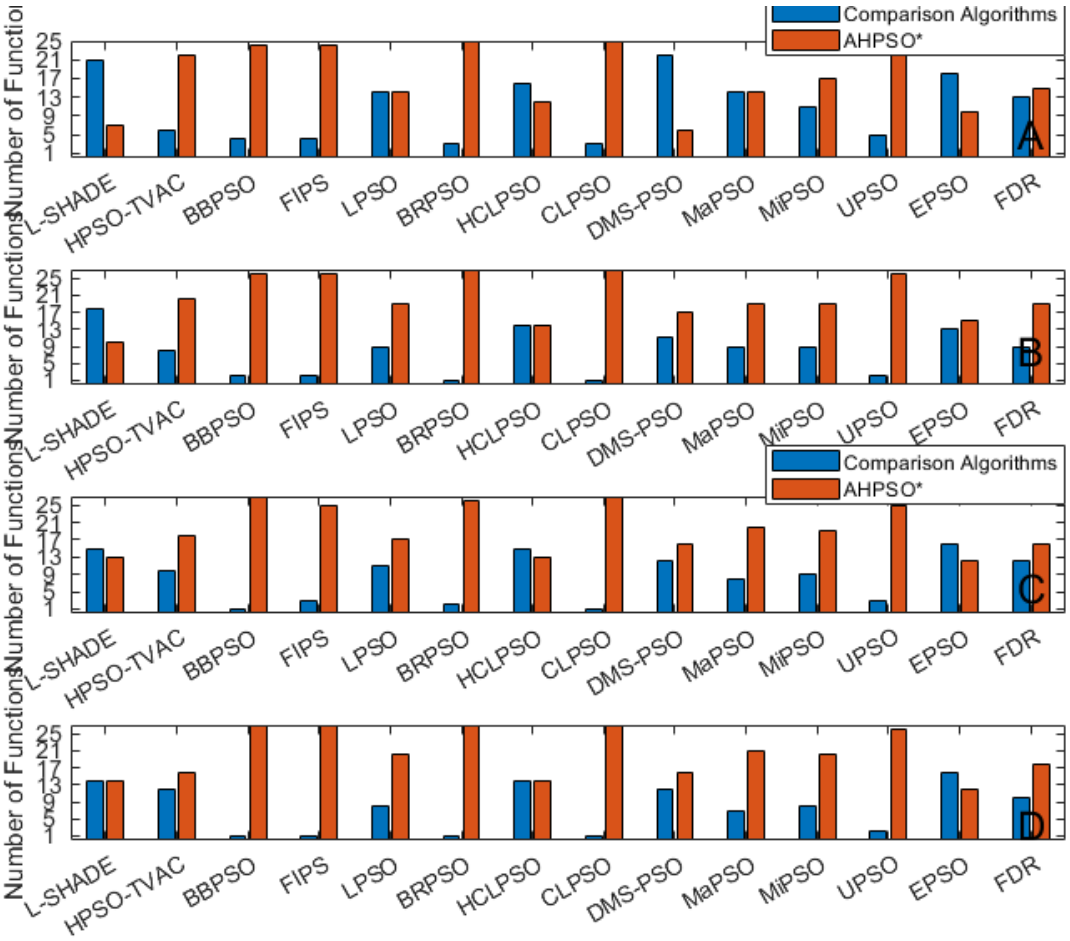


Figure 9-38-Pairwise mean performance comparison of AHPSO vs. 14 test algorithms on the 10(A), 30(B), 50(C) and 100(D) dimensional CEC'13 problems

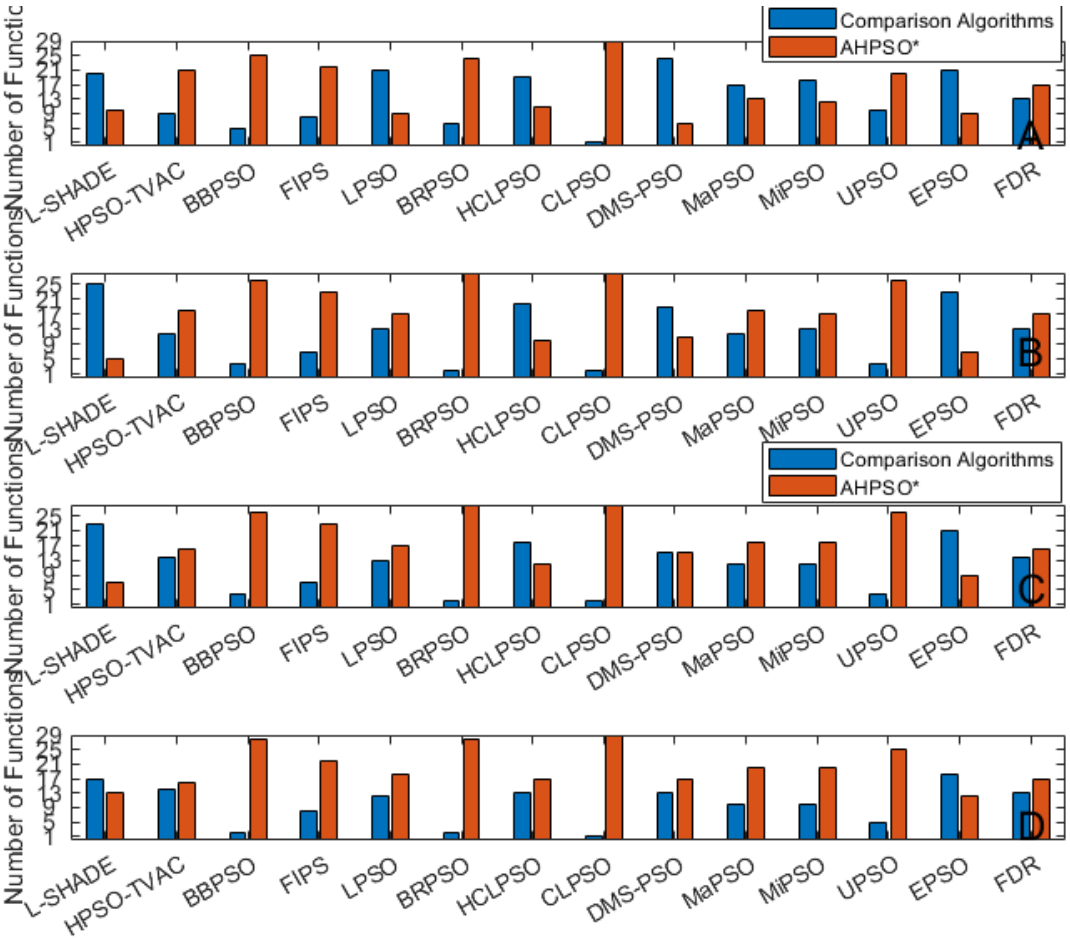


Figure 9-39- Pairwise mean performance comparison of AHPSO vs. 14 test algorithms on the 10(A), 30(B), 50(C) and 100(D) dimensional CEC'14 problems

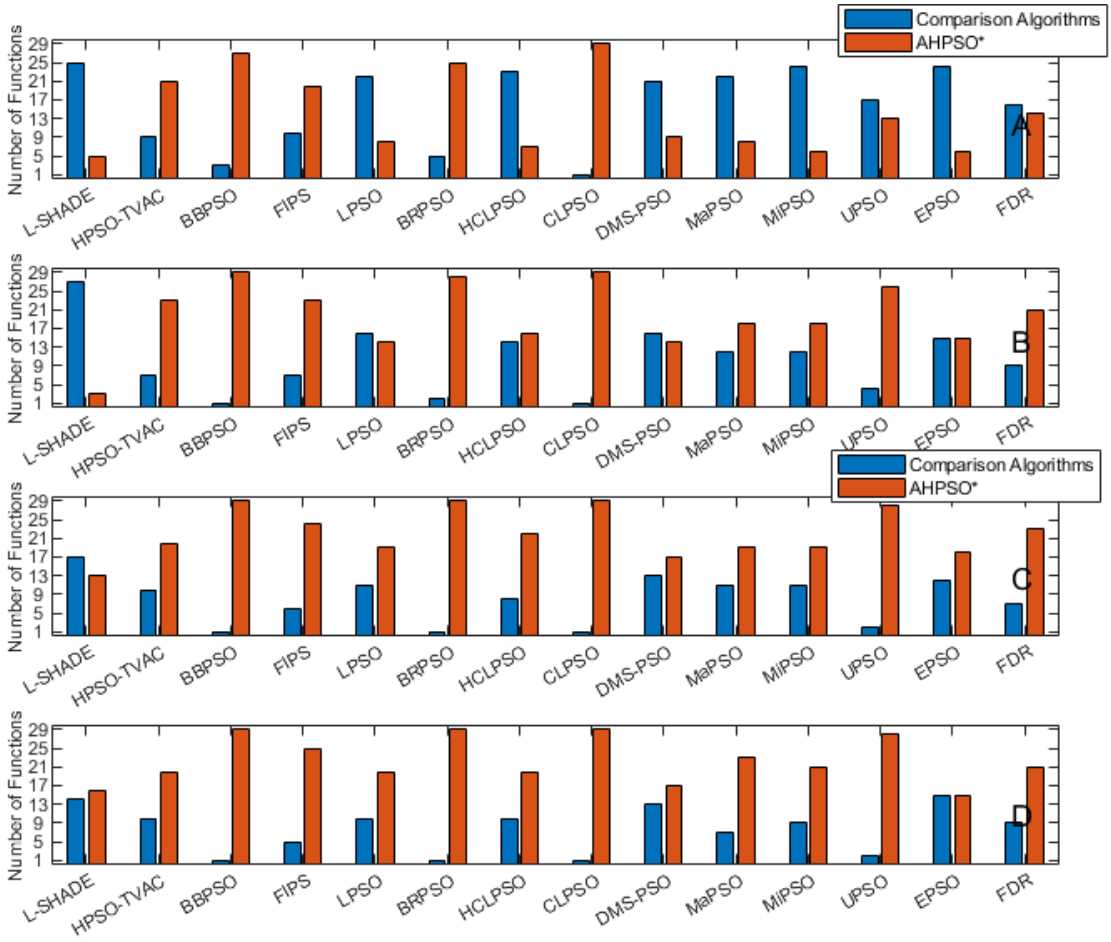


Figure 9-40- Pairwise mean performance comparison of AHPSO vs. 14 test algorithms on the 10(A), 30(B), 50(C) and 100(D) dimensional CEC'17 problems

9.7 Discussions

The performance evaluation of the proposed AHPSO algorithm was conducted on the 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 test suites. The problems considered to verify the performance of AHPSO comprised 88 benchmark problems that consist of different types of functions including unimodal, multimodal, hybrid, composition, low and high dimensional functions. The performance of the AHPSO algorithm was compared with 13 well-known PSO variants plus the CEC'14 competition winner L-SHADE.

Overall, on the 10-dimensional functions, the proposed algorithm AHPSO performed best on 4 functions and second-best on 9 functions. On the 30-dimensional problems AHPSO performed best on 8 problems and obtained the second-best in 19 cases. The performance on the overall 50-dimensional functions indicates that AHPSO obtained the best performance on 19, on the other hand, for 100-dimensinoal functions, AHPSO performed best on 29 functions and attained the second-best performance for 8 problems. Once again, this corroborates that AHPSO particularly exhibits a very good performance towards higher dimensional functions.

The performance of the AHPSO over the comparison algorithm is also validated using the Wilcoxon signed rank test which revealed that AHPSO is significantly better than most comparison algorithms in high dimensional (50-d and 100-d) functions. Moreover, the convergence rate comparison revealed that AHPSO exhibits better, or comparable convergence trend to the top

ranked algorithms including L-SHADE in most cases. The pairwise mean performance comparison revealed that AHPSO obtained lower mean error values for the majority of functions compared to most of the test algorithm employed in the experiments. Once again, a more distinguished performance is observed on the higher dimensional problems.

The AHPSO algorithm's general performance on the low dimensional functions is observed to be insufficient when compared to its superior performance on the high dimensional functions. On different function types, for example, on the 100-dimensional CEC'13 multimodal functions, AHPSO attained the lowest mean error for more functions compared all PSO variants (13 algorithms) (except L-SHADE), and on the composition functions, once again, AHPSO obtained the lowest mean error for majority of the composition functions compared to all comparison algorithms. We also observe that the proposed algorithm AHPSO exhibits similarly superior performance on the 100-dimensional CEC'14 and CEC'17 multimodal, hybrid and composition problems.

We have identified two primary limitations on the performance of the AHPSO algorithm. The low dimensional (e.g., 10-d) functions are easier to solve for most contemporary algorithms as lower dimensionality tend to correlate with lesser complexity. Hence, when compared with many competitive algorithms, as previously mentioned, the AHPSO algorithm's performance does not stand out on the low dimensional problems. (See the final chapter of this thesis for further discussion and explanation of this point). However, we clearly observe an opposite effect on multiple test suites with more complex high dimensional functions. The second limitation is the performance of AHPSO on the unimodal functions. We consistently observe that AHPSO is unable to exhibit the expected performance on the unimodal functions compared to some of its peers despite the employment of different parametric settings. However, this can be justified by the No Free Lunch Theorem (NFLT), which states that no single algorithm can be expected to be superior to another algorithm on all types of problems. Hence, it is theoretically impossible for an algorithm to solve all types of optimisation problems.

9.8 Conclusion

In this chapter, the altruistic heterogenous PSO algorithm is presented. The evolutionary mechanism of altruism inspires the AHPSO algorithm's behaviour. AHPSO introduces two behavioural models: the altruistic particle model and the paired particle model (PPM). The first model comprises the primarily adopted movement strategies whilst the PPM is used to prevent the loss of population diversity to avoid premature convergence. The effectiveness of the PPM mechanism has been verified over different experiments, and the evidence corroborates that the loss of population diversity is prevented significantly whilst the algorithm converges equally well and even faster in some cases. The performance of the AHPSO algorithm has been verified by conducting experiments on multiple test suites and various dimensions. The results indicated that the proposed algorithm AHPSO outperformed the majority of the comparison algorithms.

Chapter 10 : Performance Comparison and Analysis of the Proposed Algorithms

In this chapter, we compare the performance of the six proposed algorithms AHPSO, BEPSO, HIDMS-PSO, FFQ-HIDMS-PSO, A-HIDMS-PSO and GA-HIDMS-PSO on three test benchmark test suites, namely, CEC'13, CEC'14 and CEC'17 at 10-D, 30-D, 50-D and 100-D. The results are ranked based on the mean performance and analysis was conducted to identify statistical significance among the mean performance of the proposed algorithms. Subsequently, we compare the convergence behaviour of the six algorithms on various dimensions of the three test suite problems.

10.1 Comparative Results on the CEC'13 Benchmark Problems

10.1.1 Results on 10-dimensional problems

The results for the 10-dimensional unimodal functions (F1~F5) indicate that AHPSO and BEPSO obtained the best mean error values for 2 functions F2,F5 and F1,F4, respectively whilst HIDMS-PSO attained the best mean performance for F3. On the multimodal functions, HIDMS-PSO exhibit better performance for majority of the functions and achieved the lowest mean error values on F6, F8, F12, F15, F16 and F18 followed by the second-best overall mean performance of GA-HIDMS-PSO which performed best on F11, F14, F17 and F19. Once again, AHPSO and BEPSO solved 2 functions F9,F20 and F10,F13 whilst FFQ-HIDMS-PSO obtained the best performance solely for F7. On the 10-dimensional composition functions only BEPSO performed best on 3 functions including F21, F23 and F26 whilst rest of the algorithms only solved a single function, namely AHPSO (F25), HIDMS-PSO (F27), A-HIDMS-PSO (F28), FFQ-HIDMS-PSO (F24) and GA-HIDMS-PSO (F22).

10.1.2 Results on 30 dimensional problems

The comparison on the 30-dimensional functions reveals that for unimodal F1~F5, GA-HIDMS-PSO performed best on F3, F4, AHPSO, BEPSO and A-HIDMS-PSO attained the lowest mean values on F5, F1 and F2, respectively. On the multimodal functions F6~F20, we observe that the AHPSO algorithm solved more multimodal functions compared to rest of the algorithms including F9, F17, F18 and F19 whilst BEPSO performed best on F12, F13 and F15. Further, A-HIDMS-PSO (F6, F16), FFQ-HIDMS-PSO (F7, F20) and GA-HIDMS-PSO (F11, F14) achieved the best mean performance on 2 functions. On the composition functions (F21, F28), HIDMS-PSO exhibited better mean performance on more functions compared to rest of the algorithms including F24, F25, F26 and F27 whilst AHPSO, BEPSO, A-HIDMS-PSO and GA-HIDMS-PSO performed best on a single function, namely F28, F23, F21 and F22, respectively.

10.1.3 Results on 50 dimensional problems

The comparison on the 50-dimensional CEC'13 problems indicate that for unimodal F1~F5, the BEPSO algorithm and GA-HIDMS-PSO performed best on two functions F1, F5 and F2, F3, respectively whilst HIDMS-PSO achieved the lowest mean error for F4. The results on the multimodal functions F6~F20 yields that HIDMS-PSO exhibited better mean performance on more multimodal functions including F6, F9, F10, F16 and F20 whilst AHPSO (F17, F18, F19) and FFQ-HIDMS-PSO (F7, F8, F11) performed best on 3 functions. Further, BEPSO and GA-HIDMS-PSO attained the best mean performance on F12, F13 and F14, F15, respectively. The performances on the composition functions (F21~F28) reveal that BEPSO (F23, F28), HIDMS-PSO (F24, F27) and GA-HIDMS-PSO (F22, F25) performed best on 2 functions whilst A-HIDMS-PSO (F21) and FFQ-HIDMS-PSO (F26) attained the lowest mean error on a single function.

10.1.4 Results on 100 dimensional problems

The results for the six algorithms on the 100-dimensional CEC'13 problems reveal that for unimodal F1~F5, BEPSO and A-HIDMS-PSO performed best on two functions namely, F1, F5, and F2, F4, respectively, whilst GA-HIDMS-PSO attained the lowest mean error on F3. On the multimodal functions (F6~F20), AHPSO (F17, F18, F19, 20) and HIDMS-PSO (F6, F10, F15, F16) exhibited the best performance on 3 functions whereas BEPSO (F12, F13), FFQ-HIDMS-PSO (F7, F11) and GA-HIDMS-PSO (F8, F14) solved 2 functions. Furthermore, the results on the composition functions yield that HIDMS-PSO exhibited the best performance on F24, F25, F27, F28 and GA-HIDMS-PSO solved F21, F22, F23 whilst A-HIDMS-PSO achieved the best mean performance on F26.

Table 10-1 presents the final ranks calculated based on the mean error values obtained on the CEC'13 problems. In summary, the final ranks yield that HIDMS-PSO and GA-HIDMS-PSO maintain the 1st and 2nd rank consistently across all dimensions investigated, hence on average, both algorithms outperformed rest of the algorithms on the CEC'13 benchmark problems.

Table 10-1 The Ranks of the Proposed Algorithms for CEC2013 problems

Algorithm / Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO
10-d	3	2	1	5	4	2
30-d	4	3	2	6	5	1
50-d	6	4	1	5	3	2
100-d	4	5	1	6	3	2

Wilcoxon signed rank test is conducted to determine significant difference in the mean performance of the algorithms under consideration. In summary, we observe significance in performance on high dimensional (100-d) problems and on the contrary, in most cases the performances in insignificant for the CEC'13 benchmark problems. The details of the statistical test are displayed in Table 10-2:10-4 for each algorithm.

Table 10-2 Wilcoxon Signed Rank Test Results for AHPSO and BEPSO vs Other Algorithms with a Significance Level of $\alpha = 0.05$ for CEC2013 problems

AHPSO vs.						
Dimension	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	−	=	=	=	=	0/4/1
30	=	=	=	=	=	0/5/0
50	=	=	−	=	=	0/4/1
100	=	=	−	=	−	0/3/2

BEPSO vs.					
AHPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
+	=	=	=	=	1/4/0
=	=	=	=	=	0/5/0
=	=	−	=	=	0/4/1
=	=	=	=	=	0/5/0

Table 10-3 Wilcoxon Signed Rank Test Results for HIDMS-PSO and FFQ-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2013 problems

HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	=	=	=	=	0/5/0
30	=	=	=	=	=	0/5/0
50	=	=	−	=	=	0/4/1
100	=	=	=	=	−	0/4/1

FFQ-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
+	+	+	+	=	4/1/0
+	=	=	+	=	2/3/0

Table 10-4 Wilcoxon Signed Rank Test Results for A-HIDMS-PSO and GA-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2013 problems

A-HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	=	=	=	=	0/5/0
30	=	=	=	=	=	0/5/0
50	=	=	=	−	=	0/4/1
100	=	=	=	−	−	0/3/2

GA-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
+	=	+	=	+	3/2/0

Figure 10-1 displays the convergence rate comparison among the proposed 6 algorithms on 100-dimensional unimodal, multimodal and composition functions from the CEC'13 test suite. We can observe that GA-HIDMS-PSO consistently discovers better solution and converges faster within the initial stages of the search and however in most cases A-HIDMS-PSO and HIDMS-PSO converges to better solution compared to GA-HIDMS-PSO. In addition, we also observe an emergent pattern in the convergence behaviour where in most cases, AHPSO, BEPSO, HIDMS-PSO, A-HIDMS-PSO and FFQ-HIDMS-PSO exhibit comparable convergence trend approximately for half of the search process (see F11, F12, F14, F22, F23). Subsequently, once again, in most cases, we observe HIDMS-PSO and A-HIDMS-PSO branching out and converging with similar pace and pattern whilst AHPSO, BEPSO and FFQ-HIDMS-PSO exhibit comparable convergence characteristics which tends to be slower and delayed, that mostly ensues towards the end of the search process. However, this is the outcome of the additional mechanisms employed by AHPSO and FFQ-HIDMS-PSO for the prevention of depletion of population diversity.

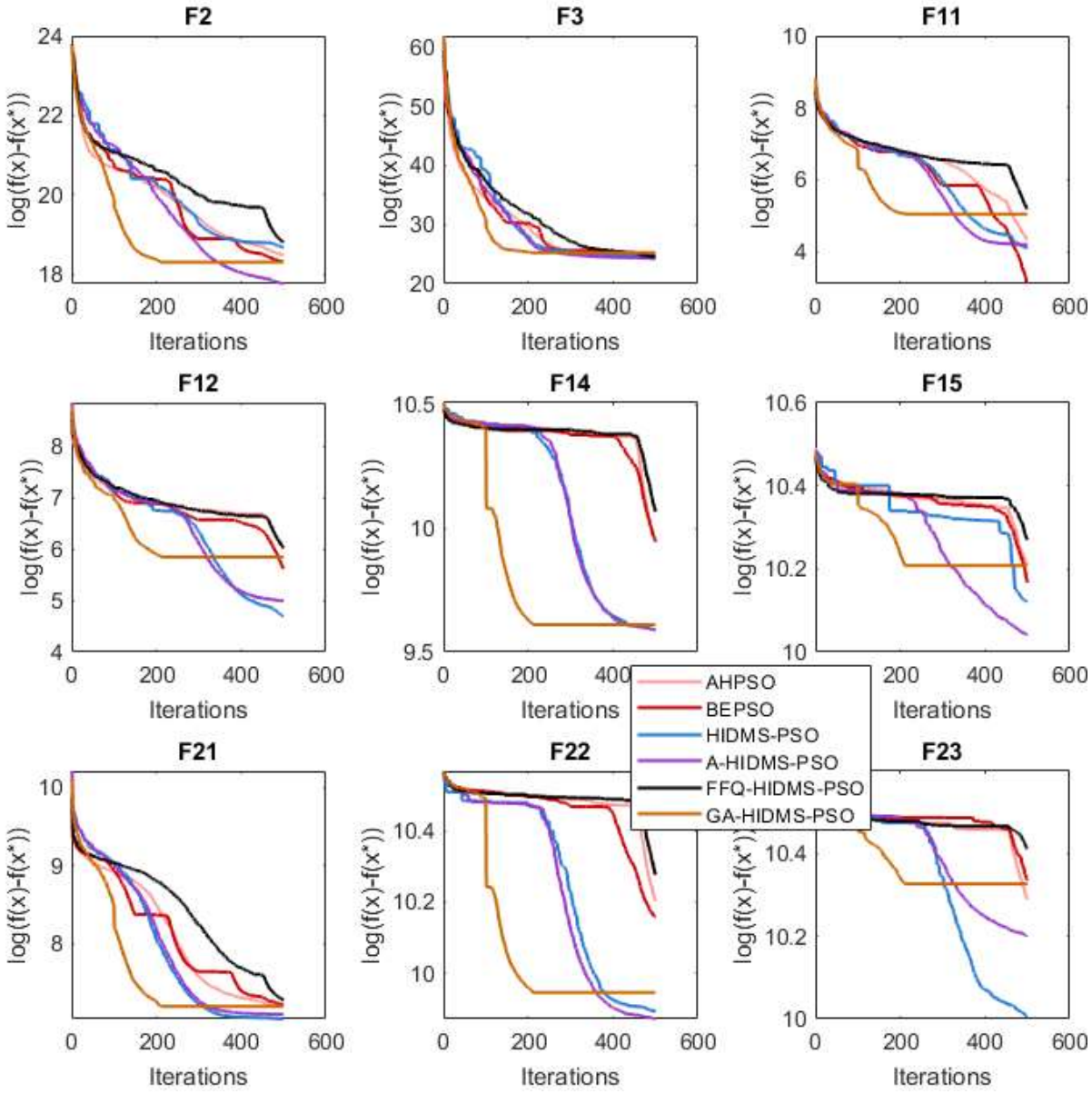


Figure 10-1 Convergence rate comparison of the six proposed algorithms on 100-dimensional CEC'13 Unimodal F2,F3, multimodal F11, F12, F14, F15, and composition F21, F22, F23.

10.2 Comparative Results on the CEC'14 Benchmark Problems

10.2.1 Results on 10 dimensional problems

The results for the 10-dimensional CEC'14 problems indicate that for the unimodal functions (F1~F3), A-HIDMS-PSO performed best on F1, F2 and AHPSO achieved the lowest mean error on F3. On the multimodal functions (F4~F16), GA-HIDMS-PSO attained better mean performance for most of the multimodal functions including F5, F8, F10, F12, F15 and F16 whilst BEPSO exhibited the best mean performance on F7, F9, F11 and F13. On the other hand, for F4 and F6, HIDMS-PSO showed the best mean performance, and AHPSO performed best on F14. On the hybrid functions (F17~F22), once again, GA-HIDMS-PSO exhibited better mean performance for most of the functions including F19,

F20, F21 and F22 whilst A-HIDMS-PSO and AHPSO performed best individually on F17 and F18, respectively. The results on the 8 composition functions (F23~F30), the HIDMS-PSO algorithm exhibit superior mean performance compared to rest of the algorithms and attained better mean performance on F24, F26, F27 and F29 whilst BEPSO, FFQ-HIDMS-PSO, AHPSO and GA-HIDMS-PSO individually achieved the lowest mean error value for F23, F25, F28 and F30, respectively.

10.2.2 Results on 30 dimensional problems

The results obtained on the 30-dimensional CEC'14 problems indicate that the best mean performance for unimodal F1, F2 and F3 are attained by HIDMS-PSO, BEPSO and A-HIDMS-PSO, respectively. The lowest mean error values for majority of the multimodal functions are obtained by GA-HIDMS-PSO including F8, F10, F11, F12 and F16 whilst HIDMS-PSO performed best on F5, F14 and F15. On the other hand, BEPSO and A-HIDMS-PSO performed best on two functions namely, F9, F13 and F4, F6, respectively and the FFQ-HIDMS-PSO exhibited superior mean performance on F7. Once again, GA-HIDMS-PSO exhibited superior mean performance on most of the hybrid functions including F20, F21 and F22 whilst the best mean performance for the remaining F17, F18 and F19 are achieved by BEPSO, A-HIDMS-PSO and FFQ-HIDMS-PSO, respectively. The experimental results on the composition functions yield that HIDMS-SPO and AHPSO solved two functions F28, F29 and F24, F25, respectively whilst the lowest mean error values for F23, F26, F27 and F30 are achieved individually by BEPSO, A-HIDMS-PSO, GA-HIDMS-PSO and FFQ-HIDMS-PSO, respectively.

10.2.3 Results on 50 dimensional problems

The mean error values obtained for the 50-dimensional CEC'14 problems suggest that on the unimodal functions, the best mean performances are exhibited by A-HIDMS-PSO (F1), AHPSO (F2) and HIDMS-PSO (F3). We observe that HIDMS-PSO performed best on majority of the multimodal functions including F4~F7, F11, F14 and F15 whilst FFQ-HIDMS-PSO (F8, F16) and GA-HIDMS-PSO (F10, F12) solved two of the multimodal functions. Further, the lowest mean error values for F9 and F13 are obtained by BEPSO and AHPSO, respectively. On the hybrid functions, the results indicate that A-HIDMS-PSO (F17, F18) and GA-HIDMS-PSO (F20, F21) obtained the lowest mean error values on two functions. Moreover, the best mean performances on F19 and F22 are achieved by FFQ-HIDMS-PSO and HIDMS-PSO, respectively. Furthermore, HIDMS-PSO exhibited the best performance on most of the composition functions including F27, F28 and F29 whilst AHPSO (F24, F25) and A-HIDMS-PSO (F23, F26) solved two functions and GA-HIDMS-PSO exhibited the best mean performance solely on F30.

10.2.4 Results on 100 dimensional problems

The experimental results on the 100-dimensional CEC'14 problems reveal that for unimodal F1, F3, the A-HIDMS-PSO algorithm performed best, and for F2, the best mean performance is exhibited by FFQ-HIDMS-PSO. On the multimodal functions, the HIDMS-PSO algorithm obtained better mean errors for most functions including F5, F12, F14 and F15 followed by the performance of FFQ-HIDMS-PSO on F6, F8 and F16. Moreover, AHPSO exhibited the top mean performance on F11 and F13 whilst the lowest mean error values for F9 and F7 are attained by BEPSO and A-HIDMS-PSO, respectively. The HIDMS-PSO and A-HIDMS-PSO algorithm showed the best mean performance for two hybrid functions, F19, F22 and F17, F21, respectively whilst FFQ-HIDMS-PSO and GA-HIDMS-PSO exhibited the best performance individually on F18 and F20, respectively. The results on the composition functions yield that the AHPSO algorithm appears to attain the best mean performance for most of the functions including F24, F25 and F26 whilst A-HIDMS-PSO solved F23 and F30. Moreover, the lowest mean error values for F27, F28 and F29 are achieved individually by HIDMS-PSO, GA-HIDMS-PSO and FFQ-HIDMS-PSO, respectively.

The proposed algorithms are ranked according to their mean performances and the final ranks are presented in Table 10-5. In summary, for 10 and 30-dimensinal CEC'14 problems, GA-HIDMS-PSO algorithm obtained the 1st rank among the proposed algorithms. For 50 and 100-dimensional CEC'14 problems, HDIMS-PSO and A-HIDMS-PSO exhibited comparable mean performances and attained the top ranks. Further, for the CEC'14 problems, we have conducted statistical analysis to identify the significance in the mean performance between the proposed algorithms. The results of the Wilcoxon signed rank test are displayed in Table 10-6:10-8. Typically, for lower dimensional problems (10-d, 30-d), in most cases we observe no significant difference in the performance of the algorithms. In summary, HIDMS-PSO's performance is significantly better than AHPSO, BEPSO and FFQ-HIDMS-PSO at 50 and 100 dimensions. Similarly, A-HIDMS-PSO exhibited statistically significantly superior performance compared to AHPSO and BEPSO for 100-dimensional CEC'14 problems and finally, for 10-dimensional problems, the performance of GA-HIDMS-PSO is significantly better than AHPSO. For 30-dimensional CEC'14 problems, GA-HIDMS-PSO exhibited statistically significantly superior mean performance compared to AHPSO, FFQ-HIDMS-PSO and A-HIDMS-PSO.

Table 10-5 The Ranks of the Proposed Algorithms for CEC2014 Problems

Algorithm / Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO
10-d	6	4	2	5	3	1
30-d	5	3	3	4	2	1
50-d	5	4	1	4	2	3
100-d	5	6	2	4	1	3

Table 10-6 Wilcoxon Signed Rank Test Results for AHPSO vs Other Algorithms with a Significance Level of $\alpha = 0.05$ for CEC2014 problems

AHPSO vs.						
Dimension	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	−	=	=	−	0/3/2
30	=	=	=	=	−	0/4/1
50	=	−	=	=	=	0/4/1
100	=	−	=	−	=	0/3/2

BEPSO vs.					
AHPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
=	−	=	=	=	0/4/1
=	−	=	−	−	0/2/3

Table 10-7 Wilcoxon Signed Rank Test Results for HIDMS-PSO and FFQ-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2014 problems

HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	+	=	=	=	=	1/4/0
30	=	=	=	=	=	0/5/0
50	+	+	+	=	=	3/2/0
100	+	+	=	=	=	2/3/0

FFQ-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	−	0/4/1
=	=	−	=	=	0/4/1
=	=	=	=	=	0/5/0

Table 10-8 Wilcoxon Signed Rank Test Results for A-HIDMS-PSO and GA-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2014 problems

A-HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	=	=	=	=	0/5/0
30	=	=	=	=	−	0/4/1
50	=	=	=	=	=	0/5/0
100	+	+	=	=	=	2/3/0

GA-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	+/=/−
+	=	=	=	=	1/4/0
+	=	=	+	+	3/2/0
=	=	=	=	=	0/5/0
=	+	=	=	=	1/4/0

Figure 10-2 exhibits the convergence rate comparison of the six proposed algorithms under consideration on various 100-dimensional unimodal, multimodal, hybrid and composition functions selected from the CEC'14 benchmark problems. The convergence characteristics of the proposed algorithms exhibit consistency with the previously experiments on the CEC'13 problems. Once again, in general, the GA-HIDMS-PSO algorithm exhibits faster initial convergence compared to all algorithms in hand. However, GA-HIDMS-PSO is unable to discover better solution in all cases compared to other algorithms. In terms of the solution quality, generally for all types of functions in the CEC'14 test suite, we consistently observe that A-HIDMS-PSO and HIDMS-PSO converges to better solutions compared to the rest of the algorithms.

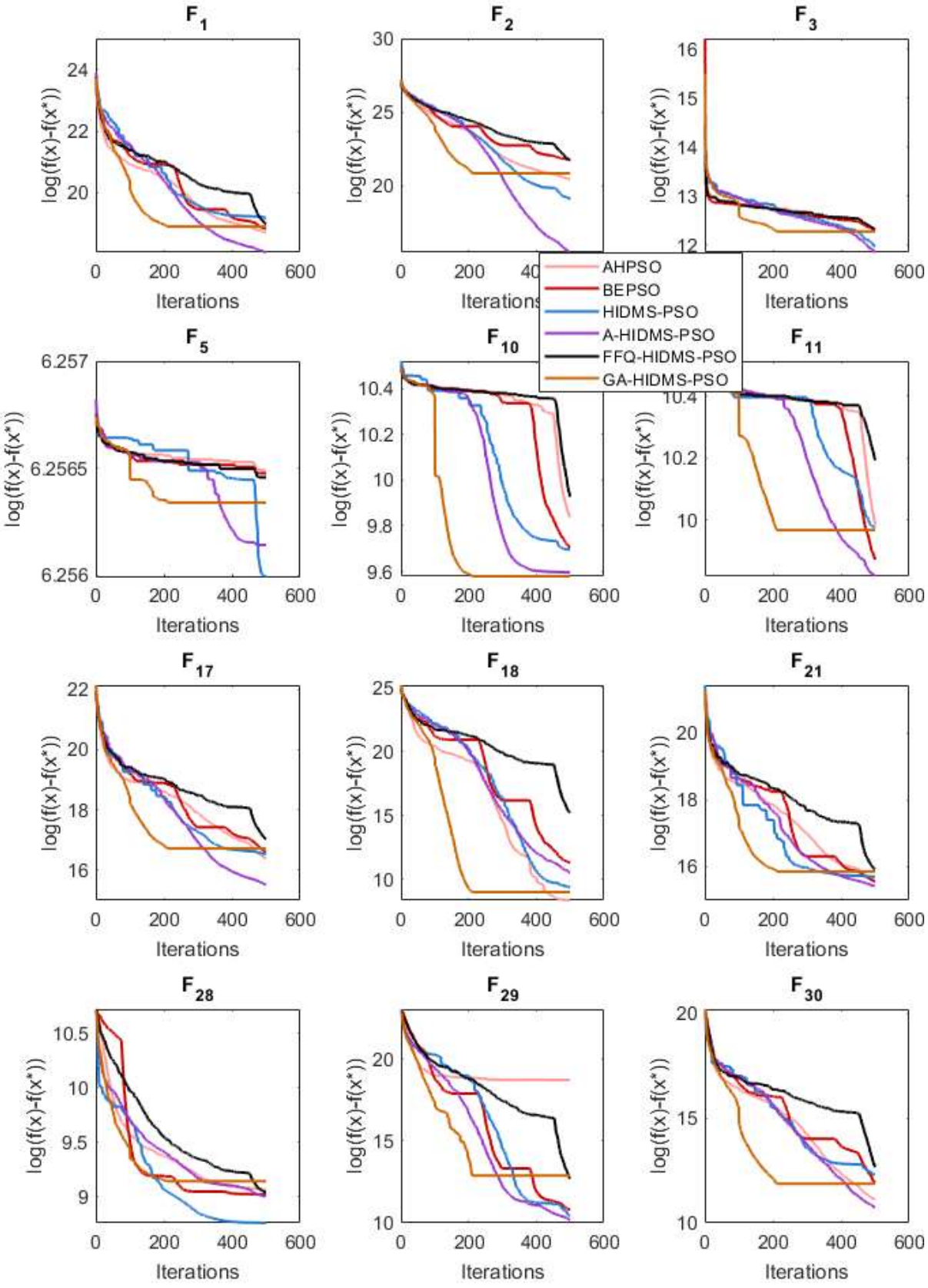


Figure 10-2 Convergence rate comparison of the six proposed algorithms on 100-dimensional CEC'14 unimodal F1 F2 F3, multimodal F5, F10, F11, hybrid F17, F18, F21, composition F28, F29, F30

10.3 Comparative Results on the CEC'17 Benchmark Problems

10.3.1 Results on 10 dimensional problems

The experimental results on the 10-dimensinal 29 CEC'17 problems show that for the unimodal F1, HIDMS-PSO exhibited the best mean performance. For the 7 multimodal functions, A-HIDMS-PSO (F3, F4), BEPSO (F5, F8) and GA-HIDMS-PSO (F6, F7) performed best on two functions whilst HIDMS-PSO obtained the lowest mean error value on F9. The AHPSO (F14, F15, F19) and BEPSO (F10, F11, F12) algorithms performed best on 3 functions whilst A-HIDMS-PSO (F13, F16) and GA-HIDMS-PSO (F17, F18) exhibited the best mean performance on 2 functions. The results on the composition functions yield that FFQ-HIDMS-PSO exhibited better mean performance for more functions compared to rest of the algorithms including F24, F25, F27 and F29 whilst A-HIDMS-PSO and GA-HIDMS-PSO achieved the best mean performances for F21, F22, F30 and F20, F23, F26, respectively. Moreover, the lowest mean error values for F8 are attained solely by the HIDMS-PSO algorithm.

10.3.2 Results on 30 dimensional problems

The mean error values obtained on the 30-dimensional CEC'17 problems show that FFQ-HIDMS-PSO exhibited the best mean performance on the only unimodal function F1. On the multimodal functions, we observe that only BEPSO (F5, F7, F8), HIDMS-PSO (F9) and A-HIDMS-PSO (F3, F4, F6) obtained the lowest mean error values for the 7 multimodal functions. For hybrid functions, the FFQ-HIDMS-PSO algorithm performed best on F11, F12 and F15 whilst BEPSO (F16, F19) and A-HIDMS-PSO (F13, F17) solved 2 functions and the lowest mean error values for F10, F14 and F18 are obtained by GA-HIDMS-PSO, HIDMS-PSO and AHPSO, respectively. The results on the composition functions indicate that the HIMS-PSO algorithm exhibited better performance on more functions compared to rest of the algorithms employed and performed best on F22, F24, F25 and F28 whilst GA-HIDMS-PSO (F20, F29), BEPSO (F21, F23) and AHPSO (F26, F30) obtained the lowest mean error value for two composition functions.

10.3.3 Results on 50 dimensional problems

The experimental results on the 50-dimensional CEC'17 test problems indicate that once again, the FFQ-HIDMS-PSO algorithm performed best on the unimodal F1. For the multimodal functions, FFQ-HIDMS-PSO and BEPSO performed best on F7,F9 and F5, F8 whilst rest of the algorithms, HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO obtained the best mean performance for a single function, F3, F4 and F6, respectively. The experimental results on the hybrid functions yield that the GA-HIDMS-PSO algorithm performed best on F10, F16, F17 and F19 whilst FFQ-HIDMS-PSO and HIDMS-PSO solved F11, F13 and F15, F18, respectively. Moreover, the lowest mean error values for F12 and F14 are obtained by BEPSO and A-HIDMS-PSO, respectively. On the composition functions, we observe that FFQ-HIDMS-PSO exhibited superior mean performance for most of the composition functions including F2, F25, F27, F28 and F30. On the other hand, BEPSO (F21, F23) and GA-HIDMS-PSO (F20, F29) performed best on two problems and the lowest mean errors for F24 and F26 are attained by HIDMS-PSO and AHPSO, respectively.

10.3.4 Results on 100 dimensional problems

The results obtained on the 100-dimensional CEC'17 problems show that the best mean performance on unimodal F1 is exhibited by GA-HIDMS-PSO. For multimodal F6, F7 and F9, the FFQ-HIDMS-PSO algorithm obtained the best performance whilst GA-HIDMS-PSO (F3, F4) and BEPSO (F5, F8) exhibited the best mean performance on two functions. The results on the hybrid functions yield that the best mean performance for most of the functions are exhibited by GA-HIDMS-PSO including

F10, F11, F12, F14 and F18 whilst FFQ-HIDMS-PSO performed best on F13, F15, F16 and F17. Moreover, the lowest mean error for F19 is attained by AHPSO. On the composition functions, we observe that AHPSO (F22, F26), BEPSO (F21, F24), HIDMS-PSO (F25, F27), FFQ-HIDMS-PSO (F23, F29) and GA-HIDMS-PSO (F20, F30) performed best on two functions whilst A-HIDMS-PSO exhibited the best mean performance for F28.

The final ranks obtained on the CEC'17 problems reveal that for 10-dimensional functions BEPSO and A-HIDMS-PSO ranked 1st and 2nd, respectively. At 30 dimensions, FFQ-HIDMS-PSO attained the 1st and HIDMS-PSO achieved the 2nd rank. For 50 and 100-dimensional problems, in the first case, the FFQ-HIDMS-PSO algorithm obtained the 1st rank and GA-HIDMS-PSO 2nd rank. In the latter case, GA-HIDMS-PSO ranked 1st and FFQ-HIDMS-PSO obtained the 2nd rank. Further details of the final ranks obtained on the CEC'17 problems are shown in Table 10-9.

Table 10-9 The Ranks of the Proposed Algorithms for CEC2017 problems

Algorithm / Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO
10-d	6	1	4	3	2	5
30-d	6	4	2	1	5	3
50-d	5	3	4	1	6	2
100-d	5	3	4	2	6	1

Further, the Wilcoxon signed rank test was conducted to determine statistical significance on the mean performance of the six proposed algorithms. The details of the Wilcoxon signed rank test are displayed in Table 10-10:10-12. From the results of the test, we determine two specific cases worth highlighting. The performance of the FFQ-HIDMS-PSO algorithm at 50 dimensions yield that it is statistically significantly better than the performance of AHPSO, BEPSO and its two variants HIDMS-PSO and A-HIDMS-PSO. In addition, on 100-dimensional CEC'17 problems, the performance of GA-HIDMS-PSO is significantly superior compared to AHPSO, HIDMS-PSO and A-HIDMS-PSO.

Table 10-10 Wilcoxon Signed Rank Test Results for AHPSO and BEPSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2017 problems

AHPSO vs.						
Dimension	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	−	=	=	=	=	0/4/1
30	=	=	=	=	=	0/5/0
50	=	=	−	=	=	0/4/1
100	=	=	−	=	−	0/3/2

BEPSO vs.						
AHPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
+	=	=	=	=	1/4/0
=	=	=	=	=	0/5/0
=	=	−	=	=	0/4/1
=	=	=	=	=	0/5/0

Table 10-11 Wilcoxon Signed Rank Test Results for HIDMS-PSO and FFQ-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2017 problems

HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	=	=	=	=	0/5/0
30	=	=	=	=	=	0/5/0
50	=	=	−	=	=	0/4/1
100	=	=	=	=	−	0/4/1

FFQ-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
+	+	+	+	=	4/1/0
+	=	=	+	=	2/3/0

Table 10-12 Wilcoxon Signed Rank Test Results for A-HIDMS-PSO and GA-HIDMS-PSO vs Other Algorithms with a Significance Level of α = 0.05 for CEC2017 problems

A-HIDMS-PSO vs.						
Dimension	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	GA-HIDMS-PSO	+/=/−
10	=	=	=	=	=	0/5/0
30	=	=	=	=	=	0/5/0
50	=	=	=	−	=	0/4/1
100	=	=	=	−	−	0/3/2

GA-HIDMS-PSO vs.					
AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	+/=/−
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
=	=	=	=	=	0/5/0
+	=	+	=	+	3/2/0

Figure 10-3 exhibits the convergence rate of the proposed algorithms on various 100-dimensional CEC'17 problems. It is not possible to distinguish a single algorithm that exhibits fast convergence to better mean solutions. However, as observed on the CEC'13 and CEC'14 problems, the AHPSO and FFQ-HIMS-PSO algorithms appear to exhibit relatively similar convergence patterns. In addition, we also observe comparable convergence pattern for A-HIDMS-PSO and HIDMS-PSO. In addition, it is also worth mentioning that the GA-HIDMS-PSO algorithm consistently exhibits faster convergence trend however as apparent, the GA-HIDMS-PSO algorithm tend to converge to inferior solutions in most cases. We presume that the issue is caused by the limited number of iterations used to record the convergence behaviour. It is highly probable that the GA-HIDMS-PSO algorithm is unable to revamp the population diversity within a short period of time to continue searching the problem space to discover better solutions hence, when executed for relatively short periods, the performance may not be as effective as expected. Thus, we presume this is also reflected on the convergence plot as it contradicts with the results of multiple experiments conducted on different test suites that yield the superior performance of GA-HIDMS-PSO.

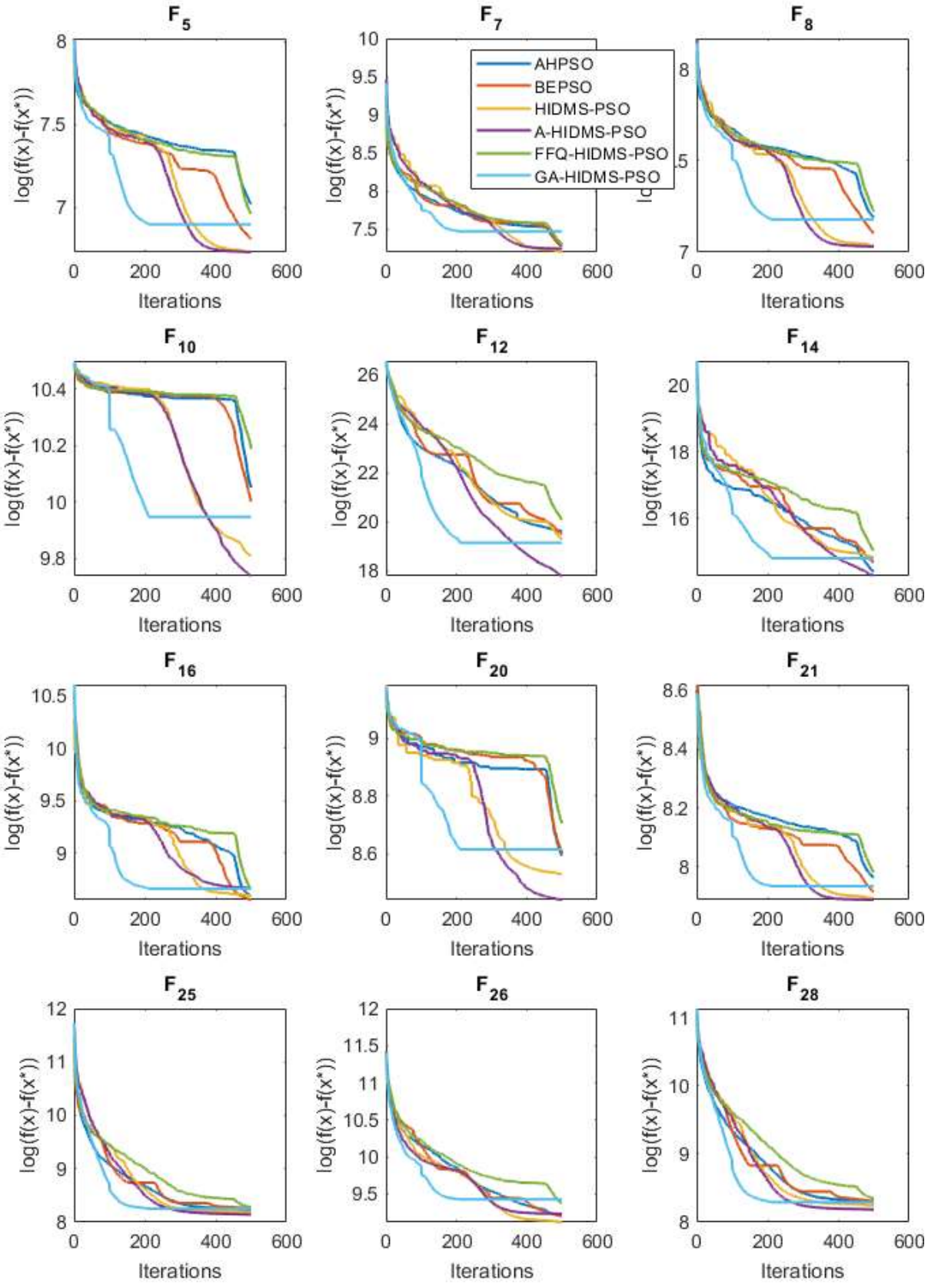


Figure 10-3 Convergence rate comparison of the six proposed algorithms on 100-dimensional CEC'17 multimodal F5, F7, F8, hybrid F10, F12, F14, F16 and composition F20, F21, F25, F26, F28

10.4 Analysis of the Proposed Algorithms

In this section, we analyse different aspects of the proposed algorithms with regards to population diversity, the impact of $pbest$ and $gbest$ on agents' fitness trend, contour maps, and consistency.

10.4.1 Analysis of the Population Diversity

The experimental results and the convergence rates obtained on different test suites reaffirmed the strong performance and the convergence behaviour of the six proposed algorithm. Further, maintaining the diversity of the population for extended periods can minimise the problems associated with premature convergence and the balance of exploration and exploitation. Hence, in Figure 10-4:10-9, the population diversity trend of the six proposed algorithms is compared with the PSO algorithm on six representative 100-dimensional multimodal problems selected from the CEC'17 test suite. The population diversity is calculated using equations (10.1) and (10.2) [251] with a particle population size of 40. As observed from Figure 10-4:10-9, the comparison verifies the population diversity maintaining capabilities of the proposed algorithms on high dimensional multimodal problems. It is worth highlighting that among the proposed algorithm, the FFQ-HIDMS-PSO exhibit superior capacity to maintain diversity of the population for extended periods compared to both PSO and the other proposed algorithms.

$$Diversity\big(S(t)\big) = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\sum_{d=1}^{D}\big(x_i^d - \overline{x^d}\big)^2} \tag{10.1}$$

$$\overline{x^d} = \frac{\sum_{i=1}^{n} x_i^d}{n} \tag{10.2}$$

where n is the swarm size, D is the dimension of the problem, x_i^d is the d dimensional value of the i^{th} particle and $\overline{x^d}$ is the average of d^{th} dimension over all particles.

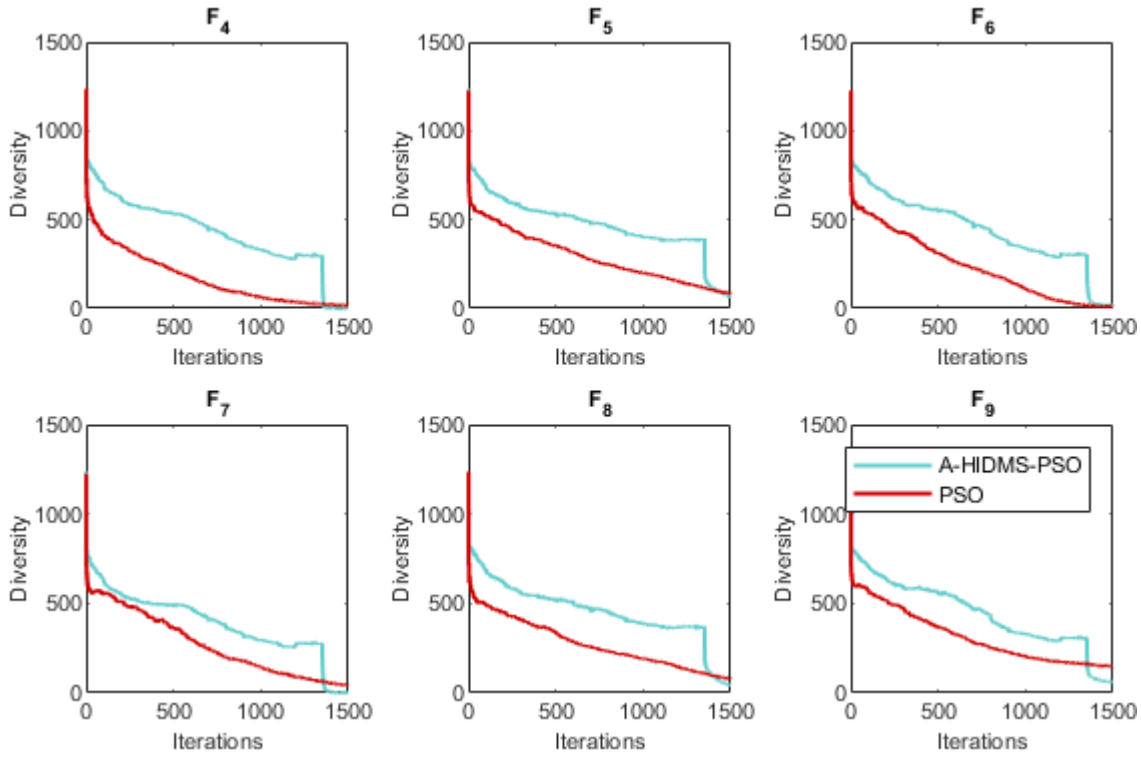


Figure 10-4 Population diversity comparison of A-HIDMS-PSO and PSO on the 100-dimensional multimodal CEC'17 problems

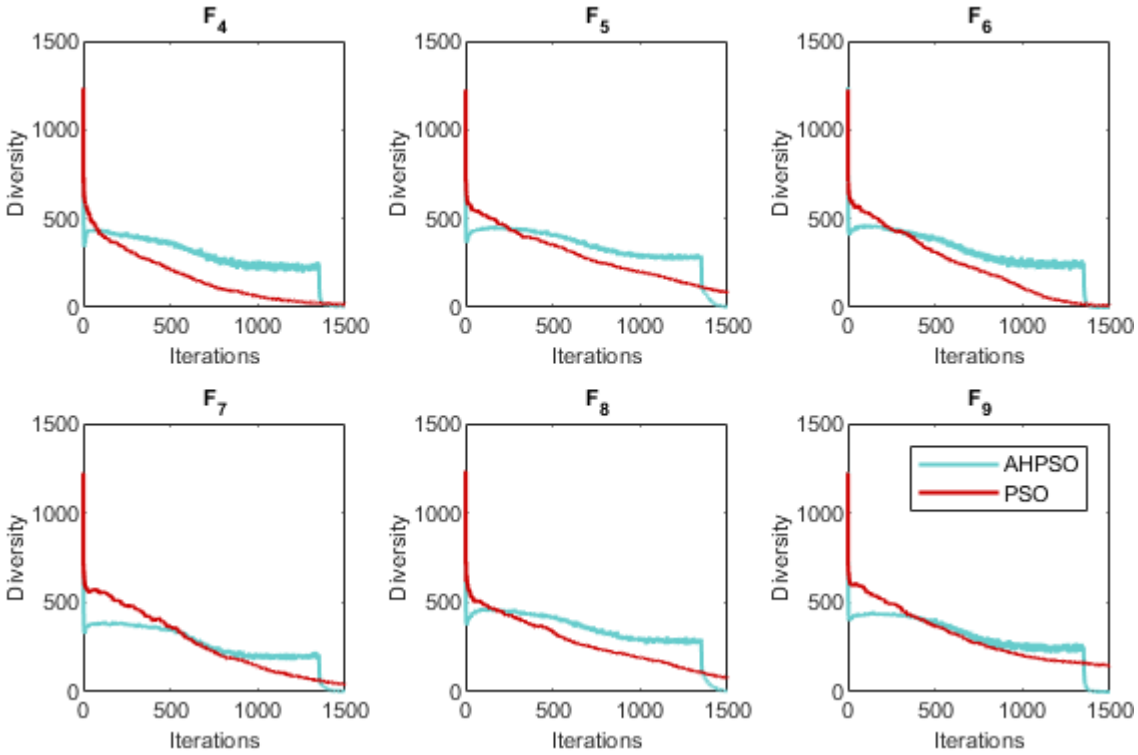


Figure 10-5 Population diversity comparison of AHPSO and PSO on the 100-dimensional multimodal CEC'17 problems

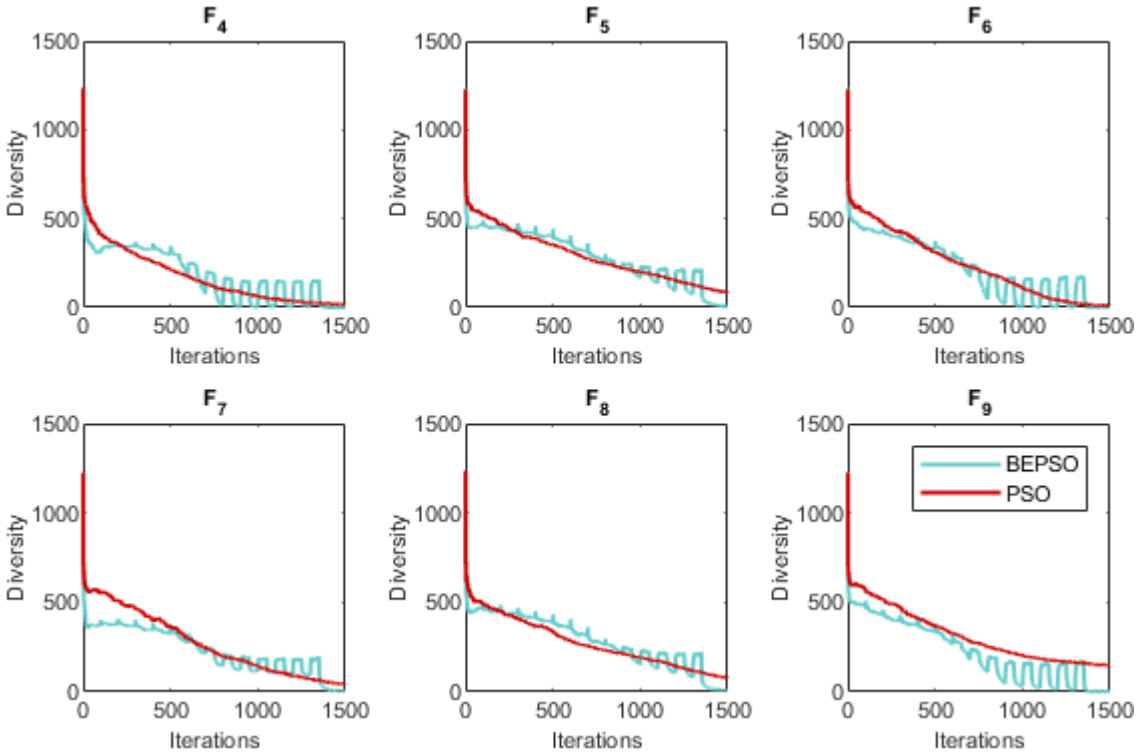


Figure 10-6 Population diversity comparison of BEPSO and PSO on the 100-dimensional multimodal CEC'17 problems

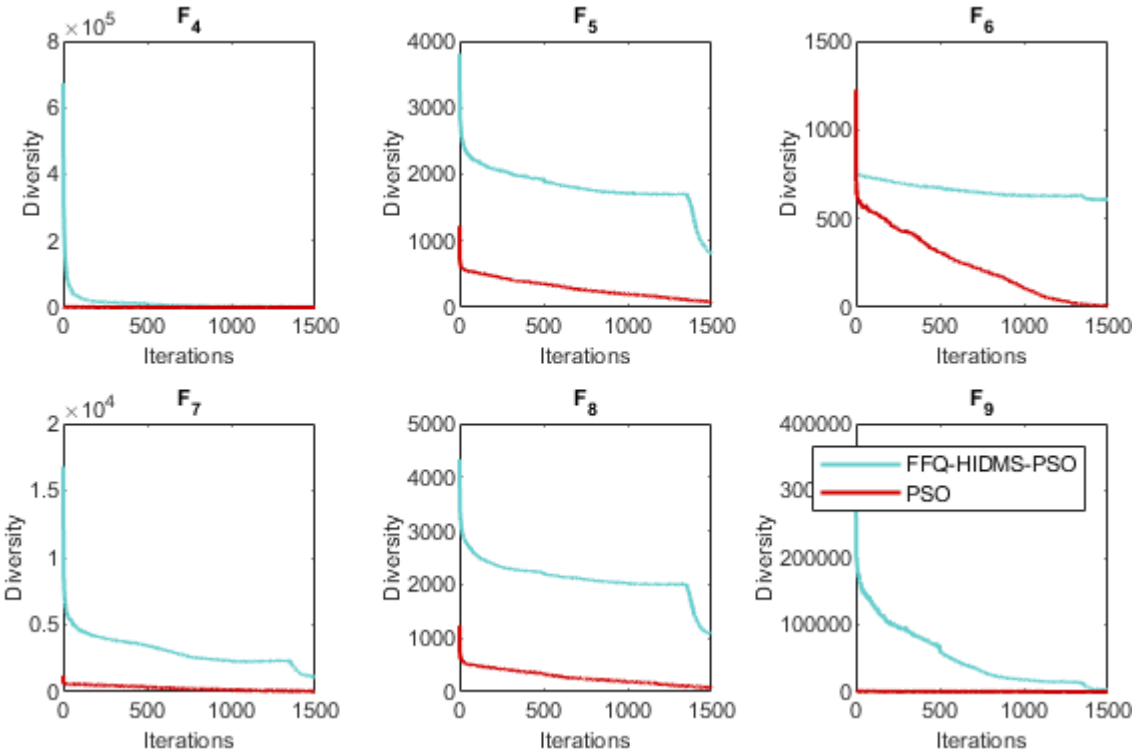


Figure 10-7 Population diversity comparison of FFQ-HIDMS-PSO and PSO on the 100-dimensional multimodal CEC'17 problems

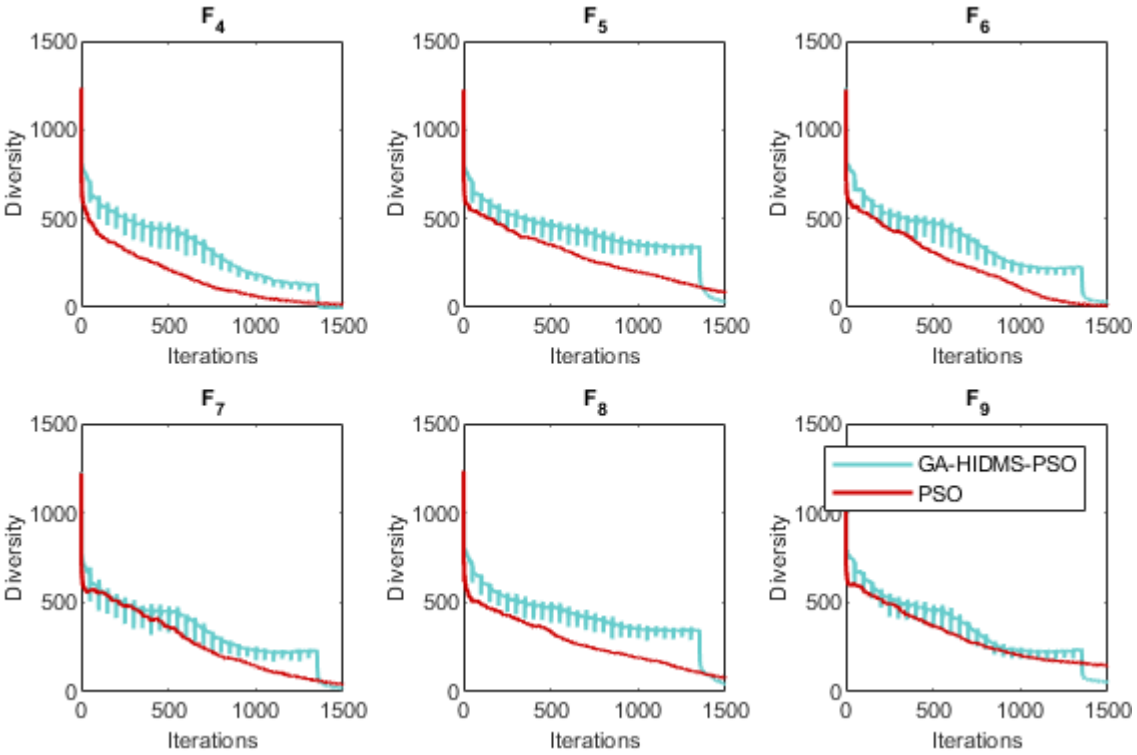


Figure 10-8 Population diversity comparison of GA-HIDMS-PSO and PSO on the 100-dimensional multimodal CEC'17 problems

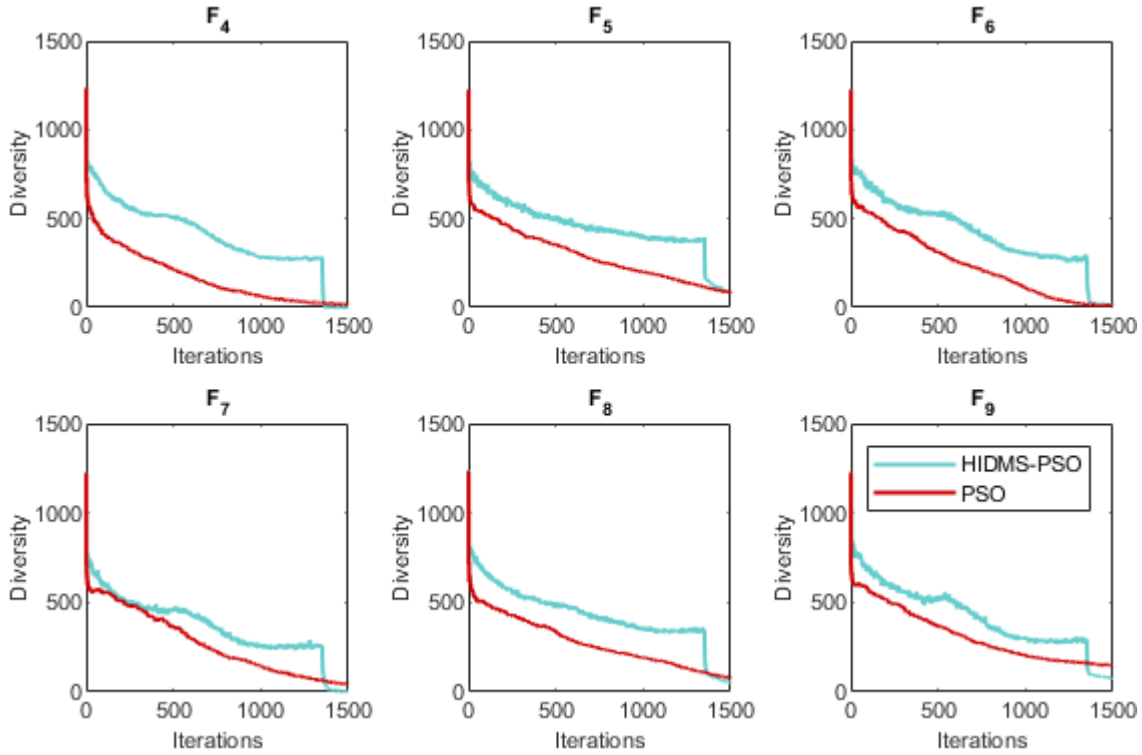


Figure 10-9 Population diversity comparison of HIDMS-PSO and PSO on the 100-dimensional multimodal CEC'17 problems

10.4.2 Analysis on the Impact of *Gbest*

In addition to the assessment of population diversity, we briefly investigated a related component to comprehend the precise impact of $gbest$ on a particle's fitness. Hence, using the same 100-dimensional multimodal problem set (F4~F9), we compared the fitness trend of a randomly selected particle to the trend of $gbest$. A very similar particle fitness and $gbest$ trend can be an indication of high influence of $gbest$ on particle's trajectory and may result in particles to prematurely converge to local optima and on the contrary, a highly uncorrelated fitness trend may indicate a wandering behaviour which may prevent particles to efficiently search and exploit the discovered solution.

In Figure 10-10:10-16, the fitness trend of the six proposed algorithms and PSO on the $gbest$ exemplar is displayed. Among the proposed algorithms, we generally observe a reasonable balance in the fitness trend that correlates with the trend of $gbest$ but concurrently the fitness of the particle fluctuates throughout the search phase. On the contrary, we observe that the particle's behaviour in the PSO algorithm is typically highly influenced by the $gbest$ exemplar to the extent that the particle's fitness trend curve irrecoverably unifies with the trend of $gbest$ within the first half of the search phase in most cases.

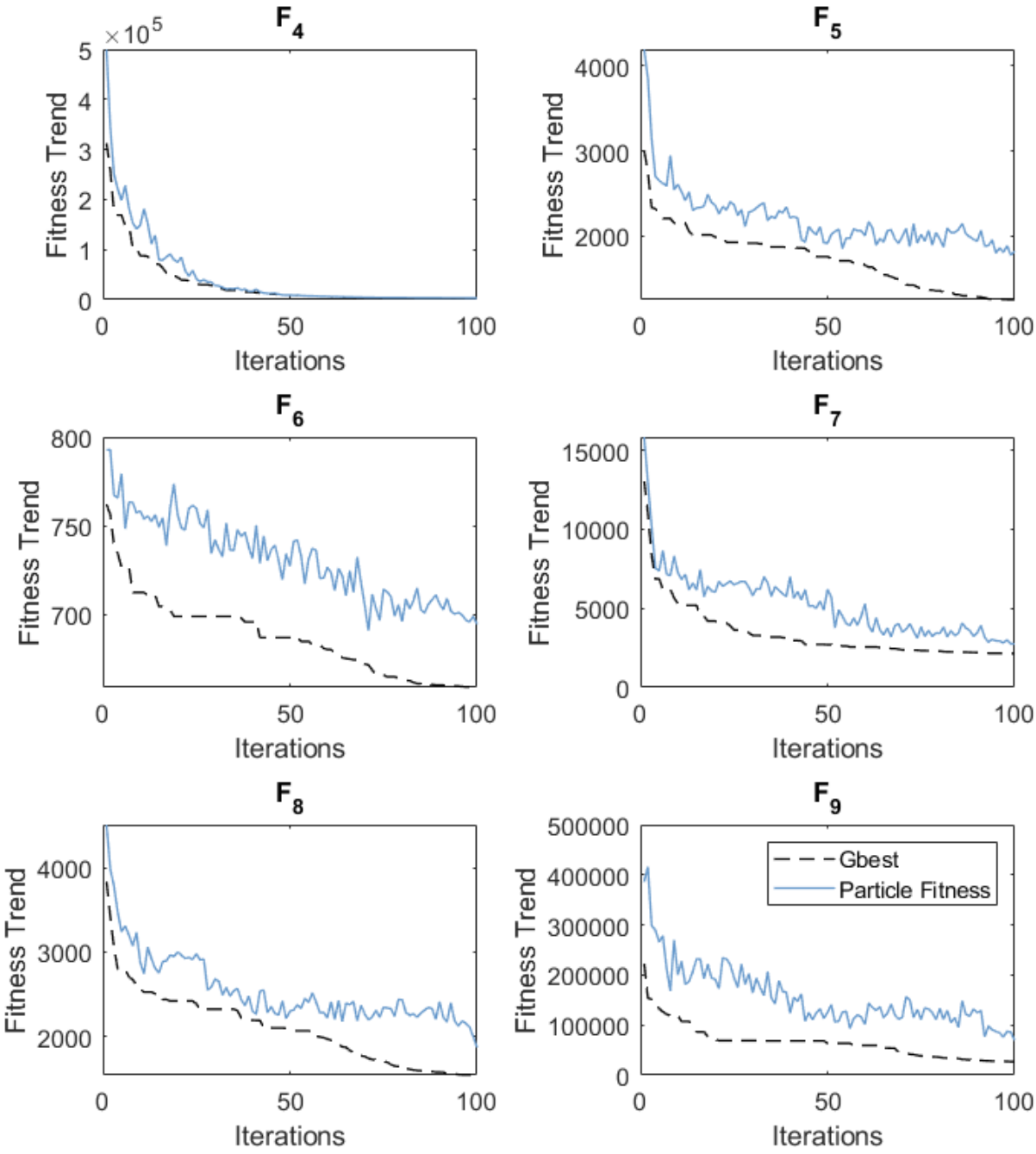


Figure 10-10 The comparison of gbest and fitness trend of a random agent in the HIDMS-PSO algorithm on 100-dimensonal multimodal CEC'17 problems

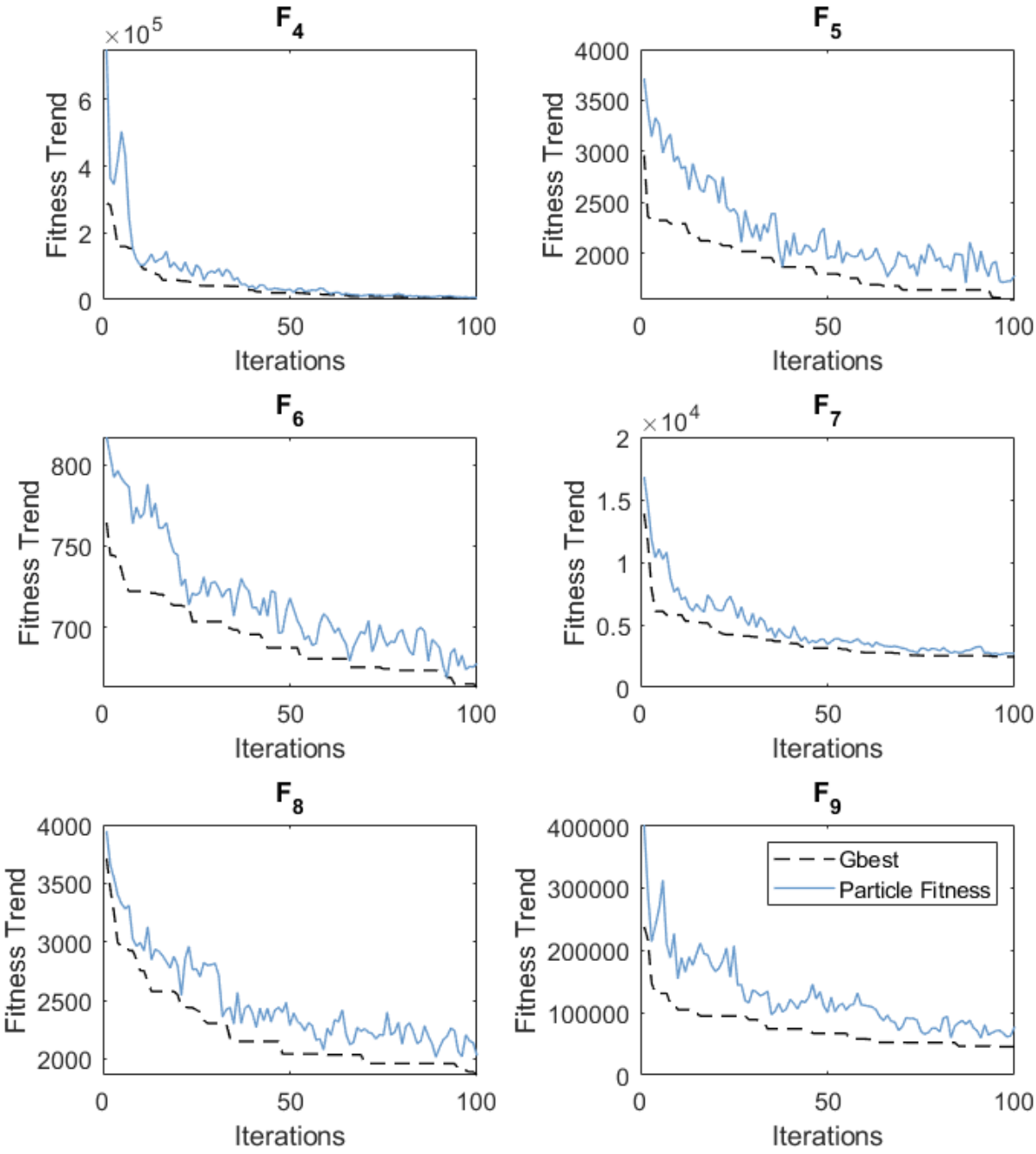


Figure 10-11 The comparison of gbest and fitness trend of a random agent in the FFQ-HIDMS-PSO algorithm on 100-dimensonal multimodal CEC'17 problems

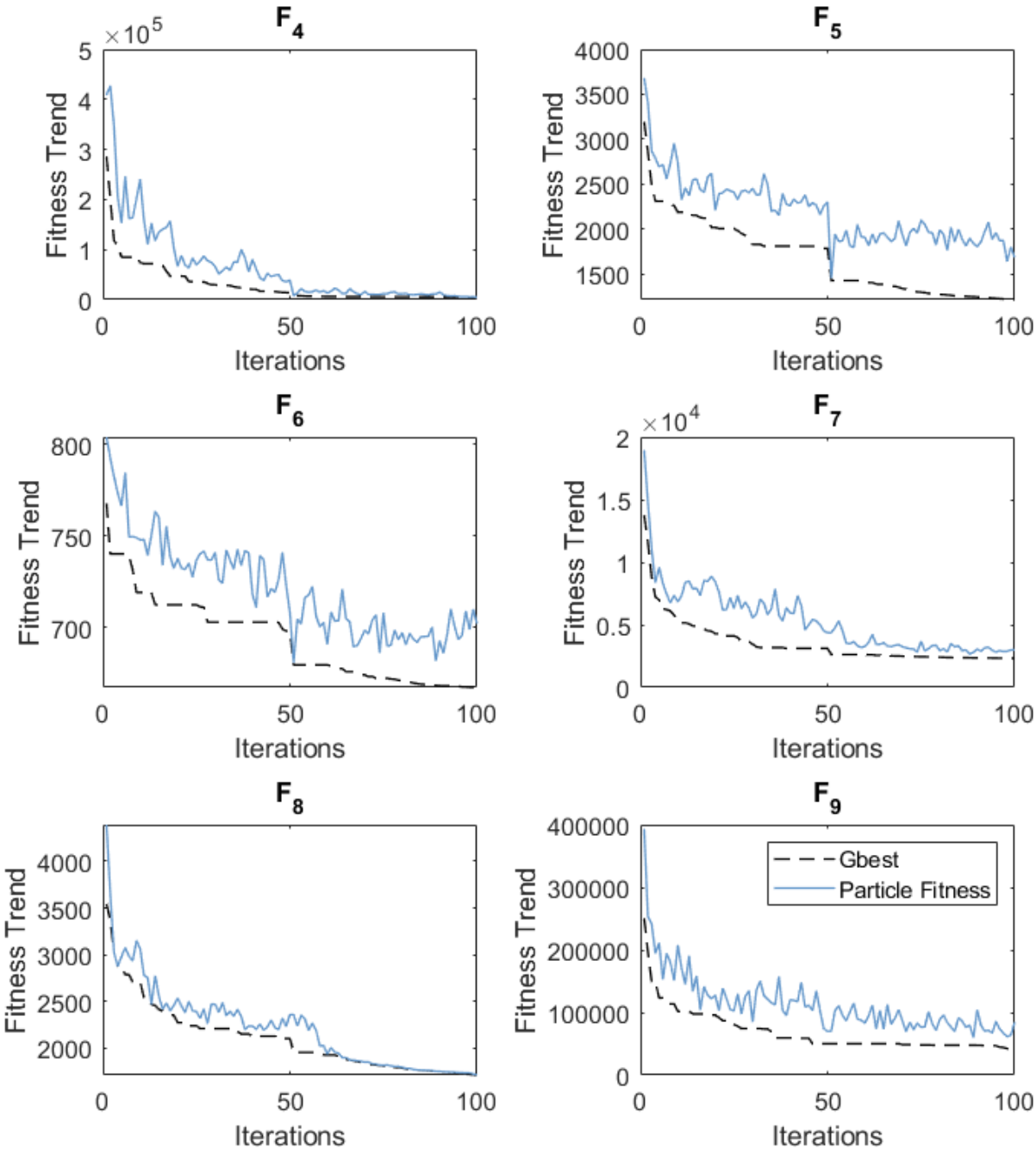


Figure 10-12 The comparison of gbest and fitness trend of a random agent in the GA-HIDMS-PSO algorithm on 100-dimensonal multimodal CEC'17 problems

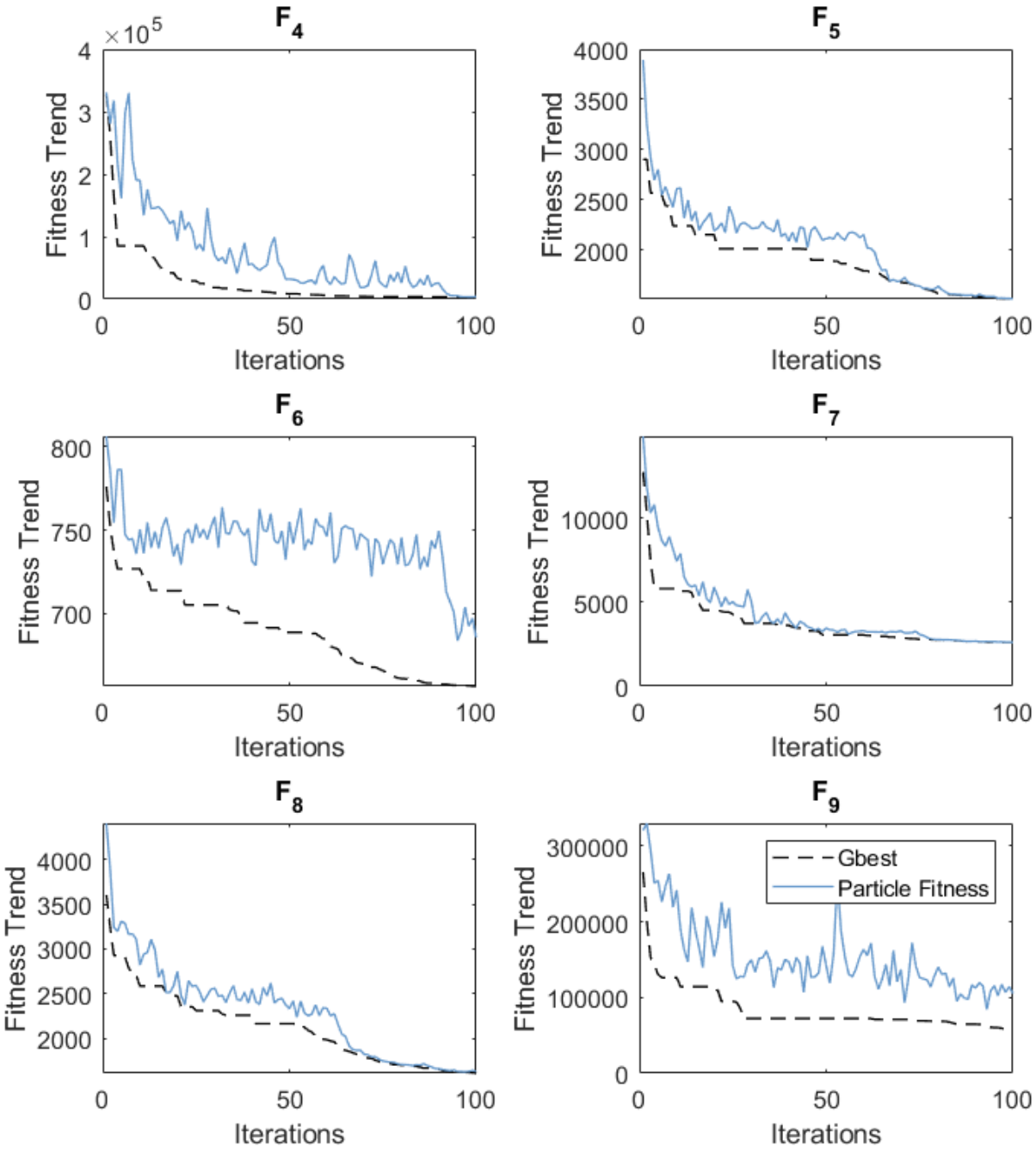


Figure 10-13 The comparison of gbest and fitness trend of a random agent in the A-HIDMS-PSO algorithm on 100-dimensonal multimodal CEC'17 problems

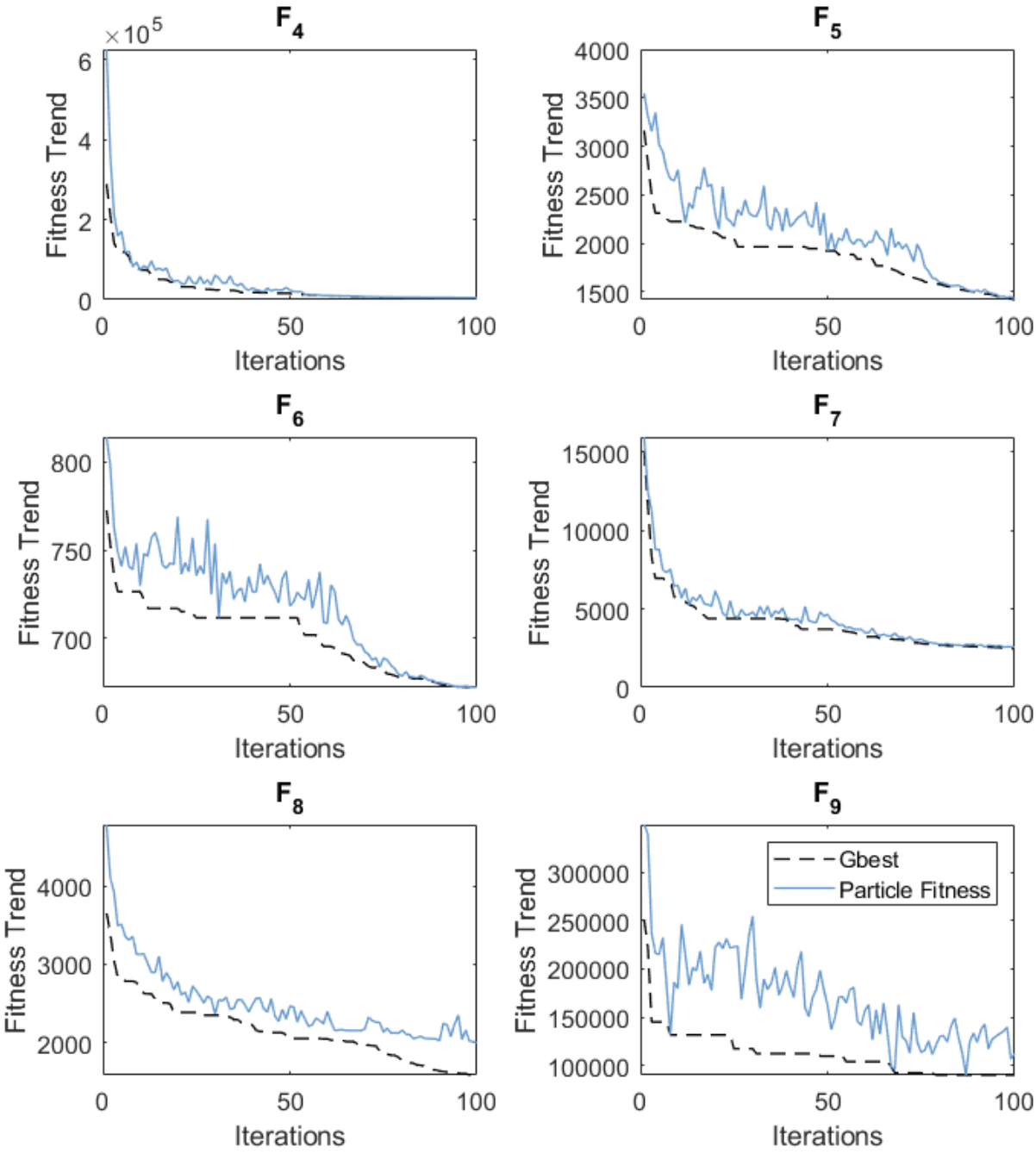


Figure 10-14 The comparison of gbest and fitness trend of a random agent in the BEPSO algorithm on 100-dimensonal multimodal CEC'17 problems

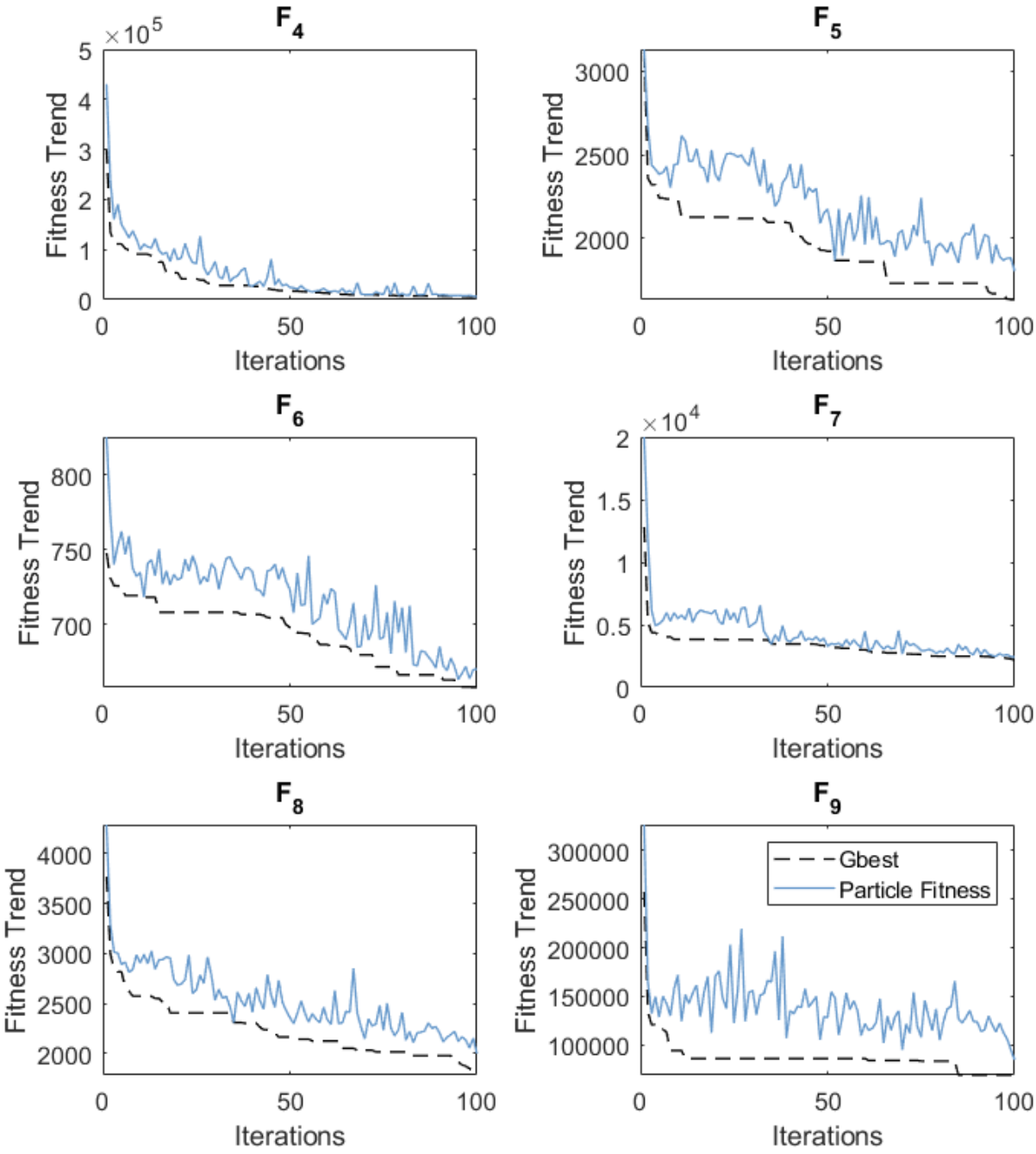


Figure 10-15 The comparison of gbest and fitness trend of a random agent in the AHPSO algorithm on 100-dimensonal multimodal CEC'17 problems

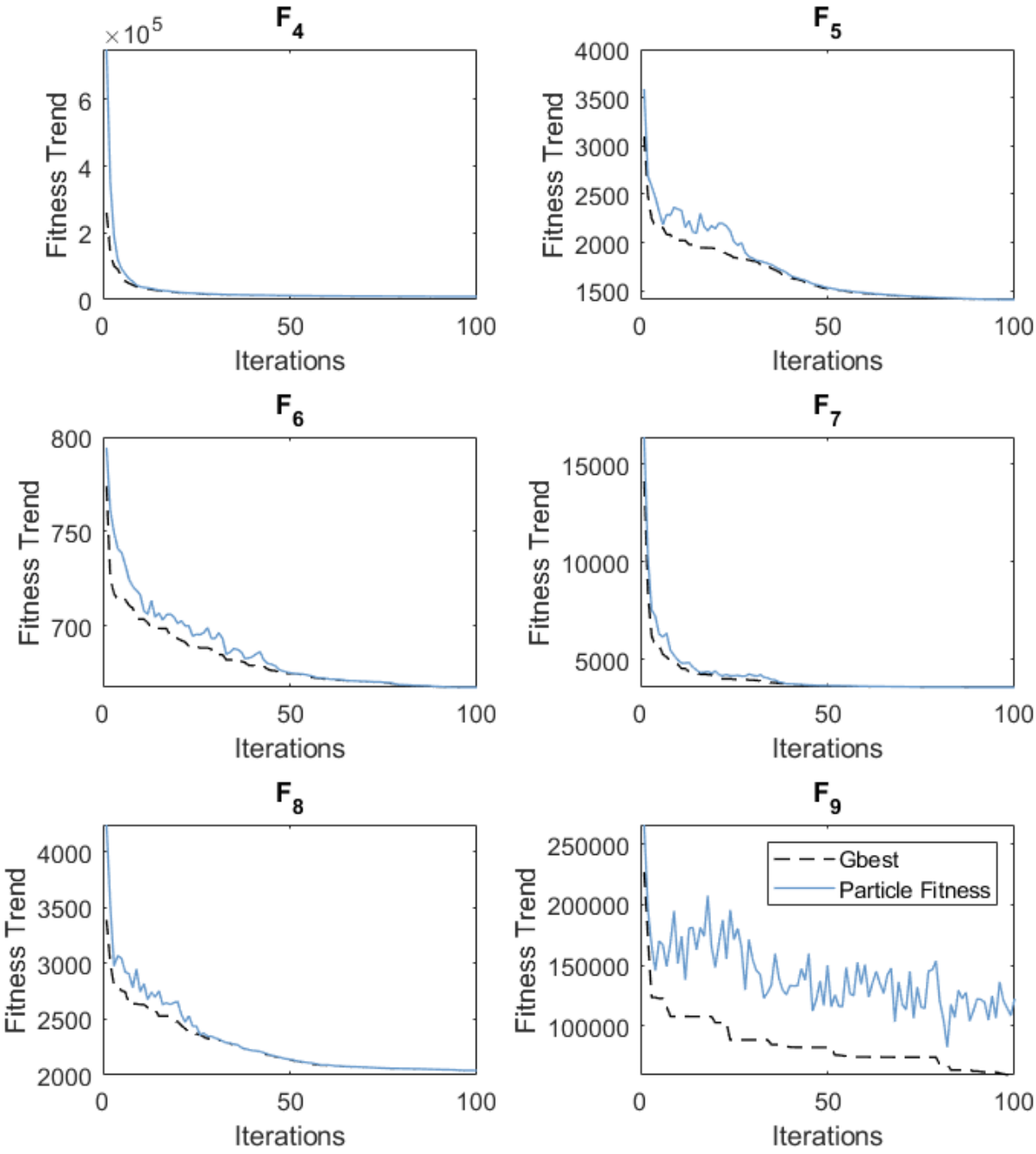


Figure 10-16 The comparison of gbest and fitness trend of a random agent in the PSO algorithm on 100-dimensonal multimodal CEC'17 problems

10.4.3 Contour Map Analysis

After the verification of the impact of $gbest$ on the fitness trend of particles, we examine the impact of $pbest$ on the position of particles. In order to precisely exhibit the stages of how the swarm identifies promising regions, we must observe the focus of $pbest$ positions ($pbest$ *shown as *, x shown as *)* as well as the distribution of $pbest$ positions in the search space, and the degree of constraint $pbest$ positions place on particles' current positions in the search space (e.g., how distant particles can move away from their $pbest$s). Figure 10-17:10-24 compares the contour maps of various 2-D multimodal problems for AHPSO, HIDMS-PSO, A-HIDMS-PSO and the PSO algorithm. The

comparison reveals that in all cases, the agents in the PSO algorithms rapidly cluster around $pbest$ positions within the initial 25-100 iterations without efficiently exploring the search space, even in small problem spaces. In addition, although $pbest$ positions determine the trajectory of agents, it appears that the agents' exploration capability is also significantly restricted which prevents agents from discovering other potential solutions which may be more optimal. On the contrary, in the case of AHPSO, HIDMS-PSO and A-HIDMS-PSO, we consistently observe that agents naturally focus on the region of $pbest$ positions; however, even at later stages of the search, agents continue to efficiently explore the search space. Hence, as opposed to the case observed in the canonical PSO algorithm, the agents in the proposed algorithms are more loosely influenced and as observed on the contour maps, agents appear to be directed towards feasible regions instead. Further, we also consistently observe that the swarm behaviour in the proposed algorithms is significantly superior at identifying and maintaining multiple feasible solutions concurrently even at later stages of the search. This type of emerging swarm behaviour is likely to be the crux for the performance observed on high-dimensional problems detailed in previous chapters.

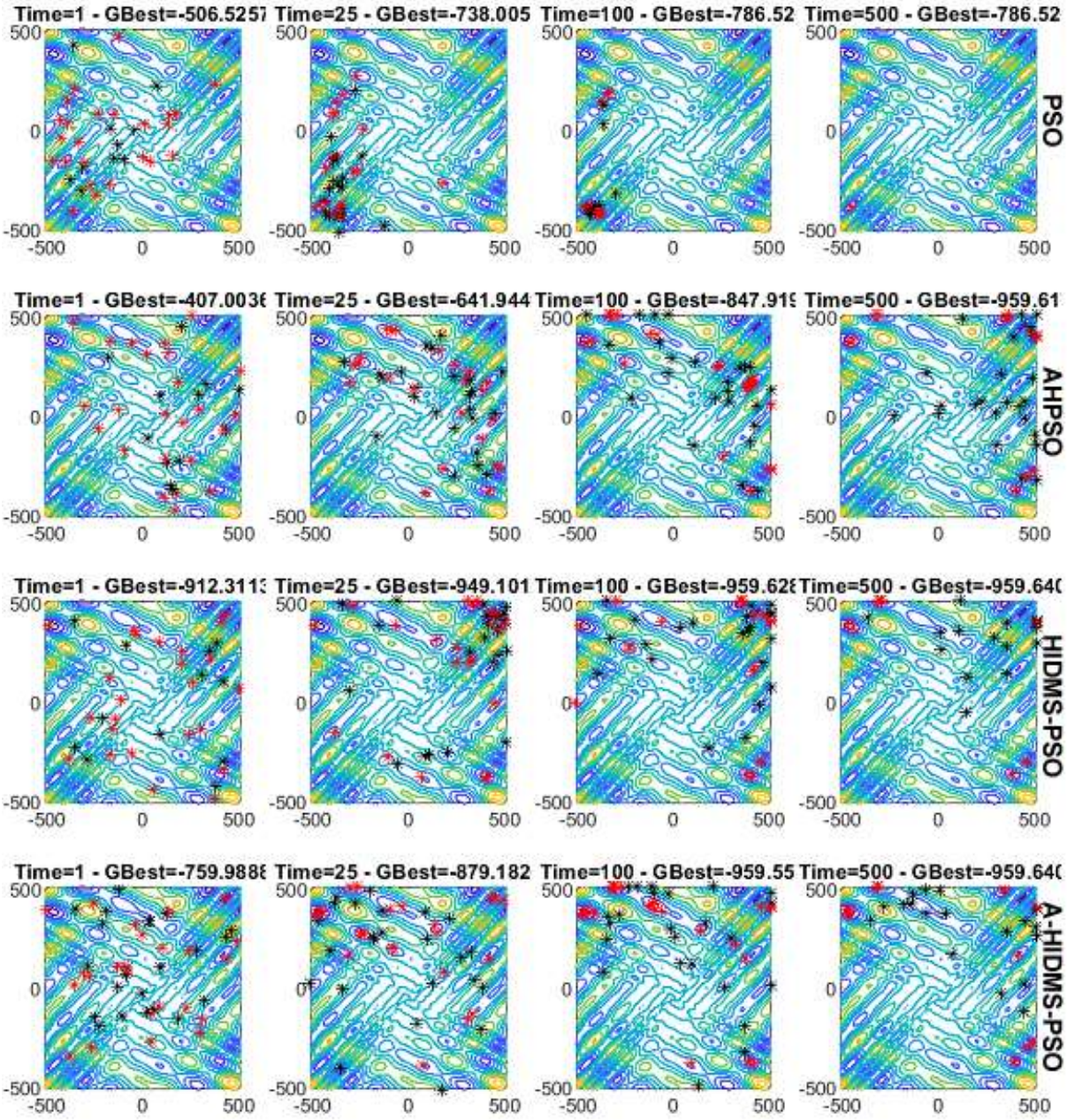


Figure 10-17 The contour map of Egg holder function iteratively showing the pbest and current particle positions

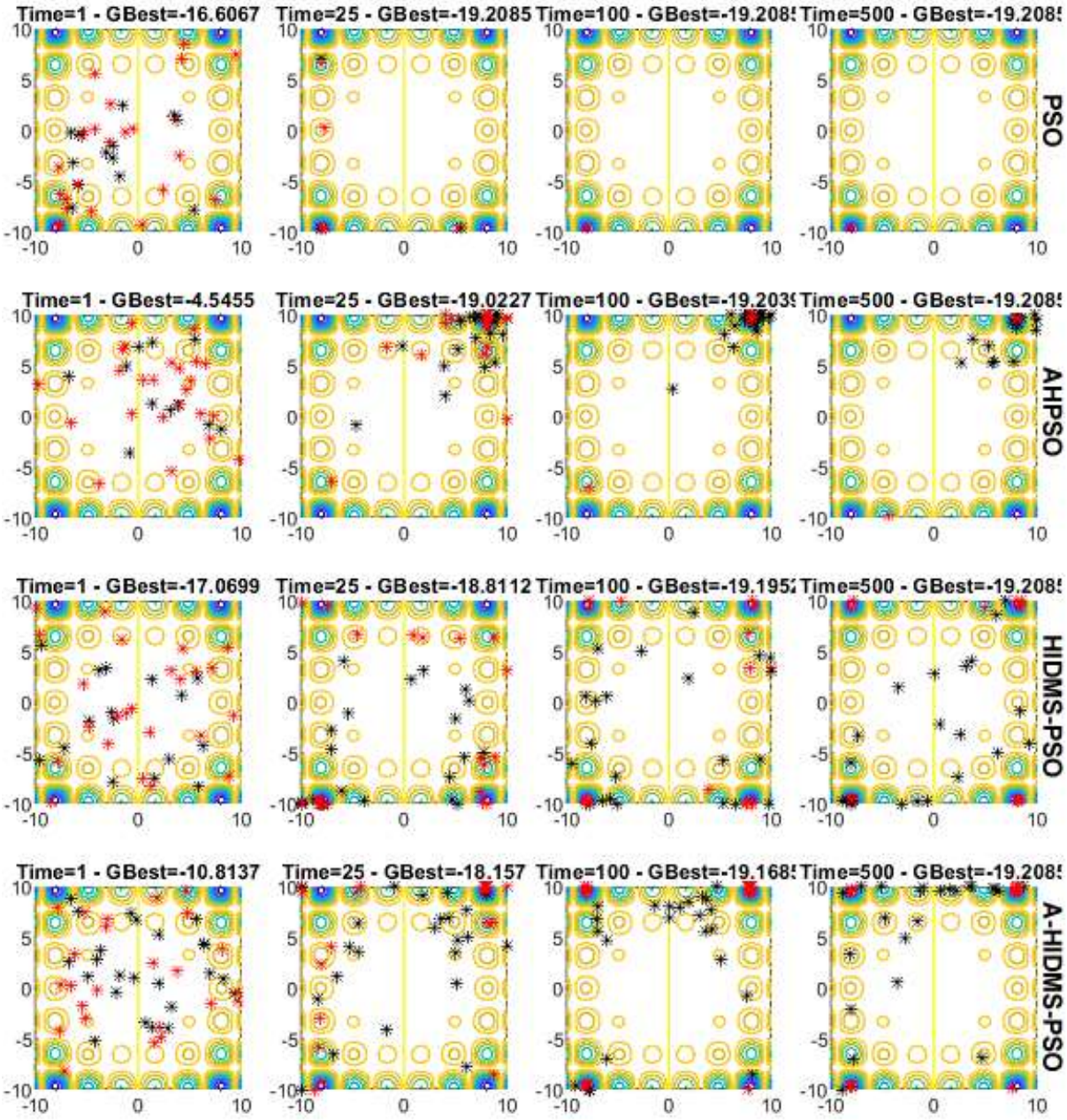


Figure 10-18 The contour map of Levy function iteratively showing the pbest and current particle positions

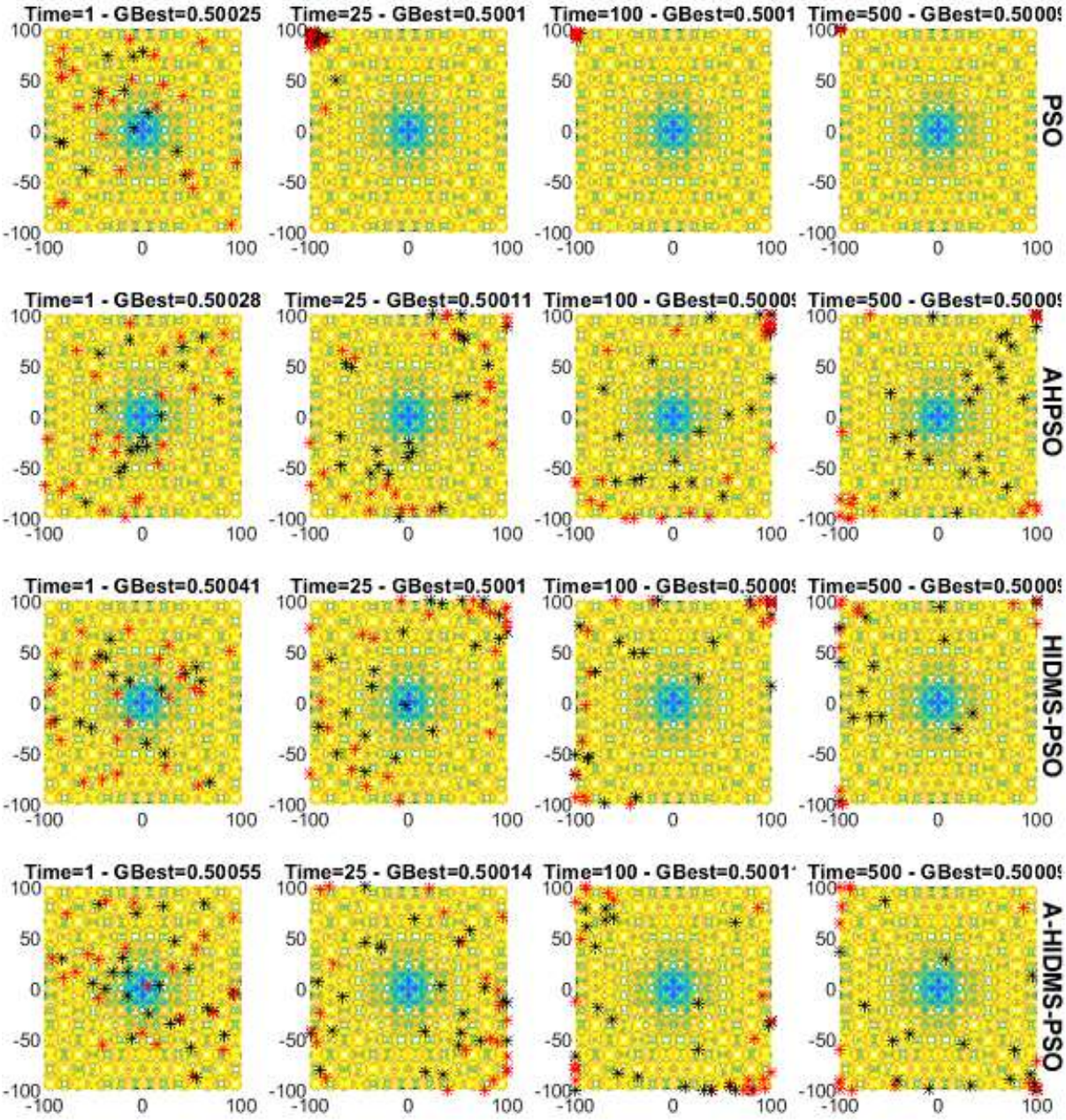


Figure 10-19 The contour map of Schaffer function iteratively showing the pbest and current particle positions

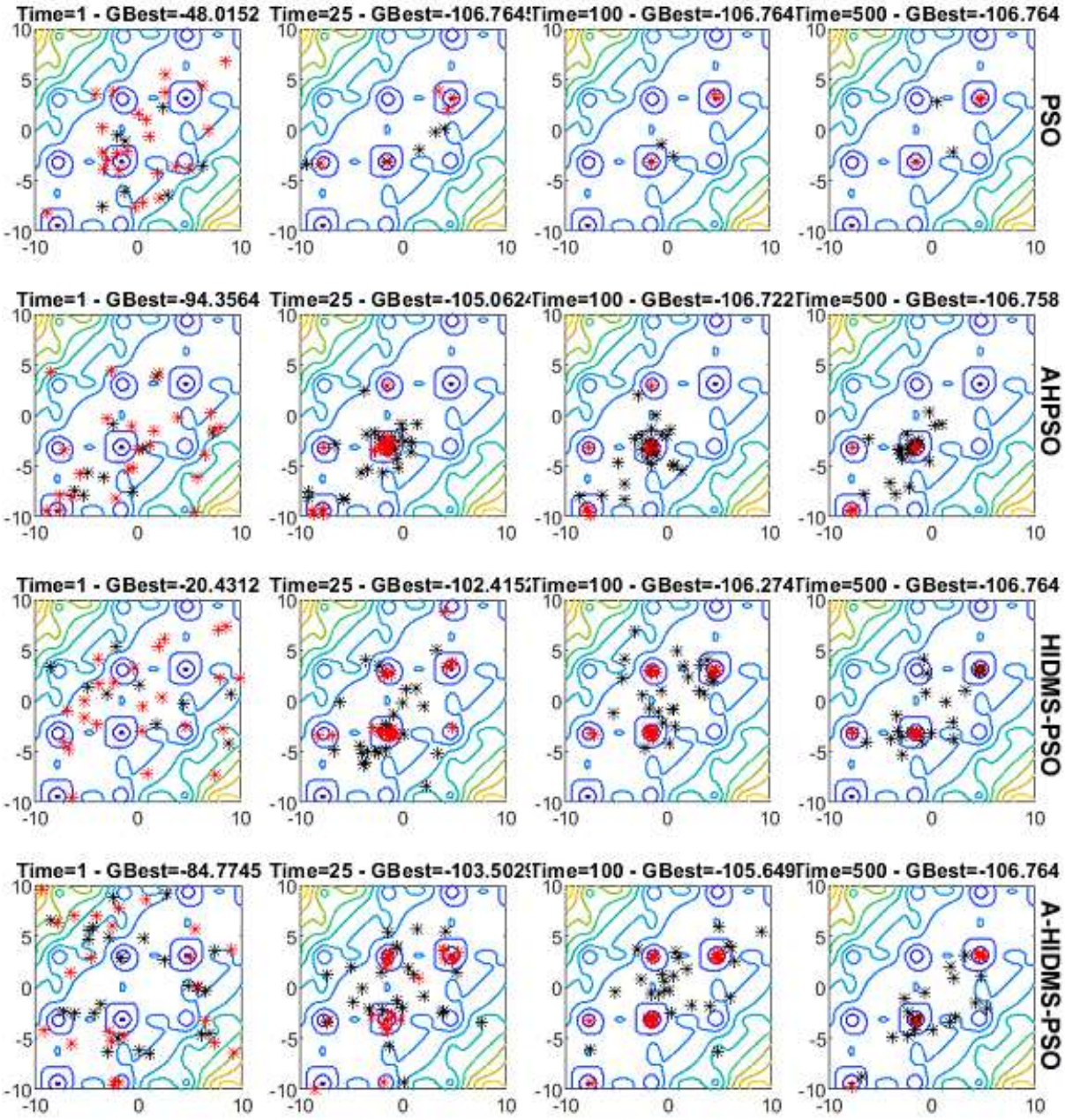


Figure 10-20 The contour map of Bird function iteratively showing the pbest and current particle positions

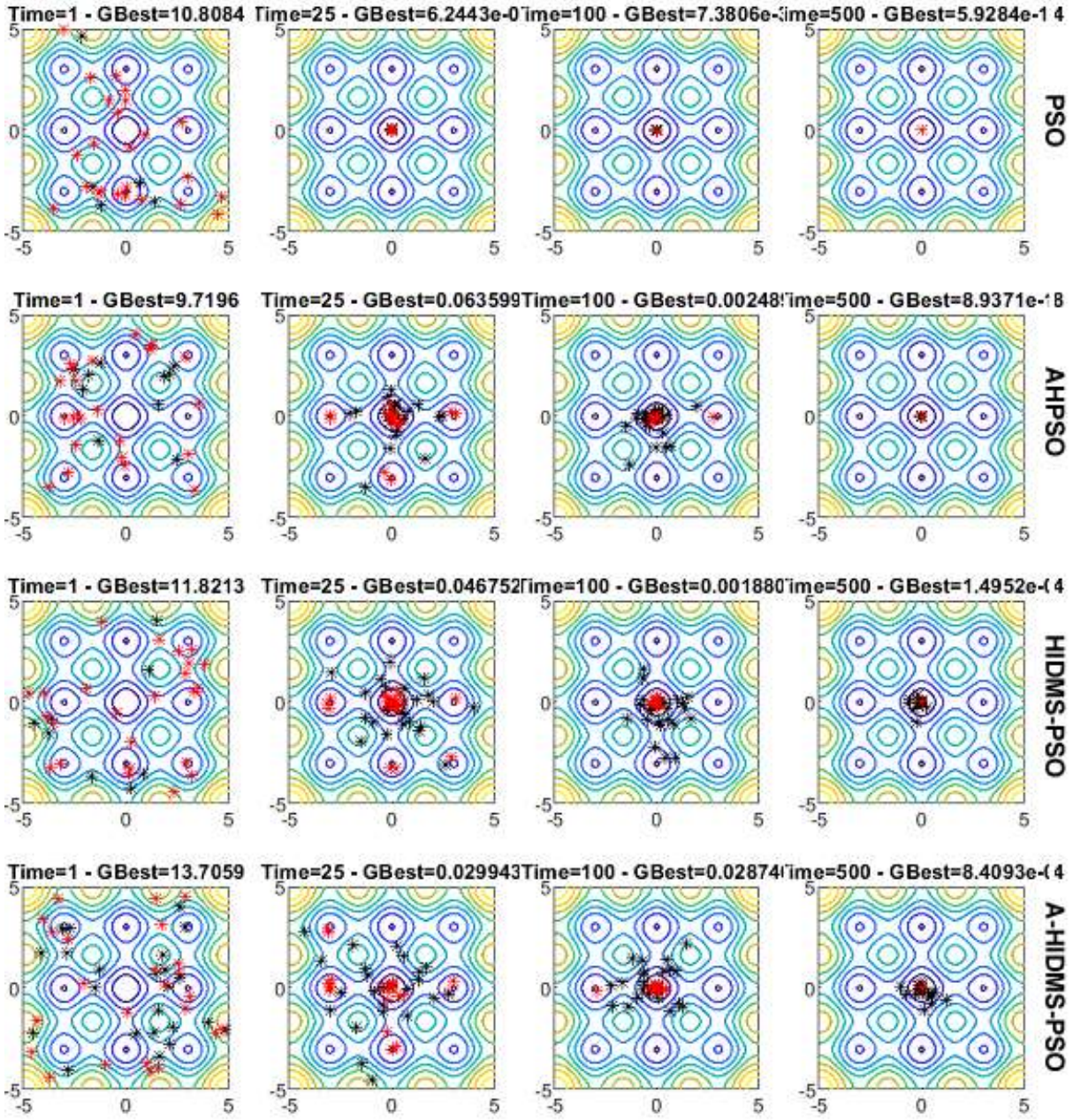


Figure 10-21 The contour map of Egg Crate function iteratively showing the pbest and current particle positions

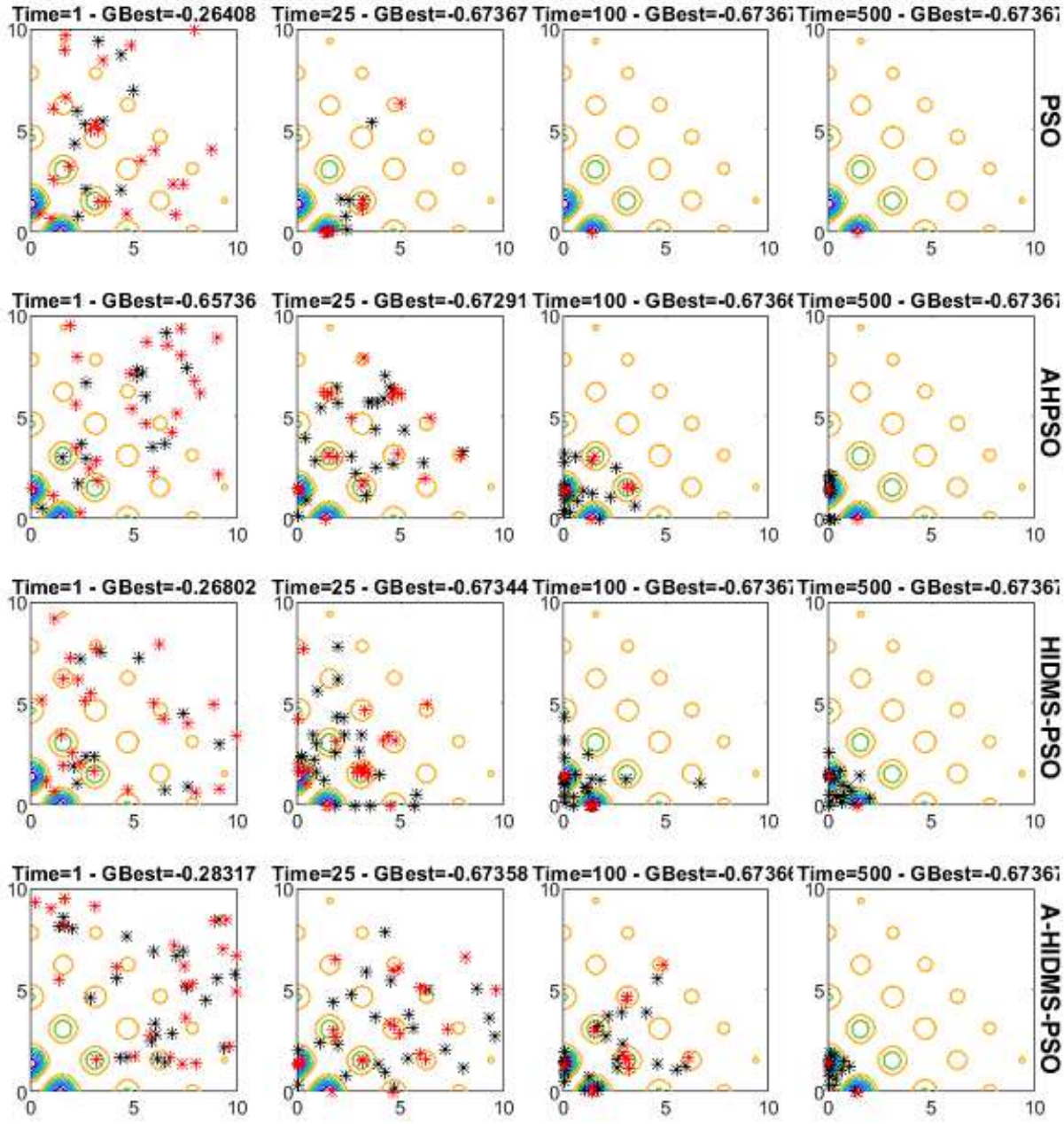


Figure 10-22 The contour map of Keane function iteratively showing the pbest and current particle positions

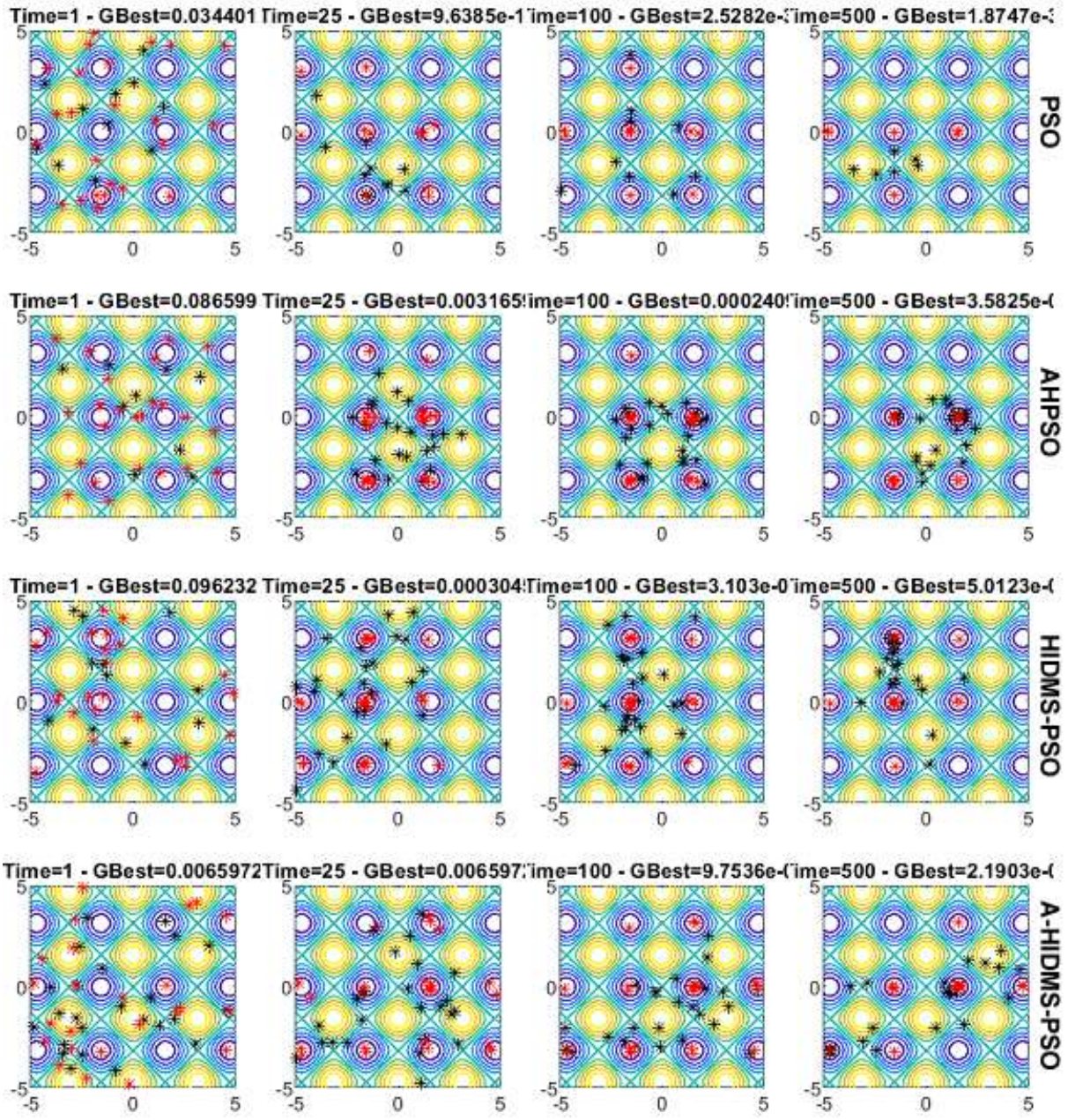


Figure 10-23 The contour map of Parsopoulos' function iteratively showing the pbest and current particle positions

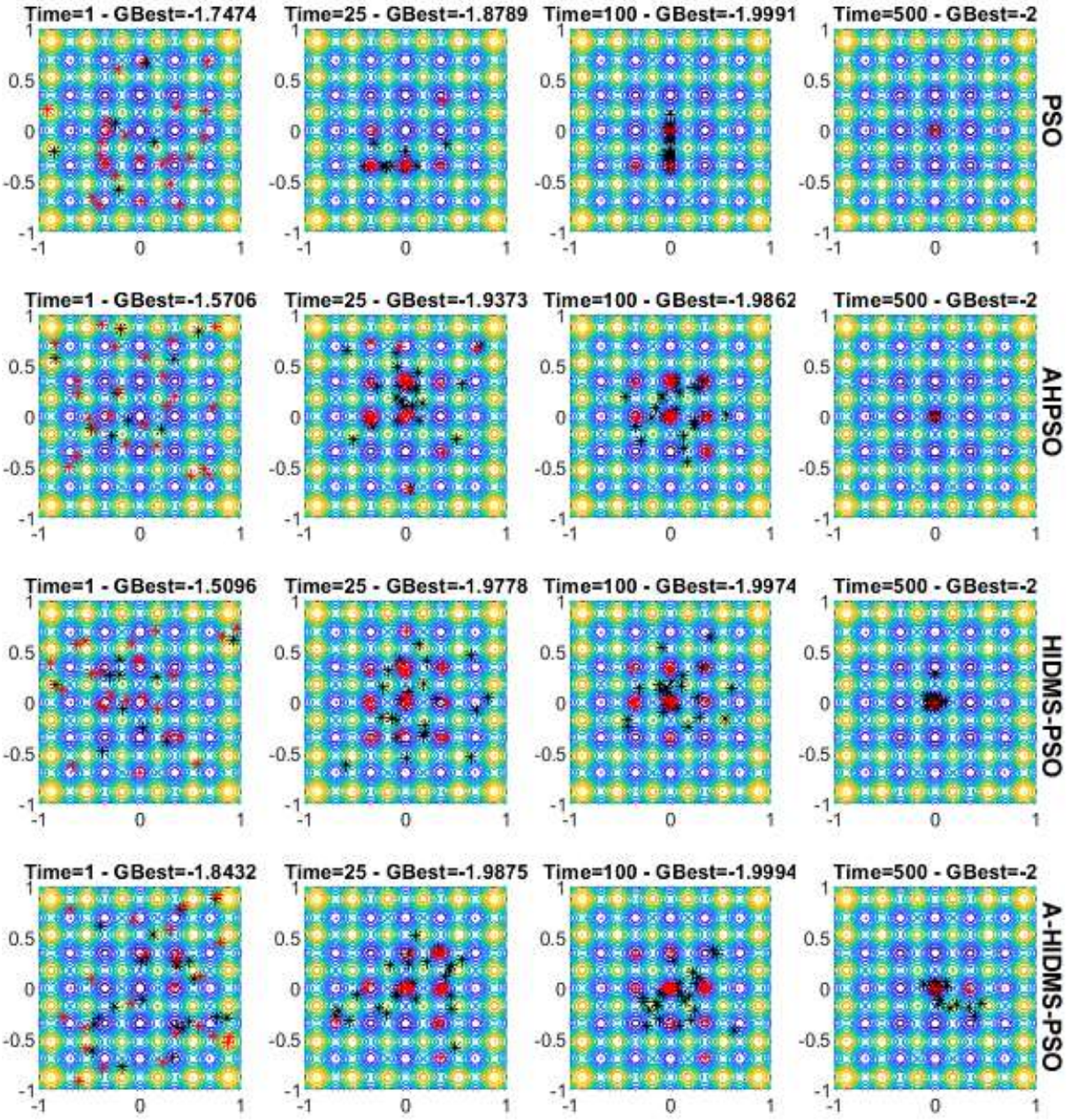


Figure 10-24 The contour map of Tsoulos' function iteratively showing the pbest and current particle positions

10.4.4 Box-Plot Analysis

In this section, the consistency of the six proposed algorithm is compared with L-SHADE, EPSO, HCLPSO and DMS-PSO on various 30 and 100-dimensional multimodal, hybrid and composition problems selected from the CEC'14 test suite.

As displayed in Figure 10-25-10-26, we cannot highlight a specific algorithm that continuously exhibited inconsistency across all problems and dimensions. Some algorithms exhibit high consistency on various problems whilst we observe the same algorithms exhibit slight inconsistency on other problems. It is reasonable to state that overall, all 10 algorithms assessed here show relatively similar consistencies. However, individual assessment of each problem reveals

that on the 30-dimensional F6, F13 and F15, the variation of the error values obtained by the 10 algorithms appear insignificant. On the other hand, for F9 and F20, the consistency of error values attained by AHPSO appear better compared to other algorithms. In addition, for 30-dimensional F27, HCLPSO and EPSO appear more consistent.

In addition, we assessed the impact of dimensionality on the consistency of the algorithm. Overall, the error values obtained by all algorithms exhibits higher variations over increased problem dimensions. However, in summary, for F6 the least and most effected algorithms from the increased dimensions are AHPSO (least) and GA-HIDMS-PSO (most), respectively. For F9 and F15, BEPSO and A-HIDMS-PSO were the most consistent algorithms with increased dimensionality whilst for both functions HIDMS-PSO exhibited higher variation in error values compared to other algorithms. Similarly, for F20 and F27, GA-HIDMS-PSO and A-HIDMS-PSO appear more consistent to change of dimensions, and for the same two problems, the variation of error values for L-SHADE (F20) and EPSO (F27) appear to rise more than the comparison algorithms with increased dimensions.

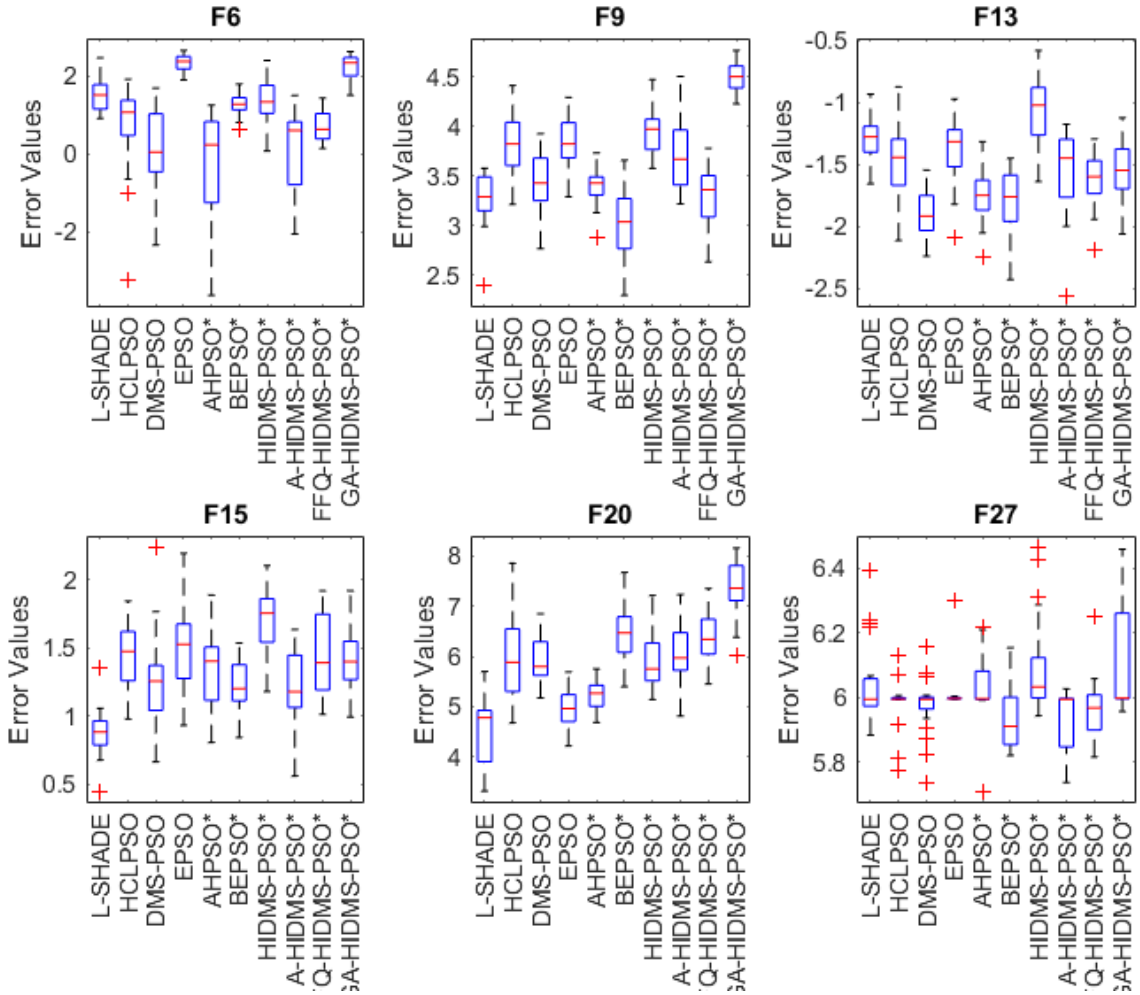


Figure 10-25 Box-plot graphs for various 30-d CEC'14 problems

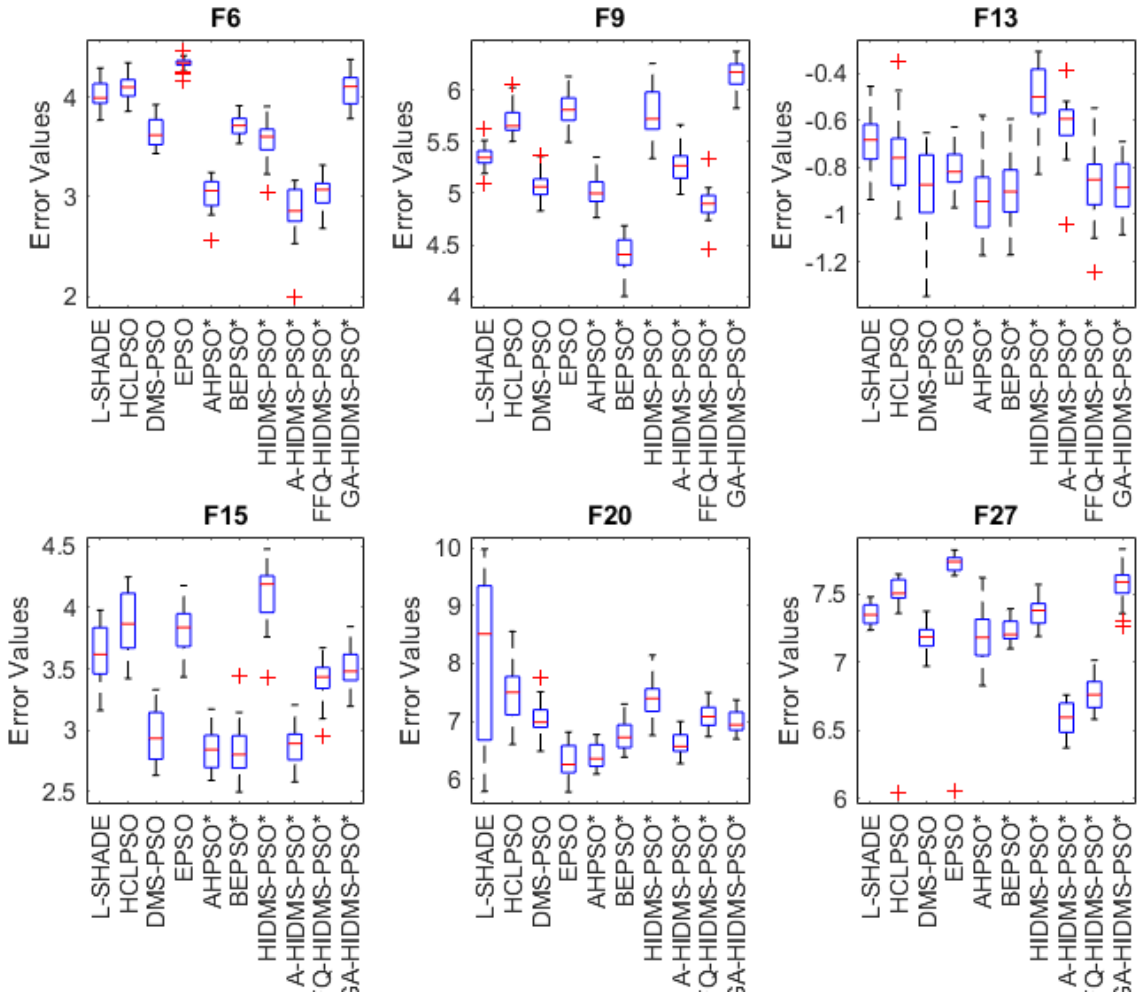


Figure 10-26 Box-plot graphs for various 100-d CEC'14 problems

10.4.5 The Impact of Velocity Initialisation Strategies

Although various theoretical and empirical aspects of particle behaviour have been extensively studied in the literature [117], [252], [253], a definitive way in which the initial velocities must be sampled is still unsettled. Typically, an initial velocity of zero is predominantly utilised as a popular strategy. In addition, small uniform distributions and initialisation of velocities tailored to the domain of the problem are also popular strategies advocated in the literature. In this section, we assess the impact of four different velocity initialisation strategies on the six proposed algorithms using the 10 and 100-dimensional CEC'17 benchmark problems. Generally, the maximum possible particle velocity is clamped (referred to as V_{max}) to avoid significantly large jumps. However, as suggested by [254], V_{max} is problem dependent, hence, the velocity clamping mechanism was not utilised for the work presented in this section. The impact of the following four common velocity initialisation strategies was employed:

- *Strategy 1*: Velocities are initialised at zero, i.e., $\vec{v}_i(0) = 0, \text{for all } i = 1, \ldots, n$, where n is the swarm size. This is the strategy adopted by all of the proposed algorithms in this thesis. However, although the strategy may be common, it is controversial at the same time, considering the implications of *zero* velocities on the initial exploration capability of the swarm, especially in more complex search spaces.

- *Strategy 2*: Velocities are initialised as random values within the upper and lower bounds of the given problem, i.e. $\vec{v}_i(0) \sim U(\vec{x}_{min}, \vec{x}_{max})$ where $\vec{x}_{min}$ and $\vec{x}_{max}$ are d-dimensional vectors defining the upper and lower bounds for each dimension. As opposed to the limitation of the first strategy in relation to the initial exploration, this strategy addresses the said issue with larger step sizes and better initial diversity. However, in this approach, particles tend to violate the boundaries of the problem space due to initial large step sizes.
- *Strategy 3*: Velocities are initialised within small uniformly distributed random values, i.e. $\vec{v}_i(0) \sim U(-0.1, 0.1)$. This strategy tends to address the issues suffered by the first two strategies. The randomly assigned small velocities enable some initial diversity whilst preventing the boundary violation problem that is associated with large step sizes.
- *Strategy 4:* $\vec{v}_i(0) \sim \vec{r} * (\vec{x}_{max} - \vec{x}_{min}) + \vec{x}_{min}) * 0.2$, where $\vec{r}$ is a d-dimensional vector drawn from the uniform distribution in the range (0,1).

Table 10-13 displays the mean ranks obtained on the 10 and 100-dimensional CEC'17 problems utilising the four different velocity initialisation strategies. In summary, the two experiments conducted in this section on the 10 and 100-dimensional problems suggest that at 10 dimensions, velocity initialisation strategy 3 (*for FFQ-HIDMS-PSO, GA-HIDMS-PSO, AHPSO)* and strategy 4 (*for HIDMS-PSO and A-HIDMS-PSO*) consistently render better mean performance. In the 100-dimensional case, FFQ-HIDMS-PSO, BEPSO and AHPSO obtained the best mean performance using strategy 2, A-HIDMS-PSO using strategy 3, and HIDMS-PSO and GA-HIDMS-PSO with the velocity initialisation using strategy 4. It is worth noting that at 100 dimensions, most of the proposed algorithms (except FFQ-HIDMS-PSO and GA-HIDMS-PSO) exhibited the worst mean performance with strategy 1 for velocity initialisation. Although the preliminary results suggest that there is no clear best strategy, we observe that the results for *strategy 1* is slightly worse and considering that this is the velocity initialisation strategy utilised by all of the algorithms proposed in this thesis, we suggest further exploration of this matter in more detail in future work.

Table 10-13 Mean ranks obtained by the proposed algorithms on 10 and 100-dimensional CEC'17 problems using different initial velocity distribution strategies

	10-dimensional			
	Str.1	**Str.2**	**Str.3**	**Str.4**
HIDMS-PSO	2.50	2.66	2.43	**2.20**
FFQ-HIDMS-PSO	2.36	2.50	**2.26**	2.66
GA-HIDMS-PSO	2.56	2.83	**2.10**	2.30
A-HIDMS-PSO	2.36	2.56	2.73	**2.13**
BEPSO	**2.03**	2.43	2.66	2.66
AHPSO	2.53	2.33	**2.30**	2.43

	100-dimensional			
	Str.1	**Str.2**	**Str.3**	**Str.4**
HIDMS-PSO	2.56	2.50	2.50	**2.23**
FFQ-HIDMS-PSO	2.40	**2.20**	2.66	2.53
GA-HIDMS-PSO	2.40	2.83	2.43	**2.13**
A-HIDMS-PSO	2.86	2.26	**1.93**	2.73
BEPSO	2.70	**2.23**	2.36	2.50
AHPSO	3.06	**2.20**	**2.20**	2.33

10.4.6 The Impact of Position Initialisation Strategies

So far, we have assessed various aspects of the proposed algorithms however, despite the rigorous research on tuning PSO algorithms, one of the most overlooked aspects is the impact of different initial distribution strategies of particle positions on the general performance and the convergence of the algorithms. Mostly, PSO and majority of the metaheuristics (including the six proposed algorithms in this thesis) employ a uniform distribution to randomly generate the initial solutions in the search space. [255] analyses the impact of alternative initialisation distributions (`uniform`, `normal`, `logarithmic`, and `lognormal`) and in this section, we replicate the same study on the six proposed algorithms to validate the algorithms' performance using different distribution strategies and observe the sensitivity of the algorithms to the said distribution strategies in complex

search spaces. The assessment of the performance for the six proposed algorithm is carried out on the 6 100-dimensional multimodal CEC'17 problems (F4~F9) using the following distributions:

- `uniform`: *d*-th dimension is generated with a uniform distribution in $\left[B_d^{min}, B_d^{max}\right]$;
- `logarithmic`: *d*-th dimension is set as $\exp\left(\ln\left(B_d^{min}\right) + \ln(B_d^{max}) \cdot rnd\right)$, where rnd is a uniformly distributed random number in the range $[0,1]$.
- `normal`: each dimension of the particle position is sampled with the truncated normal distribution with (mean) $\mu = \frac{B_d^{min}+B_d^{max}}{2}$, and using different standard deviation values $\sigma = 1, \sigma = 3, \sigma = 5$.
- `lognormal`: each dimension is sampled using a lognormal distribution with (mean) $\mu = \frac{\ln(B_d^{min})+\ln(B_d^{max})}{2}$, using the standard deviation values $\sigma = 1, \sigma = 3, \sigma = 5$.

The results for the HIDMS-PSO (Figure 10-27) on the 100-dimensional F4~F9 suggest that the normal distribution strategy renders the best results in terms of convergence speed in all cases. However, we observed different σ values were best for different functions, case in point, when $\sigma = 3$ for F5, F6, F8 and F9 and $\sigma = 1$ for F7. In addition, the uniform and lognormal distributions also exhibited similar convergence trends to the normal distribution strategy. The worst convergence rate is consistently observed in the case of logarithmic (and lognormal $\sigma = 3$ and $\sigma = 5$) for all problems considered. In the case of FFQ-HIDMS-PSO, as shown in Figure 10-28, we observe the best convergence rate with normal ($\sigma = 1, \sigma = 3, \sigma = 5$), uniform and lognormal ($\sigma = 1$) distributions whilst the worst convergence rates are consistently rendered by the logarithmic and lognormal ($\sigma = 3$ and $\sigma = 5$) distributions. For the GA-HIDMS-PSO (Figure 10-29), unlike the other algorithms observed so far, the initial distribution strategies render marginal difference in the convergence of the algorithm. Hence, the initial distribution strategy exhibited minimal impact on the GA-HIDMS-PSO algorithm. The results on the A-HIDMS-PSO (Figure 10-30) suggest that for F5, F6 and F9, the best results in terms of convergence rate were obtained by the normal distribution ($\sigma = 1, \sigma = 3, \sigma = 5$) along with uniform (second-best) and lognormal ($\sigma = 1$) (third best) distributions. For F7 and F8, all distributions exhibit similar convergence trends however the best convergence rate is attained by the uniform distribution. The worst performance was consistently observed by the logarithmic ($\sigma = 3$, $\sigma = 5$) and lognormal distributions. The results for the BEPSO algorithm on the 100-dimensional multimodal F4~F9 suggest (Figure 10-31) that normal ($\sigma = 3$, $\sigma = 5$) and lognormal ($\sigma = 3$, $\sigma = 5$) distributions render best and second-best performance in terms of the convergence rates whilst the logarithmic exhibit the worst performance for F4, F5, F6, F8 and uniform distribution for F5 and F9. Similarly, as shown in Figure 10-32, in the case of the AHPSO algorithm, we observe that normal ($\sigma = 1, \sigma = 3, \sigma = 5$) (best) and uniform (second-best) distributions reveal the best convergence rates for all functions and the worst performances are attained by logarithmic and lognormal ($\sigma = 3$, $\sigma = 5$) distributions.

Overall, the most interesting results are observed in the case of the GA-HIDMS-PSO algorithm where the hybrid variant has approximately equal performance for all the initial particle distributions. Hence, GA-HIDMS-PSO appears much less sensitive to different initial distribution strategies. On the contrary, BEPSO and AHPSO appear to be significantly more sensitive to the initial positions of the particles to the extent that in some cases (e.g., logarithmic) the BEPSO algorithm exhibit no improvement throughout the search process which suggests that the algorithm gets stuck in a local optimum. It is worth noting that all the proposed algorithms utilised the uniform distribution to sample the initial particle positions in the main studies in this thesis. From the preliminary results presented in this section, we observe that the uniform distribution does not

appear to be the best strategy. Hence, although this preliminary analysis is too small scale to allow firm conclusions to be drawn, it does suggest that the performances of the proposed algorithms might be further improved by adopting more suitable distribution strategies for initial particle positions, which may be specific to the individual algorithms. This is an area well worth exploring in future work.

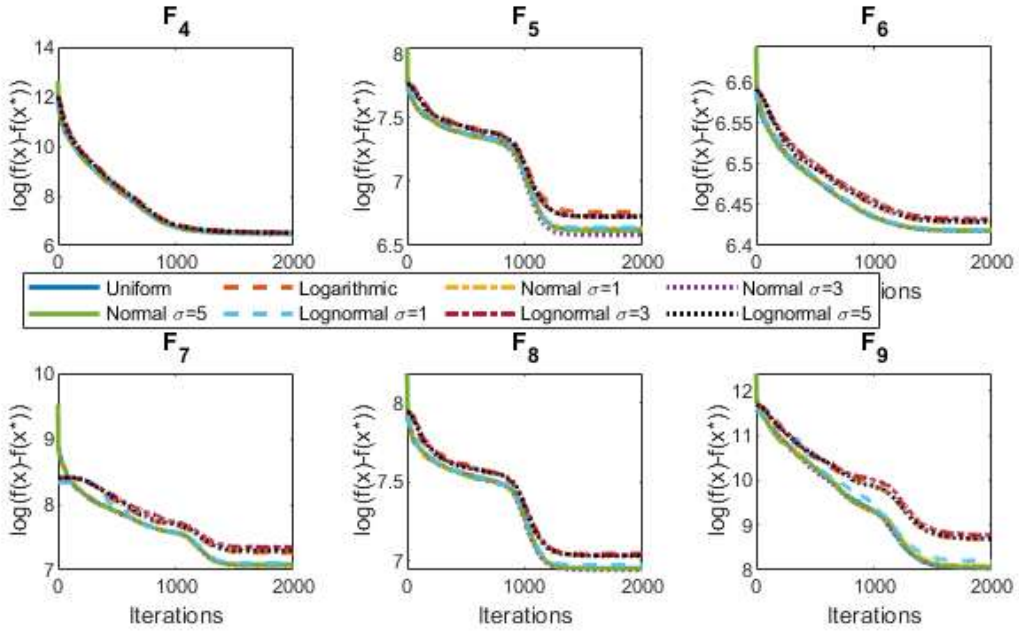


Figure 10-27 Impact of different initial position distributions on the HIDMS-PSO algorithm for 100-dimensional multimodal CEC'17 problems

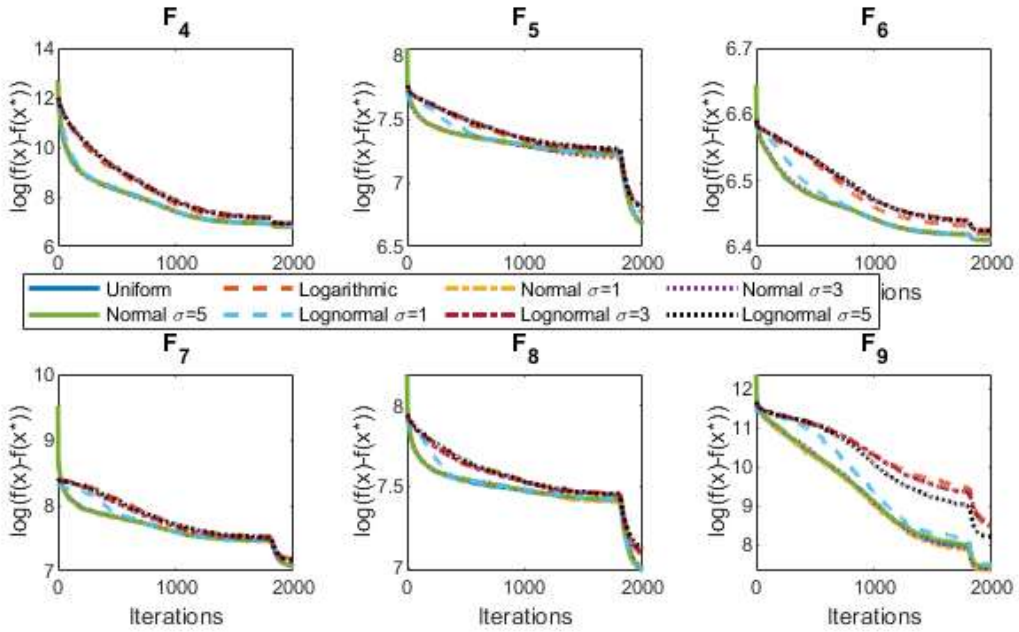


Figure 10-28 Impact of different initial position distributions on the FFQ-HIDMS-PSO algorithm for 100-dimensional multimodal CEC'17 problems

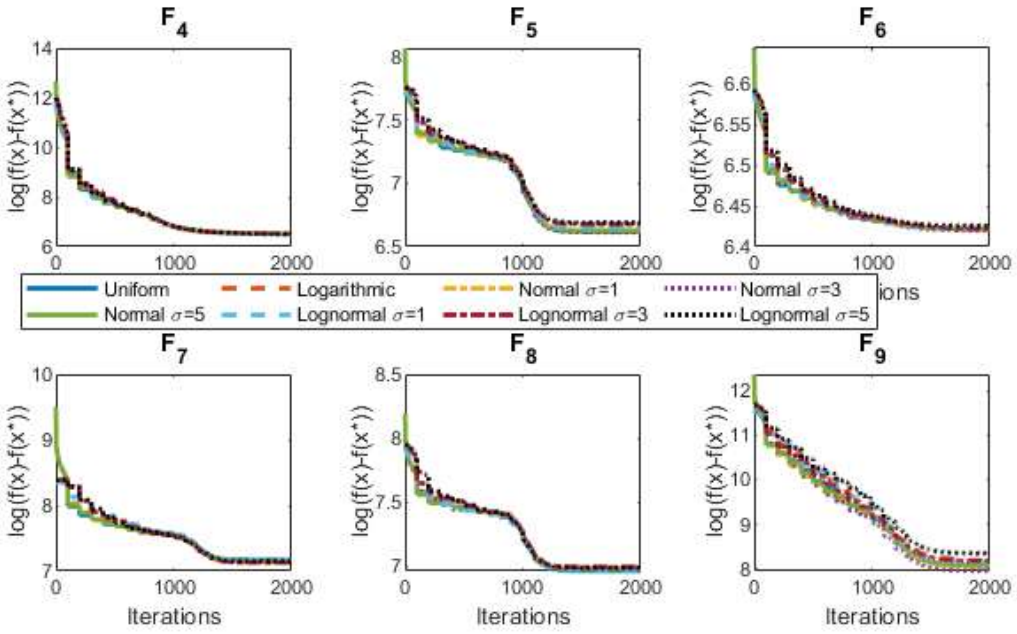


Figure 10-29 Impact of different initial position distributions on the GA-HIDMS-PSO algorithm for 100-dimensional multimodal CEC'17 problems

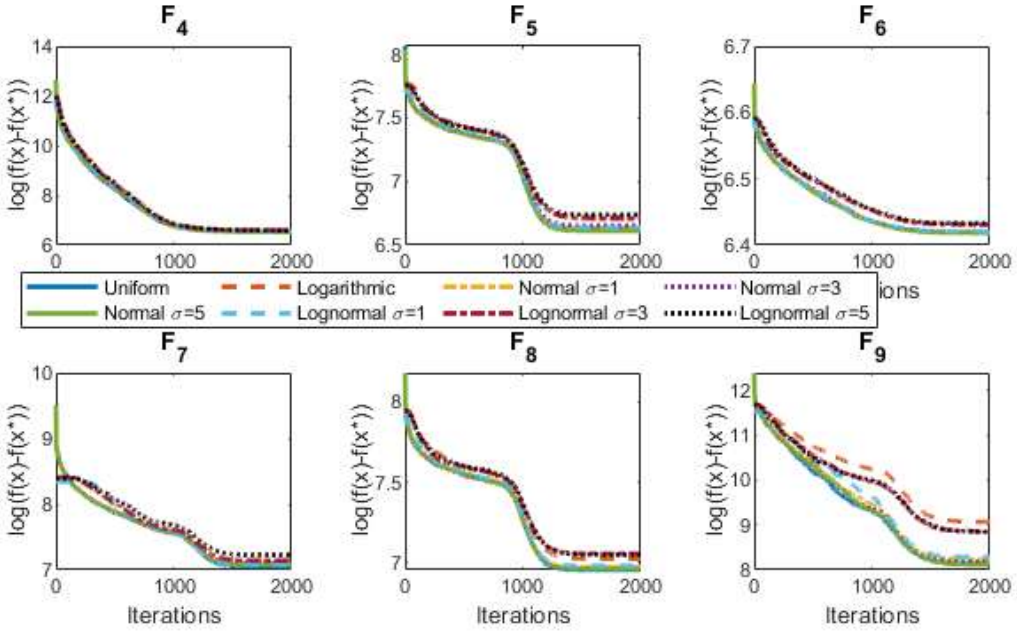


Figure 10-30 Impact of different initial position distributions on the A-HIDMS-PSO algorithm for 100-dimensional multimodal CEC'17 problems

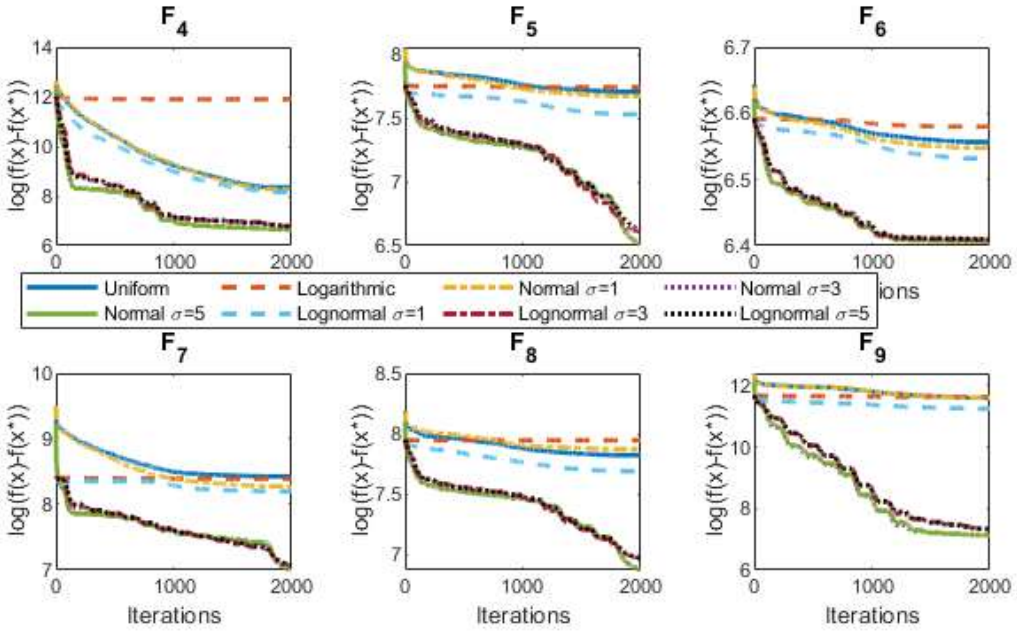


Figure 10-31 Impact of different initial position distributions on the BEPSO algorithm for 100-dimensional multimodal CEC'17 problems

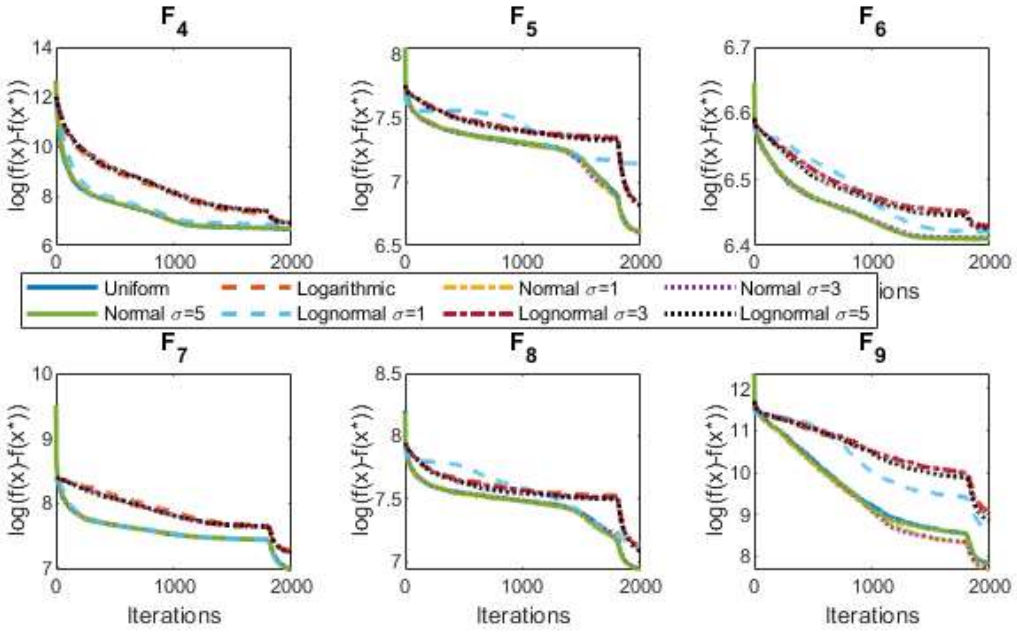


Figure 10-32 Impact of different initial position distributions on the AHPSO algorithm for 100-dimensional multimodal CEC'17 problems

10.4.7 Algorithm Complexity and Run Time Comparison

Further, Figure 10-33:10-34 displays the comparison of the average run time of the 14 test algorithms and the six proposed algorithms. The average run time is obtained over 10 individual consecutive runs on the 30 10-dimensional and 100-dimensional CEC'17 problems. All algorithms were vectorised, implemented, and executed using MATLAB on a machine with AMD Ryzen Threadripper 3990x 2.90 GHz 64-core processor.

The outcome reveals that, as predicted, for 10 and 100-dimensional problems, LPSO (*original PSO with linearly decreasing parameters*) exhibited the most optimal time efficiency. Among the proposed algorithms, for 10-dimensional CEC'17 problems, AHPSO is more time efficient and on the 100-dimensional problems FFQ-HIDMS-PSO and AHPSO are comparably more time efficient in comparison to the rest of the proposed algorithms. Moreover, in the 10-dimensional case, in general, BEPSO, HIDMS-PSO, FFQ-HIDMS-PSO, A-HIDMS-PSO and GA-HIDMS-PSO exhibited comparable average execution time to BBPSO, BRPSO, MiPSO, EPSO and FDR. The details of the execution times for each problem is presented in Appendix 7.

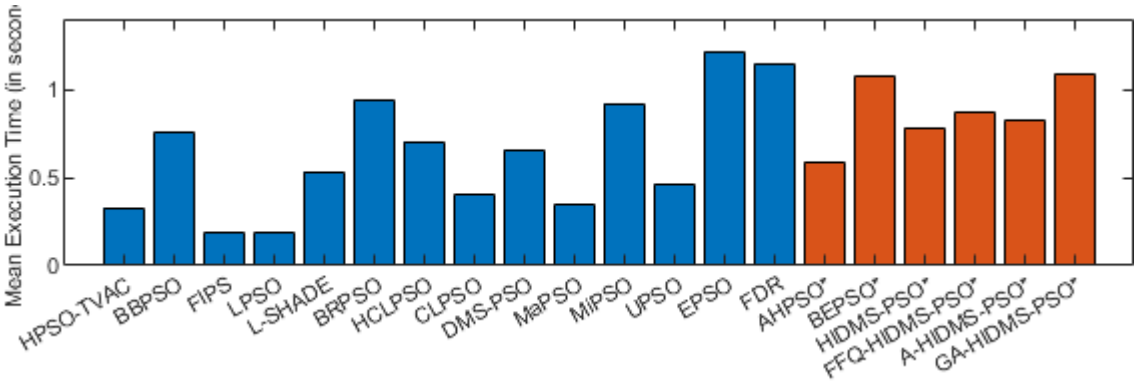


Figure 10-33 Comparison of average computational time for 10-dimensional CEC'17 problems.

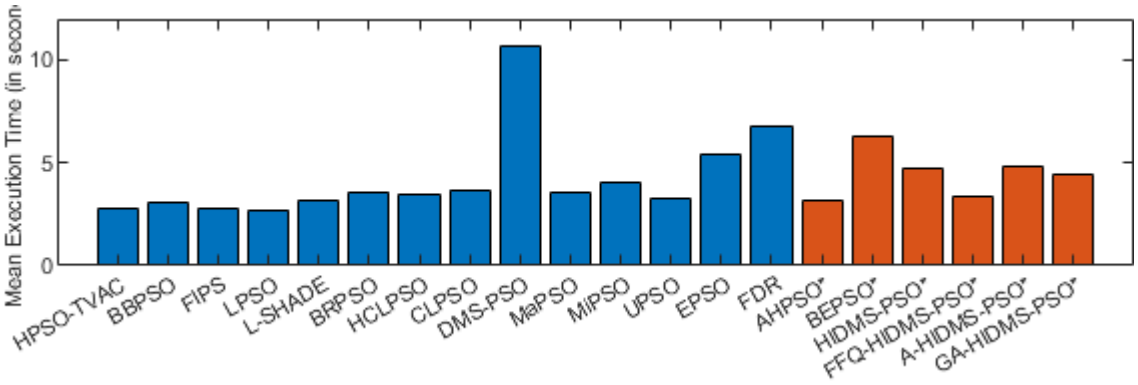


Figure 10-34 Comparison of average computational time for 100-dimensional CEC'17 problems.

Further, we examine the computation complexity of the six proposed algorithms and compare them with the 14 test algorithms employed in previous experiments. The computational complexity testing method [212] is adopted and the following steps are used to carry out the test:

Step A – Calculate the given code (see [212]) and record the computation time as T_0;
Step B – Calculate the computation time just for F_{14}. For $10^4 * 20$ function evaluations on dimension D as T_1;
Step C – Calculate the complete computation time for F_{14} with $10^4 * 20$ function evaluations on the same dimension as T_2;

Step D – Repeat *step C* 5 times and attain 5 individual T_2 values, $\overline{T_2}$=$mean(T_2)$.

Finally, the time complexity T_c is calculated as $T_c = \overline{T_2} - \frac{T_1}{T_0}$. The results for 10, 30, 50 and 100-dimensional problems are tabulated in Table 10-14. The results are also visualised in Figure 10-35 for each dimension for a more convenient comparison. In summary, the results corroborate with the comparison previously discussed in Figure 10-33:10-34. The lowest T_c values for 10, 30, 50 and 100-dimensional cases are attained by MaPSO and LPSO. On the contrary, the highest T_c values were obtained by EPSO (10-d) and DMS-PSO (30,50,100-d). Among the proposed algorithms, AHPSO consistently exhibited the lowest T_c values across all dimensions. We also notice that the difference in T_c values between LPSO and all proposed algorithms decrease with increased complexity. Case in point, LPSO obtained $T_c = 8.04$ and $T_c = 86.33$ for 10 and 100-dimensional problems whereas the calculations reveal AHPSO obtained $T_c = 22.30$ and $T_c = 94.93$ for the same dimensions. Hence, the computation complexity of LPSO increased 975% whereas the computation complexity of AHPSO exhibited 327% increase between 10-d and 100-dimensional cases. In general, we can see that the computational complexity and run time cost of the proposed algorithms compares well with the other test algorithms.

Table 10-14 Computational complexity values calculated at different dimensions

	T_c			
Algorithm	**10-d**	**30-d**	**50-d**	**100-d**
HPSO-TVAC	11.763	28.305	43.848	84.288
BBPSO	27.298	43.213	59.859	99.344
FIPS	8.253	25.816	44.311	88.579
LPSO	8.042	25.213	43.383	86.334
L-SHADE	18.926	34.853	54.291	100.676
BRPSO	32.888	46.892	68.843	111.462
HCLPSO	25.238	40.532	62.464	101.921
CLPSO	14.640	38.267	70.884	143.511
DMS-PSO	29.958	95.403	170.408	335.188
MaPSO	7.121	31.745	52.174	104.346
MiPSO	16.377	45.218	68.656	125.121
UPSO	15.910	35.698	55.273	100.492
EPSO	45.385	74.843	109.274	157.193
FDR	40.854	65.727	99.463	226.660
AHPSO*	22.302	36.296	57.048	94.930
HIDMS-PSO*	28.699	52.041	82.168	161.988
FFQ-HIDMS-PSO*	32.300	48.124	64.887	104.148
A-HIDMS-PSO*	43.940	65.162	94.263	186.176
GA-HIDMS-PSO*	36.047	71.363	91.887	136.668
BEPSO*	37.541	64.834	92.384	138.186

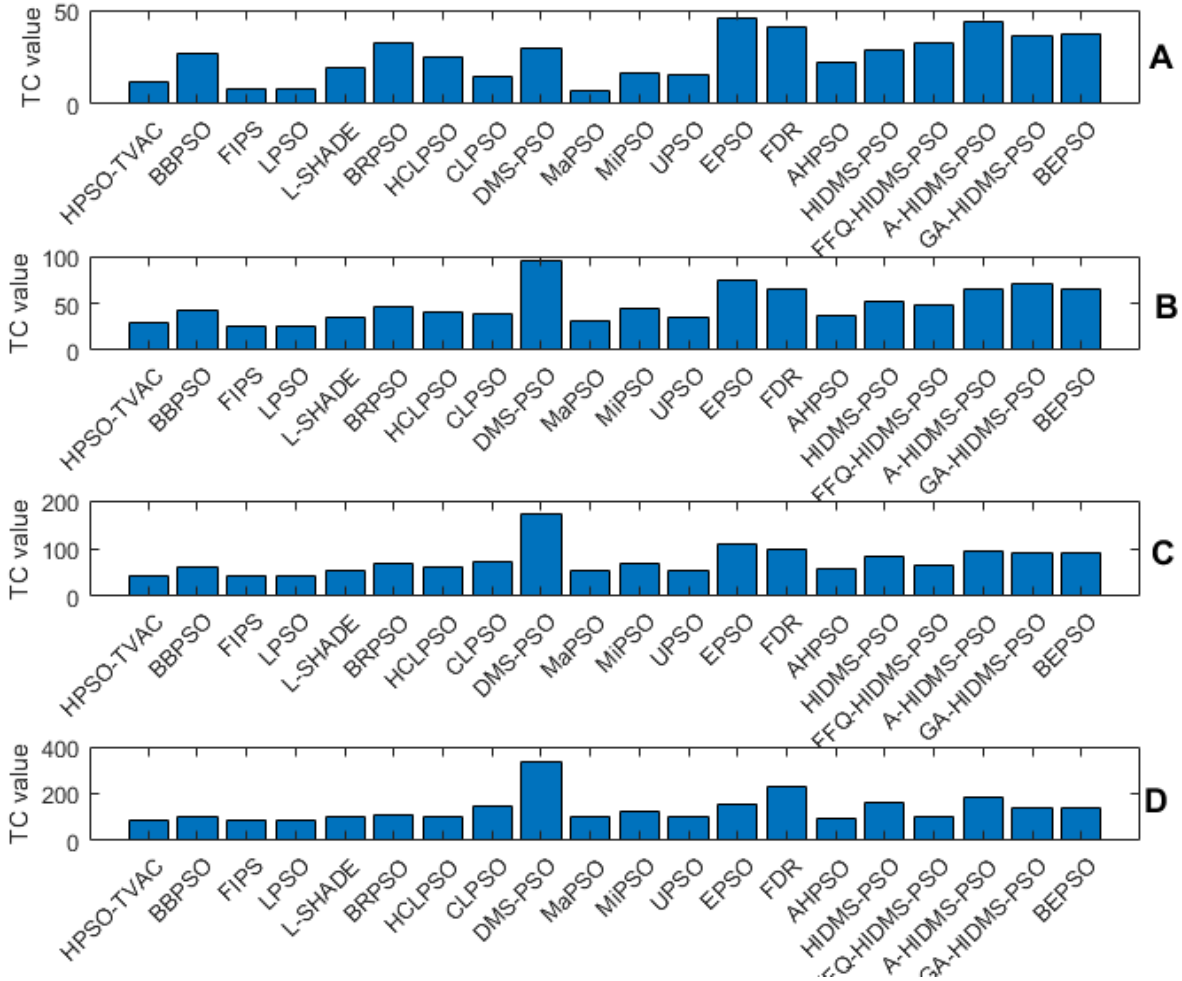


Figure 10-35 T_c values calculated for 10-d (A), 30-d (B), 50-d (C) and 100-d (D) problems

In addition to the brief analysis presented in this section, as part of future research, the phase transitions of the proposed algorithms may be explored. Related studies [256], [257] examine the varying behavioural instability of several canonical swarm-based algorithms (including PSO) to identify the likely factory that affects the phase transitions. The findings indicated that the algorithm under consideration was the key factor in phase transitions; however, it is unknown if these findings also hold for behaviourally more complex PSO variants.

10.5 Discussions

The comparison of the six proposed algorithms AHPSO, BEPSO, HIDMS-PSO, FFQ-HIDMS-PSO, GA-HIDMS-PSO and A-HDIMS-PSO were conducted on the 10, 30, 50 and 100-dimensional CEC'13, CEC'14 and CEC'17 problems (total of 87 problems). The final ranks obtained on the mentioned problems indicate that for the CEC'13 problems HIDMS-PSO obtained the 1st rank across 10, 50 and 100 dimensions and the 2nd rank on the 30-dimensional problems. Hence, we could conclude that the HIDMS-PSO algorithm exhibited superior average performance on the overall CEC'13 problems against the other 5 algorithms. The ranks obtained on the CEC'14 problems reveal that the GA-HIDMS-PSO algorithm exhibited superior overall mean performance on the low and medium size (10-d, 30-d) problems whilst HIDMS-PSO obtained the 1st rank on 50-d and A-HIDMS-PSO on 100-d problems. Finally, the final ranks on the CEC'17 problems indicate that BEPSO exhibited better overall mean performance compared to rest of the proposed algorithms on 10 and 30 dimensions

whilst for 50 and 100-dimensional problems, the FFQ-HIDMS-PSO and GA-HIDMS-PSO demonstrated better mean performance.

Furthermore, Figure 10-29 displays the total number of problems where the best and second-best solutions discovered by each algorithm across different dimensions of the 87 problems. As we can observe from the figure, HIDMS-PSO, GA-HIDMS-PSO and FFQ-HIDMS-PSO consistently obtained solutions with superior quality across all dimensions. It is worth noting that A-HIDMS-PSO also exhibited a very consistent performance and discovered the second-best solutions for more functions compared to any other algorithm under consideration. Figure 10-4 also indicates that the proposed algorithms are not significantly affected by problem dimensionality, this can be considered as an important trait, as naturally, most algorithms are expected to demonstrate less efficient performance with increased problem dimensions.

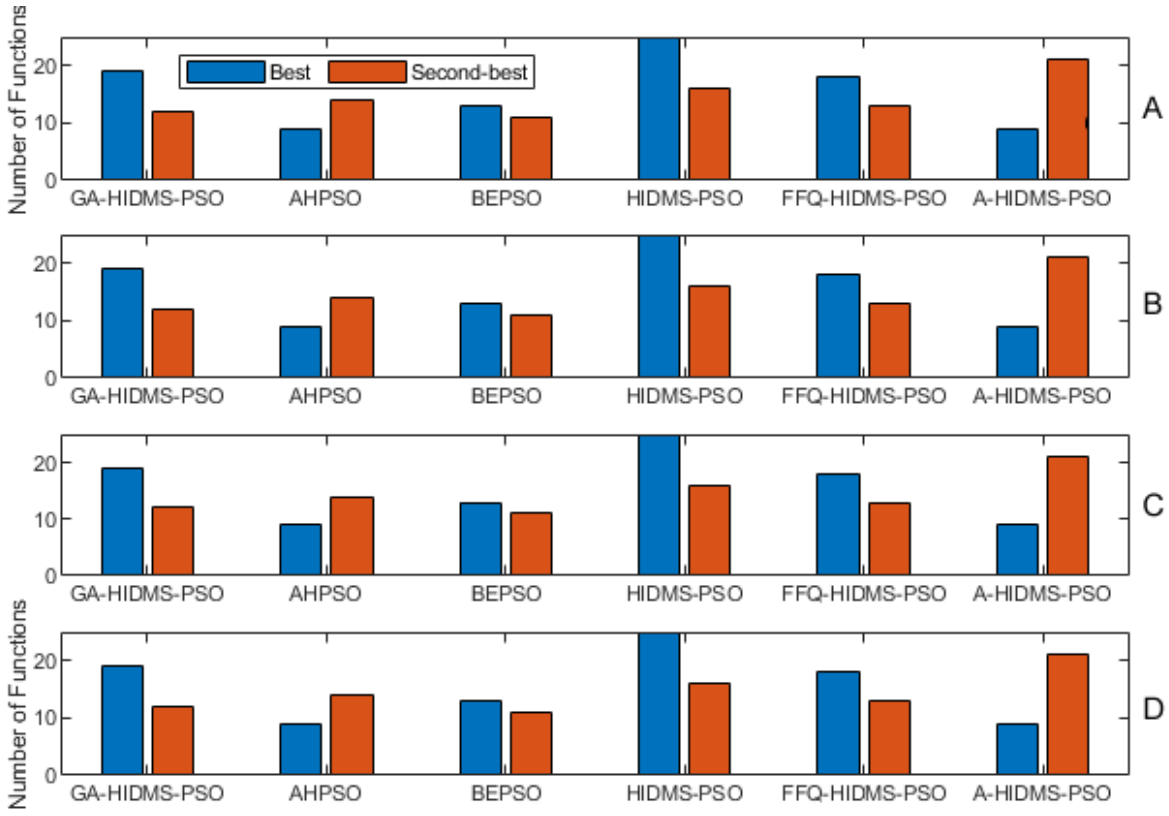


Figure 10-36 Total number of functions each algorithm attained the best and second-best performance across CEC'13, CEC'14, CEC'17 test suites at 10 (A), 30 (B), 50 (C) and 100 dimensions (D)

Moreover, Figure 10-30 display the performance of the proposed algorithms on different types of functions at various dimensions. In summary, we can observe at in general, GA-HIDMS-PSO exhibits better performance on multimodal, hybrid and composition functions across all experimented dimensions. On the other hand, for low dimensional problems, AHPSO appear to exhibit sufficient performance on unimodal, multimodal, hybrid and composition functions however with increased problems dimensions the algorithm continue to solve solely multimodal and composition functions. Similarly, the BEPSO algorithm mostly maintains its performance for unimodal, multimodal and composition functions. For low dimensional problems, the BEPSO algorithm appear more effective on the hybrid function. The FFQ-HIDMS-PSO algorithm is unable to exhibit a comparable trend to its peers for 10 dimensional problems. Besides a decent performance on the composition function, the algorithm only solves a small number of multimodal functions. However, the algorithm appears to

exhibit better performance on unimodal, multimodal and hybrid functions with higher dimensional problems. The performance of the HIDMS-PSO algorithm does not change significantly on different type of functions across four dimensions. We can observe that HIDMS-PSO can mostly solve multimodal and composition functions.

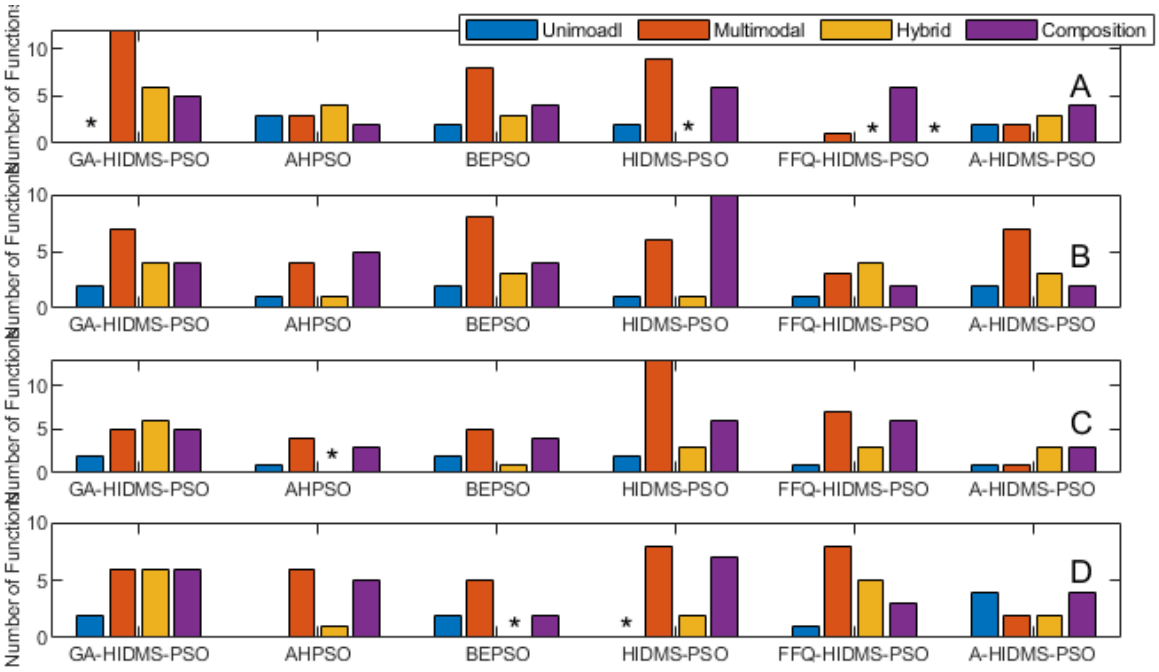


Figure 10-37 Performance of the proposed algorithms on the 10-D (A), 30-D (B), 50-D (C) and 100-D (D) unimodal, multimodal, hybrid and composition functions.

Overall, based on the conclusive evidence presented here and in other chapters of this thesis, we once again verify the degree of efficacy of the proposed algorithms on different types of functions and dimensions.

10.6 Conclusion

In this chapter, the six proposed algorithms are compared on the CEC'13, CEC'14 and CEC'17 problems based on the mean performance, their capability to cope with different types of functions and dimensions. In addition, the statistical significance was calculated, and the convergence characteristics compared. As a result, the comparison did not yield a single algorithm that demonstrated superior performance in all aspects of the comparison which is also in accordance with the NFL theorem. However, as discussed in the previous section, all proposed algorithms offer various desirable traits required (*e.g., fast convergence, coping with dimensionality*) in general-purpose optimisers. Overall, these algorithms have been shown to be highly competitive.

Chapter 11 : Applications to Constrained Real-world Optimisation Problems

So far, we have evaluated the novel algorithms proposed in this thesis on large, widely used test suites of unconstrained problems that are suitably challenging. In this chapter, we expand the investigation by comparing the performance of the six proposed, and 14 comparative test algorithms, on 29 constrained real-world problems [258]. The details of the problems are displayed in Table 11-1. Each problem was tested 30 times, and the number of function evaluations for each problem are determined using the following criteria:

$$Max_{FEs} = \begin{cases} 1 \times 10^5, & \text{if } D \leq 10 \\ 2 \times 10^5, & \text{if } 10 < D \leq 30 \\ 4 \times 10^5, & \text{if } 30 < D \leq 50 \end{cases}$$

Where Max_{FEs} is the maximum number of function evaluations, and D is the dimension of the problem.

Considering the objective of this thesis, that the proposed algorithms are intended as *general-purpose optimisers*, we expect them to cope or at least exhibit acceptable performance on a number of problems with diverse characteristics, including on non-convex constrained real-world problems, hence, in the experiments described in this chapter we maintain the same parametric settings previously used for all algorithms.

Table 11-1 Details of the 29 Constrained Real-World Optimisation Problems. D is the total number of decision variables, g and h are the number of inequality and equality constraints, respectively and $f(\overrightarrow{x^*})$ is the best known objective function value.

Prob	Name	D	g	h	$f(\overrightarrow{x^*})$
Industrial Chemical Processes					
RC01	Optimal Operation of Alkylation Unit	7	14	0	$-4.5291197395\text{E}+03$
Process Synthesis and Design Problems					
RC02	Process synthesis problem	2	2	0	$2.0000000000\text{E}+00$
RC03	Process synthesis and design problem	3	1	1	$2.5576545740\text{E}+00$
RC04	Process flow sheeting problem	3	3	0	$1.0765430833\text{E}+00$
RC05	Two-reactor problem	7	4	4	$9.9238463653\text{E}+01$
RC06	Process synthesis problem	7	9	0	$2.9248305537\text{E}+00$
RC07	Process design Problem	5	3	0	$2.6887000000\text{E}+04$
RC08	Multi-product batch plant	10	10	0	$5.3638942722\text{E}+04$
	Mechanical Engineering Problem				
RC09	Weight Minimization of a Speed Reducer	7	11	0	$2.9944244658\text{E}+03$
RC10	Optimal Design of Industrial refrigeration System	14	15	0	$3.2213000814\text{E}-02$
RC11	Tension/compression spring design (case 1)	3	3	0	$1.2665232788\text{E}-02$
RC12	Pressure vessel design	4	4	0	$5.8853327736\text{E}+03$
RC13	Welded beam design	4	5	0	$1.6702177263\text{E}+00$
RC14	Three-bar truss design problem	2	3	0	$2.6389584338\text{E}+02$
RC15	Multiple disk clutch brake design problem	5	7	0	$2.3524245790\text{E}-01$
RC16	Step-cone pulley problem	5	8	3	$1.6069868725\text{E}+01$
RC17	Robot gripper problem	7	7	0	$2.5287918415\text{E}+00$
RC18	Hydro-static thrust bearing design problem	4	7	0	$1.6254428092\text{E}+03$
RC19	10-bar truss design	10	3	0	$5.2445076066\text{E}+02$
RC20	Rolling element bearing	10	9	0	$1.4614135715\text{E}+04$
RC21	Gas Transmission Compressor Design (GTCD)	4	1	0	$2.9648954173\text{E}+06$
RC22	Gear train design Problem	4	1	1	$0.0000000000\text{E}+00$
RC23	Himmelblau's Function	5	6	0	$-3.0665538672\text{E}+04$
Power Electronic Problems					
RC24	SOPWM for 3-level Inverters	25	24	1	$3.8029250566\text{E}-02$
RC25	SOPWM for 5-level Inverters	25	24	1	$2.1215000000\text{E}-02$

RC26	SOPWM for 7-level Inverters	25	24	1	$1.5164538375E-02$
RC27	SOPWM for 8-level Inverters	30	29	1	$1.6787535766E-02$
RC28	SOPWM for 11-level Inverters	30	29	1	$9.3118741800E-03$
RC29	SOPWM for 13-level Inverters	30	29	1	$1.5096451396E-02$

11.1 Discussion of Results

In this section, the results obtained on the constrained real-world problems are summarised. The obtained results are assessed based on the mean performance, the best and the second-best solutions obtained for each problem.

For the first category, the industrial chemical process problems, the lowest mean error value for RC01 is obtained by EPSO, and the best solution is attained by DMS-PSO.

The results for the seven process synthesis and design problems indicate that for RC02, all test algorithms obtained identical mean error values and the best solution. The LPSO algorithm obtained the lowest mean error and the best solution for RC03 whilst the second-best solutions is provided by FIPS. For RC04, EPSO and the proposed algorithm HIDMS-PSO obtained the same lowest mean error values however all test algorithms provided an equal performance in the discovered best solution. The LPSO algorithm exhibited the lowest mean error value on RC05 whilst discovering the best solution for it, the second-best solution for the same problem is obtained by the proposed FFQ-HIDMS-PSO algorithm. The results obtained on the RC06 indicate that the lowest mean error value is obtained by the proposed HIDMS-PSO algorithm whilst the best and second-best solutions are provided by L-SHADE and BBPSO, respectively. For RC07, all algorithms obtained the same mean error values and discovered the identical best solutions. EPSO exhibited best mean performance and discovered the best solution on RC08 whilst the second-best solution is attained by FIPS.

The results obtained on the 15 mechanical engineering problems show that for RC09, L—SHADE, HCLPSO, EPSO and the proposed algorithm BEPSO obtained the lowest mean error values whilst the same best solution is provided by BBPSO, FIPS, LPSO, L-SHADE, HCLPSO, DMS-PSO, UPSO, EPSO, FDR, AHPSO, BEPSO and FFQ-HIDMS-PSO. For the optimal design of industrial refrigeration system (RC10), the EPSO algorithm exhibited the best mean performance whilst the best solution of provided by L-SHADE and for RC11 the lowest mean error and the best solution is attained by L-SHADE whilst the second-best solution is discovered by the proposed AHPSO algorithm. For the problem RC12, EPSO achieved the lowest mean error whilst the same best solution is discovered by BBPSO, LPSO, L-SHADE, HCLPSO, DMS-PSO, UPSO and EPSO. Similarly, for RC13, EPSO attained the lowest mean error value, and the same best solution is provided by BBPSO, L-SHADE, HCLPSO, DMS-PSO, UPSO, EPSO and FDR. The results on the RC14 indicates that the lowest mean error value is obtained by the proposed HIDMS-PSO algorithm and, the best and the second-best solutions are discovered only by L-SHADE and DMS-PSO, respectively, whilst for RC15 all considered algorithms achieved the same mean error and best solutions. For RC16 and RC17, L-SHADE obtained the top mean performance and discovered the best solution for the two real-world problems whilst for the same two problems the second-best solutions are attained by BBPSO and EPSO, respectively. The AHPSO algorithm exhibited the best mean performance for RC18 whilst L-SHADE provided the best solution and the HIDMS-PSO algorithm obtained the second-best solution. Furthermore, the HIDMS-PSO algorithm also obtained the lowest mean error value for RC19, and L-SHADE provided the best solution for the same problem whilst the second-best solution for RC19 is attained by FFQ-HIDMS-PSO. FIPS, L-SHADE, EPSO, A-HIDMS-PSO, HIDMS-PSO, FFQ-HIDMS-PSO and GA-HIDMS-PSO achieved the lowest mean error values for RC20 whilst all test algorithms discovered the same best solution for this problem. L-SHADE obtained the lowest mean error and the second-best solution on RC21,

and the best solution for the same problem is provided by DMS-PSO. Similarly, DMS-PSO, LPSO and EPSO attained the best mean performance on RC22 and the same best solution is found by all test algorithms employed in the experiment. The results obtained on the last mechanical engineering problems RC23, the top mean performance is exhibited by FIPS, LPSO, L-SHADE, DMS-PSO, UPSO and EPSO whilst the same best solution is provided by multiple algorithms including BBPSO, FIPS, LPSO, L-SHADE, HCLPSO, DMS-PSO, UPSO, EPSO, FDR, AHPSO, BEPSO and FFQ-HIDMS-PSO.

The experimental results obtained on the six power electronic problems RC24~RC29 indicate that for RC24, the lowest mean error value and the best solution is attained by L-SHADE whilst the second-best solution is achieved by GA-HIDMS-PSO. The proposed AHPSO algorithm managed to attain the best mean performance for RC25, for the same problem, the best and the second-best solutions are obtained by L-SHADE and UPSO, respectively. The proposed A-HIDMS-PSO algorithm exhibited the best mean performance on RC26 whilst FFQ-HIDMS-PSO discovered the best solution and the LPSO algorithm obtained the second-best solution for it. The results on the RC27 indicate that the proposed HIDMS-PSO algorithm exhibited the best mean performance and, the best and second-best solutions are attained by FDR and BEPSO, respectively. Once again on the RC28, the proposed AHPSO algorithm obtained the lowest mean error values whilst EPSO provided the best solution. The same second-best solution for RC28 is found by BBPSO and AHPSO. Lastly, the results on the RC29 yields that, once again AHPSO exhibited the best mean performance. In addition, the proposed algorithm AHPSO also discovered the second-best solution for this problem whilst the best solution is provided by L-SHADE.

Figure 11-1:11-4 display the total number of constrained real-world problems each algorithm exhibited the best mean performance for. As it can be observed from the figures that for the seven process synthesis and design problems, the six proposed algorithms exhibited comparable mean performance on number of problems in comparison to most of the algorithms employed in the experiments. On the mechanical engineering problems (*see Figure 11-2*), L-SHADE, EPSO and HIDMS-PSO exhibited better mean performance for more problems compared to rest of the algorithms however, L-SHADE attained lower mean error values for majority of the problems. Lastly, as shown in Figure 11-3, only 4 of the algorithms stand out for the power electronic problems. For the 5 of the 6 power electronic problems, the best mean performances are exhibited by the proposed algorithms AHPSO, A-HIDMS-PSO and HIDMS-PSO.

Overall, Figure 11:4 display the total number of problems each algorithm obtained the lowest mean error value for the entire constrained problem set. Although, all six proposed algorithms exhibited acceptable performance on the constrained real-world problems, the mean performance of HIDMS-PSO and AHPSO appear to be superior with respect to better mean performance exhibited on more problems. Moreover, as shown in Table 11-31, the mean and final ranks calculated based on the mean error values indicate that the HIDMS-PSO algorithm outranked all test algorithms and achieved the 1^{st} rank on the 29 non-convex constrained real-world problems.

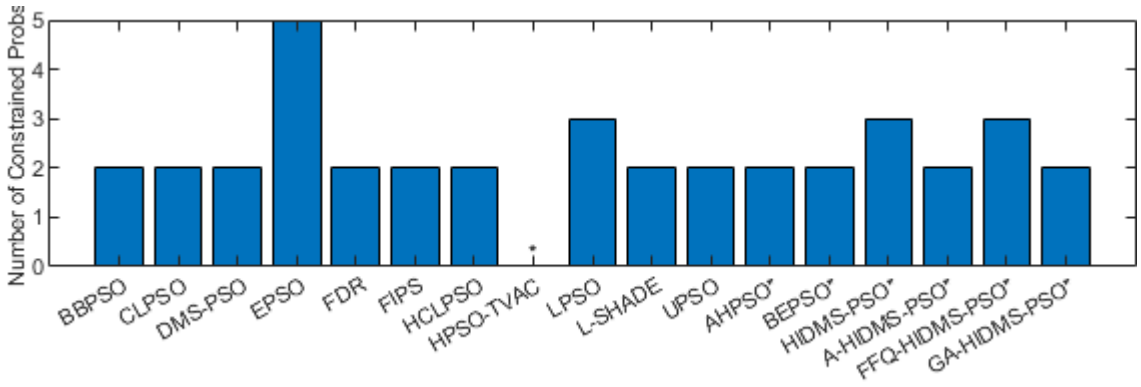


Figure 11-1 Number of process synthesis and design problems solved by each algorithm (* = zero functions).

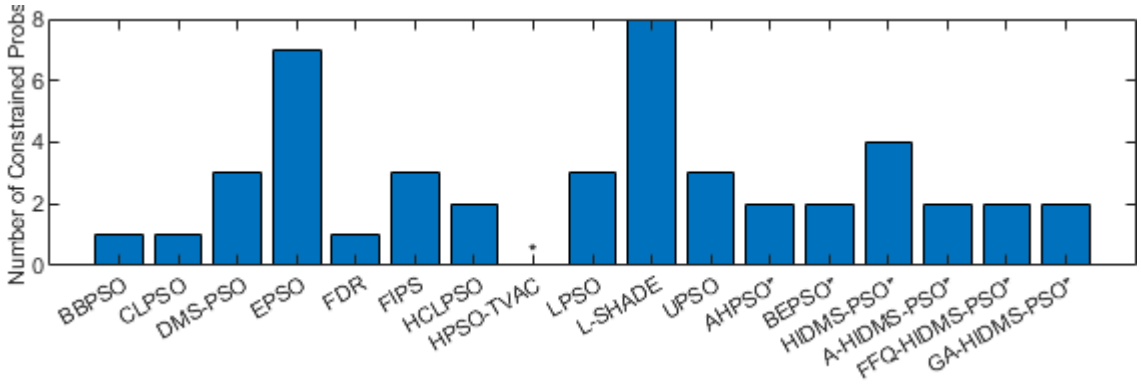


Figure 11-2 Number of mechanical engineering problems solved by each algorithm (* = zero functions).

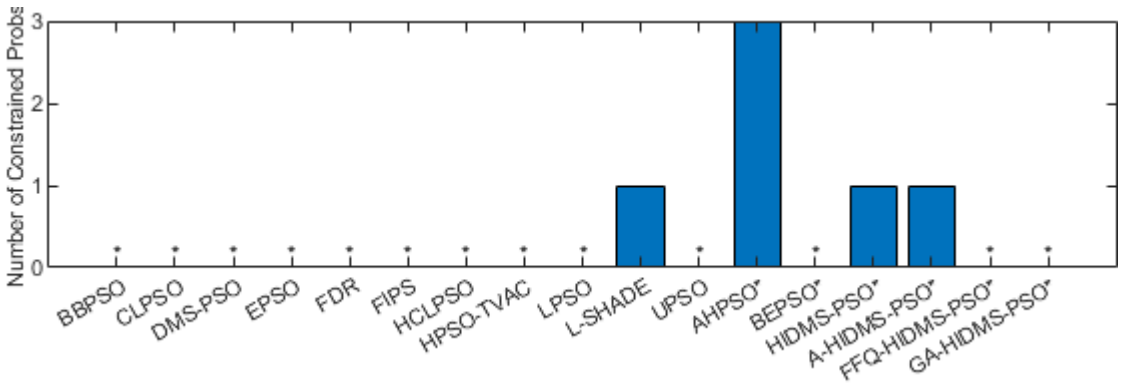


Figure 11-3 Number of power electronic problems solved by each algorithm (* = zero functions).

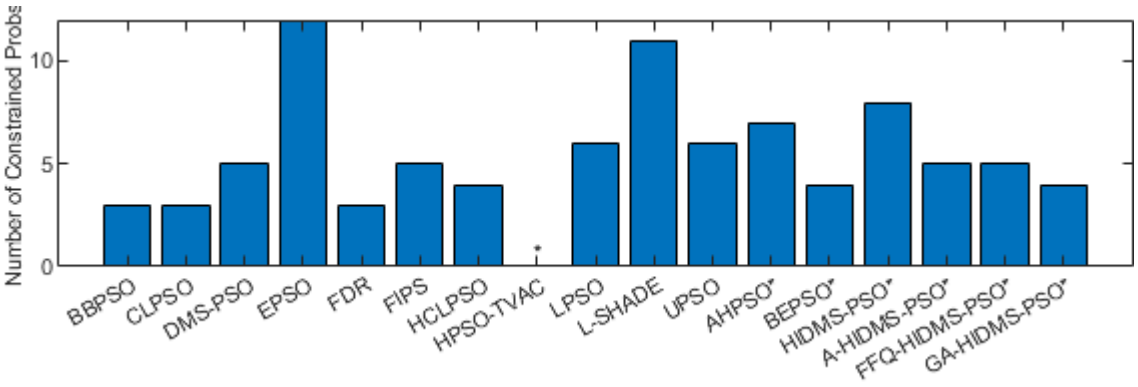


Figure 11-4 Total number of constrained real-world problems solved by each algorithm (* = zero functions).

Table 11-2 Mean and final ranks obtained on the 29 constrained real-world problems.

	BBPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	UPSO	AHPSO*	BEPSO*	HIDMS-PSO*	A-HIDMS-PSO*	FFQ-HIDMS-PSO*	GA-HIDMS-PSO
Mean Rank	10.97	11.59	11.24	4.48	8.17	8.00	7.45	14.72	6.14	5.17	5.48	6.62	5.59	4.41	6.17	7.38	5.93
Final Rank	14	16	15	2	13	12	11	17	7	3	4	9	5	1	8	10	6

Overall, for the majority of the real-world problems, despite not achieving the best performance, the proposed algorithms exhibited comparable performance in comparison to most of the algorithms employed in the experiment. Tables of the detailed results of the experiments can be found in Appendix 6.

11.2 Conclusions

In this chapter, the proposed algorithms AHPSO, BEPSO, HIDMS-PSO, A-HIDMS-PSO, FFQ-HIDMS-PSO and GA-HIDMS-PSO along with 11 PSO variants and L-SHADE were tested on 29 real-world constrained problems. The 29 problems are comprised of four categories including a single industrial chemical process problem, 7 process synthesis and design, 15 mechanical engineering and 6 power electronic problems. According to [258], the entire list of problems displayed in Table 11-1 are challenging real-world problems . More specifically, based on the analysis on the outcomes [258], the hardest problems are identified as RC01, RC08, RC18, RC24, RC25, RC26, RC27, RC28 and RC29. As shown in Table 11-1 and Figures 11-1:11-4, the proposed algorithms exhibited comparable performance in comparison to the other algorithms employed in the experiments. In addition, for the aforementioned 9 hardest problems, the proposed algorithms AHPSO (RC18, RC25, RC28, RC29), A-HIDMS-PSO (RC26) and HIDMS-PSO (RC27) exhibited better mean performance compared to the rest of the algorithms on 6 of the 9 most difficult problems. Hence, this verifies the effectiveness of the proposed algorithms on different types of problems. It is also worth noting that the proposed algorithms' capability to adapt and effectively attempt to solve different types of problems even without tuning any parameters is a convenient user interface for the tuning process for a given problem. Hence, this further demonstrates their suitability to be applied to constrained real-world problems as general-purpose optimisers. Furthermore, it is also worth recalling that that the proposed algorithms have previously exhibited highly competitive performance on various unconstrained problems.

Chapter 12 : Performance Comparison of the Proposed Algorithms with Additional Experimental Results

12.1 Introduction

In this chapter, we reaffirm the performance of the proposed algorithms by comparing their results against the experimental results published in other research by other authors [220], [259]. The previous chapters have mainly concentrated on comparisons with other PSO algorithms, here we also look at a range of other popular bio-inspired search methods. For the first and second experiments, the results of the proposed algorithms are compared with 8 PSO variants already employed in the previously conducted experiments (HCLPSO, PSO, FIPS, UPSO, CLPSO, HPSO-TVAC and sHPSO) on the 30-dimensonal CEC'05 and CEC'14 problems. In the third experiment, the results of the six proposed algorithms are compared on the 30-dimensional CEC'17 problems against 10 commonly used bio-inspired algorithms: bat algorithm (BA) [260], grey wolf optimisation (GWO) [261], butterfly optimisation algorithm (BOA) [262], whale optimisation algorithm (WOA) [263], moth flame optimisation (MFO) [264], artificial bee colony (ABC) [265] and ABC with different natural selection methods (G/T/L/E) with new versions of onlooker bee [266].

It is worth noting that for the first experiment on the CEC'05 benchmark problems, we replicated the experiment detailed in [220], and as with other experiments conducted in previous chapters, the parameter settings of the proposed algorithms are untuned (*see Table 3-1*). For the second and third experiment (on CEC'14 and CEC'17), we use the previously generated experimental results for the proposed algorithms as the experimental setup detailed in [259] is identical to the setup utilised in this thesis.

12.2 Performance Comparison with PSO variants on 30-dimensional CEC'05 Problems

The experimental results on the 30-dimensional CEC'05 problems for HIDMS-PSO, FFQ-HIDMS-PSO, GA-HIDMS-PSO, A-HIDMS-PSO, AHPSO and BEPSO against HCLPSO, PSO, FIPS, UPSO, CLPSO, OLPSO, HPSO-TVAC and sHPSO are presented in Appendix 2.

12.2.1 The Comparison of HIDMS-PSO

In summary, the results for the HIDMS-PSO algorithm indicate that for unimodal F1~F5, the proposed algorithm obtained the lowest mean error values on F4, F6, HCLPSO performed best on F1, F3 and UPSO on F2. For multimodal F6~F14, HCLPSO exhibited the best performance on most multimodal problems including F6, F9, F12, F13 and F14 whilst the proposed algorithm HIDMS-PSO attained the lowest mean error values on F10 and F11. On the other hand, OLPSO and sHPSO performed best on F7 and F8, respectively. For hybrid composition problems F16~F25, HCLPSO exhibited the best mean performance on F16, F21, F23~F25, FIPS on F18~F20, F22 whilst CLPSO and HIDMS-PSO performed best on F15 and F17, respectively.

Figure 12-1 displays the total number of problems the best mean performance exhibited by each algorithm. As apparent, HCLPSO exhibited the best mean performance for the majority of 30-diensional CEC'05 problems followed by the performance of the proposed algorithm HIDMS-PSO.

Moreover, the mean and final ranks suggest that the proposed algorithm HIDMS-PSO ranked 2nd with the average rank of 3.36 whilst HCLPSO attained the 1st rank with the average rank of 2.16.

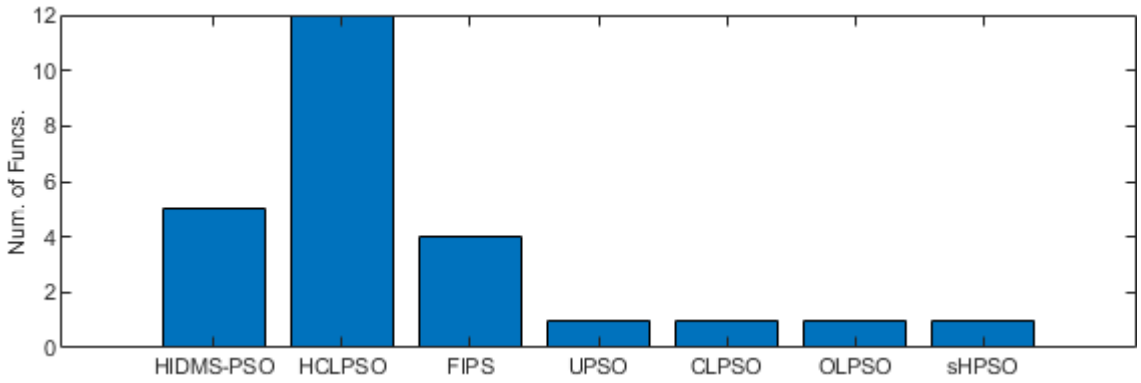


Figure 12-1 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.2.2 The Comparison of FFQ-HIDMS-PSO

The comparison of FFQ-HIDMSPSO against eight PSO variants on 30-dimensional CEC'05 problems reveal that on unimodal problems (F1~F5), FFQ-HIDMS-PSO (F4, F5) and HCLPSO (F1, F3) each performed best on two problems whilst UPSO attained the lowest mean error value on F2. For multimodal problems (F6~F14), HCLPSO exhibited the best mean performance for four problems including F6, F9, F12 and F13 whereas the proposed algorithm FFQ-HIDMS-PSO performed best on three problems F10, F11 and F14. Moreover, the lowest mean errors for F7 and F8 are obtained by OLPSO and sHPSO, respectively. For hybrid composition problems (F15~F25), the HCLPSO algorithm demonstrated the best mean performance for majority of the hybrid composition problems including F16, F17, F21, F23~F25. On the other hand, FIPS and CLPSO obtained the best performance for F18~F20, F22, and F15, respectively.

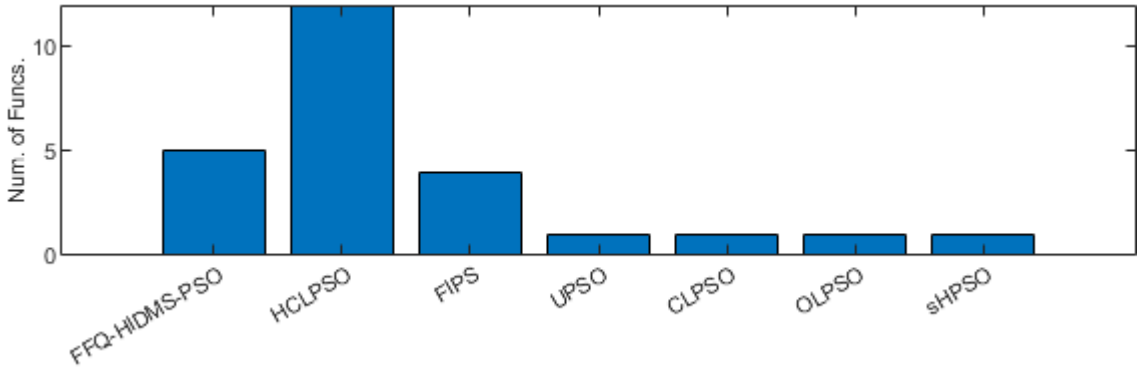


Figure 12-2 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

As shown in Figure 12-2, the HCLPSO algorithm performed best on the majority (12) of problems. On the other hand, the proposed algorithm FFQ-HIDMS-PSO exhibited the best mean performance on second-most (5) problems. In summary, the FFQ-HIDMS-PSO algorithm rank 3rd with the mean rank of 3.92. On the other hand, CLPSO and HCLPSO ranked 2nd and 1st with mean ranks of 3.88 and 2.12, respectively.

12.2.3 The Comparison of GA-HIDMS-PSO

The results comparison of GA-HIDMS-PSO and eight PSO variants indicate that on the unimodal problems F1~F5, the overall superior performance is exhibited by GA-HIDMS-PSO including achieving the lowest meane errors on F2, F4 and F5. On the other hand, HCLPSO exhibited superior mean performance for F1 and F3. For multimodal problems F6~F14, HCLPSO performed best on F6, F9, F12 and F13 whereas GA-HIDMS-PSO attained the lowest mean errors on F10, F11 and F14. Moreover, OLPSO and sHPSO exhibited the best mean performances for a single problem, F7 and F8, respectively. The results on the hybrid composition problems (F15~F25) yield that the best mean performance for F17, F21, F23~F25 are achieved by HCLPSO, and FIPS exhibited the best performance on F18~F20, F22 whilst CLPSO and GA-HIDMS-PSO individually solved a single problem F15 and F16, respectively.

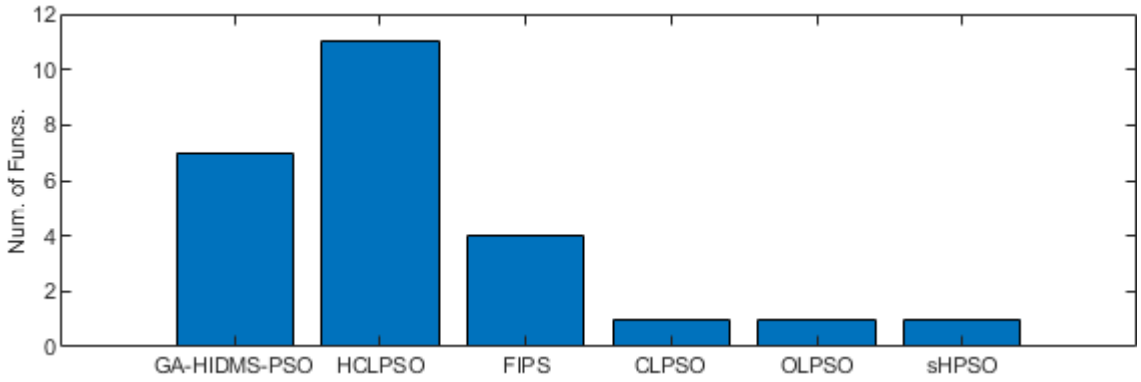


Figure 12-3 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

As shown in Figure 12-3, HCLPSO algorithm's overall mean performance appears superior with regards to best performance exhibited on more problems (11) compared to other algorithms. For the same problem set, the proposed algorithm GA-HIDMS-PSO exhibited superior mean performance for more problems (7) compared to all PSO variants except HCLPSO. Further, the mean and final ranks on the 30-dimensional CEC'05 problems indicate that the proposed algorithm GA-HIDMS-PSO obtained the 2nd final rank with the average of 3.16 whilst HCLPSO ranked 1st with the average rank of 2.24.

12.2.4 The Comparison of A-HIDMS-PSO

The comparison of the results for the A-HIDMS-PSO algorithm suggests that on the unimodal problems (F1~F5), HCLPSO and A-HIDMS-PSO each performed best on two unimodal problems F1, F3 and F4, F5, respectively, whilst UPSO attained the lowest mean error on F2. On the multimodal problems (F6~F14), the best mean performance for F6, F9, F12, F13 are achieved by HCLPSO. Similarly, for F10, F11 and F14, the best performance is attained by the proposed algorithm A-HIDMS-PSO, and finally, OLPSO and sHPSO exhibited the best mean performance on F7 and F8, respectively. For 11 hybrid composition problems (F15~F25), HCLPSO and FIPS performed best on F16, F21, F23~F25 and F18~F20, F22, respectively, whilst CLPSO and A-HIDMS-PSO attained the lowest mean errors for F15 and F17, respectively. Overall, as shown in Figure 12-4, the A-HIDMS-PSO algorithm performed best on 6 of the 25 problems and ranked 2nd (avg.=3.28) behind HCLPSO (1st with avg. rank=2.2).

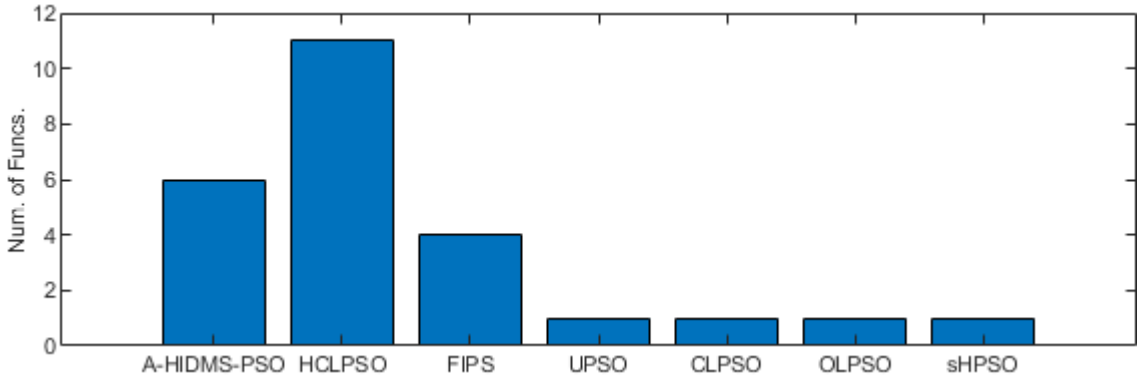


Figure 12-4 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.2.5 The Comparison of BEPSO

The comparison of BEPSO's performance against the eight PSO variants yields that for unimodal problems (F1~F5), HCLPSO (F1, F3) and BEPSO (F4, F5) obtained the best performance on two unimodal problems whilst UPSO attained the lowest mean error for F2. For the nine multimodal problems (F6~F14), HCLPSO performed best on F6, F9, F12 and F13 and BEPSO exhibited the best mean performance on F10, F11, F14 whereas OLDPSO and sHPSO achieved the lowest mean error values on F7 and F8, respectively. Lastly, for the 11 hybrid composition problems (F15~F25), HCLPSO exhibited superior mean performance on F16, F17, F21, F23~F25, FIPS on F18~F20, F22 and CLPSO on a single problem F15. We observe from Figure 12-5 that the proposed algorithm BEPSO exhibited the second-best overall mean performance in the number of problems it exhibited the best performance for and attained the 4th final rank on the 30-dimensional CEC'05 problems (*BEPSO ranked 4th, CLPSO ranked 3rd, OLPSO ranked 2nd and HCLPSO ranked 1st with avg. ranks 4.12, 3.92, 3.84 and 2.12, respectively*).

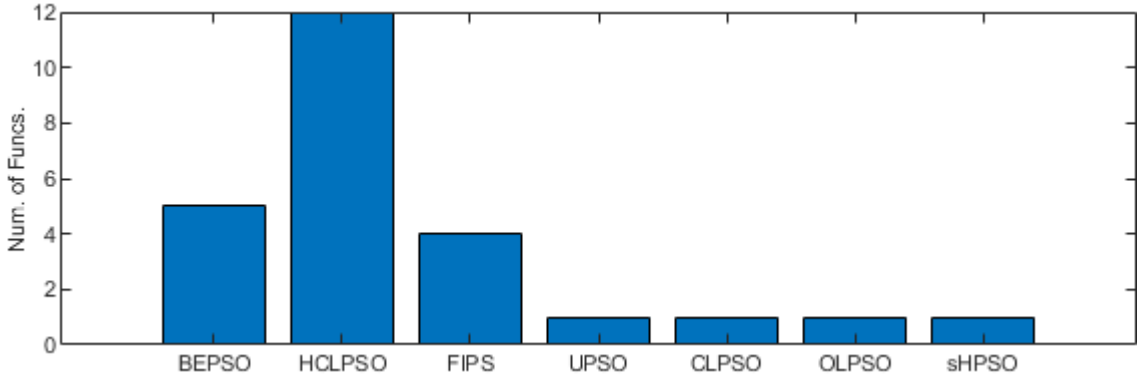


Figure 12-5 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.2.6 The Comparison of AHPSO

The comparison of results for the AHPSO algorithm and eight PSO variants on the 30-dimensional CEC'05 problems suggest that on the five unimodal problems (F1~F5), HCLPSO performed best on F1, F3 whilst UPSO, AHPSO and FIPS exhibited the best mean performance for F2, F4 and F5, respectively. On the multimodal problems (F6~F14), HCLPSO (F6, F9, F12, F13) and AHPSO (F10, F11, F14) attained the lowest mean values for majority of multimodal problems. On the other hand, the best mean performances for F7 and F8 are obtained by OLPSO and sHPSO, respectively. Lastly, on the eleven hybrid composition problems (F15~F25), the best mean error values for F16, F17, F21,

F23~F25 are achieved by HCLPSO whilst FIP performed best on F18~F20, F22, and CLPSO on F15. Overall, as shown in Figure 12-6, the AHPSO algorithm exhibited the best performance on 4 and second-best mean performance on 3 of the 25 problems. The final ranks reveal that AHPSO ranked 4th, CLPSO 3rd, OLPSO 2nd and HCLPSO 1st with avg. ranks 4.24, 3.96, 3.76 and 2.21, respectively.

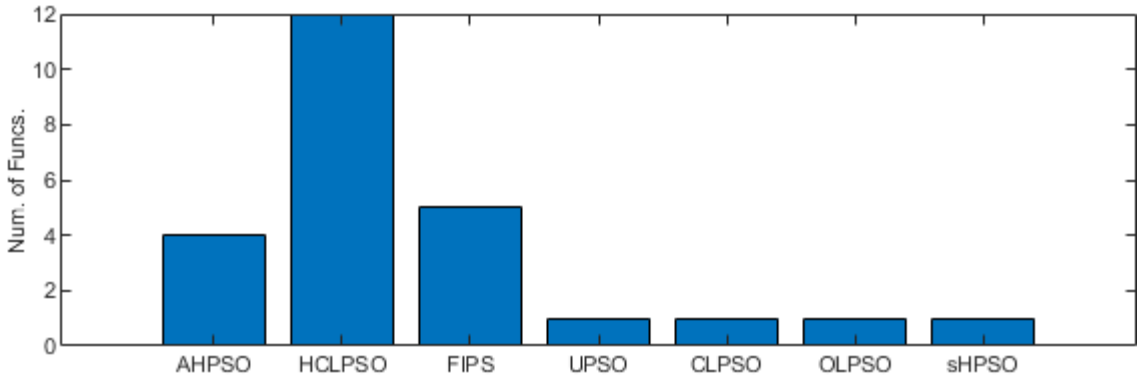


Figure 12-6 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

Further, the Wilcoxon Signed rank test results are tabulated in Table 12-1 which suggests that HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO's mean performances were significantly better than PSO, HPSO-TVAC and sHPSO. On the other hand, FFQ-HIDMS-PSO, BEPSO and AHPSO exhibited statically significantly better performance solely against HPSO-TVAC. It is worth noting that none of the algorithms were significantly better than HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO. In addition, for FFQ-HIDMS-PSO, BEPSO and AHPSO, only HCLPSO's mean performance appear to be significantly better.

Table 12-1 Wilcoxon Signed Rank Test Results with a Significance Level of $\alpha = 0.05$ for 30-dimensional CEC2005 problems

	HCLPSO	PSO	FIPS	UPSO	CLPSO	OLPSO	HPSO-TVAC	sHPSO	+/=/−
HIDMS-PSO	=	+	=	=	=	=	+	+	3/5/0
FFQ-HIDMS-PSO	−	=	=	=	=	=	=	+	1/6/1
GA-HIDMS-PSO	=	+	=	=	=	=	+	+	3/5/0
A-HIDMS-PSO	=	+	=	=	=	=	+	+	3/5/0
BEPSO	−	=	=	=	=	=	=	+	1/6/1
AHPSO	−	=	=	=	=	=	=	+	1/6/1

12.3 Performance Comparison with PSO variants on the 30-dimensional CEC'14 Problems

12.3.1 The Comparison of HIDMS-PSO

The comparison of results on the 30-dimensional CEC'14 problems suggest that the best performances for the three unimodal problems (F1~F3) are exhibited by HIDMS-PSO (F1, F3) and PSO (F2). On the multimodal problems, both HIDMS-PSO and HCLPSO obtained the best mean

performances for four problems, namely F6, F13~F15 and F8, F11, F12, F16, respectively. On the other hand, the lowest mean error values for F7, F9 and F10 are achieved by OLPSO whilst UPSO and HPSO-TVAC performed best on F4 and F5, respectively. The results on the six hybrid problems (F17~F22) reveal that the proposed algorithm HIDMS-PSO exhibited the best mean performance for F17, F20 and F21 whilst HCLPSO performed best on F18, F19 and CLPSO on F22. Lastly, the results on the eight composition functions (F23~F30), HIDMS-PSO (F24, F28), HCLPSO (F25, F26) and HPSO-TVAC (F29, F30) individually attained the lowest mean error values for two problems whilst sHPSO and OLPSO exhibited superior mean performance on F23 and F27, respectively.

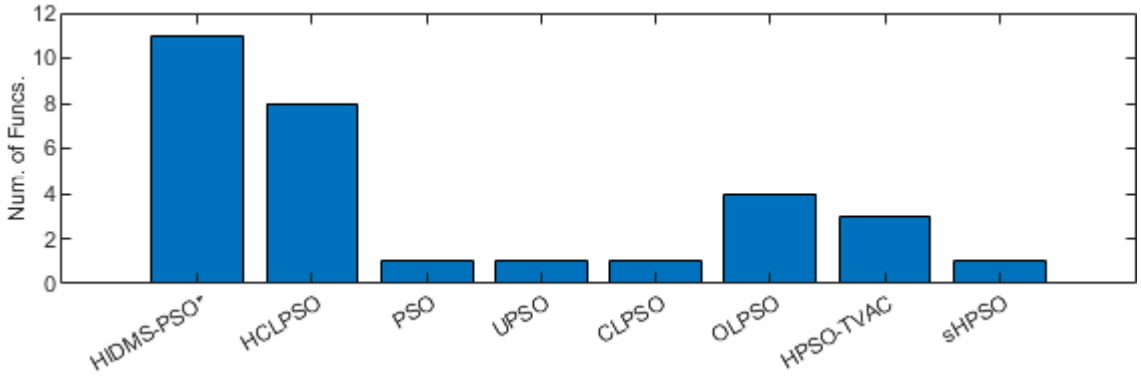


Figure 12-7 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.3.2 The Comparison of FFQ-HIDMS-PSO

The results on the 30-dimensional CEC'14 problems for FFQ-HIDMS-PSO and the eight PSO variants suggest that on the unimodal problems (F1~F3), HCLPSO performed best on F1, F3 whilst PSO exhibited the best mean performance on F2. On the multimodal problems (F4~F16), the best mean performance for F8, F11, F12, F14 and F16 are obtained by HCLPSO, and FFQ-HIDMS-PSO performed best on F6, F9, F13 and F15. On the other hand, the OLPSO exhibited superior mean performance for F7, 10 whilst UPSO and HPSO-TVAC obtained the lowest mean error values on F4 and F5, respectively. On the hybrid problems (F17~F22), we observe that the proposed algorithm FFQ-HIDMS-PSO exhibited superior performance on majority of the hybrid problems including F17, F19, F21 whilst HCLPSO, PSO and CLPSO performed best on F18, F20 and F22, respectively. The results on the eight composition problems (F23~F30) reveal that HPSO-TVAC obtained the lowest mean error values for majority of the composition problems including F28, F29, F30 whilst HCLPSO performed best on F25 and F26. Moreover, sHPSO, FIPS and OLPSO exhibited superior mean performance on F23, F24 and F27, respectively.

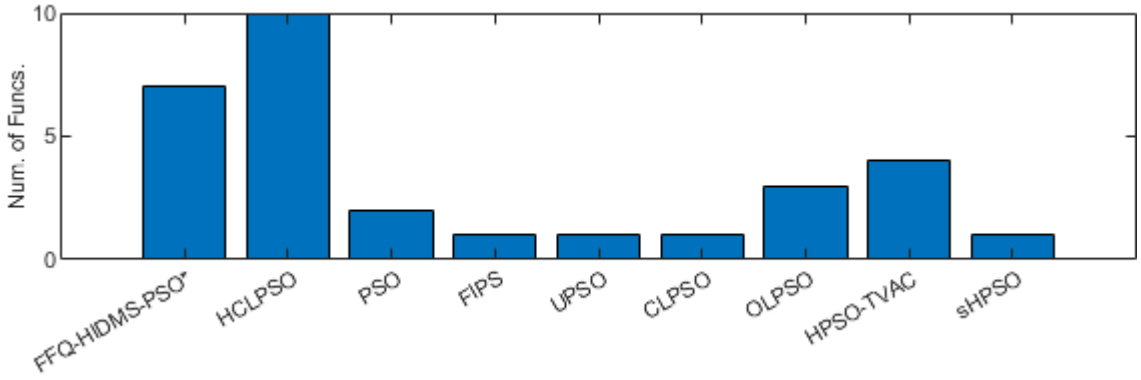


Figure 12-8 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.3.3 The Comparison of GA-HIDMS-PSO

The results on the 30-dimensional CEC'14 problems show that for the unimodal problems (F1~F3), HCLPSO, PSO and GA-HIDMS-PSO exhibited the best mean performance on F1, F2 and F3, respectively. For the 13 multimodal problems (F4~F16), the best mean performances for F8, F11~F14, F16 are obtained by HCLPSO whereas the proposed algorithm GA-HIDMS-PSO performed best on F4, F6, F9 and F15. Further, OLPSO attained the lowest mean error values on F7, 10 whilst HPSOT-TVAC on F5. The results on the 6 hybrid problems (F17~F22) indicate that the proposed algorithm GA-HIDMS-PSO exhibited the best performance for majority of the hybrid problems including F17, F2-0, F21, F22 whilst HCLPSO performed best on F18 and F19. Lastly, for the 8 composition problems (F23~F30), HPSO-TVAC achieved the top mean error values for F28, F29 and F30 whilst sHPSO, FIPS, GA-HIDMS-PSO, HCLPSO and OLPSO individually obtained the best performance on F23, F24, F25, F26 and F27, respectively.

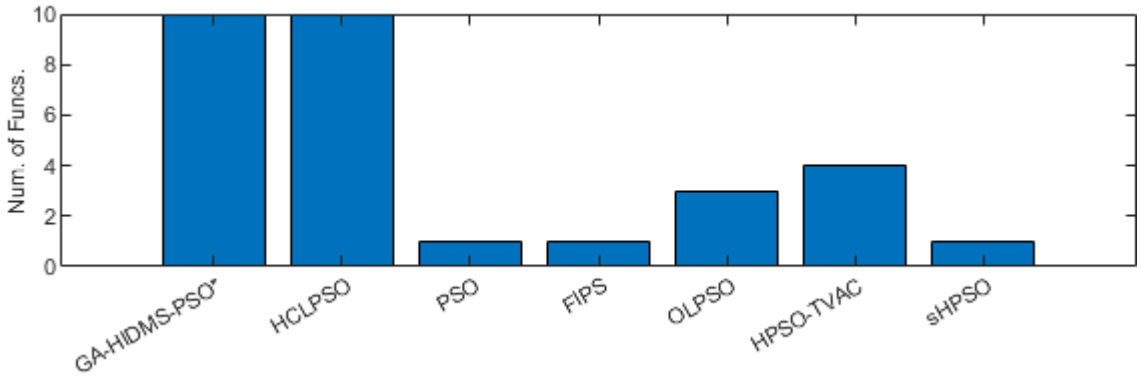


Figure 12-9 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.3.4 The Comparison of A-HIDMS-PSO

The results for the comparison of GA-HIDMS-PSO and eight PSO variants reveal that for the unimodal problems (F1~F3), the proposed algorithm A-HIDMS-PSO performed best on F1, F3 whilst PSO on F2. For the multimodal problems (F4~F16), HCLPSO exhibited the best mean performance on F8, F11, F12, F14, F16 and A-HIDMS-PSO attained the lowest mean error values on F4, F6, F13 and F15. The best performance on F7, F9, 10 are achieved by OLPSO, whereas HPSO-TVAC performed best on F5. The results on the hybrid problems (F17~F22) suggest that the proposed algorithm A-HIDMS-PSO (F17, F21) and HCLPSO (F18, F19) achieved the best performance on two problems whilst PSO and CLPSO individually attained the lowest mean error values on F20 and F22,

respectively. The results on the composition problems yield that HPSO-TVAC solved three of the eight composition problems including F28, F29 and F30 whereas the proposed algorithm A-HIDMS-PSO exhibited the best performance on F24 and F25. Moreover, the best mean performances for F23, F26 and F27 are achieved by sHPSO, HCLPSO and OLPSO, respectively.

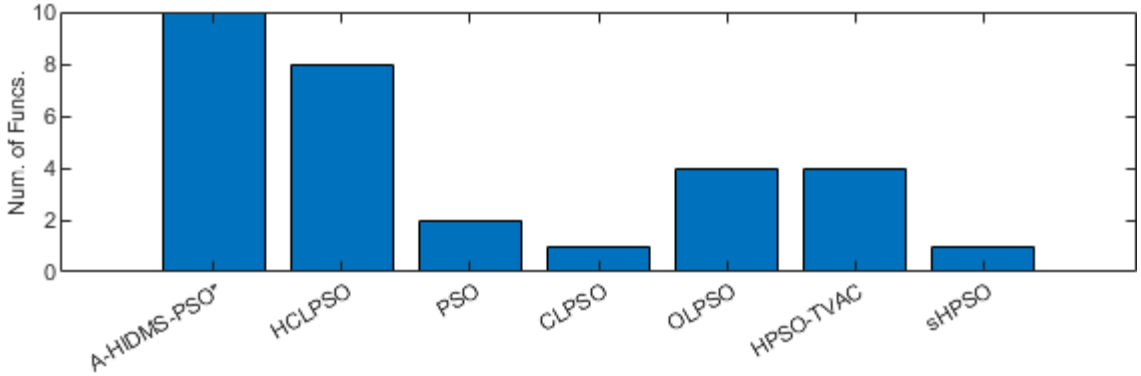


Figure 12-10 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.3.5 The Comparison of BEPSO

The performance of the BEPSO algorithm on the 30-dimensional CEC'14 problems against eight PSO variants indicate that on the unimodal problems (F1~F3), HCLPSO, PSO and BEPSO obtained the best performance for F1, F2, and F3, respectively. The results on the multimodal problems (F4~F16) show that the HCLPSO algorithm attained the lowest mean error values on F8, F11, F12, F14 and F16 whilst the proposed algorithm BEPSO achieved better mean performance for F6, F9, F13 and F15. On the other hand, OLPSO performed best on F7, F10, and the best mean solutions for F4 and F5 are obtained by UPSO and HPSO-TVAC, respectively. For the hybrid problems (F17~F22), the proposed algorithm BEPSO attained the best performance for majority of the hybrid problems including F17, F19 and F21 whilst HCLPSO, PSO and CLPSO individually exhibited the best performance on a single problem, F18, F20 and F22, respectively. Lastly, the results for the composition problems (F23~F30) yield that HPSO-TVAC solved majority of the composition problems including F28, F29, F30 whereas HCLPSO performed best on F25 and F25. Moreover, the best mean performances for F23, F24 and F27 are achieved by sHPSO, FIPS and OLPSO, respectively.

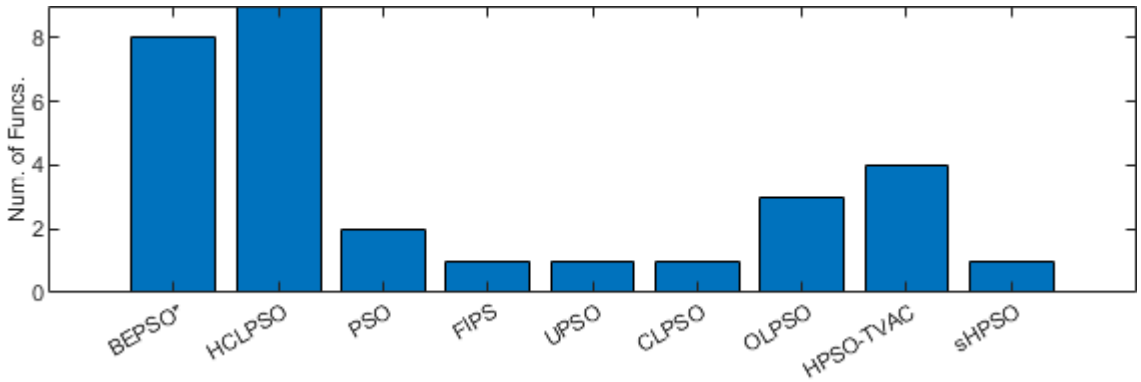


Figure 12-11 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

12.3.6 The Comparison of AHPSO

The mean performance comparison of the AHPSO algorithm against PSO variants on the 30-dimensional CEC'14 problems yield that on the three unimodal problems (F1~F3), the best mean performance for F1, F2 and F3 are attained by HCLPS, PSO and AHPSO, respectively. For 13 multimodal problems (F4~F16), we observe that majority of the multimodal problems are solved by HCLPSO and AHPSO where HCLPSO obtained the best performances for F8, F11, F12, F15, F16 and AHPSO for F6, F9, F13 and F14. Moreover, OLPSO achieved the lowest mean error values for F7, F10 whilst UPSO and HPSO-TVAC performed best on F4 and F6, respectively. For the six hybrid problems (F17~F22), AHPSO exhibited better overall performance for hybrid problems compared to other algorithms and attained the lowest mean error values for F17, F20 and F21 whilst HCLPSO performed best on F18, F19 and CLPSO on F22. Lastly, the results on the eight composition problems (F23~F30) reveal that HPSO-TVAC solved majority of the problems including F28, F29 and F30 whilst AHPSO performed best on F24 and F25. Further, the best mean performances for F23, F26 and F27 are achieved by sHPSO, HCLPSO and OLPSO, respectively.

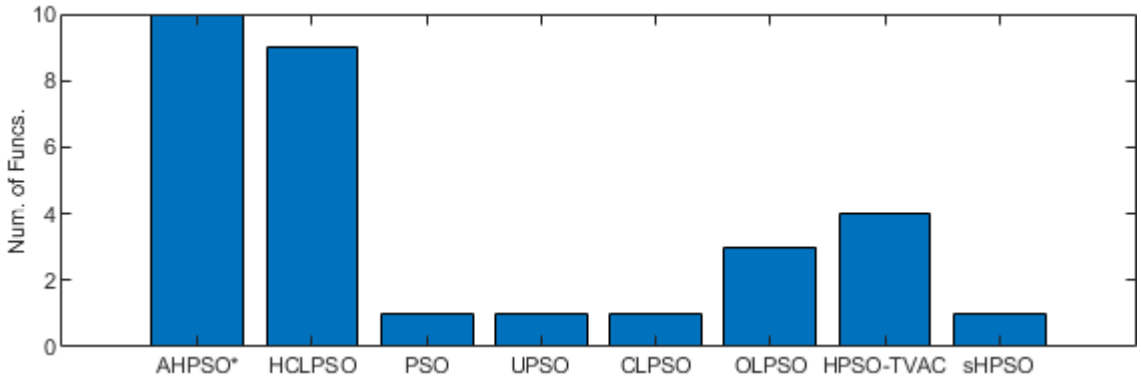


Figure 12-12 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'05 problems

Overall, HCLPSO ranked 1st in all comparisons and in each case, the proposed algorithms attained the 2nd rank. Further, as shown in Table 12-2, the Wilcoxon Signed rank test reveals that HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO are significantly better than 6 of the 8 algorithms and no other algorithms are significantly better than them. Moreover, BEPSO and AHPSO's mean performance is statistically significantly better than 3, and FFQ-HIDMS-PSO's performance is significantly better compared to 3 of the 8 algorithms used in the comparisons.

Table 12-2 Wilcoxon Signed Rank Test Results with a Significance Level of α = 0.05 for 30-dimensional CEC2014 problems

	HCLPSO	PSO	FIPS	UPSO	CLPSO	OLPSO	HPSO-TVAC	sHPSO	+/=/−
HIDMS-PSO	=	+	+	+	+	=	+	+	6/2/0
FFQ-HIDMS-PSO	−	+	+	+	=	=	=	+	4/3/1
GA-HIDMS-PSO	=	+	+	+	+	=	+	+	6/2/0
A-HIDMS-PSO	=	+	+	+	+	=	+	+	6/2/0
BEPSO	−	+	=	+	=	=	=	+	3/4/1
AHPSO	−	+	=	+	=	=	=	+	3/4/1

12.4 Performance Comparison with 10 Metaheuristics on 30-dimensional CEC'17 Problems

12.4.1 The Comparison of HIDMS-PSO

The results for the comparison of HIDMS-PSO and 10 non-PSO variant algorithms on the 30-dimensional CEC'17 problems indicate that, in summary, the ABC algorithm exhibited the best performance for unimodal F1. The best performance for majority of the multimodal problems (F2~F8) are obtained by the proposed algorithm HIDMS-PSO including F2, F4, F6, F7 and F8 whilst TABC and ABC individually attained the lowest mean error values on F3 and F5, respectively. For the hybrid problems (F9~F18), HIDMS-PSO performed best on F10, F11, F13, F16 and F17 whilst ABC exhibited the best mean performance for F12, F14 and F8. Moreover, TABC and GABC achieved the lowest mean error values on F9 and F15, respectively. Further, the results on the composition problems (F19~F29) suggest that the proposed algorithm HIDMS-PSO exhibited superior mean performance on F20, F21, F22, F26, F27 and F28 whilst ABC (F24, F29) and TABC (F23, F25) performed best on two composition functions, and GABC on a single problem F19. As shown in Figure 12-13, the HIDMS-PSO algorithm exhibited the best mean performance for most of the 30-dimensional problems. Moreover, HIDMS-PSO and TABC ranked 1st and 2nd, respectively with the average ranks of 2.44 and 3.24.

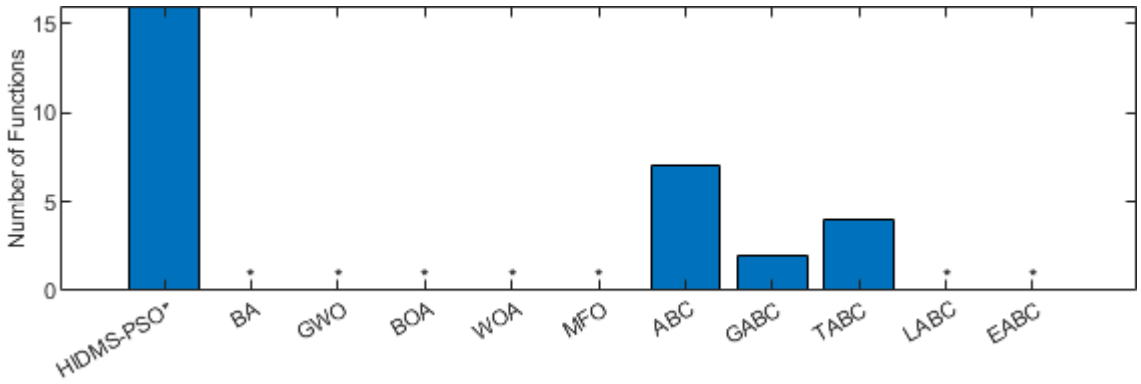


Figure 12-13 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

12.4.2 The Comparison of FFQ-HIDMS-PSO

The comparison of results for the FFQ-HIDMS-PSO algorithm and the 10 metaheuristics indicate that on the unimodal F1, ABC obtained the best performance. On the other hand, for multimodal (F2~F8), the proposed algorithm FFQ-HIDMS-PSO exhibited the best mean performance on F2, F4, F6, F7 and F8 whilst TABC and ABC performed best on F3 and F5, respectively. For 30-dimensional hybrid problems (F9~F18), the best mean performances for F10, F11, F13, F16 and F17 are obtained by the proposed algorithm FFQ-HIDMS-PSO whereas ABC performed best on F12, F14 and F18. Moreover, TABC and GABC achieved the lowest mean error values on F9 and F15, respectively. Lastly, on the 11 composition problems (F19~F29), once again, the FFQ-HIDMS-PSO algorithm exhibited the best mean performance for majority of the problems including F20, F21, F22, F26, F27, F28 and F29 whilst TABC performed best on F23 and F25. On other hand, the lowest mean error values for F19 and F24 are obtained by GABC and ABC, respectively. The FFQ-HIDMS-PSO achieved the best performance

for 17 of the 30 problems (Figure 12-14) and attained the 1st rank (avg=2.24) whilst TABC obtained the 2nd rank (avg=3.31).

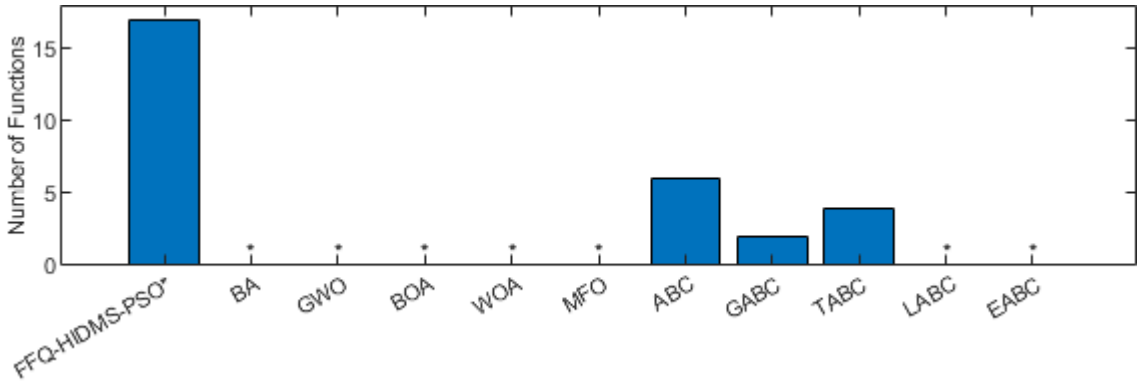


Figure 12-14 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

12.4.3 The Comparison of GA-HIDMS-PSO

The comparison of results on the 30-dimensional CEC'17 problems for the GA-HIDMS-PSO algorithm against 10 non-PSO variants reveal that ABC performed best on the unimodal F1 and for multimodal problems (F2~F8), the proposed algorithm GA-HIDMS-PSO exhibited the best mean performance on F2, F4, F6, F7 and F8 whilst TABC and ABC provided the lowest mean error values for F3 and F5, respectively. For hybrid problems (F9~F18), the proposed algorithm GA-HIDMS-PSO achieved the best performance on F10, F11, F13, F15, F16 and F17 whilst ABC provided better mean error values on F12, F14 and F18. Moreover, for F9, the best mean performance is achieved by TABC. Lastly, on the 10 composition problems (F19~F29), the best performance for F19~F22, F26~F29 is provided by the proposed algorithm GA-HIDMS-PSO, whereas TABC obtained the lowest mean error values on F23, F25 whilst ABC solved F24. Overall, the GA-HIDMS-PSO performed best on most of the problems in this problem set (see Figure 12-15) and ranked 1st (avg rank=2.20), on the other hand, TABC ranked 2nd with the average rank of 3.27.

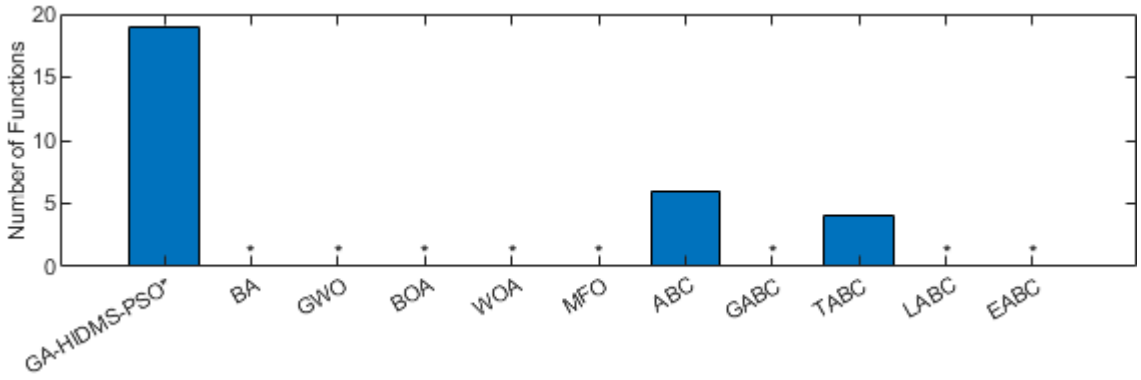


Figure 12-15 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

12.4.4 The Comparison of A-HIDMS-PSO

The experimental results on the 30-dimensional CEC'17 reveal that for unimodal F1, ABC provided the best performance and for multimodal problems (F2~F8), the proposed algorithm A-HIDMS-PSO solved F2, F4, F6, F7 and F8 whilst TABC and ABC attained the lowest mean error values for F3 and F5, respectively. For the 10 hybrid problems (F9~F18), the results indicate that the proposed

algorithm A-HIDMS-PSO provided the best mean performance for F10, F11, F13, F15, F16 and F17 whilst ABC performed best on F12, F14, F18 and TABC on F9. Finally, for the composition problems (F19~F29), the proposed algorithm A-HIDMS-PSO solved F20, F21, F22, F26, F27 and F28 whereas ABC and TABC provided the best mean performance for two problems F24, F9 and F23, F25, respectively, whilst the lowest mean error value for F19 is attained by GABC. As shown in Figure 12-16, A-HIDMSPSO exhibited the best mean performance in most cases for the 30-dimensional CEC'17 problems. In addition, the A-HIDMS-PSO algorithm ranked 1st in this comparison (avg=2.41) whilst the 2nd rank is attained by TABC (avg=3.24).

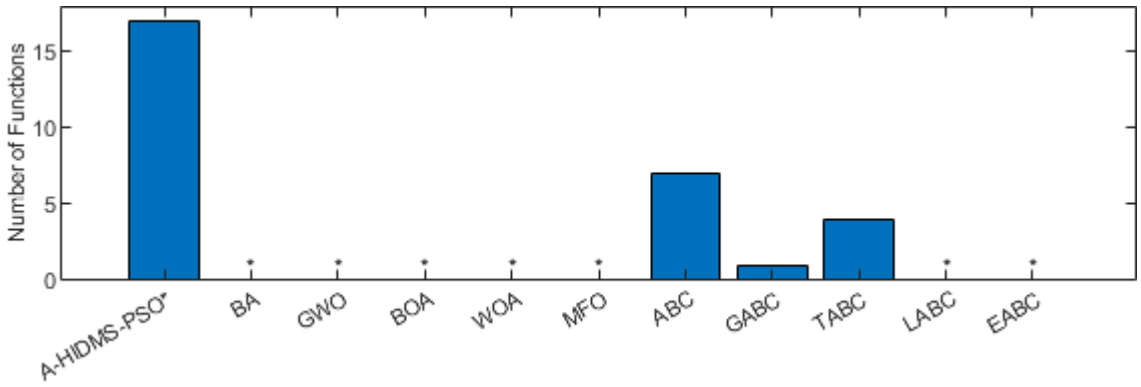


Figure 12-16 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

12.4.5 The Comparison of BEPSO

The results indicate that for unimodal F1, the best mean performance is exhibited by ABC. The 7 multimodal problems (F2~F8) reveal that the BEPSO algorithm achieved the lowest mean error values for majority of problems including F2, F4, F6, F7 and F8 whereas the remainder two problems F3 and F5 are solved TABC and ABC, respectively. Similarly, most of the hybrid problems including F10, F11, F13, F15, F16 and F17 are solved by the proposed algorithm BEPSO whilst ABC performed best on F12, F14, F18, and TABC on F9. Lastly, for the 11 composition problems (F19~F29), once again the proposed algorithm BEPSO exhibited superior performance for most of the problems including F20, F21, F22, F26, F28, F29 whilst ABC (F24, F27) and TABC (F23, F25) solved two problems each and GABC attained the lowest mean error value on F19.

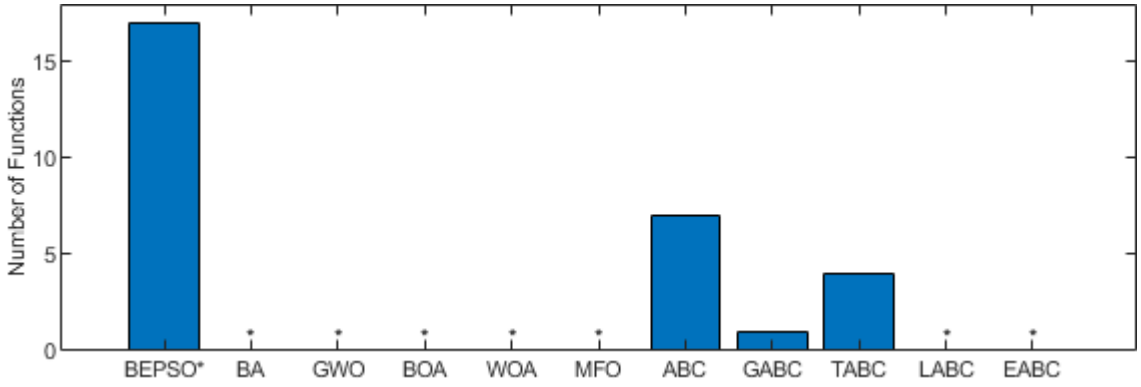


Figure 12-17 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

We observe from Figure 12-17 that BEPSO performed best on 17 of the 30 problems and obtained the 1st rank (avg=2.34) whereas the 2nd rank is attained by TABC with the average rank of 3.27.

12.4.6 The Comparison of AHPSO

In summary, the best mean performance for unimodal F1 is achieved by ABC. For multimodal problems (F2~F8), AHPSO obtained the best mean performance on most of the multimodal problems including F2, F4, F6, F7 and F8 whilst TABC and ABC performed best on F3. For the 10 hybrid problems (F9~F18), once again the proposed algorithm AHPSO solved most of the problems in this class including F10, F11, F11, F15, F16 and F17. On the other hand, ABC attained the lowest mean error values for F12, F14 and F18 whilst TABC performed best on F9. Moreover, the proposed algorithm AHPSO exhibited superior mean performance on F20, F21, F22, F27, F29 whilst TABC (F23, F25) and EABC (F26, F28) solved two problems. Similarly, GABC and ABC also exhibited the best mean performances for a single composition problem F19 and F24, respectively.

As shown in Figure 12-18, AHPSO algorithm obtained the best mean solution for most of the 30-dimensional problems. In addition, the mean and final ranks reveal that AHPSO ranked 1st with the average rank of 2.41 whilst TABC attained the 2nd rank with the average rank of 3.24.

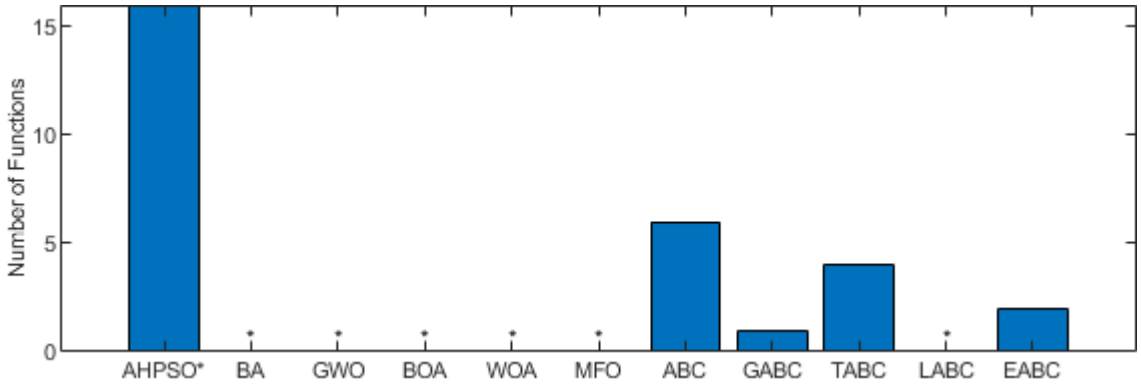


Figure 12-18 The total number of best performances achieved by each algorithm with respect to mean error values on the 30-dimensional CEC'17 problems

Table 12-3 displays the results for the Wilcoxon Signed rank test on the 30-dimensional CEC'17 problems. In summary, FFQ-HIDMS-PSO, GA-HIDMS-PSO, A-HIDMS-PSO and BEPSO were significantly better than 9 of the 10 algorithms (except TABC) whereas HIDMS-PSO and AHPSO's mean performance were significantly better compared to 8 of the 10 algorithms (except TABC and GABC). It is worth highlighting that none of the 10 comparison algorithms' performances were significantly better compared to any of the proposed algorithms.

Table 12-3 Wilcoxon Signed Rank Test Results with a Significance Level of $\alpha = 0.05$ for 30-dimensional CEC2017 problems

	BA	GWO	BOA	WOA	MFO	ABC	GABC	TABC	LABC	EABC	+/=/−
HIDMS-PSO	+	+	+	+	+	+	=	=	+	+	8/2/0
FFQ-HIDMS-PSO	+	+	+	+	+	+	+	=	+	+	9/1/0
GA-HIDMS-PSO	+	+	+	+	+	+	+	=	+	+	9/1/0
A-HIDMS-PSO	+	+	+	+	+	+	+	=	+	+	9/1/0
BEPSO	+	+	+	+	+	+	+	=	+	+	9/1/0

AHPSO	+	+	+	+	+	+	=	=	+	+	8/2/0

12.5 Discussions

The performance comparison of the six proposed algorithms with the results of 8 PSO variants on 30-dimensional CEC'05, CEC'14 and 10 bio-inspired metaheuristics on 30-dimensional CEC'17 problems yielded that overall, the comparison on the 25 30-dimensonal CEC'05 problems indicated that all proposed algorithms exhibited superior performance against the majority of the algorithms and obtained better mean performance for more problems compared to 7 of the 8 PSO variants (except HCLPSO). The mean and final ranks obtained on the mean error values showed that HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO ranked 2nd, FFQ-HIDMS-PSO ranked 3rd, AHPSO and BEPSO obtained the 4th ranks.

The second comparison on the CEC'13 problems revealed that HIDMS-PSO, A-HIDMS-PSO and AHPSO exhibited superior mean performance on more problems compared to all PSO variants considered in this experiment. In addition, all of the proposed algorithms consistently attained the 2nd rank for the 30-dimensional CEC'14 problems. Further, the Wilcoxon signed rank test conducted on the final ranks yield that the mean performance of HIDMS-PSO, GA-HIDMS-PSO and A-HIDMS-PSO's is statistically significantly better compared to 6 of the 8 PSO variants used in this comparison.

The third comparison on the 30-dimensional CEC'17 problems against 10 bio-inspired metaheuristics indicated that all proposed algorithms attained the best mean performance for the majority of the problems in comparison to all the test algorithms. Moreover, each proposed algorithm attained the 1st rank on the 30-dimensional CEC'17 problems and the Wilcoxon signed rank test revealed that the proposed algorithms exhibited statistically significantly superior mean performance against most of the comparative algorithms while the result indicated that none of the comparison algorithms' performances were statistically significant against any of the proposed algorithms.

12.6 Conclusions

In this chapter, the performance of the six proposed algorithms is compared with the results published in other author's work to revalidate the performance of the proposed algorithms. The results observed on the three distinct comparisons on 30-dimensional CEC'05, CEC'14 and CEC'17 benchmark problems suggested that overall, the results acquired from the comparisons conducted in this chapter on three different test suites using well known PSO variants and 10 bio-inspired metaheuristics corroborate the performance observed and the experimental results obtained in previous chapters of this thesis, once again verifying the comparative effectiveness of the proposed algorithms.

Chapter 13 : Conclusions and Future Works

13.1 Conclusions

The PSO algorithm is a population-based swarm intelligence algorithm, originally designed to solve real-valued optimisation problems. In essence, particles utilise their memory, and the best known solution discovered by the swarm, to ascertain their trajectories at each time step to conduct search in the problem space. PSO endures *PSO-specific* limitations such as premature convergence typically caused by the rapid loss of population diversity due to overexploitation of the globally best known solution, and more general issues that are common to all metaheuristics such as effectively balancing exploration and exploitation. Hence, in Chapters 4-9, six novel PSO variants, namely HIDMS-PSO, FFQ-HIDMS-PSO, GA-HIDMS-PSO, A-HIDMS-PSO, BEPSO and AHPSO were introduced to explore a variety of ways to address the problem of loss of population diversity, along with other aspects including the balance of exploration and exploitation.

The performance of the proposed six algorithms was verified over three real-valued single objective benchmark test suites, CEC'13, CEC'14 and CEC'17, on 10, 30, 50 and 100 dimensions comparing against 13 well-known PSO variants and L-SHADE. Non-parametric statistical analysis indicated that the proposed algorithms are significantly better than most of the test algorithms, especially on high dimensional (50-D and 100-D) problem sets.

In Chapter 10, a performance comparison of the six proposed algorithms was conducted. In summary, the comparison revealed individual algorithm's performance improvement and deterioration on different function types and dimensions. For instance, A-HIDMS-PSO and GA-HIDMS-PSO appeared to exhibit competitive performance across all dimensions for unimodal, multimodal, hybrid and composition functions whilst HIDMS-PSO primarily exhibited superior performance on multimodal and composition functions. This outcome complies with the NFL theorem in the sense that an algorithm may outperform another on a specific set of problems but may be inferior on a different kind of optimisation problems.

In Chapter 11, besides the experiments conducted on the unconstrained problems of the CEC'13, CEC'14 and CEC'17 benchmark test suites, the applicability, and the performance of the proposed six algorithms was evaluated on 29 constrained real-world problems comprised of industrial chemical processes, process synthesis and design, mechanical engineering and power electronic problems. The results obtained on a wide variety of constrained real-world optimisation problems suggested that the proposed algorithms either obtained the best or second-best solution along with other algorithms, or exhibited comparable performance to the majority of the algorithms in most cases.

In Chapter 12, the performance of the proposed algorithms was compared on the 30-dimensional CEC'05, CEC'14 and CEC'17 problems with the results published elsewhere (*see Chapter 12 for details*) using various PSO algorithms and bio-inspired metaheuristics. Once again, the comparisons conducted in this chapter corroborated with the results attained in previous chapters of the thesis.

Hence, the experimental results on unconstrained CEC'13, CEC'14, CEC'17 benchmark problems at 10, 30, 50 and 100 dimensions, and 29 non-convex constrained real-world problems

verify the spectrum of efficacy of the proposed algorithms on unconstrained problems as well as their suitability for engineering problems, and potential applicability on other real-world problems.

13.2 Discussion

The experimental results presented in Chapters 4-9 uncover a consistently observed limitation in all of the proposed algorithms. The algorithms exhibit relatively poor performance on low dimensional (specifically 10-d) problems, and the overall performance of the proposed algorithms significantly improves with increased problem dimension and complexity.

Our initial intuition was that the said limitation is associated with high agent-level heterogeneity, which significantly aids the algorithms in high-dimensional and complex search spaces, while being ineffectual in low dimensional spaces. Hence, to verify this, various alterations were made on different components of the HIDMS-PSO algorithm to reduce the agent-level heterogeneity by constricting the available velocity update equations, and subsequent experiments were conducted using each *simplified variant* on the 10-dimensional CEC'13 and CEC'14 problems. The following modifications to the agent behaviour improved the algorithm's performance on 10-dimensional problems:

The master particle resumes utilising the two learning strategies (*inward-oriented and outward-oriented*), however, with a single fixed velocity update equation from each strategy (Eq. 4.2 and Eq. 4.4). On the other hand, the slave particles solely employ the inward-oriented learning strategy (Eq. 4.5). Hence, in this *simplified HIDMS-PSO variant*, the algorithm employs three velocity update equations. Essentially, the altered or simplified agent behaviour contains more explicit roles for each particle type. Hence, the employment of the mentioned update equations renders a two-way exploring master particle that is either directed towards the average position of another unit or the most dissimilar slave within its own unit. On the contrary, the slave particle adopts the role of exploitation and is solely guided by its memory and the unit master. Consequently, this simpler behaviour model and the two complementing behaviours (exploring and exploiting) yield more efficient search for 10-dimensional problems.

Overall, the experimental results obtained on the 10-dimensional CEC'13, CEC'14 and CEC'17 problems (details included in appendix 2) revealed that the *simplified HIDMS-PSO* ranked 3rd, 4th and 3rd on the 10-dimensional CEC'13, CEC'14 and CEC'17 problems whereas the original HIDMS-PSO ranked 4th, 7th, 8th, respectively. However, when the problem dimension is increased to 100-d, the simplified HIDMS-PSO ranked 4th and 6th on the CEC'13 and CEC'14 problems, whereas the original HIDMS-PSO ranked 1st and 3rd for the same problems. Hence, we clearly observe the algorithm with *reduced heterogeneity* consistently exhibited better mean performance at low dimensions and deteriorating with increased dimensions. In addition, it is also worth noting that the Wilcoxon Signed rank test indicated the *simplified* HIDMS-PSO algorithm's mean performance was statistically significantly better than the original HIDMS-PSO algorithm on the 10-dimensional CEC'17 problems.

Considering the outcome presented here, the findings obtained from the investigation carried out for the efficacy of HIDMS-PSO on low dimensional problems suggest a suitable initial point for future work on HIDMS-PSO. Perhaps a new control parameter may be introduced to the HIDMS-PSO algorithm to adaptively control the agent-level heterogeneity with consideration of problem size to enable a more efficient regulation of heterogeneity.

The analysis of the HIDMS-PSO search dynamics emphasised the way in which various components of the algorithm tend to complement each other and in combination maintain diversity, balance exploration and exploitation and enable efficient high-quality search. However, these multiple components do increase the overall complexity of the algorithm. But it is worth noting that many recent variants of metaheuristics, such as [267], [268], have a much more complex structure than their canonical versions. These variants are discernibly more efficient and are able to handle

more diverse problems. Hence the algorithms proposed in this thesis are not unusual in comprising a more complex structure and set of interaction mechanisms in comparison to canonical algorithms. However, their individual elements are all simple. Indeed, the slightly more complex framework of the proposed algorithms is inspired by the observation that biological degeneracy plays a vital role in boosting evolvability in nature, and therefore can improve the efficiency of search processes [269]. Biological degeneracy – whereby multiple interacting mechanisms enable multiple different ways of producing an outcome – is an ubiquitous property of biological systems at all levels of organisation [270]. [271] reveals that systems with simple redundancy have considerably lower evolvability than degenerate (e.g., highly versatile) systems with selectable changes of behaviour which enable compensatory actions to occur within the system (exactly what happens at the core of the new algorithms presented in this thesis).

13.3 Future Work

The research undertaken here can be extended in many directions. A summary of potential further work is discussed in this section.

Although PSO is a very efficient algorithm for many small-scale optimisation problems, as with many metaheuristics, it suffers from the curse of dimensionality. Although the performance of the algorithms presented in this thesis were much less impacted with dimensionality compared with other algorithms used in the experiments, various other limitations were identified for future improvements with regards to the inferior performance observed for different types of problems by the presented algorithms. It is clear that the heterogeneous nature of the presented algorithms provides various advantages for the general process of optimisation, but the consistent less efficient performance on unimodal problems could be improved with suitable mechanisms to further extend the scope of the proposed algorithms. The unimodal problems do not require such a careful balance between exploration and exploitation. Typically, intensive exploitation is expected to dominate the search process for such problems and considering the heterogeneous characteristics of the proposed algorithms, this optimal exploitation stage for unimodal problems is not present in the proposed algorithms. As well as broadening the algorithms' spectrum of efficacy, the opposite approach to *specialise* the algorithms to efficiently solve a specific type of function [272]–[274] is also a potential direction of future research.

Various other improvements can be made with emphasis on homogeneity and heterogeneity of the population, topological structure of units, mechanisms that control the information flow and learning strategies. Furthermore, with respect to HIDMS-PSO, the hybrid variant GA-HIDMS-PSO acts as a successful model of an improved hybrid extension. Hence, hybridisation with other algorithms, especially by exploiting the heterogeneous nature of some algorithms, could be another fruitful direction for future work.

Considering the efficacy of the presented algorithms on high dimensional problems, further investigation on large-scale problem can be done to affirm if the performances hold for further higher dimensional cases, potentially looking at very high dimensional problems. Similarly, as presented in Chapter 11, considering their performances on constrained real-world problems, the algorithms possess significant potential to be applied to more complex real-world problems. Hence, further investigation, adaptation and improvements can be carried out in this area. More specially, AHPSO exhibited superior performance on constrained power electronic problems (*see Chapter 11*), a further investigation of this efficacy, potential applications to other problems in this domain and other problem classes can be a topic for future research.

Finally, the work presented in this thesis lacks various aspects of approaches found in the PSO literature, including exploring different types of solution initialisations and comprehensive parameter tuning, which could also be important areas for future work.

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Appendix 1: Definitions of Unconstrained Benchmark Problems

Summary of the CEC'13 Test Functions

	No.	Functions	$f_i^* = f_i(x^*)$
Unimodal Functions	1	Sphere Function	-1400
	2	Rotated High Conditioned Elliptic Function	-1300
	3	Rotated Bent Cigar Function	-1200
	4	Rotated Discus Function	-1100
	5	Different Powers Function	-1000
Basic Multimodal Functions	6	Rotated Rosenbrock's Function	-900
	7	Rotated Schaffers F7 Function	-800
	8	Rotated Ackley's Function	-700
	9	Rotated Weierstrass Function	-600
	10	Rotated Griewank's Function	-500
	11	Rastrigin's Function	-400
	12	Rotated Rastrigin's Function	-300
	13	Non-Continuous Rotated Rastrigin's Function	-200
	14	Schwefel's Function	-100
	15	Rotated Schwefel's Function	100
	16	Rotated Katsuura Function	200
	17	Lunacek Bi_Rastrigin Function	300
	18	Rotated Lunacek Bi_Rastrigin Function	400
	19	Expanded Griewank's plus Rosenbrock's Function	500
	20	Expanded Scaffer's F6 Function	600
Composition Functions	21	Composition Function 1 (n=5,Rotated)	700
	22	Composition Function 2 (n=3,Unrotated)	800
	23	Composition Function 3 (n=3,Rotated)	900
	24	Composition Function 4 (n=3,Rotated)	1000
	25	Composition Function 5 (n=3,Rotated)	1100
	26	Composition Function 6 (n=5,Rotated)	1200
	27	Composition Function 7 (n=5,Rotated)	1300
	28	Composition Function 8 (n=5,Rotated)	1400
Search Range: $[-100,100]^D$			

Summary of the CEC'14 Test Functions

	No.	Functions	$f_i^* = f_i(x^*)$
Unimodal Functions	1	Rotated High Conditioned Elliptic Function	100
	2	Rotated Bent Cigar Function	200
	3	Rotated Discus Function	300

Simple Multimodal Functions	4	Shifted and Rotated Rosenbrock's Function	400
	5	Shifted and Rotated Ackley's Function	500
	6	Shifted and Rotated Weierstrass Function	600
	7	Shifted and Rotated Griewank's Function	700
	8	Shifted Rastrigin's Function	800
	9	Shifted and Rotated Rastrigin's Function	900
	10	Shifted Schwefel's Function	1000
	11	Shifted and Rotated Schwefel's Function	1100
	12	Shifted and Rotated Katsuura Function	1200
	13	Shifted and Rotated HappyCat Function	1300
	14	Shifted and Rotated HGBat Function	1400
	15	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	1500
	16	Shifted and Rotated Expanded Scaffer's F6 Function	1600
Hybrid Function 1	17	Hybrid Function 1 (N=3)	1700
	18	Hybrid Function 2 (N=3)	1800
	19	Hybrid Function 3 (N=4)	1900
	20	Hybrid Function 4 (N=4)	2000
	21	Hybrid Function 5 (N=5)	2100
	22	Hybrid Function 6 (N=5)	2200
Composition Functions	23	Composition Function 1 (N=5)	2300
	24	Composition Function 2 (N=3)	2400
	25	Composition Function 3 (N=3)	2500
	26	Composition Function 4 (N=5)	2600
	27	Composition Function 5 (N=5)	2700
	28	Composition Function 6 (N=5)	2800
	29	Composition Function 7 (N=3)	2900
	30	Composition Function 8 (N=3)	3000
Search Range: $[-100,100]^D$			

Summary of the CEC'17 Test Functions

	No.	Functions	$f_i^* = f_i(x^*)$
Unimodal Functions	1	Shifted and Rotated Bent Cigar Function	100
	2	Shifted and Rotated Zakharov Function	200
Simple	3	Shifted and Rotated Rosenbrock's Function	300
	4	Shifted and Rotated Rastrigin's Function	400

Multimodal Functions	5	Shifted and Rotated Expanded Scaffer's F6 Function	500
	6	Shifted and Rotated Lunacek Bi_Rastrigin Function	600
	7	Shifted and Rotated Non-Continuous Rastrigin's Function	700
	8	Shifted and Rotated Levy Function	800
	9	Shifted and Rotated Schwefel's Function	900
Hybrid Function	10	Hybrid Function 1 (N=3)	1000
	11	Hybrid Function 2 (N=3)	1100
	12	Hybrid Function 3 (N=3)	1200
	13	Hybrid Function 4 (N=4)	1300
	14	Hybrid Function 5 (N=4)	1400
	15	Hybrid Function 6 (N=4)	1500
	16	Hybrid Function 6 (N=5)	1600
	17	Hybrid Function 6 (N=5)	1700
	18	Hybrid Function 6 (N=5)	1800
	19	Hybrid Function 6 (N=6)	1900
Composition Functions	20	Composition Function 1 (N=3)	2000
	21	Composition Function 2 (N=3)	2100
	22	Composition Function 3 (N=4)	2200
	23	Composition Function 4 (N=4)	2300
	24	Composition Function 5 (N=5)	2400
	25	Composition Function 6 (N=5)	2500
	26	Composition Function 7 (N=6)	2600
	27	Composition Function 8 (N=6)	2700
	28	Composition Function 9 (N=3)	2800
	29	Composition Function 10 (N=3)	2900
Search Range: $[-100,100]^D$			

Appendix 2: Experimental Results on the Unconstrained Benchmark Problems

Statistical results on 10-dimensional CEC'13 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.5E-14	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	9.3E-14	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	3	7	5	9	1	1	2	6	1	4	1	1	1	1	8
F_2	mean	1.9E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	1.4E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	2	11	8	15	7	3	4	12	6	5	10	1	13	14	9
F_3	mean	2.7E+02	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	7.0E+02	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	2	13	12	15	5	7	9	6	3	14	4	1	10	11	8
F_4	mean	1.2E+03	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	7.0E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	7	9	2	14	5	3	6	15	4	8	10	1	12	13	11
F_5	mean	2.5E-08	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	3.2E-08	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	9	12	10	14	1	3	7	11	4	8	2	1	5	6	13
F_6	mean	3.0E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	4.6E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	3	13	12	14	1	2	4	9	6	10	5	8	7	11	15
F_7	mean	2.0E-01	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	4.9E-01	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	1	13	11	15	2	7	6	8	4	12	5	3	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	9.3E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	4	7	6	8	2	11	3	12	9	10	5	1	14	13	15
F_9	mean	1.7E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	8.1E-01	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	2	9	12	14	1	6	4	13	5	11	3	7	10	8	15
F_{10}	mean	1.4E-01	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	6.4E-02	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	2	13	12	14	3	7	6	8	5	11	4	1	10	9	15
F_{11}	mean	3.6E+00	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	1.7E+00	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	10	12	13	14	7	2	9	11	5	6	8	1	3	4	15
F_{12}	mean	6.1E+00	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	1.9E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	2	10	12	14	3	4	5	11	7	13	6	1	9	8	15
F_{13}	mean	1.2E+01	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	5.5E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	3	11	13	14	2	5	7	9	6	12	4	1	10	8	15
F_{14}	mean	4.2E+02	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	1.4E+02	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	13	12	14	9	7	4	10	15	5	8	6	1	3	2	11
F_{15}	mean	3.8E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.1E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	1	13	10	14	2	6	4	15	5	7	9	3	11	12	8
F_{16}	mean	4.4E-01	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	2.4E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	2	13	4	10	6	11	9	15	8	5	7	1	12	14	3
F_{17}	mean	1.5E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	2.8E+00	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	10	9	14	15	7	5	8	13	4	6	12	1	3	2	11
F_{18}	mean	1.7E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	4.0E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	2	8	10	15	5	4	7	12	3	11	13	1	14	9	6
F_{19}	mean	8.1E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	3.3E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	11	9	14	15	7	5	8	13	4	12	10	3	2	1	6
F_{20}	mean	2.1E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	5.9E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	3	12	14	15	2	4	6	11	1	9	5	7	10	13	8
F_{21}	mean	4.0E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	1.7E-13	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	9	3	11	12	9	2	9	8	7	10	5	9	6	4	1
F_{22}	mean	4.1E+02	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	2.4E+02	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	11	12	15	13	6	4	9	14	5	7	8	1	2	3	10
F_{23}	mean	6.2E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	1.8E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	4	10	13	15	1	2	7	14	6	9	5	3	12	11	8
F_{24}	mean	1.8E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	4.0E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	5	10	14	15	8	1	9	13	4	12	6	7	3	2	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	8.2E-01	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	6	13	12	15	7	2	9	14	5	10	3	8	1	4	11
F_{26}	mean	1.2E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	2.8E+01	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	2	13	15	14	10	1	11	12	8	9	3	4	7	5	6
F_{27}	mean	3.3E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	4.3E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	3	14	10	15	9	4	2	12	6	11	8	1	5	7	13
F_{28}	mean	3.0E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	5.1E+01	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	5	12	13	15	8	2	11	6	4	14	1	7	10	9	3
Mean rank		4.89	10.82	11.11	13.61	4.79	4.21	6.82	11.36	5.00	9.43	6.00	3.04	7.68	7.61	10.14
Final rank		4	12	13	15	3	2	7	14	5	10	6	1	9	8	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	4.1E-13	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	1.2E-13	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	5	12	9	10	1	3	6	8	4	7	3	2	3	3	11
F_2	mean	3.8E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	8.6E+04	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	4	11	9	14	7	2	3	15	6	5	10	1	12	13	8
F_3	mean	2.2E+06	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	3.7E+06	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	1	13	14	15	3	8	4	7	5	10	6	2	9	11	12
F_4	mean	1.2E+03	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	4.0E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	3	11	6	14	2	1	4	15	5	8	9	7	13	12	10
F_5	mean	3.8E-06	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	1.8E-06	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	10	13	12	15	1	3	6	11	5	9	8	2	7	4	14
F_6	mean	2.1E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	2.2E-01	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	4	13	12	14	3	2	5	8	7	9	6	1	10	11	15
F_7	mean	9.9E+00	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	3.2E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	2	12	13	15	1	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	7.4E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	2	5	9	12	7	10	4	8	6	11	3	1	14	13	15
F_9	mean	1.3E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	4.5E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	2	7	12	14	1	6	3	13	5	9	4	8	11	10	15
F_{10}	mean	5.8E-02	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	2.3E-02	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	2	13	12	14	4	5	3	11	6	7	8	1	10	9	15
F_{11}	mean	3.8E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	7.7E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	10	12	13	15	9	2	7	11	6	5	8	1	4	3	14
F_{12}	mean	4.3E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	9.1E+00	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	3	11	14	15	2	5	6	8	4	13	7	1	9	10	12
F_{13}	mean	1.0E+02	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.0E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	3	12	14	15	2	4	5	8	6	13	7	1	10	11	9
F_{14}	mean	2.1E+03	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	5.9E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	10	12	14	9	11	2	7	15	6	5	8	1	4	3	13
F_{15}	mean	3.1E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	6.5E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	1	11	9	14	6	3	5	15	4	7	10	2	12	13	8
F_{16}	mean	1.5E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	6.5E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	5	12	4	11	2	6	10	13	7	3	8	1	14	15	9
F_{17}	mean	6.4E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	8.0E+00	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	7	12	14	15	8	4	9	13	5	6	10	1	2	3	11
F_{18}	mean	7.8E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	3.6E+01	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	3.0E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	8.4E-01	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	6	15	13	14	8	3	7	12	5	10	9	1	4	2	11
F_{20}	mean	1.0E+01	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	8.3E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.3E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	5.9E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	9	14	13	15	11	2	5	4	3	12	1	10	7	8	6
F_{22}	mean	2.3E+03	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	6.3E+02	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	3.6E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	8.4E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	2	8	11	14	1	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.0E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	1.4E+00	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	1	10	14	15	3	6	5	13	2	12	7	4	8	9	11
F_{25}	mean	2.0E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	7.5E-01	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	1	8	14	15	2	7	6	11	4	9	3	5	12	13	10
F_{26}	mean	2.0E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	9.5E-03	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	1	13	14	11	10	4	12	15	5	9	6	8	3	2	7
F_{27}	mean	3.9E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	4.0E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	1	10	13	15	3	6	5	14	2	12	4	7	9	8	11
F_{28}	mean	3.0E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	2.1E-08	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	5	12	13	15	9	1	10	6	4	14	2	3	8	7	11
Mean rank		4.04	11.11	11.96	13.82	4.75	4.18	6.11	11.11	4.71	9.04	6.64	3.11	8.57	8.64	11.04
Final rank		2	12	13	14	5	3	6	12	4	10	7	1	8	9	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	6.9E-13	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	1.2E-13	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	8	15	14	12	1	2	9	11	6	10	7	3	4	5	13
F_2	mean	5.9E+05	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	1.3E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	3	11	10	14	7	2	4	15	6	5	9	1	12	13	8
F_3	mean	2.6E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	1.9E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	2	15	14	13	3	6	5	11	4	8	7	1	9	10	12
F_4	mean	5.3E+02	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	1.9E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	2	11	9	14	3	1	4	15	5	6	8	7	12	13	10
F_5	mean	1.6E-05	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	6.9E-06	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	10	15	13	12	1	4	7	11	6	8	9	3	5	2	14
F_6	mean	4.4E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	9.2E-01	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	4	14	13	12	2	1	7	10	5	11	6	3	8	9	15
F_7	mean	1.7E+01	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	5.6E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	6.5E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	2	8	6	11	4	12	3	10	9	7	5	1	14	13	15
F_9	mean	2.2E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	4.4E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	1	7	12	14	2	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	4.7E-02	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	2.3E-02	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	1	15	13	12	5	4	3	11	6	7	9	2	10	8	14
F_{11}	mean	8.6E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	9.9E+00	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	10	12	14	15	8	2	7	11	5	6	9	1	3	4	13
F_{12}	mean	9.2E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	1.5E+01	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	2	12	14	15	3	6	5	9	4	13	8	1	10	11	7
F_{13}	mean	2.2E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	3.1E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	3	12	14	15	1	6	4	9	5	13	7	2	11	10	8
F_{14}	mean	4.3E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	7.6E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	10	12	14	9	11	2	7	15	5	6	8	1	3	4	13
F_{15}	mean	6.6E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	6.8E+02	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	1	9	10	14	7	3	4	15	5	6	11	2	12	13	8
F_{16}	mean	1.5E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	7.4E-01	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	2	12	7	11	5	3	10	13	4	6	8	1	14	15	9
F_{17}	mean	1.2E+02	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	1.9E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	7	13	14	15	8	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.1E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	1.7E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	1	12	14	15	5	3	6	8	4	13	9	2	11	10	7
F_{19}	mean	5.6E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	1.2E+00	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	4	15	14	13	5	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.8E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	9.9E-01	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	8.9E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	2.1E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	9	15	14	13	8	2	7	3	5	10	1	6	11	12	4
F_{22}	mean	5.2E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	8.1E+02	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	7.8E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	1.1E+03	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	2	7	11	15	3	5	6	14	4	9	10	1	12	13	8
F_{24}	mean	2.2E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	4.8E+00	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.3E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	5.9E+00	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	3	7	14	15	1	9	6	10	5	8	2	4	12	13	11
F_{26}	mean	2.3E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	5.8E+01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	3	14	13	9	7	4	10	15	5	12	6	11	2	1	8
F_{27}	mean	6.4E+02	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	6.9E+01	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	4.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	8.9E-09	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	3	13	14	15	9	2	11	5	1	12	4	10	8	7	6
Mean rank		3.82	11.57	12.61	13.39	4.57	4.11	6.18	11.46	4.57	8.86	7.11	3.46	8.89	8.89	10.50
Final rank		2	11	12	13	4	3	5	10	4	7	6	1	8	8	9
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.7E-12	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	2.6E-13	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	7	15	14	11	1	4	9	12	6	8	10	5	3	2	13
F_2	mean	2.0E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	3.9E+05	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	3	13	14	10	7	2	5	15	6	4	9	1	12	11	8
F_3	mean	2.3E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	1.7E+08	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	1	14	13	11	2	6	5	15	4	9	7	3	10	8	12
F_4	mean	2.3E+02	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	9.6E+01	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	2	11	10	14	3	1	5	15	6	4	7	9	13	12	8
F_5	mean	5.5E-05	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	1.6E-05	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	9	15	13	14	1	5	8	11	6	7	10	4	3	2	12
F_6	mean	1.0E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.2E+02	9.0E+02
	std	7.1E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	2	15	14	12	3	4	8	5	10	6	1	7	11	9	13
F_7	mean	3.7E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	5.5E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	2.5E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	7	4	8	12	6	3	9	10	5	11	2	1	14	13	15
F_9	mean	6.7E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	9.5E+00	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	1	7	12	14	2	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	6.0E-02	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	3.3E-02	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	2	15	14	11	5	6	3	12	4	7	10	1	9	8	13
F_{11}	mean	2.5E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	2.9E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	10	13	14	15	8	1	7	12	3	4	9	2	5	6	11
F_{12}	mean	2.7E+02	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	4.5E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	5.4E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	7.4E+01	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	9.9E+03	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	8.4E+02	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	9	13	14	10	11	2	7	15	5	6	8	1	3	4	12
F_{15}	mean	1.3E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	1.2E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	1	11	9	12	7	3	4	15	5	6	10	2	14	13	8
F_{16}	mean	1.9E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	9.6E-01	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	2	11	6	12	5	3	10	13	4	9	7	1	15	14	8
F_{17}	mean	2.9E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	3.2E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	7	13	15	14	8	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	2.7E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	2.7E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.4E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	2.3E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	1.8E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	1	9	2	9	7	5	9	9	4	3	8	6	9	9	9
F_{21}	mean	4.0E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	5.1E-08	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	9	15	14	12	4	3	5	13	2	8	10	6	11	7	1
F_{22}	mean	1.4E+04	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	1.8E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	10	11	14	13	9	2	7	15	5	6	8	1	3	4	12
F_{23}	mean	1.6E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	1.8E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	2.7E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	1.1E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.1E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	1.2E+02	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	1	5	14	15	2	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.0E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	1.2E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	2	9	13	15	5	1	8	14	4	12	7	10	3	6	11
F_{27}	mean	1.2E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	8.8E+01	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	2.6E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	2.4E+01	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	1	12	14	15	3	6	7	9	2	13	4	5	10	11	8
Mean rank		3.50	11.50	12.50	13.11	4.43	4.21	6.21	11.79	4.54	8.32	7.29	3.86	8.82	8.96	10.21
Final rank		1	12	14	15	4	3	6	13	5	8	7	2	9	10	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'13 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	2.0E-13	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	1.3E-13	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	3	7	5	9	1	1	2	6	1	4	1	1	1	1	8
F_2	mean	2.1E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	1.8E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	2	11	8	15	7	3	4	12	6	5	10	1	13	14	9
F_3	mean	3.4E+02	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	7.8E+02	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	2	13	12	15	5	7	9	6	3	14	4	1	10	11	8
F_4	mean	1.8E+03	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	8.4E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	7	9	2	14	5	3	6	15	4	8	10	1	12	13	11
F_5	mean	5.4E-13	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	1.4E-12	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	8	12	10	14	1	3	7	11	4	9	2	1	5	6	13
F_6	mean	9.3E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	2.1E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	11	13	12	14	1	2	3	8	5	9	4	7	6	10	15
F_7	mean	1.9E-01	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	1.3E-01	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	1	13	11	15	2	7	6	8	4	12	5	3	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	6.2E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	8	6	5	7	2	11	3	12	9	10	4	1	14	13	15
F_9	mean	2.3E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	7.5E-01	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	3	9	12	14	1	6	4	13	5	11	2	7	10	8	15
F_{10}	mean	1.5E-01	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	8.6E-02	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	2	13	12	14	3	7	6	8	5	11	4	1	10	9	15
F_{11}	mean	4.5E+00	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	1.9E+00	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	10	12	13	14	7	2	9	11	5	6	8	1	3	4	15
F_{12}	mean	8.6E+00	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	4.1E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	3	10	12	14	2	4	5	11	7	13	6	1	9	8	15
F_{13}	mean	1.2E+01	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	5.2E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	3	11	13	14	2	5	7	9	6	12	4	1	10	8	15
F_{14}	mean	4.6E+02	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	1.4E+02	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	13	12	14	9	7	4	10	15	5	8	6	1	3	2	11
F_{15}	mean	4.7E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.0E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	2	13	10	14	1	6	4	15	5	7	9	3	11	12	8
F_{16}	mean	1.0E+00	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	2.7E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	13	12	3	9	5	10	8	15	7	4	6	1	11	14	2
F_{17}	mean	1.7E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	3.6E+00	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	12	9	14	15	7	5	8	13	4	6	11	1	3	2	10
F_{18}	mean	2.3E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	8.9E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	5	8	10	15	4	3	7	12	2	11	13	1	14	9	6
F_{19}	mean	8.2E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	3.5E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	11	9	14	15	7	5	8	13	4	12	10	3	2	1	6
F_{20}	mean	2.0E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	4.8E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	3	12	14	15	2	4	6	11	1	9	5	7	10	13	8
F_{21}	mean	4.0E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	2.0E-13	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	10	3	12	13	9	2	9	8	7	11	5	9	6	4	1
F_{22}	mean	5.3E+02	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	3.0E+02	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	12	11	15	13	6	4	9	14	5	7	8	1	2	3	10
F_{23}	mean	5.3E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	1.9E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	2	10	13	15	1	3	7	14	6	9	5	4	12	11	8
F_{24}	mean	1.4E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	3.9E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	2	10	14	15	8	1	9	13	5	12	6	7	4	3	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	1.7E+00	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	6	13	12	15	7	2	9	14	5	10	3	8	1	4	11
F_{26}	mean	1.1E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	8.0E+00	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	1	13	15	14	10	2	11	12	8	9	3	4	7	5	6
F_{27}	mean	3.7E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	4.0E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	9	14	10	15	8	3	2	12	5	11	7	1	4	6	13
F_{28}	mean	3.0E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	2.6E-13	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	7	12	13	15	8	2	11	5	4	14	1	6	10	9	3
Mean rank		6.11	10.71	11.07	13.57	4.61	4.18	6.75	11.29	4.89	9.43	5.79	3.00	7.61	7.57	10.07
Final rank		6	12	13	15	3	2	7	14	4	10	5	1	9	8	11
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	6.6E-10	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	1.7E-09	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	7	12	9	10	1	3	5	8	4	6	3	2	3	3	11
F_2	mean	7.5E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	2.8E+05	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	6	11	9	14	7	2	3	15	5	4	10	1	12	13	8
F_3	mean	5.4E+06	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	4.3E+06	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	2	13	14	15	3	8	4	7	5	10	6	1	9	11	12
F_4	mean	2.2E+03	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	9.0E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	5	11	6	14	2	1	3	15	4	8	9	7	13	12	10
F_5	mean	2.3E-11	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	4.8E-11	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	9	13	12	15	1	3	6	11	5	10	8	2	7	4	14
F_6	mean	1.8E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	6.2E-01	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	3	13	12	14	4	2	5	8	7	9	6	1	10	11	15
F_7	mean	4.2E+00	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	1.7E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	1	12	13	15	2	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	6.3E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	6	4	9	12	7	10	3	8	5	11	2	1	14	13	15
F_9	mean	1.9E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	5.6E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	4	7	12	14	1	6	2	13	5	9	3	8	11	10	15
F_{10}	mean	1.3E-01	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	8.5E-02	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	3	13	12	14	4	5	2	11	6	7	8	1	10	9	15
F_{11}	mean	2.5E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	6.9E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	7	12	13	15	10	2	8	11	6	5	9	1	4	3	14
F_{12}	mean	3.4E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	2.1E+01	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	2	11	14	15	3	5	6	8	4	13	7	1	9	10	12
F_{13}	mean	7.8E+01	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.9E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	3	12	14	15	2	4	5	8	6	13	7	1	10	11	9
F_{14}	mean	3.2E+03	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	6.9E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	13	11	14	9	10	2	7	15	6	5	8	1	4	3	12
F_{15}	mean	3.5E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	6.8E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	3	11	9	14	6	2	5	15	4	7	10	1	12	13	8
F_{16}	mean	2.3E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	2.5E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	10	12	4	11	2	5	9	13	6	3	7	1	14	15	8
F_{17}	mean	6.0E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	1.1E+01	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	7	12	14	15	8	4	9	13	5	6	10	1	2	3	11
F_{18}	mean	7.3E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	2.4E+01	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	3.4E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	9.8E-01	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	8	15	13	14	7	3	6	12	5	10	9	1	4	2	11
F_{20}	mean	9.8E+00	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	8.0E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.4E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	7.4E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	10	14	13	15	11	2	5	4	3	12	1	9	7	8	6
F_{22}	mean	2.0E+03	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	5.4E+02	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	3.5E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	7.6E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	2	8	11	14	1	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.2E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	7.1E+00	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	1	10	14	15	3	6	5	13	2	12	7	4	8	9	11
F_{25}	mean	2.6E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	1.4E+01	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	3	8	14	15	1	7	6	11	4	9	2	5	12	13	10
F_{26}	mean	2.1E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	2.5E+01	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	3	13	14	11	10	4	12	15	5	9	6	8	2	1	7
F_{27}	mean	4.3E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	5.9E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	1	10	13	15	3	6	5	14	2	12	4	7	9	8	11
F_{28}	mean	2.8E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	6.2E+01	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	2	12	13	15	9	1	10	6	5	14	3	4	8	7	11
Mean rank		4.79	11.04	11.96	13.82	4.79	4.11	5.89	11.11	4.61	9.00	6.57	3.04	8.54	8.61	10.96
Final rank		4	11	13	14	4	2	5	12	3	9	6	1	7	8	10
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	9.8E-08	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	3.9E-07	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	10	15	14	12	1	2	8	11	6	9	7	3	4	5	13
F_2	mean	1.3E+06	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	2.7E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	5	11	10	14	7	2	3	15	6	4	9	1	12	13	8
F_3	mean	1.7E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	2.7E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	1	15	14	13	3	6	5	11	4	8	7	2	9	10	12
F_4	mean	2.1E+03	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	4.2E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	5	11	9	14	2	1	3	15	4	6	8	7	12	13	10
F_5	mean	3.3E-06	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	1.5E-05	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	10	15	13	12	1	4	7	11	6	8	9	3	5	2	14
F_6	mean	4.7E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	5.1E-01	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	7	14	13	12	2	1	6	10	4	11	5	3	8	9	15
F_7	mean	9.1E+00	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	1.7E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	5.8E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	2	8	6	11	4	12	3	10	9	7	5	1	14	13	15
F_9	mean	2.7E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	3.9E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	1	7	12	14	2	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	4.5E-01	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	4.1E-01	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	6	15	13	12	4	3	2	11	5	7	9	1	10	8	14
F_{11}	mean	4.4E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	1.0E+01	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	7	12	14	15	9	2	8	11	5	6	10	1	3	4	13
F_{12}	mean	5.3E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	1.3E+01	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	1	12	14	15	3	6	5	9	4	13	8	2	10	11	7
F_{13}	mean	1.4E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	5.3E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	1	12	14	15	2	6	4	9	5	13	7	3	11	10	8
F_{14}	mean	5.5E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	7.8E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	11	12	14	9	10	2	7	15	5	6	8	1	3	4	13
F_{15}	mean	8.3E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	2.1E+03	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	6	9	10	14	7	2	3	15	4	5	11	1	12	13	8
F_{16}	mean	3.0E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	4.5E-01	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	10	12	6	11	4	2	9	13	3	5	7	1	14	15	8
F_{17}	mean	1.1E+02	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	1.7E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	7	13	14	15	8	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.2E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	3.7E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	1	12	14	15	5	3	6	8	4	13	9	2	11	10	7
F_{19}	mean	7.1E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	1.5E+00	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	5	15	14	13	4	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.9E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	1.1E+00	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	8.5E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	1.9E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	7	15	14	13	9	2	8	3	5	10	1	6	11	12	4
F_{22}	mean	5.5E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	9.1E+02	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	7.7E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	1.7E+03	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	1	7	11	15	3	5	6	14	4	9	10	2	12	13	8
F_{24}	mean	2.5E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	1.2E+01	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.1E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	8.5E+00	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	3	7	14	15	1	9	6	10	5	8	2	4	12	13	11
F_{26}	mean	2.0E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	1.3E-01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	1	14	13	9	7	4	10	15	5	12	6	11	3	2	8
F_{27}	mean	8.7E+02	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	1.6E+02	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	4.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	4.4E-03	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	4	13	14	15	9	2	11	5	1	12	3	10	8	7	6
Mean rank		4.50	11.57	12.57	13.39	4.50	4.00	6.00	11.46	4.39	8.71	7.04	3.54	8.93	8.93	10.46
Final rank		4	11	12	13	4	2	5	10	3	7	6	1	8	8	9
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	7.8E-03	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	1.9E-02	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	10	15	14	11	1	4	8	12	6	7	9	5	3	2	13
F_2	mean	6.1E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	1.2E+06	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	6	13	14	10	7	2	4	15	5	3	9	1	12	11	8
F_3	mean	2.8E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	2.1E+08	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	1	14	13	11	2	6	5	15	4	9	7	3	10	8	12
F_4	mean	4.2E+03	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	7.6E+02	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	6	11	10	14	2	1	4	15	5	3	7	9	13	12	8
F_5	mean	1.4E-02	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	2.7E-02	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	10	15	13	14	1	5	8	11	6	7	9	4	3	2	12
F_6	mean	2.5E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.1E+02	9.0E+02
	std	2.9E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	11	15	14	12	2	3	7	4	9	5	1	6	10	8	13
F_7	mean	1.7E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	2.6E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	2.1E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	9	4	7	12	6	3	8	10	5	11	2	1	14	13	15
F_9	mean	7.5E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	8.9E+00	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	1	7	12	14	2	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	7.2E+00	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	3.2E+00	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	7	15	14	11	4	5	2	12	3	6	10	1	9	8	13
F_{11}	mean	1.0E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	1.4E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	7	13	14	15	9	1	8	12	3	4	10	2	5	6	11
F_{12}	mean	1.2E+02	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	2.2E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	3.1E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	9.2E+01	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	1.3E+04	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	1.5E+03	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	11	13	14	9	10	2	7	15	5	6	8	1	3	4	12
F_{15}	mean	1.9E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	5.1E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	7	11	9	12	6	2	3	15	4	5	10	1	14	13	8
F_{16}	mean	3.8E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	2.4E-01	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	10	11	5	12	4	2	9	13	3	8	6	1	15	14	7
F_{17}	mean	2.1E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	2.9E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	7	13	15	14	8	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	2.4E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	5.4E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.5E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	1.8E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	2.0E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	1	9	2	9	7	5	9	9	4	3	8	6	9	9	9
F_{21}	mean	4.3E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	2.7E-02	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	11	15	14	12	4	3	5	13	2	8	9	6	10	7	1
F_{22}	mean	1.3E+04	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	2.1E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	10	11	14	13	9	2	7	15	5	6	8	1	3	4	12
F_{23}	mean	1.6E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	2.4E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	3.4E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	2.8E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.3E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	1.7E+01	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	1	5	14	15	2	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.2E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	2.4E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	2	9	13	15	5	1	8	14	4	12	7	10	3	6	11
F_{27}	mean	1.7E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	2.3E+02	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	3.1E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	9.2E+02	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	1	12	14	15	3	6	7	9	2	13	4	5	10	11	8
Mean rank		5.00	11.50	12.43	13.07	4.25	4.07	5.96	11.75	4.32	8.07	7.18	3.79	8.75	8.93	10.18
Final rank		5	12	14	15	3	2	6	13	4	8	7	1	9	10	11
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'13 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	4.5E-14	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	9.3E-14	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	3	7	5	9	1	1	2	6	1	4	1	1	1	1	8
F_2	mean	2.1E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	1.7E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	2	11	8	15	7	3	4	12	6	5	10	1	13	14	9
F_3	mean	6.2E+02	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	1.4E+03	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	2	13	12	15	5	7	9	6	3	14	4	1	10	11	8
F_4	mean	7.7E+02	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	4.9E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	5	9	2	14	6	3	7	15	4	8	10	1	12	13	11
F_5	mean	2.4E-10	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	5.6E-10	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	9	12	10	14	1	3	7	11	4	8	2	1	5	6	13
F_6	mean	9.5E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	1.2E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	11	13	12	14	1	2	3	8	5	9	4	7	6	10	15
F_7	mean	2.9E-01	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	2.5E-01	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	2	13	11	15	1	7	6	8	4	12	5	3	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	5.7E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	5	7	6	8	2	11	3	12	9	10	4	1	14	13	15
F_9	mean	2.2E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	1.0E+00	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	3	9	12	14	1	6	4	13	5	11	2	7	10	8	15
F_{10}	mean	2.8E-01	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	1.8E-01	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	5	13	12	14	2	7	6	8	4	11	3	1	10	9	15
F_{11}	mean	2.0E-14	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	2.8E-14	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	3	12	13	14	8	2	10	11	6	7	9	1	4	5	15
F_{12}	mean	9.1E+00	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	3.8E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	3	10	12	14	2	4	5	11	7	13	6	1	9	8	15
F_{13}	mean	1.3E+01	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	6.4E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	3	11	13	14	2	5	7	9	6	12	4	1	10	8	15
F_{14}	mean	6.2E+01	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	7.0E+01	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	5	13	14	10	8	4	11	15	6	9	7	1	3	2	12
F_{15}	mean	5.4E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.2E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	3	13	10	14	1	6	4	15	5	7	9	2	11	12	8
F_{16}	mean	8.0E-01	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	3.5E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	8	13	3	10	5	11	9	15	7	4	6	1	12	14	2
F_{17}	mean	1.1E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	4.1E-01	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	4	10	14	15	8	6	9	13	5	7	12	1	3	2	11
F_{18}	mean	1.8E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	4.4E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	2	8	10	15	5	4	7	12	3	11	13	1	14	9	6
F_{19}	mean	4.4E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	1.0E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	4	10	14	15	8	6	9	13	5	12	11	3	2	1	7
F_{20}	mean	2.1E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	4.8E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	3	12	14	15	2	4	6	11	1	9	5	7	10	13	8
F_{21}	mean	4.0E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	1.8E-13	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	9	3	11	12	9	2	9	8	7	10	5	9	6	4	1
F_{22}	mean	6.2E+01	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	8.6E+01	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	5	12	15	13	7	4	10	14	6	8	9	1	2	3	11
F_{23}	mean	6.2E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	2.5E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	4	10	13	15	1	2	7	14	6	9	5	3	12	11	8
F_{24}	mean	1.8E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	3.8E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	5	10	14	15	8	1	9	13	4	12	6	7	3	2	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	1.6E+01	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	6	13	12	15	7	2	9	14	5	10	3	8	1	4	11
F_{26}	mean	1.1E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	2.1E+01	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	1	13	15	14	10	2	11	12	8	9	3	4	7	5	6
F_{27}	mean	3.3E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	4.2E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	3	14	10	15	9	4	2	12	6	11	8	1	5	7	13
F_{28}	mean	3.0E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	9.8E-09	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	7	12	13	15	8	2	11	5	4	14	1	6	10	9	3
Mean rank		4.46	10.93	11.07	13.64	4.82	4.32	7.00	11.29	5.07	9.50	5.96	2.93	7.68	7.61	10.21
Final rank		3	12	13	15	4	2	7	14	5	10	6	1	9	8	11
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.7E-13	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	1.4E-13	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	5	12	9	10	1	3	6	8	4	7	3	2	3	3	11
F_2	mean	2.4E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	1.1E+05	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	4	11	9	14	7	2	3	15	6	5	10	1	12	13	8
F_3	mean	1.5E+06	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	2.5E+06	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	1	13	14	15	3	8	4	7	5	10	6	2	9	11	12
F_4	mean	9.0E+02	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	2.6E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	2	11	6	14	3	1	4	15	5	8	9	7	13	12	10
F_5	mean	1.9E-08	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	1.1E-08	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	10	13	12	15	1	3	6	11	5	9	8	2	7	4	14
F_6	mean	1.7E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	5.8E+00	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	3	13	12	14	4	2	5	8	7	9	6	1	10	11	15
F_7	mean	6.9E+00	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	3.3E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	1	12	13	15	2	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	6.3E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	3	5	9	12	7	10	4	8	6	11	2	1	14	13	15
F_9	mean	1.4E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	3.0E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	2	7	12	14	1	6	3	13	5	9	4	8	11	10	15
F_{10}	mean	1.2E-01	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	1.1E-01	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	2	13	12	14	4	5	3	11	6	7	8	1	10	9	15
F_{11}	mean	1.8E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	4.4E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	7	12	13	15	10	2	8	11	6	5	9	1	4	3	14
F_{12}	mean	5.0E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	1.5E+01	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	3	11	14	15	2	5	6	8	4	13	7	1	9	10	12
F_{13}	mean	1.0E+02	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.9E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	3	12	14	15	2	4	5	8	6	13	7	1	10	11	9
F_{14}	mean	9.3E+02	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	2.5E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	7	12	14	10	11	2	8	15	6	5	9	1	4	3	13
F_{15}	mean	3.3E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	5.4E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	2	11	9	14	6	3	5	15	4	7	10	1	12	13	8
F_{16}	mean	1.3E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	8.2E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	2	12	5	11	3	6	10	13	7	4	8	1	14	15	9
F_{17}	mean	5.5E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	5.5E+00	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	7	12	14	15	8	4	9	13	5	6	10	1	2	3	11
F_{18}	mean	7.2E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	9.1E+00	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	2.9E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	8.2E-01	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	6	15	13	14	8	3	7	12	5	10	9	1	4	2	11
F_{20}	mean	1.0E+01	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	5.2E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.3E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	7.0E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	9	14	13	15	11	2	5	4	3	12	1	10	7	8	6
F_{22}	mean	7.4E+02	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	1.5E+02	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	7	11	14	12	10	4	8	15	5	6	9	1	3	2	13
F_{23}	mean	3.4E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	5.9E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	2	8	11	14	1	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.2E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	6.1E+00	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	1	10	14	15	3	6	5	13	2	12	7	4	8	9	11
F_{25}	mean	2.6E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	1.0E+01	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	3	8	14	15	1	7	6	11	4	9	2	5	12	13	10
F_{26}	mean	2.2E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	4.5E+01	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	5	13	14	11	10	3	12	15	4	9	6	8	2	1	7
F_{27}	mean	5.2E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	8.1E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	1	10	13	15	3	6	5	14	2	12	4	7	9	8	11
F_{28}	mean	3.0E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	5.1E-08	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	5	12	13	15	9	1	10	6	4	14	2	3	8	7	11
Mean rank		3.79	11.11	12.00	13.86	4.93	4.14	6.21	11.11	4.68	9.07	6.68	3.07	8.54	8.61	11.04
Final rank		2	12	13	14	5	3	6	12	4	10	7	1	8	9	11
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	7.0E-13	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	1.3E-13	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	8	15	14	12	1	2	9	11	6	10	7	3	4	5	13
F_2	mean	5.5E+05	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	1.6E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	3	11	10	14	7	2	4	15	6	5	9	1	12	13	8
F_3	mean	1.5E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	1.2E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	1	15	14	13	3	6	5	11	4	8	7	2	9	10	12
F_4	mean	6.7E+02	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	2.2E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	2	11	9	14	3	1	4	15	5	6	8	7	12	13	10
F_5	mean	2.8E-07	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	2.8E-07	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	10	15	13	12	1	4	7	11	6	8	9	3	5	2	14
F_6	mean	4.4E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	1.8E+00	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	4	14	13	12	2	1	7	10	5	11	6	3	8	9	15
F_7	mean	1.6E+01	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	4.8E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	5.3E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	6	8	5	11	3	12	2	10	9	7	4	1	14	13	15
F_9	mean	2.7E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	5.3E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	1	7	12	14	2	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	1.3E-01	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	6.9E-02	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	2	15	13	12	5	4	3	11	6	7	9	1	10	8	14
F_{11}	mean	5.8E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	1.2E+01	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	7	12	14	15	9	2	8	11	5	6	10	1	3	4	13
F_{12}	mean	8.3E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	2.3E+01	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	1	12	14	15	3	6	5	9	4	13	8	2	10	11	7
F_{13}	mean	1.9E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	3.7E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	1	12	14	15	2	6	4	9	5	13	7	3	11	10	8
F_{14}	mean	2.2E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	5.2E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	8	12	14	10	11	2	7	15	5	6	9	1	3	4	13
F_{15}	mean	6.5E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	1.0E+03	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	1	9	10	14	7	3	4	15	5	6	11	2	12	13	8
F_{16}	mean	2.0E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	1.1E+00	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	5	12	7	11	4	2	10	13	3	6	8	1	14	15	9
F_{17}	mean	1.3E+02	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	1.6E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	7	13	14	15	8	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.4E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	2.1E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	2	12	14	15	5	3	6	8	4	13	9	1	11	10	7
F_{19}	mean	6.4E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	1.5E+00	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	4	15	14	13	5	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.9E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	1.0E+00	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	8.3E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	3.0E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	7	15	14	13	9	2	8	3	5	10	1	6	11	12	4
F_{22}	mean	2.1E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	4.9E+02	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	7	11	14	12	10	4	8	15	5	6	9	1	3	2	13
F_{23}	mean	7.0E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	9.0E+02	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	1	7	11	15	3	5	6	14	4	9	10	2	12	13	8
F_{24}	mean	2.5E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	1.1E+01	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.0E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	9.8E+00	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	2	7	14	15	1	9	6	10	5	8	3	4	12	13	11
F_{26}	mean	2.8E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	7.8E+01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	6	14	13	9	7	3	10	15	4	12	5	11	2	1	8
F_{27}	mean	9.1E+02	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	1.4E+02	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	4.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	3.0E-08	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	3	13	14	15	9	2	11	5	1	12	4	10	8	7	6
Mean rank		3.68	11.57	12.57	13.43	4.64	4.04	6.25	11.46	4.50	8.86	7.18	3.54	8.89	8.89	10.50
Final rank		2	12	13	14	5	3	6	11	4	8	7	1	9	9	10
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	1.7E-12	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	2.4E-13	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	7	15	14	11	1	4	9	12	6	8	10	5	3	2	13
F_2	mean	2.0E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	5.8E+05	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	3	13	14	10	7	2	5	15	6	4	9	1	12	11	8
F_3	mean	1.3E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	6.6E+07	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	1	14	13	11	2	6	5	15	4	9	7	3	10	8	12
F_4	mean	3.2E+02	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	9.9E+01	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	2	11	10	14	3	1	5	15	6	4	7	9	13	12	8
F_5	mean	3.7E-06	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	2.5E-06	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	9	15	13	14	1	5	8	11	6	7	10	4	3	2	12
F_6	mean	2.2E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.1E+02	9.0E+02
	std	2.6E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	11	15	14	12	2	3	7	4	9	5	1	6	10	8	13
F_7	mean	3.5E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	6.0E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	3.8E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	6	4	8	12	7	3	9	10	5	11	2	1	14	13	15
F_9	mean	7.1E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	1.1E+01	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	1	7	12	14	2	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	1.5E-01	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	8.9E-02	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	6	15	14	11	4	5	2	12	3	7	10	1	9	8	13
F_{11}	mean	1.8E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	2.6E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	7	13	14	15	9	1	8	12	3	4	10	2	5	6	11
F_{12}	mean	2.3E+02	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	3.6E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	4.1E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	5.9E+01	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	7.1E+03	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	1.0E+03	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	8	13	14	10	11	2	7	15	5	6	9	1	3	4	12
F_{15}	mean	1.3E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	1.7E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	1	11	9	12	7	3	4	15	5	6	10	2	14	13	8
F_{16}	mean	2.2E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	1.3E+00	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	3	11	6	12	5	2	10	13	4	9	7	1	15	14	8
F_{17}	mean	3.3E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	4.5E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	7	13	15	14	8	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	3.3E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	4.3E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.5E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	2.7E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	1.5E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	1	9	2	9	7	5	9	9	4	3	8	6	9	9	9
F_{21}	mean	3.8E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	4.1E+01	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	4	15	14	12	5	3	6	13	2	9	10	7	11	8	1
F_{22}	mean	8.0E+03	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	1.1E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	8	11	14	13	10	2	7	15	5	6	9	1	3	4	12
F_{23}	mean	1.5E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	1.3E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	3.2E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	1.7E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.4E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	1.3E+01	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	2	5	14	15	1	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.3E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	5.7E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	2	9	13	15	5	1	8	14	4	12	7	10	3	6	11
F_{27}	mean	1.7E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	1.9E+02	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	3.6E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	1.1E+03	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	3	12	14	15	2	6	7	9	1	13	4	5	10	11	8
Mean rank		3.68	11.50	12.50	13.11	4.43	4.11	6.21	11.75	4.43	8.32	7.39	3.86	8.79	8.96	10.21
Final rank		1	11	13	14	4	3	5	12	4	7	6	2	8	9	10
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'13 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	6.8E-14	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	1.1E-13	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	3	7	5	9	1	1	2	6	1	4	1	1	1	1	8
F_2	mean	2.4E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	2.0E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	2	11	8	15	7	3	4	12	6	5	10	1	13	14	9
F_3	mean	4.0E+04	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	7.6E+04	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	4	13	12	15	5	7	9	6	2	14	3	1	10	11	8
F_4	mean	8.5E+02	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	5.6E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	5	9	2	14	6	3	7	15	4	8	10	1	12	13	11
F_5	mean	1.4E-10	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	2.4E-10	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	9	12	10	14	1	3	7	11	4	8	2	1	5	6	13
F_6	mean	8.8E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	3.0E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	11	13	12	14	1	2	3	8	5	9	4	7	6	10	15
F_7	mean	4.3E-01	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	4.2E-01	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	2	13	11	15	1	7	6	8	4	12	5	3	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	9.7E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	5	7	6	8	2	11	3	12	9	10	4	1	14	13	15
F_9	mean	2.5E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	1.3E+00	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	5	9	12	14	1	6	3	13	4	11	2	7	10	8	15
F_{10}	mean	1.6E-01	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	1.4E-01	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	3	13	12	14	2	7	6	8	5	11	4	1	10	9	15
F_{11}	mean	5.4E+00	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	2.8E+00	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	10	12	13	14	7	2	9	11	5	6	8	1	3	4	15
F_{12}	mean	1.0E+01	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	4.3E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	4	10	12	14	2	3	5	11	7	13	6	1	9	8	15
F_{13}	mean	1.5E+01	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	6.5E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	5	11	13	14	2	4	7	9	6	12	3	1	10	8	15
F_{14}	mean	3.2E+02	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	1.6E+02	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	11	13	14	9	7	4	10	15	5	8	6	1	3	2	12
F_{15}	mean	5.7E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.6E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	3	13	10	14	1	6	4	15	5	7	9	2	11	12	8
F_{16}	mean	6.3E-01	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	3.4E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	6	13	3	10	5	11	9	15	8	4	7	1	12	14	2
F_{17}	mean	1.8E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	3.9E+00	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	12	9	14	15	7	5	8	13	4	6	11	1	3	2	10
F_{18}	mean	2.0E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	4.6E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	3	8	10	15	5	4	7	12	2	11	13	1	14	9	6
F_{19}	mean	7.2E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	2.0E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	9	10	14	15	7	5	8	13	4	12	11	3	2	1	6
F_{20}	mean	2.1E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	4.7E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	3	12	14	15	2	4	6	11	1	9	5	7	10	13	8
F_{21}	mean	3.9E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	4.5E+01	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	9	3	12	13	10	2	10	8	7	11	5	10	6	4	1
F_{22}	mean	2.6E+02	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	1.7E+02	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	9	12	15	13	6	4	10	14	5	7	8	1	2	3	11
F_{23}	mean	5.9E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	3.3E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	3	10	13	15	1	2	7	14	6	9	5	4	12	11	8
F_{24}	mean	1.6E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	4.2E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	4	10	14	15	8	1	9	13	5	12	6	7	3	2	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	2.0E+00	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	6	13	12	15	7	2	9	14	5	10	3	8	1	4	11
F_{26}	mean	1.1E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	7.4E+00	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	2	13	15	14	10	1	11	12	8	9	3	4	7	5	6
F_{27}	mean	3.3E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	3.5E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	4	14	10	15	9	3	2	12	6	11	8	1	5	7	13
F_{28}	mean	2.9E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	4.5E+01	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	5	12	13	15	8	2	11	6	4	14	1	7	10	9	3
Mean rank		5.61	10.89	11.11	13.64	4.68	4.11	6.86	11.32	4.89	9.39	5.82	3.04	7.64	7.57	10.14
Final rank		5	12	13	15	3	2	7	14	4	10	6	1	9	8	11
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.2E-13	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	8.3E-14	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	5	12	9	10	1	3	6	8	4	7	3	2	3	3	11
F_2	mean	2.0E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	1.2E+05	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	3	11	9	14	7	2	4	15	6	5	10	1	12	13	8
F_3	mean	7.4E+06	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	7.4E+06	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	2	13	14	15	3	8	4	7	5	10	6	1	9	11	12
F_4	mean	1.3E+03	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	7.6E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	4	11	6	14	2	1	3	15	5	8	9	7	13	12	10
F_5	mean	6.7E-08	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	2.4E-07	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	10	13	12	15	1	3	6	11	5	9	8	2	7	4	14
F_6	mean	1.3E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	3.1E+00	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	3	13	12	14	4	2	5	8	7	9	6	1	10	11	15
F_7	mean	1.1E+01	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	4.4E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	2	12	13	15	1	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	6.3E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	7	4	9	12	6	10	3	8	5	11	2	1	14	13	15
F_9	mean	1.3E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	4.1E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	2	7	12	14	1	6	3	13	5	9	4	8	11	10	15
F_{10}	mean	8.8E-02	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	3.8E-02	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	2	13	12	14	4	5	3	11	6	7	8	1	10	9	15
F_{11}	mean	3.8E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	7.6E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	10	12	13	15	9	2	7	11	6	5	8	1	4	3	14
F_{12}	mean	5.0E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	1.3E+01	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	3	11	14	15	2	5	6	8	4	13	7	1	9	10	12
F_{13}	mean	1.3E+02	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.6E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	6	12	14	15	2	3	4	8	5	13	7	1	10	11	9
F_{14}	mean	2.1E+03	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	6.1E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	10	12	14	9	11	2	7	15	6	5	8	1	4	3	13
F_{15}	mean	3.4E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	4.7E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	2	11	9	14	6	3	5	15	4	7	10	1	12	13	8
F_{16}	mean	1.3E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	7.3E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	2	12	5	11	3	6	10	13	7	4	8	1	14	15	9
F_{17}	mean	7.9E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	1.3E+01	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	9	12	14	15	7	4	8	13	5	6	10	1	2	3	11
F_{18}	mean	7.7E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	1.4E+01	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	3.1E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	1.0E+00	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	8	15	13	14	7	3	6	12	5	10	9	1	4	2	11
F_{20}	mean	1.0E+01	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	7.4E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.1E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	8.0E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	6	14	13	15	11	2	5	4	3	12	1	10	8	9	7
F_{22}	mean	1.9E+03	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	6.3E+02	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	3.6E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	6.3E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	2	8	11	14	1	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.2E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	7.9E+00	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	1	10	14	15	3	6	5	13	2	12	7	4	8	9	11
F_{25}	mean	2.6E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	6.3E+00	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	3	8	14	15	1	7	6	11	4	9	2	5	12	13	10
F_{26}	mean	2.0E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	2.0E-02	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	1	13	14	11	10	4	12	15	5	9	6	8	3	2	7
F_{27}	mean	4.6E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	7.4E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	1	10	13	15	3	6	5	14	2	12	4	7	9	8	11
F_{28}	mean	3.0E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	2.2E-08	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	5	12	13	15	9	1	10	6	4	14	2	3	8	7	11
Mean rank		4.36	11.07	12.00	13.82	4.68	4.14	5.96	11.11	4.64	9.07	6.57	3.04	8.61	8.68	11.07
Final rank		3	11	13	14	5	2	6	12	4	10	7	1	8	9	11
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	7.4E-13	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	1.3E-13	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	8	15	14	12	1	2	9	11	6	10	7	3	4	5	13
F_2	mean	6.5E+05	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	1.8E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	3	11	10	14	7	2	4	15	6	5	9	1	12	13	8
F_3	mean	3.5E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	5.2E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	2	15	14	13	3	6	5	11	4	8	7	1	9	10	12
F_4	mean	6.0E+02	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	1.9E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	2	11	9	14	3	1	4	15	5	6	8	7	12	13	10
F_5	mean	3.2E-07	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	4.3E-07	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	10	15	13	12	1	4	7	11	6	8	9	3	5	2	14
F_6	mean	4.5E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	2.0E+00	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	5	14	13	12	2	1	7	10	4	11	6	3	8	9	15
F_7	mean	1.8E+01	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	4.6E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	4.2E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	4	8	6	11	3	12	2	10	9	7	5	1	14	13	15
F_9	mean	2.4E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	4.6E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	1	7	12	14	2	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	9.5E-02	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	3.9E-02	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	2	15	13	12	5	4	3	11	6	7	9	1	10	8	14
F_{11}	mean	8.0E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	1.5E+01	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	8	12	14	15	9	2	7	11	5	6	10	1	3	4	13
F_{12}	mean	9.1E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	2.0E+01	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	2	12	14	15	3	6	5	9	4	13	8	1	10	11	7
F_{13}	mean	2.2E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	5.3E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	3	12	14	15	1	6	4	9	5	13	7	2	11	10	8
F_{14}	mean	3.9E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	7.5E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	9	12	14	10	11	2	7	15	5	6	8	1	3	4	13
F_{15}	mean	7.0E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	9.4E+02	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	3	9	10	14	7	2	4	15	5	6	11	1	12	13	8
F_{16}	mean	1.8E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	1.1E+00	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	2	12	7	11	5	3	10	13	4	6	8	1	14	15	9
F_{17}	mean	1.4E+02	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	1.8E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	8	13	14	15	7	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.5E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	2.1E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	2	12	14	15	5	3	6	8	4	13	9	1	11	10	7
F_{19}	mean	6.2E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	9.2E-01	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	4	15	14	13	5	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.9E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	9.3E-01	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	7.9E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	3.7E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	7	15	14	13	9	2	8	3	5	10	1	6	11	12	4
F_{22}	mean	4.4E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	8.7E+02	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	9	11	14	12	10	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	7.2E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	9.1E+02	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	1	7	11	15	3	5	6	14	4	9	10	2	12	13	8
F_{24}	mean	2.5E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	1.4E+01	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.1E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	1.2E+01	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	3	7	14	15	1	9	6	10	5	8	2	4	12	13	11
F_{26}	mean	2.1E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	3.4E+01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	3	14	13	9	7	4	10	15	5	12	6	11	2	1	8
F_{27}	mean	8.3E+02	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	1.4E+02	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	4.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	1.4E-08	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	3	13	14	15	9	2	11	5	1	12	4	10	8	7	6
Mean rank		3.86	11.57	12.61	13.43	4.61	4.07	6.18	11.46	4.54	8.86	7.14	3.39	8.89	8.89	10.50
Final rank		2	12	13	14	5	3	6	11	4	8	7	1	9	9	10
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.6E-12	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	1.4E-13	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	7	15	14	11	1	4	9	12	6	8	10	5	3	2	13
F_2	mean	1.8E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	4.9E+05	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	3	13	14	10	7	2	5	15	6	4	9	1	12	11	8
F_3	mean	1.5E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	1.5E+08	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	1	14	13	11	2	6	5	15	4	9	7	3	10	8	12
F_4	mean	1.9E+02	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	6.7E+01	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	2	11	10	14	3	1	5	15	6	4	7	9	13	12	8
F_5	mean	1.7E-06	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	9.5E-07	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	9	15	13	14	1	5	8	11	6	7	10	4	3	2	12
F_6	mean	2.3E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.1E+02	9.0E+02
	std	2.9E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	11	15	14	12	2	3	7	4	9	5	1	6	10	8	13
F_7	mean	3.8E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	3.9E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	2.7E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	9	4	7	12	6	3	8	10	5	11	2	1	14	13	15
F_9	mean	6.3E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	7.5E+00	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	1	7	12	14	2	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	1.0E-01	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	5.7E-02	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	4	15	14	11	5	6	2	12	3	7	10	1	9	8	13
F_{11}	mean	2.3E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	4.1E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	8	13	14	15	9	1	7	12	3	4	10	2	5	6	11
F_{12}	mean	2.4E+02	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	3.9E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	4.7E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	6.3E+01	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	9.8E+03	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	1.4E+03	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	8	13	14	10	11	2	7	15	5	6	9	1	3	4	12
F_{15}	mean	1.3E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	2.1E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	1	11	9	12	7	3	4	15	5	6	10	2	14	13	8
F_{16}	mean	2.0E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	1.1E+00	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	2	11	6	12	5	3	10	13	4	9	7	1	15	14	8
F_{17}	mean	4.0E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	5.7E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	8	13	15	14	7	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	3.7E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	3.5E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.6E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	3.1E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	2.0E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	1	9	2	9	7	5	9	9	4	3	8	6	9	9	9
F_{21}	mean	4.0E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	1.0E-07	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	9	15	14	12	4	3	5	13	2	8	10	6	11	7	1
F_{22}	mean	1.2E+04	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	2.6E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	9	11	14	13	10	2	7	15	5	6	8	1	3	4	12
F_{23}	mean	1.6E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	1.9E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	3.1E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	1.6E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.3E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	2.5E+01	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	2	5	14	15	1	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.0E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	7.0E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	2	9	13	15	5	1	8	14	4	12	7	10	3	6	11
F_{27}	mean	1.5E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	2.7E+02	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	2.7E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	9.0E+01	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	1	12	14	15	3	6	7	9	2	13	4	5	10	11	8
Mean rank		3.89	11.50	12.46	13.11	4.39	4.18	6.11	11.75	4.46	8.29	7.36	3.82	8.79	8.93	10.21
Final rank		2	12	14	15	4	3	6	13	5	8	7	1	9	10	11
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'13 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	0.0E+00	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	0.0E+00	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	1	6	4	8	1	1	2	5	1	3	1	1	1	1	7
F_2	mean	7.2E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	5.8E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	8	11	7	15	6	2	3	12	5	4	10	1	13	14	9
F_3	mean	1.1E+07	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	3.9E+07	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	8	13	12	15	4	6	9	5	2	14	3	1	10	11	7
F_4	mean	5.1E+02	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	5.3E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	4	9	2	14	6	3	7	15	5	8	10	1	12	13	11
F_5	mean	7.5E-12	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	2.0E-11	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	9	12	10	14	1	3	7	11	4	8	2	1	5	6	13
F_6	mean	6.0E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	4.8E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	9	13	12	14	1	2	3	8	5	10	4	7	6	11	15
F_7	mean	2.0E+00	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	4.6E+00	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	5	13	11	15	1	7	6	8	3	12	4	2	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	8.5E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	7	6	5	8	2	11	3	12	9	10	4	1	14	13	15
F_9	mean	1.9E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	7.6E-01	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	3	9	12	14	1	6	4	13	5	11	2	7	10	8	15
F_{10}	mean	6.7E-02	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	5.0E-02	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	2	13	12	14	3	7	6	8	5	11	4	1	10	9	15
F_{11}	mean	2.7E+00	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	1.6E+00	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	7	12	13	14	8	2	10	11	5	6	9	1	3	4	15
F_{12}	mean	6.4E+00	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	2.7E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	3	10	12	14	2	4	5	11	7	13	6	1	9	8	15
F_{13}	mean	9.1E+00	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	5.9E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	2	11	13	14	3	5	7	9	6	12	4	1	10	8	15
F_{14}	mean	2.5E+02	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	1.1E+02	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	11	13	14	9	7	4	10	15	5	8	6	1	3	2	12
F_{15}	mean	5.0E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.7E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	2	13	10	14	1	6	4	15	5	7	9	3	11	12	8
F_{16}	mean	9.2E-01	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	2.0E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	9	13	3	10	5	11	8	15	7	4	6	1	12	14	2
F_{17}	mean	1.5E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	2.5E+00	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	10	9	14	15	7	5	8	13	4	6	12	1	3	2	11
F_{18}	mean	2.1E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	4.0E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	3	8	10	15	5	4	7	12	2	11	13	1	14	9	6
F_{19}	mean	8.0E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	2.7E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	11	9	14	15	7	5	8	13	4	12	10	3	2	1	6
F_{20}	mean	2.3E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	7.6E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	4	12	14	15	2	3	6	11	1	9	5	7	10	13	8
F_{21}	mean	3.5E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	1.1E+02	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	3	4	12	13	10	2	10	9	8	11	6	10	7	5	1
F_{22}	mean	2.6E+02	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	2.3E+02	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	9	12	15	13	6	4	10	14	5	7	8	1	2	3	11
F_{23}	mean	4.2E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	2.3E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	2	10	13	15	1	3	7	14	6	9	5	4	12	11	8
F_{24}	mean	1.9E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	3.8E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	5	10	14	15	8	1	9	13	4	12	6	7	3	2	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	2.5E+01	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	5	13	12	15	7	2	9	14	6	10	3	8	1	4	11
F_{26}	mean	1.1E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	7.2E+00	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	1	13	15	14	10	2	11	12	8	9	3	4	7	5	6
F_{27}	mean	3.5E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	6.4E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	7	14	10	15	9	3	2	12	5	11	8	1	4	6	13
F_{28}	mean	3.1E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	3.5E+01	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	9	12	13	15	7	2	11	5	4	14	1	6	10	8	3
Mean rank		5.68	10.82	11.00	13.61	4.68	4.14	6.86	11.25	4.86	9.36	5.86	3.00	7.64	7.57	10.11
Final rank		5	12	13	15	3	2	7	14	4	10	6	1	9	8	11
Algorithms		BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.2E-13	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	9.0E-14	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	3	12	9	10	1	4	6	8	5	7	4	2	4	4	11
F_2	mean	7.9E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	2.9E+05	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	6	11	9	14	7	2	3	15	5	4	10	1	12	13	8
F_3	mean	8.5E+06	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	7.9E+06	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	2	13	14	15	3	8	4	7	5	10	6	1	9	11	12
F_4	mean	1.8E+03	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	6.2E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	5	11	6	14	2	1	3	15	4	8	9	7	13	12	10
F_5	mean	2.4E-11	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	3.7E-11	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	9	13	12	15	1	3	6	11	5	10	8	2	7	4	14
F_6	mean	2.4E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	1.2E+01	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	4	13	12	14	3	2	5	8	7	9	6	1	10	11	15
F_7	mean	5.1E+00	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	2.0E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	1	12	13	15	2	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	5.3E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	3	5	9	12	7	10	4	8	6	11	2	1	14	13	15
F_9	mean	1.9E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	4.1E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	4	7	12	14	1	6	2	13	5	9	3	8	11	10	15
F_{10}	mean	2.3E-01	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	7.9E-02	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	6	13	12	14	3	4	2	11	5	7	8	1	10	9	15
F_{11}	mean	2.9E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	8.2E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	7	12	13	15	10	2	8	11	6	5	9	1	4	3	14
F_{12}	mean	2.1E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	5.8E+00	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	1	11	14	15	3	5	6	8	4	13	7	2	9	10	12
F_{13}	mean	5.7E+01	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.4E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	1	12	14	15	3	4	5	8	6	13	7	2	10	11	9
F_{14}	mean	2.4E+03	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	5.4E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	11	12	14	9	10	2	7	15	6	5	8	1	4	3	13
F_{15}	mean	2.9E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	7.3E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	1	11	9	14	6	3	5	15	4	7	10	2	12	13	8
F_{16}	mean	2.2E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	3.4E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	10	12	4	11	2	5	9	13	6	3	7	1	14	15	8
F_{17}	mean	5.5E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	8.4E+00	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	7	12	14	15	8	4	9	13	5	6	10	1	2	3	11
F_{18}	mean	7.5E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	3.9E+01	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	3.4E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	8.1E-01	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	8	15	13	14	7	3	6	12	5	10	9	1	4	2	11
F_{20}	mean	1.0E+01	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	8.0E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.4E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	6.4E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	10	14	13	15	11	2	5	4	3	12	1	9	7	8	6
F_{22}	mean	1.9E+03	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	5.1E+02	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	3.0E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	5.9E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	1	8	11	14	2	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.2E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	7.1E+00	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	1	10	14	15	3	6	5	13	2	12	7	4	8	9	11
F_{25}	mean	2.6E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	5.7E+00	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	3	8	14	15	1	7	6	11	4	9	2	5	12	13	10
F_{26}	mean	2.0E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	2.6E-02	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	1	13	14	11	10	4	12	15	5	9	6	8	3	2	7
F_{27}	mean	4.9E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	5.6E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	1	10	13	15	3	6	5	14	2	12	4	7	9	8	11
F_{28}	mean	3.0E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	9.4E-10	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	5	12	13	15	9	1	10	6	4	14	2	3	8	7	11
Mean rank		4.43	11.11	11.96	13.82	4.79	4.14	5.96	11.11	4.61	9.04	6.57	3.11	8.61	8.68	11.00
Final rank		3	12	13	14	5	2	6	12	4	10	7	1	8	9	11
Algorithms		BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	3.5E-13	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	1.2E-13	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	3	15	14	12	1	2	9	11	7	10	8	4	5	6	13
F_2	mean	1.4E+06	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	5.1E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	5	11	10	14	7	2	3	15	6	4	9	1	12	13	8
F_3	mean	2.2E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	2.6E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	1	15	14	13	3	6	5	11	4	8	7	2	9	10	12
F_4	mean	1.6E+03	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	3.0E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	4	11	9	14	2	1	3	15	5	6	8	7	12	13	10
F_5	mean	1.7E-09	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	5.2E-09	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	9	15	13	12	1	4	7	11	6	8	10	3	5	2	14
F_6	mean	4.7E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	1.0E+00	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	7	14	13	12	2	1	6	10	4	11	5	3	8	9	15
F_7	mean	9.7E+00	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	2.0E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	4.3E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	6	8	5	11	3	12	2	10	9	7	4	1	14	13	15
F_9	mean	3.4E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	5.4E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	2	7	12	14	1	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	2.2E-01	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	1.2E-01	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	6	15	13	12	4	3	2	11	5	7	9	1	10	8	14
F_{11}	mean	5.6E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	1.4E+01	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	7	12	14	15	9	2	8	11	5	6	10	1	3	4	13
F_{12}	mean	4.4E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	9.3E+00	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	1	12	14	15	3	6	5	9	4	13	8	2	10	11	7
F_{13}	mean	1.2E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	3.3E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	1	12	14	15	2	6	4	9	5	13	7	3	11	10	8
F_{14}	mean	4.9E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	6.7E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	11	12	14	9	10	2	7	15	5	6	8	1	3	4	13
F_{15}	mean	7.7E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	1.8E+03	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	5	9	10	14	7	2	3	15	4	6	11	1	12	13	8
F_{16}	mean	2.9E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	4.8E-01	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	10	12	6	11	4	2	9	13	3	5	7	1	14	15	8
F_{17}	mean	1.0E+02	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	1.2E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	7	13	14	15	8	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.2E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	4.6E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	1	12	14	15	5	3	6	8	4	13	9	2	11	10	7
F_{19}	mean	6.5E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	1.4E+00	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	5	15	14	13	4	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.9E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	1.1E+00	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	9.4E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	1.4E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	10	15	14	13	8	2	7	3	5	9	1	6	11	12	4
F_{22}	mean	4.4E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	1.1E+03	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	9	11	14	12	10	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	6.4E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	1.2E+03	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	1	7	11	15	3	5	6	14	4	9	10	2	12	13	8
F_{24}	mean	2.7E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	1.2E+01	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.1E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	1.2E+01	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	3	7	14	15	1	9	6	10	5	8	2	4	12	13	11
F_{26}	mean	2.2E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	4.7E+01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	3	14	13	9	7	4	10	15	5	12	6	11	2	1	8
F_{27}	mean	1.0E+03	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	1.8E+02	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	4.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	4.6E-08	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	3	13	14	15	9	2	11	5	1	12	4	10	8	7	6
Mean rank		4.43	11.57	12.54	13.39	4.43	4.00	5.96	11.46	4.46	8.75	7.11	3.57	8.93	8.93	10.46
Final rank		3	11	12	13	3	2	5	10	4	7	6	1	8	8	9
Algorithms		BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	8.1E-13	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	1.6E-13	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	4	15	14	11	1	5	9	12	7	8	10	6	3	2	13
F_2	mean	5.0E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	1.8E+06	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	6	13	14	10	7	2	4	15	5	3	9	1	12	11	8
F_3	mean	5.6E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	6.7E+08	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	1	14	13	11	2	6	5	15	4	9	7	3	10	8	12
F_4	mean	3.4E+03	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	4.9E+02	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	6	11	10	14	2	1	4	15	5	3	7	9	13	12	8
F_5	mean	5.1E-08	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	5.1E-08	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	9	15	13	14	1	5	8	11	6	7	10	4	3	2	12
F_6	mean	2.3E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.1E+02	9.0E+02
	std	2.2E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	11	15	14	12	2	3	7	4	9	5	1	6	10	8	13
F_7	mean	1.9E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	3.4E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	2.8E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	9	4	7	12	6	3	8	10	5	11	2	1	14	13	15
F_9	mean	9.6E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	8.7E+00	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	2	7	12	14	1	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	7.9E-01	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	4.5E-01	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	7	15	14	11	4	5	2	12	3	6	10	1	9	8	13
F_{11}	mean	1.4E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	2.4E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	7	13	14	15	9	1	8	12	3	4	10	2	5	6	11
F_{12}	mean	8.5E+01	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	2.2E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	2.8E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	1.1E+02	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	1.1E+04	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	1.2E+03	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	9	13	14	10	11	2	7	15	5	6	8	1	3	4	12
F_{15}	mean	1.8E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	4.4E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	6	11	9	12	7	2	3	15	4	5	10	1	14	13	8
F_{16}	mean	3.8E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	2.5E-01	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	10	11	5	12	4	2	9	13	3	8	6	1	15	14	7
F_{17}	mean	2.5E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	2.7E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	7	13	15	14	8	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	2.5E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	5.8E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.6E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	2.5E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	1.7E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	3	9	1	9	7	5	9	9	4	2	8	6	9	9	9
F_{21}	mean	4.0E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	1.7E+00	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	9	15	14	12	4	3	5	13	2	8	10	6	11	7	1
F_{22}	mean	1.2E+04	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	1.7E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	9	11	14	13	10	2	7	15	5	6	8	1	3	4	12
F_{23}	mean	1.5E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	1.9E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	3.6E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	1.7E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.6E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	2.4E+01	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	2	5	14	15	1	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.7E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	1.9E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	4	9	13	15	5	1	8	14	3	12	7	10	2	6	11
F_{27}	mean	2.2E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	2.3E+02	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	2.9E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	7.0E+02	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	1	12	14	15	3	6	7	9	2	13	4	5	10	11	8
Mean rank		4.75	11.50	12.39	13.11	4.29	4.11	6.00	11.75	4.32	8.07	7.29	3.82	8.75	8.93	10.18
Final rank		5	12	14	15	3	2	6	13	4	8	7	1	9	10	11
Algorithms		BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'13 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.4E-13	4.7E+01	1.2E-12	1.5E+03	0.0E+00	0.0E+00	2.3E-14	3.2E-03	0.0E+00	6.7E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.2E+03
	std	1.2E-13	2.1E+02	1.2E-12	7.3E+02	0.0E+00	0.0E+00	7.0E-14	9.0E-03	0.0E+00	7.6E-13	0.0E+00	0.0E+00	0.0E+00	0.0E+00	1.9E+02
	rank	3	7	5	9	1	1	2	6	1	4	1	1	1	1	8
F_2	mean	1.8E+04	1.0E+06	4.7E+04	8.9E+06	4.5E+04	2.5E+04	2.7E+04	1.6E+06	4.4E+04	3.7E+04	2.1E+05	0.0E+00	1.8E+06	2.4E+06	2.0E+05
	std	1.2E+04	1.6E+06	1.1E+05	4.8E+06	3.2E+04	1.6E+04	1.9E+04	6.3E+05	4.1E+04	4.3E+04	1.2E+05	0.0E+00	8.6E+05	1.2E+06	1.3E+05
	rank	2	11	8	15	7	3	4	12	6	5	10	1	13	14	9
F_3	mean	3.3E+06	8.2E+07	5.7E+07	4.3E+09	6.7E+04	5.2E+05	1.3E+07	5.2E+05	1.7E+04	1.4E+08	1.7E+04	4.7E-01	4.0E+07	4.4E+07	9.9E+06
	std	8.3E+06	1.5E+08	7.4E+07	4.6E+09	2.1E+05	5.7E+05	4.6E+07	8.6E+05	2.7E+04	2.8E+08	3.6E+04	1.5E+00	4.3E+07	3.3E+07	3.1E+07
	rank	7	13	12	15	4	6	9	5	2	14	3	1	10	11	8
F_4	mean	1.0E+03	4.1E+03	2.8E+01	1.3E+04	8.5E+02	2.3E+02	9.5E+02	2.1E+04	7.3E+02	3.2E+03	4.2E+03	1.4E-02	8.2E+03	9.2E+03	6.7E+03
	std	5.0E+02	2.8E+03	5.4E+01	3.7E+03	9.0E+02	1.4E+02	1.0E+03	9.8E+03	7.6E+02	2.4E+03	1.5E+03	6.1E-02	2.1E+03	4.1E+03	3.5E+03
	rank	7	9	2	14	5	3	6	15	4	8	10	1	12	13	11
F_5	mean	1.5E-13	1.2E+01	1.8E-03	8.7E+02	0.0E+00	2.3E-14	8.5E-14	2.1E-02	4.0E-14	4.3E-12	5.7E-15	0.0E+00	5.1E-14	5.7E-14	8.3E+02
	std	9.3E-14	4.1E+01	3.2E-03	1.1E+03	0.0E+00	4.7E-14	5.1E-14	4.4E-02	5.6E-14	5.2E-12	2.5E-14	0.0E+00	6.9E-14	5.8E-14	1.4E+02
	rank	8	12	10	14	1	3	7	11	4	9	2	1	5	6	13
F_6	mean	7.2E+00	1.7E+01	1.1E+01	1.4E+02	9.4E-01	1.2E+00	3.5E+00	5.7E+00	4.7E+00	6.1E+00	3.6E+00	5.4E+00	5.2E+00	8.1E+00	5.3E+02
	std	4.4E+00	3.0E+01	1.6E+01	7.2E+01	1.5E+00	3.0E+00	4.8E+00	1.8E+00	4.8E+00	1.6E+01	1.6E+00	5.0E+00	3.3E+00	1.5E+01	1.1E+02
	rank	10	13	12	14	1	2	3	8	5	9	4	7	6	11	15
F_7	mean	5.6E-01	5.2E+01	5.0E+01	1.0E+03	2.3E-01	3.9E+00	3.1E+00	4.8E+00	6.8E-01	5.0E+01	1.7E+00	5.7E-01	3.4E+01	3.1E+01	5.7E+02
	std	1.1E+00	3.4E+01	3.8E+01	1.5E+03	2.2E-01	3.7E+00	5.0E+00	6.5E+00	1.1E+00	2.1E+01	1.0E+00	2.3E+00	8.1E+00	9.2E+00	1.3E+02
	rank	2	13	11	15	1	7	6	8	4	12	5	3	10	9	14
F_8	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	5.3E-02	8.7E-02	8.3E-02	7.4E-02	8.4E-02	7.5E-02	6.8E-02	7.0E-02	6.5E-02	7.0E-02	7.7E-02	1.8E-01	9.1E-02	7.5E-02	1.1E-01
	rank	8	6	5	7	2	11	3	12	9	10	4	1	14	13	15
F_9	mean	1.7E+00	5.7E+00	6.7E+00	9.0E+00	1.4E+00	3.0E+00	2.5E+00	8.6E+00	2.5E+00	5.8E+00	1.8E+00	3.7E+00	5.8E+00	5.3E+00	1.4E+01
	std	1.0E+00	1.4E+00	1.3E+00	1.0E+00	7.2E-01	1.1E+00	8.8E-01	1.1E+00	1.2E+00	1.0E+00	6.0E-01	1.8E+00	6.6E-01	8.5E-01	9.6E-01
	rank	2	9	12	14	1	6	4	13	5	11	3	7	10	8	15
F_{10}	mean	1.3E-01	1.2E+01	8.6E+00	2.9E+02	1.5E-01	5.7E-01	3.0E-01	9.2E-01	2.8E-01	5.0E+00	2.6E-01	1.3E-02	1.3E+00	1.3E+00	4.0E+02
	std	6.0E-02	2.0E+01	2.0E+01	1.5E+02	8.2E-02	3.8E-01	2.1E-01	4.5E-01	1.7E-01	2.1E+00	1.2E-01	1.3E-02	8.8E-01	6.4E-01	8.2E+01
	rank	2	13	12	14	3	7	6	8	5	11	4	1	10	9	15
F_{11}	mean	4.2E+00	7.1E+00	4.0E+01	1.4E+02	2.8E+00	5.7E-15	3.4E+00	6.8E+00	9.9E-01	1.5E+00	2.9E+00	0.0E+00	5.0E-02	9.9E-02	1.6E+02
	std	1.9E+00	4.7E+00	2.0E+01	2.8E+01	1.5E+00	1.7E-14	3.0E+00	5.7E+00	1.0E+00	9.9E-01	8.4E-01	0.0E+00	2.2E-01	3.1E-01	3.8E+01
	rank	10	12	13	14	7	2	9	11	5	6	8	1	3	4	15
F_{12}	mean	8.0E+00	2.5E+01	3.7E+01	1.1E+02	6.4E+00	9.3E+00	1.1E+01	2.8E+01	1.2E+01	3.8E+01	1.2E+01	5.2E+00	2.0E+01	1.9E+01	2.0E+02
	std	5.1E+00	1.3E+01	1.7E+01	3.2E+01	1.8E+00	3.2E+00	3.7E+00	1.2E+01	3.7E+00	1.3E+01	4.0E+00	2.0E+00	4.4E+00	4.9E+00	8.2E+01
	rank	3	10	12	14	2	4	5	11	7	13	6	1	9	8	15
F_{13}	mean	1.5E+01	3.5E+01	5.2E+01	1.1E+02	9.7E+00	1.5E+01	1.8E+01	2.3E+01	1.6E+01	4.8E+01	1.4E+01	7.4E+00	2.6E+01	2.3E+01	1.7E+02
	std	8.2E+00	1.1E+01	1.7E+01	3.3E+01	5.0E+00	5.8E+00	7.8E+00	7.2E+00	7.6E+00	1.3E+01	4.3E+00	4.0E+00	7.7E+00	6.7E+00	3.5E+00
	rank	5	11	13	14	2	4	7	9	6	12	3	1	10	8	15
F_{14}	mean	4.3E+02	4.0E+02	8.1E+02	2.3E+02	1.8E+02	1.6E+01	2.4E+02	8.2E+02	1.4E+02	2.3E+02	1.5E+02	6.2E-02	1.2E+01	6.0E+00	3.4E+02
	std	1.9E+02	2.5E+02	2.5E+02	1.1E+02	1.3E+02	3.1E+01	1.3E+02	1.8E+02	9.2E+01	1.2E+02	1.0E+02	5.4E-02	3.6E+01	2.6E+01	1.3E+02
	rank	13	12	14	9	7	4	10	15	5	8	6	1	3	2	11
F_{15}	mean	3.9E+02	9.9E+02	7.9E+02	1.3E+03	3.8E+02	6.1E+02	5.9E+02	1.5E+03	5.9E+02	6.5E+02	7.2E+02	5.3E+02	8.3E+02	9.3E+02	6.9E+02
	std	2.0E+02	2.5E+02	1.8E+02	1.9E+02	1.5E+02	2.5E+02	2.3E+02	1.4E+02	2.8E+02	2.0E+02	1.9E+02	1.8E+02	1.8E+02	1.3E+02	1.6E+02
	rank	2	13	10	14	1	6	4	15	5	7	9	3	11	12	8
F_{16}	mean	1.0E+00	9.6E-01	5.4E-01	9.5E-01	5.7E-01	9.5E-01	8.9E-01	1.2E+00	7.8E-01	5.6E-01	6.8E-01	3.1E-01	9.6E-01	1.0E+00	5.2E-01
	std	3.1E-01	3.3E-01	1.6E-01	2.2E-01	1.4E-01	2.5E-01	2.2E-01	1.8E-01	2.6E-01	2.5E-01	1.4E-01	1.3E-01	2.1E-01	1.8E-01	2.0E-01
	rank	13	12	3	9	5	10	8	15	7	4	6	1	11	14	2
F_{17}	mean	1.5E+01	1.5E+01	2.8E+01	1.4E+02	1.3E+01	1.2E+01	1.4E+01	2.4E+01	1.1E+01	1.2E+01	1.7E+01	9.6E+00	1.0E+01	1.0E+01	1.6E+01
	std	2.8E+00	3.1E+00	5.7E+00	4.2E+01	1.2E+00	2.8E+00	2.1E+00	3.3E+00	2.0E+00	1.7E+00	3.5E+00	2.3E+00	8.8E-02	4.7E-02	3.5E+00
	rank	9	10	14	15	7	5	8	13	4	6	12	1	3	2	11
F_{18}	mean	1.9E+01	2.7E+01	3.2E+01	1.6E+02	2.2E+01	2.2E+01	2.6E+01	3.3E+01	1.9E+01	3.2E+01	3.3E+01	1.6E+01	3.4E+01	3.2E+01	2.5E+01
	std	4.8E+00	6.8E+00	1.0E+01	4.5E+01	2.9E+00	5.8E+00	7.0E+00	4.0E+00	4.8E+00	1.0E+01	5.2E+00	3.3E+00	5.6E+00	3.8E+00	6.4E+00
	rank	3	8	10	15	5	4	7	12	2	11	13	1	14	9	6
F_{19}	mean	7.2E-01	7.2E-01	1.5E+00	2.1E+02	6.6E-01	5.6E-01	6.7E-01	1.3E+00	5.4E-01	1.1E+00	7.7E-01	2.5E-01	8.2E-02	7.4E-02	5.6E-01
	std	2.6E-01	2.1E-01	8.7E-01	2.6E+02	2.1E-01	1.8E-01	1.9E-01	2.6E-01	1.6E-01	6.0E-01	1.6E-01	3.8E-02	1.0E-01	7.7E-02	2.4E-01
	rank	9	10	14	15	7	5	8	13	4	12	11	3	2	1	6
F_{20}	mean	1.9E+00	3.2E+00	3.6E+00	4.2E+00	1.9E+00	2.2E+00	2.5E+00	3.1E+00	1.8E+00	3.0E+00	2.5E+00	2.7E+00	3.0E+00	3.2E+00	2.9E+00
	std	4.5E-01	6.0E-01	4.7E-01	4.3E-01	3.4E-01	4.3E-01	6.8E-01	2.9E-01	5.2E-01	4.9E-01	3.0E-01	5.0E-01	3.1E-01	2.6E-01	4.1E-01
	rank	3	12	14	15	2	4	6	11	1	9	5	7	10	13	8
F_{21}	mean	4.0E+02	3.6E+02	4.0E+02	4.3E+02	4.0E+02	3.1E+02	4.0E+02	3.9E+02	3.9E+02	4.0E+02	3.8E+02	4.0E+02	3.8E+02	3.8E+02	3.1E+02
	std	2.6E-13	9.5E+01	8.3E-13	2.7E+01	1.7E-13	1.2E+02	1.7E-13	3.9E+01	6.7E+01	1.0E-12	6.2E+01	1.7E-13	4.5E+01	7.9E+01	1.1E+02
	rank	10	3	12	13	9	2	9	8	7	11	5	9	6	4	1
F_{22}	mean	2.8E+02	4.7E+02	1.3E+03	6.0E+02	1.9E+02	3.8E+01	3.1E+02	1.0E+03	1.0E+02	2.0E+02	2.3E+02	1.2E+00	2.4E+01	2.7E+01	3.9E+02
	std	2.2E+02	2.8E+02	3.5E+02	3.0E+02	1.9E+02	2.8E+01	1.6E+02	2.0E+02	8.1E+01	1.0E+02	9.9E+01	2.9E+00	4.9E+01	4.9E+01	1.5E+02
	rank	9	12	15	13	6	4	10	14	5	7	8	1	2	3	11
F_{23}	mean	4.8E+02	1.1E+03	1.4E+03	1.6E+03	2.9E+02	5.6E+02	7.3E+02	1.6E+03	7.1E+02	1.1E+03	6.4E+02	6.0E+02	1.3E+03	1.2E+03	8.1E+02
	std	2.3E+02	4.3E+02	2.9E+02	2.0E+02	2.2E+02	2.8E+02	2.5E+02	1.8E+02	3.2E+02	3.2E+02	2.6E+02	3.0E+02	2.1E+02	1.9E+02	2.7E+02
	rank	2	10	13	15	1	3	7	14	6	9	5	4	12	11	8
F_{24}	mean	1.7E+02	2.1E+02	2.3E+02	2.4E+02	2.1E+02	1.4E+02	2.1E+02	2.2E+02	1.7E+02	2.2E+02	2.0E+02	2.0E+02	1.6E+02	1.6E+02	2.1E+02
	std	4.4E+01	2.1E+01	5.3E+00	1.5E+01	2.5E+00	3.9E+01	5.7E+00	1.6E+00	3.6E+01	4.8E+00	2.1E+01	5.2E+00	2.2E+01	2.5E+01	1.2E+01
	rank	4	10	14	15	8	1	9	13	5	12	6	7	3	2	11
F_{25}	mean	2.0E+02	2.2E+02	2.2E+02	2.3E+02	2.0E+02	1.9E+02	2.1E+02	2.2E+02	2.0E+02	2.1E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.2E+02
	std	1.7E+01	2.8E+00	1.6E+01	1.8E+01	2.9E+00	3.0E+01	6.1E+00	1.0E+00	1.3E+01	1.7E+01	2.7E+01	4.8E+00	1.9E+01	1.9E+01	2.1E+00
	rank	3	13	12	15	7	2	9	14	6	10	4	8	1	5	11
F_{26}	mean	1.3E+02	2.0E+02	2.2E+02	2.2E+02	1.7E+02	1.1E+02	1.9E+02	2.0E+02	1.4E+02	1.6E+02	1.2E+02	1.2E+02	1.4E+02	1.3E+02	1.3E+02
	std	3.8E+01	6.7E+01	8.0E+01	4.6E+01	6.4E+01	6.9E+00	8.3E+01	1.2E+01	4.4E+01	3.3E+01	2.0E+01	3.9E+01	1.4E+01	1.0E+01	2.6E+01
	rank	4	13	15	14	10	1	11	12	8	9	2	3	7	5	6
F_{27}	mean	3.5E+02	5.4E+02	4.2E+02	6.3E+02	3.6E+02	3.3E+02	3.2E+02	5.3E+02	3.5E+02	4.5E+02	3.5E+02	3.1E+02	3.4E+02	3.5E+02	5.3E+02
	std	5.0E+01	4.1E+01	6.4E+01	1.5E+02	3.9E+01	2.9E+01	3.4E+01	1.7E+01	5.0E+01	6.3E+01	3.2E+01	4.6E+01	2.7E+01	2.9E+01	4.4E+01
	rank	8	14	10	15	9	3	2	12	5	11	7	1	4	6	13
F_{28}	mean	3.5E+02	3.7E+02	5.5E+02	8.4E+02	3.1E+02	2.5E+02	3.2E+02	3.0E+02	2.9E+02	6.8E+02	2.5E+02	3.0E+02	3.1E+02	3.1E+02	2.7E+02
	std	5.2E+01	1.7E+02	2.4E+02	1.4E+02	2.2E+01	8.9E+01	5.8E+01	1.4E+01	4.5E+01	1.4E+02	8.9E+01	0.0E+00	1.1E+02	1.3E+02	7.3E+01
	rank	11	12	13	15	7	2	10	5	4	14	1	6	9	8	3
Mean rank		6.14	10.82	11.07	13.57	4.57	4.11	6.75	11.25	4.89	9.43	5.82	2.96	7.54	7.57	10.14
Final rank		6	12	13	15	3	2	7	14	4	10	5	1	8	9	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'13 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	1.1E-10	1.9E+03	5.4E+02	6.9E+02	2.3E-14	2.3E-13	4.8E-13	1.1E+00	2.5E-13	3.3E-12	2.3E-13	1.5E-13	2.3E-13	2.3E-13	1.3E+03
	std	3.5E-10	1.7E+03	4.9E+02	6.4E+02	7.0E-14	0.0E+00	1.3E-13	2.0E+00	7.0E-14	1.5E-12	0.0E+00	1.1E-13	0.0E+00	0.0E+00	7.4E+01
	rank	7	12	9	10	1	3	5	8	4	6	3	2	3	3	11
F_2	mean	7.3E+05	1.4E+07	7.8E+06	7.4E+07	1.1E+06	1.3E+05	2.3E+05	8.0E+07	6.9E+05	6.8E+05	9.4E+06	2.5E+04	1.9E+07	2.0E+07	6.5E+06
	std	2.6E+05	1.2E+07	6.0E+06	4.6E+07	7.1E+05	4.9E+04	8.0E+04	1.9E+07	2.6E+05	2.5E+05	3.7E+06	6.8E+04	4.7E+06	6.6E+06	4.9E+06
	rank	6	11	9	14	7	2	3	15	5	4	10	1	12	13	8
F_3	mean	1.0E+07	1.7E+10	2.2E+10	2.2E+10	1.8E+07	1.3E+08	2.6E+07	1.0E+08	2.9E+07	9.8E+08	3.9E+07	3.6E+06	8.6E+08	1.7E+09	2.2E+09
	std	1.3E+07	1.3E+10	2.1E+10	9.6E+09	1.7E+07	2.5E+08	3.3E+07	6.7E+07	4.4E+07	1.0E+09	3.1E+07	7.8E+06	5.8E+08	2.5E+09	2.2E+09
	rank	2	13	14	15	3	8	4	7	5	10	6	1	9	11	12
F_4	mean	1.2E+03	1.9E+04	4.0E+03	7.3E+04	9.3E+02	9.3E+01	1.3E+03	1.3E+05	1.8E+03	5.7E+03	8.9E+03	4.8E+03	5.2E+04	4.7E+04	1.8E+04
	std	4.1E+02	1.2E+04	1.6E+03	1.5E+04	5.6E+02	7.3E+01	7.2E+02	2.9E+04	9.0E+02	2.4E+03	2.0E+03	1.9E+04	1.2E+04	1.4E+04	3.8E+03
	rank	3	11	6	14	2	1	4	15	5	8	9	7	13	12	10
F_5	mean	8.4E-12	5.9E+02	1.4E+02	2.0E+03	1.1E-13	2.1E-13	4.4E-13	1.8E+00	3.3E-13	2.4E-11	2.1E-12	1.9E-13	2.0E-12	2.3E-13	9.5E+02
	std	1.5E-11	6.4E+02	5.6E+01	7.1E+03	2.5E-14	6.7E-14	1.1E-13	4.1E+00	7.3E-14	3.2E-11	1.5E-12	6.7E-14	5.0E-12	3.7E-14	4.9E+01
	rank	9	13	12	15	1	3	6	11	5	10	8	2	7	4	14
F_6	mean	3.6E+01	1.7E+02	1.0E+02	2.4E+02	2.1E+01	8.7E+00	2.5E+01	2.8E+01	2.7E+01	2.9E+01	2.6E+01	4.0E+00	5.4E+01	6.3E+01	8.1E+02
	std	2.7E+01	1.7E+02	4.3E+01	3.9E+01	4.0E+00	6.7E+00	2.6E+01	2.1E+00	2.4E+01	2.7E+01	9.5E-01	9.7E+00	2.5E+01	1.9E+01	8.3E+01
	rank	9	13	12	14	3	2	4	7	6	8	5	1	10	11	15
F_7	mean	8.6E+00	1.5E+02	1.7E+02	2.4E+06	8.9E+00	3.4E+01	2.4E+01	1.2E+02	1.7E+01	1.2E+02	3.3E+01	2.9E+01	9.3E+01	9.1E+01	6.3E+02
	std	9.2E+00	4.8E+01	5.2E+01	3.1E+06	4.7E+00	1.4E+01	1.2E+01	3.2E+01	7.8E+00	2.6E+01	9.1E+00	1.6E+01	1.3E+01	1.3E+01	1.2E+02
	rank	1	12	13	15	2	7	4	11	3	10	6	5	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	6.0E-02	4.7E-02	4.0E-02	4.1E-02	3.5E-02	4.5E-02	6.0E-02	4.4E-02	6.7E-02	3.0E-02	6.1E-02	1.2E-01	4.1E-02	7.4E-02	3.0E-02
	rank	7	4	9	12	6	10	3	8	5	11	2	1	14	13	15
F_9	mean	1.1E+01	3.0E+01	3.5E+01	4.0E+01	1.2E+01	2.1E+01	1.7E+01	4.0E+01	2.0E+01	3.1E+01	1.8E+01	3.1E+01	3.2E+01	3.2E+01	5.0E+01
	std	2.9E+00	3.1E+00	3.2E+00	2.0E+00	3.2E+00	2.7E+00	3.1E+00	8.6E-01	5.1E+00	2.7E+00	2.5E+00	1.4E+00	1.7E+00	1.9E+00	1.5E+00
	rank	1	7	12	14	2	6	3	13	5	9	4	8	11	10	15
F_{10}	mean	9.5E-02	4.1E+02	1.8E+02	4.4E+02	1.5E-01	1.5E-01	1.3E-01	9.7E+00	1.6E-01	7.8E-01	9.1E-01	4.5E-02	2.3E+00	1.8E+00	4.9E+02
	std	4.1E-02	3.6E+02	1.0E+02	1.6E+02	5.2E-02	1.1E-01	5.0E-02	1.6E+01	6.3E-02	4.9E-01	4.5E-01	3.1E-02	1.3E+00	4.4E-01	5.4E+00
	rank	2	13	12	14	4	5	3	11	6	7	8	1	10	9	15
F_{11}	mean	2.3E+01	1.3E+02	3.2E+02	7.1E+02	3.4E+01	7.5E-01	3.0E+01	8.2E+01	9.0E+00	8.3E+00	3.4E+01	1.4E-13	3.1E+00	2.6E+00	3.9E+02
	std	8.6E+00	3.2E+01	6.8E+01	2.1E+02	9.5E+00	9.1E-01	7.0E+00	2.5E+01	3.8E+00	5.5E+00	6.0E+00	2.9E-14	2.0E+00	1.8E+00	9.4E+00
	rank	7	12	13	15	10	2	8	11	6	5	9	1	4	3	14
F_{12}	mean	3.1E+01	1.9E+02	3.5E+02	6.2E+02	3.8E+01	5.4E+01	5.6E+01	1.7E+02	5.4E+01	3.0E+02	1.4E+02	2.8E+01	1.7E+02	1.8E+02	2.9E+02
	std	1.7E+01	7.2E+01	6.0E+01	1.9E+02	1.5E+01	1.6E+01	2.1E+01	3.4E+01	1.5E+01	5.3E+01	2.6E+01	7.8E+00	3.2E+01	3.0E+01	9.3E+00
	rank	2	11	14	15	3	5	6	8	4	13	7	1	9	10	12
F_{13}	mean	7.7E+01	2.3E+02	4.2E+02	8.7E+02	7.4E+01	1.2E+02	1.2E+02	1.8E+02	1.2E+02	3.8E+02	1.5E+02	6.6E+01	2.1E+02	2.1E+02	2.0E+02
	std	2.8E+01	6.6E+01	8.3E+01	2.1E+02	2.2E+01	2.6E+01	2.6E+01	1.2E+01	2.2E+01	5.5E+01	2.6E+01	1.6E+01	3.8E+01	3.0E+01	3.4E+00
	rank	3	12	14	15	2	4	5	8	6	13	7	1	10	11	9
F_{14}	mean	2.9E+03	2.8E+03	4.7E+03	1.7E+03	2.2E+03	5.3E+01	1.3E+03	7.1E+03	8.2E+02	8.1E+02	1.5E+03	9.9E-02	8.1E+01	6.3E+01	3.1E+03
	std	5.3E+02	3.2E+02	7.0E+02	8.4E+02	5.9E+02	4.1E+01	3.4E+02	3.6E+02	2.0E+02	2.0E+02	3.8E+02	4.2E-02	9.6E+01	9.0E+01	4.4E+02
	rank	12	11	14	9	10	2	7	15	6	5	8	1	4	3	13
F_{15}	mean	3.4E+03	5.5E+03	5.1E+03	7.3E+03	4.1E+03	3.5E+03	3.8E+03	8.1E+03	3.7E+03	4.1E+03	5.4E+03	3.2E+03	6.2E+03	6.3E+03	4.3E+03
	std	6.1E+02	1.1E+03	9.2E+02	3.8E+02	5.4E+02	4.5E+02	7.9E+02	2.5E+02	8.4E+02	7.5E+02	7.5E+02	3.1E+02	6.5E+02	4.7E+02	5.6E+02
	rank	2	11	9	14	6	3	5	15	4	7	10	1	12	13	8
F_{16}	mean	2.2E+00	2.5E+00	1.5E+00	2.3E+00	1.4E+00	1.6E+00	1.9E+00	2.5E+00	1.6E+00	1.4E+00	1.7E+00	8.3E-01	2.6E+00	2.6E+00	1.8E+00
	std	4.5E-01	2.4E-01	3.4E-01	3.8E-01	2.5E-01	3.1E-01	3.1E-01	1.8E-01	4.0E-01	3.8E-01	2.3E-01	1.8E-01	4.7E-01	3.5E-01	3.8E-01
	rank	10	12	4	11	2	5	9	13	6	3	7	1	14	15	8
F_{17}	mean	5.1E+01	1.3E+02	3.7E+02	9.6E+02	6.5E+01	3.6E+01	6.8E+01	1.7E+02	3.8E+01	3.9E+01	8.4E+01	3.0E+01	3.1E+01	3.1E+01	1.0E+02
	std	7.0E+00	7.7E+01	7.2E+01	4.3E+02	9.7E+00	2.6E+00	1.0E+01	1.0E+01	2.8E+00	4.5E+00	1.2E+01	2.6E-14	3.8E-01	5.5E-01	2.0E+01
	rank	7	12	14	15	8	4	9	13	5	6	10	1	2	3	11
F_{18}	mean	7.1E+01	2.3E+02	3.2E+02	1.2E+03	1.3E+02	8.5E+01	1.6E+02	2.1E+02	7.9E+01	3.7E+02	2.2E+02	6.4E+01	2.3E+02	2.3E+02	1.7E+02
	std	3.0E+01	7.0E+01	6.7E+01	2.4E+02	1.3E+01	4.1E+01	5.4E+01	1.1E+01	1.0E+01	6.6E+01	2.0E+01	5.2E+00	2.3E+01	2.3E+01	3.0E+01
	rank	2	11	13	15	5	4	6	8	3	14	9	1	10	12	7
F_{19}	mean	2.8E+00	1.4E+03	4.9E+01	1.5E+02	3.1E+00	1.9E+00	3.1E+00	1.3E+01	2.2E+00	5.5E+00	4.8E+00	1.1E+00	2.1E+00	1.7E+00	6.0E+00
	std	6.8E-01	3.6E+03	2.2E+01	1.1E+02	7.9E-01	4.5E-01	7.7E-01	8.3E-01	3.5E-01	1.0E+00	1.0E+00	9.8E-02	1.2E+00	7.1E-01	1.6E+00
	rank	6	15	13	14	8	3	7	12	5	10	9	1	4	2	11
F_{20}	mean	9.9E+00	1.3E+01	1.5E+01	1.5E+01	1.1E+01	1.1E+01	1.5E+01	1.3E+01	1.1E+01	1.3E+01	1.2E+01	1.2E+01	1.5E+01	1.5E+01	1.4E+01
	std	8.1E-01	7.9E-01	2.2E-01	1.3E-01	4.6E-01	1.5E+00	1.2E+00	2.1E-01	1.1E+00	1.1E+00	3.3E-01	8.0E-01	4.4E-01	4.8E-01	1.4E+00
	rank	1	7	13	15	2	4	12	8	3	9	6	5	11	14	10
F_{21}	mean	3.7E+02	6.6E+02	4.9E+02	9.9E+02	3.5E+02	2.5E+02	3.1E+02	3.1E+02	2.9E+02	3.7E+02	2.2E+02	3.4E+02	3.2E+02	3.3E+02	3.2E+02
	std	7.4E+01	3.7E+02	6.3E+01	2.8E+02	7.9E+01	5.9E+01	1.1E+02	5.7E+01	8.6E+01	7.4E+01	4.1E+01	8.9E+01	7.1E+01	5.9E+01	9.8E+01
	rank	11	14	13	15	10	2	5	4	3	12	1	9	7	8	6
F_{22}	mean	1.9E+03	3.1E+03	6.1E+03	3.2E+03	1.9E+03	1.4E+02	1.4E+03	7.4E+03	5.3E+02	6.8E+02	1.8E+03	9.1E+01	1.3E+02	1.1E+02	3.7E+03
	std	1.1E+03	7.1E+02	9.7E+02	1.5E+03	4.3E+02	4.9E+01	3.8E+02	2.7E+02	2.1E+02	2.3E+02	3.1E+02	3.5E+01	3.8E+01	2.3E+01	4.3E+02
	rank	10	11	14	12	9	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	3.3E+03	5.0E+03	6.4E+03	7.8E+03	3.1E+03	3.7E+03	3.8E+03	8.2E+03	4.0E+03	5.3E+03	5.6E+03	3.8E+03	7.2E+03	7.4E+03	4.8E+03
	std	4.0E+02	1.1E+03	9.2E+02	7.5E+02	6.7E+02	9.2E+02	7.9E+02	2.9E+02	8.4E+02	8.7E+02	6.0E+02	5.6E+02	5.0E+02	4.9E+02	9.1E+02
	rank	2	8	11	14	1	3	5	15	6	9	10	4	12	13	7
F_{24}	mean	2.3E+02	2.8E+02	3.1E+02	4.7E+02	2.3E+02	2.5E+02	2.5E+02	3.0E+02	2.2E+02	2.9E+02	2.5E+02	2.4E+02	2.6E+02	2.6E+02	2.9E+02
	std	1.5E+01	8.1E+00	1.9E+01	9.4E+01	8.4E+00	1.0E+01	1.6E+01	2.0E+00	6.1E+00	8.7E+00	5.7E+00	1.6E+01	9.8E+00	9.6E+00	5.9E+00
	rank	2	10	14	15	3	6	5	13	1	12	7	4	8	9	11
F_{25}	mean	2.6E+02	2.9E+02	3.3E+02	4.5E+02	2.3E+02	2.8E+02	2.8E+02	3.0E+02	2.6E+02	2.9E+02	2.5E+02	2.7E+02	3.0E+02	3.0E+02	2.9E+02
	std	9.6E+00	1.0E+01	1.2E+01	3.2E+01	8.5E+00	1.2E+01	1.8E+01	2.5E+00	1.7E+01	1.3E+01	5.3E+00	1.6E+01	7.5E+00	9.0E+00	7.1E+00
	rank	3	8	14	15	1	7	6	11	4	9	2	5	12	13	10
F_{26}	mean	2.1E+02	3.4E+02	3.7E+02	2.9E+02	2.9E+02	2.1E+02	3.1E+02	4.0E+02	2.2E+02	2.6E+02	2.3E+02	2.5E+02	2.0E+02	2.0E+02	2.4E+02
	std	3.1E+01	7.0E+01	5.8E+01	1.1E+02	6.1E+01	3.1E+01	6.3E+01	6.5E+00	4.4E+01	8.7E+01	6.1E+01	7.5E+01	6.6E-01	5.2E-01	7.6E+01
	rank	4	13	14	11	10	3	12	15	5	9	6	8	2	1	7
F_{27}	mean	5.7E+02	1.1E+03	1.2E+03	1.8E+03	6.5E+02	8.6E+02	7.9E+02	1.3E+03	5.5E+02	1.2E+03	7.9E+02	9.3E+02	1.0E+03	9.5E+02	1.1E+03
	std	9.9E+01	7.5E+01	8.2E+01	1.9E+02	6.6E+01	1.1E+02	1.1E+02	3.3E+01	9.3E+01	1.3E+02	6.1E+01	1.8E+02	1.1E+02	1.9E+02	7.2E+01
	rank	2	10	13	15	3	6	5	14	1	12	4	7	9	8	11
F_{28}	mean	2.6E+02	1.8E+03	2.9E+03	5.9E+03	3.4E+02	2.8E+02	4.2E+02	3.0E+02	3.0E+02	2.9E+03	2.9E+02	3.0E+02	3.0E+02	3.0E+02	4.2E+02
	std	8.3E+01	4.1E+02	7.8E+02	1.6E+03	2.5E+02	6.2E+01	3.6E+02	9.1E-08	3.1E-13	1.0E+03	4.5E+01	1.9E-13	3.0E-01	6.4E-03	3.8E+02
	rank	1	12	13	15	9	2	10	6	5	14	3	4	8	7	11
Mean rank		4.79	11.04	11.96	13.82	4.75	4.14	6.00	11.07	4.54	8.96	6.57	3.04	8.54	8.61	11.00
Final rank		5	12	14	15	4	2	6	13	3	10	7	1	8	9	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'13 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	7.0E-09	8.0E+03	1.6E+03	9.8E+00	2.2E-13	3.1E-13	1.0E-12	3.8E+00	4.7E-13	4.6E-12	4.9E-13	3.8E-13	4.0E-13	4.1E-13	1.4E+03
	std	1.9E-08	4.7E+03	6.3E+02	7.2E+00	5.1E-14	1.1E-13	3.3E-13	8.1E+00	5.1E-14	2.8E-12	8.3E-14	1.3E-13	1.0E-13	9.3E-14	3.8E+01
	rank	10	15	14	12	1	2	8	11	6	9	7	3	4	5	13
F_2	mean	1.3E+06	4.2E+07	4.1E+07	7.1E+07	1.7E+06	4.7E+05	7.8E+05	3.2E+08	1.5E+06	9.7E+05	2.4E+07	5.4E+04	4.8E+07	4.8E+07	1.7E+07
	std	3.2E+05	5.6E+07	1.6E+07	2.4E+07	6.3E+05	1.3E+05	2.5E+05	6.5E+07	4.5E+05	4.1E+05	1.2E+07	4.6E+04	1.5E+07	1.2E+07	1.4E+07
	rank	5	11	10	14	7	2	3	15	6	4	9	1	12	13	8
F_3	mean	5.0E+07	6.8E+10	4.2E+10	4.1E+10	7.0E+07	2.2E+08	1.7E+08	5.2E+09	9.0E+07	8.7E+08	6.3E+08	2.6E+07	1.6E+09	2.2E+09	9.2E+09
	std	4.4E+07	3.7E+10	1.6E+10	2.3E+10	5.8E+07	2.5E+08	1.7E+08	4.2E+09	7.2E+07	1.6E+09	5.2E+08	3.0E+07	1.0E+09	1.6E+09	7.8E+09
	rank	2	15	14	13	3	6	5	11	4	8	7	1	9	10	12
F_4	mean	1.7E+03	4.7E+04	1.5E+04	1.2E+05	7.9E+02	3.3E+01	1.1E+03	3.1E+05	1.8E+03	3.8E+03	1.1E+04	1.0E+04	8.0E+04	9.1E+04	1.7E+04
	std	4.8E+02	1.6E+04	4.7E+03	2.0E+04	3.0E+02	1.5E+01	5.5E+02	8.9E+04	6.3E+02	1.3E+03	1.7E+03	1.4E+04	2.0E+04	1.7E+04	3.8E+03
	rank	4	11	9	14	2	1	3	15	5	6	8	7	12	13	10
F_5	mean	2.1E-08	4.3E+03	4.4E+02	1.4E+02	2.0E-13	5.0E-13	1.2E-12	1.2E+01	6.9E-13	5.4E-10	9.7E-09	4.1E-13	6.9E-13	3.7E-13	9.7E+02
	std	6.8E-08	4.5E+03	1.6E+02	4.5E+01	7.3E-14	1.0E-13	6.2E-13	1.9E+01	1.9E-13	2.4E-09	7.6E-09	9.2E-14	1.4E-12	7.3E-14	2.9E+01
	rank	10	15	13	12	1	4	7	11	6	8	9	3	5	2	14
F_6	mean	5.5E+01	4.4E+02	3.0E+02	2.3E+02	4.2E+01	4.2E+01	4.6E+01	5.7E+01	4.4E+01	6.6E+01	4.6E+01	4.4E+01	4.9E+01	5.4E+01	8.7E+02
	std	2.0E+01	2.3E+02	1.0E+02	5.2E+01	1.7E+00	7.8E+00	1.1E+01	2.4E+01	8.4E-01	2.6E+01	1.4E+00	1.3E+00	1.1E+01	1.7E+01	3.2E+01
	rank	9	14	13	12	2	1	6	10	4	11	5	3	7	8	15
F_7	mean	1.5E+01	1.9E+02	2.1E+02	1.1E+05	1.8E+01	5.3E+01	4.4E+01	1.9E+02	3.9E+01	1.3E+02	6.4E+01	6.2E+01	1.1E+02	1.1E+02	6.5E+02
	std	3.9E+00	3.1E+01	8.8E+01	1.8E+05	5.3E+00	1.4E+01	1.3E+01	3.5E+01	1.0E+01	2.5E+01	1.1E+01	1.5E+01	1.0E+01	1.2E+01	1.0E+02
	rank	1	11	13	15	2	5	4	12	3	10	7	6	9	8	14
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01
	std	2.4E-02	3.4E-02	4.1E-02	3.1E-02	3.9E-02	2.4E-02	4.1E-02	3.1E-02	3.5E-02	4.6E-02	3.5E-02	9.2E-02	2.9E-02	3.0E-02	5.2E-02
	rank	10	7	5	11	3	12	2	9	8	6	4	1	14	13	15
F_9	mean	2.6E+01	5.7E+01	6.6E+01	7.4E+01	2.8E+01	4.5E+01	3.8E+01	7.3E+01	3.9E+01	6.0E+01	4.2E+01	5.9E+01	6.2E+01	6.2E+01	8.6E+01
	std	2.7E+00	4.6E+00	3.9E+00	2.2E+00	5.0E+00	3.6E+00	5.7E+00	9.8E-01	6.8E+00	2.5E+00	3.2E+00	2.7E+00	3.7E+00	3.2E+00	2.8E+00
	rank	1	7	12	14	2	6	3	13	4	9	5	8	11	10	15
F_{10}	mean	2.9E-01	1.8E+03	5.0E+02	2.7E+02	1.8E-01	1.5E-01	1.4E-01	4.3E+01	2.0E-01	5.6E-01	4.5E+00	5.3E-02	6.0E+00	4.3E+00	5.2E+02
	std	2.4E-01	7.0E+02	1.4E+02	8.4E+01	1.0E-01	8.7E-02	6.9E-02	4.5E+01	1.1E-01	3.9E-01	2.1E+00	6.0E-02	2.4E+00	1.4E+00	8.8E+01
	rank	6	15	13	12	4	3	2	11	5	7	9	1	10	8	14
F_{11}	mean	5.1E+01	3.1E+02	6.1E+02	9.2E+02	8.2E+01	1.6E+00	7.4E+01	1.6E+02	1.4E+01	1.7E+01	8.4E+01	9.9E-02	1.2E+01	1.2E+01	4.0E+02
	std	9.9E+00	9.4E+01	6.1E+01	3.6E+02	1.9E+01	1.5E+00	1.2E+01	3.9E+01	4.1E+00	6.3E+00	1.4E+01	3.1E-01	5.3E+00	4.3E+00	7.6E+00
	rank	7	12	14	15	9	2	8	11	5	6	10	1	3	4	13
F_{12}	mean	5.1E+01	4.5E+02	6.7E+02	8.7E+02	1.0E+02	1.3E+02	1.3E+02	3.9E+02	1.2E+02	5.5E+02	3.4E+02	8.9E+01	4.0E+02	4.3E+02	3.0E+02
	std	1.6E+01	1.2E+02	1.3E+02	1.8E+02	3.8E+01	2.5E+01	4.3E+01	1.4E+01	2.3E+01	7.1E+01	3.8E+01	1.6E+01	5.7E+01	6.1E+01	7.3E+00
	rank	1	12	14	15	3	6	5	9	4	13	8	2	10	11	7
F_{13}	mean	1.5E+02	5.8E+02	7.4E+02	1.5E+03	2.0E+02	2.6E+02	2.3E+02	3.9E+02	2.3E+02	7.1E+02	3.5E+02	2.0E+02	4.9E+02	4.6E+02	3.5E+02
	std	4.6E+01	1.0E+02	8.3E+01	2.0E+02	3.5E+01	4.8E+01	5.5E+01	2.0E+01	5.6E+01	8.7E+01	2.8E+01	3.9E+01	4.1E+01	3.1E+01	4.1E+01
	rank	1	12	14	15	2	6	4	9	5	13	7	3	11	10	8
F_{14}	mean	5.1E+03	5.6E+03	9.3E+03	4.0E+03	4.6E+03	4.7E+01	2.1E+03	1.4E+04	1.1E+03	1.2E+03	3.3E+03	1.0E-01	7.8E+01	1.4E+02	6.3E+03
	std	5.4E+02	6.7E+02	1.1E+03	1.5E+03	1.1E+03	3.2E+01	4.6E+02	3.0E+02	3.5E+02	3.3E+02	4.5E+02	2.7E-02	8.0E+01	1.1E+02	6.1E+02
	rank	11	12	14	9	10	2	7	15	5	6	8	1	3	4	13
F_{15}	mean	8.8E+03	1.1E+04	1.1E+04	1.4E+04	8.4E+03	6.7E+03	7.0E+03	1.5E+04	7.6E+03	8.2E+03	1.2E+04	6.6E+03	1.3E+04	1.4E+04	9.3E+03
	std	2.0E+03	2.4E+03	7.9E+02	5.5E+02	7.6E+02	7.2E+02	1.1E+03	3.7E+02	1.2E+03	6.9E+02	9.7E+02	5.6E+02	7.8E+02	6.4E+02	1.1E+03
	rank	7	9	10	14	6	2	3	15	4	5	11	1	12	13	8
F_{16}	mean	3.1E+00	3.2E+00	2.2E+00	3.2E+00	1.9E+00	1.8E+00	2.7E+00	3.4E+00	1.9E+00	2.1E+00	2.5E+00	1.2E+00	3.8E+00	3.9E+00	2.5E+00
	std	2.6E-01	3.5E-01	4.1E-01	4.3E-01	3.0E-01	2.9E-01	4.7E-01	2.7E-01	5.3E-01	6.2E-01	2.7E-01	1.7E-01	3.5E-01	5.0E-01	4.0E-01
	rank	10	12	6	11	4	2	9	13	3	5	7	1	14	15	8
F_{17}	mean	9.7E+01	4.9E+02	8.3E+02	1.5E+03	1.4E+02	6.5E+01	1.4E+02	3.7E+02	6.9E+01	7.8E+01	1.7E+02	5.1E+01	5.6E+01	5.5E+01	2.5E+02
	std	2.0E+01	1.8E+02	1.2E+02	4.9E+02	2.1E+01	6.5E+00	2.1E+01	2.3E+01	7.9E+00	2.9E+01	2.3E+01	5.3E-14	3.4E+00	2.8E+00	3.6E+01
	rank	7	13	14	15	8	4	9	12	5	6	10	1	3	2	11
F_{18}	mean	1.0E+02	6.7E+02	7.5E+02	2.1E+03	2.5E+02	1.6E+02	2.8E+02	4.2E+02	1.7E+02	7.3E+02	4.6E+02	1.3E+02	4.9E+02	4.9E+02	3.4E+02
	std	1.7E+01	1.8E+02	9.6E+01	2.8E+02	2.6E+01	5.1E+01	1.2E+02	1.1E+01	3.2E+01	1.4E+02	1.9E+01	1.3E+01	2.3E+01	3.9E+01	8.0E+01
	rank	1	12	14	15	5	3	6	8	4	13	9	2	11	10	7
F_{19}	mean	4.9E+00	2.4E+04	4.4E+02	1.0E+02	6.5E+00	4.2E+00	7.7E+00	3.0E+01	4.5E+00	9.5E+00	9.7E+00	2.5E+00	1.0E+01	8.4E+00	1.8E+01
	std	6.0E-01	2.8E+04	3.1E+02	1.1E+02	1.2E+00	8.9E-01	1.9E+00	1.7E+00	9.2E-01	1.6E+00	2.4E+00	4.6E-01	4.9E+00	3.3E+00	5.9E+00
	rank	4	15	14	13	5	2	6	12	3	8	9	1	10	7	11
F_{20}	mean	1.8E+01	2.2E+01	2.4E+01	2.5E+01	2.0E+01	1.9E+01	2.4E+01	2.3E+01	2.0E+01	2.3E+01	2.1E+01	2.1E+01	2.4E+01	2.4E+01	2.2E+01
	std	1.3E+00	9.4E-01	2.7E-01	1.7E-01	6.8E-01	9.1E-01	2.2E+00	2.4E-01	6.7E-01	1.2E+00	3.6E-01	9.1E-01	6.2E-01	3.5E-01	1.4E+00
	rank	1	8	13	15	3	2	11	10	4	9	6	5	12	14	7
F_{21}	mean	9.1E+02	1.7E+03	1.2E+03	1.0E+03	8.8E+02	4.3E+02	8.7E+02	6.0E+02	6.4E+02	9.2E+02	3.6E+02	7.7E+02	1.0E+03	1.0E+03	6.1E+02
	std	1.3E+02	7.2E+02	2.3E+02	4.7E+02	3.2E+02	2.6E+02	3.2E+02	3.6E+02	3.5E+02	2.8E+02	2.8E+02	4.0E+02	2.3E+02	1.3E+02	4.2E+02
	rank	9	15	14	13	8	2	7	3	5	10	1	6	11	12	4
F_{22}	mean	4.3E+03	6.4E+03	1.2E+04	6.5E+03	4.9E+03	1.8E+02	2.7E+03	1.4E+04	1.2E+03	1.5E+03	3.7E+03	2.9E+01	1.5E+02	1.1E+02	7.8E+03
	std	1.3E+03	9.4E+02	1.4E+03	1.9E+03	9.4E+02	1.2E+02	6.3E+02	3.4E+02	3.6E+02	3.8E+02	5.4E+02	5.4E+01	1.6E+02	7.8E+01	1.1E+03
	rank	9	11	14	12	10	4	7	15	5	6	8	1	3	2	13
F_{23}	mean	7.0E+03	9.5E+03	1.3E+04	1.6E+04	8.4E+03	8.5E+03	9.0E+03	1.5E+04	8.5E+03	1.0E+04	1.2E+04	7.7E+03	1.5E+04	1.5E+04	1.0E+04
	std	1.3E+03	1.4E+03	8.7E+02	4.4E+02	1.0E+03	9.4E+02	9.5E+02	3.1E+02	1.3E+03	1.3E+03	1.2E+03	8.2E+02	8.3E+02	6.9E+02	1.2E+03
	rank	1	7	11	15	3	5	6	14	4	9	10	2	12	13	8
F_{24}	mean	2.6E+02	3.5E+02	4.1E+02	7.8E+02	2.7E+02	3.1E+02	3.0E+02	3.8E+02	2.7E+02	3.8E+02	3.1E+02	3.0E+02	3.2E+02	3.3E+02	3.6E+02
	std	2.2E+01	1.4E+01	1.7E+01	2.7E+02	1.1E+01	1.4E+01	1.8E+01	5.3E+00	1.6E+01	1.6E+01	5.8E+00	2.9E+01	1.4E+01	1.2E+01	1.0E+01
	rank	1	10	14	15	3	7	4	13	2	12	6	5	8	9	11
F_{25}	mean	3.1E+02	3.7E+02	4.5E+02	6.6E+02	2.8E+02	3.8E+02	3.6E+02	3.8E+02	3.5E+02	3.8E+02	3.1E+02	3.4E+02	4.1E+02	4.1E+02	3.9E+02
	std	1.3E+01	1.4E+01	2.9E+01	5.4E+01	1.4E+01	1.2E+01	1.8E+01	3.0E+00	1.4E+01	1.4E+01	9.6E+00	3.4E+01	9.9E+00	1.0E+01	8.3E+00
	rank	3	7	14	15	1	9	6	10	5	8	2	4	12	13	11
F_{26}	mean	2.8E+02	4.3E+02	4.1E+02	3.6E+02	3.5E+02	2.3E+02	3.7E+02	4.9E+02	2.6E+02	3.9E+02	2.7E+02	3.9E+02	2.1E+02	2.0E+02	3.5E+02
	std	8.5E+01	5.6E+01	1.1E+02	1.7E+02	5.2E+01	7.6E+01	7.7E+01	3.7E+00	9.0E+01	1.3E+02	9.8E+01	9.8E+01	1.9E+00	1.4E+00	1.2E+02
	rank	6	14	13	9	7	3	10	15	4	12	5	11	2	1	8
F_{27}	mean	1.0E+03	1.7E+03	2.2E+03	3.4E+03	1.1E+03	1.4E+03	1.4E+03	2.2E+03	1.2E+03	2.1E+03	1.4E+03	1.7E+03	1.9E+03	1.9E+03	1.9E+03
	std	1.5E+02	1.1E+02	1.5E+02	4.8E+02	1.4E+02	2.7E+02	1.6E+02	3.0E+01	1.6E+02	1.3E+02	6.3E+01	1.8E+02	1.0E+02	1.0E+02	9.6E+01
	rank	1	8	14	15	2	6	4	13	3	12	5	7	10	11	9
F_{28}	mean	6.0E+02	3.8E+03	5.0E+03	9.9E+03	1.0E+03	4.0E+02	1.8E+03	4.1E+02	4.0E+02	3.1E+03	4.0E+02	1.0E+03	9.6E+02	5.8E+02	4.9E+02
	std	7.7E+02	1.6E+03	2.3E+03	2.7E+03	1.3E+03	7.0E-13	1.6E+03	3.4E+01	4.2E-13	2.6E+03	7.6E-08	1.3E+03	1.4E+03	8.1E+02	1.7E+02
	rank	7	13	14	15	9	2	11	4	1	12	3	10	8	6	5
Mean rank		5.18	11.54	12.54	13.39	4.46	3.96	5.93	11.39	4.36	8.68	6.96	3.50	8.86	8.82	10.43
Final rank		5	13	14	15	4	2	6	12	3	8	7	1	10	9	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'13 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	6.9E-03	5.4E+04	7.6E+03	6.0E-01	4.1E-13	9.0E-13	1.2E-11	1.8E+01	1.4E-12	4.9E-12	1.9E-11	1.4E-12	7.7E-13	7.5E-13	1.4E+03
	std	2.1E-02	1.4E+04	2.3E+03	5.4E-01	2.5E-13	1.7E-13	1.6E-11	4.8E+01	2.3E-13	1.8E-12	1.3E-11	5.0E-13	1.4E-13	1.1E-13	7.2E+01
	rank	10	15	14	11	1	4	8	12	6	7	9	5	3	2	13
F_2	mean	5.7E+06	1.9E+08	2.3E+08	1.3E+08	6.5E+06	1.2E+06	3.1E+06	3.5E+09	4.6E+06	3.1E+06	9.1E+07	4.4E+05	1.4E+08	1.3E+08	5.3E+07
	std	9.0E+05	1.4E+08	6.6E+07	3.1E+07	1.7E+06	2.7E+05	8.3E+05	7.7E+08	1.0E+06	8.0E+05	2.8E+07	1.7E+05	4.8E+07	2.1E+07	2.1E+07
	rank	6	13	14	10	7	2	4	15	5	3	9	1	12	11	8
F_3	mean	9.4E+08	8.2E+11	2.4E+11	8.0E+10	6.9E+08	2.7E+09	2.3E+09	2.2E+14	1.8E+09	1.3E+10	8.9E+09	1.0E+09	1.3E+10	1.1E+10	1.3E+11
	std	4.4E+08	9.6E+11	7.2E+10	1.7E+10	3.5E+08	2.5E+09	1.6E+09	9.7E+14	7.1E+08	8.1E+09	2.1E+09	4.9E+08	9.6E+09	6.1E+09	4.2E+10
	rank	2	14	13	11	1	6	5	15	4	9	7	3	10	8	12
F_4	mean	3.6E+03	1.4E+05	5.2E+04	2.9E+05	1.7E+03	8.9E+00	2.1E+03	1.0E+06	2.7E+03	2.0E+03	1.9E+04	4.5E+04	1.8E+05	1.8E+05	2.3E+04
	std	6.0E+02	3.5E+04	1.3E+04	3.5E+04	4.9E+02	5.5E+00	7.6E+02	4.1E+05	1.0E+03	6.4E+02	1.5E+03	3.2E+04	3.8E+04	3.9E+04	3.1E+03
	rank	6	11	10	14	2	1	4	15	5	3	7	9	13	12	8
F_5	mean	1.1E-02	1.7E+04	2.7E+03	5.5E+03	7.6E-13	1.9E-12	2.3E-11	3.3E+01	2.3E-12	1.4E-11	5.8E-05	1.2E-12	1.1E-12	7.8E-13	1.1E+03
	std	3.8E-02	1.1E+04	7.4E+02	2.4E+04	3.2E-13	3.0E-13	6.7E-11	3.0E+01	3.9E-13	4.3E-12	3.4E-05	3.9E-13	8.9E-13	1.2E-13	3.1E+02
	rank	10	15	13	14	1	5	8	11	6	7	9	4	3	2	12
F_6	mean	2.2E+02	3.1E+03	1.3E+03	4.4E+02	1.1E+02	1.2E+02	1.9E+02	1.2E+02	2.1E+02	1.5E+02	9.6E+01	1.6E+02	2.2E+02	2.1E+02	9.0E+02
	std	4.2E+01	1.9E+03	2.4E+02	8.2E+01	3.4E+01	6.7E+01	4.0E+01	4.4E+01	4.5E+01	5.4E+01	1.3E+00	6.1E+01	2.7E+01	3.2E+01	3.4E+00
	rank	10	15	14	12	2	3	7	4	9	5	1	6	11	8	13
F_7	mean	2.7E+01	1.6E+03	7.1E+04	9.2E+07	4.7E+01	1.0E+02	9.4E+01	3.9E+02	1.0E+02	2.9E+04	1.3E+02	1.1E+02	1.4E+02	1.4E+02	7.6E+02
	std	4.0E+00	1.9E+03	7.3E+04	9.2E+07	1.0E+01	1.6E+01	1.4E+01	5.9E+01	1.9E+01	3.2E+04	2.0E+01	1.9E+01	1.6E+01	1.7E+01	2.5E+01
	rank	1	12	14	15	2	5	3	10	4	13	7	6	8	9	11
F_8	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.2E+01
	std	2.2E-02	3.7E-02	2.9E-02	2.4E-02	3.6E-02	5.7E-02	2.3E-02	2.0E-02	3.5E-02	1.8E-02	3.8E-02	5.3E-02	2.9E-02	2.0E-02	2.7E-02
	rank	7	4	8	12	6	3	9	10	5	11	2	1	14	13	15
F_9	mean	6.7E+01	1.4E+02	1.5E+02	1.6E+02	8.0E+01	1.1E+02	1.0E+02	1.6E+02	1.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	1.5E+02	1.8E+02
	std	4.9E+00	8.2E+00	6.0E+00	4.1E+00	1.1E+01	6.5E+00	8.2E+00	1.3E+00	1.3E+01	6.1E+00	5.0E+00	3.1E+00	3.6E+00	4.0E+00	2.9E+00
	rank	1	7	12	14	2	5	3	13	6	8	4	9	10	11	15
F_{10}	mean	5.4E+00	7.5E+03	2.7E+03	4.6E+02	1.0E-01	1.2E-01	8.6E-02	6.7E+02	1.0E-01	2.3E-01	5.6E+01	5.3E-02	1.3E+01	8.2E+00	9.5E+02
	std	1.8E+00	1.8E+03	4.5E+02	2.0E+02	4.7E-02	9.3E-02	4.7E-02	8.1E+01	5.2E-02	1.5E-01	1.9E+01	3.1E-02	5.1E+00	1.2E+00	3.5E+02
	rank	7	15	14	11	4	5	2	12	3	6	10	1	9	8	13
F_{11}	mean	1.1E+02	1.3E+03	1.9E+03	1.9E+03	2.3E+02	3.7E+00	2.1E+02	7.6E+02	3.1E+01	3.9E+01	2.4E+02	1.2E+01	6.9E+01	8.2E+01	6.2E+02
	std	1.4E+01	2.0E+02	1.6E+02	7.3E+02	4.0E+01	1.6E+00	3.6E+01	3.7E+01	8.2E+00	1.1E+01	2.6E+01	2.3E+01	2.1E+01	2.2E+01	9.1E+01
	rank	7	13	14	15	9	1	8	12	3	4	10	2	5	6	11
F_{12}	mean	1.1E+02	1.5E+03	2.0E+03	2.0E+03	3.0E+02	4.0E+02	3.2E+02	9.7E+02	3.9E+02	1.8E+03	7.9E+02	3.0E+02	1.2E+03	1.2E+03	6.8E+02
	std	1.8E+01	2.9E+02	2.3E+02	4.0E+02	1.0E+02	7.0E+01	5.5E+01	3.1E+01	6.8E+01	2.0E+02	9.3E+01	5.8E+01	8.3E+01	5.8E+01	1.2E+02
	rank	1	12	14	15	2	6	4	9	5	13	8	3	10	11	7
F_{13}	mean	3.3E+02	1.8E+03	2.2E+03	4.1E+03	5.5E+02	6.8E+02	5.4E+02	9.8E+02	6.7E+02	2.1E+03	9.2E+02	6.3E+02	1.2E+03	1.2E+03	9.2E+02
	std	6.1E+01	3.2E+02	1.9E+02	4.7E+02	9.0E+01	8.7E+01	7.0E+01	3.1E+01	1.1E+02	2.4E+02	4.3E+01	6.0E+01	4.1E+01	5.4E+01	7.9E+01
	rank	1	12	14	15	3	6	2	9	5	13	7	4	10	11	8
F_{14}	mean	1.3E+04	1.6E+04	2.3E+04	1.2E+04	1.2E+04	6.5E+01	5.5E+03	3.3E+04	2.2E+03	2.8E+03	9.8E+03	2.4E+01	6.3E+02	7.5E+02	1.6E+04
	std	1.7E+03	2.0E+03	1.3E+03	5.1E+03	1.7E+03	5.2E+01	6.5E+02	5.6E+02	4.1E+02	4.4E+02	1.3E+03	4.9E+01	2.3E+02	2.5E+02	1.0E+03
	rank	11	13	14	9	10	2	7	15	5	6	8	1	3	4	12
F_{15}	mean	1.5E+04	2.9E+04	2.3E+04	3.1E+04	1.8E+04	1.4E+04	1.5E+04	3.3E+04	1.5E+04	1.6E+04	2.6E+04	1.4E+04	3.1E+04	3.1E+04	2.1E+04
	std	4.0E+03	4.1E+03	1.4E+03	7.2E+02	9.3E+02	1.0E+03	1.6E+03	4.0E+02	1.4E+03	1.4E+03	1.2E+03	6.4E+02	6.4E+02	7.3E+02	2.4E+03
	rank	3	11	9	12	7	2	4	15	5	6	10	1	14	13	8
F_{16}	mean	3.8E+00	4.0E+00	3.0E+00	4.0E+00	2.7E+00	2.1E+00	3.6E+00	4.0E+00	2.5E+00	3.2E+00	3.0E+00	1.7E+00	4.5E+00	4.5E+00	3.2E+00
	std	2.8E-01	3.0E-01	3.4E-01	3.8E-01	2.5E-01	3.4E-01	3.7E-01	2.1E-01	5.9E-01	5.3E-01	2.6E-01	1.7E-01	4.1E-01	3.8E-01	3.2E-01
	rank	10	11	5	12	4	2	9	13	3	8	6	1	15	14	7
F_{17}	mean	2.1E+02	2.0E+03	2.7E+03	2.5E+03	3.7E+02	1.4E+02	4.0E+02	9.9E+02	1.3E+02	1.4E+02	4.5E+02	1.0E+02	1.3E+02	1.3E+02	7.5E+02
	std	2.1E+01	4.5E+02	3.2E+02	9.9E+02	6.1E+01	1.6E+01	4.0E+01	3.4E+01	9.9E+00	9.8E+00	3.8E+01	1.6E-01	9.0E+00	8.5E+00	1.0E+02
	rank	7	13	15	14	8	5	9	12	3	6	10	1	2	4	11
F_{18}	mean	2.2E+02	2.1E+03	2.4E+03	4.9E+03	6.3E+02	3.9E+02	7.1E+02	1.1E+03	4.7E+02	2.4E+03	1.2E+03	4.4E+02	1.2E+03	1.2E+03	9.3E+02
	std	2.7E+01	5.9E+02	2.5E+02	5.8E+02	3.5E+01	4.9E+01	2.9E+02	2.3E+01	6.3E+01	2.3E+02	3.4E+01	4.0E+01	4.7E+01	4.6E+01	1.8E+02
	rank	1	12	14	15	5	2	6	8	4	13	9	3	10	11	7
F_{19}	mean	1.2E+01	1.7E+06	1.5E+04	1.5E+02	2.5E+01	1.1E+01	2.8E+01	8.5E+01	1.1E+01	2.6E+01	3.5E+01	1.0E+01	7.2E+01	7.6E+01	1.4E+02
	std	2.8E+00	9.6E+05	1.1E+04	6.2E+01	6.9E+00	2.0E+00	7.6E+00	2.7E+00	2.2E+00	3.9E+00	5.5E+00	1.7E+00	2.0E+01	1.7E+01	1.5E+02
	rank	4	15	14	13	5	2	7	11	3	6	8	1	9	10	12
F_{20}	mean	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01	5.0E+01
	std	2.5E-01	0.0E+00	1.5E-01	0.0E+00	8.4E-10	1.6E-02	0.0E+00	0.0E+00	1.1E-01	1.5E-01	2.9E-10	3.0E-07	0.0E+00	0.0E+00	0.0E+00
	rank	1	9	2	9	7	5	9	9	4	3	8	6	9	9	9
F_{21}	mean	4.3E+02	5.1E+03	1.2E+03	4.3E+02	3.9E+02	3.7E+02	3.9E+02	4.4E+02	3.6E+02	4.0E+02	4.1E+02	3.9E+02	4.2E+02	3.9E+02	3.1E+02
	std	1.6E-01	1.3E+03	6.0E+02	3.2E+00	3.7E+01	4.7E+01	3.7E+01	1.6E+02	5.0E+01	2.2E+01	3.4E+01	3.1E+01	1.1E+01	3.2E+01	3.1E+01
	rank	11	15	14	12	4	3	5	13	2	8	9	6	10	7	1
F_{22}	mean	1.1E+04	1.7E+04	2.9E+04	2.1E+04	1.2E+04	1.6E+02	7.1E+03	3.3E+04	2.4E+03	3.4E+03	1.0E+04	5.4E+01	5.2E+02	5.9E+02	1.9E+04
	std	2.3E+03	1.3E+03	1.6E+03	5.0E+03	1.2E+03	9.3E+01	1.3E+03	5.4E+02	5.8E+02	7.1E+02	1.6E+03	5.0E+01	2.3E+02	2.7E+02	2.0E+03
	rank	9	11	14	13	10	2	7	15	5	6	8	1	3	4	12
F_{23}	mean	1.7E+04	2.7E+04	2.8E+04	3.4E+04	1.9E+04	2.1E+04	2.1E+04	3.4E+04	1.8E+04	2.4E+04	2.7E+04	1.8E+04	3.4E+04	3.4E+04	2.4E+04
	std	2.3E+03	5.1E+03	1.7E+03	6.8E+02	1.9E+03	1.9E+03	3.6E+03	6.9E+02	3.4E+03	2.3E+03	1.8E+03	1.7E+03	8.6E+02	6.0E+02	2.9E+03
	rank	1	9	11	13	4	5	6	14	3	7	10	2	12	15	8
F_{24}	mean	4.0E+02	5.4E+02	7.0E+02	3.0E+03	4.2E+02	5.1E+02	4.8E+02	6.1E+02	4.6E+02	6.1E+02	4.9E+02	4.5E+02	4.9E+02	4.9E+02	5.7E+02
	std	2.1E+01	2.5E+01	4.4E+01	1.1E+03	2.5E+01	2.1E+01	3.4E+01	5.9E+00	2.7E+01	2.3E+01	1.5E+01	3.3E+01	2.0E+01	2.4E+01	1.2E+01
	rank	1	10	14	15	2	9	5	13	4	12	6	3	7	8	11
F_{25}	mean	4.7E+02	5.9E+02	7.6E+02	1.4E+03	4.3E+02	6.4E+02	6.3E+02	6.0E+02	5.9E+02	5.9E+02	4.9E+02	5.1E+02	6.8E+02	6.8E+02	6.5E+02
	std	2.5E+01	2.0E+01	5.0E+01	1.3E+02	3.5E+01	2.3E+01	2.3E+01	4.0E+00	3.0E+01	1.9E+01	1.0E+01	4.0E+01	1.6E+01	1.9E+01	2.2E+01
	rank	2	5	14	15	1	10	9	8	6	7	3	4	13	12	11
F_{26}	mean	4.4E+02	6.4E+02	7.0E+02	1.2E+03	5.1E+02	3.3E+02	5.4E+02	7.1E+02	4.6E+02	6.8E+02	5.4E+02	6.4E+02	4.5E+02	5.2E+02	6.5E+02
	std	6.9E+01	2.0E+01	1.7E+01	6.4E+02	2.2E+01	1.9E+02	8.4E+01	4.3E+00	1.6E+02	2.0E+01	1.1E+02	2.0E+01	1.7E+02	1.8E+02	1.3E+01
	rank	2	9	13	15	5	1	8	14	4	12	7	10	3	6	11
F_{27}	mean	2.2E+03	3.6E+03	4.7E+03	8.3E+03	2.5E+03	3.4E+03	3.1E+03	4.4E+03	2.9E+03	4.3E+03	3.2E+03	3.5E+03	3.6E+03	3.7E+03	4.0E+03
	std	2.5E+02	3.3E+02	2.4E+02	2.4E+03	2.9E+02	2.0E+02	2.1E+02	6.4E+01	2.5E+02	2.2E+02	1.1E+02	4.9E+02	2.9E+02	1.3E+02	9.8E+01
	rank	1	9	14	15	2	6	4	13	3	12	5	7	8	10	11
F_{28}	mean	2.8E+03	1.2E+04	1.8E+04	2.8E+04	3.6E+03	4.1E+03	4.3E+03	4.7E+03	3.5E+03	1.7E+04	3.7E+03	3.9E+03	5.0E+03	6.0E+03	4.5E+03
	std	5.5E+02	2.0E+03	2.2E+03	4.7E+03	1.1E+03	9.6E+02	1.2E+03	8.1E+02	1.1E+03	1.6E+03	1.0E+03	1.2E+03	2.1E+03	2.7E+03	1.4E+03
	rank	1	12	14	15	3	6	7	9	2	13	4	5	10	11	8
Mean rank		4.79	11.50	12.46	13.07	4.25	4.07	6.04	11.75	4.36	8.11	7.18	3.79	8.79	8.93	10.18
Final rank		5	12	14	15	3	2	6	13	4	8	7	1	9	10	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.2E+04	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	2.2E+04	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.8E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	5.3E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	5	13	12	15	11	6	9	14	7	8	4	1	2	3	10
F_3	mean	8.2E+01	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	1.4E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.1E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.7E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	11	10	14	15	3	1	12	6	9	5	7	13	4	2	8
F_5	mean	2.0E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	1.0E-01	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	11	14	8	13	2	1	4	15	10	3	6	9	7	5	12
F_6	mean	3.3E-03	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	6.4E-03	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	1	12	13	15	2	7	6	14	3	11	4	5	8	9	10
F_7	mean	1.3E-01	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.5E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	10	14	12	15	7	8	11	6	9	13	5	1	2	3	4
F_8	mean	6.0E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	2.1E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	6.3E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	2.4E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	5	13	14	15	3	7	9	10	6	11	2	1	4	8	12
F_{10}	mean	2.9E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.6E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	13	12	15	10	6	2	8	14	7	9	5	1	4	3	11
F_{11}	mean	2.8E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	1.9E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	8	12	13	14	3	4	9	15	5	10	2	1	6	7	11
F_{12}	mean	1.9E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.7E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	7	4	12	14	13	8	6	15	3	2	11	1	9	10	5
F_{13}	mean	6.2E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	3.2E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	2	12	13	15	1	7	5	8	4	14	3	6	11	9	10
F_{14}	mean	9.9E-02	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	5.1E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	3	10	13	15	1	5	4	9	2	14	6	7	11	12	8
F_{15}	mean	8.0E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.2E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	5	7	12	15	2	4	9	13	6	14	11	1	10	8	3
F_{16}	mean	2.0E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.2E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	7	10	15	13	1	3	5	14	2	11	9	4	6	8	12
F_{17}	mean	8.0E+02	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	4.9E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	4.0E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	2.3E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	1.5E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	1.5E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.7E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E+01	3.2E+01	3.8E+01	2.4E+02
	std	1.8E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	7	14	5	15	9	2	11	12	6	13	8	1	3	4	10
F_{21}	mean	4.4E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.2E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	2.9E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	2.7E+01	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	11	10	15	14	7	3	13	9	2	12	8	1	4	6	5
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	2.6E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	6	10	9	11	3	4	7	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	5.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	1	12	14	15	4	8	6	3	7	13	5	2	9	10	11
F_{25}	mean	1.5E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.3E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	3	13	15	14	5	1	10	11	6	12	2	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	2.0E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	6.3E+01	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.3E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	1	12	14	13	8	3	10	15	6	9	5	7	2	4	11
F_{28}	mean	3.0E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	1.4E+02	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	1	13	14	15	4	10	9	3	8	7	2	12	6	5	11
F_{29}	mean	3.4E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	5.2E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	8	12	14	15	2	7	13	3	9	4	1	11	6	5	10
F_{30}	mean	1.5E+03	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	3.3E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	13	11	14	15	3	8	12	1	9	10	2	6	4	5	7
Mean rank		6.43	11.70	11.50	14.23	5.00	4.43	8.30	10.03	5.57	9.80	5.50	3.67	6.40	6.80	8.80
Final rank		7	14	13	15	3	2	9	12	5	11	4	1	6	8	10
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.6E+05	1.6E+07	1.5E+07	1.1E+08	3.8E+05	5.5E+04	1.1E+05	5.6E+07	2.9E+05	1.9E+05	2.5E+06	9.9E+02	8.3E+06	1.0E+07	4.9E+06
	std	8.0E+04	1.7E+07	1.2E+07	4.1E+07	2.3E+05	4.0E+04	7.1E+04	2.6E+07	2.2E+05	1.2E+05	1.1E+06	2.5E+03	3.6E+06	6.1E+06	4.7E+06
	rank	4	13	12	15	7	2	3	14	6	5	8	1	10	11	9
F_2	mean	3.7E+03	2.9E+09	4.8E+08	9.6E+08	4.9E+01	1.8E-03	5.6E+01	1.6E+04	3.0E+01	2.9E+03	3.5E+03	3.5E-14	3.4E+03	4.9E+03	1.4E+07
	std	1.7E+03	3.1E+09	3.8E+08	8.7E+08	8.0E+01	3.9E-03	1.1E+02	1.4E+04	3.9E+01	1.7E+03	5.7E+03	1.6E-14	3.0E+03	3.5E+03	7.8E+07
	rank	9	15	13	14	4	2	5	11	3	6	8	1	7	10	12
F_3	mean	3.9E+01	2.0E+04	3.1E+03	4.5E+04	2.4E+01	1.2E-02	2.3E+01	3.7E+04	2.6E+01	3.7E+02	4.9E+02	9.7E-14	2.3E+02	3.0E+02	3.0E+02
	std	4.2E+01	2.0E+04	2.2E+03	3.1E+04	3.5E+01	3.0E-02	2.4E+01	2.6E+04	2.7E+01	3.3E+02	3.5E+02	8.7E-14	2.6E+02	2.7E+02	3.7E+02
	rank	6	13	12	15	4	2	3	14	5	10	11	1	7	8	9
F_4	mean	3.6E+01	2.0E+02	1.9E+02	3.7E+02	2.6E+01	3.7E+00	4.3E+00	2.6E+01	4.2E+01	4.5E+01	2.5E+01	1.3E-12	8.8E+01	7.5E+01	7.9E+01
	std	2.8E+01	1.6E+02	5.6E+01	1.4E+02	1.8E+01	1.5E+01	1.6E+01	4.0E-01	3.6E+01	2.9E+01	1.1E+00	1.6E-12	2.7E+01	2.5E+01	3.3E+01
	rank	7	14	13	15	6	2	3	5	8	9	4	1	12	10	11
F_5	mean	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	2.3E-01	3.0E-01	7.5E-02	1.6E-01	7.2E-02	5.9E-02	3.4E-01	4.9E-02	1.6E-02	1.9E-04	6.0E-02	5.2E-03	6.8E-02	8.3E-02	1.4E-01
	rank	4	11	5	14	7	6	12	15	2	1	13	3	8	9	10
F_6	mean	3.7E+00	1.9E+01	3.0E+01	3.6E+01	3.7E+00	1.1E+01	6.6E+00	3.9E+01	3.6E+00	2.0E+01	8.3E+00	5.5E+00	1.5E+01	1.6E+01	2.1E+01
	std	1.4E+00	3.1E+00	3.8E+00	5.0E+00	2.3E+00	2.2E+00	2.3E+00	1.4E+00	2.0E+00	2.5E+00	1.6E+00	2.6E+00	1.8E+00	2.0E+00	2.9E+00
	rank	3	10	13	14	2	7	5	15	1	11	6	4	8	9	12
F_7	mean	1.2E-02	2.2E+01	8.0E+00	1.5E+01	7.9E-03	4.7E-03	1.6E-02	5.4E-05	5.3E-03	1.3E-02	1.4E-03	3.2E-03	7.3E-03	6.5E-04	1.8E-03
	std	9.4E-03	1.8E+01	5.4E+00	1.6E+01	1.1E-02	7.7E-03	1.9E-02	2.2E-04	8.2E-03	1.7E-02	4.3E-03	6.1E-03	1.3E-02	1.3E-03	4.6E-03
	rank	10	15	13	14	9	6	12	1	7	11	3	5	8	2	4
F_8	mean	4.2E+01	8.0E+01	1.4E+02	1.5E+02	3.3E+01	9.9E-02	3.1E+01	7.2E+01	9.0E+00	8.8E+00	3.3E+01	2.7E-13	2.1E+00	1.8E+00	6.7E+01
	std	7.2E+00	2.0E+01	2.4E+01	4.6E+01	7.4E+00	3.1E-01	9.6E+00	1.4E+01	3.0E+00	4.3E+00	6.5E+00	5.7E-14	2.1E+00	1.4E+00	1.1E+01
	rank	10	13	14	15	8	2	7	12	6	5	9	1	4	3	11
F_9	mean	4.2E+01	1.4E+02	1.6E+02	2.3E+02	3.7E+01	5.0E+01	5.5E+01	1.6E+02	4.5E+01	1.2E+02	5.9E+01	2.4E+01	6.7E+01	7.1E+01	9.4E+01
	std	1.0E+01	3.1E+01	2.3E+01	7.6E+01	8.4E+00	1.5E+01	1.4E+01	1.5E+01	1.2E+01	2.3E+01	1.2E+01	5.4E+00	1.2E+01	1.2E+01	2.0E+01
	rank	3	12	13	15	2	5	6	14	4	11	7	1	8	9	10
F_{10}	mean	1.7E+03	2.6E+03	4.1E+03	9.7E+02	1.1E+03	5.8E+00	6.6E+02	6.3E+03	3.8E+02	5.1E+02	8.6E+02	2.4E-01	3.6E+01	4.3E+00	2.7E+03
	std	5.5E+02	4.1E+02	7.2E+02	4.7E+02	5.1E+02	1.6E+00	3.7E+02	3.5E+02	1.9E+02	1.7E+02	3.2E+02	7.6E-01	6.3E+01	2.2E+01	5.0E+02
	rank	11	12	14	9	10	3	7	15	5	6	8	1	4	2	13
F_{11}	mean	2.7E+03	3.8E+03	4.4E+03	6.0E+03	2.6E+03	2.0E+03	2.7E+03	7.6E+03	2.3E+03	3.0E+03	2.7E+03	1.7E+03	3.1E+03	3.4E+03	3.4E+03
	std	6.5E+02	6.5E+02	7.5E+02	1.2E+03	4.6E+02	3.3E+02	5.5E+02	2.9E+02	5.2E+02	6.6E+02	4.6E+02	2.1E+02	4.8E+02	4.5E+02	4.5E+02
	rank	5	12	13	14	4	2	6	15	3	8	7	1	9	11	10
F_{12}	mean	4.2E-01	3.1E-01	1.3E+00	1.8E+00	8.6E-01	1.7E-01	6.2E-01	2.5E+00	1.9E-01	3.2E-01	9.4E-01	1.2E-01	6.5E-01	6.1E-01	5.4E-01
	std	3.7E-01	1.1E-01	3.9E-01	4.5E-01	1.9E-01	8.0E-02	3.5E-01	2.2E-01	7.3E-02	1.3E-01	3.2E-01	1.9E-02	1.3E-01	1.3E-01	1.5E-01
	rank	6	4	13	14	11	2	9	15	3	5	12	1	10	8	7
F_{13}	mean	1.8E-01	7.3E-01	5.6E-01	4.7E-01	1.4E-01	2.6E-01	2.9E-01	3.4E-01	2.2E-01	3.7E-01	3.5E-01	3.0E-01	2.8E-01	2.9E-01	4.1E-01
	std	4.7E-02	5.4E-01	1.3E-01	1.1E-01	2.6E-02	6.3E-02	5.9E-02	5.0E-02	6.6E-02	1.0E-01	4.5E-02	4.8E-02	3.6E-02	3.8E-02	6.8E-02
	rank	2	15	14	13	1	4	6	9	3	11	10	8	5	7	12
F_{14}	mean	1.7E-01	8.4E+00	2.0E+00	2.4E+00	3.0E-01	2.6E-01	2.3E-01	3.0E-01	2.4E-01	2.6E-01	2.4E-01	3.0E-01	2.4E-01	2.3E-01	5.4E-01
	std	3.3E-02	1.3E+01	3.6E+00	4.8E+00	1.6E-01	2.5E-02	3.5E-02	3.9E-02	5.0E-02	5.2E-02	2.6E-02	1.4E-01	3.2E-02	2.9E-02	2.4E-01
	rank	1	15	13	14	9	8	2	10	6	7	4	11	5	3	12
F_{15}	mean	3.4E+00	4.7E+02	9.4E+01	1.5E+02	4.4E+00	5.0E+00	5.4E+00	1.5E+01	4.1E+00	4.2E+01	8.8E+00	2.5E+00	1.1E+01	1.1E+01	6.9E+00
	std	1.1E+00	6.8E+02	9.5E+01	1.3E+02	1.7E+00	1.0E+00	1.5E+00	9.6E-01	1.0E+00	1.6E+01	2.2E+00	4.0E-01	2.7E+00	2.1E+00	1.8E+00
	rank	2	15	13	14	4	5	6	11	3	12	8	1	10	9	7
F_{16}	mean	1.0E+01	1.1E+01	1.2E+01	1.3E+01	9.7E+00	9.5E+00	1.0E+01	1.3E+01	1.0E+01	1.1E+01	1.1E+01	9.4E+00	1.1E+01	1.1E+01	1.2E+01
	std	6.0E-01	5.9E-01	5.3E-01	4.6E-01	7.2E-01	8.9E-01	8.8E-01	2.4E-01	7.2E-01	6.3E-01	3.4E-01	3.9E-01	5.0E-01	4.2E-01	4.2E-01
	rank	5	10	13	14	3	2	6	15	4	9	11	1	8	7	12
F_{17}	mean	4.0E+04	9.4E+05	6.5E+05	5.2E+06	7.7E+04	2.2E+04	2.6E+04	1.4E+06	6.3E+04	5.8E+04	2.9E+05	1.4E+03	1.8E+06	1.6E+06	2.6E+05
	std	3.7E+04	1.3E+06	8.9E+05	3.8E+06	5.7E+04	1.7E+04	1.8E+04	6.7E+05	4.7E+04	2.8E+04	1.7E+05	6.5E+02	1.5E+06	1.0E+06	2.9E+05
	rank	4	11	10	15	7	2	3	12	6	5	9	1	14	13	8
F_{18}	mean	6.6E+02	8.4E+04	1.5E+03	1.5E+03	2.7E+03	3.3E+02	3.2E+03	2.1E+05	8.6E+02	1.3E+03	3.8E+02	1.4E+02	9.5E+01	1.1E+02	1.6E+03
	std	1.0E+03	3.0E+05	5.1E+03	2.3E+03	5.4E+03	2.9E+02	4.5E+03	1.6E+05	1.1E+03	1.5E+03	2.2E+02	4.3E+01	2.1E+01	9.8E+01	2.3E+03
	rank	6	14	9	10	12	4	13	15	7	8	5	3	1	2	11
F_{19}	mean	8.3E+00	3.2E+01	2.5E+01	7.2E+01	7.2E+00	6.9E+00	6.8E+00	1.8E+01	5.1E+00	1.4E+01	1.2E+01	4.9E+00	1.2E+01	1.1E+01	9.5E+00
	std	1.4E+00	3.5E+01	1.5E+01	1.0E+02	2.3E+00	1.5E+00	1.3E+00	8.5E-01	1.1E+00	3.8E+00	9.7E-01	9.2E-01	2.0E+00	2.0E+00	1.8E+00
	rank	6	14	13	15	5	4	3	12	2	11	10	1	9	8	7
F_{20}	mean	3.2E+02	5.2E+03	8.1E+02	6.3E+04	3.8E+02	1.8E+02	7.4E+02	2.0E+04	4.9E+02	2.0E+03	1.5E+03	1.1E+02	4.5E+03	5.1E+03	3.2E+03
	std	1.3E+02	8.9E+03	5.7E+02	4.0E+04	2.1E+02	6.9E+01	5.2E+02	7.8E+03	4.3E+02	1.3E+03	6.2E+02	6.1E+01	2.3E+03	2.8E+03	1.8E+03
	rank	3	13	7	15	4	2	6	14	5	9	8	1	11	12	10
F_{21}	mean	2.5E+04	2.1E+05	4.2E+04	2.2E+06	2.3E+04	1.4E+04	2.4E+04	4.9E+05	3.1E+04	4.0E+04	7.3E+04	6.6E+02	8.7E+04	9.0E+04	1.1E+05
	std	1.1E+04	2.3E+05	5.7E+04	6.4E+06	1.6E+04	1.0E+04	1.5E+04	2.6E+05	3.3E+04	2.6E+04	5.5E+04	2.5E+02	6.5E+04	8.0E+04	9.8E+04
	rank	5	13	8	15	3	2	4	14	6	7	9	1	10	11	12
F_{22}	mean	2.3E+02	4.9E+02	7.1E+02	8.4E+02	2.3E+02	2.6E+02	3.1E+02	4.1E+02	2.3E+02	5.2E+02	1.5E+02	2.1E+02	2.6E+02	2.7E+02	2.9E+02
	std	9.6E+01	2.2E+02	2.4E+02	2.1E+02	8.7E+01	1.2E+02	1.2E+02	1.5E+02	1.0E+02	1.6E+02	6.7E+01	9.5E+01	9.0E+01	7.9E+01	1.2E+02
	rank	3	12	14	15	5	6	10	11	4	13	1	2	7	8	9
F_{23}	mean	3.2E+02	3.3E+02	3.2E+02	3.3E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02
	std	3.4E-12	1.2E+01	6.5E+00	9.8E+00	6.0E-13	4.3E-13	1.6E-12	1.2E-06	2.2E-12	1.8E-11	8.1E-08	2.2E-13	3.1E-02	9.6E-03	6.5E-01
	rank	11	15	13	14	1	8	9	4	10	2	3	7	6	5	12
F_{24}	mean	2.1E+02	2.5E+02	2.5E+02	2.4E+02	2.3E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.3E+02	2.3E+02
	std	9.2E+00	1.0E+01	9.1E+00	3.8E+01	7.0E+00	1.5E+00	1.6E+00	1.3E+00	3.8E+00	3.5E+00	9.9E-01	5.5E+00	8.1E-01	8.3E-01	7.1E+00
	rank	1	14	15	13	9	8	5	2	7	11	3	10	4	6	12
F_{25}	mean	2.1E+02	2.1E+02	2.2E+02	2.2E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02	2.1E+02	2.2E+02	2.0E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02
	std	2.2E+00	4.2E+00	5.9E+00	4.7E+00	1.2E-01	1.9E+00	2.5E+00	2.1E+00	1.5E+00	4.0E+00	3.4E-01	1.5E+00	1.8E+00	2.1E+00	2.2E+00
	rank	9	10	15	14	1	8	5	7	4	13	2	3	12	11	6
F_{26}	mean	1.3E+02	1.1E+02	1.8E+02	1.9E+02	1.4E+02	1.0E+02	1.8E+02	1.0E+02	1.1E+02	1.2E+02	1.0E+02	1.0E+02	1.1E+02	1.1E+02	1.0E+02
	std	4.7E+01	3.0E+01	4.2E+01	2.8E+01	5.0E+01	7.6E-02	4.4E+01	3.4E-02	2.5E+01	3.6E+01	4.9E-02	5.3E-02	3.4E+01	3.0E+01	1.3E-01
	rank	11	8	14	15	12	2	13	4	6	10	3	1	9	7	5
F_{27}	mean	3.9E+02	8.2E+02	8.1E+02	1.3E+03	4.4E+02	4.0E+02	4.8E+02	1.3E+03	4.0E+02	7.1E+02	4.2E+02	4.1E+02	4.3E+02	4.3E+02	6.8E+02
	std	3.5E+01	1.9E+02	3.8E+02	5.2E+02	4.8E+01	3.2E+00	8.6E+01	3.5E+01	2.1E+01	2.9E+02	3.8E+01	5.7E+01	2.0E+01	1.8E+01	2.7E+02
	rank	1	13	12	15	8	3	9	14	2	11	5	4	7	6	10
F_{28}	mean	3.9E+02	1.2E+03	3.9E+03	5.1E+03	4.1E+02	9.6E+02	1.5E+03	3.9E+02	9.0E+02	7.7E+02	3.9E+02	8.7E+02	8.8E+02	8.7E+02	1.2E+03
	std	1.9E+02	2.1E+02	1.2E+03	1.7E+03	2.7E+01	4.8E+01	7.2E+02	4.6E+00	5.6E+01	3.3E+02	4.0E+00	4.3E+01	3.1E+02	2.3E+02	2.5E+02
	rank	3	11	14	15	4	10	13	2	9	5	1	6	8	7	12
F_{29}	mean	1.2E+03	3.1E+06	1.3E+07	4.7E+06	2.1E+02	9.7E+02	1.4E+03	2.2E+02	1.3E+03	2.2E+02	2.1E+02	7.7E+02	1.1E+03	1.1E+03	4.1E+06
	std	2.0E+02	4.0E+06	1.8E+07	1.0E+07	1.2E+00	2.6E+02	4.9E+02	1.2E+00	3.1E+02	1.6E+01	1.5E+00	3.2E+01	1.9E+02	1.9E+02	3.7E+06
	rank	9	12	15	14	1	6	11	4	10	3	2	5	8	7	13
F_{30}	mean	2.5E+03	2.9E+04	1.8E+04	8.3E+04	7.7E+02	2.1E+03	1.9E+03	9.4E+02	1.8E+03	8.4E+02	6.3E+02	2.4E+03	6.1E+02	6.5E+02	1.4E+04
	std	3.6E+02	3.3E+04	4.2E+04	8.5E+04	1.8E+02	6.6E+02	6.1E+02	1.8E+02	6.1E+02	2.4E+02	1.5E+02	1.0E+03	1.3E+02	1.5E+02	2.6E+04
	rank	11	14	13	15	4	9	8	6	7	5	2	10	1	3	12
Mean rank		5.70	12.43	12.43	14.10	5.57	4.47	7.00	10.40	5.13	8.00	6.40	3.23	7.67	7.47	10.00
Final rank		5	13	13	14	4	2	7	12	3	10	6	1	9	8	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	8.4E+05	1.0E+08	5.6E+07	8.0E+07	1.2E+06	3.4E+05	4.2E+05	3.3E+08	7.9E+05	6.5E+05	9.8E+06	3.0E+04	1.6E+07	1.4E+07	2.2E+07
	std	2.5E+05	9.7E+07	5.4E+07	3.4E+07	5.5E+05	1.3E+05	2.0E+05	8.9E+07	2.4E+05	1.6E+05	3.6E+06	7.8E+04	7.6E+06	4.1E+06	1.9E+07
	rank	6	14	12	13	7	2	3	15	5	4	8	1	10	9	11
F_2	mean	7.2E+03	1.3E+10	3.4E+09	2.7E+08	9.9E+03	3.0E+03	6.3E+03	1.5E+06	5.4E+03	5.0E+03	1.1E+03	2.3E-10	1.5E+03	2.5E+03	4.9E+03
	std	5.0E+03	5.4E+09	1.6E+09	2.9E+08	1.0E-04	3.1E+03	6.1E+03	1.6E+06	5.6E+03	6.5E+03	1.5E+03	4.9E-10	1.4E+03	2.5E+03	8.1E+03
	rank	10	15	14	13	11	5	9	12	8	7	2	1	3	4	6
F_3	mean	1.8E+02	7.7E+04	2.6E+04	1.8E+05	5.4E+02	6.9E+00	6.0E+02	2.9E+05	1.0E+03	2.4E+03	3.4E+03	1.5E+00	3.9E+03	3.5E+03	3.6E+03
	std	8.1E+01	3.7E+04	1.1E+04	3.8E+04	4.7E+02	7.9E+00	5.4E+02	9.8E+04	8.1E+02	1.2E+03	1.2E+03	5.2E+00	2.4E+03	2.0E+03	2.2E+03
	rank	3	13	12	14	4	2	5	15	6	7	8	1	11	9	10
F_4	mean	9.2E+01	1.2E+03	4.7E+02	4.2E+02	4.3E+01	4.1E+01	5.7E+01	4.6E+01	8.8E+01	8.6E+01	4.4E+01	1.4E+01	1.5E+02	1.4E+02	1.2E+02
	std	3.3E+01	9.6E+02	1.4E+02	8.0E+01	1.1E+01	3.4E+01	2.9E+01	8.5E-01	2.5E+01	4.0E+01	1.7E+00	3.2E+01	2.3E+01	3.1E+01	4.6E+01
	rank	9	15	14	13	3	2	6	5	8	7	4	1	12	11	10
F_5	mean	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	3.7E-01	3.0E-01	1.2E-01	1.2E-01	5.7E-02	4.9E-02	3.3E-01	4.4E-02	4.5E-03	2.6E-04	6.3E-02	3.6E-03	7.3E-02	6.8E-02	2.0E-01
	rank	5	13	7	14	6	4	11	15	2	1	12	3	8	10	9
F_6	mean	3.5E+00	4.0E+01	6.0E+01	7.1E+01	1.1E+01	2.6E+01	1.9E+01	7.1E+01	1.3E+01	4.1E+01	2.3E+01	1.5E+01	3.4E+01	3.5E+01	4.8E+01
	std	1.5E+00	4.0E+00	3.1E+00	5.1E+00	3.4E+00	4.1E+00	5.2E+00	1.5E+00	4.1E+00	4.4E+00	2.2E+00	4.3E+00	4.8E+00	4.5E+00	4.3E+00
	rank	1	10	13	14	2	7	5	15	3	11	6	4	8	9	12
F_7	mean	2.7E-03	1.2E+02	3.4E+01	2.6E+00	9.2E-03	1.1E-02	5.3E-03	6.9E-05	5.3E-03	8.9E-03	3.1E-03	4.3E-03	1.1E-02	8.8E-03	5.3E-01
	std	6.0E-03	7.9E+01	1.8E+01	1.1E+00	9.3E-03	1.4E-02	6.8E-03	2.8E-04	7.0E-03	6.8E-03	8.1E-03	6.8E-03	5.6E-03	6.9E-03	2.9E+00
	rank	2	15	14	13	9	11	6	1	5	8	3	4	10	7	12
F_8	mean	1.0E+02	2.0E+02	3.2E+02	3.0E+02	7.1E+01	9.5E-01	7.3E+01	2.2E+02	1.5E+01	2.2E+01	8.2E+01	1.7E-01	8.7E+00	9.7E+00	1.7E+02
	std	1.2E+01	3.8E+01	3.7E+01	7.9E+01	1.3E+01	8.2E-01	2.0E+01	2.0E+01	4.9E+00	6.3E+00	1.5E+01	5.9E-01	4.2E+00	4.3E+00	2.9E+01
	rank	10	12	15	14	7	2	8	13	5	6	9	1	3	4	11
F_9	mean	9.0E+01	3.1E+02	3.9E+02	3.9E+02	7.6E+01	1.0E+02	1.3E+02	3.6E+02	9.5E+01	2.5E+02	1.5E+02	6.5E+01	1.9E+02	2.2E+02	2.0E+02
	std	1.1E+01	5.4E+01	3.9E+01	1.5E+02	1.7E+01	1.9E+01	3.2E+01	1.7E+01	2.0E+01	3.7E+01	3.3E+01	1.4E+01	3.1E+01	2.8E+01	3.0E+01
	rank	3	12	15	14	2	5	6	13	4	11	7	1	8	10	9
F_{10}	mean	3.8E+03	5.8E+03	8.3E+03	3.2E+03	3.1E+03	2.2E+01	1.6E+03	1.3E+04	8.5E+02	1.1E+03	2.8E+03	4.0E+00	9.3E+01	8.9E+01	6.0E+03
	std	8.9E+02	8.5E+02	9.8E+02	1.6E+03	8.7E+02	4.4E+01	4.4E+02	4.4E+02	2.2E+02	2.4E+02	4.5E+02	2.2E+01	1.0E+02	8.7E+01	7.2E+02
	rank	11	12	14	10	9	2	7	15	5	6	8	1	4	3	13
F_{11}	mean	4.7E+03	7.6E+03	9.1E+03	1.2E+04	5.2E+03	4.0E+03	5.9E+03	1.4E+04	5.0E+03	5.6E+03	6.1E+03	3.9E+03	8.8E+03	8.7E+03	6.5E+03
	std	1.1E+03	7.9E+02	9.6E+02	2.3E+03	7.6E+02	4.9E+02	9.6E+02	3.5E+02	8.7E+02	6.5E+02	9.8E+02	3.4E+02	8.5E+02	7.0E+02	5.8E+02
	rank	3	10	13	14	5	2	7	15	4	6	8	1	12	11	9
F_{12}	mean	6.3E-01	4.1E-01	2.0E+00	2.6E+00	1.2E+00	1.4E-01	9.6E-01	3.4E+00	2.0E-01	3.2E-01	1.5E+00	1.1E-01	1.0E+00	1.0E+00	8.5E-01
	std	5.5E-01	1.5E-01	4.1E-01	1.2E+00	1.8E-01	4.8E-02	3.5E-01	2.1E-01	5.4E-02	9.9E-02	2.8E-01	1.4E-02	2.2E-01	1.6E-01	1.9E-01
	rank	6	5	13	14	11	2	8	15	3	4	12	1	9	10	7
F_{13}	mean	3.4E-01	1.1E+00	6.4E-01	5.5E-01	2.4E-01	3.9E-01	4.5E-01	4.7E-01	3.8E-01	4.9E-01	4.9E-01	3.7E-01	4.7E-01	4.8E-01	4.6E-01
	std	5.2E-02	9.1E-01	1.2E-01	8.0E-02	5.2E-02	7.5E-02	6.9E-02	4.6E-02	9.3E-02	6.8E-02	4.2E-02	6.5E-02	4.1E-02	4.6E-02	6.8E-02
	rank	2	15	14	13	1	5	6	9	4	11	12	3	8	10	7
F_{14}	mean	2.2E-01	3.0E+01	3.7E+00	3.0E-01	3.8E-01	3.0E-01	4.1E-01	3.5E-01	3.1E-01	3.1E-01	2.7E-01	3.6E-01	3.3E-01	3.2E-01	4.5E-01
	std	2.6E-02	2.4E+01	6.3E+00	4.2E-02	2.1E-01	4.1E-02	2.4E-01	3.9E-02	9.6E-02	3.4E-02	2.4E-02	1.4E-01	2.7E-02	2.3E-02	1.9E-01
	rank	1	15	14	4	11	3	12	9	6	5	2	10	8	7	13
F_{15}	mean	6.4E+00	3.3E+04	8.8E+02	5.8E+02	8.5E+00	1.3E+01	1.2E+01	3.2E+01	1.1E+01	1.1E+02	2.2E+01	7.6E+00	3.2E+01	2.9E+01	2.4E+01
	std	1.9E+00	4.8E+04	8.6E+02	8.3E+02	3.4E+00	3.7E+00	3.4E+00	1.3E+00	3.5E+00	3.2E+01	4.5E+00	1.1E+00	5.8E+00	4.9E+00	7.6E+00
	rank	1	15	14	13	3	6	5	11	4	12	7	2	10	9	8
F_{16}	mean	1.9E+01	2.1E+01	2.2E+01	2.2E+01	1.9E+01	1.8E+01	1.9E+01	2.3E+01	1.9E+01	2.0E+01	2.1E+01	1.8E+01	2.1E+01	2.1E+01	2.1E+01
	std	7.8E-01	5.4E-01	5.6E-01	4.7E-01	8.8E-01	5.1E-01	6.9E-01	2.1E-01	8.5E-01	7.4E-01	4.6E-01	3.8E-01	3.7E-01	4.9E-01	6.0E-01
	rank	5	8	13	14	4	2	6	15	3	7	11	1	10	9	12
F_{17}	mean	7.0E+04	7.8E+06	1.9E+06	8.1E+06	2.4E+05	4.4E+04	5.1E+04	1.1E+07	1.1E+05	1.2E+05	1.6E+06	4.0E+03	2.7E+06	2.4E+06	1.0E+06
	std	2.8E+04	1.3E+07	3.6E+06	5.5E+06	1.5E+05	1.8E+04	2.3E+04	3.8E+06	7.1E+04	7.4E+04	6.3E+05	6.6E+03	3.1E+06	2.1E+06	9.0E+05
	rank	4	13	10	14	7	2	3	15	5	6	9	1	12	11	8
F_{18}	mean	9.7E+02	1.6E+08	1.9E+07	2.2E+05	6.3E+03	3.8E+02	1.5E+03	6.2E+05	6.9E+02	2.0E+03	7.5E+02	2.6E+02	8.6E+02	9.3E+02	2.6E+03
	std	8.1E+02	4.8E+08	7.2E+07	9.6E+05	7.9E+03	2.2E+02	1.0E+03	1.1E+06	6.8E+02	2.5E+03	3.9E+02	2.6E+02	8.2E+02	7.1E+02	1.4E+03
	rank	7	15	14	12	11	2	8	13	3	9	4	1	5	6	10
F_{19}	mean	1.6E+01	7.9E+01	8.0E+01	2.4E+02	2.1E+01	1.5E+01	1.4E+01	3.3E+01	1.2E+01	3.8E+01	2.5E+01	1.2E+01	2.4E+01	2.1E+01	2.8E+01
	std	2.3E+00	5.8E+01	2.3E+01	7.2E+02	2.7E+00	2.8E+00	2.6E+00	6.1E-01	2.3E+00	2.6E+01	1.7E+00	2.8E+00	1.6E+01	1.2E+01	7.5E+00
	rank	5	13	14	15	6	4	3	11	2	12	9	1	8	7	10
F_{20}	mean	3.3E+02	2.1E+04	6.0E+03	2.3E+05	5.0E+02	3.0E+02	6.7E+02	7.1E+04	7.0E+02	1.7E+03	2.2E+03	3.5E+02	1.3E+04	1.3E+04	3.9E+03
	std	7.1E+01	2.5E+04	5.0E+03	2.5E+05	1.8E+02	8.9E+01	2.4E+02	3.5E+04	5.3E+02	1.5E+03	9.6E+02	1.5E+02	5.0E+03	5.7E+03	2.9E+03
	rank	2	13	10	15	4	1	5	14	6	7	8	3	11	12	9
F_{21}	mean	8.9E+04	2.0E+06	6.0E+05	7.7E+06	9.4E+04	3.8E+04	3.9E+04	4.4E+06	1.3E+05	9.2E+04	6.4E+05	1.8E+03	1.5E+06	1.4E+06	2.5E+05
	std	5.1E+04	2.9E+06	8.4E+05	7.2E+06	4.9E+04	2.5E+04	1.6E+04	1.5E+06	7.1E+04	7.4E+04	3.3E+05	1.6E+03	9.4E+05	1.1E+06	1.6E+05
	rank	4	13	9	15	6	2	3	14	7	5	10	1	12	11	8
F_{22}	mean	4.3E+02	1.3E+03	1.7E+03	1.8E+03	4.9E+02	8.3E+02	7.5E+02	1.7E+03	7.3E+02	1.4E+03	5.4E+02	5.9E+02	7.8E+02	7.4E+02	9.0E+02
	std	1.7E+02	3.1E+02	4.3E+02	5.4E+02	2.6E+02	2.2E+02	2.5E+02	1.4E+02	2.5E+02	4.1E+02	1.6E+02	1.6E+02	1.7E+02	1.7E+02	2.4E+02
	rank	1	11	14	15	2	9	7	13	5	12	3	4	8	6	10
F_{23}	mean	3.4E+02	3.9E+02	3.7E+02	3.6E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02
	std	1.7E-11	4.1E+01	1.5E+01	6.3E+00	1.2E-12	6.8E-13	5.0E-12	3.4E-06	8.7E-13	7.6E-11	5.4E-07	5.0E-13	2.0E-02	5.2E-03	6.9E-01
	rank	11	15	14	13	1	8	10	4	9	2	3	7	6	5	12
F_{24}	mean	2.6E+02	3.3E+02	3.3E+02	2.9E+02	2.8E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.8E+02	2.6E+02	2.6E+02	2.8E+02
	std	1.2E+00	3.8E+01	1.4E+01	2.1E+01	2.7E+00	5.5E+00	5.7E+00	6.6E-06	4.5E+00	4.8E+00	4.5E+00	3.3E+00	4.5E+00	4.6E+00	3.9E+00
	rank	1	15	14	13	11	3	8	6	7	5	9	10	4	2	12
F_{25}	mean	2.1E+02	2.2E+02	2.5E+02	2.4E+02	2.0E+02	2.1E+02	2.2E+02	2.3E+02	2.1E+02	2.4E+02	2.0E+02	2.1E+02	2.3E+02	2.3E+02	2.2E+02
	std	9.0E+00	1.3E+01	9.1E+00	1.4E+01	7.8E-02	3.0E+00	4.5E+00	5.2E+00	3.6E+00	6.5E+00	4.5E-01	5.4E+00	3.3E+00	4.0E+00	5.5E+00
	rank	3	9	15	14	1	6	8	10	5	13	2	4	12	11	7
F_{26}	mean	2.0E+02	1.9E+02	2.0E+02	2.0E+02	1.6E+02	1.2E+02	1.7E+02	1.0E+02	1.5E+02	1.8E+02	1.1E+02	1.0E+02	1.5E+02	1.5E+02	1.1E+02
	std	1.7E-02	1.2E+02	1.0E+00	4.9E+00	5.0E+01	3.7E+01	6.3E+01	5.0E-02	5.0E+01	4.1E+01	3.1E+01	2.0E-01	5.0E+01	5.1E+01	2.6E+01
	rank	13	12	14	15	9	5	10	1	6	11	4	2	8	7	3
F_{27}	mean	3.6E+02	1.4E+03	2.0E+03	3.1E+03	6.4E+02	9.1E+02	8.5E+02	2.1E+03	6.1E+02	1.5E+03	9.2E+02	6.9E+02	9.8E+02	9.6E+02	1.6E+03
	std	3.4E+01	1.1E+02	3.2E+02	9.2E+02	7.5E+01	2.7E+02	1.4E+02	3.3E+01	1.1E+02	1.4E+02	6.1E+01	7.6E+01	1.4E+02	1.4E+02	1.3E+02
	rank	1	10	13	15	3	6	5	14	2	11	7	4	9	8	12
F_{28}	mean	9.1E+02	2.1E+03	8.3E+03	1.1E+04	3.9E+02	1.8E+03	2.7E+03	4.0E+02	1.7E+03	1.6E+03	3.9E+02	1.3E+03	7.2E+02	9.7E+02	2.3E+03
	std	7.4E+02	5.7E+02	2.5E+03	4.0E+03	1.5E+01	1.9E+02	8.2E+02	1.9E+01	3.7E+02	1.4E+03	1.3E+01	2.2E+02	4.7E+02	9.3E+02	7.6E+02
	rank	5	11	14	15	1	10	13	3	9	8	2	7	4	6	12
F_{29}	mean	1.8E+03	3.0E+07	6.3E+07	1.3E+08	2.2E+02	1.4E+03	1.4E+03	2.4E+02	1.3E+03	2.3E+02	2.3E+02	8.9E+02	1.1E+03	1.1E+03	4.6E+07
	std	5.4E+02	1.9E+07	8.0E+07	5.9E+08	1.7E+01	3.1E+02	3.3E+02	4.9E+00	3.8E+02	1.6E+01	1.6E+00	1.0E+02	1.9E+02	2.9E+02	1.6E+07
	rank	11	12	14	15	1	9	10	4	8	3	2	5	6	7	13
F_{30}	mean	1.7E+04	5.9E+04	1.5E+05	2.1E+05	1.2E+03	9.7E+03	1.2E+04	2.1E+03	1.1E+04	1.8E+03	1.1E+03	1.0E+04	1.3E+03	1.3E+03	3.5E+04
	std	2.0E+03	4.5E+04	2.0E+05	1.2E+05	2.9E+02	9.9E+02	1.2E+03	2.0E+02	1.2E+03	3.4E+02	1.8E+02	1.5E+03	2.6E+02	2.3E+02	2.5E+04
	rank	11	13	14	15	2	7	10	6	9	5	1	8	3	4	12
Mean rank		5.23	12.47	13.13	13.50	5.33	4.53	7.10	10.70	5.27	7.40	6.40	3.13	8.07	7.83	9.90
Final rank		3	13	14	15	5	2	7	12	4	8	6	1	10	9	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	3.5E+06	4.5E+08	3.2E+08	1.7E+08	2.7E+06	9.8E+05	1.1E+06	3.6E+09	4.7E+06	2.8E+06	6.7E+07	2.6E+05	1.6E+08	1.6E+08	1.1E+08
	std	4.5E+05	2.4E+08	1.2E+08	8.8E+07	1.0E+06	3.0E+05	5.6E+05	8.1E+08	1.5E+06	7.1E+05	2.7E+07	1.9E+05	3.3E+07	4.0E+07	7.3E+07
	rank	6	14	13	12	4	2	3	15	7	5	8	1	10	11	9
F_2	mean	2.0E+04	7.3E+10	1.4E+10	8.2E+06	1.9E+04	2.0E+04	1.6E+04	4.1E+06	1.2E+04	1.4E+04	1.1E+04	3.0E-09	1.2E+04	1.4E+04	3.0E+04
	std	1.2E+04	2.2E+10	3.4E+09	1.1E+07	1.9E+04	1.5E+04	1.5E+04	1.3E+07	1.2E+04	1.1E+04	1.7E+04	5.1E-09	6.9E+03	6.5E+03	2.9E+04
	rank	9	15	14	13	8	10	7	12	4	5	2	1	3	6	11
F_3	mean	9.4E+01	1.8E+05	1.0E+05	3.8E+05	7.4E+02	1.9E-01	3.8E+02	1.1E+06	1.7E+03	2.3E+03	4.1E+03	9.2E-01	9.7E+03	8.0E+03	3.0E+03
	std	4.9E+01	8.1E+04	2.0E+04	1.3E+05	9.0E+02	2.5E-01	3.7E+02	2.8E+05	1.6E+03	1.5E+03	1.4E+03	2.9E+00	5.0E+03	3.4E+03	2.2E+03
	rank	3	13	12	14	5	1	4	15	6	7	9	2	11	10	8
F_4	mean	2.3E+02	8.5E+03	2.0E+03	5.0E+02	1.0E+02	1.4E+02	1.8E+02	9.8E+01	2.1E+02	2.0E+02	9.5E+01	1.3E+02	3.3E+02	3.4E+02	3.3E+02
	std	5.8E+01	4.4E+03	4.3E+02	5.9E+01	2.0E+01	5.2E+01	3.7E+01	9.2E-01	4.7E+01	4.9E+01	2.2E+00	4.7E+01	5.6E+01	4.0E+01	1.1E+02
	rank	9	15	14	13	3	5	6	2	8	7	1	4	10	12	11
F_5	mean	2.0E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	4.2E-01	9.0E-02	9.6E-02	3.2E-01	4.0E-02	1.0E-01	2.8E-01	3.2E-02	3.2E-04	3.5E-01	3.9E-02	3.9E-03	3.6E-02	4.6E-02	1.0E-01
	rank	5	14	7	13	6	3	11	15	1	4	9	2	12	10	8
F_6	mean	2.5E+01	9.9E+01	1.4E+02	1.5E+02	4.8E+01	7.6E+01	6.5E+01	1.6E+02	6.1E+01	1.0E+02	7.7E+01	5.8E+01	1.1E+02	1.1E+02	1.2E+02
	std	3.3E+00	6.9E+00	6.5E+00	9.4E+00	7.2E+00	6.9E+00	7.2E+00	1.5E+00	6.8E+00	4.3E+00	4.6E+00	6.6E+00	8.8E+00	7.9E+00	5.8E+00
	rank	1	8	13	14	2	6	5	15	4	9	7	3	11	10	12
F_7	mean	1.2E-03	7.4E+02	1.2E+02	1.1E+00	4.9E-03	9.8E-03	3.8E-03	2.2E-02	2.5E-03	1.2E-03	2.2E-03	9.8E-03	1.1E-02	8.0E-03	5.6E-12
	std	3.0E-03	2.4E+02	3.5E+01	1.8E-01	9.0E-03	2.7E-02	7.3E-03	7.4E-03	4.6E-03	3.0E-03	2.6E-03	1.4E-02	1.8E-02	1.1E-02	3.1E-12
	rank	2	15	14	13	7	9	6	12	5	3	4	10	11	8	1
F_8	mean	2.7E+02	6.2E+02	8.3E+02	6.6E+02	2.0E+02	9.0E-01	2.2E+02	7.6E+02	3.2E+01	7.7E+01	2.4E+02	9.6E-01	4.0E+01	4.4E+01	4.8E+02
	std	2.2E+01	6.2E+01	6.4E+01	8.8E+01	3.7E+01	9.2E-01	5.0E+01	3.0E+01	1.1E+01	3.1E+01	2.6E+01	1.9E+00	9.1E+00	9.3E+00	5.5E+01
	rank	10	12	15	13	7	1	8	14	3	6	9	2	4	5	11
F_9	mean	2.6E+02	8.8E+02	1.0E+03	1.0E+03	1.9E+02	3.5E+02	3.1E+02	9.6E+02	2.9E+02	6.7E+02	4.9E+02	2.1E+02	8.3E+02	8.3E+02	5.6E+02
	std	2.8E+01	1.1E+02	6.5E+01	3.6E+02	4.8E+01	6.0E+01	4.9E+01	2.6E+01	5.1E+01	5.1E+01	1.3E+02	4.0E+01	6.5E+01	6.5E+01	6.1E+01
	rank	3	12	14	15	1	6	5	13	4	9	7	2	11	10	8
F_{10}	mean	1.0E+04	1.5E+04	2.1E+04	1.0E+04	8.9E+03	2.9E+01	4.7E+03	3.1E+04	1.9E+03	2.4E+03	7.6E+03	2.8E+01	3.9E+02	4.3E+02	1.4E+04
	std	8.6E+02	1.8E+03	1.1E+03	3.2E+03	2.0E+03	4.8E+01	6.4E+02	7.9E+02	4.0E+02	5.3E+02	1.1E+03	6.0E+01	2.0E+02	1.8E+02	1.3E+03
	rank	10	13	14	11	9	2	7	15	5	6	8	1	3	4	12
F_{11}	mean	1.0E+04	1.7E+04	2.2E+04	2.8E+04	1.3E+04	1.1E+04	1.3E+04	3.3E+04	1.2E+04	1.3E+04	1.8E+04	1.1E+04	2.6E+04	2.6E+04	1.5E+04
	std	1.3E+03	1.3E+03	1.3E+03	4.6E+03	1.5E+03	6.7E+02	1.7E+03	6.1E+02	1.3E+03	1.1E+03	2.1E+03	5.5E+02	9.9E+02	1.1E+03	1.4E+03
	rank	1	9	11	14	5	3	7	15	4	6	10	2	12	13	8
F_{12}	mean	1.0E+00	1.7E+00	2.8E+00	3.7E+00	1.8E+00	2.1E-01	1.6E+00	4.0E+00	2.8E-01	4.5E-01	2.1E+00	1.8E-01	2.3E+00	2.3E+00	1.5E+00
	std	9.6E-01	1.4E+00	4.8E-01	7.6E-01	2.1E-01	5.5E-02	4.7E-01	2.2E-01	5.9E-02	8.9E-02	2.5E-01	1.6E-02	2.5E-01	3.2E-01	2.8E-01
	rank	5	8	13	14	9	2	7	15	3	4	10	1	11	12	6
F_{13}	mean	4.6E-01	3.7E+00	6.0E-01	5.4E-01	4.2E-01	4.6E-01	6.4E-01	6.9E-01	5.0E-01	5.0E-01	6.1E-01	5.0E-01	5.0E-01	5.1E-01	5.5E-01
	std	5.5E-02	1.2E+00	6.6E-02	6.4E-02	5.4E-02	5.4E-02	6.4E-02	5.7E-02	6.9E-02	5.6E-02	6.0E-02	7.6E-02	4.2E-02	4.4E-02	5.3E-02
	rank	3	15	11	9	1	2	13	14	5	4	12	6	7	8	10
F_{14}	mean	2.3E-01	2.1E+02	4.6E+01	3.5E-01	4.3E-01	3.4E-01	3.5E-01	4.2E-01	3.2E-01	3.2E-01	3.2E-01	3.6E-01	3.4E-01	3.4E-01	6.3E-01
	std	2.5E-02	7.6E+01	1.7E+01	3.5E-02	2.0E-01	2.7E-02	1.3E-01	1.3E-01	3.5E-02	3.0E-02	2.8E-02	1.1E-01	2.4E-02	2.8E-02	2.2E-01
	rank	1	15	14	9	12	5	8	11	2	4	3	10	6	7	13
F_{15}	mean	1.5E+01	1.5E+06	2.1E+04	7.2E+02	2.6E+01	4.3E+01	4.4E+01	8.4E+01	4.5E+01	3.3E+02	7.4E+01	3.9E+01	1.0E+02	1.1E+02	1.8E+02
	std	2.9E+00	1.5E+06	1.4E+04	1.0E+03	1.1E+01	6.7E+00	7.9E+00	3.2E+00	1.1E+01	5.2E+01	1.1E+01	7.5E+00	1.6E+01	1.2E+01	1.1E+02
	rank	1	15	14	13	2	4	5	8	6	12	7	3	9	10	11
F_{16}	mean	4.2E+01	4.4E+01	4.5E+01	4.7E+01	4.2E+01	4.1E+01	4.3E+01	4.8E+01	4.1E+01	4.3E+01	4.5E+01	4.0E+01	4.6E+01	4.6E+01	4.5E+01
	std	1.3E+00	1.3E+00	6.9E-01	5.2E-01	9.9E-01	7.5E-01	1.2E+00	2.0E-01	1.2E+00	8.3E-01	5.2E-01	7.2E-01	4.4E-01	5.2E-01	5.8E-01
	rank	5	8	11	14	4	2	6	15	3	7	10	1	12	13	9
F_{17}	mean	4.9E+05	2.4E+07	1.7E+07	3.5E+07	1.0E+06	1.9E+05	4.0E+05	1.1E+08	7.3E+05	4.6E+05	9.3E+06	3.5E+04	1.7E+07	1.9E+07	7.3E+06
	std	3.2E+05	2.7E+07	5.8E+06	2.1E+07	3.4E+05	6.0E+04	1.8E+05	2.4E+07	2.9E+05	1.7E+05	4.5E+06	3.2E+04	6.6E+06	8.3E+06	5.0E+06
	rank	5	13	10	14	7	2	3	15	6	4	9	1	11	12	8
F_{18}	mean	9.4E+02	9.5E+08	2.1E+08	1.4E+03	3.1E+03	6.9E+02	2.3E+03	4.2E+06	2.1E+03	1.5E+03	1.7E+03	1.5E+03	6.4E+02	1.0E+03	4.7E+03
	std	3.1E+02	1.6E+09	1.6E+08	1.5E+03	3.6E+03	3.1E+02	2.3E+03	4.7E+06	2.4E+03	1.4E+03	8.0E+02	1.6E+03	4.6E+02	1.1E+03	4.5E+03
	rank	3	15	14	5	11	2	10	13	9	6	8	7	1	4	12
F_{19}	mean	8.0E+01	5.6E+02	3.4E+02	1.8E+02	6.3E+01	7.1E+01	8.2E+01	7.4E+01	1.0E+02	9.1E+01	5.9E+01	9.5E+01	1.4E+02	1.3E+02	1.2E+02
	std	2.1E+01	1.5E+02	1.2E+02	7.6E+01	1.9E+01	2.7E+01	3.6E+01	1.9E+00	1.6E+01	3.5E+01	2.4E+00	2.4E+01	2.7E+01	2.5E+01	1.1E+01
	rank	5	15	14	13	2	3	6	4	9	7	1	8	12	11	10
F_{20}	mean	5.7E+02	1.1E+05	3.7E+04	1.1E+06	1.1E+03	5.2E+02	1.4E+03	5.3E+05	1.8E+03	2.6E+03	5.0E+03	6.8E+03	5.2E+04	4.9E+04	6.8E+03
	std	7.3E+01	6.0E+04	1.9E+04	8.9E+05	3.7E+02	7.8E+01	5.6E+02	1.2E+05	8.0E+02	7.9E+02	1.4E+03	5.3E+03	1.7E+04	1.4E+04	2.1E+03
	rank	2	13	10	15	3	1	4	14	5	6	7	8	12	11	9
F_{21}	mean	2.0E+05	6.4E+06	5.6E+06	1.7E+07	4.2E+05	8.0E+04	1.1E+05	4.6E+07	3.1E+05	2.2E+05	4.4E+06	1.8E+04	8.4E+06	8.4E+06	2.1E+06
	std	4.7E+04	5.4E+06	3.0E+06	2.0E+07	1.8E+05	2.8E+04	6.7E+04	1.2E+07	1.5E+05	5.7E+04	1.9E+06	2.9E+04	9.5E+06	7.5E+06	2.3E+06
	rank	4	11	10	14	7	2	3	15	6	5	9	1	13	12	8
F_{22}	mean	1.2E+03	3.5E+03	3.9E+03	5.0E+03	1.4E+03	1.9E+03	2.3E+03	5.4E+03	2.1E+03	3.0E+03	2.0E+03	1.7E+03	1.8E+03	1.8E+03	2.5E+03
	std	3.3E+02	5.8E+02	8.1E+02	1.1E+03	3.4E+02	3.5E+02	4.7E+02	2.3E+02	4.6E+02	4.7E+02	4.8E+02	2.5E+02	3.3E+02	3.1E+02	4.1E+02
	rank	1	12	13	14	2	6	9	15	8	11	7	3	5	4	10
F_{23}	mean	3.5E+02	6.4E+02	5.0E+02	3.6E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02
	std	2.1E-05	1.8E+02	6.4E+01	2.8E+00	3.8E-11	2.1E-11	5.4E-10	2.3E-01	2.8E-10	1.6E-10	4.7E-03	2.8E-12	8.9E-03	1.3E-02	7.8E+00
	rank	11	15	14	13	1	8	10	6	9	2	3	7	4	5	12
F_{24}	mean	3.7E+02	6.7E+02	5.9E+02	4.2E+02	3.9E+02	3.6E+02	3.8E+02	3.6E+02	3.8E+02	3.4E+02	3.8E+02	4.2E+02	3.7E+02	3.7E+02	4.1E+02
	std	4.8E+00	8.3E+01	4.1E+01	2.1E+01	5.6E+00	7.5E+00	5.0E+00	2.3E+00	3.6E+00	1.2E+01	3.9E+00	8.1E+00	3.3E+00	2.8E+00	8.2E+00
	rank	6	15	14	13	10	2	8	3	7	1	9	12	4	5	11
F_{25}	mean	2.0E+02	2.8E+02	3.5E+02	2.8E+02	2.1E+02	2.6E+02	2.8E+02	5.2E+02	2.7E+02	3.0E+02	2.1E+02	2.8E+02	2.8E+02	2.8E+02	2.6E+02
	std	2.1E-09	2.8E+01	2.3E+01	2.1E+01	2.8E+01	5.2E+00	1.1E+01	6.2E+01	1.2E+01	1.1E+01	1.2E+00	1.2E+01	2.0E+01	9.8E+00	1.7E+01
	rank	1	10	14	12	3	4	8	15	6	13	2	11	7	9	5
F_{26}	mean	2.0E+02	2.3E+02	2.1E+02	2.1E+02	2.0E+02	2.0E+02	2.0E+02	1.0E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.1E+02	2.1E+02	1.9E+02
	std	5.2E-02	2.0E+01	4.7E+00	6.6E+00	6.2E-02	2.6E-02	9.3E-02	5.6E-02	9.9E-02	2.9E-02	3.8E+01	9.5E-03	3.0E+00	2.0E+00	3.7E+01
	rank	6	15	14	13	9	5	8	1	10	7	2	4	12	11	3
F_{27}	mean	5.8E+02	2.9E+03	4.4E+03	6.7E+03	1.6E+03	2.1E+03	2.0E+03	4.4E+03	1.8E+03	3.1E+03	2.3E+03	1.6E+03	1.6E+03	1.7E+03	3.5E+03
	std	6.7E+01	1.6E+02	2.2E+02	1.7E+03	2.0E+02	7.4E+02	2.3E+02	5.1E+01	4.5E+02	2.0E+02	1.2E+02	1.4E+02	3.0E+02	2.7E+02	1.4E+02
	rank	1	10	13	15	2	8	7	14	6	11	9	3	4	5	12
F_{28}	mean	4.9E+03	5.0E+03	2.0E+04	3.0E+04	7.6E+02	5.3E+03	7.9E+03	2.6E+03	6.4E+03	7.1E+03	6.7E+02	3.3E+03	7.1E+03	6.2E+03	6.3E+03
	std	2.0E+03	7.9E+02	5.3E+03	9.7E+03	2.6E+02	6.4E+02	1.4E+03	6.8E+02	1.1E+03	2.8E+03	1.0E+02	6.2E+02	4.2E+03	4.1E+03	7.8E+02
	rank	5	6	14	15	2	7	13	3	10	11	1	4	12	8	9
F_{29}	mean	3.8E+03	9.8E+07	4.9E+08	5.0E+05	2.6E+02	1.5E+03	1.6E+03	3.0E+02	1.8E+03	2.9E+02	2.6E+02	1.5E+03	8.1E+02	8.4E+02	2.2E+08
	std	8.2E+02	4.5E+07	4.4E+08	1.7E+06	3.4E+00	2.7E+02	2.6E+02	2.8E+01	2.7E+02	4.9E+01	2.5E+00	1.7E+02	6.5E+02	5.1E+02	5.8E+07
	rank	11	13	15	12	2	8	9	4	10	3	1	7	5	6	14
F_{30}	mean	1.2E+04	6.6E+05	7.1E+05	5.3E+05	3.0E+03	9.4E+03	1.2E+04	6.8E+03	1.0E+04	4.0E+03	2.8E+03	9.9E+03	3.0E+03	3.2E+03	5.1E+05
	std	1.2E+03	4.7E+05	4.6E+05	8.6E+05	4.5E+02	1.0E+03	3.4E+03	3.3E+02	1.4E+03	3.7E+02	4.8E+02	1.3E+03	4.2E+02	4.0E+02	3.8E+05
	rank	10	14	15	13	2	7	11	6	9	5	1	8	3	4	12
Mean rank		4.83	12.60	12.93	12.83	4.93	4.27	7.23	11.13	6.00	6.33	6.17	4.50	8.27	8.53	9.43
Final rank		3	13	15	14	4	1	8	12	5	7	6	2	9	10	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.6E+04	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	1.6E+04	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.5E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	5.6E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	4	13	12	15	11	6	9	14	7	8	5	1	2	3	10
F_3	mean	6.5E+01	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	8.9E+01	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	5	13	2	15	8	3	10	14	4	12	9	1	7	6	11
F_4	mean	2.7E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.5E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	11	10	14	15	3	1	12	6	9	5	7	13	4	2	8
F_5	mean	1.7E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	7.4E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	2	14	9	13	3	1	5	15	11	4	7	10	8	6	12
F_6	mean	1.3E-01	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	7.4E-02	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	3	12	13	15	1	7	6	14	2	11	4	5	8	9	10
F_7	mean	4.8E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	2.9E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	4	14	12	15	8	9	11	7	10	13	6	1	2	3	5
F_8	mean	3.8E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.9E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	5.6E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.2E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	4	13	14	15	3	7	9	10	6	11	2	1	5	8	12
F_{10}	mean	3.0E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.3E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	13	12	15	10	6	2	8	14	7	9	5	1	4	3	11
F_{11}	mean	2.9E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	1.8E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	8	12	13	14	3	4	9	15	5	10	2	1	6	7	11
F_{12}	mean	6.9E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	3.5E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	13	4	11	14	12	7	6	15	3	2	10	1	8	9	5
F_{13}	mean	7.3E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	3.9E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.0E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	2.9E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	8.0E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	2.7E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	5	7	12	15	2	4	9	13	6	14	11	1	10	8	3
F_{16}	mean	2.0E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	3.4E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	7	10	15	13	1	3	5	14	2	11	9	4	6	8	12
F_{17}	mean	8.1E+02	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	5.4E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	5.0E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	3.1E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	10	15	2	12	11	3	13	9	8	14	6	1	4	7	5
F_{19}	mean	1.4E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	2.2E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.8E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	1.6E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	2.9E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.2E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	8	12	9	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	6.6E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	5.2E+01	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	12	10	15	14	7	3	13	9	2	11	8	1	4	6	5
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	5.6E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.2E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	1	12	14	15	4	8	6	3	7	13	5	2	9	10	11
F_{25}	mean	1.2E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.1E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	1	13	15	14	5	2	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	1.9E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.4E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	4	12	14	13	8	2	10	15	6	9	5	7	1	3	11
F_{28}	mean	3.8E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.3E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	4.0E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	7.6E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	1.1E+03	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	4.1E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	12	11	14	15	3	8	13	1	9	10	2	6	4	5	7
Mean rank		6.53	11.70	11.53	14.23	4.97	4.43	8.27	10.00	5.53	9.77	5.50	3.70	6.40	6.77	8.83
Final rank		7	14	13	15	3	2	9	12	5	11	4	1	6	8	10
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.5E+05	1.6E+07	1.5E+07	1.1E+08	3.8E+05	5.5E+04	1.1E+05	5.6E+07	2.9E+05	1.9E+05	2.5E+06	9.9E+02	8.3E+06	1.0E+07	4.9E+06
	std	2.3E+05	1.7E+07	1.2E+07	4.1E+07	2.3E+05	4.0E+04	7.1E+04	2.6E+07	2.2E+05	1.2E+05	1.1E+06	2.5E+03	3.6E+06	6.1E+06	4.7E+06
	rank	5	13	12	15	7	2	3	14	6	4	8	1	10	11	9
F_2	mean	1.8E+03	2.9E+09	4.8E+08	9.6E+08	4.9E+01	1.8E-03	5.6E+01	1.6E+04	3.0E+01	2.9E+03	3.5E+03	3.5E-14	3.4E+03	4.9E+03	1.4E+07
	std	1.1E+03	3.1E+09	3.8E+08	8.7E+08	8.0E+01	3.9E-03	1.1E+02	1.4E+04	3.9E+01	1.7E+03	5.7E+03	1.6E-14	3.0E+03	3.5E+03	7.8E+07
	rank	6	15	13	14	4	2	5	11	3	7	9	1	8	10	12
F_3	mean	1.1E+02	2.0E+04	3.1E+03	4.5E+04	2.4E+01	1.2E-02	2.3E+01	3.7E+04	2.6E+01	3.7E+02	4.9E+02	9.7E-14	2.3E+02	3.0E+02	3.0E+02
	std	9.2E+01	2.0E+04	2.2E+03	3.1E+04	3.5E+01	3.0E-02	2.4E+01	2.6E+04	2.7E+01	3.3E+02	3.5E+02	8.7E-14	2.6E+02	2.7E+02	3.7E+02
	rank	6	13	12	15	4	2	3	14	5	10	11	1	7	8	9
F_4	mean	7.2E+01	2.0E+02	1.9E+02	3.7E+02	2.6E+01	3.7E+00	4.3E+00	2.6E+01	4.2E+01	4.5E+01	2.5E+01	1.3E-12	8.8E+01	7.5E+01	7.9E+01
	std	2.6E+00	1.6E+02	5.6E+01	1.4E+02	1.8E+01	1.5E+01	1.6E+01	4.0E-01	3.6E+01	2.9E+01	1.1E+00	1.6E-12	2.7E+01	2.5E+01	3.3E+01
	rank	9	14	13	15	6	2	3	5	7	8	4	1	12	10	11
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	3.9E-02	3.0E-01	7.5E-02	1.6E-01	7.2E-02	5.9E-02	3.4E-01	4.9E-02	1.6E-02	1.9E-04	6.0E-02	5.2E-03	6.8E-02	8.3E-02	1.4E-01
	rank	14	10	4	13	6	5	11	15	2	1	12	3	7	8	9
F_6	mean	2.2E+00	1.9E+01	3.0E+01	3.6E+01	3.7E+00	1.1E+01	6.6E+00	3.9E+01	3.6E+00	2.0E+01	8.3E+00	5.5E+00	1.5E+01	1.6E+01	2.1E+01
	std	1.2E+00	3.1E+00	3.8E+00	5.0E+00	2.3E+00	2.2E+00	2.3E+00	1.4E+00	2.0E+00	2.5E+00	1.6E+00	2.6E+00	1.8E+00	2.0E+00	2.9E+00
	rank	1	10	13	14	3	7	5	15	2	11	6	4	8	9	12
F_7	mean	7.4E-04	2.2E+01	8.0E+00	1.5E+01	7.9E-03	4.7E-03	1.6E-02	5.4E-05	5.3E-03	1.3E-02	1.4E-03	3.2E-03	7.3E-03	6.5E-04	1.8E-03
	std	2.3E-03	1.8E+01	5.4E+00	1.6E+01	1.1E-02	7.7E-03	1.9E-02	2.2E-04	8.2E-03	1.7E-02	4.3E-03	6.1E-03	1.3E-02	1.3E-03	4.6E-03
	rank	3	15	13	14	10	7	12	1	8	11	4	6	9	2	5
F_8	mean	1.9E+01	8.0E+01	1.4E+02	1.5E+02	3.3E+01	9.9E-02	3.1E+01	7.2E+01	9.0E+00	8.8E+00	3.3E+01	2.7E-13	2.1E+00	1.8E+00	6.7E+01
	std	4.3E+00	2.0E+01	2.4E+01	4.6E+01	7.4E+00	3.1E-01	9.6E+00	1.4E+01	3.0E+00	4.3E+00	6.5E+00	5.7E-14	2.1E+00	1.4E+00	1.1E+01
	rank	7	13	14	15	9	2	8	12	6	5	10	1	4	3	11
F_9	mean	2.6E+01	1.4E+02	1.6E+02	2.3E+02	3.7E+01	5.0E+01	5.5E+01	1.6E+02	4.5E+01	1.2E+02	5.9E+01	2.4E+01	6.7E+01	7.1E+01	9.4E+01
	std	9.0E+00	3.1E+01	2.3E+01	7.6E+01	8.4E+00	1.5E+01	1.4E+01	1.5E+01	1.2E+01	2.3E+01	1.2E+01	5.4E+00	1.2E+01	1.2E+01	2.0E+01
	rank	2	12	13	15	3	5	6	14	4	11	7	1	8	9	10
F_{10}	mean	1.7E+03	2.6E+03	4.1E+03	9.7E+02	1.1E+03	5.8E+00	6.6E+02	6.3E+03	3.8E+02	5.1E+02	8.6E+02	2.4E-01	3.6E+01	4.3E+00	2.7E+03
	std	5.2E+02	4.1E+02	7.2E+02	4.7E+02	5.1E+02	1.6E+00	3.7E+02	3.5E+02	1.9E+02	1.7E+02	3.2E+02	7.6E-01	6.3E+01	2.2E+01	5.0E+02
	rank	11	12	14	9	10	3	7	15	5	6	8	1	4	2	13
F_{11}	mean	2.8E+03	3.8E+03	4.4E+03	6.0E+03	2.6E+03	2.0E+03	2.7E+03	7.6E+03	2.3E+03	3.0E+03	2.7E+03	1.7E+03	3.1E+03	3.4E+03	3.4E+03
	std	6.8E+02	6.5E+02	7.5E+02	1.2E+03	4.6E+02	3.3E+02	5.5E+02	2.9E+02	5.2E+02	6.6E+02	4.6E+02	2.1E+02	4.8E+02	4.5E+02	4.5E+02
	rank	7	12	13	14	4	2	5	15	3	8	6	1	9	11	10
F_{12}	mean	1.9E+00	3.1E-01	1.3E+00	1.8E+00	8.6E-01	1.7E-01	6.2E-01	2.5E+00	1.9E-01	3.2E-01	9.4E-01	1.2E-01	6.5E-01	6.1E-01	5.4E-01
	std	5.0E-01	1.1E-01	3.9E-01	4.5E-01	1.9E-01	8.0E-02	3.5E-01	2.2E-01	7.3E-02	1.3E-01	3.2E-01	1.9E-02	1.3E-01	1.3E-01	1.5E-01
	rank	14	4	12	13	10	2	8	15	3	5	11	1	9	7	6
F_{13}	mean	1.9E-01	7.3E-01	5.6E-01	4.7E-01	1.4E-01	2.6E-01	2.9E-01	3.4E-01	2.2E-01	3.7E-01	3.5E-01	3.0E-01	2.8E-01	2.9E-01	4.1E-01
	std	4.4E-02	5.4E-01	1.3E-01	1.1E-01	2.6E-02	6.3E-02	5.9E-02	5.0E-02	6.6E-02	1.0E-01	4.5E-02	4.8E-02	3.6E-02	3.8E-02	6.8E-02
	rank	2	15	14	13	1	4	6	9	3	11	10	8	5	7	12
F_{14}	mean	2.5E-01	8.4E+00	2.0E+00	2.4E+00	3.0E-01	2.6E-01	2.3E-01	3.0E-01	2.4E-01	2.6E-01	2.4E-01	3.0E-01	2.4E-01	2.3E-01	5.4E-01
	std	3.2E-02	1.3E+01	3.6E+00	4.8E+00	1.6E-01	2.5E-02	3.5E-02	3.9E-02	5.0E-02	5.2E-02	2.6E-02	1.4E-01	3.2E-02	2.9E-02	2.4E-01
	rank	6	15	13	14	9	8	1	10	5	7	3	11	4	2	12
F_{15}	mean	3.4E+00	4.7E+02	9.4E+01	1.5E+02	4.4E+00	5.0E+00	5.4E+00	1.5E+01	4.1E+00	4.2E+01	8.8E+00	2.5E+00	1.1E+01	1.1E+01	6.9E+00
	std	7.3E-01	6.8E+02	9.5E+01	1.3E+02	1.7E+00	1.0E+00	1.5E+00	9.6E-01	1.0E+00	1.6E+01	2.2E+00	4.0E-01	2.7E+00	2.1E+00	1.8E+00
	rank	2	15	13	14	4	5	6	11	3	12	8	1	10	9	7
F_{16}	mean	1.0E+01	1.1E+01	1.2E+01	1.3E+01	9.7E+00	9.5E+00	1.0E+01	1.3E+01	1.0E+01	1.1E+01	1.1E+01	9.4E+00	1.1E+01	1.1E+01	1.2E+01
	std	4.9E-01	5.9E-01	5.3E-01	4.6E-01	7.2E-01	8.9E-01	8.8E-01	2.4E-01	7.2E-01	6.3E-01	3.4E-01	3.9E-01	5.0E-01	4.2E-01	4.2E-01
	rank	6	10	13	14	3	2	5	15	4	9	11	1	8	7	12
F_{17}	mean	5.3E+04	9.4E+05	6.5E+05	5.2E+06	7.7E+04	2.2E+04	2.6E+04	1.4E+06	6.3E+04	5.8E+04	2.9E+05	1.4E+03	1.8E+06	1.6E+06	2.6E+05
	std	2.8E+04	1.3E+06	8.9E+05	3.8E+06	5.7E+04	1.7E+04	1.8E+04	6.7E+05	4.7E+04	2.8E+04	1.7E+05	6.5E+02	1.5E+06	1.0E+06	2.9E+05
	rank	4	11	10	15	7	2	3	12	6	5	9	1	14	13	8
F_{18}	mean	9.7E+02	8.4E+04	1.5E+03	1.5E+03	2.7E+03	3.3E+02	3.2E+03	2.1E+05	8.6E+02	1.3E+03	3.8E+02	1.4E+02	9.5E+01	1.1E+02	1.6E+03
	std	1.9E+03	3.0E+05	5.1E+03	2.3E+03	5.4E+03	2.9E+02	4.5E+03	1.6E+05	1.1E+03	1.5E+03	2.2E+02	4.3E+01	2.1E+01	9.8E+01	2.3E+03
	rank	7	14	9	10	12	4	13	15	6	8	5	3	1	2	11
F_{19}	mean	5.1E+00	3.2E+01	2.5E+01	7.2E+01	7.2E+00	6.9E+00	6.8E+00	1.8E+01	5.1E+00	1.4E+01	1.2E+01	4.9E+00	1.2E+01	1.1E+01	9.5E+00
	std	1.2E+00	3.5E+01	1.5E+01	1.0E+02	2.3E+00	1.5E+00	1.3E+00	8.5E-01	1.1E+00	3.8E+00	9.7E-01	9.2E-01	2.0E+00	2.0E+00	1.8E+00
	rank	3	14	13	15	6	5	4	12	2	11	10	1	9	8	7
F_{20}	mean	5.7E+02	5.2E+03	8.1E+02	6.3E+04	3.8E+02	1.8E+02	7.4E+02	2.0E+04	4.9E+02	2.0E+03	1.5E+03	1.1E+02	4.5E+03	5.1E+03	3.2E+03
	std	3.2E+02	8.9E+03	5.7E+02	4.0E+04	2.1E+02	6.9E+01	5.2E+02	7.8E+03	4.3E+02	1.3E+03	6.2E+02	6.1E+01	2.3E+03	2.8E+03	1.8E+03
	rank	5	13	7	15	3	2	6	14	4	9	8	1	11	12	10
F_{21}	mean	2.9E+04	2.1E+05	4.2E+04	2.2E+06	2.3E+04	1.4E+04	2.4E+04	4.9E+05	3.1E+04	4.0E+04	7.3E+04	6.6E+02	8.7E+04	9.0E+04	1.1E+05
	std	1.9E+04	2.3E+05	5.7E+04	6.4E+06	1.6E+04	1.0E+04	1.5E+04	2.6E+05	3.3E+04	2.6E+04	5.5E+04	2.5E+02	6.5E+04	8.0E+04	9.8E+04
	rank	5	13	8	15	3	2	4	14	6	7	9	1	10	11	12
F_{22}	mean	2.0E+02	4.9E+02	7.1E+02	8.4E+02	2.3E+02	2.6E+02	3.1E+02	4.1E+02	2.3E+02	5.2E+02	1.5E+02	2.1E+02	2.6E+02	2.7E+02	2.9E+02
	std	5.6E+01	2.2E+02	2.4E+02	2.1E+02	8.7E+01	1.2E+02	1.2E+02	1.5E+02	1.0E+02	1.6E+02	6.7E+01	9.5E+01	9.0E+01	7.9E+01	1.2E+02
	rank	2	12	14	15	5	6	10	11	4	13	1	3	7	8	9
F_{23}	mean	3.2E+02	3.3E+02	3.2E+02	3.3E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02
	std	2.2E-08	1.2E+01	6.5E+00	9.8E+00	6.0E-13	4.3E-13	1.6E-12	1.2E-06	2.2E-12	1.8E-11	8.1E-08	2.2E-13	3.1E-02	9.6E-03	6.5E-01
	rank	11	15	13	14	1	8	9	4	10	2	3	7	6	5	12
F_{24}	mean	2.2E+02	2.5E+02	2.5E+02	2.4E+02	2.3E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.3E+02	2.3E+02
	std	1.4E+00	1.0E+01	9.1E+00	3.8E+01	7.0E+00	1.5E+00	1.6E+00	1.3E+00	3.8E+00	3.5E+00	9.9E-01	5.5E+00	8.1E-01	8.3E-01	7.1E+00
	rank	1	14	15	13	9	8	5	2	7	11	3	10	4	6	12
F_{25}	mean	2.1E+02	2.1E+02	2.2E+02	2.2E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02	2.1E+02	2.2E+02	2.0E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02
	std	1.9E+00	4.2E+00	5.9E+00	4.7E+00	1.2E-01	1.9E+00	2.5E+00	2.1E+00	1.5E+00	4.0E+00	3.4E-01	1.5E+00	1.8E+00	2.1E+00	2.2E+00
	rank	5	10	15	14	1	9	6	8	4	13	2	3	12	11	7
F_{26}	mean	1.8E+02	1.1E+02	1.8E+02	1.9E+02	1.4E+02	1.0E+02	1.8E+02	1.0E+02	1.1E+02	1.2E+02	1.0E+02	1.0E+02	1.1E+02	1.1E+02	1.0E+02
	std	4.4E+01	3.0E+01	4.2E+01	2.8E+01	5.0E+01	7.6E-02	4.4E+01	3.4E-02	2.5E+01	3.6E+01	4.9E-02	5.3E-02	3.4E+01	3.0E+01	1.3E-01
	rank	12	8	14	15	11	2	13	4	6	10	3	1	9	7	5
F_{27}	mean	3.9E+02	8.2E+02	8.1E+02	1.3E+03	4.4E+02	4.0E+02	4.8E+02	1.3E+03	4.0E+02	7.1E+02	4.2E+02	4.1E+02	4.3E+02	4.3E+02	6.8E+02
	std	5.0E+01	1.9E+02	3.8E+02	5.2E+02	4.8E+01	3.2E+00	8.6E+01	3.5E+01	2.1E+01	2.9E+02	3.8E+01	5.7E+01	2.0E+01	1.8E+01	2.7E+02
	rank	1	13	12	15	8	3	9	14	2	11	5	4	7	6	10
F_{28}	mean	8.7E+02	1.2E+03	3.9E+03	5.1E+03	4.1E+02	9.6E+02	1.5E+03	3.9E+02	9.0E+02	7.7E+02	3.9E+02	8.7E+02	8.8E+02	8.7E+02	1.2E+03
	std	4.0E+01	2.1E+02	1.2E+03	1.7E+03	2.7E+01	4.8E+01	7.2E+02	4.6E+00	5.6E+01	3.3E+02	4.0E+00	4.3E+01	3.1E+02	2.3E+02	2.5E+02
	rank	7	11	14	15	3	10	13	2	9	4	1	5	8	6	12
F_{29}	mean	1.8E+03	3.1E+06	1.3E+07	4.7E+06	2.1E+02	9.7E+02	1.4E+03	2.2E+02	1.3E+03	2.2E+02	2.1E+02	7.7E+02	1.1E+03	1.1E+03	4.1E+06
	std	5.0E+02	4.0E+06	1.8E+07	1.0E+07	1.2E+00	2.6E+02	4.9E+02	1.2E+00	3.1E+02	1.6E+01	1.5E+00	3.2E+01	1.9E+02	1.9E+02	3.7E+06
	rank	11	12	15	14	1	6	10	4	9	3	2	5	8	7	13
F_{30}	mean	1.8E+03	2.9E+04	1.8E+04	8.3E+04	7.7E+02	2.1E+03	1.9E+03	9.4E+02	1.8E+03	8.4E+02	6.3E+02	2.4E+03	6.1E+02	6.5E+02	1.4E+04
	std	4.5E+02	3.3E+04	4.2E+04	8.5E+04	1.8E+02	6.6E+02	6.1E+02	1.8E+02	6.1E+02	2.4E+02	1.5E+02	1.0E+03	1.3E+02	1.5E+02	2.6E+04
	rank	8	14	13	15	4	10	9	6	7	5	2	11	1	3	12
Mean rank		6.10	12.40	12.37	14.03	5.57	4.57	6.93	10.43	5.03	7.93	6.37	3.30	7.63	7.33	10.00
Final rank		5	14	13	15	4	2	7	12	3	10	6	1	9	8	11
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.0E+06	1.0E+08	5.6E+07	8.0E+07	1.2E+06	3.4E+05	4.2E+05	3.3E+08	7.9E+05	6.5E+05	9.8E+06	3.0E+04	1.6E+07	1.4E+07	2.2E+07
	std	3.4E+05	9.7E+07	5.4E+07	3.4E+07	5.5E+05	1.3E+05	2.0E+05	8.9E+07	2.4E+05	1.6E+05	3.6E+06	7.8E+04	7.6E+06	4.1E+06	1.9E+07
	rank	6	14	12	13	7	2	3	15	5	4	8	1	10	9	11
F_2	mean	9.9E+03	1.3E+10	3.4E+09	2.7E+08	9.9E+03	3.0E+03	6.3E+03	1.5E+06	5.4E+03	5.0E+03	1.1E+03	2.3E-10	1.5E+03	2.5E+03	4.9E+03
	std	7.5E+03	5.4E+09	1.6E+09	2.9E+08	1.0E+04	3.1E+03	6.1E+03	1.6E+06	5.6E+03	6.5E+03	1.5E+03	4.9E-10	1.4E+03	2.5E+03	8.1E+03
	rank	10	15	14	13	11	5	9	12	8	7	2	1	3	4	6
F_3	mean	9.9E+02	7.7E+04	2.6E+04	1.8E+05	5.4E+02	6.9E+00	6.0E+02	2.9E+05	1.0E+03	2.4E+03	3.4E+03	1.5E+00	3.9E+03	3.5E+03	3.6E+03
	std	5.2E+02	3.7E+04	1.1E+04	3.8E+04	4.7E+02	7.9E+00	5.4E+02	9.8E+04	8.1E+02	1.2E+03	1.2E+03	5.2E+00	2.4E+03	2.0E+03	2.2E+03
	rank	5	13	12	14	3	2	4	15	6	7	8	1	11	9	10
F_4	mean	9.8E+01	1.2E+03	4.7E+02	4.2E+02	4.3E+01	4.1E+01	5.7E+01	4.6E+01	8.8E+01	8.6E+01	4.4E+01	1.4E+01	1.5E+02	1.4E+02	1.2E+02
	std	4.3E-01	9.6E+02	1.4E+02	8.0E+01	1.1E+01	3.4E+01	2.9E+01	8.5E-01	2.5E+01	4.0E+01	1.7E+00	3.2E+01	2.3E+01	3.1E+01	4.6E+01
	rank	9	15	14	13	3	2	6	5	8	7	4	1	12	11	10
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	4.0E-02	3.0E-01	1.2E-01	1.2E-01	5.7E-02	4.9E-02	3.3E-01	4.4E-02	4.5E-03	2.6E-04	6.3E-02	3.6E-03	7.3E-02	6.8E-02	2.0E-01
	rank	15	12	6	13	5	4	10	14	2	1	11	3	7	9	8
F_6	mean	4.4E+00	4.0E+01	6.0E+01	7.1E+01	1.1E+01	2.6E+01	1.9E+01	7.1E+01	1.3E+01	4.1E+01	2.3E+01	1.5E+01	3.4E+01	3.5E+01	4.8E+01
	std	8.1E-01	4.0E+00	3.1E+00	5.1E+00	3.4E+00	4.1E+00	5.2E+00	1.5E+00	4.1E+00	4.4E+00	2.2E+00	4.3E+00	4.8E+00	4.5E+00	4.3E+00
	rank	1	10	13	14	2	7	5	15	3	11	6	4	8	9	12
F_7	mean	3.0E-03	1.2E+02	3.4E+01	2.6E+00	9.2E-03	1.1E-02	5.3E-03	6.9E-05	5.3E-03	8.9E-03	3.1E-03	4.3E-03	1.1E-02	8.8E-03	5.3E-01
	std	5.0E-03	7.9E+01	1.8E+01	1.1E+00	9.3E-03	1.4E-02	6.8E-03	2.8E-04	7.0E-03	6.8E-03	8.1E-03	6.8E-03	5.6E-03	6.9E-03	2.9E+00
	rank	2	15	14	13	9	11	6	1	5	8	3	4	10	7	12
F_8	mean	3.9E+01	2.0E+02	3.2E+02	3.0E+02	7.1E+01	9.5E-01	7.3E+01	2.2E+02	1.5E+01	2.2E+01	8.2E+01	1.7E-01	8.7E+00	9.7E+00	1.7E+02
	std	1.1E+01	3.8E+01	3.7E+01	7.9E+01	1.3E+01	8.2E-01	2.0E+01	2.0E+01	4.9E+00	6.3E+00	1.5E+01	5.9E-01	4.2E+00	4.3E+00	2.9E+01
	rank	7	12	15	14	8	2	9	13	5	6	10	1	3	4	11
F_9	mean	4.7E+01	3.1E+02	3.9E+02	3.9E+02	7.6E+01	1.0E+02	1.3E+02	3.6E+02	9.5E+01	2.5E+02	1.5E+02	6.5E+01	1.9E+02	2.2E+02	2.0E+02
	std	8.7E+00	5.4E+01	3.9E+01	1.5E+02	1.7E+01	1.9E+01	3.2E+01	1.7E+01	2.0E+01	3.7E+01	3.3E+01	1.4E+01	3.1E+01	2.8E+01	3.0E+01
	rank	1	12	15	14	3	5	6	13	4	11	7	2	8	10	9
F_{10}	mean	3.9E+03	5.8E+03	8.3E+03	3.2E+03	3.1E+03	2.2E+01	1.6E+03	1.3E+04	8.5E+02	1.1E+03	2.8E+03	4.0E+00	9.3E+01	8.9E+01	6.0E+03
	std	9.5E+02	8.5E+02	9.8E+02	1.6E+03	8.7E+02	4.4E+01	4.4E+02	4.4E+02	2.2E+02	2.4E+02	4.5E+02	2.2E+01	1.0E+02	8.7E+01	7.2E+02
	rank	11	12	14	10	9	2	7	15	5	6	8	1	4	3	13
F_{11}	mean	5.8E+03	7.6E+03	9.1E+03	1.2E+04	5.2E+03	4.0E+03	5.9E+03	1.4E+04	5.0E+03	5.6E+03	6.1E+03	3.9E+03	8.8E+03	8.7E+03	6.5E+03
	std	9.9E+02	7.9E+02	9.6E+02	2.3E+03	7.6E+02	4.9E+02	9.6E+02	3.5E+02	8.7E+02	6.5E+02	9.8E+02	3.4E+02	8.5E+02	7.0E+02	5.8E+02
	rank	6	10	13	14	4	2	7	15	3	5	8	1	12	11	9
F_{12}	mean	2.7E+00	4.1E-01	2.0E+00	2.6E+00	1.2E+00	1.4E-01	9.6E-01	3.4E+00	2.0E-01	3.2E-01	1.5E+00	1.1E-01	1.0E+00	1.0E+00	8.5E-01
	std	7.2E-01	1.5E-01	4.1E-01	1.2E+00	1.8E-01	4.8E-02	3.5E-01	2.1E-01	5.4E-02	9.9E-02	2.8E-01	1.4E-02	2.2E-01	1.6E-01	1.9E-01
	rank	14	5	12	13	10	2	7	15	3	4	11	1	8	9	6
F_{13}	mean	2.9E-01	1.1E+00	6.4E-01	5.5E-01	2.4E-01	3.9E-01	4.5E-01	4.7E-01	3.8E-01	4.9E-01	4.9E-01	3.7E-01	4.7E-01	4.8E-01	4.6E-01
	std	4.9E-02	9.1E-01	1.2E-01	8.0E-02	5.2E-02	7.5E-02	6.9E-02	4.6E-02	9.3E-02	6.8E-02	4.2E-02	6.5E-02	4.1E-02	4.6E-02	6.8E-02
	rank	2	15	14	13	1	5	6	9	4	11	12	3	8	10	7
F_{14}	mean	3.1E-01	3.0E+01	3.7E+00	3.0E-01	3.8E-01	3.0E-01	4.1E-01	3.5E-01	3.1E-01	3.1E-01	2.7E-01	3.6E-01	3.3E-01	3.2E-01	4.5E-01
	std	1.1E-01	2.4E+01	6.3E+00	4.2E-02	2.1E-01	4.1E-02	2.4E-01	3.9E-02	9.6E-02	3.4E-02	2.4E-02	1.4E-01	2.7E-02	2.3E-02	1.9E-01
	rank	4	15	14	3	11	2	12	9	6	5	1	10	8	7	13
F_{15}	mean	6.5E+00	3.3E+04	8.8E+02	5.8E+02	8.5E+00	1.3E+01	1.2E+01	3.2E+01	1.1E+01	1.1E+02	2.2E+01	7.6E+00	3.2E+01	2.9E+01	2.4E+01
	std	1.8E+00	4.8E+04	8.6E+02	8.3E+02	3.4E+00	3.7E+00	3.4E+00	1.3E+00	3.5E+00	3.2E+01	4.5E+00	1.1E+00	5.8E+00	4.9E+00	7.6E+00
	rank	1	15	14	13	3	6	5	11	4	12	7	2	10	9	8
F_{16}	mean	1.8E+01	2.1E+01	2.2E+01	2.2E+01	1.9E+01	1.8E+01	1.9E+01	2.3E+01	1.9E+01	2.0E+01	2.1E+01	1.8E+01	2.1E+01	2.1E+01	2.1E+01
	std	7.8E-01	5.4E-01	5.6E-01	4.7E-01	8.8E-01	5.1E-01	6.9E-01	2.1E-01	8.5E-01	7.4E-01	4.6E-01	3.8E-01	3.7E-01	4.9E-01	6.0E-01
	rank	3	8	13	14	5	2	6	15	4	7	11	1	10	9	12
F_{17}	mean	1.3E+05	7.8E+06	1.9E+06	8.1E+06	2.4E+05	4.4E+04	5.1E+04	1.1E+07	1.1E+05	1.2E+05	1.6E+06	4.0E+03	2.7E+06	2.4E+06	1.0E+06
	std	2.3E+05	1.3E+07	3.6E+06	5.5E+06	1.5E+05	1.8E+04	2.3E+04	3.8E+06	7.1E+04	7.4E+04	6.3E+05	6.6E+03	3.1E+06	2.1E+06	9.0E+05
	rank	6	13	10	14	7	2	3	15	4	5	9	1	12	11	8
F_{18}	mean	2.1E+03	1.6E+08	1.9E+07	2.2E+05	6.3E+03	3.8E+02	1.5E+03	6.2E+05	6.9E+02	2.0E+03	7.5E+02	2.6E+02	8.6E+02	9.3E+02	2.6E+03
	std	1.1E+03	4.8E+08	7.2E+07	9.6E+05	7.9E+03	2.2E+02	1.0E+03	1.1E+06	6.8E+02	2.5E+03	3.9E+02	2.6E+02	8.2E+02	7.1E+02	1.4E+03
	rank	9	15	14	12	11	2	7	13	3	8	4	1	5	6	10
F_{19}	mean	1.5E+01	7.9E+01	8.0E+01	2.4E+02	2.1E+01	1.5E+01	1.4E+01	3.3E+01	1.2E+01	3.8E+01	2.5E+01	1.2E+01	2.4E+01	2.1E+01	2.8E+01
	std	1.7E+00	5.8E+01	2.3E+01	7.2E+02	2.7E+00	2.8E+00	2.6E+00	6.1E-01	2.3E+00	2.6E+01	1.7E+00	2.8E+00	1.6E+01	1.2E+01	7.5E+00
	rank	4	13	14	15	6	5	3	11	2	12	9	1	8	7	10
F_{20}	mean	4.8E+02	2.1E+04	6.0E+03	2.3E+05	5.0E+02	3.0E+02	6.7E+02	7.1E+04	7.0E+02	1.7E+03	2.2E+03	3.5E+02	1.3E+04	1.3E+04	3.9E+03
	std	1.4E+02	2.5E+04	5.0E+03	2.5E+05	1.8E+02	8.9E+01	2.4E+02	3.5E+04	5.3E+02	1.5E+03	9.6E+02	1.5E+02	5.0E+03	5.7E+03	2.9E+03
	rank	3	13	10	15	4	1	5	14	6	7	8	2	11	12	9
F_{21}	mean	1.0E+05	2.0E+06	6.0E+05	7.7E+06	9.4E+04	3.8E+04	3.9E+04	4.4E+06	1.3E+05	9.2E+04	6.4E+05	1.8E+03	1.5E+06	1.4E+06	2.5E+05
	std	3.2E+04	2.9E+06	8.4E+05	7.2E+06	4.9E+04	2.5E+04	1.6E+04	1.5E+06	7.1E+04	7.4E+04	3.3E+05	1.6E+03	9.4E+05	1.1E+06	1.6E+05
	rank	6	13	9	15	5	2	3	14	7	4	10	1	12	11	8
F_{22}	mean	4.5E+02	1.3E+03	1.7E+03	1.8E+03	4.9E+02	8.3E+02	7.5E+02	1.7E+03	7.3E+02	1.4E+03	5.4E+02	5.9E+02	7.8E+02	7.4E+02	9.0E+02
	std	2.4E+02	3.1E+02	4.3E+02	5.4E+02	2.6E+02	2.2E+02	2.5E+02	1.4E+02	2.5E+02	4.1E+02	1.6E+02	1.6E+02	1.7E+02	1.7E+02	2.4E+02
	rank	1	11	14	15	2	9	7	13	5	12	3	4	8	6	10
F_{23}	mean	3.4E+02	3.9E+02	3.7E+02	3.6E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02
	std	7.2E-07	4.1E+01	1.5E+01	6.3E+00	1.2E-12	6.8E-13	5.0E-12	3.4E-06	8.7E-13	7.6E-11	5.4E-07	5.0E-13	2.0E-02	5.2E-03	6.9E-01
	rank	11	15	14	13	1	8	10	4	9	2	3	7	6	5	12
F_{24}	mean	2.7E+02	3.3E+02	3.3E+02	2.9E+02	2.8E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.8E+02	2.6E+02	2.6E+02	2.8E+02
	std	3.2E+00	3.8E+01	1.4E+01	2.1E+01	2.7E+00	5.5E+00	5.7E+00	6.6E-06	4.5E+00	4.8E+00	4.5E+00	3.3E+00	4.5E+00	4.6E+00	3.9E+00
	rank	6	15	14	13	11	2	8	5	7	4	9	10	3	1	12
F_{25}	mean	2.2E+02	2.2E+02	2.5E+02	2.4E+02	2.0E+02	2.1E+02	2.2E+02	2.3E+02	2.1E+02	2.4E+02	2.0E+02	2.1E+02	2.3E+02	2.3E+02	2.2E+02
	std	3.1E+00	1.3E+01	9.1E+00	1.4E+01	7.8E-02	3.0E+00	4.5E+00	5.2E+00	3.6E+00	6.5E+00	4.5E-01	5.4E+00	3.3E+00	4.0E+00	5.5E+00
	rank	8	9	15	14	1	5	7	10	4	13	2	3	12	11	6
F_{26}	mean	1.9E+02	1.9E+02	2.0E+02	2.0E+02	1.6E+02	1.2E+02	1.7E+02	1.0E+02	1.5E+02	1.8E+02	1.1E+02	1.0E+02	1.5E+02	1.5E+02	1.1E+02
	std	3.1E+01	1.2E+02	1.0E+00	4.9E+00	5.0E+01	3.7E+01	6.3E+01	5.0E-02	5.0E+01	4.1E+01	3.1E+01	2.0E-01	5.0E+01	5.1E+01	2.6E+01
	rank	13	12	14	15	9	5	10	1	6	11	4	2	8	7	3
F_{27}	mean	4.9E+02	1.4E+03	2.0E+03	3.1E+03	6.4E+02	9.1E+02	8.5E+02	2.1E+03	6.1E+02	1.5E+03	9.2E+02	6.9E+02	9.8E+02	9.6E+02	1.6E+03
	std	6.8E+01	1.1E+02	3.2E+02	9.2E+02	7.5E+01	2.7E+02	1.4E+02	3.3E+01	1.1E+02	1.4E+02	6.1E+01	7.6E+01	1.4E+02	1.4E+02	1.3E+02
	rank	1	10	13	15	3	6	5	14	2	11	7	4	9	8	12
F_{28}	mean	1.3E+03	2.1E+03	8.3E+03	1.1E+04	3.9E+02	1.8E+03	2.7E+03	4.0E+02	1.7E+03	1.6E+03	3.9E+02	1.3E+03	7.2E+02	9.7E+02	2.3E+03
	std	1.3E+02	5.7E+02	2.5E+03	4.0E+03	1.5E+01	1.9E+02	8.2E+02	1.9E+01	3.7E+02	1.4E+03	1.3E+01	2.2E+02	4.7E+02	9.3E+02	7.6E+02
	rank	6	11	14	15	1	10	13	3	9	8	2	7	4	5	12
F_{29}	mean	2.9E+03	3.0E+07	6.3E+07	1.3E+08	2.2E+02	1.4E+03	1.4E+03	2.4E+02	1.3E+03	2.3E+02	2.3E+02	8.9E+02	1.1E+03	1.1E+03	4.6E+07
	std	5.7E+02	1.9E+07	8.0E+07	5.9E+08	1.7E+01	3.1E+02	3.3E+02	4.9E+00	3.8E+02	1.6E+01	1.6E+00	1.0E+02	1.9E+02	2.9E+02	1.6E+07
	rank	11	12	14	15	1	9	10	4	8	3	2	5	6	7	13
F_{30}	mean	1.1E+04	5.9E+04	1.5E+05	2.1E+05	1.2E+03	9.7E+03	1.2E+04	2.1E+03	1.1E+04	1.8E+03	1.1E+03	1.0E+04	1.3E+03	1.3E+03	3.5E+04
	std	6.4E+02	4.5E+04	2.0E+05	1.2E+05	2.9E+02	9.9E+02	1.2E+03	2.0E+02	1.2E+03	3.4E+02	1.8E+02	1.5E+03	2.6E+02	2.3E+02	2.5E+04
	rank	9	13	14	15	2	7	11	6	10	5	1	8	3	4	12
Mean rank		6.37	12.43	13.07	13.40	5.27	4.47	7.00	10.63	5.23	7.23	6.33	3.10	7.97	7.70	9.80
Final rank		6	13	14	15	4	2	7	12	3	8	5	1	10	9	11
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	6.7E+06	4.5E+08	3.2E+08	1.7E+08	2.7E+06	9.8E+05	1.1E+06	3.6E+09	4.7E+06	2.8E+06	6.7E+07	2.6E+05	1.6E+08	1.6E+08	1.1E+08
	std	3.2E+06	2.4E+08	1.2E+08	8.8E+07	1.0E+06	3.0E+05	5.6E+05	8.1E+08	1.5E+06	7.1E+05	2.7E+07	1.9E+05	3.3E+07	4.0E+07	7.3E+07
	rank	7	14	13	12	4	2	3	15	6	5	8	1	10	11	9
F_2	mean	1.7E+04	7.3E+10	1.4E+10	8.2E+06	1.9E+04	2.0E+04	1.6E+04	4.1E+06	1.2E+04	1.4E+04	1.1E+04	3.0E-09	1.2E+04	1.4E+04	3.0E+04
	std	1.4E+04	2.2E+10	3.4E+09	1.1E+07	1.9E+04	1.5E+04	1.5E+04	1.3E+07	1.2E+04	1.1E+04	1.7E+04	5.1E-09	6.9E+03	6.5E+03	2.9E+04
	rank	8	15	14	13	9	10	7	12	4	5	2	1	3	6	11
F_3	mean	1.6E+03	1.8E+05	1.0E+05	3.8E+05	7.4E+02	1.9E-01	3.8E+02	1.1E+06	1.7E+03	2.3E+03	4.1E+03	9.2E-01	9.7E+03	8.0E+03	3.0E+03
	std	9.7E+02	8.1E+04	2.0E+04	1.3E+05	9.0E+02	2.5E-01	3.7E+02	2.8E+05	1.6E+03	1.5E+03	1.4E+03	2.9E+00	5.0E+03	3.4E+03	2.2E+03
	rank	5	13	12	14	4	1	3	15	6	7	9	2	11	10	8
F_4	mean	2.3E+02	8.5E+03	2.0E+03	5.0E+02	1.0E+02	1.4E+02	1.8E+02	9.8E+01	2.1E+02	2.0E+02	9.5E+01	1.3E+02	3.3E+02	3.4E+02	3.3E+02
	std	2.8E+01	4.4E+03	4.3E+02	5.9E+01	2.0E+01	5.2E+01	3.7E+01	9.2E-01	4.7E+01	4.9E+01	2.2E+00	4.7E+01	5.6E+01	4.0E+01	1.1E+02
	rank	9	15	14	13	3	5	6	2	8	7	1	4	10	12	11
F_5	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	2.6E-02	9.0E-02	9.6E-02	3.2E-01	4.0E-02	1.0E-01	2.8E-01	3.2E-02	3.2E-04	3.5E-01	3.9E-02	3.9E-03	3.6E-02	4.6E-02	1.0E-01
	rank	14	13	6	12	5	3	10	15	1	4	8	2	11	9	7
F_6	mean	1.8E+01	9.9E+01	1.4E+02	1.5E+02	4.8E+01	7.6E+01	6.5E+01	1.6E+02	6.1E+01	1.0E+02	7.7E+01	5.8E+01	1.1E+02	1.1E+02	1.2E+02
	std	3.0E+00	6.9E+00	6.5E+00	9.4E+00	7.2E+00	6.9E+00	7.2E+00	1.5E+00	6.8E+00	4.3E+00	4.6E+00	6.6E+00	8.8E+00	7.9E+00	5.8E+00
	rank	1	8	13	14	2	6	5	15	4	9	7	3	11	10	12
F_7	mean	1.2E-02	7.4E+02	1.2E+02	1.1E+00	4.9E-03	9.8E-03	3.8E-03	2.2E-02	2.5E-03	1.2E-03	2.2E-03	9.8E-03	1.1E-02	8.0E-03	5.6E-12
	std	1.8E-02	2.4E+02	3.5E+01	1.8E-01	9.0E-03	2.7E-02	7.3E-03	7.4E-03	4.6E-03	3.0E-03	2.6E-03	1.4E-02	1.8E-02	1.1E-02	3.1E-12
	rank	11	15	14	13	6	8	5	12	4	2	3	9	10	7	1
F_8	mean	8.9E+01	6.2E+02	8.3E+02	6.6E+02	2.0E+02	9.0E-01	2.2E+02	7.6E+02	3.2E+01	7.7E+01	2.4E+02	9.6E-01	4.0E+01	4.4E+01	4.8E+02
	std	1.5E+01	6.2E+01	6.4E+01	8.8E+01	3.7E+01	9.2E-01	5.0E+01	3.0E+01	1.1E+01	3.1E+01	2.6E+01	1.9E+00	9.1E+00	9.3E+00	5.5E+01
	rank	7	12	15	13	8	1	9	14	3	6	10	2	4	5	11
F_9	mean	1.0E+02	8.8E+02	1.0E+03	1.0E+03	1.9E+02	3.5E+02	3.1E+02	9.6E+02	2.9E+02	6.7E+02	4.9E+02	2.1E+02	8.3E+02	8.3E+02	5.6E+02
	std	1.5E+01	1.1E+02	6.5E+01	3.6E+02	4.8E+01	6.0E+01	4.9E+01	2.6E+01	5.1E+01	5.1E+01	1.3E+02	4.0E+01	6.5E+01	6.5E+01	6.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_{10}	mean	1.0E+04	1.5E+04	2.1E+04	1.0E+04	8.9E+03	2.9E+01	4.7E+03	3.1E+04	1.9E+03	2.4E+03	7.6E+03	2.8E+01	3.9E+02	4.3E+02	1.4E+04
	std	1.8E+03	1.8E+03	1.1E+03	3.2E+03	2.0E+03	4.8E+01	6.4E+02	7.9E+02	4.0E+02	5.3E+02	1.1E+03	6.0E+01	2.0E+02	1.8E+02	1.3E+03
	rank	10	13	14	11	9	2	7	15	5	6	8	1	3	4	12
F_{11}	mean	1.2E+04	1.7E+04	2.2E+04	2.8E+04	1.3E+04	1.1E+04	1.3E+04	3.3E+04	1.2E+04	1.3E+04	1.8E+04	1.1E+04	2.6E+04	2.6E+04	1.5E+04
	std	1.4E+03	1.3E+03	1.3E+03	4.6E+03	1.5E+03	6.7E+02	1.7E+03	6.1E+02	1.3E+03	1.1E+03	2.1E+03	5.5E+02	9.9E+02	1.1E+03	1.4E+03
	rank	3	9	11	14	5	2	7	15	4	6	10	1	12	13	8
F_{12}	mean	3.6E+00	1.7E+00	2.8E+00	3.7E+00	1.8E+00	2.1E-01	1.6E+00	4.0E+00	2.8E-01	4.5E-01	2.1E+00	1.8E-01	2.3E+00	2.3E+00	1.5E+00
	std	3.7E-01	1.4E+00	4.8E-01	7.6E-01	2.1E-01	5.5E-02	4.7E-01	2.2E-01	5.9E-02	8.9E-02	2.5E-01	1.6E-02	2.5E-01	3.2E-01	2.8E-01
	rank	13	7	12	14	8	2	6	15	3	4	9	1	10	11	5
F_{13}	mean	4.4E-01	3.7E+00	6.0E-01	5.4E-01	4.2E-01	4.6E-01	6.4E-01	6.9E-01	5.0E-01	5.0E-01	6.1E-01	5.0E-01	5.0E-01	5.1E-01	5.5E-01
	std	3.5E-02	1.2E+00	6.6E-02	6.4E-02	5.4E-02	5.4E-02	6.4E-02	5.7E-02	6.9E-02	5.6E-02	6.0E-02	7.6E-02	4.2E-02	4.4E-02	5.3E-02
	rank	2	15	11	9	1	3	13	14	5	4	12	6	7	8	10
F_{14}	mean	3.5E-01	2.1E+02	4.6E+01	3.5E-01	4.3E-01	3.4E-01	3.5E-01	4.2E-01	3.2E-01	3.2E-01	3.2E-01	3.6E-01	3.4E-01	3.4E-01	6.3E-01
	std	1.2E-01	7.6E+01	1.7E+01	3.5E-02	2.0E-01	2.7E-02	1.3E-01	1.3E-01	3.5E-02	3.0E-02	2.8E-02	1.1E-01	2.4E-02	2.8E-02	2.2E-01
	rank	8	15	14	9	12	4	7	11	1	3	2	10	5	6	13
F_{15}	mean	1.5E+01	1.5E+06	2.1E+04	7.2E+02	2.6E+01	4.3E+01	4.4E+01	8.4E+01	4.5E+01	3.3E+02	7.4E+01	3.9E+01	1.0E+02	1.1E+02	1.8E+02
	std	2.6E+00	1.5E+06	1.4E+04	1.0E+03	1.1E+01	6.7E+00	7.9E+00	3.2E+00	1.1E+01	5.2E+01	1.1E+01	7.5E+00	1.6E+01	1.2E+01	1.1E+02
	rank	1	15	14	13	2	4	5	8	6	12	7	3	9	10	11
F_{16}	mean	4.1E+01	4.4E+01	4.5E+01	4.7E+01	4.2E+01	4.1E+01	4.3E+01	4.8E+01	4.1E+01	4.3E+01	4.5E+01	4.0E+01	4.6E+01	4.6E+01	4.5E+01
	std	1.7E+00	1.3E+00	6.9E-01	5.2E-01	9.9E-01	7.5E-01	1.2E+00	2.0E-01	1.2E+00	8.3E-01	5.2E-01	7.2E-01	4.4E-01	5.2E-01	5.8E-01
	rank	3	8	11	14	5	2	6	15	4	7	10	1	12	13	9
F_{17}	mean	6.9E+05	2.4E+07	1.7E+07	3.5E+07	1.0E+06	1.9E+05	4.0E+05	1.1E+08	7.3E+05	4.6E+05	9.3E+06	3.5E+04	1.7E+07	1.9E+07	7.3E+06
	std	1.7E+05	2.7E+07	5.8E+06	2.1E+07	3.4E+05	6.0E+04	1.8E+05	2.4E+07	2.9E+05	1.7E+05	4.5E+06	3.2E+04	6.6E+06	8.3E+06	5.0E+06
	rank	5	13	10	14	7	2	3	15	6	4	9	1	11	12	8
F_{18}	mean	9.2E+02	9.5E+08	2.1E+08	1.4E+03	3.1E+03	6.9E+02	2.3E+03	4.2E+06	2.1E+03	1.5E+03	1.7E+03	1.5E+03	6.4E+02	1.0E+03	4.7E+03
	std	7.4E+02	1.6E+09	1.6E+08	1.5E+03	3.6E+03	3.1E+02	2.3E+03	4.7E+06	2.4E+03	1.4E+03	8.0E+02	1.6E+03	4.6E+02	1.1E+03	4.5E+03
	rank	3	15	14	5	11	2	10	13	9	6	8	7	1	4	12
F_{19}	mean	1.0E+02	5.6E+02	3.4E+02	1.8E+02	6.3E+01	7.1E+01	8.2E+01	7.4E+01	1.0E+02	9.1E+01	5.9E+01	9.5E+01	1.4E+02	1.3E+02	1.2E+02
	std	2.3E+00	1.5E+02	1.2E+02	7.6E+01	1.9E+01	2.7E+01	3.6E+01	1.9E+00	1.6E+01	3.5E+01	2.4E+00	2.4E+01	2.7E+01	2.5E+01	1.1E+01
	rank	9	15	14	13	2	3	5	4	8	6	1	7	12	11	10
F_{20}	mean	8.7E+02	1.1E+05	3.7E+04	1.1E+06	1.1E+03	5.2E+02	1.4E+03	5.3E+05	1.8E+03	2.6E+03	5.0E+03	6.8E+03	5.2E+04	4.9E+04	6.8E+03
	std	2.4E+02	6.0E+04	1.9E+04	8.9E+05	3.7E+02	7.8E+01	5.6E+02	1.2E+05	8.0E+02	7.9E+02	1.4E+03	5.3E+03	1.7E+04	1.4E+04	2.1E+03
	rank	2	13	10	15	3	1	4	14	5	6	7	8	12	11	9
F_{21}	mean	3.6E+05	6.4E+06	5.6E+06	1.7E+07	4.2E+05	8.0E+04	1.1E+05	4.6E+07	3.1E+05	2.2E+05	4.4E+06	1.8E+04	8.4E+06	8.4E+06	2.1E+06
	std	1.2E+05	5.4E+06	3.0E+06	2.0E+07	1.8E+05	2.8E+04	6.7E+04	1.2E+07	1.5E+05	5.7E+04	1.9E+06	2.9E+04	9.5E+06	7.5E+06	2.3E+06
	rank	6	11	10	14	7	2	3	15	5	4	9	1	13	12	8
F_{22}	mean	1.4E+03	3.5E+03	3.9E+03	5.0E+03	1.4E+03	1.9E+03	2.3E+03	5.4E+03	2.1E+03	3.0E+03	2.0E+03	1.7E+03	1.8E+03	1.8E+03	2.5E+03
	std	4.6E+02	5.8E+02	8.1E+02	1.1E+03	3.4E+02	3.5E+02	4.7E+02	2.3E+02	4.6E+02	4.7E+02	4.8E+02	2.5E+02	3.3E+02	3.1E+02	4.1E+02
	rank	2	12	13	14	1	6	9	15	8	11	7	3	5	4	10
F_{23}	mean	3.5E+02	6.4E+02	5.0E+02	3.6E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02
	std	5.6E-03	1.8E+02	6.4E+01	2.8E+00	3.8E-11	2.1E-11	5.4E-10	2.3E-01	2.8E-10	1.6E-10	4.7E-03	2.8E-12	8.9E-03	1.3E-02	7.8E+00
	rank	11	15	14	13	1	8	10	6	9	2	3	7	4	5	12
F_{24}	mean	3.8E+02	6.7E+02	5.9E+02	4.2E+02	3.9E+02	3.6E+02	3.8E+02	3.6E+02	3.8E+02	3.4E+02	3.8E+02	4.2E+02	3.7E+02	3.7E+02	4.1E+02
	std	5.1E+00	8.3E+01	4.1E+01	2.1E+01	5.6E+00	7.5E+00	5.0E+00	2.3E+00	3.6E+00	1.2E+01	3.9E+00	8.1E+00	3.3E+00	2.8E+00	8.2E+00
	rank	6	15	14	13	10	2	8	3	7	1	9	12	4	5	11
F_{25}	mean	2.2E+02	2.8E+02	3.5E+02	2.8E+02	2.1E+02	2.6E+02	2.8E+02	5.2E+02	2.7E+02	3.0E+02	2.1E+02	2.8E+02	2.8E+02	2.8E+02	2.6E+02
	std	1.9E+01	2.8E+01	2.3E+01	2.1E+01	2.8E+01	5.2E+00	1.1E+01	6.2E+01	1.2E+01	1.1E+01	1.2E+00	1.2E+01	2.0E+01	9.8E+00	1.7E+01
	rank	3	10	14	12	2	4	8	15	6	13	1	11	7	9	5
F_{26}	mean	2.0E+02	2.3E+02	2.1E+02	2.1E+02	2.0E+02	2.0E+02	2.0E+02	1.0E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.1E+02	2.1E+02	1.9E+02
	std	4.0E-02	2.0E+01	4.7E+00	6.6E+00	6.2E-02	2.6E-02	9.3E-02	5.6E-02	9.9E-02	2.9E-02	3.8E+01	9.5E-03	3.0E+00	2.0E+00	3.7E+01
	rank	7	15	14	13	9	5	8	1	10	6	2	4	12	11	3
F_{27}	mean	8.4E+02	2.9E+03	4.4E+03	6.7E+03	1.6E+03	2.1E+03	2.0E+03	4.4E+03	1.8E+03	3.1E+03	2.3E+03	1.6E+03	1.6E+03	1.7E+03	3.5E+03
	std	1.0E+02	1.6E+02	2.2E+02	1.7E+03	2.0E+02	7.4E+02	2.3E+02	5.1E+01	4.5E+02	2.0E+02	1.2E+02	1.4E+02	3.0E+02	2.7E+02	1.4E+02
	rank	1	10	13	15	2	8	7	14	6	11	9	3	4	5	12
F_{28}	mean	2.5E+03	5.0E+03	2.0E+04	3.0E+04	7.6E+02	5.3E+03	7.9E+03	2.6E+03	6.4E+03	7.1E+03	6.7E+02	3.3E+03	7.1E+03	6.2E+03	6.3E+03
	std	3.8E+02	7.9E+02	5.3E+03	9.7E+03	2.6E+02	6.4E+02	1.4E+03	6.8E+02	1.1E+03	2.8E+03	1.0E+02	6.2E+02	4.2E+03	4.1E+03	7.8E+02
	rank	3	6	14	15	2	7	13	4	10	11	1	5	12	8	9
F_{29}	mean	2.2E+03	9.8E+07	4.9E+08	5.0E+05	2.6E+02	1.5E+03	1.6E+03	3.0E+02	1.8E+03	2.9E+02	2.6E+02	1.5E+03	8.1E+02	8.4E+02	2.2E+08
	std	1.7E+02	4.5E+07	4.4E+08	1.7E+06	3.4E+00	2.7E+02	2.6E+02	2.8E+01	2.7E+02	4.9E+01	2.5E+00	1.7E+02	6.5E+02	5.1E+02	5.8E+07
	rank	11	13	15	12	2	8	9	4	10	3	1	7	5	6	14
F_{30}	mean	9.6E+03	6.6E+05	7.1E+05	5.3E+05	3.0E+03	9.4E+03	1.2E+04	6.8E+03	1.0E+04	4.0E+03	2.8E+03	9.9E+03	3.0E+03	3.2E+03	5.1E+05
	std	1.5E+03	4.7E+05	4.6E+05	8.6E+05	4.5E+02	1.0E+03	3.4E+03	3.3E+02	1.4E+03	3.7E+02	4.8E+02	1.3E+03	4.2E+02	4.0E+02	3.8E+05
	rank	8	14	15	13	2	7	11	6	10	5	1	9	3	4	12
Mean rank		6.00	12.53	12.87	12.80	4.87	4.20	7.07	11.17	5.90	6.17	6.03	4.50	8.13	8.40	9.37
Final rank		5	13	15	14	3	1	8	12	4	7	6	2	9	10	11
Algorithms		FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.3E+04	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	1.2E+04	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	1.2E+03	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	1.9E+03	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	9	13	12	15	11	5	8	14	6	7	4	1	2	3	10
F_3	mean	1.4E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	2.4E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.4E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.6E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	11	10	14	15	3	1	12	6	9	5	7	13	4	2	8
F_5	mean	1.6E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	8.2E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	1	14	9	13	3	2	5	15	11	4	7	10	8	6	12
F_6	mean	1.3E-01	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	1.8E-01	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	3	12	13	15	1	7	6	14	2	11	4	5	8	9	10
F_7	mean	9.7E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.0E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	9	14	12	15	7	8	11	6	10	13	5	1	2	3	4
F_8	mean	6.8E-14	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	5.7E-14	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	2	12	13	14	9	1	8	7	5	6	10	1	4	3	11
F_9	mean	5.1E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	1.6E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	2	13	14	15	4	7	9	10	6	11	3	1	5	8	12
F_{10}	mean	9.9E+01	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	9.5E+01	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	7	13	15	11	6	2	9	14	8	10	5	1	4	3	12
F_{11}	mean	2.1E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	1.7E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	5	12	13	14	3	4	9	15	6	10	2	1	7	8	11
F_{12}	mean	1.7E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.2E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	4	5	12	14	13	8	7	15	3	2	11	1	9	10	6
F_{13}	mean	7.6E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	3.0E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.1E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	3.9E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	7.1E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	2.5E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	2	7	12	15	3	5	9	13	6	14	11	1	10	8	4
F_{16}	mean	1.5E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.3E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	3	10	15	13	1	4	6	14	2	11	9	5	7	8	12
F_{17}	mean	1.1E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	1.3E+03	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	5	11	2	15	4	3	9	12	7	10	6	1	13	14	8
F_{18}	mean	3.7E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	3.9E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	7.4E-01	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.0E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	6	12	14	15	7	2	9	13	3	11	8	1	5	4	10
F_{20}	mean	1.3E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	2.7E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	7	14	5	15	9	2	11	12	6	13	8	1	3	4	10
F_{21}	mean	1.8E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	1.3E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	6	12	9	15	4	2	5	11	3	10	7	1	13	14	8
F_{22}	mean	5.6E+00	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	8.9E+00	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	2	11	15	14	8	4	13	10	3	12	9	1	5	7	6
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	2.0E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	6	9	8	10	3	4	6	3	4	7	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	3.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	4	12	14	15	3	8	6	2	7	13	5	1	9	10	11
F_{25}	mean	1.5E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	3.6E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	6	13	15	14	4	1	10	11	5	12	2	3	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.4E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	3	12	14	15	1	7	6	9	4	13	2	5	11	10	8
F_{27}	mean	1.3E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	3	12	14	13	8	2	10	15	6	9	5	7	1	4	11
F_{28}	mean	4.0E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	5.1E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	10	13	14	15	3	9	8	2	7	6	1	12	5	4	11
F_{29}	mean	3.8E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	1.4E+02	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	6.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	1.3E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	8	12	14	15	3	9	13	1	10	11	2	6	4	5	7
Mean rank		5.37	11.80	11.53	14.23	5.03	4.47	8.40	10.03	5.63	9.83	5.60	3.67	6.53	6.90	8.97
Final rank		4	14	13	15	3	2	9	12	6	11	5	1	7	8	10
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.2E+05	1.6E+07	1.5E+07	1.1E+08	3.8E+05	5.5E+04	1.1E+05	5.6E+07	2.9E+05	1.9E+05	2.5E+06	9.9E+02	8.3E+06	1.0E+07	4.9E+06
	std	1.6E+05	1.7E+07	1.2E+07	4.1E+07	2.3E+05	4.0E+04	7.1E+04	2.6E+07	2.2E+05	1.2E+05	1.1E+06	2.5E+03	3.6E+06	6.1E+06	4.7E+06
	rank	5	13	12	15	7	2	3	14	6	4	8	1	10	11	9
F_2	mean	3.5E+03	2.9E+09	4.8E+08	9.6E+08	4.9E+01	1.8E-03	5.6E+01	1.6E+04	3.0E+01	2.9E+03	3.5E+03	3.5E-14	3.4E+03	4.9E+03	1.4E+07
	std	2.5E+03	3.1E+09	3.8E+08	8.7E+08	8.0E+01	3.9E-03	1.1E+02	1.4E+04	3.9E+01	1.7E+03	5.7E+03	1.6E-14	3.0E+03	3.5E+03	7.8E+07
	rank	8	15	13	14	4	2	5	11	3	6	9	1	7	10	12
F_3	mean	2.5E+01	2.0E+04	3.1E+03	4.5E+04	2.4E+01	1.2E-02	2.3E+01	3.7E+04	2.6E+01	3.7E+02	4.9E+02	9.7E-14	2.3E+02	3.0E+02	3.0E+02
	std	2.6E+01	2.0E+04	2.2E+03	3.1E+04	3.5E+01	3.0E-02	2.4E+01	2.6E+04	2.7E+01	3.3E+02	3.5E+02	8.7E-14	2.6E+02	2.7E+02	3.7E+02
	rank	5	13	12	15	4	2	3	14	6	10	11	1	7	8	9
F_4	mean	1.1E+01	2.0E+02	1.9E+02	3.7E+02	2.6E+01	3.7E+00	4.3E+00	2.6E+01	4.2E+01	4.5E+01	2.5E+01	1.3E-12	8.8E+01	7.5E+01	7.9E+01
	std	2.4E+01	1.6E+02	5.6E+01	1.4E+02	1.8E+01	1.5E+01	1.6E+01	4.0E-01	3.6E+01	2.9E+01	1.1E+00	1.6E-12	2.7E+01	2.5E+01	3.3E+01
	rank	4	14	13	15	7	2	3	6	8	9	5	1	12	10	11
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	7.0E-02	3.0E-01	7.5E-02	1.6E-01	7.2E-02	5.9E-02	3.4E-01	4.9E-02	1.6E-02	1.9E-04	6.0E-02	5.2E-03	6.8E-02	8.3E-02	1.4E-01
	rank	10	11	4	14	6	5	12	15	2	1	13	3	7	8	9
F_6	mean	2.7E+00	1.9E+01	3.0E+01	3.6E+01	3.7E+00	1.1E+01	6.6E+00	3.9E+01	3.6E+00	2.0E+01	8.3E+00	5.5E+00	1.5E+01	1.6E+01	2.1E+01
	std	1.2E+00	3.1E+00	3.8E+00	5.0E+00	2.3E+00	2.2E+00	2.3E+00	1.4E+00	2.0E+00	2.5E+00	1.6E+00	2.6E+00	1.8E+00	2.0E+00	2.9E+00
	rank	1	10	13	14	3	7	5	15	2	11	6	4	8	9	12
F_7	mean	7.6E-03	2.2E+01	8.0E+00	1.5E+01	7.9E-03	4.7E-03	1.6E-02	5.4E-05	5.3E-03	1.3E-02	1.4E-03	3.2E-03	7.3E-03	6.5E-04	1.8E-03
	std	9.2E-03	1.8E+01	5.4E+00	1.6E+01	1.1E-02	7.7E-03	1.9E-02	2.2E-04	8.2E-03	1.7E-02	4.3E-03	6.1E-03	1.3E-02	1.3E-03	4.6E-03
	rank	9	15	13	14	10	6	12	1	7	11	3	5	8	2	4
F_8	mean	1.4E+01	8.0E+01	1.4E+02	1.5E+02	3.3E+01	9.9E-02	3.1E+01	7.2E+01	9.0E+00	8.8E+00	3.3E+01	2.7E-13	2.1E+00	1.8E+00	6.7E+01
	std	4.3E+00	2.0E+01	2.4E+01	4.6E+01	7.4E+00	3.1E-01	9.6E+00	1.4E+01	3.0E+00	4.3E+00	6.5E+00	5.7E-14	2.1E+00	1.4E+00	1.1E+01
	rank	7	13	14	15	9	2	8	12	6	5	10	1	4	3	11
F_9	mean	4.0E+01	1.4E+02	1.6E+02	2.3E+02	3.7E+01	5.0E+01	5.5E+01	1.6E+02	4.5E+01	1.2E+02	5.9E+01	2.4E+01	6.7E+01	7.1E+01	9.4E+01
	std	1.2E+01	3.1E+01	2.3E+01	7.6E+01	8.4E+00	1.5E+01	1.4E+01	1.5E+01	1.2E+01	2.3E+01	1.2E+01	5.4E+00	1.2E+01	1.2E+01	2.0E+01
	rank	3	12	13	15	2	5	6	14	4	11	7	1	8	9	10
F_{10}	mean	3.9E+02	2.6E+03	4.1E+03	9.7E+02	1.1E+03	5.8E+00	6.6E+02	6.3E+03	3.8E+02	5.1E+02	8.6E+02	2.4E-01	3.6E+01	4.3E+00	2.7E+03
	std	1.8E+02	4.1E+02	7.2E+02	4.7E+02	5.1E+02	1.6E+00	3.7E+02	3.5E+02	1.9E+02	1.7E+02	3.2E+02	7.6E-01	6.3E+01	2.2E+01	5.0E+02
	rank	6	12	14	10	11	3	8	15	5	7	9	1	4	2	13
F_{11}	mean	2.3E+03	3.8E+03	4.4E+03	6.0E+03	2.6E+03	2.0E+03	2.7E+03	7.6E+03	2.3E+03	3.0E+03	2.7E+03	1.7E+03	3.1E+03	3.4E+03	3.4E+03
	std	4.3E+02	6.5E+02	7.5E+02	1.2E+03	4.6E+02	3.3E+02	5.5E+02	2.9E+02	5.2E+02	6.6E+02	4.6E+02	2.1E+02	4.8E+02	4.5E+02	4.5E+02
	rank	4	12	13	14	5	2	6	15	3	8	7	1	9	11	10
F_{12}	mean	3.6E-01	3.1E-01	1.3E+00	1.8E+00	8.6E-01	1.7E-01	6.2E-01	2.5E+00	1.9E-01	3.2E-01	9.4E-01	1.2E-01	6.5E-01	6.1E-01	5.4E-01
	std	2.1E-01	1.1E-01	3.9E-01	4.5E-01	1.9E-01	8.0E-02	3.5E-01	2.2E-01	7.3E-02	1.3E-01	3.2E-01	1.9E-02	1.3E-01	1.3E-01	1.5E-01
	rank	6	4	13	14	11	2	9	15	3	5	12	1	10	8	7
F_{13}	mean	2.7E-01	7.3E-01	5.6E-01	4.7E-01	1.4E-01	2.6E-01	2.9E-01	3.4E-01	2.2E-01	3.7E-01	3.5E-01	3.0E-01	2.8E-01	2.9E-01	4.1E-01
	std	6.1E-02	5.4E-01	1.3E-01	1.1E-01	2.6E-02	6.3E-02	5.9E-02	5.0E-02	6.6E-02	1.0E-01	4.5E-02	4.8E-02	3.6E-02	3.8E-02	6.8E-02
	rank	4	15	14	13	1	3	6	9	2	11	10	8	5	7	12
F_{14}	mean	2.5E-01	8.4E+00	2.0E+00	2.4E+00	3.0E-01	2.6E-01	2.3E-01	3.0E-01	2.4E-01	2.6E-01	2.4E-01	3.0E-01	2.4E-01	2.3E-01	5.4E-01
	std	4.2E-02	1.3E+01	3.6E+00	4.8E+00	1.6E-01	2.5E-02	3.5E-02	3.9E-02	5.0E-02	5.2E-02	2.6E-02	1.4E-01	3.2E-02	2.9E-02	2.4E-01
	rank	6	15	13	14	9	8	1	10	5	7	3	11	4	2	12
F_{15}	mean	4.2E+00	4.7E+02	9.4E+01	1.5E+02	4.4E+00	5.0E+00	5.4E+00	1.5E+01	4.1E+00	4.2E+01	8.8E+00	2.5E+00	1.1E+01	1.1E+01	6.9E+00
	std	1.2E+00	6.8E+02	9.5E+01	1.3E+02	1.7E+00	1.0E+00	1.5E+00	9.6E-01	1.0E+00	1.6E+01	2.2E+00	4.0E-01	2.7E+00	2.1E+00	1.8E+00
	rank	3	15	13	14	4	5	6	11	2	12	8	1	10	9	7
F_{16}	mean	1.0E+01	1.1E+01	1.2E+01	1.3E+01	9.7E+00	9.5E+00	1.0E+01	1.3E+01	1.0E+01	1.1E+01	1.1E+01	9.4E+00	1.1E+01	1.1E+01	1.2E+01
	std	7.8E-01	5.9E-01	5.3E-01	4.6E-01	7.2E-01	8.9E-01	8.8E-01	2.4E-01	7.2E-01	6.3E-01	3.4E-01	3.9E-01	5.0E-01	4.2E-01	4.2E-01
	rank	4	10	13	14	3	2	6	15	5	9	11	1	8	7	12
F_{17}	mean	3.9E+04	9.4E+05	6.5E+05	5.2E+06	7.7E+04	2.2E+04	2.6E+04	1.4E+06	6.3E+04	5.8E+04	2.9E+05	1.4E+03	1.8E+06	1.6E+06	2.6E+05
	std	3.4E+04	1.3E+06	8.9E+05	3.8E+06	5.7E+04	1.7E+04	1.8E+04	6.7E+05	4.7E+04	2.8E+04	1.7E+05	6.5E+02	1.5E+06	1.0E+06	2.9E+05
	rank	4	11	10	15	7	2	3	12	6	5	9	1	14	13	8
F_{18}	mean	2.1E+03	8.4E+04	1.5E+03	1.5E+03	2.7E+03	3.3E+02	3.2E+03	2.1E+05	8.6E+02	1.3E+03	3.8E+02	1.4E+02	9.5E+01	1.1E+02	1.6E+03
	std	2.8E+03	3.0E+05	5.1E+03	2.3E+03	5.4E+03	2.9E+02	4.5E+03	1.6E+05	1.1E+03	1.5E+03	2.2E+02	4.3E+01	2.1E+01	9.8E+01	2.3E+03
	rank	11	14	8	9	12	4	13	15	6	7	5	3	1	2	10
F_{19}	mean	7.3E+00	3.2E+01	2.5E+01	7.2E+01	7.2E+00	6.9E+00	6.8E+00	1.8E+01	5.1E+00	1.4E+01	1.2E+01	4.9E+00	1.2E+01	1.1E+01	9.5E+00
	std	1.1E+00	3.5E+01	1.5E+01	1.0E+02	2.3E+00	1.5E+00	1.3E+00	8.5E-01	1.1E+00	3.8E+00	9.7E-01	9.2E-01	2.0E+00	2.0E+00	1.8E+00
	rank	6	14	13	15	5	4	3	12	2	11	10	1	9	8	7
F_{20}	mean	2.0E+02	5.2E+03	8.1E+02	6.3E+04	3.8E+02	1.8E+02	7.4E+02	2.0E+04	4.9E+02	2.0E+03	1.5E+03	1.1E+02	4.5E+03	5.1E+03	3.2E+03
	std	1.5E+02	8.9E+03	5.7E+02	4.0E+04	2.1E+02	6.9E+01	5.2E+02	7.8E+03	4.3E+02	1.3E+03	6.2E+02	6.1E+01	2.3E+03	2.8E+03	1.8E+03
	rank	3	13	7	15	4	2	6	14	5	9	8	1	11	12	10
F_{21}	mean	1.4E+04	2.1E+05	4.2E+04	2.2E+06	2.3E+04	1.4E+04	2.4E+04	4.9E+05	3.1E+04	4.0E+04	7.3E+04	6.6E+02	8.7E+04	9.0E+04	1.1E+05
	std	9.7E+03	2.3E+05	5.7E+04	6.4E+06	1.6E+04	1.0E+04	1.5E+04	2.6E+05	3.3E+04	2.6E+04	5.5E+04	2.5E+02	6.5E+04	8.0E+04	9.8E+04
	rank	3	13	8	15	4	2	5	14	6	7	9	1	10	11	12
F_{22}	mean	1.6E+02	4.9E+02	7.1E+02	8.4E+02	2.3E+02	2.6E+02	3.1E+02	4.1E+02	2.3E+02	5.2E+02	1.5E+02	2.1E+02	2.6E+02	2.7E+02	2.9E+02
	std	8.1E+01	2.2E+02	2.4E+02	2.1E+02	8.7E+01	1.2E+02	1.2E+02	1.5E+02	1.0E+02	1.6E+02	6.7E+01	9.5E+01	9.0E+01	7.9E+01	1.2E+02
	rank	2	12	14	15	5	6	10	11	4	13	1	3	7	8	9
F_{23}	mean	3.2E+02	3.3E+02	3.2E+02	3.3E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02
	std	2.2E-12	1.2E+01	6.5E+00	9.8E+00	6.0E-13	4.3E-13	1.6E-12	1.2E-06	2.2E-12	1.8E-11	8.1E-08	2.2E-13	3.1E-02	9.6E-03	6.5E-01
	rank	11	15	13	14	1	8	9	4	10	2	3	7	6	5	12
F_{24}	mean	2.2E+02	2.5E+02	2.5E+02	2.4E+02	2.3E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.3E+02	2.3E+02
	std	1.5E+00	1.0E+01	9.1E+00	3.8E+01	7.0E+00	1.5E+00	1.6E+00	1.3E+00	3.8E+00	3.5E+00	9.9E-01	5.5E+00	8.1E-01	8.3E-01	7.1E+00
	rank	2	14	15	13	9	8	5	1	7	11	3	10	4	6	12
F_{25}	mean	2.0E+02	2.1E+02	2.2E+02	2.2E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02	2.1E+02	2.2E+02	2.0E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02
	std	8.0E-01	4.2E+00	5.9E+00	4.7E+00	1.2E-01	1.9E+00	2.5E+00	2.1E+00	1.5E+00	4.0E+00	3.4E-01	1.5E+00	1.8E+00	2.1E+00	2.2E+00
	rank	3	10	15	14	1	9	6	8	5	13	2	4	12	11	7
F_{26}	mean	1.1E+02	1.1E+02	1.8E+02	1.9E+02	1.4E+02	1.0E+02	1.8E+02	1.0E+02	1.1E+02	1.2E+02	1.0E+02	1.0E+02	1.1E+02	1.1E+02	1.0E+02
	std	3.4E+01	3.0E+01	4.2E+01	2.8E+01	5.0E+01	7.6E-02	4.4E+01	3.4E-02	2.5E+01	3.6E+01	4.9E-02	5.3E-02	3.4E+01	3.0E+01	1.3E-01
	rank	9	8	14	15	12	2	13	4	6	11	3	1	10	7	5
F_{27}	mean	3.8E+02	8.2E+02	8.1E+02	1.3E+03	4.4E+02	4.0E+02	4.8E+02	1.3E+03	4.0E+02	7.1E+02	4.2E+02	4.1E+02	4.3E+02	4.3E+02	6.8E+02
	std	3.1E+01	1.9E+02	3.8E+02	5.2E+02	4.8E+01	3.2E+00	8.6E+01	3.5E+01	2.1E+01	2.9E+02	3.8E+01	5.7E+01	2.0E+01	1.8E+01	2.7E+02
	rank	1	13	12	15	8	3	9	14	2	11	5	4	7	6	10
F_{28}	mean	8.1E+02	1.2E+03	3.9E+03	5.1E+03	4.1E+02	9.6E+02	1.5E+03	3.9E+02	9.0E+02	7.7E+02	3.9E+02	8.7E+02	8.8E+02	8.7E+02	1.2E+03
	std	6.0E+01	2.1E+02	1.2E+03	1.7E+03	2.7E+01	4.8E+01	7.2E+02	4.6E+00	5.6E+01	3.3E+02	4.0E+00	4.3E+01	3.1E+02	2.3E+02	2.5E+02
	rank	5	11	14	15	3	10	13	2	9	4	1	6	8	7	12
F_{29}	mean	1.7E+03	3.1E+06	1.3E+07	4.7E+06	2.1E+02	9.7E+02	1.4E+03	2.2E+02	1.3E+03	2.2E+02	2.1E+02	7.7E+02	1.1E+03	1.1E+03	4.1E+06
	std	5.5E+02	4.0E+06	1.8E+07	1.0E+07	1.2E+00	2.6E+02	4.9E+02	1.2E+00	3.1E+02	1.6E+01	1.5E+00	3.2E+01	1.9E+02	1.9E+02	3.7E+06
	rank	11	12	15	14	1	6	10	4	9	3	2	5	8	7	13
F_{30}	mean	2.3E+03	2.9E+04	1.8E+04	8.3E+04	7.7E+02	2.1E+03	1.9E+03	9.4E+02	1.8E+03	8.4E+02	6.3E+02	2.4E+03	6.1E+02	6.5E+02	1.4E+04
	std	5.9E+02	3.3E+04	4.2E+04	8.5E+04	1.8E+02	6.6E+02	6.1E+02	1.8E+02	6.1E+02	2.4E+02	1.5E+02	1.0E+03	1.3E+02	1.5E+02	2.6E+04
	rank	10	14	13	15	4	9	8	6	7	5	2	11	1	3	12
Mean rank		5.53	12.43	12.37	14.10	5.73	4.43	7.07	10.43	5.10	7.97	6.50	3.33	7.63	7.40	9.97
Final rank		4	14	13	15	5	2	7	12	3	10	6	1	9	8	11
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	6.9E+05	1.0E+08	5.6E+07	8.0E+07	1.2E+06	3.4E+05	4.2E+05	3.3E+08	7.9E+05	6.5E+05	9.8E+06	3.0E+04	1.6E+07	1.4E+07	2.2E+07
	std	2.4E+05	9.7E+07	5.4E+07	3.4E+07	5.5E+05	1.3E+05	2.0E+05	8.9E+07	2.4E+05	1.6E+05	3.6E+06	7.8E+04	7.6E+06	4.1E+06	1.9E+07
	rank	5	14	12	13	7	2	3	15	6	4	8	1	10	9	11
F_2	mean	1.0E+04	1.3E+10	3.4E+09	2.7E+08	9.9E+03	3.0E+03	6.3E+03	1.5E+06	5.4E+03	5.0E+03	1.1E+03	2.3E-10	1.5E+03	2.5E+03	4.9E+03
	std	8.5E+03	5.4E+09	1.6E+09	2.9E+08	1.0E+04	3.1E+03	6.1E+03	1.6E+06	5.6E+03	6.5E+03	1.5E+03	4.9E-10	1.4E+03	2.5E+03	8.1E+03
	rank	11	15	14	13	10	5	9	12	8	7	2	1	3	4	6
F_3	mean	2.6E+02	7.7E+04	2.6E+04	1.8E+05	5.4E+02	6.9E+00	6.0E+02	2.9E+05	1.0E+03	2.4E+03	3.4E+03	1.5E+00	3.9E+03	3.5E+03	3.6E+03
	std	2.4E+02	3.7E+04	1.1E+04	3.8E+04	4.7E+02	7.9E+00	5.4E+02	9.8E+04	8.1E+02	1.2E+03	1.2E+03	5.2E+00	2.4E+03	2.0E+03	2.2E+03
	rank	3	13	12	14	4	2	5	15	6	7	8	1	11	9	10
F_4	mean	9.6E+01	1.2E+03	4.7E+02	4.2E+02	4.3E+01	4.1E+01	5.7E+01	4.6E+01	8.8E+01	8.6E+01	4.4E+01	1.4E+01	1.5E+02	1.4E+02	1.2E+02
	std	3.9E+00	9.6E+02	1.4E+02	8.0E+01	1.1E+01	3.4E+01	2.9E+01	8.5E-01	2.5E+01	4.0E+01	1.7E+00	3.2E+01	2.3E+01	3.1E+01	4.6E+01
	rank	9	15	14	13	3	2	6	5	8	7	4	1	12	11	10
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	5.0E-02	3.0E-01	1.2E-01	1.2E-01	5.7E-02	4.9E-02	3.3E-01	4.4E-02	4.5E-03	2.6E-04	6.3E-02	3.6E-03	7.3E-02	6.8E-02	2.0E-01
	rank	10	13	6	14	5	4	11	15	2	1	12	3	7	9	8
F_6	mean	6.8E+00	4.0E+01	6.0E+01	7.1E+01	1.1E+01	2.6E+01	1.9E+01	7.1E+01	1.3E+01	4.1E+01	2.3E+01	1.5E+01	3.4E+01	3.5E+01	4.8E+01
	std	2.3E+00	4.0E+00	3.1E+00	5.1E+00	3.4E+00	4.1E+00	5.2E+00	1.5E+00	4.1E+00	4.4E+00	2.2E+00	4.3E+00	4.8E+00	4.5E+00	4.3E+00
	rank	1	10	13	14	2	7	5	15	3	11	6	4	8	9	12
F_7	mean	8.9E-03	1.2E+02	3.4E+01	2.6E+00	9.2E-03	1.1E-02	5.3E-03	6.9E-05	5.3E-03	8.9E-03	3.1E-03	4.3E-03	1.1E-02	8.8E-03	5.3E-01
	std	9.4E-03	7.9E+01	1.8E+01	1.1E+00	9.3E-03	1.4E-02	6.8E-03	2.8E-04	7.0E-03	6.8E-03	8.1E-03	6.8E-03	5.6E-03	6.9E-03	2.9E+00
	rank	7	15	14	13	9	11	5	1	4	8	2	3	10	6	12
F_8	mean	4.9E+01	2.0E+02	3.2E+02	3.0E+02	7.1E+01	9.5E-01	7.3E+01	2.2E+02	1.5E+01	2.2E+01	8.2E+01	1.7E-01	8.7E+00	9.7E+00	1.7E+02
	std	9.9E+00	3.8E+01	3.7E+01	7.9E+01	1.3E+01	8.2E-01	2.0E+01	2.0E+01	4.9E+00	6.3E+00	1.5E+01	5.9E-01	4.2E+00	4.3E+00	2.9E+01
	rank	7	12	15	14	8	2	9	13	5	6	10	1	3	4	11
F_9	mean	7.7E+01	3.1E+02	3.9E+02	3.9E+02	7.6E+01	1.0E+02	1.3E+02	3.6E+02	9.5E+01	2.5E+02	1.5E+02	6.5E+01	1.9E+02	2.2E+02	2.0E+02
	std	1.7E+01	5.4E+01	3.9E+01	1.5E+02	1.7E+01	1.9E+01	3.2E+01	1.7E+01	2.0E+01	3.7E+01	3.3E+01	1.4E+01	3.1E+01	2.8E+01	3.0E+01
	rank	3	12	15	14	2	5	6	13	4	11	7	1	8	10	9
F_{10}	mean	1.6E+03	5.8E+03	8.3E+03	3.2E+03	3.1E+03	2.2E+01	1.6E+03	1.3E+04	8.5E+02	1.1E+03	2.8E+03	4.0E+00	9.3E+01	8.9E+01	6.0E+03
	std	3.3E+02	8.5E+02	9.8E+02	1.6E+03	8.7E+02	4.4E+01	4.4E+02	4.4E+02	2.2E+02	2.4E+02	4.5E+02	2.2E+01	1.0E+02	8.7E+01	7.2E+02
	rank	7	12	14	11	10	2	8	15	5	6	9	1	4	3	13
F_{11}	mean	4.9E+03	7.6E+03	9.1E+03	1.2E+04	5.2E+03	4.0E+03	5.9E+03	1.4E+04	5.0E+03	5.6E+03	6.1E+03	3.9E+03	8.8E+03	8.7E+03	6.5E+03
	std	8.8E+02	7.9E+02	9.6E+02	2.3E+03	7.6E+02	4.9E+02	9.6E+02	3.5E+02	8.7E+02	6.5E+02	9.8E+02	3.4E+02	8.5E+02	7.0E+02	5.8E+02
	rank	3	10	13	14	5	2	7	15	4	6	8	1	12	11	9
F_{12}	mean	6.0E-01	4.1E-01	2.0E+00	2.6E+00	1.2E+00	1.4E-01	9.6E-01	3.4E+00	2.0E-01	3.2E-01	1.5E+00	1.1E-01	1.0E+00	1.0E+00	8.5E-01
	std	2.9E-01	1.5E-01	4.1E-01	1.2E+00	1.8E-01	4.8E-02	3.5E-01	2.1E-01	5.4E-02	9.9E-02	2.8E-01	1.4E-02	2.2E-01	1.6E-01	1.9E-01
	rank	6	5	13	14	11	2	8	15	3	4	12	1	9	10	7
F_{13}	mean	3.7E-01	1.1E+00	6.4E-01	5.5E-01	2.4E-01	3.9E-01	4.5E-01	4.7E-01	3.8E-01	4.9E-01	4.9E-01	3.7E-01	4.7E-01	4.8E-01	4.6E-01
	std	8.2E-02	9.1E-01	1.2E-01	8.0E-02	5.2E-02	7.5E-02	6.9E-02	4.6E-02	9.3E-02	6.8E-02	4.2E-02	6.5E-02	4.1E-02	4.6E-02	6.8E-02
	rank	3	15	14	13	1	5	6	9	4	11	12	2	8	10	7
F_{14}	mean	3.2E-01	3.0E+01	3.7E+00	3.0E-01	3.8E-01	3.0E-01	4.1E-01	3.5E-01	3.1E-01	3.1E-01	2.7E-01	3.6E-01	3.3E-01	3.2E-01	4.5E-01
	std	1.5E-01	2.4E+01	6.3E+00	4.2E-02	2.1E-01	4.1E-02	2.4E-01	3.9E-02	9.6E-02	3.4E-02	2.4E-02	1.4E-01	2.7E-02	2.3E-02	1.9E-01
	rank	7	15	14	3	11	2	12	9	5	4	1	10	8	6	13
F_{15}	mean	7.8E+00	3.3E+04	8.8E+02	5.8E+02	8.5E+00	1.3E+01	1.2E+01	3.2E+01	1.1E+01	1.1E+02	2.2E+01	7.6E+00	3.2E+01	2.9E+01	2.4E+01
	std	2.2E+00	4.8E+04	8.6E+02	8.3E+02	3.4E+00	3.7E+00	3.4E+00	1.3E+00	3.5E+00	3.2E+01	4.5E+00	1.1E+00	5.8E+00	4.9E+00	7.6E+00
	rank	2	15	14	13	3	6	5	11	4	12	7	1	10	9	8
F_{16}	mean	1.9E+01	2.1E+01	2.2E+01	2.2E+01	1.9E+01	1.8E+01	1.9E+01	2.3E+01	1.9E+01	2.0E+01	2.1E+01	1.8E+01	2.1E+01	2.1E+01	2.1E+01
	std	7.8E-01	5.4E-01	5.6E-01	4.7E-01	8.8E-01	5.1E-01	6.9E-01	2.1E-01	8.5E-01	7.4E-01	4.6E-01	3.8E-01	3.7E-01	4.9E-01	6.0E-01
	rank	5	8	13	14	4	2	6	15	3	7	11	1	10	9	12
F_{17}	mean	8.3E+04	7.8E+06	1.9E+06	8.1E+06	2.4E+05	4.4E+04	5.1E+04	1.1E+07	1.1E+05	1.2E+05	1.6E+06	4.0E+03	2.7E+06	2.4E+06	1.0E+06
	std	5.0E+04	1.3E+07	3.6E+06	5.5E+06	1.5E+05	1.8E+04	2.3E+04	3.8E+06	7.1E+04	7.4E+04	6.3E+05	6.6E+03	3.1E+06	2.1E+06	9.0E+05
	rank	4	13	10	14	7	2	3	15	5	6	9	1	12	11	8
F_{18}	mean	1.9E+03	1.6E+08	1.9E+07	2.2E+05	6.3E+03	3.8E+02	1.5E+03	6.2E+05	6.9E+02	2.0E+03	7.5E+02	2.6E+02	8.6E+02	9.3E+02	2.6E+03
	std	1.6E+03	4.8E+08	7.2E+07	9.6E+05	7.9E+03	2.2E+02	1.0E+03	1.1E+06	6.8E+02	2.5E+03	3.9E+02	2.6E+02	8.2E+02	7.1E+02	1.4E+03
	rank	8	15	14	12	11	2	7	13	3	9	4	1	5	6	10
F_{19}	mean	1.7E+01	7.9E+01	8.0E+01	2.4E+02	2.1E+01	1.5E+01	1.4E+01	3.3E+01	1.2E+01	3.8E+01	2.5E+01	1.2E+01	2.4E+01	2.1E+01	2.8E+01
	std	1.9E+00	5.8E+01	2.3E+01	7.2E+02	2.7E+00	2.8E+00	2.6E+00	6.1E-01	2.3E+00	2.6E+01	1.7E+00	2.8E+00	1.6E+01	1.2E+01	7.5E+00
	rank	5	13	14	15	6	4	3	11	2	12	9	1	8	7	10
F_{20}	mean	2.6E+02	2.1E+04	6.0E+03	2.3E+05	5.0E+02	3.0E+02	6.7E+02	7.1E+04	7.0E+02	1.7E+03	2.2E+03	3.5E+02	1.3E+04	1.3E+04	3.9E+03
	std	6.8E+01	2.5E+04	5.0E+03	2.5E+05	1.8E+02	8.9E+01	2.4E+02	3.5E+04	5.3E+02	1.5E+03	9.6E+02	1.5E+02	5.0E+03	5.7E+03	2.9E+03
	rank	1	13	10	15	4	2	5	14	6	7	8	3	11	12	9
F_{21}	mean	5.4E+04	2.0E+06	6.0E+05	7.7E+06	9.4E+04	3.8E+04	3.9E+04	4.4E+06	1.3E+05	9.2E+04	6.4E+05	1.8E+03	1.5E+06	1.4E+06	2.5E+05
	std	2.9E+04	2.9E+06	8.4E+05	7.2E+06	4.9E+04	2.5E+04	1.6E+04	1.5E+06	7.1E+04	7.4E+04	3.3E+05	1.6E+03	9.4E+05	1.1E+06	1.6E+05
	rank	4	13	9	15	6	2	3	14	7	5	10	1	12	11	8
F_{22}	mean	5.2E+02	1.3E+03	1.7E+03	1.8E+03	4.9E+02	8.3E+02	7.5E+02	1.7E+03	7.3E+02	1.4E+03	5.4E+02	5.9E+02	7.8E+02	7.4E+02	9.0E+02
	std	2.3E+02	3.1E+02	4.3E+02	5.4E+02	2.6E+02	2.2E+02	2.5E+02	1.4E+02	2.5E+02	4.1E+02	1.6E+02	1.6E+02	1.7E+02	1.7E+02	2.4E+02
	rank	2	11	14	15	1	9	7	13	5	12	3	4	8	6	10
F_{23}	mean	3.4E+02	3.9E+02	3.7E+02	3.6E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02
	std	7.3E-12	4.1E+01	1.5E+01	6.3E+00	1.2E-12	6.8E-13	5.0E-12	3.4E-06	8.7E-13	7.6E-11	5.4E-07	5.0E-13	2.0E-02	5.2E-03	6.9E-01
	rank	11	15	14	13	1	8	10	4	9	2	3	7	6	5	12
F_{24}	mean	2.7E+02	3.3E+02	3.3E+02	2.9E+02	2.8E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.8E+02	2.6E+02	2.6E+02	2.8E+02
	std	4.4E+00	3.8E+01	1.4E+01	2.1E+01	2.7E+00	5.5E+00	5.7E+00	6.6E-06	4.5E+00	4.8E+00	4.5E+00	3.3E+00	4.5E+00	4.6E+00	3.9E+00
	rank	6	15	14	13	11	2	8	5	7	4	9	10	3	1	12
F_{25}	mean	2.1E+02	2.2E+02	2.5E+02	2.4E+02	2.0E+02	2.1E+02	2.2E+02	2.3E+02	2.1E+02	2.4E+02	2.0E+02	2.1E+02	2.3E+02	2.3E+02	2.2E+02
	std	2.1E+00	1.3E+01	9.1E+00	1.4E+01	7.8E-02	3.0E+00	4.5E+00	5.2E+00	3.6E+00	6.5E+00	4.5E-01	5.4E+00	3.3E+00	4.0E+00	5.5E+00
	rank	3	9	15	14	1	6	8	10	5	13	2	4	12	11	7
F_{26}	mean	1.8E+02	1.9E+02	2.0E+02	2.0E+02	1.6E+02	1.2E+02	1.7E+02	1.0E+02	1.5E+02	1.8E+02	1.1E+02	1.0E+02	1.5E+02	1.5E+02	1.1E+02
	std	4.3E+01	1.2E+02	1.0E+00	4.9E+00	5.0E+01	3.7E+01	6.3E+01	5.0E-02	5.0E+01	4.1E+01	3.1E+01	2.0E-01	5.0E+01	5.1E+01	2.6E+01
	rank	11	13	14	15	9	5	10	1	6	12	4	2	8	7	3
F_{27}	mean	5.0E+02	1.4E+03	2.0E+03	3.1E+03	6.4E+02	9.1E+02	8.5E+02	2.1E+03	6.1E+02	1.5E+03	9.2E+02	6.9E+02	9.8E+02	9.6E+02	1.6E+03
	std	5.7E+01	1.1E+02	3.2E+02	9.2E+02	7.5E+01	2.7E+02	1.4E+02	3.3E+01	1.1E+02	1.4E+02	6.1E+01	7.6E+01	1.4E+02	1.4E+02	1.3E+02
	rank	1	10	13	15	3	6	5	14	2	11	7	4	9	8	12
F_{28}	mean	1.1E+03	2.1E+03	8.3E+03	1.1E+04	3.9E+02	1.8E+03	2.7E+03	4.0E+02	1.7E+03	1.6E+03	3.9E+02	1.3E+03	7.2E+02	9.7E+02	2.3E+03
	std	5.8E+01	5.7E+02	2.5E+03	4.0E+03	1.5E+01	1.9E+02	8.2E+02	1.9E+01	3.7E+02	1.4E+03	1.3E+01	2.2E+02	4.7E+02	9.3E+02	7.6E+02
	rank	6	11	14	15	1	10	13	3	9	8	2	7	4	5	12
F_{29}	mean	2.5E+03	3.0E+07	6.3E+07	1.3E+08	2.2E+02	1.4E+03	1.4E+03	2.4E+02	1.3E+03	2.3E+02	2.3E+02	8.9E+02	1.1E+03	1.1E+03	4.6E+07
	std	6.0E+02	1.9E+07	8.0E+07	5.9E+08	1.7E+01	3.1E+02	3.3E+02	4.9E+00	3.8E+02	1.6E+01	1.6E+00	1.0E+02	1.9E+02	2.9E+02	1.6E+07
	rank	11	12	14	15	1	9	10	4	8	3	2	5	6	7	13
F_{30}	mean	9.4E+03	5.9E+04	1.5E+05	2.1E+05	1.2E+03	9.7E+03	1.2E+04	2.1E+03	1.1E+04	1.8E+03	1.1E+03	1.0E+04	1.3E+03	1.3E+03	3.5E+04
	std	6.7E+02	4.5E+04	2.0E+05	1.2E+05	2.9E+02	9.9E+02	1.2E+03	2.0E+02	1.2E+03	3.4E+02	1.8E+02	1.5E+03	2.6E+02	2.3E+02	2.5E+04
	rank	7	13	14	15	2	8	11	6	10	5	1	9	3	4	12
Mean rank		5.63	12.50	13.10	13.50	5.30	4.53	7.13	10.67	5.27	7.37	6.40	3.07	8.00	7.67	9.87
Final rank		5	13	14	15	4	2	7	12	3	8	6	1	10	9	11
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	2.0E+06	4.5E+08	3.2E+08	1.7E+08	2.7E+06	9.8E+05	1.1E+06	3.6E+09	4.7E+06	2.8E+06	6.7E+07	2.6E+05	1.6E+08	1.6E+08	1.1E+08
	std	4.1E+05	2.4E+08	1.2E+08	8.8E+07	1.0E+06	3.0E+05	5.6E+05	8.1E+08	1.5E+06	7.1E+05	2.7E+07	1.9E+05	3.3E+07	4.0E+07	7.3E+07
	rank	4	14	13	12	5	2	3	15	7	6	8	1	10	11	9
F_2	mean	2.4E+04	7.3E+10	1.4E+10	8.2E+06	1.9E+04	2.0E+04	1.6E+04	4.1E+06	1.2E+04	1.4E+04	1.1E+04	3.0E-09	1.2E+04	1.4E+04	3.0E+04
	std	3.0E+04	2.2E+10	3.4E+09	1.1E+07	1.9E+04	1.5E+04	1.5E+04	1.3E+07	1.2E+04	1.1E+04	1.7E+04	5.1E-09	6.9E+03	6.5E+03	2.9E+04
	rank	10	15	14	13	8	9	7	12	4	5	2	1	3	6	11
F_3	mean	1.5E+02	1.8E+05	1.0E+05	3.8E+05	7.4E+02	1.9E-01	3.8E+02	1.1E+06	1.7E+03	2.3E+03	4.1E+03	9.2E-01	9.7E+03	8.0E+03	3.0E+03
	std	9.6E+01	8.1E+04	2.0E+04	1.3E+05	9.0E+02	2.5E-01	3.7E+02	2.8E+05	1.6E+03	1.5E+03	1.4E+03	2.9E+00	5.0E+03	3.4E+03	2.2E+03
	rank	3	13	12	14	5	1	4	15	6	7	9	2	11	10	8
F_4	mean	1.8E+02	8.5E+03	2.0E+03	5.0E+02	1.0E+02	1.4E+02	1.8E+02	9.8E+01	2.1E+02	2.0E+02	9.5E+01	1.3E+02	3.3E+02	3.4E+02	3.3E+02
	std	3.3E+01	4.4E+03	4.3E+02	5.9E+01	2.0E+01	5.2E+01	3.7E+01	9.2E-01	4.7E+01	4.9E+01	2.2E+00	4.7E+01	5.6E+01	4.0E+01	1.1E+02
	rank	6	15	14	13	3	5	7	2	9	8	1	4	10	12	11
F_5	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	1.9E-01	9.0E-02	9.6E-02	3.2E-01	4.0E-02	1.0E-01	2.8E-01	3.2E-02	3.2E-04	3.5E-01	3.9E-02	3.9E-03	3.6E-02	4.6E-02	1.0E-01
	rank	8	14	6	13	5	3	11	15	1	4	9	2	12	10	7
F_6	mean	2.1E+01	9.9E+01	1.4E+02	1.5E+02	4.8E+01	7.6E+01	6.5E+01	1.6E+02	6.1E+01	1.0E+02	7.7E+01	5.8E+01	1.1E+02	1.1E+02	1.2E+02
	std	3.9E+00	6.9E+00	6.5E+00	9.4E+00	7.2E+00	6.9E+00	7.2E+00	1.5E+00	6.8E+00	4.3E+00	4.6E+00	6.6E+00	8.8E+00	7.9E+00	5.8E+00
	rank	1	8	13	14	2	6	5	15	4	9	7	3	11	10	12
F_7	mean	3.7E-03	7.4E+02	1.2E+02	1.1E+00	4.9E-03	9.8E-03	3.8E-03	2.2E-02	2.5E-03	1.2E-03	2.2E-03	9.8E-03	1.1E-02	8.0E-03	5.6E-12
	std	5.5E-03	2.4E+02	3.5E+01	1.8E-01	9.0E-03	2.7E-02	7.3E-03	7.4E-03	4.6E-03	3.0E-03	2.6E-03	1.4E-02	1.8E-02	1.1E-02	3.1E-12
	rank	5	15	14	13	7	9	6	12	4	2	3	10	11	8	1
F_8	mean	1.6E+02	6.2E+02	8.3E+02	6.6E+02	2.0E+02	9.0E-01	2.2E+02	7.6E+02	3.2E+01	7.7E+01	2.4E+02	9.6E-01	4.0E+01	4.4E+01	4.8E+02
	std	1.7E+01	6.2E+01	6.4E+01	8.8E+01	3.7E+01	9.2E-01	5.0E+01	3.0E+01	1.1E+01	3.1E+01	2.6E+01	1.9E+00	9.1E+00	9.3E+00	5.5E+01
	rank	7	12	15	13	8	1	9	14	3	6	10	2	4	5	11
F_9	mean	1.8E+02	8.8E+02	1.0E+03	1.0E+03	1.9E+02	3.5E+02	3.1E+02	9.6E+02	2.9E+02	6.7E+02	4.9E+02	2.1E+02	8.3E+02	8.3E+02	5.6E+02
	std	3.9E+01	1.1E+02	6.5E+01	3.6E+02	4.8E+01	6.0E+01	4.9E+01	2.6E+01	5.1E+01	5.1E+01	1.3E+02	4.0E+01	6.5E+01	6.5E+01	6.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_{10}	mean	6.7E+03	1.5E+04	2.1E+04	1.0E+04	8.9E+03	2.9E+01	4.7E+03	3.1E+04	1.9E+03	2.4E+03	7.6E+03	2.8E+01	3.9E+02	4.3E+02	1.4E+04
	std	1.0E+03	1.8E+03	1.1E+03	3.2E+03	2.0E+03	4.8E+01	6.4E+02	7.9E+02	4.0E+02	5.3E+02	1.1E+03	6.0E+01	2.0E+02	1.8E+02	1.3E+03
	rank	8	13	14	11	10	2	7	15	5	6	9	1	3	4	12
F_{11}	mean	1.2E+04	1.7E+04	2.2E+04	2.8E+04	1.3E+04	1.1E+04	1.3E+04	3.3E+04	1.2E+04	1.3E+04	1.8E+04	1.1E+04	2.6E+04	2.6E+04	1.5E+04
	std	1.5E+03	1.3E+03	1.3E+03	4.6E+03	1.5E+03	6.7E+02	1.7E+03	6.1E+02	1.3E+03	1.1E+03	2.1E+03	5.5E+02	9.9E+02	1.1E+03	1.4E+03
	rank	3	9	11	14	5	2	7	15	4	6	10	1	12	13	8
F_{12}	mean	1.0E+00	1.7E+00	2.8E+00	3.7E+00	1.8E+00	2.1E-01	1.6E+00	4.0E+00	2.8E-01	4.5E-01	2.1E+00	1.8E-01	2.3E+00	2.3E+00	1.5E+00
	std	4.8E-01	1.4E+00	4.8E-01	7.6E-01	2.1E-01	5.5E-02	4.7E-01	2.2E-01	5.9E-02	8.9E-02	2.5E-01	1.6E-02	2.5E-01	3.2E-01	2.8E-01
	rank	5	8	13	14	9	2	7	15	3	4	10	1	11	12	6
F_{13}	mean	5.3E-01	3.7E+00	6.0E-01	5.4E-01	4.2E-01	4.6E-01	6.4E-01	6.9E-01	5.0E-01	5.0E-01	6.1E-01	5.0E-01	5.0E-01	5.1E-01	5.5E-01
	std	5.4E-02	1.2E+00	6.6E-02	6.4E-02	5.4E-02	5.4E-02	6.4E-02	5.7E-02	6.9E-02	5.6E-02	6.0E-02	7.6E-02	4.2E-02	4.4E-02	5.3E-02
	rank	8	15	11	9	1	2	13	14	4	3	12	5	6	7	10
F_{14}	mean	4.6E-01	2.1E+02	4.6E+01	3.5E-01	4.3E-01	3.4E-01	3.5E-01	4.2E-01	3.2E-01	3.2E-01	3.2E-01	3.6E-01	3.4E-01	3.4E-01	6.3E-01
	std	2.7E-01	7.6E+01	1.7E+01	3.5E-02	2.0E-01	2.7E-02	1.3E-01	1.3E-01	3.5E-02	3.0E-02	2.8E-02	1.1E-01	2.4E-02	2.8E-02	2.2E-01
	rank	12	15	14	8	11	4	7	10	1	3	2	9	5	6	13
F_{15}	mean	1.9E+01	1.5E+06	2.1E+04	7.2E+02	2.6E+01	4.3E+01	4.4E+01	8.4E+01	4.5E+01	3.3E+02	7.4E+01	3.9E+01	1.0E+02	1.1E+02	1.8E+02
	std	3.3E+00	1.5E+06	1.4E+04	1.0E+03	1.1E+01	6.7E+00	7.9E+00	3.2E+00	1.1E+01	5.2E+01	1.1E+01	7.5E+00	1.6E+01	1.2E+01	1.1E+02
	rank	1	15	14	13	2	4	5	8	6	12	7	3	9	10	11
F_{16}	mean	4.1E+01	4.4E+01	4.5E+01	4.7E+01	4.2E+01	4.1E+01	4.3E+01	4.8E+01	4.1E+01	4.3E+01	4.5E+01	4.0E+01	4.6E+01	4.6E+01	4.5E+01
	std	1.1E+00	1.3E+00	6.9E-01	5.2E-01	9.9E-01	7.5E-01	1.2E+00	2.0E-01	1.2E+00	8.3E-01	5.2E-01	7.2E-01	4.4E-01	5.2E-01	5.8E-01
	rank	3	8	11	14	5	2	6	15	4	7	10	1	12	13	9
F_{17}	mean	3.5E+05	2.4E+07	1.7E+07	3.5E+07	1.0E+06	1.9E+05	4.0E+05	1.1E+08	7.3E+05	4.6E+05	9.3E+06	3.5E+04	1.7E+07	1.9E+07	7.3E+06
	std	1.2E+05	2.7E+07	5.8E+06	2.1E+07	3.4E+05	6.0E+04	1.8E+05	2.4E+07	2.9E+05	1.7E+05	4.5E+06	3.2E+04	6.6E+06	8.3E+06	5.0E+06
	rank	3	13	10	14	7	2	4	15	6	5	9	1	11	12	8
F_{18}	mean	3.4E+03	9.5E+08	2.1E+08	1.4E+03	3.1E+03	6.9E+02	2.3E+03	4.2E+06	2.1E+03	1.5E+03	1.7E+03	1.5E+03	6.4E+02	1.0E+03	4.7E+03
	std	2.6E+03	1.6E+09	1.6E+08	1.5E+03	3.6E+03	3.1E+02	2.3E+03	4.7E+06	2.4E+03	1.4E+03	8.0E+02	1.6E+03	4.6E+02	1.1E+03	4.5E+03
	rank	11	15	14	4	10	2	9	13	8	5	7	6	1	3	12
F_{19}	mean	1.0E+02	5.6E+02	3.4E+02	1.8E+02	6.3E+01	7.1E+01	8.2E+01	7.4E+01	1.0E+02	9.1E+01	5.9E+01	9.5E+01	1.4E+02	1.3E+02	1.2E+02
	std	3.1E+00	1.5E+02	1.2E+02	7.6E+01	1.9E+01	2.7E+01	3.6E+01	1.9E+00	1.6E+01	3.5E+01	2.4E+00	2.4E+01	2.7E+01	2.5E+01	1.1E+01
	rank	9	15	14	13	2	3	5	4	8	6	1	7	12	11	10
F_{20}	mean	5.6E+02	1.1E+05	3.7E+04	1.1E+06	1.1E+03	5.2E+02	1.4E+03	5.3E+05	1.8E+03	2.6E+03	5.0E+03	6.8E+03	5.2E+04	4.9E+04	6.8E+03
	std	1.2E+02	6.0E+04	1.9E+04	8.9E+05	3.7E+02	7.8E+01	5.6E+02	1.2E+05	8.0E+02	7.9E+02	1.4E+03	5.3E+03	1.7E+04	1.4E+04	2.1E+03
	rank	2	13	10	15	3	1	4	14	5	6	7	8	12	11	9
F_{21}	mean	1.4E+05	6.4E+06	5.6E+06	1.7E+07	4.2E+05	8.0E+04	1.1E+05	4.6E+07	3.1E+05	2.2E+05	4.4E+06	1.8E+04	8.4E+06	8.4E+06	2.1E+06
	std	6.5E+04	5.4E+06	3.0E+06	2.0E+07	1.8E+05	2.8E+04	6.7E+04	1.2E+07	1.5E+05	5.7E+04	1.9E+06	2.9E+04	9.5E+06	7.5E+06	2.3E+06
	rank	4	11	10	14	7	2	3	15	6	5	9	1	13	12	8
F_{22}	mean	1.5E+03	3.5E+03	3.9E+03	5.0E+03	1.4E+03	1.9E+03	2.3E+03	5.4E+03	2.1E+03	3.0E+03	2.0E+03	1.7E+03	1.8E+03	1.8E+03	2.5E+03
	std	4.6E+02	5.8E+02	8.1E+02	1.1E+03	3.4E+02	3.5E+02	4.7E+02	2.3E+02	4.6E+02	4.7E+02	4.8E+02	2.5E+02	3.3E+02	3.1E+02	4.1E+02
	rank	2	12	13	14	1	6	9	15	8	11	7	3	5	4	10
F_{23}	mean	3.5E+02	6.4E+02	5.0E+02	3.6E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02
	std	4.5E-08	1.8E+02	6.4E+01	2.8E+00	3.8E-11	2.1E-11	5.4E-10	2.3E-01	2.8E-10	1.6E-10	4.7E-03	2.8E-12	8.9E-03	1.3E-02	7.8E+00
	rank	11	15	14	13	1	8	10	6	9	2	3	7	4	5	12
F_{24}	mean	3.8E+02	6.7E+02	5.9E+02	4.2E+02	3.9E+02	3.6E+02	3.8E+02	3.6E+02	3.8E+02	3.4E+02	3.8E+02	4.2E+02	3.7E+02	3.7E+02	4.1E+02
	std	4.8E+00	8.3E+01	4.1E+01	2.1E+01	5.6E+00	7.5E+00	5.0E+00	2.3E+00	3.6E+00	1.2E+01	3.9E+00	8.1E+00	3.3E+00	2.8E+00	8.2E+00
	rank	8	15	14	13	10	2	7	3	6	1	9	12	4	5	11
F_{25}	mean	2.5E+02	2.8E+02	3.5E+02	2.8E+02	2.1E+02	2.6E+02	2.8E+02	5.2E+02	2.7E+02	3.0E+02	2.1E+02	2.8E+02	2.8E+02	2.8E+02	2.6E+02
	std	7.2E+00	2.8E+01	2.3E+01	2.1E+01	2.8E+01	5.2E+00	1.1E+01	6.2E+01	1.2E+01	1.1E+01	1.2E+00	1.2E+01	2.0E+01	9.8E+00	1.7E+01
	rank	3	10	14	12	2	4	8	15	6	13	1	11	7	9	5
F_{26}	mean	2.0E+02	2.3E+02	2.1E+02	2.1E+02	2.0E+02	2.0E+02	2.0E+02	1.0E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.1E+02	2.1E+02	1.9E+02
	std	3.6E-02	2.0E+01	4.7E+00	6.6E+00	6.2E-02	2.6E-02	9.3E-02	5.6E-02	9.9E-02	2.9E-02	3.8E+01	9.5E-03	3.0E+00	2.0E+00	3.7E+01
	rank	7	15	14	13	9	5	8	1	10	6	2	4	12	11	3
F_{27}	mean	8.2E+02	2.9E+03	4.4E+03	6.7E+03	1.6E+03	2.1E+03	2.0E+03	4.4E+03	1.8E+03	3.1E+03	2.3E+03	1.6E+03	1.6E+03	1.7E+03	3.5E+03
	std	1.3E+02	1.6E+02	2.2E+02	1.7E+03	2.0E+02	7.4E+02	2.3E+02	5.1E+01	4.5E+02	2.0E+02	1.2E+02	1.4E+02	3.0E+02	2.7E+02	1.4E+02
	rank	1	10	13	15	2	8	7	14	6	11	9	3	4	5	12
F_{28}	mean	2.1E+03	5.0E+03	2.0E+04	3.0E+04	7.6E+02	5.3E+03	7.9E+03	2.6E+03	6.4E+03	7.1E+03	6.7E+02	3.3E+03	7.1E+03	6.2E+03	6.3E+03
	std	2.6E+02	7.9E+02	5.3E+03	9.7E+03	2.6E+02	6.4E+02	1.4E+03	6.8E+02	1.1E+03	2.8E+03	1.0E+02	6.2E+02	4.2E+03	4.1E+03	7.8E+02
	rank	3	6	14	15	2	7	13	4	10	11	1	5	12	8	9
F_{29}	mean	2.3E+03	9.8E+07	4.9E+08	5.0E+05	2.6E+02	1.5E+03	1.6E+03	3.0E+02	1.8E+03	2.9E+02	2.6E+02	1.5E+03	8.1E+02	8.4E+02	2.2E+08
	std	2.0E+02	4.5E+07	4.4E+08	1.7E+06	3.4E+00	2.7E+02	2.6E+02	2.8E+01	2.7E+02	4.9E+01	2.5E+00	1.7E+02	6.5E+02	5.1E+02	5.8E+07
	rank	11	13	15	12	2	8	9	4	10	3	1	7	5	6	14
F_{30}	mean	9.7E+03	6.6E+05	7.1E+05	5.3E+05	3.0E+03	9.4E+03	1.2E+04	6.8E+03	1.0E+04	4.0E+03	2.8E+03	9.9E+03	3.0E+03	3.2E+03	5.1E+05
	std	1.1E+03	4.7E+05	4.6E+05	8.6E+05	4.5E+02	1.0E+03	3.4E+03	3.3E+02	1.4E+03	3.7E+02	4.8E+02	1.3E+03	4.2E+02	4.0E+02	3.8E+05
	rank	8	14	15	13	2	7	11	6	10	5	1	9	3	4	12
Mean rank		5.60	12.60	12.90	12.77	4.93	4.17	7.20	11.13	5.90	6.23	6.10	4.43	8.20	8.43	9.40
Final rank		4	13	15	14	3	1	8	12	5	7	6	2	9	10	11
Algorithms		GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.9E+03	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	7.0E+03	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.4E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	4.8E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	4	13	12	15	11	6	9	14	7	8	5	1	2	3	10
F_3	mean	1.3E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	1.7E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.8E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.4E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	12	10	14	15	3	1	11	6	9	5	7	13	4	2	8
F_5	mean	1.9E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	4.5E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	3	14	9	13	2	1	5	15	11	4	7	10	8	6	12
F_6	mean	3.6E-02	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	9.3E-02	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	2	12	13	15	1	7	6	14	3	11	4	5	8	9	10
F_7	mean	9.5E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.5E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	9	14	12	15	7	8	11	6	10	13	5	1	2	3	4
F_8	mean	4.9E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.8E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	9.3E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.3E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	10	13	14	15	3	6	8	9	5	11	2	1	4	7	12
F_{10}	mean	1.6E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.1E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	11	13	15	10	6	2	8	14	7	9	5	1	4	3	12
F_{11}	mean	3.3E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	2.6E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	9	12	13	14	3	4	8	15	5	10	2	1	6	7	11
F_{12}	mean	2.1E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.4E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	7	4	12	14	13	8	6	15	3	2	11	1	9	10	5
F_{13}	mean	7.8E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.6E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.1E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	4.2E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	9.5E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.8E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	6	7	12	15	2	4	9	13	5	14	11	1	10	8	3
F_{16}	mean	1.6E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.7E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	4	10	15	13	1	3	6	14	2	11	9	5	7	8	12
F_{17}	mean	1.0E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	6.6E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	3.6E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	2.6E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	1.3E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.1E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.9E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	3.3E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	3.1E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.6E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	1.9E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	8.5E+00	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	5	11	15	14	8	3	13	10	2	12	9	1	4	7	6
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	2.3E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	9	12	14	15	3	7	5	2	6	13	4	1	8	10	11
F_{25}	mean	1.4E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.7E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	2	13	15	14	5	1	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.1E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.2E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	2	12	14	13	8	3	10	15	6	9	5	7	1	4	11
F_{28}	mean	3.9E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.9E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	3.8E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	8.5E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	7.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	1.7E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	9	12	14	15	3	8	13	1	10	11	2	6	4	5	7
Mean rank		6.60	11.80	11.53	14.23	4.93	4.37	8.17	9.97	5.50	9.83	5.50	3.70	6.37	6.80	8.87
Final rank		6	13	12	14	3	2	8	11	4	10	4	1	5	7	9
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.9E+03	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	7.0E+03	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.4E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	4.8E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	4	13	12	15	11	6	9	14	7	8	5	1	2	3	10
F_3	mean	1.3E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	1.7E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.8E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.4E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	12	10	14	15	3	1	11	6	9	5	7	13	4	2	8
F_5	mean	1.9E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	4.5E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	3	14	9	13	2	1	5	15	11	4	7	10	8	6	12
F_6	mean	3.6E-02	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	9.3E-02	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	2	12	13	15	1	7	6	14	3	11	4	5	8	9	10
F_7	mean	9.5E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.5E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	9	14	12	15	7	8	11	6	10	13	5	1	2	3	4
F_8	mean	4.9E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.8E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	9.3E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.3E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	10	13	14	15	3	6	8	9	5	11	2	1	4	7	12
F_{10}	mean	1.6E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.1E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	11	13	15	10	6	2	8	14	7	9	5	1	4	3	12
F_{11}	mean	3.3E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	2.6E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	9	12	13	14	3	4	8	15	5	10	2	1	6	7	11
F_{12}	mean	2.1E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.4E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	7	4	12	14	13	8	6	15	3	2	11	1	9	10	5
F_{13}	mean	7.8E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.6E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.1E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	4.2E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	9.5E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.8E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	6	7	12	15	2	4	9	13	5	14	11	1	10	8	3
F_{16}	mean	1.6E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.7E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	4	10	15	13	1	3	6	14	2	11	9	5	7	8	12
F_{17}	mean	1.0E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	6.6E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	3.6E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	2.6E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	1.3E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.1E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.9E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	3.3E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	3.1E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.6E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	1.9E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	8.5E+00	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	5	11	15	14	8	3	13	10	2	12	9	1	4	7	6
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02
	std	2.3E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	9	12	14	15	3	7	5	2	6	13	4	1	8	10	11
F_{25}	mean	1.4E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.7E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	2	13	15	14	5	1	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.1E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.2E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	2	12	14	13	8	3	10	15	6	9	5	7	1	4	11
F_{28}	mean	3.9E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.9E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	3.8E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	8.5E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	7.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	1.7E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	9	12	14	15	3	8	13	1	10	11	2	6	4	5	7
Mean rank		6.60	11.80	11.53	14.23	4.93	4.37	8.17	9.97	5.50	9.83	5.50	3.70	6.37	6.80	8.87
Final rank		6	13	12	14	3	2	8	11	4	10	4	1	5	7	9
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.9E+03	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	7.0E+03	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.4E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	4.8E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	4	13	12	15	11	6	9	14	7	8	5	1	2	3	10
F_3	mean	1.3E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	1.7E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.8E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.4E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	12	10	14	15	3	1	11	6	9	5	7	13	4	2	8
F_5	mean	1.9E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	4.5E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	3	14	9	13	2	1	5	15	11	4	7	10	8	6	12
F_6	mean	3.6E-02	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	9.3E-02	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	2	12	13	15	1	7	6	14	3	11	4	5	8	9	10
F_7	mean	9.5E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.5E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	9	14	12	15	7	8	11	6	10	13	5	1	2	3	4
F_8	mean	4.9E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.8E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	9.3E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.3E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	10	13	14	15	3	6	8	9	5	11	2	1	4	7	12
F_{10}	mean	1.6E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.1E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	11	13	15	10	6	2	8	14	7	9	5	1	4	3	12
F_{11}	mean	3.3E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	2.6E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	9	12	13	14	3	4	8	15	5	10	2	1	6	7	11
F_{12}	mean	2.1E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.4E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	7	4	12	14	13	8	6	15	3	2	11	1	9	10	5
F_{13}	mean	7.8E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.6E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.1E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	4.2E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	9.5E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.8E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	6	7	12	15	2	4	9	13	5	14	11	1	10	8	3
F_{16}	mean	1.6E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.7E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	4	10	15	13	1	3	6	14	2	11	9	5	7	8	12
F_{17}	mean	1.0E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	6.6E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	3.6E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	2.6E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	1.3E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.1E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.9E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	3.3E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	3.1E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.6E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	1.9E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	8.5E+00	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	5	11	15	14	8	3	13	10	2	12	9	1	4	7	6
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	2.3E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	9	12	14	15	3	7	5	2	6	13	4	1	8	10	11
F_{25}	mean	1.4E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.7E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	2	13	15	14	5	1	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.1E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.2E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	2	12	14	13	8	3	10	15	6	9	5	7	1	4	11
F_{28}	mean	3.9E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.9E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	3.8E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	8.5E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	7.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	1.7E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	9	12	14	15	3	8	13	1	10	11	2	6	4	5	7
Mean rank		6.60	11.80	11.53	14.23	4.93	4.37	8.17	9.97	5.50	9.83	5.50	3.70	6.37	6.80	8.87
Final rank		6	13	12	14	3	2	8	11	4	10	4	1	5	7	9
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	9.9E+03	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	7.0E+03	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	3.4E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	4.8E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	4	13	12	15	11	6	9	14	7	8	5	1	2	3	10
F_3	mean	1.3E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	1.7E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	7	13	2	15	8	3	10	14	4	12	9	1	6	5	11
F_4	mean	2.8E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.4E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	12	10	14	15	3	1	11	6	9	5	7	13	4	2	8
F_5	mean	1.9E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	4.5E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	3	14	9	13	2	1	5	15	11	4	7	10	8	6	12
F_6	mean	3.6E-02	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	9.3E-02	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	2	12	13	15	1	7	6	14	3	11	4	5	8	9	10
F_7	mean	9.5E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	5.5E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	9	14	12	15	7	8	11	6	10	13	5	1	2	3	4
F_8	mean	4.9E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E+01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.8E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	9.3E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.3E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	10	13	14	15	3	6	8	9	5	11	2	1	4	7	12
F_{10}	mean	1.6E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	1.1E+02	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	11	13	15	10	6	2	8	14	7	9	5	1	4	3	12
F_{11}	mean	3.3E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	2.6E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	9	12	13	14	3	4	8	15	5	10	2	1	6	7	11
F_{12}	mean	2.1E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.4E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	7	4	12	14	13	8	6	15	3	2	11	1	9	10	5
F_{13}	mean	7.8E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.6E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	3	12	13	15	1	7	5	8	4	14	2	6	11	9	10
F_{14}	mean	1.1E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	4.2E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	9.5E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.8E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	6	7	12	15	2	4	9	13	5	14	11	1	10	8	3
F_{16}	mean	1.6E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.7E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	4	10	15	13	1	3	6	14	2	11	9	5	7	8	12
F_{17}	mean	1.0E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	6.6E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	3.6E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	2.6E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	9	15	2	12	11	3	13	10	8	14	6	1	4	7	5
F_{19}	mean	1.3E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.1E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.9E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	3.3E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	3.1E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.6E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	1.9E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	8.5E+00	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	5	11	15	14	8	3	13	10	2	12	9	1	4	7	6
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02
	std	2.3E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E-01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.0E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	9	12	14	15	3	7	5	2	6	13	4	1	8	10	11
F_{25}	mean	1.4E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	2.7E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	2	13	15	14	5	1	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.1E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.2E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.6E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	2	12	14	13	8	3	10	15	6	9	5	7	1	4	11
F_{28}	mean	3.9E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.9E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	3.8E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	8.5E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	7.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	1.7E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	9	12	14	15	3	8	13	1	10	11	2	6	4	5	7
Mean rank		6.60	11.80	11.53	14.23	4.93	4.37	8.17	9.97	5.50	9.83	5.50	3.70	6.37	6.80	8.87
Final rank		6	13	12	14	3	2	8	11	4	10	4	1	5	7	9
Algorithms		A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.4E+04	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	1.4E+04	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	5	14	10	15	7	2	3	13	4	6	8	1	11	12	9
F_2	mean	7.5E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	1.0E+03	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	6	13	12	15	11	5	9	14	7	8	4	1	2	3	10
F_3	mean	2.4E+02	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	2.8E+02	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	9	13	2	15	7	3	10	14	4	12	8	1	6	5	11
F_4	mean	3.5E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	3.3E-14	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	14	10	13	15	3	1	11	6	9	5	7	12	4	2	8
F_5	mean	1.8E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	6.3E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	3	14	9	13	2	1	5	15	11	4	7	10	8	6	12
F_6	mean	1.6E-01	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	1.8E-01	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	4	12	13	15	1	7	6	14	2	11	3	5	8	9	10
F_7	mean	1.6E-02	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	1.4E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	1	14	12	15	8	9	11	7	10	13	6	2	3	4	5
F_8	mean	3.1E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	1.5E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	9	12	13	14	8	1	7	6	4	5	10	1	3	2	11
F_9	mean	3.6E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	1.9E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	1	13	14	15	4	7	9	10	6	11	3	2	5	8	12
F_{10}	mean	2.3E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	9.7E+01	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	12	13	15	10	6	2	8	14	7	9	5	1	4	3	11
F_{11}	mean	2.0E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	1.9E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	4	12	13	14	3	5	9	15	6	10	2	1	7	8	11
F_{12}	mean	5.9E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	2.9E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	13	4	11	14	12	7	6	15	3	2	10	1	8	9	5
F_{13}	mean	5.6E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.4E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	2	12	13	15	1	7	5	8	4	14	3	6	11	9	10
F_{14}	mean	1.0E-01	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	3.3E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	4	10	13	15	1	5	3	9	2	14	6	7	11	12	8
F_{15}	mean	7.6E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	2.8E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	4	7	12	15	2	5	9	13	6	14	11	1	10	8	3
F_{16}	mean	1.9E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	4.7E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	7	10	15	13	1	3	5	14	2	11	9	4	6	8	12
F_{17}	mean	1.2E+03	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	1.1E+03	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	5	11	2	15	4	3	9	12	7	10	6	1	13	14	8
F_{18}	mean	5.9E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	5.2E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	11	15	2	12	10	3	13	9	8	14	6	1	4	7	5
F_{19}	mean	1.3E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	3.8E-01	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	2.7E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	3.4E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	10	14	5	15	8	2	11	12	6	13	7	1	3	4	9
F_{21}	mean	2.7E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	2.0E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	8	12	9	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	4.1E+01	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	4.5E+01	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	11	10	15	14	7	3	13	9	2	12	8	1	4	6	5
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	2.5E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	6	10	9	11	3	4	7	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	2.9E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	1	12	14	15	4	8	6	3	7	13	5	2	9	10	11
F_{25}	mean	1.6E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	3.7E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	6	13	15	14	4	1	10	11	5	12	2	3	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	3.2E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	1.3E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.8E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	3	12	14	13	8	2	10	15	6	9	5	7	1	4	11
F_{28}	mean	3.9E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	3.3E+01	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	9	13	14	15	3	10	8	2	7	6	1	12	5	4	11
F_{29}	mean	4.0E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	8.4E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	9	12	14	15	2	7	13	3	8	4	1	11	6	5	10
F_{30}	mean	7.4E+02	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	2.5E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02
	rank	9	12	14	15	3	8	13	1	10	11	2	6	4	5	7
Mean rank		6.57	11.77	11.50	14.23	4.80	4.40	8.27	10.00	5.57	9.83	5.47	3.70	6.43	6.83	8.80
Final rank		7	14	13	15	3	2	9	12	5	11	4	1	6	8	10

Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.7E+05	1.6E+07	1.5E+07	1.1E+08	3.8E+05	5.5E+04	1.1E+05	5.6E+07	2.9E+05	1.9E+05	2.5E+06	9.9E+02	8.3E+06	1.0E+07	4.9E+06
	std	3.2E+05	1.7E+07	1.2E+07	4.1E+07	2.3E+05	4.0E+04	7.1E+04	2.6E+07	2.2E+05	1.2E+05	1.1E+06	2.5E+03	3.6E+06	6.1E+06	4.7E+06
	rank	5	13	12	15	7	2	3	14	6	4	8	1	10	11	9
F_2	mean	1.0E+03	2.9E+09	4.8E+08	9.6E+08	4.9E+01	1.8E-03	5.6E+01	1.6E+04	3.0E+01	2.9E+03	3.5E+03	3.5E-14	3.4E+03	4.9E+03	1.4E+07
	std	1.1E+03	3.1E+09	3.8E+08	8.7E+08	8.0E+01	3.9E-03	1.1E+02	1.4E+04	3.9E+01	1.7E+03	5.7E+03	1.6E-14	3.0E+03	3.5E+03	7.8E+07
	rank	6	15	13	14	4	2	5	11	3	7	9	1	8	10	12
F_3	mean	7.3E+01	2.0E+04	3.1E+03	4.5E+04	2.4E+01	1.2E-02	2.3E+01	3.7E+04	2.6E+01	3.7E+02	4.9E+02	9.7E-14	2.3E+02	3.0E+02	3.0E+02
	std	6.3E+01	2.0E+04	2.2E+03	3.1E+04	3.5E+01	3.0E-02	2.4E+01	2.6E+04	2.7E+01	3.3E+02	3.5E+02	8.7E-14	2.6E+02	2.7E+02	3.7E+02
	rank	6	13	12	15	4	2	3	14	5	10	11	1	7	8	9
F_4	mean	7.0E+01	2.0E+02	1.9E+02	3.7E+02	2.6E+01	3.7E+00	4.3E+00	2.6E+01	4.2E+01	4.5E+01	2.5E+01	1.3E-12	8.8E+01	7.5E+01	7.9E+01
	std	4.3E+00	1.6E+02	5.6E+01	1.4E+02	1.8E+01	1.5E+01	1.6E+01	4.0E-01	3.6E+01	2.9E+01	1.1E+00	1.6E-12	2.7E+01	2.5E+01	3.3E+01
	rank	9	14	13	15	6	2	3	5	7	8	4	1	12	10	11
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	5.4E-02	3.0E-01	7.5E-02	1.6E-01	7.2E-02	5.9E-02	3.4E-01	4.9E-02	1.6E-02	1.9E-04	6.0E-02	5.2E-03	6.8E-02	8.3E-02	1.4E-01
	rank	14	10	4	13	6	5	11	15	2	1	12	3	7	8	9
F_6	mean	2.5E+00	1.9E+01	3.0E+01	3.6E+01	3.7E+00	1.1E+01	6.6E+00	3.9E+01	3.6E+00	2.0E+01	8.3E+00	5.5E+00	1.5E+01	1.6E+01	2.1E+01
	std	7.7E-01	3.1E+00	3.8E+00	5.0E+00	2.3E+00	2.2E+00	2.3E+00	1.4E+00	2.0E+00	2.5E+00	1.6E+00	2.6E+00	1.8E+00	2.0E+00	2.9E+00
	rank	1	10	13	14	3	7	5	15	2	11	6	4	8	9	12
F_7	mean	1.0E-02	2.2E+01	8.0E+00	1.5E+01	7.9E-03	4.7E-03	1.6E-02	5.4E-05	5.3E-03	1.3E-02	1.4E-03	3.2E-03	7.3E-03	6.5E-04	1.8E-03
	std	2.2E-02	1.8E+01	5.4E+00	1.6E+01	1.1E-02	7.7E-03	1.9E-02	2.2E-04	8.2E-03	1.7E-02	4.3E-03	6.1E-03	1.3E-02	1.3E-03	4.6E-03
	rank	10	15	13	14	9	6	12	1	7	11	3	5	8	2	4
F_8	mean	2.4E+01	8.0E+01	1.4E+02	1.5E+02	3.3E+01	9.9E-02	3.1E+01	7.2E+01	9.0E+00	8.8E+00	3.3E+01	2.7E-13	2.1E+00	1.8E+00	6.7E+01
	std	5.8E+00	2.0E+01	2.4E+01	4.6E+01	7.4E+00	3.1E-01	9.6E+00	1.4E+01	3.0E+00	4.3E+00	6.5E+00	5.7E-14	2.1E+00	1.4E+00	1.1E+01
	rank	7	13	14	15	9	2	8	12	6	5	10	1	4	3	11
F_9	mean	1.9E+01	1.4E+02	1.6E+02	2.3E+02	3.7E+01	5.0E+01	5.5E+01	1.6E+02	4.5E+01	1.2E+02	5.9E+01	2.4E+01	6.7E+01	7.1E+01	9.4E+01
	std	4.4E+00	3.1E+01	2.3E+01	7.6E+01	8.4E+00	1.5E+01	1.4E+01	1.5E+01	1.2E+01	2.3E+01	1.2E+01	5.4E+00	1.2E+01	1.2E+01	2.0E+01
	rank	1	12	13	15	3	5	6	14	4	11	7	2	8	9	10
F_{10}	mean	1.4E+03	2.6E+03	4.1E+03	9.7E+02	1.1E+03	5.8E+00	6.6E+02	6.3E+03	3.8E+02	5.1E+02	8.6E+02	2.4E-01	3.6E+01	4.3E+00	2.7E+03
	std	3.5E+02	4.1E+02	7.2E+02	4.7E+02	5.1E+02	1.6E+00	3.7E+02	3.5E+02	1.9E+02	1.7E+02	3.2E+02	7.6E-01	6.3E+01	2.2E+01	5.0E+02
	rank	11	12	14	9	10	3	7	15	5	6	8	1	4	2	13
F_{11}	mean	2.4E+03	3.8E+03	4.4E+03	6.0E+03	2.6E+03	2.0E+03	2.7E+03	7.6E+03	2.3E+03	3.0E+03	2.7E+03	1.7E+03	3.1E+03	3.4E+03	3.4E+03
	std	5.3E+02	6.5E+02	7.5E+02	1.2E+03	4.6E+02	3.3E+02	5.5E+02	2.9E+02	5.2E+02	6.6E+02	4.6E+02	2.1E+02	4.8E+02	4.5E+02	4.5E+02
	rank	4	12	13	14	5	2	6	15	3	8	7	1	9	11	10
F_{12}	mean	1.9E+00	3.1E-01	1.3E+00	1.8E+00	8.6E-01	1.7E-01	6.2E-01	2.5E+00	1.9E-01	3.2E-01	9.4E-01	1.2E-01	6.5E-01	6.1E-01	5.4E-01
	std	3.8E-01	1.1E-01	3.9E-01	4.5E-01	1.9E-01	8.0E-02	3.5E-01	2.2E-01	7.3E-02	1.3E-01	3.2E-01	1.9E-02	1.3E-01	1.3E-01	1.5E-01
	rank	14	4	12	13	10	2	8	15	3	5	11	1	9	7	6
F_{13}	mean	1.7E-01	7.3E-01	5.6E-01	4.7E-01	1.4E-01	2.6E-01	2.9E-01	3.4E-01	2.2E-01	3.7E-01	3.5E-01	3.0E-01	2.8E-01	2.9E-01	4.1E-01
	std	4.9E-02	5.4E-01	1.3E-01	1.1E-01	2.6E-02	6.3E-02	5.9E-02	5.0E-02	6.6E-02	1.0E-01	4.5E-02	4.8E-02	3.6E-02	3.8E-02	6.8E-02
	rank	2	15	14	13	1	4	6	9	3	11	10	8	5	7	12
F_{14}	mean	2.5E-01	8.4E+00	2.0E+00	2.4E+00	3.0E-01	2.6E-01	2.3E-01	3.0E-01	2.4E-01	2.6E-01	2.4E-01	3.0E-01	2.4E-01	2.3E-01	5.4E-01
	std	4.9E-02	1.3E+01	3.6E+00	4.8E+00	1.6E-01	2.5E-02	3.5E-02	3.9E-02	5.0E-02	5.2E-02	2.6E-02	1.4E-01	3.2E-02	2.9E-02	2.4E-01
	rank	6	15	13	14	9	8	1	10	5	7	3	11	4	2	12
F_{15}	mean	3.6E+00	4.7E+02	9.4E+01	1.5E+02	4.4E+00	5.0E+00	5.4E+00	1.5E+01	4.1E+00	4.2E+01	8.8E+00	2.5E+00	1.1E+01	1.1E+01	6.9E+00
	std	1.1E+00	6.8E+02	9.5E+01	1.3E+02	1.7E+00	1.0E+00	1.5E+00	9.6E-01	1.0E+00	1.6E+01	2.2E+00	4.0E-01	2.7E+00	2.1E+00	1.8E+00
	rank	2	15	13	14	4	5	6	11	3	12	8	1	10	9	7
F_{16}	mean	1.0E+01	1.1E+01	1.2E+01	1.3E+01	9.7E+00	9.5E+00	1.0E+01	1.3E+01	1.0E+01	1.1E+01	1.1E+01	9.4E+00	1.1E+01	1.1E+01	1.2E+01
	std	7.2E-01	5.9E-01	5.3E-01	4.6E-01	7.2E-01	8.9E-01	8.8E-01	2.4E-01	7.2E-01	6.3E-01	3.4E-01	3.9E-01	5.0E-01	4.2E-01	4.2E-01
	rank	5	10	13	14	3	2	6	15	4	9	11	1	8	7	12
F_{17}	mean	3.9E+04	9.4E+05	6.5E+05	5.2E+06	7.7E+04	2.2E+04	2.6E+04	1.4E+06	6.3E+04	5.8E+04	2.9E+05	1.4E+03	1.8E+06	1.6E+06	2.6E+05
	std	2.3E+04	1.3E+06	8.9E+05	3.8E+06	5.7E+04	1.7E+04	1.8E+04	6.7E+05	4.7E+04	2.8E+04	1.7E+05	6.5E+02	1.5E+06	1.0E+06	2.9E+05
	rank	4	11	10	15	7	2	3	12	6	5	9	1	14	13	8
F_{18}	mean	8.1E+02	8.4E+04	1.5E+03	1.5E+03	2.7E+03	3.3E+02	3.2E+03	2.1E+05	8.6E+02	1.3E+03	3.8E+02	1.4E+02	9.5E+01	1.1E+02	1.6E+03
	std	9.3E+02	3.0E+05	5.1E+03	2.3E+03	5.4E+03	2.9E+02	4.5E+03	1.6E+05	1.1E+03	1.5E+03	2.2E+02	4.3E+01	2.1E+01	9.8E+01	2.3E+03
	rank	6	14	9	10	12	4	13	15	7	8	5	3	1	2	11
F_{19}	mean	5.4E+00	3.2E+01	2.5E+01	7.2E+01	7.2E+00	6.9E+00	6.8E+00	1.8E+01	5.1E+00	1.4E+01	1.2E+01	4.9E+00	1.2E+01	1.1E+01	9.5E+00
	std	1.5E+00	3.5E+01	1.5E+01	1.0E+02	2.3E+00	1.5E+00	1.3E+00	8.5E-01	1.1E+00	3.8E+00	9.7E-01	9.2E-01	2.0E+00	2.0E+00	1.8E+00
	rank	3	14	13	15	6	5	4	12	2	11	10	1	9	8	7
F_{20}	mean	5.4E+02	5.2E+03	8.1E+02	6.3E+04	3.8E+02	1.8E+02	7.4E+02	2.0E+04	4.9E+02	2.0E+03	1.5E+03	1.1E+02	4.5E+03	5.1E+03	3.2E+03
	std	2.7E+02	8.9E+03	5.7E+02	4.0E+04	2.1E+02	6.9E+01	5.2E+02	7.8E+03	4.3E+02	1.3E+03	6.2E+02	6.1E+01	2.3E+03	2.8E+03	1.8E+03
	rank	5	13	7	15	3	2	6	14	4	9	8	1	11	12	10
F_{21}	mean	2.1E+04	2.1E+05	4.2E+04	2.2E+06	2.3E+04	1.4E+04	2.4E+04	4.9E+05	3.1E+04	4.0E+04	7.3E+04	6.6E+02	8.7E+04	9.0E+04	1.1E+05
	std	8.8E+03	2.3E+05	5.7E+04	6.4E+06	1.6E+04	1.0E+04	1.5E+04	2.6E+05	3.3E+04	2.6E+04	5.5E+04	2.5E+02	6.5E+04	8.0E+04	9.8E+04
	rank	3	13	8	15	4	2	5	14	6	7	9	1	10	11	12
F_{22}	mean	2.8E+02	4.9E+02	7.1E+02	8.4E+02	2.3E+02	2.6E+02	3.1E+02	4.1E+02	2.3E+02	5.2E+02	1.5E+02	2.1E+02	2.6E+02	2.7E+02	2.9E+02
	std	9.5E+01	2.2E+02	2.4E+02	2.1E+02	8.7E+01	1.2E+02	1.2E+02	1.5E+02	1.0E+02	1.6E+02	6.7E+01	9.5E+01	9.0E+01	7.9E+01	1.2E+02
	rank	8	12	14	15	4	5	10	11	3	13	1	2	6	7	9
F_{23}	mean	3.2E+02	3.3E+02	3.2E+02	3.3E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02
	std	3.7E-12	1.2E+01	6.5E+00	9.8E+00	6.0E-13	4.3E-13	1.6E-12	1.2E-06	2.2E-12	1.8E-11	8.1E-08	2.2E-13	3.1E-02	9.6E-03	6.5E-01
	rank	11	15	13	14	1	8	9	4	10	2	3	7	6	5	12
F_{24}	mean	2.3E+02	2.5E+02	2.5E+02	2.4E+02	2.3E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.3E+02	2.3E+02
	std	3.8E+00	1.0E+01	9.1E+00	3.8E+01	7.0E+00	1.5E+00	1.6E+00	1.3E+00	3.8E+00	3.5E+00	9.9E-01	5.5E+00	8.1E-01	8.3E-01	7.1E+00
	rank	7	14	15	13	9	8	4	1	6	11	2	10	3	5	12
F_{25}	mean	2.1E+02	2.1E+02	2.2E+02	2.2E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02	2.1E+02	2.2E+02	2.0E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02
	std	1.3E+00	4.2E+00	5.9E+00	4.7E+00	1.2E-01	1.9E+00	2.5E+00	2.1E+00	1.5E+00	4.0E+00	3.4E-01	1.5E+00	1.8E+00	2.1E+00	2.2E+00
	rank	8	10	15	14	1	9	5	7	4	13	2	3	12	11	6
F_{26}	mean	1.5E+02	1.1E+02	1.8E+02	1.9E+02	1.4E+02	1.0E+02	1.8E+02	1.0E+02	1.1E+02	1.2E+02	1.0E+02	1.0E+02	1.1E+02	1.1E+02	1.0E+02
	std	5.1E+01	3.0E+01	4.2E+01	2.8E+01	5.0E+01	7.6E-02	4.4E+01	3.4E-02	2.5E+01	3.6E+01	4.9E-02	5.3E-02	3.4E+01	3.0E+01	1.3E-01
	rank	12	8	14	15	11	2	13	4	6	10	3	1	9	7	5
F_{27}	mean	3.9E+02	8.2E+02	8.1E+02	1.3E+03	4.4E+02	4.0E+02	4.8E+02	1.3E+03	4.0E+02	7.1E+02	4.2E+02	4.1E+02	4.3E+02	4.3E+02	6.8E+02
	std	4.9E+01	1.9E+02	3.8E+02	5.2E+02	4.8E+01	3.2E+00	8.6E+01	3.5E+01	2.1E+01	2.9E+02	3.8E+01	5.7E+01	2.0E+01	1.8E+01	2.7E+02
	rank	1	13	12	15	8	3	9	14	2	11	5	4	7	6	10
F_{28}	mean	9.9E+02	1.2E+03	3.9E+03	5.1E+03	4.1E+02	9.6E+02	1.5E+03	3.9E+02	9.0E+02	7.7E+02	3.9E+02	8.7E+02	8.8E+02	8.7E+02	1.2E+03
	std	1.8E+02	2.1E+02	1.2E+03	1.7E+03	2.7E+01	4.8E+01	7.2E+02	4.6E+00	5.6E+01	3.3E+02	4.0E+00	4.3E+01	3.1E+02	2.3E+02	2.5E+02
	rank	10	11	14	15	3	9	13	2	8	4	1	5	7	6	12
F_{29}	mean	2.5E+03	3.1E+06	1.3E+07	4.7E+06	2.1E+02	9.7E+02	1.4E+03	2.2E+02	1.3E+03	2.2E+02	2.1E+02	7.7E+02	1.1E+03	1.1E+03	4.1E+06
	std	1.7E+03	4.0E+06	1.8E+07	1.0E+07	1.2E+00	2.6E+02	4.9E+02	1.2E+00	3.1E+02	1.6E+01	1.5E+00	3.2E+01	1.9E+02	1.9E+02	3.7E+06
	rank	11	12	15	14	1	6	10	4	9	3	2	5	8	7	13
F_{30}	mean	2.3E+03	2.9E+04	1.8E+04	8.3E+04	7.7E+02	2.1E+03	1.9E+03	9.4E+02	1.8E+03	8.4E+02	6.3E+02	2.4E+03	6.1E+02	6.5E+02	1.4E+04
	std	5.3E+02	3.3E+04	4.2E+04	8.5E+04	1.8E+02	6.6E+02	6.1E+02	1.8E+02	6.1E+02	2.4E+02	1.5E+02	1.0E+03	1.3E+02	1.5E+02	2.6E+04
	rank	10	14	13	15	4	9	8	6	7	5	2	11	1	3	12
Mean rank		6.73	12.40	12.37	14.03	5.57	4.43	6.93	10.37	4.93	7.93	6.33	3.27	7.50	7.27	9.93

Final rank	6	14	13	15	4	2	7	12	3	10	5	1	9	8	11
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	7.6E+05	1.0E+08	5.6E+07	8.0E+07	1.2E+06	3.4E+05	4.2E+05	3.3E+08	7.9E+05	6.5E+05	9.8E+06	3.0E+04	1.6E+07	1.4E+07	2.2E+07
	std	3.9E+05	9.7E+07	5.4E+07	3.4E+07	5.5E+05	1.3E+05	2.0E+05	8.9E+07	2.4E+05	1.6E+05	3.6E+06	7.8E+04	7.6E+06	4.1E+06	1.9E+07
	rank	5	14	12	13	7	2	3	15	6	4	8	1	10	9	11
F_2	mean	9.4E+03	1.3E+10	3.4E+09	2.7E+08	9.9E+03	3.0E+03	6.3E+03	1.5E+06	5.4E+03	5.0E+03	1.1E+03	2.3E-10	1.5E+03	2.5E+03	4.9E+03
	std	8.0E+03	5.4E+09	1.6E+09	2.9E+08	1.0E+04	3.1E+03	6.1E+03	1.6E+06	5.6E+03	6.5E+03	1.5E+03	4.9E-10	1.4E+03	2.5E+03	8.1E+03
	rank	10	15	14	13	11	5	9	12	8	7	2	1	3	4	6
F_3	mean	6.2E+02	7.7E+04	2.6E+04	1.8E+05	5.4E+02	6.9E+00	6.0E+02	2.9E+05	1.0E+03	2.4E+03	3.4E+03	1.5E+00	3.9E+03	3.5E+03	3.6E+03
	std	2.6E+02	3.7E+04	1.1E+04	3.8E+04	4.7E+02	7.9E+00	5.4E+02	9.8E+04	8.1E+02	1.2E+03	1.2E+03	5.2E+00	2.4E+03	2.0E+03	2.2E+03
	rank	5	13	12	14	3	2	4	15	6	7	8	1	11	9	10
F_4	mean	9.8E+01	1.2E+03	4.7E+02	4.2E+02	4.3E+01	4.1E+01	5.7E+01	4.6E+01	8.8E+01	8.6E+01	4.4E+01	1.4E+01	1.5E+02	1.4E+02	1.2E+02
	std	1.5E+00	9.6E+02	1.4E+02	8.0E+01	1.1E+01	3.4E+01	2.9E+01	8.5E-01	2.5E+01	4.0E+01	1.7E+00	3.2E+01	2.3E+01	3.1E+01	4.6E+01
	rank	9	15	14	13	3	2	6	5	8	7	4	1	12	11	10
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	4.6E-02	3.0E-01	1.2E-01	1.2E-01	5.7E-02	4.9E-02	3.3E-01	4.4E-02	4.5E-03	2.6E-04	6.3E-02	3.6E-03	7.3E-02	6.8E-02	2.0E-01
	rank	14	12	6	13	5	4	10	15	2	1	11	3	7	9	8
F_6	mean	7.4E+00	4.0E+01	6.0E+01	7.1E+01	1.1E+01	2.6E+01	1.9E+01	7.1E+01	1.3E+01	4.1E+01	2.3E+01	1.5E+01	3.4E+01	3.5E+01	4.8E+01
	std	1.5E+00	4.0E+00	3.1E+00	5.1E+00	3.4E+00	4.1E+00	5.2E+00	1.5E+00	4.1E+00	4.4E+00	2.2E+00	4.3E+00	4.8E+00	4.5E+00	4.3E+00
	rank	1	10	13	14	2	7	5	15	3	11	6	4	8	9	12
F_7	mean	6.7E-03	1.2E+02	3.4E+01	2.6E+00	9.2E-03	1.1E-02	5.3E-03	6.9E-05	5.3E-03	8.9E-03	3.1E-03	4.3E-03	1.1E-02	8.8E-03	5.3E-01
	std	1.0E-02	7.9E+01	1.8E+01	1.1E+00	9.3E-03	1.4E-02	6.8E-03	2.8E-04	7.0E-03	6.8E-03	8.1E-03	6.8E-03	5.6E-03	6.9E-03	2.9E+00
	rank	6	15	14	13	9	11	5	1	4	8	2	3	10	7	12
F_8	mean	4.6E+01	2.0E+02	3.2E+02	3.0E+02	7.1E+01	9.5E-01	7.3E+01	2.2E+02	1.5E+01	2.2E+01	8.2E+01	1.7E-01	8.7E+00	9.7E+00	1.7E+02
	std	1.0E+01	3.8E+01	3.7E+01	7.9E+01	1.3E+01	8.2E-01	2.0E+01	2.0E+01	4.9E+00	6.3E+00	1.5E+01	5.9E-01	4.2E+00	4.3E+00	2.9E+01
	rank	7	12	15	14	8	2	9	13	5	6	10	1	3	4	11
F_9	mean	3.9E+01	3.1E+02	3.9E+02	3.9E+02	7.6E+01	1.0E+02	1.3E+02	3.6E+02	9.5E+01	2.5E+02	1.5E+02	6.5E+01	1.9E+02	2.2E+02	2.0E+02
	std	1.0E+01	5.4E+01	3.9E+01	1.5E+02	1.7E+01	1.9E+01	3.2E+01	1.7E+01	2.0E+01	3.7E+01	3.3E+01	1.4E+01	3.1E+01	2.8E+01	3.0E+01
	rank	1	12	15	14	3	5	6	13	4	11	7	2	8	10	9
F_{10}	mean	3.8E+03	5.8E+03	8.3E+03	3.2E+03	3.1E+03	2.2E+01	1.6E+03	1.3E+04	8.5E+02	1.1E+03	2.8E+03	4.0E+00	9.3E+01	8.9E+01	6.0E+03
	std	6.7E+02	8.5E+02	9.8E+02	1.6E+03	8.7E+02	4.4E+01	4.4E+02	4.4E+02	2.2E+02	2.4E+02	4.5E+02	2.2E+01	1.0E+02	8.7E+01	7.2E+02
	rank	11	12	14	10	9	2	7	15	5	6	8	1	4	3	13
F_{11}	mean	5.0E+03	7.6E+03	9.1E+03	1.2E+04	5.2E+03	4.0E+03	5.9E+03	1.4E+04	5.0E+03	5.6E+03	6.1E+03	3.9E+03	8.8E+03	8.7E+03	6.5E+03
	std	8.5E+02	7.9E+02	9.6E+02	2.3E+03	7.6E+02	4.9E+02	9.6E+02	3.5E+02	8.7E+02	6.5E+02	9.8E+02	3.4E+02	8.5E+02	7.0E+02	5.8E+02
	rank	4	10	13	14	5	2	7	15	3	6	8	1	12	11	9
F_{12}	mean	2.6E+00	4.1E-01	2.0E+00	2.6E+00	1.2E+00	1.4E-01	9.6E-01	3.4E+00	2.0E-01	3.2E-01	1.5E+00	1.1E-01	1.0E+00	1.0E+00	8.5E-01
	std	5.8E-01	1.5E-01	4.1E-01	1.2E+00	1.8E-01	4.8E-02	3.5E-01	2.1E-01	5.4E-02	9.9E-02	2.8E-01	1.4E-02	2.2E-01	1.6E-01	1.9E-01
	rank	14	5	12	13	10	2	7	15	3	4	11	1	8	9	6
F_{13}	mean	2.7E-01	1.1E+00	6.4E-01	5.5E-01	2.4E-01	3.9E-01	4.5E-01	4.7E-01	3.8E-01	4.9E-01	4.9E-01	3.7E-01	4.7E-01	4.8E-01	4.6E-01
	std	4.5E-02	9.1E-01	1.2E-01	8.0E-02	5.2E-02	7.5E-02	6.9E-02	4.6E-02	9.3E-02	6.8E-02	4.2E-02	6.5E-02	4.1E-02	4.6E-02	6.8E-02
	rank	2	15	14	13	1	5	6	9	4	11	12	3	8	10	7
F_{14}	mean	2.9E-01	3.0E+01	3.7E+00	3.0E-01	3.8E-01	3.0E-01	4.1E-01	3.5E-01	3.1E-01	3.1E-01	2.7E-01	3.6E-01	3.3E-01	3.2E-01	4.5E-01
	std	1.1E-01	2.4E+01	6.3E+00	4.2E-02	2.1E-01	4.1E-02	2.4E-01	3.9E-02	9.6E-02	3.4E-02	2.4E-02	1.4E-01	2.7E-02	2.3E-02	1.9E-01
	rank	2	15	14	4	11	3	12	9	6	5	1	10	8	7	13
F_{15}	mean	6.4E+00	3.3E+04	8.8E+02	5.8E+02	8.5E+00	1.3E+01	1.2E+01	3.2E+01	1.1E+01	1.1E+02	2.2E+01	7.6E+00	3.2E+01	2.9E+01	2.4E+01
	std	1.8E+00	4.8E+04	8.6E+02	8.3E+02	3.4E+00	3.7E+00	3.4E+00	1.3E+00	3.5E+00	3.2E+01	4.5E+00	1.1E+00	5.8E+00	4.9E+00	7.6E+00
	rank	1	15	14	13	3	6	5	11	4	12	7	2	10	9	8
F_{16}	mean	1.9E+01	2.1E+01	2.2E+01	2.2E+01	1.9E+01	1.8E+01	1.9E+01	2.3E+01	1.9E+01	2.0E+01	2.1E+01	1.8E+01	2.1E+01	2.1E+01	2.1E+01
	std	7.1E-01	5.4E-01	5.6E-01	4.7E-01	8.8E-01	5.1E-01	6.9E-01	2.1E-01	8.5E-01	7.4E-01	4.6E-01	3.8E-01	3.7E-01	4.9E-01	6.0E-01
	rank	5	8	13	14	4	2	6	15	3	7	11	1	10	9	12
F_{17}	mean	6.8E+04	7.8E+06	1.9E+06	8.1E+06	2.4E+05	4.4E+04	5.1E+04	1.1E+07	1.1E+05	1.2E+05	1.6E+06	4.0E+03	2.7E+06	2.4E+06	1.0E+06
	std	3.5E+04	1.3E+07	3.6E+06	5.5E+06	1.5E+05	1.8E+04	2.3E+04	3.8E+06	7.1E+04	7.4E+04	6.3E+05	6.6E+03	3.1E+06	2.1E+06	9.0E+05
	rank	4	13	10	14	7	2	3	15	5	6	9	1	12	11	8
F_{18}	mean	2.3E+03	1.6E+08	1.9E+07	2.2E+05	6.3E+03	3.8E+02	1.5E+03	6.2E+05	6.9E+02	2.0E+03	7.5E+02	2.6E+02	8.6E+02	9.3E+02	2.6E+03
	std	1.3E+03	4.8E+08	7.2E+07	9.6E+05	7.9E+03	2.2E+02	1.0E+03	1.1E+06	6.8E+02	2.5E+03	3.9E+02	2.6E+02	8.2E+02	7.1E+02	1.4E+03
	rank	9	15	14	12	11	2	7	13	3	8	4	1	5	6	10
F_{19}	mean	2.0E+01	7.9E+01	8.0E+01	2.4E+02	2.1E+01	1.5E+01	1.4E+01	3.3E+01	1.2E+01	3.8E+01	2.5E+01	1.2E+01	2.4E+01	2.1E+01	2.8E+01
	std	4.5E+00	5.8E+01	2.3E+01	7.2E+02	2.7E+00	2.8E+00	2.6E+00	6.1E-01	2.3E+00	2.6E+01	1.7E+00	2.8E+00	1.6E+01	1.2E+01	7.5E+00
	rank	5	13	14	15	6	4	3	11	2	12	9	1	8	7	10
F_{20}	mean	4.2E+02	2.1E+04	6.0E+03	2.3E+05	5.0E+02	3.0E+02	6.7E+02	7.1E+04	7.0E+02	1.7E+03	2.2E+03	3.5E+02	1.3E+04	1.3E+04	3.9E+03
	std	1.1E+02	2.5E+04	5.0E+03	2.5E+05	1.8E+02	8.9E+01	2.4E+02	3.5E+04	5.3E+02	1.5E+03	9.6E+02	1.5E+02	5.0E+03	5.7E+03	2.9E+03
	rank	3	13	10	15	4	1	5	14	6	7	8	2	11	12	9
F_{21}	mean	6.7E+04	2.0E+06	6.0E+05	7.7E+06	9.4E+04	3.8E+04	3.9E+04	4.4E+06	1.3E+05	9.2E+04	6.4E+05	1.8E+03	1.5E+06	1.4E+06	2.5E+05
	std	3.3E+04	2.9E+06	8.4E+05	7.2E+06	4.9E+04	2.5E+04	1.6E+04	1.5E+06	7.1E+04	7.4E+04	3.3E+05	1.6E+03	9.4E+05	1.1E+06	1.6E+05
	rank	4	13	9	15	6	2	3	14	7	5	10	1	12	11	8
F_{22}	mean	4.3E+02	1.3E+03	1.7E+03	1.8E+03	4.9E+02	8.3E+02	7.5E+02	1.7E+03	7.3E+02	1.4E+03	5.4E+02	5.9E+02	7.8E+02	7.4E+02	9.0E+02
	std	2.3E+02	3.1E+02	4.3E+02	5.4E+02	2.6E+02	2.2E+02	2.5E+02	1.4E+02	2.5E+02	4.1E+02	1.6E+02	1.6E+02	1.7E+02	1.7E+02	2.4E+02
	rank	1	11	14	15	2	9	7	13	5	12	3	4	8	6	10
F_{23}	mean	3.4E+02	3.9E+02	3.7E+02	3.6E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02
	std	2.8E-02	4.1E+01	1.5E+01	6.3E+00	1.2E-12	6.8E-13	5.0E-12	3.4E-06	8.7E-13	7.6E-11	5.4E-07	5.0E-13	2.0E-02	5.2E-03	6.9E-01
	rank	11	15	14	13	1	8	10	4	9	2	3	7	6	5	12
F_{24}	mean	2.7E+02	3.3E+02	3.3E+02	2.9E+02	2.8E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.8E+02	2.6E+02	2.6E+02	2.8E+02
	std	3.0E+00	3.8E+01	1.4E+01	2.1E+01	2.7E+00	5.5E+00	5.7E+00	6.6E-06	4.5E+00	4.8E+00	4.5E+00	3.3E+00	4.5E+00	4.6E+00	3.9E+00
	rank	9	15	14	13	11	2	7	5	6	4	8	10	3	1	12
F_{25}	mean	2.2E+02	2.2E+02	2.5E+02	2.4E+02	2.0E+02	2.1E+02	2.2E+02	2.3E+02	2.1E+02	2.4E+02	2.0E+02	2.1E+02	2.3E+02	2.3E+02	2.2E+02
	std	2.5E+00	1.3E+01	9.1E+00	1.4E+01	7.8E-02	3.0E+00	4.5E+00	5.2E+00	3.6E+00	6.5E+00	4.5E-01	5.4E+00	3.3E+00	4.0E+00	5.5E+00
	rank	8	9	15	14	1	5	7	10	4	13	2	3	12	11	6
F_{26}	mean	2.0E+02	1.9E+02	2.0E+02	2.0E+02	1.6E+02	1.2E+02	1.7E+02	1.0E+02	1.5E+02	1.8E+02	1.1E+02	1.0E+02	1.5E+02	1.5E+02	1.1E+02
	std	2.2E+01	1.2E+02	1.0E+00	4.9E+00	5.0E+01	3.7E+01	6.3E+01	5.0E-02	5.0E+01	4.1E+01	3.1E+01	2.0E-01	5.0E+01	5.1E+01	2.6E+01
	rank	13	12	14	15	9	5	10	1	6	11	4	2	8	7	3
F_{27}	mean	6.1E+02	1.4E+03	2.0E+03	3.1E+03	6.4E+02	9.1E+02	8.5E+02	2.1E+03	6.1E+02	1.5E+03	9.2E+02	6.9E+02	9.8E+02	9.6E+02	1.6E+03
	std	8.4E+01	1.1E+02	3.2E+02	9.2E+02	7.5E+01	2.7E+02	1.4E+02	3.3E+01	1.1E+02	1.4E+02	6.1E+01	7.6E+01	1.4E+02	1.4E+02	1.3E+02
	rank	1	10	13	15	3	6	5	14	2	11	7	4	9	8	12
F_{28}	mean	1.6E+03	2.1E+03	8.3E+03	1.1E+04	3.9E+02	1.8E+03	2.7E+03	4.0E+02	1.7E+03	1.6E+03	3.9E+02	1.3E+03	7.2E+02	9.7E+02	2.3E+03
	std	2.9E+02	5.7E+02	2.5E+03	4.0E+03	1.5E+01	1.9E+02	8.2E+02	1.9E+01	3.7E+02	1.4E+03	1.3E+01	2.2E+02	4.7E+02	9.3E+02	7.6E+02
	rank	7	11	14	15	1	10	13	3	9	8	2	6	4	5	12
F_{29}	mean	1.1E+04	3.0E+07	6.3E+07	1.3E+08	2.2E+02	1.4E+03	1.4E+03	2.4E+02	1.3E+03	2.3E+02	2.3E+02	8.9E+02	1.1E+03	1.1E+03	4.6E+07
	std	1.2E+04	1.9E+07	8.0E+07	5.9E+08	1.7E+01	3.1E+02	3.3E+02	4.9E+00	3.8E+02	1.6E+01	1.6E+00	1.0E+02	1.9E+02	2.9E+02	1.6E+07
	rank	11	12	14	15	1	9	10	4	8	3	2	5	6	7	13
F_{30}	mean	1.1E+04	5.9E+04	1.5E+05	2.1E+05	1.2E+03	9.7E+03	1.2E+04	2.1E+03	1.1E+04	1.8E+03	1.1E+03	1.0E+04	1.3E+03	1.3E+03	3.5E+04
	std	9.7E+02	4.5E+04	2.0E+05	1.2E+05	2.9E+02	9.9E+02	1.2E+03	2.0E+02	1.2E+03	3.4E+02	1.8E+02	1.5E+03	2.6E+02	2.3E+02	2.5E+04
	rank	9	13	14	15	2	7	11	6	10	5	1	8	3	4	12

Mean rank	6.40	12.43	13.07	13.43	5.30	4.47	6.93	10.67	5.20	7.33	6.27	3.03	7.97	7.70	9.80
Final rank	6	13	14	15	4	2	7	12	3	8	5	1	10	9	11
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	2.9E+06	4.5E+08	3.2E+08	1.7E+08	2.7E+06	9.8E+05	1.1E+06	3.6E+09	4.7E+06	2.8E+06	6.7E+07	2.6E+05	1.6E+08	1.6E+08	1.1E+08
	std	6.7E+05	2.4E+08	1.2E+08	8.8E+07	1.0E+06	3.0E+05	5.6E+05	8.1E+08	1.5E+06	7.1E+05	2.7E+07	1.9E+05	3.3E+07	4.0E+07	7.3E+07
	rank	6	14	13	12	4	2	3	15	7	5	8	1	10	11	9
F_2	mean	1.8E+04	7.3E+10	1.4E+10	8.2E+06	1.9E+04	2.0E+04	1.6E+04	4.1E+06	1.2E+04	1.4E+04	1.1E+04	3.0E-09	1.2E+04	1.4E+04	3.0E+04
	std	1.7E+04	2.2E+10	3.4E+09	1.1E+07	1.9E+04	1.5E+04	1.5E+04	1.3E+07	1.2E+04	1.1E+04	1.7E+04	5.1E-09	6.9E+03	6.5E+03	2.9E+04
	rank	8	15	14	13	9	10	7	12	4	5	2	1	3	6	11
F_3	mean	9.0E+02	1.8E+05	1.0E+05	3.8E+05	7.4E+02	1.9E-01	3.8E+02	1.1E+06	1.7E+03	2.3E+03	4.1E+03	9.2E-01	9.7E+03	8.0E+03	3.0E+03
	std	3.9E+02	8.1E+04	2.0E+04	1.3E+05	9.0E+02	2.5E-01	3.7E+02	2.8E+05	1.6E+03	1.5E+03	1.4E+03	2.9E+00	5.0E+03	3.4E+03	2.2E+03
	rank	5	13	12	14	4	1	3	15	6	7	9	2	11	10	8
F_4	mean	2.1E+02	8.5E+03	2.0E+03	5.0E+02	1.0E+02	1.4E+02	1.8E+02	9.8E+01	2.1E+02	2.0E+02	9.5E+01	1.3E+02	3.3E+02	3.4E+02	3.3E+02
	std	2.6E+01	4.4E+03	4.3E+02	5.9E+01	2.0E+01	5.2E+01	3.7E+01	9.2E-01	4.7E+01	4.9E+01	2.2E+00	4.7E+01	5.6E+01	4.0E+01	1.1E+02
	rank	9	15	14	13	3	5	6	2	8	7	1	4	10	12	11
F_5	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	3.3E-02	9.0E-02	9.6E-02	3.2E-01	4.0E-02	1.0E-01	2.8E-01	3.2E-02	3.2E-04	3.5E-01	3.9E-02	3.9E-03	3.6E-02	4.6E-02	1.0E-01
	rank	14	13	6	12	5	3	10	15	1	4	8	2	11	9	7
F_6	mean	3.1E+01	9.9E+01	1.4E+02	1.5E+02	4.8E+01	7.6E+01	6.5E+01	1.6E+02	6.1E+01	1.0E+02	7.7E+01	5.8E+01	1.1E+02	1.1E+02	1.2E+02
	std	4.5E+00	6.9E+00	6.5E+00	9.4E+00	7.2E+00	6.9E+00	7.2E+00	1.5E+00	6.8E+00	4.3E+00	4.6E+00	6.6E+00	8.8E+00	7.9E+00	5.8E+00
	rank	1	8	13	14	2	6	5	15	4	9	7	3	11	10	12
F_7	mean	5.3E-01	7.4E+02	1.2E+02	1.1E+00	4.9E-03	9.8E-03	3.8E-03	2.2E-02	2.5E-03	1.2E-03	2.2E-03	9.8E-03	1.1E-02	8.0E-03	5.6E-12
	std	1.6E+00	2.4E+02	3.5E+01	1.8E-01	9.0E-03	2.7E-02	7.3E-03	7.4E-03	4.6E-03	3.0E-03	2.6E-03	1.4E-02	1.8E-02	1.1E-02	3.1E-12
	rank	12	15	14	13	6	8	5	11	4	2	3	9	10	7	1
F_8	mean	1.1E+02	6.2E+02	8.3E+02	6.6E+02	2.0E+02	9.0E-01	2.2E+02	7.6E+02	3.2E+01	7.7E+01	2.4E+02	9.6E-01	4.0E+01	4.4E+01	4.8E+02
	std	1.9E+01	6.2E+01	6.4E+01	8.8E+01	3.7E+01	9.2E-01	5.0E+01	3.0E+01	1.1E+01	3.1E+01	2.6E+01	1.9E+00	9.1E+00	9.3E+00	5.5E+01
	rank	7	12	15	13	8	1	9	14	3	6	10	2	4	5	11
F_9	mean	9.3E+01	8.8E+02	1.0E+03	1.0E+03	1.9E+02	3.5E+02	3.1E+02	9.6E+02	2.9E+02	6.7E+02	4.9E+02	2.1E+02	8.3E+02	8.3E+02	5.6E+02
	std	1.4E+01	1.1E+02	6.5E+01	3.6E+02	4.8E+01	6.0E+01	4.9E+01	2.6E+01	5.1E+01	5.1E+01	1.3E+02	4.0E+01	6.5E+01	6.5E+01	6.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_{10}	mean	1.1E+04	1.5E+04	2.1E+04	1.0E+04	8.9E+03	2.9E+01	4.7E+03	3.1E+04	1.9E+03	2.4E+03	7.6E+03	2.8E+01	3.9E+02	4.3E+02	1.4E+04
	std	1.1E+03	1.8E+03	1.1E+03	3.2E+03	2.0E+03	4.8E+01	6.4E+02	7.9E+02	4.0E+02	5.3E+02	1.1E+03	6.0E+01	2.0E+02	1.8E+02	1.3E+03
	rank	11	13	14	10	9	2	7	15	5	6	8	1	3	4	12
F_{11}	mean	1.1E+04	1.7E+04	2.2E+04	2.8E+04	1.3E+04	1.1E+04	1.3E+04	3.3E+04	1.2E+04	1.3E+04	1.8E+04	1.1E+04	2.6E+04	2.6E+04	1.5E+04
	std	1.3E+03	1.3E+03	1.3E+03	4.6E+03	1.5E+03	6.7E+02	1.7E+03	6.1E+02	1.3E+03	1.1E+03	2.1E+03	5.5E+02	9.9E+02	1.1E+03	1.4E+03
	rank	3	9	11	14	5	2	7	15	4	6	10	1	12	13	8
F_{12}	mean	3.5E+00	1.7E+00	2.8E+00	3.7E+00	1.8E+00	2.1E-01	1.6E+00	4.0E+00	2.8E-01	4.5E-01	2.1E+00	1.8E-01	2.3E+00	2.3E+00	1.5E+00
	std	4.0E-01	1.4E+00	4.8E-01	7.6E-01	2.1E-01	5.5E-02	4.7E-01	2.2E-01	5.9E-02	8.9E-02	2.5E-01	1.6E-02	2.5E-01	3.2E-01	2.8E-01
	rank	13	7	12	14	8	2	6	15	3	4	9	1	10	11	5
F_{13}	mean	3.9E-01	3.7E+00	6.0E-01	5.4E-01	4.2E-01	4.6E-01	6.4E-01	6.9E-01	5.0E-01	5.0E-01	6.1E-01	5.0E-01	5.0E-01	5.1E-01	5.5E-01
	std	6.2E-02	1.2E+00	6.6E-02	6.4E-02	5.4E-02	5.4E-02	6.4E-02	5.7E-02	6.9E-02	5.6E-02	6.0E-02	7.6E-02	4.2E-02	4.4E-02	5.3E-02
	rank	1	15	11	9	2	3	13	14	5	4	12	6	7	8	10
F_{14}	mean	4.2E-01	2.1E+02	4.6E+01	3.5E-01	4.3E-01	3.4E-01	3.5E-01	4.2E-01	3.2E-01	3.2E-01	3.2E-01	3.6E-01	3.4E-01	3.4E-01	6.3E-01
	std	1.8E-01	7.6E+01	1.7E+01	3.5E-02	2.0E-01	2.7E-02	1.3E-01	1.3E-01	3.5E-02	3.0E-02	2.8E-02	1.1E-01	2.4E-02	2.8E-02	2.2E-01
	rank	10	15	14	8	12	4	7	11	1	3	2	9	5	6	13
F_{15}	mean	1.7E+01	1.5E+06	2.1E+04	7.2E+02	2.6E+01	4.3E+01	4.4E+01	8.4E+01	4.5E+01	3.3E+02	7.4E+01	3.9E+01	1.0E+02	1.1E+02	1.8E+02
	std	2.8E+00	1.5E+06	1.4E+04	1.0E+03	1.1E+01	6.7E+00	7.9E+00	3.2E+00	1.1E+01	5.2E+01	1.1E+01	7.5E+00	1.6E+01	1.2E+01	1.1E+02
	rank	1	15	14	13	2	4	5	8	6	12	7	3	9	10	11
F_{16}	mean	4.2E+01	4.4E+01	4.5E+01	4.7E+01	4.2E+01	4.1E+01	4.3E+01	4.8E+01	4.1E+01	4.3E+01	4.5E+01	4.0E+01	4.6E+01	4.6E+01	4.5E+01
	std	1.7E+00	1.3E+00	6.9E-01	5.2E-01	9.9E-01	7.5E-01	1.2E+00	2.0E-01	1.2E+00	8.3E-01	5.2E-01	7.2E-01	4.4E-01	5.2E-01	5.8E-01
	rank	4	8	11	14	5	2	6	15	3	7	10	1	12	13	9
F_{17}	mean	5.4E+05	2.4E+07	1.7E+07	3.5E+07	1.0E+06	1.9E+05	4.0E+05	1.1E+08	7.3E+05	4.6E+05	9.3E+06	3.5E+04	1.7E+07	1.9E+07	7.3E+06
	std	2.2E+05	2.7E+07	5.8E+06	2.1E+07	3.4E+05	6.0E+04	1.8E+05	2.4E+07	2.9E+05	1.7E+05	4.5E+06	3.2E+04	6.6E+06	8.3E+06	5.0E+06
	rank	5	13	10	14	7	2	3	15	6	4	9	1	11	12	8
F_{18}	mean	1.7E+03	9.5E+08	2.1E+08	1.4E+03	3.1E+03	6.9E+02	2.3E+03	4.2E+06	2.1E+03	1.5E+03	1.7E+03	1.5E+03	6.4E+02	1.0E+03	4.7E+03
	std	1.3E+03	1.6E+09	1.6E+08	1.5E+03	3.6E+03	3.1E+02	2.3E+03	4.7E+06	2.4E+03	1.4E+03	8.0E+02	1.6E+03	4.6E+02	1.1E+03	4.5E+03
	rank	8	15	14	4	11	2	10	13	9	5	7	6	1	3	12
F_{19}	mean	1.1E+02	5.6E+02	3.4E+02	1.8E+02	6.3E+01	7.1E+01	8.2E+01	7.4E+01	1.0E+02	9.1E+01	5.9E+01	9.5E+01	1.4E+02	1.3E+02	1.2E+02
	std	4.3E+00	1.5E+02	1.2E+02	7.6E+01	1.9E+01	2.7E+01	3.6E+01	1.9E+00	1.6E+01	3.5E+01	2.4E+00	2.4E+01	2.7E+01	2.5E+01	1.1E+01
	rank	9	15	14	13	2	3	5	4	8	6	1	7	12	11	10
F_{20}	mean	8.0E+02	1.1E+05	3.7E+04	1.1E+06	1.1E+03	5.2E+02	1.4E+03	5.3E+05	1.8E+03	2.6E+03	5.0E+03	6.8E+03	5.2E+04	4.9E+04	6.8E+03
	std	1.6E+02	6.0E+04	1.9E+04	8.9E+05	3.7E+02	7.8E+01	5.6E+02	1.2E+05	8.0E+02	7.9E+02	1.4E+03	5.3E+03	1.7E+04	1.4E+04	2.1E+03
	rank	2	13	10	15	3	1	4	14	5	6	7	8	12	11	9
F_{21}	mean	2.5E+05	6.4E+06	5.6E+06	1.7E+07	4.2E+05	8.0E+04	1.1E+05	4.6E+07	3.1E+05	2.2E+05	4.4E+06	1.8E+04	8.4E+06	8.4E+06	2.1E+06
	std	9.7E+04	5.4E+06	3.0E+06	2.0E+07	1.8E+05	2.8E+04	6.7E+04	1.2E+07	1.5E+05	5.7E+04	1.9E+06	2.9E+04	9.5E+06	7.5E+06	2.3E+06
	rank	5	11	10	14	7	2	3	15	6	4	9	1	13	12	8
F_{22}	mean	1.3E+03	3.5E+03	3.9E+03	5.0E+03	1.4E+03	1.9E+03	2.3E+03	5.4E+03	2.1E+03	3.0E+03	2.0E+03	1.7E+03	1.8E+03	1.8E+03	2.5E+03
	std	3.7E+02	5.8E+02	8.1E+02	1.1E+03	3.4E+02	3.5E+02	4.7E+02	2.3E+02	4.6E+02	4.7E+02	4.8E+02	2.5E+02	3.3E+02	3.1E+02	4.1E+02
	rank	1	12	13	14	2	6	9	15	8	11	7	3	5	4	10
F_{23}	mean	3.5E+02	6.4E+02	5.0E+02	3.6E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02
	std	1.8E+00	1.8E+02	6.4E+01	2.8E+00	3.8E-11	2.1E-11	5.4E-10	2.3E-01	2.8E-10	1.6E-10	4.7E-03	2.8E-12	8.9E-03	1.3E-02	7.8E+00
	rank	11	15	14	13	1	8	10	6	9	2	3	7	4	5	12
F_{24}	mean	3.9E+02	6.7E+02	5.9E+02	4.2E+02	3.9E+02	3.6E+02	3.8E+02	3.6E+02	3.8E+02	3.4E+02	3.8E+02	4.2E+02	3.7E+02	3.7E+02	4.1E+02
	std	4.5E+00	8.3E+01	4.1E+01	2.1E+01	5.6E+00	7.5E+00	5.0E+00	2.3E+00	3.6E+00	1.2E+01	3.9E+00	8.1E+00	3.3E+00	2.8E+00	8.2E+00
	rank	9	15	14	13	10	2	7	3	6	1	8	12	4	5	11
F_{25}	mean	2.6E+02	2.8E+02	3.5E+02	2.8E+02	2.1E+02	2.6E+02	2.8E+02	5.2E+02	2.7E+02	3.0E+02	2.1E+02	2.8E+02	2.8E+02	2.8E+02	2.6E+02
	std	4.3E+00	2.8E+01	2.3E+01	2.1E+01	2.8E+01	5.2E+00	1.1E+01	6.2E+01	1.2E+01	1.1E+01	1.2E+00	1.2E+01	2.0E+01	9.8E+00	1.7E+01
	rank	4	10	14	12	2	3	8	15	6	13	1	11	7	9	5
F_{26}	mean	2.0E+02	2.3E+02	2.1E+02	2.1E+02	2.0E+02	2.0E+02	2.0E+02	1.0E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.1E+02	2.1E+02	1.9E+02
	std	1.1E-01	2.0E+01	4.7E+00	6.6E+00	6.2E-02	2.6E-02	9.3E-02	5.6E-02	9.9E-02	2.9E-02	3.8E+01	9.5E-03	3.0E+00	2.0E+00	3.7E+01
	rank	7	15	14	13	9	5	8	1	10	6	2	4	12	11	3
F_{27}	mean	1.2E+03	2.9E+03	4.4E+03	6.7E+03	1.6E+03	2.1E+03	2.0E+03	4.4E+03	1.8E+03	3.1E+03	2.3E+03	1.6E+03	1.6E+03	1.7E+03	3.5E+03
	std	1.7E+02	1.6E+02	2.2E+02	1.7E+03	2.0E+02	7.4E+02	2.3E+02	5.1E+01	4.5E+02	2.0E+02	1.2E+02	1.4E+02	3.0E+02	2.7E+02	1.4E+02
	rank	1	10	13	15	2	8	7	14	6	11	9	3	4	5	12
F_{28}	mean	3.8E+03	5.0E+03	2.0E+04	3.0E+04	7.6E+02	5.3E+03	7.9E+03	2.6E+03	6.4E+03	7.1E+03	6.7E+02	3.3E+03	7.1E+03	6.2E+03	6.3E+03
	std	6.2E+02	7.9E+02	5.3E+03	9.7E+03	2.6E+02	6.4E+02	1.4E+03	6.8E+02	1.1E+03	2.8E+03	1.0E+02	6.2E+02	4.2E+03	4.1E+03	7.8E+02
	rank	5	6	14	15	2	7	13	3	10	11	1	4	12	8	9
F_{29}	mean	1.1E+04	9.8E+07	4.9E+08	5.0E+05	2.6E+02	1.5E+03	1.6E+03	3.0E+02	1.8E+03	2.9E+02	2.6E+02	1.5E+03	8.1E+02	8.4E+02	2.2E+08
	std	9.8E+03	4.5E+07	4.4E+08	1.7E+06	3.4E+00	2.7E+02	2.6E+02	2.8E+01	2.7E+02	4.9E+01	2.5E+00	1.7E+02	6.5E+02	5.1E+02	5.8E+07
	rank	11	13	15	12	2	8	9	4	10	3	1	7	5	6	14
F_{30}	mean	1.5E+04	6.6E+05	7.1E+05	5.3E+05	3.0E+03	9.4E+03	1.2E+04	6.8E+03	1.0E+04	4.0E+03	2.8E+03	9.9E+03	3.0E+03	3.2E+03	5.1E+05
	std	4.8E+03	4.7E+05	4.6E+05	8.6E+05	4.5E+02	1.0E+03	3.4E+03	3.3E+02	1.4E+03	3.7E+02	4.8E+02	1.3E+03	4.2E+02	4.0E+02	3.8E+05

rank	11	14	15	13	2	7	10	6	9	5	1	8	3	4	12
Mean rank	6.50	12.53	12.87	12.70	4.93	4.17	7.00	11.10	5.87	6.13	5.97	4.37	8.13	8.37	9.37
Final rank	7	13	15	14	3	1	8	12	4	6	5	2	9	10	11
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'14 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	4.4E+04	1.9E+06	2.4E+05	3.2E+07	2.6E+04	1.7E+03	2.4E+03	5.5E+05	3.7E+03	1.9E+04	3.2E+04	0.0E+00	2.5E+05	4.1E+05	9.0E+04
	std	7.4E+04	7.2E+06	5.2E+05	2.0E+07	3.7E+04	1.4E+03	2.3E+03	4.8E+05	3.6E+03	2.0E+04	3.0E+04	0.0E+00	2.5E+05	4.5E+05	6.6E+04
	rank	8	14	10	15	6	2	3	13	4	5	7	1	11	12	9
F_2	mean	4.8E+02	5.0E+03	2.4E+03	4.8E+08	1.5E+03	4.8E+02	1.1E+03	8.3E+03	1.1E+03	1.1E+03	3.7E+02	0.0E+00	1.9E+01	2.9E+01	1.5E+03
	std	6.7E+02	4.6E+03	3.0E+03	3.4E+08	2.5E+03	6.6E+02	1.8E+03	1.9E+04	9.8E+02	9.9E+02	7.3E+02	0.0E+00	5.5E+01	6.6E+01	2.4E+03
	rank	5	13	12	15	11	6	9	14	7	8	4	1	2	3	10
F_3	mean	6.0E+01	1.1E+03	3.2E-01	1.2E+04	2.1E+02	2.4E+01	3.5E+02	3.8E+03	6.1E+01	5.0E+02	2.4E+02	0.0E+00	7.3E+01	7.1E+01	3.8E+02
	std	8.4E+01	3.6E+03	7.4E-01	5.8E+03	3.7E+02	4.4E+01	4.2E+02	2.8E+03	1.0E+02	8.6E+02	2.3E+02	0.0E+00	7.7E+01	8.3E+01	6.1E+02
	rank	4	13	2	15	8	3	10	14	5	12	9	1	7	6	11
F_4	mean	3.0E+01	1.6E+01	2.9E+01	5.5E+02	5.8E-01	2.8E-01	2.8E+01	4.1E+00	1.5E+01	1.9E+00	4.3E+00	2.8E+01	9.0E-01	4.0E-01	1.0E+01
	std	1.3E+01	1.6E+01	1.1E+01	5.4E+02	2.4E-01	9.6E-01	1.4E+01	2.0E-01	1.7E+01	1.4E+00	5.9E-01	1.4E+01	1.7E+00	5.9E-01	1.3E+01
	rank	14	10	13	15	3	1	11	6	9	5	7	12	4	2	8
F_5	mean	1.9E+01	2.0E+01	2.0E+01	2.0E+01	1.8E+01	1.6E+01	1.9E+01	2.0E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01	1.9E+01	1.9E+01	2.0E+01
	std	4.5E+00	1.4E-01	3.9E-04	5.1E-01	6.2E+00	8.1E+00	4.5E+00	7.2E-02	2.0E-02	3.8E+00	4.1E+00	3.0E-03	3.2E+00	2.6E+00	5.4E-02
	rank	7	14	9	13	2	1	4	15	11	3	6	10	8	5	12
F_6	mean	2.1E-01	3.5E+00	5.4E+00	7.2E+00	4.1E-03	7.0E-01	3.1E-01	6.3E+00	4.9E-02	2.8E+00	1.4E-01	2.1E-01	7.3E-01	8.6E-01	1.9E+00
	std	1.5E-01	1.4E+00	1.5E+00	1.5E+00	1.5E-02	6.1E-01	5.5E-01	1.0E+00	2.0E-01	8.4E-01	1.5E-01	3.5E-01	4.6E-01	5.6E-01	1.2E+00
	rank	4	12	13	15	1	7	6	14	2	11	3	5	8	9	10
F_7	mean	1.3E-01	5.6E-01	2.9E-01	7.2E+01	7.6E-02	7.9E-02	1.3E-01	6.7E-02	1.0E-01	4.8E-01	5.0E-02	1.7E-02	1.9E-02	1.9E-02	4.8E-02
	std	6.1E-02	1.4E+00	1.9E-01	2.8E+01	4.3E-02	4.2E-02	8.1E-02	5.6E-02	5.9E-02	2.7E-01	2.6E-02	1.9E-02	1.3E-02	9.8E-03	2.1E-02
	rank	10	14	12	15	7	8	11	6	9	13	5	1	2	3	4
F_8	mean	6.1E+00	1.0E+01	2.8E+01	2.9E+01	3.0E+00	0.0E+00	2.9E+00	1.1E+00	7.5E-01	1.0E+00	3.1E+00	0.0E+00	2.5E-01	9.9E-02	6.6E+00
	std	2.5E+00	4.2E+00	1.1E+01	1.1E+01	1.6E+00	0.0E+00	1.6E+00	6.8E-01	8.5E-01	1.1E+00	1.4E+00	0.0E+00	5.5E-01	3.1E-01	2.4E+00
	rank	10	12	13	14	8	1	7	6	4	5	9	1	3	2	11
F_9	mean	6.8E+00	1.6E+01	2.8E+01	4.0E+01	5.4E+00	6.4E+00	7.6E+00	8.2E+00	6.4E+00	1.2E+01	5.4E+00	3.7E+00	5.6E+00	6.6E+00	1.2E+01
	std	3.6E+00	8.6E+00	7.9E+00	1.2E+01	2.4E+00	2.0E+00	2.8E+00	2.8E+00	2.1E+00	6.2E+00	1.8E+00	1.2E+00	1.4E+00	1.6E+00	5.2E+00
	rank	8	13	14	15	3	6	9	10	5	11	2	1	4	7	12
F_{10}	mean	2.8E+02	2.4E+02	7.4E+02	1.6E+02	9.3E+01	2.4E+00	1.5E+02	5.1E+02	1.0E+02	1.5E+02	7.4E+01	4.4E-02	1.8E+01	6.4E+00	1.6E+02
	std	8.9E+01	1.7E+02	2.5E+02	9.4E+01	6.6E+01	3.0E+00	1.3E+02	1.9E+02	7.5E+01	1.3E+02	5.7E+01	4.6E-02	4.3E+01	2.6E+01	1.2E+02
	rank	13	12	15	10	6	2	8	14	7	9	5	1	4	3	11
F_{11}	mean	3.1E+02	5.5E+02	1.0E+03	1.0E+03	1.8E+02	2.0E+02	2.9E+02	1.3E+03	2.1E+02	3.6E+02	1.6E+02	1.5E+02	2.4E+02	2.5E+02	4.0E+02
	std	1.5E+02	3.2E+02	2.8E+02	2.7E+02	1.5E+02	1.4E+02	1.8E+02	2.0E+02	1.9E+02	2.3E+02	1.4E+02	1.1E+02	1.1E+02	9.7E+01	1.7E+02
	rank	9	12	13	14	3	4	8	15	5	10	2	1	6	7	11
F_{12}	mean	3.6E-01	1.7E-01	3.6E-01	7.0E-01	3.9E-01	2.4E-01	1.9E-01	1.1E+00	1.5E-01	1.4E-01	3.5E-01	6.6E-02	2.6E-01	2.8E-01	1.8E-01
	std	1.8E-01	8.6E-02	1.9E-01	2.9E-01	1.5E-01	1.2E-01	1.2E-01	2.7E-01	9.3E-02	8.5E-02	1.6E-01	1.9E-02	7.3E-02	6.4E-02	6.8E-02
	rank	11	4	12	14	13	7	6	15	3	2	10	1	8	9	5
F_{13}	mean	7.0E-02	2.2E-01	3.2E-01	2.2E+00	4.2E-02	1.2E-01	8.9E-02	1.4E-01	7.8E-02	4.2E-01	7.2E-02	1.0E-01	1.6E-01	1.4E-01	1.5E-01
	std	2.3E-02	7.8E-02	1.1E-01	9.9E-01	2.2E-02	3.7E-02	3.7E-02	3.6E-02	3.8E-02	1.8E-01	2.6E-02	2.8E-02	3.4E-02	3.7E-02	5.4E-02
	rank	2	12	13	15	1	7	5	8	4	14	3	6	11	9	10
F_{14}	mean	8.1E-02	2.5E-01	3.4E-01	1.3E+01	6.4E-02	1.1E-01	9.9E-02	1.8E-01	9.2E-02	4.4E-01	1.1E-01	1.2E-01	2.5E-01	2.5E-01	1.8E-01
	std	3.0E-02	1.9E-01	1.9E-01	6.9E+00	2.7E-02	4.3E-02	3.5E-02	4.4E-02	5.4E-02	2.7E-01	3.9E-02	4.4E-02	6.5E-02	6.7E-02	4.4E-02
	rank	2	10	13	15	1	5	4	9	3	14	6	7	11	12	8
F_{15}	mean	9.8E-01	9.9E-01	1.6E+00	6.5E+01	7.5E-01	8.0E-01	1.0E+00	1.7E+00	8.5E-01	2.3E+00	1.2E+00	4.2E-01	1.1E+00	1.0E+00	7.5E-01
	std	3.3E-01	3.2E-01	7.3E-01	1.1E+02	1.4E-01	2.3E-01	4.1E-01	4.0E-01	2.7E-01	1.1E+00	3.4E-01	1.0E-01	2.6E-01	1.8E-01	2.0E-01
	rank	6	7	12	15	2	4	9	13	5	14	11	1	10	8	3
F_{16}	mean	2.2E+00	2.4E+00	3.1E+00	3.0E+00	1.1E+00	1.5E+00	1.9E+00	3.0E+00	1.5E+00	2.5E+00	2.1E+00	1.8E+00	1.9E+00	2.1E+00	2.6E+00
	std	3.6E-01	5.0E-01	4.3E-01	2.9E-01	5.3E-01	4.9E-01	5.5E-01	2.0E-01	6.0E-01	3.0E-01	3.8E-01	3.3E-01	2.7E-01	3.1E-01	2.4E-01
	rank	9	10	15	13	1	3	5	14	2	11	8	4	6	7	12
F_{17}	mean	6.8E+02	5.2E+03	5.9E+02	2.0E+05	1.1E+03	1.1E+03	2.0E+03	7.6E+03	1.3E+03	2.6E+03	1.2E+03	7.6E+01	3.0E+04	5.1E+04	1.7E+03
	std	4.3E+02	3.3E+03	2.8E+02	2.3E+05	1.3E+03	8.8E+02	2.0E+03	4.9E+03	1.4E+03	2.4E+03	1.0E+03	9.0E+01	5.5E+04	6.1E+04	1.9E+03
	rank	3	11	2	15	5	4	9	12	7	10	6	1	13	14	8
F_{18}	mean	4.6E+03	1.6E+04	1.4E+02	6.9E+03	5.1E+03	1.7E+02	8.1E+03	4.4E+03	2.2E+03	8.1E+03	4.4E+02	1.0E+00	2.9E+02	5.1E+02	4.0E+02
	std	1.6E+03	1.4E+04	8.4E+01	4.8E+03	5.9E+03	2.0E+02	6.8E+03	4.0E+03	3.4E+03	6.0E+03	6.6E+02	1.2E+00	4.9E+02	7.0E+02	3.8E+02
	rank	10	15	2	12	11	3	13	9	8	14	6	1	4	7	5
F_{19}	mean	1.5E+00	2.4E+00	4.1E+00	1.8E+01	7.4E-01	5.0E-01	1.2E+00	2.5E+00	5.7E-01	2.0E+00	1.1E+00	5.0E-01	6.6E-01	5.8E-01	1.6E+00
	std	4.7E-02	1.1E+00	1.2E+00	1.6E+01	4.5E-01	4.0E-01	6.2E-01	3.9E-01	4.7E-01	7.1E-01	3.2E-01	5.6E-01	3.1E-01	3.3E-01	5.2E-01
	rank	9	12	14	15	6	2	8	13	3	11	7	1	5	4	10
F_{20}	mean	1.7E+02	1.8E+03	6.1E+01	3.5E+03	2.3E+02	2.3E+01	8.3E+02	1.0E+03	7.0E+01	1.2E+03	1.7E+02	3.2E-01	3.2E+01	3.8E+01	2.4E+02
	std	1.5E+02	7.2E+03	5.4E+01	3.4E+03	3.3E+02	1.9E+01	1.1E+03	7.9E+02	1.1E+02	1.5E+03	1.4E+02	3.0E-01	4.1E+01	6.5E+01	1.9E+02
	rank	8	14	5	15	9	2	11	12	6	13	7	1	3	4	10
F_{21}	mean	4.9E+02	1.7E+03	3.1E+02	5.5E+04	1.2E+02	6.0E+01	1.3E+02	1.3E+03	6.2E+01	9.6E+02	2.4E+02	2.0E+01	2.0E+03	2.6E+03	2.6E+02
	std	3.6E+02	6.6E+03	1.5E+02	8.2E+04	1.2E+02	6.6E+01	1.1E+02	7.5E+02	6.7E+01	9.3E+02	1.3E+02	3.8E+01	2.3E+03	2.5E+03	1.8E+02
	rank	9	12	8	15	4	2	5	11	3	10	6	1	13	14	7
F_{22}	mean	1.1E+02	2.9E+01	1.5E+02	1.3E+02	2.2E+01	1.0E+01	7.1E+01	2.4E+01	9.4E+00	5.8E+01	2.3E+01	2.2E+00	1.6E+01	2.0E+01	2.0E+01
	std	5.1E+01	1.5E+01	6.1E+01	4.7E+01	3.0E+01	9.8E+00	6.0E+01	1.9E+00	1.0E+01	5.2E+01	3.0E+01	6.2E+00	3.6E+01	4.3E+01	8.5E+00
	rank	13	10	15	14	7	3	12	9	2	11	8	1	4	6	5
F_{23}	mean	3.3E+02	3.3E+02	3.3E+02	3.8E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.3E+02	3.0E+02	3.3E+02	3.3E+02
	std	8.1E-13	6.5E+00	4.9E+00	5.8E+01	1.7E-13	2.2E-13	1.7E-13	1.7E-13	2.5E-13	3.4E-12	1.7E-13	1.7E-13	8.0E+01	1.8E+01	2.6E-13
	rank	7	10	9	11	3	4	6	3	4	8	3	3	1	2	5
F_{24}	mean	1.1E+02	1.3E+02	1.7E+02	1.9E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.1E+02	1.5E+02	1.1E+02	1.1E+02	1.1E+02	1.2E+02	1.2E+02
	std	4.4E+00	2.0E+01	3.0E+01	1.8E+01	2.4E+00	3.7E+00	3.8E+00	3.4E+00	4.0E+00	3.1E+01	2.4E+00	2.5E+00	3.4E+00	3.2E+00	4.6E+00
	rank	1	12	14	15	4	8	6	3	7	13	5	2	9	10	11
F_{25}	mean	1.3E+02	1.9E+02	2.0E+02	1.9E+02	1.5E+02	1.3E+02	1.7E+02	1.9E+02	1.5E+02	1.9E+02	1.4E+02	1.5E+02	1.6E+02	1.6E+02	1.6E+02
	std	1.5E+01	2.0E+01	1.5E+00	8.7E+00	4.1E+01	1.4E+01	3.4E+01	2.3E+01	3.8E+01	1.9E+01	3.2E+01	3.9E+01	1.3E+01	1.6E+01	2.9E+01
	rank	1	13	15	14	5	2	10	11	6	12	3	4	7	8	9
F_{26}	mean	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02	1.0E+02
	std	2.1E-02	1.4E-01	1.1E-01	4.0E+00	2.1E-02	4.2E-02	4.6E-02	2.5E-02	4.2E-02	1.3E-01	2.5E-02	2.0E-02	5.2E-02	4.7E-02	5.5E-02
	rank	2	12	14	15	1	7	6	9	4	13	3	5	11	10	8
F_{27}	mean	2.0E+02	3.7E+02	3.9E+02	3.9E+02	2.1E+02	1.2E+02	3.1E+02	5.0E+02	1.9E+02	2.6E+02	1.8E+02	2.1E+02	8.3E+01	1.4E+02	3.4E+02
	std	1.9E+02	1.3E+02	9.9E+01	1.7E+02	1.8E+02	1.7E+02	1.4E+02	4.6E+01	2.0E+02	1.9E+02	1.7E+02	1.8E+02	1.5E+02	1.6E+02	1.2E+02
	rank	6	12	14	13	8	2	10	15	5	9	4	7	1	3	11
F_{28}	mean	2.8E+02	4.2E+02	8.1E+02	1.1E+03	3.3E+02	3.9E+02	3.8E+02	3.1E+02	3.7E+02	3.5E+02	3.1E+02	4.2E+02	3.5E+02	3.4E+02	4.1E+02
	std	1.7E+02	6.2E+01	2.5E+02	2.2E+02	4.3E+01	3.4E+01	1.5E+02	1.5E-02	1.3E+02	7.0E+01	1.3E-02	5.3E+01	5.5E+01	4.3E+01	1.9E+01
	rank	1	13	14	15	4	10	9	3	8	7	2	12	6	5	11
F_{29}	mean	3.5E+02	8.7E+04	3.3E+05	8.9E+06	2.0E+02	2.8E+02	1.1E+05	2.1E+02	3.5E+02	2.2E+02	2.0E+02	8.6E+04	2.6E+02	2.5E+02	5.1E+02
	std	9.9E+01	3.9E+05	1.1E+06	1.5E+07	2.9E+00	5.6E+01	4.8E+05	1.1E+00	8.6E+01	3.7E+01	5.9E-01	3.9E+05	2.2E+01	1.8E+01	1.1E+02
	rank	8	12	14	15	2	7	13	3	9	4	1	11	6	5	10
F_{30}	mean	1.7E+03	9.7E+02	1.8E+03	5.7E+03	3.5E+02	6.8E+02	1.1E+03	2.7E+02	7.6E+02	9.7E+02	3.2E+02	5.0E+02	4.3E+02	4.4E+02	5.7E+02
	std	2.4E+02	3.9E+02	7.1E+02	1.2E+04	9.9E+01	1.3E+02	4.2E+02	1.8E+01	1.9E+02	5.1E+02	6.3E+01	4.4E+01	4.3E+01	7.8E+01	1.3E+02

	rank	13	11	14	15	3	8	12	1	9	10	2	6	4	5	7
Mean rank		7.10	11.70	11.50	14.23	4.93	4.37	8.17	10.00	5.53	9.73	5.33	3.67	6.40	6.70	8.80
Final rank		8	14	13	15	3	2	9	12	5	11	4	1	6	7	10
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'14 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.1E+05	1.6E+07	1.5E+07	1.1E+08	3.8E+05	5.5E+04	1.1E+05	5.6E+07	2.9E+05	1.9E+05	2.5E+06	9.9E+02	8.3E+06	1.0E+07	4.9E+06
	std	3.7E+05	1.7E+07	1.2E+07	4.1E+07	2.3E+05	4.0E+04	7.1E+04	2.6E+07	2.2E+05	1.2E+05	1.1E+06	2.5E+03	3.6E+06	6.1E+06	4.7E+06
	rank	7	13	12	15	6	2	3	14	5	4	8	1	10	11	9
F_2	mean	2.7E+03	2.9E+09	4.8E+08	9.6E+08	4.9E+01	1.8E-03	5.6E+01	1.6E+04	3.0E+01	2.9E+03	3.5E+03	3.5E-14	3.4E+03	4.9E+03	1.4E+07
	std	2.2E+03	3.1E+09	3.8E+08	8.7E+08	8.0E+01	3.9E-03	1.1E+02	1.4E+04	3.9E+01	1.7E+03	5.7E+03	1.6E-14	3.0E+03	3.5E+03	7.8E+07
	rank	6	15	13	14	4	2	5	11	3	7	9	1	8	10	12
F_3	mean	5.7E+01	2.0E+04	3.1E+03	4.5E+04	2.4E+01	1.2E-02	2.3E+01	3.7E+04	2.6E+01	3.7E+02	4.9E+02	9.7E-14	2.3E+02	3.0E+02	3.0E+02
	std	7.5E+01	2.0E+04	2.2E+03	3.1E+04	3.5E+01	3.0E-02	2.4E+01	2.6E+04	2.7E+01	3.3E+02	3.5E+02	8.7E-14	2.6E+02	2.7E+02	3.7E+02
	rank	6	13	12	15	4	2	3	14	5	10	11	1	7	8	9
F_4	mean	7.6E+01	2.0E+02	1.9E+02	3.7E+02	2.6E+01	3.7E+00	4.3E+00	2.6E+01	4.2E+01	4.5E+01	2.5E+01	1.3E-12	8.8E+01	7.5E+01	7.9E+01
	std	1.6E+01	1.6E+02	5.6E+01	1.4E+02	1.8E+01	1.5E+01	1.6E+01	4.0E-01	3.6E+01	2.9E+01	1.1E+00	1.6E-12	2.7E+01	2.5E+01	3.3E+01
	rank	10	14	13	15	6	2	3	5	7	8	4	1	12	9	11
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.0E+01	2.1E+01
	std	6.9E-02	3.0E-01	7.5E-02	1.6E-01	7.2E-02	5.9E-02	3.4E-01	4.9E-02	1.6E-02	1.9E-04	6.0E-02	5.2E-03	6.8E-02	8.3E-02	1.4E-01
	rank	14	10	4	13	6	5	11	15	2	1	12	3	7	8	9
F_6	mean	3.5E+00	1.9E+01	3.0E+01	3.6E+01	3.7E+00	1.1E+01	6.6E+00	3.9E+01	3.6E+00	2.0E+01	8.3E+00	5.5E+00	1.5E+01	1.6E+01	2.1E+01
	std	1.1E+00	3.1E+00	3.8E+00	5.0E+00	2.3E+00	2.2E+00	2.3E+00	1.4E+00	2.0E+00	2.5E+00	1.6E+00	2.6E+00	1.8E+00	2.0E+00	2.9E+00
	rank	1	10	13	14	3	7	5	15	2	11	6	4	8	9	12
F_7	mean	1.3E-02	2.2E+01	8.0E+00	1.5E+01	7.9E-03	4.7E-03	1.6E-02	5.4E-05	5.3E-03	1.3E-02	1.4E-03	3.2E-03	7.3E-03	6.5E-04	1.8E-03
	std	1.1E-02	1.8E+01	5.4E+00	1.6E+01	1.1E-02	7.7E-03	1.9E-02	2.2E-04	8.2E-03	1.7E-02	4.3E-03	6.1E-03	1.3E-02	1.3E-03	4.6E-03
	rank	11	15	13	14	9	6	12	1	7	10	3	5	8	2	4
F_8	mean	2.5E+01	8.0E+01	1.4E+02	1.5E+02	3.3E+01	9.9E-02	3.1E+01	7.2E+01	9.0E+00	8.8E+00	3.3E+01	2.7E-13	2.1E+00	1.8E+00	6.7E+01
	std	5.8E+00	2.0E+01	2.4E+01	4.6E+01	7.4E+00	3.1E-01	9.6E+00	1.4E+01	3.0E+00	4.3E+00	6.5E+00	5.7E-14	2.1E+00	1.4E+00	1.1E+01
	rank	7	13	14	15	9	2	8	12	6	5	10	1	4	3	11
F_9	mean	3.0E+01	1.4E+02	1.6E+02	2.3E+02	3.7E+01	5.0E+01	5.5E+01	1.6E+02	4.5E+01	1.2E+02	5.9E+01	2.4E+01	6.7E+01	7.1E+01	9.4E+01
	std	9.1E+00	3.1E+01	2.3E+01	7.6E+01	8.4E+00	1.5E+01	1.4E+01	1.5E+01	1.2E+01	2.3E+01	1.2E+01	5.4E+00	1.2E+01	1.2E+01	2.0E+01
	rank	2	12	13	15	3	5	6	14	4	11	7	1	8	9	10
F_{10}	mean	1.4E+03	2.6E+03	4.1E+03	9.7E+02	1.1E+03	5.8E+00	6.6E+02	6.3E+03	3.8E+02	5.1E+02	8.6E+02	2.4E-01	3.6E+01	4.3E+00	2.7E+03
	std	4.0E+02	4.1E+02	7.2E+02	4.7E+02	5.1E+02	1.6E+00	3.7E+02	3.5E+02	1.9E+02	1.7E+02	3.2E+02	7.6E-01	6.3E+01	2.2E+01	5.0E+02
	rank	11	12	14	9	10	3	7	15	5	6	8	1	4	2	13
F_{11}	mean	2.6E+03	3.8E+03	4.4E+03	6.0E+03	2.6E+03	2.0E+03	2.7E+03	7.6E+03	2.3E+03	3.0E+03	2.7E+03	1.7E+03	3.1E+03	3.4E+03	3.4E+03
	std	6.4E+02	6.5E+02	7.5E+02	1.2E+03	4.6E+02	3.3E+02	5.5E+02	2.9E+02	5.2E+02	6.6E+02	4.6E+02	2.1E+02	4.8E+02	4.5E+02	4.5E+02
	rank	4	12	13	14	5	2	6	15	3	8	7	1	9	11	10
F_{12}	mean	1.4E+00	3.1E-01	1.3E+00	1.8E+00	8.6E-01	1.7E-01	6.2E-01	2.5E+00	1.9E-01	3.2E-01	9.4E-01	1.2E-01	6.5E-01	6.1E-01	5.4E-01
	std	5.4E-01	1.1E-01	3.9E-01	4.5E-01	1.9E-01	8.0E-02	3.5E-01	2.2E-01	7.3E-02	1.3E-01	3.2E-01	1.9E-02	1.3E-01	1.3E-01	1.5E-01
	rank	13	4	12	14	10	2	8	15	3	5	11	1	9	7	6
F_{13}	mean	1.8E-01	7.3E-01	5.6E-01	4.7E-01	1.4E-01	2.6E-01	2.9E-01	3.4E-01	2.2E-01	3.7E-01	3.5E-01	3.0E-01	2.8E-01	2.9E-01	4.1E-01
	std	4.0E-02	5.4E-01	1.3E-01	1.1E-01	2.6E-02	6.3E-02	5.9E-02	5.0E-02	6.6E-02	1.0E-01	4.5E-02	4.8E-02	3.6E-02	3.8E-02	6.8E-02
	rank	2	15	14	13	1	4	6	9	3	11	10	8	5	7	12
F_{14}	mean	1.9E-01	8.4E+00	2.0E+00	2.4E+00	3.0E-01	2.6E-01	2.3E-01	3.0E-01	2.4E-01	2.6E-01	2.4E-01	3.0E-01	2.4E-01	2.3E-01	5.4E-01
	std	2.9E-02	1.3E+01	3.6E+00	4.8E+00	1.6E-01	2.5E-02	3.5E-02	3.9E-02	5.0E-02	5.2E-02	2.6E-02	1.4E-01	3.2E-02	2.9E-02	2.4E-01
	rank	1	15	13	14	9	8	2	10	6	7	4	11	5	3	12
F_{15}	mean	4.6E+00	4.7E+02	9.4E+01	1.5E+02	4.4E+00	5.0E+00	5.4E+00	1.5E+01	4.1E+00	4.2E+01	8.8E+00	2.5E+00	1.1E+01	1.1E+01	6.9E+00
	std	1.4E+00	6.8E+02	9.5E+01	1.3E+02	1.7E+00	1.0E+00	1.5E+00	9.6E-01	1.0E+00	1.6E+01	2.2E+00	4.0E-01	2.7E+00	2.1E+00	1.8E+00
	rank	4	15	13	14	3	5	6	11	2	12	8	1	10	9	7
F_{16}	mean	1.1E+01	1.1E+01	1.2E+01	1.3E+01	9.7E+00	9.5E+00	1.0E+01	1.3E+01	1.0E+01	1.1E+01	1.1E+01	9.4E+00	1.1E+01	1.1E+01	1.2E+01
	std	3.6E-01	5.9E-01	5.3E-01	4.6E-01	7.2E-01	8.9E-01	8.8E-01	2.4E-01	7.2E-01	6.3E-01	3.4E-01	3.9E-01	5.0E-01	4.2E-01	4.2E-01
	rank	6	10	13	14	3	2	5	15	4	9	11	1	8	7	12
F_{17}	mean	5.0E+04	9.4E+05	6.5E+05	5.2E+06	7.7E+04	2.2E+04	2.6E+04	1.4E+06	6.3E+04	5.8E+04	2.9E+05	1.4E+03	1.8E+06	1.6E+06	2.6E+05
	std	4.7E+04	1.3E+06	8.9E+05	3.8E+06	5.7E+04	1.7E+04	1.8E+04	6.7E+05	4.7E+04	2.8E+04	1.7E+05	6.5E+02	1.5E+06	1.0E+06	2.9E+05
	rank	4	11	10	15	7	2	3	12	6	5	9	1	14	13	8
F_{18}	mean	6.1E+02	8.4E+04	1.5E+03	1.5E+03	2.7E+03	3.3E+02	3.2E+03	2.1E+05	8.6E+02	1.3E+03	3.8E+02	1.4E+02	9.5E+01	1.1E+02	1.6E+03
	std	6.9E+02	3.0E+05	5.1E+03	2.3E+03	5.4E+03	2.9E+02	4.5E+03	1.6E+05	1.1E+03	1.5E+03	2.2E+02	4.3E+01	2.1E+01	9.8E+01	2.3E+03
	rank	6	14	9	10	12	4	13	15	7	8	5	3	1	2	11
F_{19}	mean	8.0E+00	3.2E+01	2.5E+01	7.2E+01	7.2E+00	6.9E+00	6.8E+00	1.8E+01	5.1E+00	1.4E+01	1.2E+01	4.9E+00	1.2E+01	1.1E+01	9.5E+00
	std	1.7E+00	3.5E+01	1.5E+01	1.0E+02	2.3E+00	1.5E+00	1.3E+00	8.5E-01	1.1E+00	3.8E+00	9.7E-01	9.2E-01	2.0E+00	2.0E+00	1.8E+00
	rank	6	14	13	15	5	4	3	12	2	11	10	1	9	8	7
F_{20}	mean	2.5E+02	5.2E+03	8.1E+02	6.3E+04	3.8E+02	1.8E+02	7.4E+02	2.0E+04	4.9E+02	2.0E+03	1.5E+03	1.1E+02	4.5E+03	5.1E+03	3.2E+03
	std	6.6E+01	8.9E+03	5.7E+02	4.0E+04	2.1E+02	6.9E+01	5.2E+02	7.8E+03	4.3E+02	1.3E+03	6.2E+02	6.1E+01	2.3E+03	2.8E+03	1.8E+03
	rank	3	13	7	15	4	2	6	14	5	9	8	1	11	12	10
F_{21}	mean	2.1E+04	2.1E+05	4.2E+04	2.2E+06	2.3E+04	1.4E+04	2.4E+04	4.9E+05	3.1E+04	4.0E+04	7.3E+04	6.6E+02	8.7E+04	9.0E+04	1.1E+05
	std	1.6E+04	2.3E+05	5.7E+04	6.4E+06	1.6E+04	1.0E+04	1.5E+04	2.6E+05	3.3E+04	2.6E+04	5.5E+04	2.5E+02	6.5E+04	8.0E+04	9.8E+04
	rank	3	13	8	15	4	2	5	14	6	7	9	1	10	11	12
F_{22}	mean	2.8E+02	4.9E+02	7.1E+02	8.4E+02	2.3E+02	2.6E+02	3.1E+02	4.1E+02	2.3E+02	5.2E+02	1.5E+02	2.1E+02	2.6E+02	2.7E+02	2.9E+02
	std	6.3E+01	2.2E+02	2.4E+02	2.1E+02	8.7E+01	1.2E+02	1.2E+02	1.5E+02	1.0E+02	1.6E+02	6.7E+01	9.5E+01	9.0E+01	7.9E+01	1.2E+02
	rank	8	12	14	15	4	5	10	11	3	13	1	2	6	7	9
F_{23}	mean	3.2E+02	3.3E+02	3.2E+02	3.3E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.1E+02	3.2E+02
	std	1.5E-04	1.2E+01	6.5E+00	9.8E+00	6.0E-13	4.3E-13	1.6E-12	1.2E-06	2.2E-12	1.8E-11	8.1E-08	2.2E-13	3.1E-02	9.6E-03	6.5E-01
	rank	11	15	13	14	1	8	9	4	10	2	3	7	6	5	12
F_{24}	mean	2.0E+02	2.5E+02	2.5E+02	2.4E+02	2.3E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.2E+02	2.3E+02	2.3E+02	2.3E+02	2.3E+02
	std	7.1E-02	1.0E+01	9.1E+00	3.8E+01	7.0E+00	1.5E+00	1.6E+00	1.3E+00	3.8E+00	3.5E+00	9.9E-01	5.5E+00	8.1E-01	8.3E-01	7.1E+00
	rank	1	14	15	13	9	8	5	2	7	11	3	10	4	6	12
F_{25}	mean	2.0E+02	2.1E+02	2.2E+02	2.2E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02	2.1E+02	2.2E+02	2.0E+02	2.0E+02	2.1E+02	2.1E+02	2.1E+02
	std	4.2E-13	4.2E+00	5.9E+00	4.7E+00	1.2E-01	1.9E+00	2.5E+00	2.1E+00	1.5E+00	4.0E+00	3.4E-01	1.5E+00	1.8E+00	2.1E+00	2.2E+00
	rank	1	10	15	14	2	9	6	8	5	13	3	4	12	11	7
F_{26}	mean	1.6E+02	1.1E+02	1.8E+02	1.9E+02	1.4E+02	1.0E+02	1.8E+02	1.0E+02	1.1E+02	1.2E+02	1.0E+02	1.0E+02	1.1E+02	1.1E+02	1.0E+02
	std	4.9E+01	3.0E+01	4.2E+01	2.8E+01	5.0E+01	7.6E-02	4.4E+01	3.4E-02	2.5E+01	3.6E+01	4.9E-02	5.3E-02	3.4E+01	3.0E+01	1.3E-01
	rank	12	8	14	15	11	2	13	4	6	10	3	1	9	7	5
F_{27}	mean	4.2E+02	8.2E+02	8.1E+02	1.3E+03	4.4E+02	4.0E+02	4.8E+02	1.3E+03	4.0E+02	7.1E+02	4.2E+02	4.1E+02	4.3E+02	4.3E+02	6.8E+02
	std	7.1E+01	1.9E+02	3.8E+02	5.2E+02	4.8E+01	3.2E+00	8.6E+01	3.5E+01	2.1E+01	2.9E+02	3.8E+01	5.7E+01	2.0E+01	1.8E+01	2.7E+02
	rank	5	13	12	15	8	2	9	14	1	11	4	3	7	6	10
F_{28}	mean	1.2E+03	1.2E+03	3.9E+03	5.1E+03	4.1E+02	9.6E+02	1.5E+03	3.9E+02	9.0E+02	7.7E+02	3.9E+02	8.7E+02	8.8E+02	8.7E+02	1.2E+03
	std	3.7E+02	2.1E+02	1.2E+03	1.7E+03	2.7E+01	4.8E+01	7.2E+02	4.6E+00	5.6E+01	3.3E+02	4.0E+00	4.3E+01	3.1E+02	2.3E+02	2.5E+02
	rank	11	10	14	15	3	9	13	2	8	4	1	5	7	6	12
F_{29}	mean	6.3E+05	3.1E+06	1.3E+07	4.7E+06	2.1E+02	9.7E+02	1.4E+03	2.2E+02	1.3E+03	2.2E+02	2.1E+02	7.7E+02	1.1E+03	1.1E+03	4.1E+06
	std	2.4E+06	4.0E+06	1.8E+07	1.0E+07	1.2E+00	2.6E+02	4.9E+02	1.2E+00	3.1E+02	1.6E+01	1.5E+00	3.2E+01	1.9E+02	1.9E+02	3.7E+06
	rank	11	12	15	14	1	6	10	4	9	3	2	5	8	7	13
F_{30}	mean	2.4E+03	2.9E+04	1.8E+04	8.3E+04	7.7E+02	2.1E+03	1.9E+03	9.4E+02	1.8E+03	8.4E+02	6.3E+02	2.4E+03	6.1E+02	6.5E+02	1.4E+04

	std	1.0E+03	3.3E+04	4.2E+04	8.5E+04	1.8E+02	6.6E+02	6.1E+02	1.8E+02	6.1E+02	2.4E+02	1.5E+02	1.0E+03	1.3E+02	1.5E+02	2.6E+04
	rank	11	14	13	15	4	9	8	6	7	5	2	10	1	3	12
Mean rank		6.60	12.37	12.37	14.07	5.53	4.37	6.97	10.43	4.97	7.90	6.40	3.20	7.57	7.30	9.97
Final rank		6	13	13	14	4	2	7	12	3	10	5	1	9	8	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'14 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.1E+06	1.0E+08	5.6E+07	8.0E+07	1.2E+06	3.4E+05	4.2E+05	3.3E+08	7.9E+05	6.5E+05	9.8E+06	3.0E+04	1.6E+07	1.4E+07	2.2E+07
	std	4.0E+05	9.7E+07	5.4E+07	3.4E+07	5.5E+05	1.3E+05	2.0E+05	8.9E+07	2.4E+05	1.6E+05	3.6E+06	7.8E+04	7.6E+06	4.1E+06	1.9E+07
	rank	6	14	12	13	7	2	3	15	5	4	8	1	10	9	11
F_2	mean	7.0E+03	1.3E+10	3.4E+09	2.7E+08	9.9E+03	3.0E+03	6.3E+03	1.5E+06	5.4E+03	5.0E+03	1.1E+03	2.3E-10	1.5E+03	2.5E+03	4.9E+03
	std	6.6E+03	5.4E+09	1.6E+09	2.9E+08	1.0E+04	3.1E+03	6.1E+03	1.6E+06	5.6E+03	6.5E+03	1.5E+03	4.9E-10	1.4E+03	2.5E+03	8.1E+03
	rank	10	15	14	13	11	5	9	12	8	7	2	1	3	4	6
F_3	mean	3.7E+02	7.7E+04	2.6E+04	1.8E+05	5.4E+02	6.9E+00	6.0E+02	2.9E+05	1.0E+03	2.4E+03	3.4E+03	1.5E+00	3.9E+03	3.5E+03	3.6E+03
	std	2.5E+02	3.7E+04	1.1E+04	3.8E+04	4.7E+02	7.9E+00	5.4E+02	9.8E+04	8.1E+02	1.2E+03	1.2E+03	5.2E+00	2.4E+03	2.0E+03	2.2E+03
	rank	3	13	12	14	4	2	5	15	6	7	8	1	11	9	10
F_4	mean	1.0E+02	1.2E+03	4.7E+02	4.2E+02	4.3E+01	4.1E+01	5.7E+01	4.6E+01	8.8E+01	8.6E+01	4.4E+01	1.4E+01	1.5E+02	1.4E+02	1.2E+02
	std	1.1E+01	9.6E+02	1.4E+02	8.0E+01	1.1E+01	3.4E+01	2.9E+01	8.5E-01	2.5E+01	4.0E+01	1.7E+00	3.2E+01	2.3E+01	3.1E+01	4.6E+01
	rank	9	15	14	13	3	2	6	5	8	7	4	1	12	11	10
F_5	mean	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	3.8E-02	3.0E-01	1.2E-01	1.2E-01	5.7E-02	4.9E-02	3.3E-01	4.4E-02	4.5E-03	2.6E-04	6.3E-02	3.6E-03	7.3E-02	6.8E-02	2.0E-01
	rank	14	12	6	13	5	4	10	15	2	1	11	3	7	9	8
F_6	mean	8.9E+00	4.0E+01	6.0E+01	7.1E+01	1.1E+01	2.6E+01	1.9E+01	7.1E+01	1.3E+01	4.1E+01	2.3E+01	1.5E+01	3.4E+01	3.5E+01	4.8E+01
	std	1.5E+00	4.0E+00	3.1E+00	5.1E+00	3.4E+00	4.1E+00	5.2E+00	1.5E+00	4.1E+00	4.4E+00	2.2E+00	4.3E+00	4.8E+00	4.5E+00	4.3E+00
	rank	1	10	13	14	2	7	5	15	3	11	6	4	8	9	12
F_7	mean	1.5E-02	1.2E+02	3.4E+01	2.6E+00	9.2E-03	1.1E-02	5.3E-03	6.9E-05	5.3E-03	8.9E-03	3.1E-03	4.3E-03	1.1E-02	8.8E-03	5.3E-01
	std	2.1E-02	7.9E+01	1.8E+01	1.1E+00	9.3E-03	1.4E-02	6.8E-03	2.8E-04	7.0E-03	6.8E-03	8.1E-03	6.8E-03	5.6E-03	6.9E-03	2.9E+00
	rank	11	15	14	13	8	10	5	1	4	7	2	3	9	6	12
F_8	mean	4.2E+01	2.0E+02	3.2E+02	3.0E+02	7.1E+01	9.5E-01	7.3E+01	2.2E+02	1.5E+01	2.2E+01	8.2E+01	1.7E-01	8.7E+00	9.7E+00	1.7E+02
	std	8.5E+00	3.8E+01	3.7E+01	7.9E+01	1.3E+01	8.2E-01	2.0E+01	2.0E+01	4.9E+00	6.3E+00	1.5E+01	5.9E-01	4.2E+00	4.3E+00	2.9E+01
	rank	7	12	15	14	8	2	9	13	5	6	10	1	3	4	11
F_9	mean	5.5E+01	3.1E+02	3.9E+02	3.9E+02	7.6E+01	1.0E+02	1.3E+02	3.6E+02	9.5E+01	2.5E+02	1.5E+02	6.5E+01	1.9E+02	2.2E+02	2.0E+02
	std	1.0E+01	5.4E+01	3.9E+01	1.5E+02	1.7E+01	1.9E+01	3.2E+01	1.7E+01	2.0E+01	3.7E+01	3.3E+01	1.4E+01	3.1E+01	2.8E+01	3.0E+01
	rank	1	12	15	14	3	5	6	13	4	11	7	2	8	10	9
F_{10}	mean	3.7E+03	5.8E+03	8.3E+03	3.2E+03	3.1E+03	2.2E+01	1.6E+03	1.3E+04	8.5E+02	1.1E+03	2.8E+03	4.0E+00	9.3E+01	8.9E+01	6.0E+03
	std	6.5E+02	8.5E+02	9.8E+02	1.6E+03	8.7E+02	4.4E+01	4.4E+02	4.4E+02	2.2E+02	2.4E+02	4.5E+02	2.2E+01	1.0E+02	8.7E+01	7.2E+02
	rank	11	12	14	10	9	2	7	15	5	6	8	1	4	3	13
F_{11}	mean	5.3E+03	7.6E+03	9.1E+03	1.2E+04	5.2E+03	4.0E+03	5.9E+03	1.4E+04	5.0E+03	5.6E+03	6.1E+03	3.9E+03	8.8E+03	8.7E+03	6.5E+03
	std	1.0E+03	7.9E+02	9.6E+02	2.3E+03	7.6E+02	4.9E+02	9.6E+02	3.5E+02	8.7E+02	6.5E+02	9.8E+02	3.4E+02	8.5E+02	7.0E+02	5.8E+02
	rank	5	10	13	14	4	2	7	15	3	6	8	1	12	11	9
F_{12}	mean	2.0E+00	4.1E-01	2.0E+00	2.6E+00	1.2E+00	1.4E-01	9.6E-01	3.4E+00	2.0E-01	3.2E-01	1.5E+00	1.1E-01	1.0E+00	1.0E+00	8.5E-01
	std	7.4E-01	1.5E-01	4.1E-01	1.2E+00	1.8E-01	4.8E-02	3.5E-01	2.1E-01	5.4E-02	9.9E-02	2.8E-01	1.4E-02	2.2E-01	1.6E-01	1.9E-01
	rank	13	5	12	14	10	2	7	15	3	4	11	1	8	9	6
F_{13}	mean	2.5E-01	1.1E+00	6.4E-01	5.5E-01	2.4E-01	3.9E-01	4.5E-01	4.7E-01	3.8E-01	4.9E-01	4.9E-01	3.7E-01	4.7E-01	4.8E-01	4.6E-01
	std	4.0E-02	9.1E-01	1.2E-01	8.0E-02	5.2E-02	7.5E-02	6.9E-02	4.6E-02	9.3E-02	6.8E-02	4.2E-02	6.5E-02	4.1E-02	4.6E-02	6.8E-02
	rank	2	15	14	13	1	5	6	9	4	11	12	3	8	10	7
F_{14}	mean	2.4E-01	3.0E+01	3.7E+00	3.0E-01	3.8E-01	3.0E-01	4.1E-01	3.5E-01	3.1E-01	3.1E-01	2.7E-01	3.6E-01	3.3E-01	3.2E-01	4.5E-01
	std	3.8E-02	2.4E+01	6.3E+00	4.2E-02	2.1E-01	4.1E-02	2.4E-01	3.9E-02	9.6E-02	3.4E-02	2.4E-02	1.4E-01	2.7E-02	2.3E-02	1.9E-01
	rank	1	15	14	4	11	3	12	9	6	5	2	10	8	7	13
F_{15}	mean	8.8E+00	3.3E+04	8.8E+02	5.8E+02	8.5E+00	1.3E+01	1.2E+01	3.2E+01	1.1E+01	1.1E+02	2.2E+01	7.6E+00	3.2E+01	2.9E+01	2.4E+01
	std	2.0E+00	4.8E+04	8.6E+02	8.3E+02	3.4E+00	3.7E+00	3.4E+00	1.3E+00	3.5E+00	3.2E+01	4.5E+00	1.1E+00	5.8E+00	4.9E+00	7.6E+00
	rank	3	15	14	13	2	6	5	11	4	12	7	1	10	9	8
F_{16}	mean	1.9E+01	2.1E+01	2.2E+01	2.2E+01	1.9E+01	1.8E+01	1.9E+01	2.3E+01	1.9E+01	2.0E+01	2.1E+01	1.8E+01	2.1E+01	2.1E+01	2.1E+01
	std	1.0E+00	5.4E-01	5.6E-01	4.7E-01	8.8E-01	5.1E-01	6.9E-01	2.1E-01	8.5E-01	7.4E-01	4.6E-01	3.8E-01	3.7E-01	4.9E-01	6.0E-01
	rank	4	8	13	14	5	2	6	15	3	7	11	1	10	9	12
F_{17}	mean	1.1E+05	7.8E+06	1.9E+06	8.1E+06	2.4E+05	4.4E+04	5.1E+04	1.1E+07	1.1E+05	1.2E+05	1.6E+06	4.0E+03	2.7E+06	2.4E+06	1.0E+06
	std	4.5E+04	1.3E+07	3.6E+06	5.5E+06	1.5E+05	1.8E+04	2.3E+04	3.8E+06	7.1E+04	7.4E+04	6.3E+05	6.6E+03	3.1E+06	2.1E+06	9.0E+05
	rank	4	13	10	14	7	2	3	15	5	6	9	1	12	11	8
F_{18}	mean	1.8E+03	1.6E+08	1.9E+07	2.2E+05	6.3E+03	3.8E+02	1.5E+03	6.2E+05	6.9E+02	2.0E+03	7.5E+02	2.6E+02	8.6E+02	9.3E+02	2.6E+03
	std	1.3E+03	4.8E+08	7.2E+07	9.6E+05	7.9E+03	2.2E+02	1.0E+03	1.1E+06	6.8E+02	2.5E+03	3.9E+02	2.6E+02	8.2E+02	7.1E+02	1.4E+03
	rank	8	15	14	12	11	2	7	13	3	9	4	1	5	6	10
F_{19}	mean	1.8E+01	7.9E+01	8.0E+01	2.4E+02	2.1E+01	1.5E+01	1.4E+01	3.3E+01	1.2E+01	3.8E+01	2.5E+01	1.2E+01	2.4E+01	2.1E+01	2.8E+01
	std	2.4E+00	5.8E+01	2.3E+01	7.2E+02	2.7E+00	2.8E+00	2.6E+00	6.1E-01	2.3E+00	2.6E+01	1.7E+00	2.8E+00	1.6E+01	1.2E+01	7.5E+00
	rank	5	13	14	15	6	4	3	11	2	12	9	1	8	7	10
F_{20}	mean	3.3E+02	2.1E+04	6.0E+03	2.3E+05	5.0E+02	3.0E+02	6.7E+02	7.1E+04	7.0E+02	1.7E+03	2.2E+03	3.5E+02	1.3E+04	1.3E+04	3.9E+03
	std	6.7E+01	2.5E+04	5.0E+03	2.5E+05	1.8E+02	8.9E+01	2.4E+02	3.5E+04	5.3E+02	1.5E+03	9.6E+02	1.5E+02	5.0E+03	5.7E+03	2.9E+03
	rank	2	13	10	15	4	1	5	14	6	7	8	3	11	12	9
F_{21}	mean	1.0E+05	2.0E+06	6.0E+05	7.7E+06	9.4E+04	3.8E+04	3.9E+04	4.4E+06	1.3E+05	9.2E+04	6.4E+05	1.8E+03	1.5E+06	1.4E+06	2.5E+05
	std	3.5E+04	2.9E+06	8.4E+05	7.2E+06	4.9E+04	2.5E+04	1.6E+04	1.5E+06	7.1E+04	7.4E+04	3.3E+05	1.6E+03	9.4E+05	1.1E+06	1.6E+05
	rank	6	13	9	15	5	2	3	14	7	4	10	1	12	11	8
F_{22}	mean	6.1E+02	1.3E+03	1.7E+03	1.8E+03	4.9E+02	8.3E+02	7.5E+02	1.7E+03	7.3E+02	1.4E+03	5.4E+02	5.9E+02	7.8E+02	7.4E+02	9.0E+02
	std	3.0E+02	3.1E+02	4.3E+02	5.4E+02	2.6E+02	2.2E+02	2.5E+02	1.4E+02	2.5E+02	4.1E+02	1.6E+02	1.6E+02	1.7E+02	1.7E+02	2.4E+02
	rank	4	11	14	15	1	9	7	13	5	12	2	3	8	6	10
F_{23}	mean	3.4E+02	3.9E+02	3.7E+02	3.6E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02	3.4E+02
	std	1.3E-02	4.1E+01	1.5E+01	6.3E+00	1.2E-12	6.8E-13	5.0E-12	3.4E-06	8.7E-13	7.6E-11	5.4E-07	5.0E-13	2.0E-02	5.2E-03	6.9E-01
	rank	11	15	14	13	1	8	10	4	9	2	3	7	6	5	12
F_{24}	mean	2.0E+02	3.3E+02	3.3E+02	2.9E+02	2.8E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.6E+02	2.7E+02	2.8E+02	2.6E+02	2.6E+02	2.8E+02
	std	1.5E+01	3.8E+01	1.4E+01	2.1E+01	2.7E+00	5.5E+00	5.7E+00	6.6E-06	4.5E+00	4.8E+00	4.5E+00	3.3E+00	4.5E+00	4.6E+00	3.9E+00
	rank	1	15	14	13	11	3	8	6	7	5	9	10	4	2	12
F_{25}	mean	2.0E+02	2.2E+02	2.5E+02	2.4E+02	2.0E+02	2.1E+02	2.2E+02	2.3E+02	2.1E+02	2.4E+02	2.0E+02	2.1E+02	2.3E+02	2.3E+02	2.2E+02
	std	3.7E-12	1.3E+01	9.1E+00	1.4E+01	7.8E-02	3.0E+00	4.5E+00	5.2E+00	3.6E+00	6.5E+00	4.5E-01	5.4E+00	3.3E+00	4.0E+00	5.5E+00
	rank	1	9	15	14	2	6	8	10	5	13	3	4	12	11	7
F_{26}	mean	1.9E+02	1.9E+02	2.0E+02	2.0E+02	1.6E+02	1.2E+02	1.7E+02	1.0E+02	1.5E+02	1.8E+02	1.1E+02	1.0E+02	1.5E+02	1.5E+02	1.1E+02
	std	2.5E+01	1.2E+02	1.0E+00	4.9E+00	5.0E+01	3.7E+01	6.3E+01	5.0E-02	5.0E+01	4.1E+01	3.1E+01	2.0E-01	5.0E+01	5.1E+01	2.6E+01
	rank	13	12	14	15	9	5	10	1	6	11	4	2	8	7	3
F_{27}	mean	7.3E+02	1.4E+03	2.0E+03	3.1E+03	6.4E+02	9.1E+02	8.5E+02	2.1E+03	6.1E+02	1.5E+03	9.2E+02	6.9E+02	9.8E+02	9.6E+02	1.6E+03
	std	8.4E+01	1.1E+02	3.2E+02	9.2E+02	7.5E+01	2.7E+02	1.4E+02	3.3E+01	1.1E+02	1.4E+02	6.1E+01	7.6E+01	1.4E+02	1.4E+02	1.3E+02
	rank	4	10	13	15	2	6	5	14	1	11	7	3	9	8	12
F_{28}	mean	2.2E+03	2.1E+03	8.3E+03	1.1E+04	3.9E+02	1.8E+03	2.7E+03	4.0E+02	1.7E+03	1.6E+03	3.9E+02	1.3E+03	7.2E+02	9.7E+02	2.3E+03
	std	5.4E+02	5.7E+02	2.5E+03	4.0E+03	1.5E+01	1.9E+02	8.2E+02	1.9E+01	3.7E+02	1.4E+03	1.3E+01	2.2E+02	4.7E+02	9.3E+02	7.6E+02
	rank	11	10	14	15	1	9	13	3	8	7	2	6	4	5	12
F_{29}	mean	2.5E+06	3.0E+07	6.3E+07	1.3E+08	2.2E+02	1.4E+03	1.4E+03	2.4E+02	1.3E+03	2.3E+02	2.3E+02	8.9E+02	1.1E+03	1.1E+03	4.6E+07
	std	1.4E+07	1.9E+07	8.0E+07	5.9E+08	1.7E+01	3.1E+02	3.3E+02	4.9E+00	3.8E+02	1.6E+01	1.6E+00	1.0E+02	1.9E+02	2.9E+02	1.6E+07
	rank	11	12	14	15	1	9	10	4	8	3	2	5	6	7	13

F_{30}	mean	1.2E+04	5.9E+04	1.5E+05	2.1E+05	1.2E+03	9.7E+03	1.2E+04	2.1E+03	1.1E+04	1.8E+03	1.1E+03	1.0E+04	1.3E+03	1.3E+03	3.5E+04
	std	1.5E+03	4.5E+04	2.0E+05	1.2E+05	2.9E+02	9.9E+02	1.2E+03	2.0E+02	1.2E+03	3.4E+02	1.8E+02	1.5E+03	2.6E+02	2.3E+02	2.5E+04
	rank	10	13	14	15	2	7	11	6	9	5	1	8	3	4	12
Mean rank		6.40	12.40	13.07	13.47	5.20	4.47	7.03	10.70	5.13	7.30	6.33	3.00	7.97	7.70	9.83
Final rank		6	13	14	15	4	2	7	12	3	8	5	1	10	9	11
Algorithms		AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'14 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	6.1E+06	4.5E+08	3.2E+08	1.7E+08	2.7E+06	9.8E+05	1.1E+06	3.6E+09	4.7E+06	2.8E+06	6.7E+07	2.6E+05	1.6E+08	1.6E+08	1.1E+08
	std	1.0E+06	2.4E+08	1.2E+08	8.8E+07	1.0E+06	3.0E+05	5.6E+05	8.1E+08	1.5E+06	7.1E+05	2.7E+07	1.9E+05	3.3E+07	4.0E+07	7.3E+07
	rank	7	14	13	12	4	2	3	15	6	5	8	1	10	11	9
F_2	mean	8.2E+05	7.3E+10	1.4E+10	8.2E+06	1.9E+04	2.0E+04	1.6E+04	4.1E+06	1.2E+04	1.4E+04	1.1E+04	3.0E-09	1.2E+04	1.4E+04	3.0E+04
	std	2.0E+06	2.2E+10	3.4E+09	1.1E+07	1.9E+04	1.5E+04	1.5E+04	1.3E+07	1.2E+04	1.1E+04	1.7E+04	5.1E-09	6.9E+03	6.5E+03	2.9E+04
	rank	11	15	14	13	8	9	7	12	4	5	2	1	3	6	10
F_3	mean	7.2E+02	1.8E+05	1.0E+05	3.8E+05	7.4E+02	1.9E-01	3.8E+02	1.1E+06	1.7E+03	2.3E+03	4.1E+03	9.2E-01	9.7E+03	8.0E+03	3.0E+03
	std	3.0E+02	8.1E+04	2.0E+04	1.3E+05	9.0E+02	2.5E-01	3.7E+02	2.8E+05	1.6E+03	1.5E+03	1.4E+03	2.9E+00	5.0E+03	3.4E+03	2.2E+03
	rank	4	13	12	14	5	1	3	15	6	7	9	2	11	10	8
F_4	mean	2.3E+02	8.5E+03	2.0E+03	5.0E+02	1.0E+02	1.4E+02	1.8E+02	9.8E+01	2.1E+02	2.0E+02	9.5E+01	1.3E+02	3.3E+02	3.4E+02	3.3E+02
	std	3.2E+01	4.4E+03	4.3E+02	5.9E+01	2.0E+01	5.2E+01	3.7E+01	9.2E-01	4.7E+01	4.9E+01	2.2E+00	4.7E+01	5.6E+01	4.0E+01	1.1E+02
	rank	9	15	14	13	3	5	6	2	8	7	1	4	10	12	11
F_5	mean	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.0E+01	2.0E+01	2.1E+01	2.0E+01	2.1E+01	2.1E+01	2.1E+01
	std	3.2E-02	9.0E-02	9.6E-02	3.2E-01	4.0E-02	1.0E-01	2.8E-01	3.2E-02	3.2E-04	3.5E-01	3.9E-02	3.9E-03	3.6E-02	4.6E-02	1.0E-01
	rank	14	13	6	12	5	3	10	15	1	4	8	2	11	9	7
F_6	mean	3.2E+01	9.9E+01	1.4E+02	1.5E+02	4.8E+01	7.6E+01	6.5E+01	1.6E+02	6.1E+01	1.0E+02	7.7E+01	5.8E+01	1.1E+02	1.1E+02	1.2E+02
	std	2.9E+00	6.9E+00	6.5E+00	9.4E+00	7.2E+00	6.9E+00	7.2E+00	1.5E+00	6.8E+00	4.3E+00	4.6E+00	6.6E+00	8.8E+00	7.9E+00	5.8E+00
	rank	1	8	13	14	2	6	5	15	4	9	7	3	11	10	12
F_7	mean	1.8E-01	7.4E+02	1.2E+02	1.1E+00	4.9E-03	9.8E-03	3.8E-03	2.2E-02	2.5E-03	1.2E-03	2.2E-03	9.8E-03	1.1E-02	8.0E-03	5.6E-12
	std	2.1E-01	2.4E+02	3.5E+01	1.8E-01	9.0E-03	2.7E-02	7.3E-03	7.4E-03	4.6E-03	3.0E-03	2.6E-03	1.4E-02	1.8E-02	1.1E-02	3.1E-12
	rank	12	15	14	13	6	8	5	11	4	2	3	9	10	7	1
F_8	mean	1.1E+02	6.2E+02	8.3E+02	6.6E+02	2.0E+02	9.0E-01	2.2E+02	7.6E+02	3.2E+01	7.7E+01	2.4E+02	9.6E-01	4.0E+01	4.4E+01	4.8E+02
	std	1.6E+01	6.2E+01	6.4E+01	8.8E+01	3.7E+01	9.2E-01	5.0E+01	3.0E+01	1.1E+01	3.1E+01	2.6E+01	1.9E+00	9.1E+00	9.3E+00	5.5E+01
	rank	7	12	15	13	8	1	9	14	3	6	10	2	4	5	11
F_9	mean	1.3E+02	8.8E+02	1.0E+03	1.0E+03	1.9E+02	3.5E+02	3.1E+02	9.6E+02	2.9E+02	6.7E+02	4.9E+02	2.1E+02	8.3E+02	8.3E+02	5.6E+02
	std	2.0E+01	1.1E+02	6.5E+01	3.6E+02	4.8E+01	6.0E+01	4.9E+01	2.6E+01	5.1E+01	5.1E+01	1.3E+02	4.0E+01	6.5E+01	6.5E+01	6.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_{10}	mean	1.0E+04	1.5E+04	2.1E+04	1.0E+04	8.9E+03	2.9E+01	4.7E+03	3.1E+04	1.9E+03	2.4E+03	7.6E+03	2.8E+01	3.9E+02	4.3E+02	1.4E+04
	std	8.7E+02	1.8E+03	1.1E+03	3.2E+03	2.0E+03	4.8E+01	6.4E+02	7.9E+02	4.0E+02	5.3E+02	1.1E+03	6.0E+01	2.0E+02	1.8E+02	1.3E+03
	rank	10	13	14	11	9	2	7	15	5	6	8	1	3	4	12
F_{11}	mean	1.0E+04	1.7E+04	2.2E+04	2.8E+04	1.3E+04	1.1E+04	1.3E+04	3.3E+04	1.2E+04	1.3E+04	1.8E+04	1.1E+04	2.6E+04	2.6E+04	1.5E+04
	std	1.1E+03	1.3E+03	1.3E+03	4.6E+03	1.5E+03	6.7E+02	1.7E+03	6.1E+02	1.3E+03	1.1E+03	2.1E+03	5.5E+02	9.9E+02	1.1E+03	1.4E+03
	rank	1	9	11	14	5	3	7	15	4	6	10	2	12	13	8
F_{12}	mean	3.0E+00	1.7E+00	2.8E+00	3.7E+00	1.8E+00	2.1E-01	1.6E+00	4.0E+00	2.8E-01	4.5E-01	2.1E+00	1.8E-01	2.3E+00	2.3E+00	1.5E+00
	std	5.6E-01	1.4E+00	4.8E-01	7.6E-01	2.1E-01	5.5E-02	4.7E-01	2.2E-01	5.9E-02	8.9E-02	2.5E-01	1.6E-02	2.5E-01	3.2E-01	2.8E-01
	rank	13	7	12	14	8	2	6	15	3	4	9	1	10	11	5
F_{13}	mean	3.6E-01	3.7E+00	6.0E-01	5.4E-01	4.2E-01	4.6E-01	6.4E-01	6.9E-01	5.0E-01	5.0E-01	6.1E-01	5.0E-01	5.0E-01	5.1E-01	5.5E-01
	std	5.4E-02	1.2E+00	6.6E-02	6.4E-02	5.4E-02	5.4E-02	6.4E-02	5.7E-02	6.9E-02	5.6E-02	6.0E-02	7.6E-02	4.2E-02	4.4E-02	5.3E-02
	rank	1	15	11	9	2	3	13	14	5	4	12	6	7	8	10
F_{14}	mean	3.1E-01	2.1E+02	4.6E+01	3.5E-01	4.3E-01	3.4E-01	3.5E-01	4.2E-01	3.2E-01	3.2E-01	3.2E-01	3.6E-01	3.4E-01	3.4E-01	6.3E-01
	std	1.5E-01	7.6E+01	1.7E+01	3.5E-02	2.0E-01	2.7E-02	1.3E-01	1.3E-01	3.5E-02	3.0E-02	2.8E-02	1.1E-01	2.4E-02	2.8E-02	2.2E-01
	rank	1	15	14	9	12	5	8	11	2	4	3	10	6	7	13
F_{15}	mean	2.2E+01	1.5E+06	2.1E+04	7.2E+02	2.6E+01	4.3E+01	4.4E+01	8.4E+01	4.5E+01	3.3E+02	7.4E+01	3.9E+01	1.0E+02	1.1E+02	1.8E+02
	std	3.6E+00	1.5E+06	1.4E+04	1.0E+03	1.1E+01	6.7E+00	7.9E+00	3.2E+00	1.1E+01	5.2E+01	1.1E+01	7.5E+00	1.6E+01	1.2E+01	1.1E+02
	rank	1	15	14	13	2	4	5	8	6	12	7	3	9	10	11
F_{16}	mean	4.1E+01	4.4E+01	4.5E+01	4.7E+01	4.2E+01	4.1E+01	4.3E+01	4.8E+01	4.1E+01	4.3E+01	4.5E+01	4.0E+01	4.6E+01	4.6E+01	4.5E+01
	std	1.3E+00	1.3E+00	6.9E-01	5.2E-01	9.9E-01	7.5E-01	1.2E+00	2.0E-01	1.2E+00	8.3E-01	5.2E-01	7.2E-01	4.4E-01	5.2E-01	5.8E-01
	rank	3	8	11	14	5	2	6	15	4	7	10	1	12	13	9
F_{17}	mean	8.0E+05	2.4E+07	1.7E+07	3.5E+07	1.0E+06	1.9E+05	4.0E+05	1.1E+08	7.3E+05	4.6E+05	9.3E+06	3.5E+04	1.7E+07	1.9E+07	7.3E+06
	std	2.3E+05	2.7E+07	5.8E+06	2.1E+07	3.4E+05	6.0E+04	1.8E+05	2.4E+07	2.9E+05	1.7E+05	4.5E+06	3.2E+04	6.6E+06	8.3E+06	5.0E+06
	rank	6	13	10	14	7	2	3	15	5	4	9	1	11	12	8
F_{18}	mean	1.3E+03	9.5E+08	2.1E+08	1.4E+03	3.1E+03	6.9E+02	2.3E+03	4.2E+06	2.1E+03	1.5E+03	1.7E+03	1.5E+03	6.4E+02	1.0E+03	4.7E+03
	std	1.1E+03	1.6E+09	1.6E+08	1.5E+03	3.6E+03	3.1E+02	2.3E+03	4.7E+06	2.4E+03	1.4E+03	8.0E+02	1.6E+03	4.6E+02	1.1E+03	4.5E+03
	rank	4	15	14	5	11	2	10	13	9	6	8	7	1	3	12
F_{19}	mean	1.1E+02	5.6E+02	3.4E+02	1.8E+02	6.3E+01	7.1E+01	8.2E+01	7.4E+01	1.0E+02	9.1E+01	5.9E+01	9.5E+01	1.4E+02	1.3E+02	1.2E+02
	std	2.8E+00	1.5E+02	1.2E+02	7.6E+01	1.9E+01	2.7E+01	3.6E+01	1.9E+00	1.6E+01	3.5E+01	2.4E+00	2.4E+01	2.7E+01	2.5E+01	1.1E+01
	rank	9	15	14	13	2	3	5	4	8	6	1	7	12	11	10
F_{20}	mean	6.8E+02	1.1E+05	3.7E+04	1.1E+06	1.1E+03	5.2E+02	1.4E+03	5.3E+05	1.8E+03	2.6E+03	5.0E+03	6.8E+03	5.2E+04	4.9E+04	6.8E+03
	std	1.2E+02	6.0E+04	1.9E+04	8.9E+05	3.7E+02	7.8E+01	5.6E+02	1.2E+05	8.0E+02	7.9E+02	1.4E+03	5.3E+03	1.7E+04	1.4E+04	2.1E+03
	rank	2	13	10	15	3	1	4	14	5	6	7	8	12	11	9
F_{21}	mean	3.7E+05	6.4E+06	5.6E+06	1.7E+07	4.2E+05	8.0E+04	1.1E+05	4.6E+07	3.1E+05	2.2E+05	4.4E+06	1.8E+04	8.4E+06	8.4E+06	2.1E+06
	std	1.2E+05	5.4E+06	3.0E+06	2.0E+07	1.8E+05	2.8E+04	6.7E+04	1.2E+07	1.5E+05	5.7E+04	1.9E+06	2.9E+04	9.5E+06	7.5E+06	2.3E+06
	rank	6	11	10	14	7	2	3	15	5	4	9	1	13	12	8
F_{22}	mean	1.5E+03	3.5E+03	3.9E+03	5.0E+03	1.4E+03	1.9E+03	2.3E+03	5.4E+03	2.1E+03	3.0E+03	2.0E+03	1.7E+03	1.8E+03	1.8E+03	2.5E+03
	std	5.7E+02	5.8E+02	8.1E+02	1.1E+03	3.4E+02	3.5E+02	4.7E+02	2.3E+02	4.6E+02	4.7E+02	4.8E+02	2.5E+02	3.3E+02	3.1E+02	4.1E+02
	rank	2	12	13	14	1	6	9	15	8	11	7	3	5	4	10
F_{23}	mean	3.5E+02	6.4E+02	5.0E+02	3.6E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02	3.5E+02
	std	2.8E-01	1.8E+02	6.4E+01	2.8E+00	3.8E-11	2.1E-11	5.4E-10	2.3E-01	2.8E-10	1.6E-10	4.7E-03	2.8E-12	8.9E-03	1.3E-02	7.8E+00
	rank	11	15	14	13	1	8	10	6	9	2	3	7	4	5	12
F_{24}	mean	2.2E+02	6.7E+02	5.9E+02	4.2E+02	3.9E+02	3.6E+02	3.8E+02	3.6E+02	3.8E+02	3.4E+02	3.8E+02	4.2E+02	3.7E+02	3.7E+02	4.1E+02
	std	5.4E+01	8.3E+01	4.1E+01	2.1E+01	5.6E+00	7.5E+00	5.0E+00	2.3E+00	3.6E+00	1.2E+01	3.9E+00	8.1E+00	3.3E+00	2.8E+00	8.2E+00
	rank	1	15	14	13	10	3	8	4	7	2	9	12	5	6	11
F_{25}	mean	2.0E+02	2.8E+02	3.5E+02	2.8E+02	2.1E+02	2.6E+02	2.8E+02	5.2E+02	2.7E+02	3.0E+02	2.1E+02	2.8E+02	2.8E+02	2.8E+02	2.6E+02
	std	3.9E-13	2.8E+01	2.3E+01	2.1E+01	2.8E+01	5.2E+00	1.1E+01	6.2E+01	1.2E+01	1.1E+01	1.2E+00	1.2E+01	2.0E+01	9.8E+00	1.7E+01
	rank	1	10	14	12	3	4	8	15	6	13	2	11	7	9	5
F_{26}	mean	2.0E+02	2.3E+02	2.1E+02	2.1E+02	2.0E+02	2.0E+02	2.0E+02	1.0E+02	2.0E+02	2.0E+02	1.9E+02	2.0E+02	2.1E+02	2.1E+02	1.9E+02
	std	2.8E-02	2.0E+01	4.7E+00	6.6E+00	6.2E-02	2.6E-02	9.3E-02	5.6E-02	9.9E-02	2.9E-02	3.8E+01	9.5E-03	3.0E+00	2.0E+00	3.7E+01
	rank	4	15	14	13	9	6	8	1	10	7	2	5	12	11	3
F_{27}	mean	1.4E+03	2.9E+03	4.4E+03	6.7E+03	1.6E+03	2.1E+03	2.0E+03	4.4E+03	1.8E+03	3.1E+03	2.3E+03	1.6E+03	1.6E+03	1.7E+03	3.5E+03
	std	1.5E+02	1.6E+02	2.2E+02	1.7E+03	2.0E+02	7.4E+02	2.3E+02	5.1E+01	4.5E+02	2.0E+02	1.2E+02	1.4E+02	3.0E+02	2.7E+02	1.4E+02
	rank	1	10	13	15	2	8	7	14	6	11	9	3	4	5	12
F_{28}	mean	4.8E+03	5.0E+03	2.0E+04	3.0E+04	7.6E+02	5.3E+03	7.9E+03	2.6E+03	6.4E+03	7.1E+03	6.7E+02	3.3E+03	7.1E+03	6.2E+03	6.3E+03
	std	1.1E+03	7.9E+02	5.3E+03	9.7E+03	2.6E+02	6.4E+02	1.4E+03	6.8E+02	1.1E+03	2.8E+03	1.0E+02	6.2E+02	4.2E+03	4.1E+03	7.8E+02
	rank	5	6	14	15	2	7	13	3	10	11	1	4	12	8	9
F_{29}	mean	7.7E+03	9.8E+07	4.9E+08	5.0E+05	2.6E+02	1.5E+03	1.6E+03	3.0E+02	1.8E+03	2.9E+02	2.6E+02	1.5E+03	8.1E+02	8.4E+02	2.2E+08
	std	9.4E+03	4.5E+07	4.4E+08	1.7E+06	3.4E+00	2.7E+02	2.6E+02	2.8E+01	2.7E+02	4.9E+01	2.5E+00	1.7E+02	6.5E+02	5.1E+02	5.8E+07
	rank	11	13	15	12	2	8	9	4	10	3	1	7	5	6	14
F_{30}	mean	1.0E+04	6.6E+05	7.1E+05	5.3E+05	3.0E+03	9.4E+03	1.2E+04	6.8E+03	1.0E+04	4.0E+03	2.8E+03	9.9E+03	3.0E+03	3.2E+03	5.1E+05
	std	2.2E+03	4.7E+05	4.6E+05	8.6E+05	4.5E+02	1.0E+03	3.4E+03	3.3E+02	1.4E+03	3.7E+02	4.8E+02	1.3E+03	4.2E+02	4.0E+02	3.8E+05

rank	9	14	15	13	2	7	11	6	10	5	1	8	3	4	12
Mean rank	5.60	12.53	12.87	12.80	4.93	4.30	7.10	11.13	5.90	6.27	6.10	4.50	8.20	8.43	9.33
Final rank	4	13	15	14	3	1	8	12	5	7	6	2	9	10	11
Algorithms	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.7E+02	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	3.8E+02	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	3	14	11	15	12	7	8	13	6	9	4	1	5	2	10
F_3	mean	5.7E-14	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	2.6E-14	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	6	11	8	14	1	2	5	10	3	7	9	1	12	13	4
F_4	mean	1.1E+00	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	3.3E-01	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	6	14	13	15	4	2	3	12	5	9	11	1	10	8	7
F_5	mean	7.3E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	2.2E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	8	13	14	15	2	4	9	10	7	12	5	1	3	6	11
F_6	mean	2.5E-05	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	8.8E-05	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	11	13	14	15	9	1	2	5	8	12	6	10	7	4	3
F_7	mean	1.8E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	3.7E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	7	11	14	15	3	8	5	13	6	12	10	1	4	2	9
F_8	mean	5.1E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	2.4E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	4	13	14	15	2	7	10	9	3	12	8	1	5	6	11
F_9	mean	8.0E-14	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	5.3E-14	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	5	8	10	11	1	2	4	1	2	9	2	1	7	6	3
F_{10}	mean	5.0E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	2.1E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	11	12	13	14	2	3	8	15	6	9	4	1	5	7	10
F_{11}	mean	4.6E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	2.8E+00	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	10	14	13	15	2	5	11	4	3	12	7	1	8	9	6
F_{12}	mean	7.8E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	2.9E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	5	14	2	15	12	8	6	13	10	7	3	1	9	11	4
F_{13}	mean	4.7E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	3.1E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	11	14	4	15	10	7	9	12	8	13	6	1	2	3	5
F_{14}	mean	5.5E+01	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	5.3E+01	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	7	12	8	13	4	2	6	11	3	10	5	1	14	15	9
F_{15}	mean	8.6E+01	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	6.5E+01	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	7	13	8	15	6	2	10	14	3	12	9	1	4	5	11
F_{16}	mean	9.7E+00	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	2.6E+01	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	6	11	14	15	10	1	13	5	4	12	9	2	7	3	8
F_{17}	mean	2.8E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	5.8E+00	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	8	13	15	14	7	4	11	10	5	12	6	1	3	2	9
F_{18}	mean	3.6E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	2.1E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	7	11	12	15	13	4	8	14	5	9	6	1	3	2	10
F_{19}	mean	1.5E+02	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	2.3E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	12	6	8	15	7	2	11	13	5	14	9	1	4	3	10
F_{20}	mean	2.5E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	8.2E+00	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	11	14	15	13	8	4	12	9	6	10	5	2	1	3	7
F_{21}	mean	1.3E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	5.1E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	4	15	14	13	9	5	10	12	3	8	6	7	2	1	11
F_{22}	mean	9.3E+01	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	2.8E+01	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	3	14	7	15	8	1	11	10	12	13	2	6	4	5	9
F_{23}	mean	3.1E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	3.9E+00	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	7	12	14	15	4	3	9	10	5	13	8	2	6	1	11
F_{24}	mean	1.9E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	1.1E+02	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	3	12	14	15	6	5	8	11	4	10	7	9	2	1	13
F_{25}	mean	4.1E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	2.2E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	5	14	12	15	9	3	11	6	1	13	2	10	8	7	4
F_{26}	mean	2.9E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	2.8E+01	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	7	14	12	15	10	3	5	13	6	9	4	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	9.1E-01	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	7	12	13	15	2	10	11	14	8	5	1	6	4	3	9
F_{28}	mean	3.1E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	6.3E+01	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	1	12	11	14	7	2	13	10	2	4	6	8	3	5	9
F_{29}	mean	2.6E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.3E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	6	12	14	15	2	3	9	10	4	13	5	1	11	8	7
F_{30}	mean	4.3E+03	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	2.7E+03	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	7	14	13	15	1	6	12	3	5	4	2	11	9	10	8
Mean rank		6.72	12.48	11.52	14.52	5.97	4.00	8.62	10.07	5.10	10.14	5.76	3.38	5.62	5.28	8.24

Final rank	8	14	13	15	7	2	10	11	3	12	6	1	5	4	9
Algorithms	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.6E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	2.2E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	5	14	13	15	7	9	11	12	6	8	4	1	3	2	10
F_3	mean	1.8E-11	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	2.5E-11	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	2	11	10	14	6	1	3	15	7	4	9	5	13	12	8
F_4	mean	7.0E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	2.7E+01	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	8	14	13	15	5	1	2	4	11	7	3	6	10	9	12
F_5	mean	4.4E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	8.4E+00	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_6	mean	8.9E-02	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	7.7E-02	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	9	13	15	14	6	3	8	2	4	12	10	7	5	1	11
F_7	mean	6.8E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	1.5E+01	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	2	11	14	15	3	4	6	12	5	13	10	1	8	7	9
F_8	mean	3.9E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	1.1E+01	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	3	12	13	15	2	4	6	14	5	11	7	1	8	9	10
F_9	mean	4.1E-02	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	1.2E-01	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	2	13	14	15	5	7	6	1	4	12	8	3	10	9	11
F_{10}	mean	2.8E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	7.1E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	5	12	13	14	4	2	8	15	3	7	6	1	10	11	9
F_{11}	mean	6.3E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	2.2E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	4	14	13	15	1	7	8	3	5	12	2	10	11	9	6
F_{12}	mean	2.5E+05	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	1.4E+05	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	8	14	13	15	5	2	4	12	3	6	7	1	10	9	11
F_{13}	mean	1.2E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	8.8E+03	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	1.2E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	9.1E+02	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	2	14	10	15	7	3	4	12	5	6	8	1	11	13	9
F_{15}	mean	1.6E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	1.9E+03	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	6	13	11	14	9	5	10	15	7	8	4	3	1	2	12
F_{16}	mean	6.0E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	1.9E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	7	12	13	15	1	4	9	14	8	11	2	3	6	5	10
F_{17}	mean	1.1E+02	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	5.8E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	2	13	14	15	9	4	8	12	1	11	6	3	5	7	10
F_{18}	mean	6.6E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	2.8E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	2	13	7	15	11	3	4	14	5	6	12	1	8	9	10
F_{19}	mean	4.3E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	3.4E+03	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	8	14	7	15	9	4	10	13	6	11	5	1	2	3	12
F_{20}	mean	2.5E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	4.2E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	9	13	15	14	2	7	6	12	4	10	3	1	5	8	11
F_{21}	mean	2.4E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	8.0E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	3	12	14	15	2	4	7	13	5	11	6	1	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	1.9E-09	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	2	13	12	14	4	5	8	15	3	10	1	7	9	6	11
F_{23}	mean	3.8E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	1.1E+01	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.3E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	2.3E+01	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	1	11	14	15	4	3	6	12	5	13	7	2	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	1.0E-01	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	6	13	14	15	2	11	10	1	8	12	3	7	5	4	9
F_{26}	mean	5.8E+02	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	5.1E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	1	12	14	15	5	2	4	11	3	13	6	7	9	8	10
F_{27}	mean	5.1E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	6.4E+00	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	8	13	14	15	4	10	11	6	9	3	5	7	2	1	12
F_{28}	mean	3.2E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	3.6E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	1	14	13	15	7	2	5	12	6	3	8	4	10	9	11
F_{29}	mean	5.6E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	5.6E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	6	13	14	15	1	5	9	11	2	10	3	4	7	8	12
F_{30}	mean	9.7E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	4.9E+03	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	10	14	13	15	6	7	8	11	9	2	1	3	5	4	12

Mean rank	4.69	12.76	12.83	14.79	4.90	4.52	6.90	10.76	5.31	9.10	5.62	3.24	7.28	7.00	10.31
Final rank	3	13	14	15	4	2	7	12	5	10	6	1	9	8	11
Algorithms	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	3.1E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	3.2E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	6	14	13	15	10	7	8	11	9	5	4	1	3	2	12
F_3	mean	1.3E-04	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	4.0E-04	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	2	11	8	12	5	1	4	15	6	3	9	7	14	13	10
F_4	mean	8.8E+01	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	5.5E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	8	14	13	15	3	1	6	5	11	7	4	2	9	10	12
F_5	mean	9.7E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	1.1E+01	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	10	8
F_6	mean	2.4E-01	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	2.2E-01	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	7	13	15	14	8	2	9	1	4	11	10	6	5	3	12
F_7	mean	1.2E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	1.6E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	2	12	14	15	3	4	5	11	6	13	7	1	8	10	9
F_8	mean	9.1E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	1.4E+01	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_9	mean	5.3E+00	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	8.5E+00	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	4.9E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	6.5E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	3	10	13	14	5	2	6	15	4	7	8	1	11	12	9
F_{11}	mean	1.1E+02	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	1.1E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	1	14	13	15	2	5	7	8	4	6	3	9	12	11	10
F_{12}	mean	1.5E+06	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	4.1E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	7	14	13	15	4	2	3	12	5	6	10	1	8	9	11
F_{13}	mean	7.3E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	2.6E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	10	13	14	15	9	5	7	12	6	8	1	3	4	2	11
F_{14}	mean	2.0E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.4E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	4	10	12	15	7	2	3	13	6	5	9	1	11	14	8
F_{15}	mean	3.5E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	2.5E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	7	15	11	14	9	2	8	13	5	10	3	1	6	4	12
F_{16}	mean	5.2E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	1.7E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	1	12	13	14	2	7	8	15	9	10	3	4	6	5	11
F_{17}	mean	6.4E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	1.5E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	1	12	13	15	2	8	9	14	7	11	6	3	4	5	10
F_{18}	mean	5.4E+04	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	3.0E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	4	13	8	15	7	2	3	14	5	6	10	1	11	12	9
F_{19}	mean	1.5E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	5.4E+03	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	8	15	13	14	10	2	9	12	6	7	3	1	5	4	11
F_{20}	mean	4.9E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	1.5E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	2	12	13	14	1	4	9	15	5	11	6	3	8	7	10
F_{21}	mean	2.8E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	1.4E+01	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	3	12	14	15	2	6	5	13	4	11	7	1	8	9	10
F_{22}	mean	1.7E+03	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	2.5E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	5.1E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	2.1E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	2	12	15	14	3	4	6	11	5	13	7	1	8	9	10
F_{24}	mean	5.4E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	3.0E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	1	11	14	15	3	4	6	12	5	13	8	2	9	10	7
F_{25}	mean	5.2E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	2.1E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	7	14	13	15	3	8	10	1	11	4	2	12	6	5	9
F_{26}	mean	1.7E+03	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	1.0E+03	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	1	12	15	14	2	3	5	11	6	13	7	4	9	8	10
F_{27}	mean	5.5E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	1.8E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.8E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	2.1E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	4	15	12	14	1	10	9	11	5	3	2	8	7	6	13
F_{29}	mean	8.0E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.5E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	3	11	14	15	4	5	9	13	2	10	6	1	8	7	12

F_{30}	mean	8.0E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	9.3E+04	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	8	14	13	15	5	6	10	11	9	4	1	7	3	2	12
Mean rank		4.07	12.52	13.03	14.55	4.31	4.52	6.97	10.93	5.93	8.31	5.79	3.34	7.76	7.66	10.31
Final rank		2	13	14	15	3	4	7	12	6	10	5	1	9	8	11
Algorithms		HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'17 problems for HIDMS-PSO vs comparison algorithms

F	Criteria	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	9.0E+03	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	1.3E+04	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	9	15	14	13	8	10	6	12	7	5	3	1	2	4	11
F_3	mean	7.6E+03	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	2.9E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	3	12	7	11	6	1	5	15	4	2	9	8	13	14	10
F_4	mean	2.3E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	1.2E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	9	15	13	14	3	5	8	2	7	6	1	4	11	10	12
F_5	mean	2.0E+02	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	5.0E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	2	12	14	15	1	6	5	13	4	9	7	3	11	10	8
F_6	mean	8.8E-01	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	4.1E-01	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	6	13	15	14	8	1	10	4	5	11	9	7	2	3	12
F_7	mean	3.7E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	3.8E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	2	14	15	13	1	4	5	11	7	12	6	3	9	10	8
F_8	mean	1.9E+02	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.3E+02	8.5E+02	5.5E+02
	std	3.6E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	2	12	15	14	1	6	5	13	4	9	7	3	10	11	8
F_9	mean	8.5E+01	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	3.5E+01	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	1	11	12	15	2	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.1E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.1E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	3	9	11	14	6	2	5	15	4	7	10	1	13	12	8
F_{11}	mean	1.0E+03	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	1.3E+02	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	6	13	10	14	1	3	4	15	2	5	8	7	12	11	9
F_{12}	mean	6.7E+06	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	3.9E+06	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	7	15	14	13	6	2	3	12	4	5	10	1	9	8	11
F_{13}	mean	1.9E+04	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	1.1E+04	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	10	15	14	12	8	1	5	13	4	9	2	7	6	3	11
F_{14}	mean	1.3E+05	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	7.8E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	5	13	10	15	7	3	2	14	6	4	9	1	12	11	8
F_{15}	mean	5.8E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	6.8E+03	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	9	15	14	12	10	2	6	13	8	7	5	1	4	3	11
F_{16}	mean	2.2E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	6.0E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	1.6E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	3.7E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	2.4E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	1.0E+05	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	5	11	9	14	7	1	4	15	6	3	10	2	12	13	8
F_{19}	mean	8.8E+03	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	1.0E+04	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	10	15	14	11	9	1	6	13	5	7	3	8	2	4	12
F_{20}	mean	2.0E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	3.7E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	3	12	13	14	1	7	8	15	5	10	9	2	4	6	11
F_{21}	mean	4.3E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	3.1E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	2	13	15	14	1	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	1.1E+04	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	4.8E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	1	9	11	14	4	3	5	15	6	7	10	2	13	12	8
F_{23}	mean	6.9E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	2.7E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	1	11	15	14	3	4	7	12	6	13	5	2	8	9	10
F_{24}	mean	1.1E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	3.8E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	1	12	15	13	2	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	7.1E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	6.2E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	1	14	12	13	2	6	15	8	5	7	3	4	11	10	9
F_{26}	mean	5.1E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	4.2E+02	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	6.3E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	1.9E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	4	9	12	13	1	14	14	3	8	10	2	7	6	5	11
F_{28}	mean	5.4E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	1.4E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	5	14	11	13	2	7	15	3	8	4	1	6	10	9	12
F_{29}	mean	2.4E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	5.0E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	3.7E+04	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	2.1E+04	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06
	rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10
Mean rank		4.10	12.52	12.83	13.28	4.10	4.72	7.41	11.14	5.76	8.07	6.00	3.62	8.41	8.21	9.76
Final rank		2	12	13	14	2	3	6	11	4	7	5	1	9	8	10

Algorithms	HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.0E+03	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	1.3E+03	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	8	14	11	15	12	6	7	13	5	9	3	1	4	2	10
F_3	mean	1.6E-13	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	1.3E-13	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	6	11	8	14	1	2	5	10	3	7	9	1	12	13	4
F_4	mean	1.0E+00	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	9.2E-02	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	6	14	13	15	4	2	3	12	5	9	11	1	10	8	7
F_5	mean	5.7E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	2.8E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	4	13	14	15	2	5	9	10	8	12	6	1	3	7	11
F_6	mean	9.1E-03	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	7.9E-03	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	11	13	14	15	9	1	2	5	8	12	6	10	7	4	3
F_7	mean	1.9E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	4.2E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	7	11	14	15	3	8	5	13	6	12	10	1	4	2	9
F_8	mean	5.7E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	3.7E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	5	13	14	15	2	7	10	9	3	12	8	1	4	6	11
F_9	mean	1.6E-13	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	1.2E-13	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	5	8	10	11	1	2	4	1	2	9	2	1	7	6	3
F_{10}	mean	4.6E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	2.1E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	11	12	13	14	2	3	8	15	6	9	4	1	5	7	10
F_{11}	mean	1.8E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	9.6E-01	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	2	14	13	15	3	6	11	5	4	12	8	1	9	10	7
F_{12}	mean	7.7E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	4.9E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	5	14	2	15	12	8	6	13	10	7	3	1	9	11	4
F_{13}	mean	4.6E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	2.8E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	11	14	4	15	10	7	9	12	8	13	6	1	2	3	5
F_{14}	mean	5.6E+01	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	2.4E+01	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	7	12	8	13	4	2	6	11	3	10	5	1	14	15	9
F_{15}	mean	7.5E+01	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	5.8E+01	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	7	13	8	15	6	2	10	14	3	12	9	1	4	5	11
F_{16}	mean	2.6E+01	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	4.9E+01	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	10	11	14	15	9	1	13	5	4	12	8	2	6	3	7
F_{17}	mean	3.7E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	1.1E+01	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	12	13	15	14	7	4	10	9	5	11	6	1	3	2	8
F_{18}	mean	6.9E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	6.5E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	10	11	12	15	13	4	7	14	5	8	6	1	3	2	9
F_{19}	mean	9.2E+01	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	1.4E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	10	6	8	15	7	2	12	13	5	14	9	1	4	3	11
F_{20}	mean	2.6E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	7.6E+00	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	11	14	15	13	8	4	12	9	6	10	5	2	1	3	7
F_{21}	mean	1.2E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	4.5E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	3	15	14	13	9	5	10	12	4	8	6	7	2	1	11
F_{22}	mean	1.0E+02	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	3.9E-01	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	7	14	6	15	8	1	11	10	12	13	2	5	3	4	9
F_{23}	mean	3.1E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	3.7E+00	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	7	12	14	15	4	3	9	10	5	13	8	2	6	1	11
F_{24}	mean	1.5E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	9.7E+01	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	3	12	14	15	6	5	8	11	4	10	7	9	2	1	13
F_{25}	mean	4.0E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	1.4E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	4	14	12	15	9	3	11	6	1	13	2	10	8	7	5
F_{26}	mean	2.8E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	3.9E+01	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	7	14	12	15	10	3	5	13	6	9	4	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	3.7E-01	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	7	12	13	15	2	10	11	14	8	5	1	6	4	3	9
F_{28}	mean	3.8E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	1.2E+02	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	4	12	11	14	7	1	13	10	1	3	6	8	2	5	9
F_{29}	mean	2.6E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.0E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	5	12	14	15	2	3	9	10	4	13	6	1	11	8	7
F_{30}	mean	2.9E+03	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	1.7E+03	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	6	14	13	15	1	7	12	3	5	4	2	11	9	10	8
Mean rank		6.93	12.48	11.48	14.52	5.97	4.03	8.55	10.07	5.14	10.03	5.79	3.34	5.48	5.31	8.24
Final rank		8	14	13	15	7	2	10	12	3	11	6	1	5	4	9

Algorithms	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.3E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	1.6E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	5	14	13	15	7	9	11	12	6	8	4	1	3	2	10
F_3	mean	1.8E+01	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	1.9E+01	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	7	11	10	14	5	1	2	15	6	3	9	4	13	12	8
F_4	mean	8.8E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	3.4E+00	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	12	14	13	15	5	1	2	4	10	7	3	6	9	8	11
F_5	mean	2.4E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	5.8E+00	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	2	12	14	15	3	5	6	13	4	11	7	1	9	8	10
F_6	mean	2.1E-01	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	7.0E-02	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	10	13	15	14	6	3	8	2	4	12	9	7	5	1	11
F_7	mean	6.1E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	8.9E+00	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	2	11	14	15	3	4	6	12	5	13	10	1	8	7	9
F_8	mean	2.6E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	7.4E+00	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	1	12	13	15	3	4	6	14	5	11	7	2	8	9	10
F_9	mean	1.0E-01	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	1.1E-01	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	2	13	14	15	5	7	6	1	4	12	8	3	10	9	11
F_{10}	mean	3.0E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	5.1E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	5	12	13	14	4	2	8	15	3	7	6	1	10	11	9
F_{11}	mean	4.2E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	1.8E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	1	14	13	15	2	7	8	4	5	12	3	10	11	9	6
F_{12}	mean	3.0E+04	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	2.2E+04	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	5	14	13	15	6	2	4	12	3	7	8	1	10	9	11
F_{13}	mean	1.4E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	1.1E+04	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	1.9E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	2.7E+03	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	2	14	10	15	7	3	4	12	5	6	8	1	11	13	9
F_{15}	mean	1.1E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	8.5E+02	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	6	13	11	14	9	5	10	15	7	8	4	3	1	2	12
F_{16}	mean	5.6E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	2.6E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	6	12	13	15	1	4	9	14	8	11	2	3	7	5	10
F_{17}	mean	1.2E+02	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	5.9E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	3	13	14	15	9	4	8	12	1	11	6	2	5	7	10
F_{18}	mean	8.2E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	2.9E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	4	13	7	15	11	2	3	14	5	6	12	1	8	9	10
F_{19}	mean	3.7E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	2.5E+03	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	8	14	7	15	9	4	10	13	6	11	5	1	2	3	12
F_{20}	mean	2.3E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	2.9E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	9	13	15	14	2	7	6	12	4	10	3	1	5	8	11
F_{21}	mean	2.3E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	9.4E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	2	12	14	15	3	4	7	13	5	11	6	1	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	1.3E-07	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	2	13	12	14	4	5	8	15	3	10	1	7	9	6	11
F_{23}	mean	3.8E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	1.1E+01	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.5E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	1.3E+01	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	2	11	14	15	4	3	6	12	5	13	7	1	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	3.3E-01	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	8	13	14	15	2	11	10	1	7	12	3	6	5	4	9
F_{26}	mean	6.3E+02	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	4.7E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	1	12	14	15	5	2	4	11	3	13	6	7	9	8	10
F_{27}	mean	5.0E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	4.9E+00	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	7	13	14	15	4	10	11	6	9	3	5	8	2	1	12
F_{28}	mean	4.0E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	2.5E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	6	14	13	15	7	1	4	12	5	2	8	3	10	9	11
F_{29}	mean	5.5E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	6.1E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	6	13	14	15	1	5	9	11	2	10	3	4	7	8	12
F_{30}	mean	4.0E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	1.3E+03	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	7	14	13	15	6	8	9	11	10	2	1	3	5	4	12

Mean rank	4.90	12.76	12.83	14.79	5.03	4.48	6.83	10.79	5.21	9.07	5.66	3.14	7.28	6.97	10.28
Final rank	3	13	14	15	4	2	7	12	5	10	6	1	9	8	11
Algorithms	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.1E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	1.2E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	3	14	13	15	10	7	8	11	9	6	5	1	4	2	12
F_3	mean	1.9E+03	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	9.9E+02	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	6	11	8	12	4	1	3	15	5	2	9	7	14	13	10
F_4	mean	1.1E+02	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	2.8E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	11	14	13	15	3	1	6	5	10	7	4	2	8	9	12
F_5	mean	4.3E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	9.2E+00	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	10	8
F_6	mean	4.0E-01	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	1.1E-01	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	8	13	15	14	7	2	9	1	4	11	10	6	5	3	12
F_7	mean	1.1E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	1.4E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	1	12	14	15	3	4	5	11	6	13	7	2	8	10	9
F_8	mean	4.2E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	7.9E+00	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	8	10
F_9	mean	1.3E+00	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	9.6E-01	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	5.3E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	8.3E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	5	10	13	14	4	2	6	15	3	7	8	1	11	12	9
F_{11}	mean	9.7E+01	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	1.6E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	1	14	13	15	2	5	7	8	4	6	3	9	12	11	10
F_{12}	mean	5.3E+05	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	3.2E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	7	14	13	15	4	2	3	12	5	6	10	1	8	9	11
F_{13}	mean	2.3E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	1.9E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	5	13	14	15	10	6	8	12	7	9	1	3	4	2	11
F_{14}	mean	1.9E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.5E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	4	10	12	15	7	2	3	13	6	5	9	1	11	14	8
F_{15}	mean	8.6E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	5.5E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	9	15	11	14	8	2	7	13	5	10	3	1	6	4	12
F_{16}	mean	5.3E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	1.8E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	1	12	13	14	2	7	8	15	9	10	3	4	6	5	11
F_{17}	mean	6.9E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	2.3E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	1	12	13	15	2	8	9	14	7	11	6	3	4	5	10
F_{18}	mean	6.7E+04	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	4.5E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	4	13	8	15	7	2	3	14	5	6	10	1	11	12	9
F_{19}	mean	1.4E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	4.8E+03	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	8	15	13	14	10	2	9	12	6	7	3	1	5	4	11
F_{20}	mean	4.0E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	1.8E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	2	12	13	14	1	4	9	15	5	11	6	3	8	7	10
F_{21}	mean	2.4E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	1.2E+01	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	1	12	14	15	3	6	5	13	4	11	7	2	8	9	10
F_{22}	mean	3.8E+02	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	1.3E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	4.8E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	1.5E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	1	12	15	14	3	4	6	11	5	13	7	2	8	9	10
F_{24}	mean	5.7E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	3.1E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	9	10	7
F_{25}	mean	4.9E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	2.0E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	7	14	13	15	3	8	10	1	11	4	2	12	6	5	9
F_{26}	mean	6.3E+02	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	5.8E+02	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	1	12	15	14	2	3	5	11	6	13	7	4	9	8	10
F_{27}	mean	5.3E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	1.2E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.7E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	1.9E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	4	15	12	14	1	10	9	11	5	3	2	8	7	6	13
F_{29}	mean	6.4E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.1E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	1	11	14	15	4	5	9	13	3	10	6	2	8	7	12
F_{30}	mean	7.5E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	6.9E+04	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	7	14	13	15	5	6	10	11	9	4	1	8	3	2	12
Mean rank		3.86	12.52	13.03	14.55	4.31	4.55	6.93	10.93	5.90	8.34	5.83	3.55	7.76	7.62	10.31
Final rank		2	13	14	15	3	4	7	12	6	10	5	1	9	8	11

Algorithms	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 100-dimensional CEC'17 problems for FFQ-HIDMS-PSO vs comparison algorithms

F	Criteria	FFQ-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	1.2E+04	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	1.5E+04	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	9	15	14	13	8	10	6	12	7	5	3	1	2	4	11
F_3	mean	4.7E+04	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	6.6E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	6	12	7	11	5	1	4	15	3	2	9	8	13	14	10
F_4	mean	2.5E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	2.4E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	10	15	13	14	3	5	8	2	7	6	1	4	11	9	12
F_5	mean	9.9E+01	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	1.8E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_6	mean	6.4E-01	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	1.2E-01	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	6	13	15	14	8	1	10	4	5	11	9	7	2	3	12
F_7	mean	2.1E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	1.7E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	1	14	15	13	2	4	5	11	7	12	6	3	9	10	8
F_8	mean	9.9E+01	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.3E+02	8.5E+02	5.5E+02
	std	1.6E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	1	12	15	14	2	6	5	13	4	9	7	3	10	11	8
F_9	mean	1.6E+01	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	8.7E+00	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	1	11	12	15	2	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.3E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.6E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	4	9	11	14	6	2	5	15	3	7	10	1	13	12	8
F_{11}	mean	7.9E+02	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	1.0E+02	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	3	13	10	14	1	4	5	15	2	6	8	7	12	11	9
F_{12}	mean	1.4E+06	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	7.7E+05	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	7	15	14	13	6	2	3	12	4	5	10	1	9	8	11
F_{13}	mean	3.8E+03	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	3.1E+03	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	6	15	14	12	9	1	5	13	4	10	2	8	7	3	11
F_{14}	mean	9.2E+04	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	2.3E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	4	13	10	15	7	3	2	14	6	5	9	1	12	11	8
F_{15}	mean	1.1E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	1.2E+03	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	5	15	14	12	10	2	7	13	9	8	6	1	4	3	11
F_{16}	mean	1.8E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	4.9E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	1.6E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	3.5E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	2.6E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	7.7E+04	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	5	11	9	14	7	1	4	15	6	3	10	2	12	13	8
F_{19}	mean	2.1E+03	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	2.2E+03	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	7	15	14	11	10	1	6	13	5	8	3	9	2	4	12
F_{20}	mean	2.1E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	4.2E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	3	12	13	14	1	7	8	15	5	10	9	2	4	6	11
F_{21}	mean	3.3E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	2.0E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	1	13	15	14	2	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	6.6E+03	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	6.7E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	1	9	11	14	4	3	5	15	6	7	10	2	13	12	8
F_{23}	mean	6.5E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	2.3E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	1	11	15	14	3	4	7	12	6	13	5	2	8	9	10
F_{24}	mean	1.0E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	3.2E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	1	12	15	13	2	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	8.1E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	3.8E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	7	14	12	13	1	5	15	8	4	6	2	3	11	10	9
F_{26}	mean	2.5E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	1.8E+03	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	6.4E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	2.4E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	4	9	12	13	1	14	14	3	8	10	2	7	6	5	11
F_{28}	mean	5.8E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	1.5E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	8	14	11	13	2	6	15	3	7	4	1	5	10	9	12
F_{29}	mean	2.1E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	3.7E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	9.6E+03	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	3.2E+03	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06
	rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10
Mean rank		3.93	12.52	12.83	13.28	4.24	4.69	7.45	11.14	5.66	8.21	6.00	3.62	8.45	8.17	9.76

Final rank	2	13	14	15	3	4	7	12	5	9	6	1	10	8	11
Algorithms	FFC-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.1E+02	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	1.4E+03	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	6	14	11	15	12	7	8	13	5	9	3	1	4	2	10
F_3	mean	8.5E-14	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	7.0E-14	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	6	11	8	14	1	2	5	10	3	7	9	1	12	13	4
F_4	mean	1.9E+00	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	7.3E-01	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	6	14	13	15	4	2	3	12	5	9	11	1	10	8	7
F_5	mean	7.4E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	3.0E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	8	13	14	15	2	4	9	10	7	12	5	1	3	6	11
F_6	mean	1.2E-07	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	3.4E-07	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	8	13	14	15	10	1	2	5	9	12	6	11	7	4	3
F_7	mean	1.5E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	2.5E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	2	11	14	15	4	8	6	13	7	12	10	1	5	3	9
F_8	mean	5.7E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	1.7E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	5	13	14	15	2	7	10	9	3	12	8	1	4	6	11
F_9	mean	1.1E-13	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	8.6E-14	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	5	8	10	11	1	2	4	1	2	9	2	1	7	6	3
F_{10}	mean	4.3E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	1.4E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	10	12	13	14	2	3	8	15	6	9	4	1	5	7	11
F_{11}	mean	4.4E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	2.3E+00	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	10	14	13	15	2	5	11	4	3	12	7	1	8	9	6
F_{12}	mean	8.0E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	5.6E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	6	14	2	15	12	8	5	13	10	7	3	1	9	11	4
F_{13}	mean	4.5E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	3.1E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	11	14	4	15	10	7	9	12	8	13	6	1	2	3	5
F_{14}	mean	1.7E+02	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	2.7E+02	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	10	12	7	13	4	2	6	11	3	9	5	1	14	15	8
F_{15}	mean	1.3E+02	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	2.7E+02	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	10	13	7	15	6	2	9	14	3	12	8	1	4	5	11
F_{16}	mean	5.5E+01	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	8.1E+01	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	10	11	14	15	9	1	13	5	4	12	8	2	6	3	7
F_{17}	mean	1.3E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	8.9E+00	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	6	13	15	14	8	4	11	10	5	12	7	1	3	2	9
F_{18}	mean	2.2E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	1.8E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	5	11	12	15	13	4	8	14	6	9	7	1	3	2	10
F_{19}	mean	2.7E+02	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	4.0E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	12	6	8	15	7	2	11	13	5	14	9	1	4	3	10
F_{20}	mean	1.3E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	1.1E+01	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	8	14	15	13	9	4	12	10	6	11	5	2	1	3	7
F_{21}	mean	1.4E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	5.5E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	4	15	14	13	9	5	10	12	3	8	6	7	2	1	11
F_{22}	mean	1.0E+02	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	6.4E-01	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	9	14	6	15	7	1	11	10	12	13	2	5	3	4	8
F_{23}	mean	2.9E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	6.9E+01	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	1	12	14	15	5	4	9	10	6	13	8	3	7	2	11
F_{24}	mean	1.5E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	9.6E+01	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	3	12	14	15	6	5	8	11	4	10	7	9	2	1	13
F_{25}	mean	4.3E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	2.3E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	12	14	11	15	8	3	10	5	1	13	2	9	7	6	4
F_{26}	mean	2.5E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	1.1E+02	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	4	14	12	15	10	3	6	13	7	9	5	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	1.4E+00	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	7	12	13	15	2	10	11	14	8	5	1	6	4	3	9
F_{28}	mean	4.3E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	1.5E+02	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	8	12	11	14	6	1	13	10	1	3	5	7	2	4	9
F_{29}	mean	2.7E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.5E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	8	12	14	15	2	3	9	10	4	13	5	1	11	7	6
F_{30}	mean	5.4E+03	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	3.6E+03	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	7	14	13	15	1	6	12	3	5	4	2	11	9	10	8
Mean rank		7.14	12.48	11.38	14.52	6.00	4.00	8.59	10.07	5.21	10.10	5.72	3.34	5.48	5.21	8.14
Final rank		7	13	12	14	6	2	9	10	3	11	5	1	4	3	8

Algorithms	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.2E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	5.2E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	11	14	13	15	6	8	10	12	5	7	4	1	3	2	9
F_3	mean	2.0E-10	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	3.4E-10	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	2	11	10	14	6	1	3	15	7	4	9	5	13	12	8
F_4	mean	6.6E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	3.9E+01	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	8	14	13	15	5	1	2	4	11	7	3	6	10	9	12
F_5	mean	4.2E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	9.9E+00	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_6	mean	1.2E-02	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	2.3E-02	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	8	13	15	14	6	3	9	2	4	12	10	7	5	1	11
F_7	mean	6.3E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	9.3E+00	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	2	11	14	15	3	4	6	12	5	13	10	1	8	7	9
F_8	mean	4.2E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	8.5E+00	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	3	12	13	15	2	4	6	14	5	11	7	1	8	9	10
F_9	mean	8.6E-01	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	5.8E-01	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	2	13	14	15	5	7	6	1	4	12	8	3	10	9	11
F_{10}	mean	2.4E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	5.9E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	3	12	13	14	5	2	8	15	4	7	6	1	10	11	9
F_{11}	mean	5.5E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	1.6E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	4	14	13	15	1	7	8	3	5	12	2	10	11	9	6
F_{12}	mean	3.0E+05	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	1.8E+05	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	8	14	13	15	5	2	4	12	3	6	7	1	10	9	11
F_{13}	mean	1.2E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	8.4E+03	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	1.7E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	2.1E+03	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	2	14	10	15	7	3	4	12	5	6	8	1	11	13	9
F_{15}	mean	3.3E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	7.6E+03	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	7	13	11	14	9	5	10	15	6	8	4	3	1	2	12
F_{16}	mean	4.7E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	2.2E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	4	12	13	15	1	5	9	14	8	11	2	3	7	6	10
F_{17}	mean	1.3E+02	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	6.5E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	3	13	14	15	9	4	8	12	1	11	6	2	5	7	10
F_{18}	mean	6.9E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	3.4E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	2	13	7	15	11	3	4	14	5	6	12	1	8	9	10
F_{19}	mean	4.8E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	3.6E+03	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	8	14	7	15	9	4	10	13	6	11	5	1	2	3	12
F_{20}	mean	1.7E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	4.3E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	4	13	15	14	2	8	7	12	5	10	3	1	6	9	11
F_{21}	mean	2.3E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	8.3E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	3	12	14	15	2	4	7	13	5	11	6	1	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	7.6E-01	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	2	13	12	14	4	5	8	15	3	10	1	7	9	6	11
F_{23}	mean	3.8E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	1.6E+01	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.3E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	1.8E+01	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	1	11	14	15	4	3	6	12	5	13	7	2	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	4.0E+00	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	11	13	14	15	2	10	9	1	7	12	3	6	5	4	8
F_{26}	mean	4.8E+02	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	4.7E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	1	12	14	15	5	2	4	11	3	13	6	7	9	8	10
F_{27}	mean	5.1E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	7.8E+00	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	8	13	14	15	4	10	11	6	9	3	5	7	2	1	12
F_{28}	mean	3.3E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	3.8E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	1	14	13	15	7	2	5	12	6	3	8	4	10	9	11
F_{29}	mean	4.9E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	4.0E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	2	13	14	15	1	6	9	11	3	10	4	5	7	8	12
F_{30}	mean	4.6E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	1.6E+03	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	9	14	13	15	6	7	8	11	10	2	1	3	5	4	12

Mean rank	4.59	12.76	12.83	14.79	4.90	4.55	6.90	10.76	5.34	9.07	5.66	3.21	7.34	7.07	10.24
Final rank	3	13	14	15	4	2	7	12	5	10	6	1	9	8	11
Algorithms	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	1.4E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	1.6E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	4	14	13	15	10	7	8	11	9	6	5	1	3	2	12
F_3	mean	7.8E+00	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	9.5E+00	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	3	11	8	12	5	1	4	15	6	2	9	7	14	13	10
F_4	mean	7.5E+01	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	5.7E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	8	14	13	15	3	1	6	5	11	7	4	2	9	10	12
F_5	mean	9.1E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	1.6E+01	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	10	8
F_6	mean	1.1E-01	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	7.3E-02	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	7	13	15	14	8	2	9	1	4	11	10	6	5	3	12
F_7	mean	1.2E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	1.5E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	1	12	14	15	3	4	5	11	6	13	7	2	8	10	9
F_8	mean	9.0E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	1.8E+01	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_9	mean	4.1E+00	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	2.1E+00	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	4.3E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	7.3E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	3	10	13	14	5	2	6	15	4	7	8	1	11	12	9
F_{11}	mean	1.1E+02	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	2.0E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	1	14	13	15	2	5	7	8	4	6	3	9	12	11	10
F_{12}	mean	1.3E+06	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	8.8E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	7	14	13	15	4	2	3	12	5	6	10	1	8	9	11
F_{13}	mean	5.0E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	5.1E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	8	13	14	15	10	5	7	12	6	9	1	3	4	2	11
F_{14}	mean	2.3E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.3E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	4	10	12	15	7	2	3	13	6	5	9	1	11	14	8
F_{15}	mean	5.8E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	4.8E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	8	15	11	14	9	2	7	13	5	10	3	1	6	4	12
F_{16}	mean	5.1E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	1.7E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	1	12	13	14	2	7	8	15	9	10	3	4	6	5	11
F_{17}	mean	6.1E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	1.8E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	1	12	13	15	2	8	9	14	7	11	6	3	4	5	10
F_{18}	mean	9.3E+04	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	3.9E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	6	13	8	15	7	2	3	14	4	5	10	1	11	12	9
F_{19}	mean	1.3E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	3.6E+03	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	7	15	13	14	10	2	9	12	6	8	3	1	5	4	11
F_{20}	mean	3.6E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	1.5E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	1	12	13	14	2	4	9	15	5	11	6	3	8	7	10
F_{21}	mean	2.8E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	1.1E+01	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	3	12	14	15	2	6	5	13	4	11	7	1	8	9	10
F_{22}	mean	1.6E+03	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	2.4E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	5.0E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	2.2E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	1	12	15	14	3	4	6	11	5	13	7	2	8	9	10
F_{24}	mean	5.7E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	2.8E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	9	10	7
F_{25}	mean	5.5E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	4.3E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	12	14	13	15	3	7	9	1	10	4	2	11	6	5	8
F_{26}	mean	1.2E+03	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	9.0E+02	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	1	12	15	14	2	3	5	11	6	13	7	4	9	8	10
F_{27}	mean	5.5E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	2.7E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.9E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	2.2E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	7	15	12	14	1	10	9	11	4	3	2	8	6	5	13
F_{29}	mean	6.2E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.2E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	1	11	14	15	4	5	9	13	3	10	6	2	8	7	12
F_{30}	mean	8.7E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	6.5E+04	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	10	14	13	15	5	6	9	11	8	4	1	7	3	2	12

Mean rank	4.24	12.52	13.03	14.55	4.38	4.48	6.86	10.93	5.83	8.34	5.83	3.38	7.72	7.62	10.28
Final rank	2	12	13	14	3	4	6	11	5	9	5	1	8	7	10
Algorithms	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'17 problems for GA-HIDMS-PSO vs comparison algorithms

F	Criteria	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	3.4E+03	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	3.0E+03	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	3	15	14	13	9	10	7	12	8	6	4	1	2	5	11
F_3	mean	5.2E+03	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	1.3E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	2	12	7	11	6	1	5	15	4	3	9	8	13	14	10
F_4	mean	2.0E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	3.6E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	7	15	13	14	3	5	9	2	8	6	1	4	11	10	12
F_5	mean	2.1E+02	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	2.6E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	2	12	14	15	1	6	5	13	4	9	7	3	11	10	8
F_6	mean	7.0E-01	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	2.7E-01	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	6	13	15	14	8	1	10	4	5	11	9	7	2	3	12
F_7	mean	2.8E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	3.6E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	1	14	15	13	2	4	5	11	7	12	6	3	9	10	8
F_8	mean	2.4E+02	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.2E+02	8.5E+02	5.5E+02
	std	2.8E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	3	12	15	14	1	6	5	13	4	9	7	2	10	11	8
F_9	mean	4.3E+01	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	3.0E+01	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	1	11	12	15	2	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.1E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.3E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	2	9	11	14	6	3	5	15	4	7	10	1	13	12	8
F_{11}	mean	7.0E+02	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	8.9E+01	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	2	13	10	14	1	4	5	15	3	6	8	7	12	11	9
F_{12}	mean	7.2E+05	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	2.7E+05	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	4	15	14	13	7	2	3	12	5	6	10	1	9	8	11
F_{13}	mean	4.7E+03	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	1.6E+03	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	7	15	14	12	9	1	5	13	4	10	2	8	6	3	11
F_{14}	mean	5.6E+04	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	1.4E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	4	13	10	15	7	3	2	14	6	5	9	1	12	11	8
F_{15}	mean	1.2E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	7.0E+02	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	5	15	14	12	10	2	7	13	9	8	6	1	4	3	11
F_{16}	mean	2.0E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	5.8E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	1.7E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	5.2E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	1.1E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	2.1E+04	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	3	11	9	14	7	1	5	15	6	4	10	2	12	13	8
F_{19}	mean	1.7E+03	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	2.1E+03	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	5	15	14	11	10	1	7	13	6	8	3	9	2	4	12
F_{20}	mean	1.6E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	4.2E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	1	12	13	14	2	7	8	15	5	10	9	3	4	6	11
F_{21}	mean	4.3E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	4.0E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	2	13	15	14	1	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	1.2E+04	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	3.2E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	2	9	11	14	4	3	5	15	6	7	10	1	13	12	8
F_{23}	mean	7.3E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	3.4E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	2	11	15	14	3	4	7	12	6	13	5	1	8	9	10
F_{24}	mean	1.0E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	3.9E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	1	12	15	13	2	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	7.3E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	6.0E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	1	14	12	13	2	6	15	8	5	7	3	4	11	10	9
F_{26}	mean	5.4E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	4.4E+02	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	6.9E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	2.8E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	4	9	12	13	1	14	14	3	8	10	2	7	6	5	11
F_{28}	mean	5.5E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	1.7E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	7	14	11	13	2	6	15	3	8	4	1	5	10	9	12
F_{29}	mean	2.2E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	4.5E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	7.4E+03	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	2.9E+03	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06
	rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10
Mean rank		3.07	12.52	12.83	13.28	4.31	4.76	7.62	11.14	5.97	8.38	6.07	3.59	8.41	8.24	9.76
Final rank		1	13	14	15	3	4	7	12	5	9	6	2	10	8	11

Algorithms	GA-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.7E+02	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	1.6E+03	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	7	14	11	15	12	6	8	13	5	9	3	1	4	2	10
F_3	mean	4.5E-14	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	3.0E-14	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	5	10	7	13	1	2	5	9	3	6	8	1	11	12	4
F_4	mean	1.3E-01	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	1.9E-02	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	2	14	13	15	5	3	4	12	6	9	11	1	10	8	7
F_5	mean	9.7E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	4.4E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	10	13	14	15	2	4	8	9	7	12	5	1	3	6	11
F_6	mean	5.6E-06	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	2.1E-05	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	10	13	14	15	9	1	2	5	8	12	6	11	7	4	3
F_7	mean	2.1E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	5.2E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	11	10	14	15	3	7	5	13	6	12	9	1	4	2	8
F_8	mean	8.7E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	3.0E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	10	13	14	15	2	6	9	8	3	12	7	1	4	5	11
F_9	mean	9.1E-14	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	8.7E-14	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	5	8	10	11	1	2	4	1	2	9	2	1	7	6	3
F_{10}	mean	4.9E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	2.5E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	11	12	13	14	2	3	8	15	6	9	4	1	5	7	10
F_{11}	mean	3.7E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	1.9E+00	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	9	14	13	15	2	5	11	4	3	12	7	1	8	10	6
F_{12}	mean	9.0E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	6.4E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	7	14	2	15	12	8	5	13	10	6	3	1	9	11	4
F_{13}	mean	3.2E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	2.1E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	9	14	4	15	11	7	10	12	8	13	6	1	2	3	5
F_{14}	mean	4.6E+01	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	1.2E+01	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	5	12	8	13	4	2	7	11	3	10	6	1	14	15	9
F_{15}	mean	7.7E+01	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	6.6E+01	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	7	13	8	15	6	2	10	14	3	12	9	1	4	5	11
F_{16}	mean	4.9E+00	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	4.4E+00	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	4	11	14	15	10	1	13	6	5	12	9	2	7	3	8
F_{17}	mean	2.4E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	9.0E+00	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	8	13	15	14	7	4	11	10	5	12	6	1	3	2	9
F_{18}	mean	7.5E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	6.5E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	10	11	12	15	13	4	7	14	5	8	6	1	3	2	9
F_{19}	mean	1.5E+02	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	3.6E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	12	6	8	15	7	2	11	13	5	14	9	1	4	3	10
F_{20}	mean	2.5E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	6.9E+00	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	11	14	15	13	8	4	12	9	6	10	5	2	1	3	7
F_{21}	mean	1.1E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	3.4E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	1	15	14	13	9	5	10	12	4	8	6	7	3	2	11
F_{22}	mean	9.2E+01	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	2.8E+01	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	3	14	7	15	8	1	11	10	12	13	2	6	4	5	9
F_{23}	mean	3.1E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.2E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	3.4E+00	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	6	12	14	15	4	3	9	10	5	13	8	2	7	1	11
F_{24}	mean	2.1E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	1.2E+02	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	4	12	14	15	6	5	8	11	3	10	7	9	2	1	13
F_{25}	mean	4.1E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	1.7E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	4	14	12	15	9	3	11	6	1	13	2	10	8	7	5
F_{26}	mean	3.0E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	2.2E+01	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	7	14	12	15	10	3	5	13	6	9	4	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	5.9E-01	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	7	12	13	15	2	10	11	14	8	5	1	6	4	3	9
F_{28}	mean	3.7E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	1.2E+02	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	3	12	11	14	7	1	13	10	1	4	6	8	2	5	9
F_{29}	mean	2.6E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.1E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	5	12	14	15	2	3	9	10	4	13	6	1	11	8	7
F_{30}	mean	2.4E+03	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	1.3E+03	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	6	14	13	15	1	7	12	3	5	4	2	11	9	10	8
Mean rank		6.86	12.41	11.48	14.48	6.03	3.93	8.59	10.00	5.10	10.03	5.69	3.41	5.55	5.28	8.21
Final rank		8	14	13	15	7	2	10	11	3	12	6	1	5	4	9

Algorithms	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.4E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	2.4E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	5	14	13	15	7	9	11	12	6	8	4	1	3	2	10
F_3	mean	7.9E-12	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	6.8E-12	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	2	11	10	14	6	1	3	15	7	4	9	5	13	12	8
F_4	mean	5.8E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	2.3E+00	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	8	14	13	15	5	1	2	4	11	7	3	6	10	9	12
F_5	mean	4.2E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	1.0E+01	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_6	mean	1.1E-02	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	2.5E-02	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	8	13	15	14	6	3	9	2	4	12	10	7	5	1	11
F_7	mean	7.3E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	1.4E+01	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	3	11	14	15	2	4	6	12	5	13	10	1	8	7	9
F_8	mean	4.0E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	1.6E+01	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	3	12	13	15	2	4	6	14	5	11	7	1	8	9	10
F_9	mean	5.3E-01	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	7.8E-01	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	2	13	14	15	5	7	6	1	4	12	8	3	10	9	11
F_{10}	mean	2.9E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	6.8E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	5	12	13	14	4	2	8	15	3	7	6	1	10	11	9
F_{11}	mean	6.9E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	3.0E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	5	14	13	15	1	7	8	3	4	12	2	10	11	9	6
F_{12}	mean	4.6E+04	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	3.1E+04	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	5	14	13	15	6	2	4	12	3	7	8	1	10	9	11
F_{13}	mean	1.1E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	1.1E+04	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	1.5E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	1.1E+03	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	2	14	10	15	7	3	4	12	5	6	8	1	11	13	9
F_{15}	mean	2.9E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	4.0E+03	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	6	13	11	14	9	5	10	15	7	8	4	3	1	2	12
F_{16}	mean	5.0E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	2.5E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	4	12	13	15	1	5	9	14	8	11	2	3	7	6	10
F_{17}	mean	8.8E+01	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	3.8E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	1	13	14	15	9	4	8	12	2	11	6	3	5	7	10
F_{18}	mean	7.2E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	3.8E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	3	13	7	15	11	2	4	14	5	6	12	1	8	9	10
F_{19}	mean	7.8E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	1.2E+04	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	11	14	7	15	8	4	9	13	6	10	5	1	2	3	12
F_{20}	mean	2.3E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	9.1E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	9	13	15	14	2	7	6	12	4	10	3	1	5	8	11
F_{21}	mean	2.4E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	7.9E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	3	12	14	15	2	4	7	13	5	11	6	1	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	1.3E+00	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	4	13	12	14	3	5	8	15	2	10	1	7	9	6	11
F_{23}	mean	4.0E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	1.5E+01	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	3	11	14	15	2	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.6E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	1.1E+01	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	4	11	14	15	3	2	6	12	5	13	7	1	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	1.9E-01	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	6	13	14	15	2	11	10	1	8	12	3	7	5	4	9
F_{26}	mean	1.1E+03	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	5.1E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	4	12	14	15	5	1	3	11	2	13	6	7	9	8	10
F_{27}	mean	5.1E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	4.1E+00	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	7	13	14	15	4	10	11	6	9	3	5	8	2	1	12
F_{28}	mean	3.3E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	5.4E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	1	14	13	15	7	2	5	12	6	3	8	4	10	9	11
F_{29}	mean	5.5E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	4.6E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	6	13	14	15	1	5	9	11	2	10	3	4	7	8	12
F_{30}	mean	7.2E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	4.5E+03	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	10	14	13	15	6	7	8	11	9	2	1	3	5	4	12
Mean rank		4.90	12.76	12.83	14.79	4.76	4.45	6.86	10.76	5.24	9.10	5.66	3.24	7.31	7.03	10.31
Final rank		4	13	14	15	3	2	7	12	5	10	6	1	9	8	11

Algorithms	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	5.4E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	7.1E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	9	14	13	15	10	6	7	11	8	5	4	1	3	2	12
F_3	mean	2.3E-03	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	9.2E-03	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	2	11	8	12	5	1	4	15	6	3	9	7	14	13	10
F_4	mean	1.1E+02	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	5.0E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	10	14	13	15	3	1	6	5	11	7	4	2	8	9	12
F_5	mean	8.3E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	1.7E+01	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	10	8
F_6	mean	4.2E-02	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	3.8E-02	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	6	13	15	14	8	2	9	1	4	11	10	7	5	3	12
F_7	mean	1.6E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	2.6E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	3	12	14	15	2	4	5	11	6	13	7	1	8	10	9
F_8	mean	7.4E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	1.2E+01	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	3	12	14	15	2	5	6	13	4	11	7	1	9	8	10
F_9	mean	5.2E+00	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	6.5E+00	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	5.2E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	8.9E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	5	10	13	14	4	2	6	15	3	7	8	1	11	12	9
F_{11}	mean	1.6E+02	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	2.3E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	5	14	13	15	1	4	7	8	3	6	2	9	12	11	10
F_{12}	mean	8.0E+05	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	3.2E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	7	14	13	15	4	2	3	12	5	6	10	1	8	9	11
F_{13}	mean	9.2E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	3.9E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	10	13	14	15	9	5	7	12	6	8	1	3	4	2	11
F_{14}	mean	1.4E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.0E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	3	10	12	15	7	2	4	13	6	5	9	1	11	14	8
F_{15}	mean	3.9E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	3.8E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	7	15	11	14	9	2	8	13	5	10	3	1	6	4	12
F_{16}	mean	9.5E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	3.6E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	3	12	13	14	1	7	8	15	9	10	2	4	6	5	11
F_{17}	mean	7.6E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	1.8E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	3	12	13	15	1	8	9	14	7	11	6	2	4	5	10
F_{18}	mean	7.4E+04	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	3.7E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	4	13	8	15	7	2	3	14	5	6	10	1	11	12	9
F_{19}	mean	1.7E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	1.1E+04	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	9	15	13	14	10	2	8	12	6	7	3	1	5	4	11
F_{20}	mean	6.2E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	2.1E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	8	12	13	14	1	3	9	15	4	11	5	2	7	6	10
F_{21}	mean	2.8E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	1.7E+01	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	3	12	14	15	2	6	5	13	4	11	7	1	8	9	10
F_{22}	mean	1.0E+03	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	2.0E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	5.2E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	1.6E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	3	12	15	14	2	4	6	11	5	13	7	1	8	9	10
F_{24}	mean	5.8E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	2.1E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	9	10	7
F_{25}	mean	4.9E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	2.5E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	7	14	13	15	3	8	10	1	11	4	2	12	6	5	9
F_{26}	mean	2.0E+03	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	4.4E+02	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	4	12	15	14	1	2	5	11	6	13	7	3	9	8	10
F_{27}	mean	5.4E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	1.2E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.8E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	2.1E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	4	15	12	14	1	10	9	11	5	3	2	8	7	6	13
F_{29}	mean	7.2E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.7E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	2	11	14	15	4	5	9	13	3	10	6	1	8	7	12
F_{30}	mean	7.7E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	1.3E+05	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	8	14	13	15	5	6	10	11	9	4	1	7	3	2	12
Mean rank		4.93	12.52	13.03	14.55	4.07	4.38	6.93	10.93	5.83	8.31	5.69	3.24	7.69	7.59	10.31

Final rank	4	13	14	15	2	3	7	12	6	10	5	1	9	8	11
Algorithms	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'17 problems for A-HIDMS-PSO vs comparison algorithms

F	Criteria	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	9.5E+03	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	1.0E+04	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	9	15	14	13	8	10	6	12	7	5	3	1	2	4	11
F_3	mean	5.9E+03	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	1.7E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	3	12	7	11	6	1	5	15	4	2	9	8	13	14	10
F_4	mean	2.3E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	1.9E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	9	15	13	14	3	5	8	2	7	6	1	4	11	10	12
F_5	mean	2.2E+02	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	2.7E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	2	12	14	15	1	6	5	13	4	9	7	3	11	10	8
F_6	mean	8.9E-01	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	6.1E-01	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	6	13	15	14	8	1	10	4	5	11	9	7	2	3	12
F_7	mean	3.9E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	5.4E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	2	14	15	13	1	4	5	11	7	12	6	3	9	10	8
F_8	mean	2.2E+02	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.3E+02	8.5E+02	5.5E+02
	std	3.7E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	2	12	15	14	1	6	5	13	4	9	7	3	10	11	8
F_9	mean	2.0E+02	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	1.8E+02	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	2	11	12	15	1	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.2E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.8E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	4	9	11	14	6	2	5	15	3	7	10	1	13	12	8
F_{11}	mean	9.4E+02	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	1.2E+02	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	5	13	10	14	1	3	4	15	2	6	8	7	12	11	9
F_{12}	mean	9.9E+05	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	4.0E+05	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	6	15	14	13	7	2	3	12	4	5	10	1	9	8	11
F_{13}	mean	9.5E+03	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	2.8E+03	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	10	15	14	12	8	1	5	13	4	9	2	7	6	3	11
F_{14}	mean	6.1E+04	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	3.2E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	4	13	10	15	7	3	2	14	6	5	9	1	12	11	8
F_{15}	mean	3.7E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	2.5E+03	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	9	15	14	12	10	2	6	13	8	7	5	1	4	3	11
F_{16}	mean	2.5E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	6.5E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	2.0E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	5.1E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	1.6E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	8.3E+04	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	3	11	9	14	7	1	5	15	6	4	10	2	12	13	8
F_{19}	mean	1.3E+04	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	1.5E+04	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	10	15	14	11	9	1	6	13	5	7	3	8	2	4	12
F_{20}	mean	2.2E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	4.0E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	3	12	13	14	1	7	8	15	5	10	9	2	4	6	11
F_{21}	mean	4.3E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	3.5E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	2	13	15	14	1	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	1.3E+04	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	3.3E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	4	9	11	14	3	2	5	15	6	7	10	1	13	12	8
F_{23}	mean	7.3E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	3.6E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	2	11	15	14	3	4	7	12	6	13	5	1	8	9	10
F_{24}	mean	1.1E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	3.6E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	2	12	15	13	1	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	7.1E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	5.8E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	1	14	12	13	2	6	15	8	5	7	3	4	11	10	9
F_{26}	mean	5.4E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	4.5E+02	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	6.5E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	2.6E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	4	9	12	13	1	14	14	3	8	10	2	7	6	5	11
F_{28}	mean	5.3E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	2.7E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	5	14	11	13	2	7	15	3	8	4	1	6	10	9	12
F_{29}	mean	2.4E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	4.3E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	8.2E+03	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	4.5E+03	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06
	rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10

Mean rank	4.17	12.52	12.83	13.28	4.03	4.69	7.45	11.14	5.72	8.17	6.00	3.55	8.41	8.21	9.76
Final rank	3	13	14	15	2	4	7	12	5	8	6	1	10	9	11
Algorithms	A-HIDMS-PSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	7.5E+02	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	1.4E+03	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	5	14	11	15	12	7	8	13	6	9	3	1	4	2	10
F_3	mean	1.7E-13	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	1.4E-13	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	6	11	8	14	1	2	5	10	3	7	9	1	12	13	4
F_4	mean	9.8E-01	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	2.3E-01	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	6	14	13	15	4	2	3	12	5	9	11	1	10	8	7
F_5	mean	4.3E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	1.7E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	2	13	14	15	3	5	9	10	8	12	6	1	4	7	11
F_6	mean	3.9E-05	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	7.8E-05	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	11	13	14	15	9	1	2	5	8	12	6	10	7	4	3
F_7	mean	1.6E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	3.2E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	5	11	14	15	3	8	6	13	7	12	10	1	4	2	9
F_8	mean	3.3E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	2.4E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	1	13	14	15	3	7	10	9	4	12	8	2	5	6	11
F_9	mean	1.1E-13	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	1.1E-13	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	5	8	10	11	1	2	4	1	2	9	2	1	7	6	3
F_{10}	mean	3.6E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	1.7E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	9	12	13	14	2	3	8	15	6	10	4	1	5	7	11
F_{11}	mean	1.2E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	8.2E-01	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	2	14	13	15	3	6	11	5	4	12	8	1	9	10	7
F_{12}	mean	6.2E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	2.8E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	4	14	2	15	12	8	6	13	10	7	3	1	9	11	5
F_{13}	mean	3.7E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	1.9E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	9	14	4	15	11	7	10	12	8	13	6	1	2	3	5
F_{14}	mean	5.1E+01	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	2.2E+01	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	7	12	8	13	4	2	6	11	3	10	5	1	14	15	9
F_{15}	mean	8.6E+01	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	9.9E+01	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	7	13	8	15	6	2	10	14	3	12	9	1	4	5	11
F_{16}	mean	1.5E+01	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	3.7E+01	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	8	11	14	15	10	1	13	5	4	12	9	2	6	3	7
F_{17}	mean	3.4E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	8.7E+00	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	11	13	15	14	7	4	10	9	5	12	6	1	3	2	8
F_{18}	mean	6.5E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	6.7E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	10	11	12	15	13	4	7	14	5	8	6	1	3	2	9
F_{19}	mean	4.1E+02	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	6.7E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	12	6	8	15	7	2	11	13	5	14	9	1	4	3	10
F_{20}	mean	1.9E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	1.1E+01	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	10	14	15	13	8	4	12	9	6	11	5	2	1	3	7
F_{21}	mean	1.4E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	5.2E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	4	15	14	13	9	5	10	12	3	8	6	7	2	1	11
F_{22}	mean	9.8E+01	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	8.1E+00	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	6	14	7	15	8	1	11	10	12	13	2	5	3	4	9
F_{23}	mean	3.1E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	1.7E+00	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	3	12	14	15	5	4	9	10	6	13	8	2	7	1	11
F_{24}	mean	1.6E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	1.0E+02	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	3	12	14	15	6	5	8	11	4	10	7	9	2	1	13
F_{25}	mean	4.3E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	2.4E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	12	14	11	15	8	3	10	5	1	13	2	9	7	6	4
F_{26}	mean	2.9E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	3.1E+01	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	7	14	12	15	10	3	5	13	6	9	4	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	5.7E-01	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	7	12	13	15	2	10	11	14	8	5	1	6	4	3	9
F_{28}	mean	4.3E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	1.4E+02	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	8	12	11	14	6	1	13	10	1	3	5	7	2	4	9
F_{29}	mean	2.6E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.1E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	6	12	14	15	2	3	9	10	4	13	5	1	11	8	7
F_{30}	mean	3.3E+03	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	1.1E+03	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	7	14	13	15	1	6	12	3	5	4	2	11	9	10	8
Mean rank		6.66	12.48	11.48	14.52	6.07	4.07	8.59	10.03	5.24	10.14	5.76	3.31	5.55	5.24	8.24
Final rank		7	13	12	14	6	2	9	10	3	11	5	1	4	3	8

Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	2.7E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	3.0E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	5	14	13	15	7	9	11	12	6	8	4	1	3	2	10
F_3	mean	1.5E+01	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	2.5E+01	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	7	11	10	14	5	1	2	15	6	3	9	4	13	12	8
F_4	mean	8.8E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	1.4E+00	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	12	14	13	15	5	1	2	4	10	7	3	6	9	8	11
F_5	mean	2.2E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	6.4E+00	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	8	10
F_6	mean	5.3E-02	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	5.0E-02	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	8	13	15	14	6	3	9	2	4	12	10	7	5	1	11
F_7	mean	5.7E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	5.4E+00	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	2	11	14	15	3	4	6	12	5	13	10	1	8	7	9
F_8	mean	2.0E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	5.6E+00	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	1	12	13	15	3	4	6	14	5	11	7	2	8	9	10
F_9	mean	3.4E+00	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	3.2E+00	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	3	13	14	15	5	7	6	1	4	12	8	2	10	9	11
F_{10}	mean	2.9E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	4.5E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	5	12	13	14	4	2	8	15	3	7	6	1	10	11	9
F_{11}	mean	6.1E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	3.2E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	4	14	13	15	1	7	8	3	5	12	2	10	11	9	6
F_{12}	mean	4.8E+04	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	4.0E+04	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	5	14	13	15	6	2	4	12	3	7	8	1	10	9	11
F_{13}	mean	1.7E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	1.5E+04	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	2.4E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	3.6E+03	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	3	14	10	15	7	2	4	12	5	6	8	1	11	13	9
F_{15}	mean	1.6E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	1.9E+03	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	6	13	11	14	9	5	10	15	7	8	4	3	1	2	12
F_{16}	mean	4.0E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	2.7E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	2	12	13	15	1	5	9	14	8	11	3	4	7	6	10
F_{17}	mean	9.9E+01	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	5.3E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	1	13	14	15	9	4	8	12	2	11	6	3	5	7	10
F_{18}	mean	7.6E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	3.1E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	4	13	7	15	11	2	3	14	5	6	12	1	8	9	10
F_{19}	mean	2.9E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	2.3E+03	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	7	14	8	15	9	4	10	13	6	11	5	1	2	3	12
F_{20}	mean	2.3E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	3.9E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	9	13	15	14	2	7	6	12	4	10	3	1	5	8	11
F_{21}	mean	2.2E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	6.9E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	1	12	14	15	3	4	7	13	5	11	6	2	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	9.2E-01	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	2	13	12	14	4	5	8	15	3	10	1	7	9	6	11
F_{23}	mean	3.8E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	9.1E+00	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	2	11	14	15	3	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.4E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	7.5E+00	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	1	11	14	15	4	3	6	12	5	13	7	2	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	4.5E-01	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	11	13	14	15	2	10	9	1	7	12	3	6	5	4	8
F_{26}	mean	8.2E+02	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	5.7E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	2	12	14	15	5	1	4	11	3	13	6	7	9	8	10
F_{27}	mean	5.1E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	6.0E+00	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	8	13	14	15	4	10	11	6	9	3	5	7	2	1	12
F_{28}	mean	4.1E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	2.3E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	7	14	13	15	6	1	4	12	5	2	8	3	10	9	11
F_{29}	mean	5.5E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	4.2E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	6	13	14	15	1	5	9	11	2	10	3	4	7	8	12
F_{30}	mean	4.4E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	1.1E+03	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	9	14	13	15	6	7	8	11	10	2	1	3	5	4	12
Mean rank		4.93	12.76	12.86	14.79	4.97	4.38	6.79	10.76	5.24	9.07	5.69	3.24	7.28	7.00	10.24

Final rank	3	13	14	15	4	2	7	12	5	10	6	1	9	8	11
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	4.4E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	5.3E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	9	14	13	15	10	6	7	11	8	5	4	1	3	2	12
F_3	mean	1.7E+03	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	6.1E+02	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	6	11	8	12	4	1	3	15	5	2	9	7	14	13	10
F_4	mean	1.2E+02	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	3.8E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	11	14	13	15	3	1	6	5	10	7	4	2	8	9	12
F_5	mean	4.1E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	1.1E+01	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	10	8
F_6	mean	4.5E-01	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	2.7E-01	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	8	13	15	14	7	2	9	1	4	11	10	6	5	3	12
F_7	mean	1.1E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	1.7E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	1	12	14	15	3	4	5	11	6	13	7	2	8	10	9
F_8	mean	3.9E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	1.0E+01	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	8	10
F_9	mean	1.9E+01	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	9.6E+00	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	4.9E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	7.8E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	3	10	13	14	5	2	6	15	4	7	8	1	11	12	9
F_{11}	mean	1.1E+02	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	2.7E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	1	14	13	15	2	5	7	8	4	6	3	9	12	11	10
F_{12}	mean	4.1E+05	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	1.8E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	6	14	13	15	4	2	3	12	5	7	10	1	8	9	11
F_{13}	mean	7.0E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	8.1E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	10	13	14	15	9	5	7	12	6	8	1	3	4	2	11
F_{14}	mean	2.1E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.0E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	4	10	12	15	7	2	3	13	6	5	9	1	11	14	8
F_{15}	mean	8.2E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	7.0E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	9	15	11	14	8	2	7	13	5	10	3	1	6	4	12
F_{16}	mean	7.0E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	2.1E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	1	12	13	14	2	7	8	15	9	10	3	4	6	5	11
F_{17}	mean	6.9E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	2.3E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	1	12	13	15	2	8	9	14	7	11	6	3	4	5	10
F_{18}	mean	1.0E+05	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	8.6E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	6	13	8	15	7	2	3	14	4	5	10	1	11	12	9
F_{19}	mean	1.3E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	4.4E+03	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	7	15	13	14	10	2	9	12	6	8	3	1	5	4	11
F_{20}	mean	4.1E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	1.5E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	2	12	13	14	1	4	9	15	5	11	6	3	8	7	10
F_{21}	mean	2.4E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	7.3E+00	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	1	12	14	15	3	6	5	13	4	11	7	2	8	9	10
F_{22}	mean	6.7E+02	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	1.7E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	4.8E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	1.1E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	1	12	15	14	3	4	6	11	5	13	7	2	8	9	10
F_{24}	mean	5.4E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	1.5E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	1	11	14	15	3	4	6	12	5	13	8	2	9	10	7
F_{25}	mean	5.0E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	3.2E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	7	14	13	15	3	8	10	1	11	4	2	12	6	5	9
F_{26}	mean	7.9E+02	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	6.9E+02	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	1	12	15	14	2	3	5	11	6	13	7	4	9	8	10
F_{27}	mean	5.5E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	1.2E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.8E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	2.0E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	6	15	12	14	1	10	9	11	4	3	2	8	7	5	13
F_{29}	mean	6.4E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.4E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	1	11	14	15	4	5	9	13	3	10	6	2	8	7	12
F_{30}	mean	8.9E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	1.8E+05	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	10	14	13	15	5	6	9	11	8	4	1	7	3	2	12

Mean rank	4.31	12.52	13.03	14.55	4.31	4.48	6.83	10.93	5.76	8.31	5.79	3.55	7.72	7.59	10.31
Final rank	2	12	13	14	2	3	6	11	4	9	5	1	8	7	10
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	LIPSO

Statistical results on 100-dimensional CEC'17 problems for BEPSO vs comparison algorithms

F	Criteria	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	LIPSO
F_1	mean	6.1E+03	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	6.6E+03	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	6	15	14	13	9	10	7	12	8	5	3	1	2	4	11
F_3	mean	4.0E+04	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	8.0E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	6	12	7	11	5	1	4	15	3	2	9	8	13	14	10
F_4	mean	2.4E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	2.3E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	9	15	13	14	3	5	8	2	7	6	1	4	11	10	12
F_5	mean	8.8E+01	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	1.1E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_6	mean	2.7E+00	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	1.1E+00	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	7	13	15	14	8	1	10	4	5	11	9	6	2	3	12
F_7	mean	2.4E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	2.8E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	1	14	15	13	2	4	5	11	7	12	6	3	9	10	8
F_8	mean	8.9E+01	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.3E+02	8.5E+02	5.5E+02
	std	2.0E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	1	12	15	14	2	6	5	13	4	9	7	3	10	11	8
F_9	mean	1.4E+02	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	7.3E+01	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	1	11	12	15	2	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.2E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.5E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	3	9	11	14	6	2	5	15	4	7	10	1	13	12	8
F_{11}	mean	8.3E+02	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	1.3E+02	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	4	13	10	14	1	3	5	15	2	6	8	7	12	11	9
F_{12}	mean	1.3E+06	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	4.8E+05	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	7	15	14	13	6	2	3	12	4	5	10	1	9	8	11
F_{13}	mean	8.2E+03	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	6.4E+03	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	9	15	14	12	8	1	5	13	4	10	2	7	6	3	11
F_{14}	mean	1.0E+05	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	5.3E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	5	13	10	15	7	3	2	14	6	4	9	1	12	11	8
F_{15}	mean	1.7E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	1.5E+03	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	8	15	14	12	10	2	6	13	9	7	5	1	4	3	11
F_{16}	mean	2.3E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	2.8E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	1.9E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	3.7E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	3.0E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	1.0E+05	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	5	11	9	14	7	1	4	15	6	3	10	2	12	13	8
F_{19}	mean	4.2E+03	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	4.6E+03	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	10	15	14	11	9	1	6	13	5	7	3	8	2	4	12
F_{20}	mean	1.9E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	4.2E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	1	12	13	14	2	7	8	15	5	10	9	3	4	6	11
F_{21}	mean	3.1E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	1.5E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	1	13	15	14	2	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	5.2E+03	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	6.5E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	1	9	11	14	4	3	5	15	6	7	10	2	13	12	8
F_{23}	mean	6.6E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	2.1E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	1	11	15	14	3	4	7	12	6	13	5	2	8	9	10
F_{24}	mean	1.0E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	1.9E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	1	12	15	13	2	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	7.9E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	5.9E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	7	14	12	13	1	5	15	8	4	6	2	3	11	10	9
F_{26}	mean	3.7E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	1.4E+03	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	6.7E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	2.4E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	4	9	12	13	1	14	14	3	8	10	2	7	6	5	11
F_{28}	mean	5.7E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	1.3E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	8	14	11	13	2	6	15	3	7	4	1	5	10	9	12
F_{29}	mean	2.4E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	4.4E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	1.4E+04	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	1.0E+04	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06

rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10
Mean rank	4.10	12.52	12.83	13.28	4.24	4.66	7.45	11.14	5.72	8.10	5.97	3.55	8.41	8.21	9.76
Final rank	2	13	14	15	3	4	7	12	5	8	6	1	10	9	11
Algorithms	BEPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 10-dimensional CEC'17 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	5.4E+02	6.8E+07	2.1E+03	8.4E+09	3.6E+03	9.6E+02	9.7E+02	3.0E+04	8.2E+02	1.4E+03	2.9E+02	0.0E+00	6.9E+02	1.4E+02	1.8E+03
	std	7.4E+02	3.0E+08	3.2E+03	1.9E+09	5.5E+03	9.0E+02	1.0E+03	3.8E+04	1.2E+03	2.4E+03	5.7E+02	0.0E+00	1.1E+03	1.6E+02	1.8E+03
	rank	4	14	11	15	12	7	8	13	6	9	3	1	5	2	10
F_3	mean	1.4E-13	8.1E+01	1.5E-09	6.7E+03	0.0E+00	2.3E-14	4.5E-14	9.0E+00	2.6E-14	3.0E-13	1.9E-08	0.0E+00	1.6E+02	2.2E+02	2.8E-14
	std	1.1E-13	3.6E+02	6.8E-09	2.4E+03	0.0E+00	2.9E-14	3.0E-14	3.9E+01	2.9E-14	3.0E-13	4.3E-08	0.0E+00	1.6E+02	1.4E+02	2.9E-14
	rank	6	11	8	14	1	2	5	10	3	7	9	1	12	13	4
F_4	mean	8.3E-01	1.4E+01	5.1E+00	6.5E+02	9.8E-01	1.9E-01	1.9E-01	4.3E+00	9.8E-01	3.6E+00	4.2E+00	0.0E+00	3.9E+00	3.4E+00	2.8E+00
	std	2.6E-01	2.9E+01	4.4E+00	2.6E+02	2.2E-01	6.6E-02	1.9E-01	2.4E-01	3.5E-01	7.6E+00	8.4E-01	0.0E+00	2.8E+00	2.0E+00	2.5E+00
	rank	4	14	13	15	5	2	3	12	6	9	11	1	10	8	7
F_5	mean	6.5E+00	2.0E+01	3.7E+01	6.7E+01	4.3E+00	6.3E+00	7.9E+00	9.2E+00	6.6E+00	1.9E+01	6.4E+00	4.0E+00	5.7E+00	6.4E+00	1.4E+01
	std	2.9E+00	8.5E+00	1.3E+01	1.4E+01	2.1E+00	2.4E+00	2.3E+00	3.4E+00	2.4E+00	7.9E+00	1.8E+00	1.7E+00	1.6E+00	1.5E+00	3.6E+00
	rank	7	13	14	15	2	4	9	10	8	12	5	1	3	6	11
F_6	mean	6.9E-02	6.7E-01	1.9E+01	3.1E+01	3.4E-07	4.5E-14	1.0E-13	7.7E-11	1.4E-07	2.0E-01	8.7E-09	6.3E-06	2.2E-08	4.8E-12	1.1E-13
	std	6.0E-02	1.1E+00	1.0E+01	6.5E+00	1.3E-06	5.7E-14	3.5E-14	4.6E-11	4.4E-07	7.0E-01	3.0E-08	1.9E-05	9.9E-08	9.9E-12	2.5E-14
	rank	11	13	14	15	9	1	2	5	8	12	6	10	7	4	3
F_7	mean	2.0E+01	2.0E+01	3.5E+01	1.3E+02	1.6E+01	1.9E+01	1.7E+01	2.5E+01	1.8E+01	2.5E+01	2.0E+01	1.4E+01	1.6E+01	1.6E+01	1.9E+01
	std	4.7E+00	5.2E+00	8.6E+00	2.5E+01	3.8E+00	2.9E+00	3.1E+00	4.2E+00	3.1E+00	6.4E+00	3.2E+00	1.1E+00	1.6E+00	1.8E+00	3.6E+00
	rank	10	11	14	15	3	7	5	13	6	12	9	1	4	2	8
F_8	mean	8.4E+00	1.4E+01	2.5E+01	4.1E+01	5.0E+00	6.6E+00	7.7E+00	7.3E+00	5.1E+00	1.1E+01	6.8E+00	4.2E+00	5.3E+00	6.1E+00	9.9E+00
	std	3.5E+00	5.5E+00	9.4E+00	7.9E+00	2.1E+00	2.1E+00	3.3E+00	2.4E+00	2.0E+00	4.7E+00	2.2E+00	1.1E+00	1.4E+00	1.8E+00	3.2E+00
	rank	10	13	14	15	2	6	9	8	3	12	7	1	4	5	11
F_9	mean	4.5E-03	1.1E+00	7.5E+01	2.5E+02	0.0E+00	5.7E-15	5.1E-14	0.0E+00	5.7E-15	3.0E+00	5.7E-15	0.0E+00	7.9E-05	1.2E-06	1.7E-14
	std	2.0E-02	3.4E+00	1.3E+02	1.2E+02	0.0E+00	2.5E-14	5.8E-14	0.0E+00	2.5E-14	9.3E+00	2.5E-14	0.0E+00	3.5E-04	2.3E-06	4.2E-14
	rank	7	8	10	11	1	2	4	1	2	9	2	1	6	5	3
F_{10}	mean	4.8E+02	5.6E+02	9.6E+02	1.2E+03	1.4E+02	2.0E+02	3.0E+02	1.3E+03	2.6E+02	4.1E+02	2.1E+02	1.4E+02	2.4E+02	2.7E+02	4.5E+02
	std	2.3E+02	1.8E+02	3.0E+02	2.0E+02	1.1E+02	1.3E+02	1.8E+02	2.2E+02	1.9E+02	2.3E+02	1.2E+02	1.1E+02	1.2E+02	1.1E+02	1.6E+02
	rank	11	12	13	14	2	3	8	15	6	9	4	1	5	7	10
F_{11}	mean	5.4E+00	5.2E+01	3.8E+01	3.9E+02	2.1E+00	3.2E+00	4.9E+00	3.2E+00	2.5E+00	2.0E+01	3.6E+00	1.1E+00	3.6E+00	3.7E+00	3.3E+00
	std	2.6E+00	8.1E+01	2.4E+01	1.8E+02	1.6E+00	1.6E+00	2.5E+00	7.5E-01	1.6E+00	7.8E+00	1.2E+00	1.3E+00	1.5E+00	1.2E+00	1.6E+00
	rank	11	14	13	15	2	5	10	4	3	12	7	1	8	9	6
F_{12}	mean	9.0E+03	4.3E+05	3.4E+03	1.3E+08	1.5E+04	9.5E+03	7.8E+03	2.4E+05	1.3E+04	8.2E+03	5.2E+03	3.2E+02	1.1E+04	1.3E+04	7.5E+03
	std	5.1E+03	1.8E+06	3.3E+03	1.1E+08	1.3E+04	1.0E+04	7.4E+03	2.2E+05	1.3E+04	5.3E+03	5.0E+03	2.1E+02	3.6E+03	7.1E+03	1.2E+04
	rank	7	14	2	15	12	8	5	13	10	6	3	1	9	11	4
F_{13}	mean	5.7E+03	9.4E+03	5.5E+02	1.5E+04	4.5E+03	9.0E+02	3.9E+03	8.2E+03	2.3E+03	9.3E+03	7.4E+02	5.8E+00	3.7E+02	5.5E+02	7.3E+02
	std	2.0E+03	1.1E+04	8.7E+02	1.5E+04	6.8E+03	1.7E+03	4.4E+03	5.4E+03	2.8E+03	5.7E+03	6.9E+02	2.6E+00	3.9E+02	6.9E+02	5.0E+02
	rank	11	14	4	15	10	7	9	12	8	13	6	1	2	3	5
F_{14}	mean	4.2E+01	2.7E+02	6.9E+01	3.3E+02	3.4E+01	3.0E+01	4.9E+01	2.3E+02	3.3E+01	1.5E+02	4.7E+01	6.0E+00	3.5E+02	4.0E+02	7.1E+01
	std	1.1E+01	6.1E+02	3.8E+01	7.7E+02	2.1E+01	1.8E+01	2.8E+01	1.5E+02	1.4E+01	2.1E+02	2.5E+01	8.8E+00	6.8E+02	1.1E+03	4.0E+01
	rank	5	12	8	13	4	2	7	11	3	10	6	1	14	15	9
F_{15}	mean	4.4E+01	3.3E+02	1.0E+02	1.7E+03	6.2E+01	1.5E+01	1.3E+02	9.5E+02	2.0E+01	3.0E+02	1.1E+02	2.6E-01	2.1E+01	4.2E+01	1.9E+02
	std	2.7E+01	1.1E+03	9.1E+01	1.4E+03	8.8E+01	9.7E+00	1.9E+02	6.2E+02	2.0E+01	3.1E+02	8.2E+01	3.5E-01	2.4E+01	7.3E+01	8.8E+01
	rank	6	13	8	15	7	2	10	14	3	12	9	1	4	5	11
F_{16}	mean	1.8E+02	6.4E+01	2.9E+02	3.2E+02	2.1E+01	6.7E-01	1.3E+02	8.0E+00	6.7E+00	1.1E+02	1.5E+01	6.9E-01	1.0E+01	2.9E+00	1.4E+01
	std	1.1E+02	6.8E+01	1.3E+02	8.9E+01	4.4E+01	2.7E-01	1.0E+02	1.1E+01	2.6E+01	1.1E+02	3.6E+01	4.1E-01	2.6E+01	2.6E+00	3.1E+01
	rank	13	10	14	15	9	1	12	5	4	11	8	2	6	3	7
F_{17}	mean	4.6E+01	6.1E+01	8.9E+01	6.7E+01	2.1E+01	7.4E+00	3.3E+01	3.0E+01	1.2E+01	3.4E+01	1.8E+01	3.5E-01	2.3E+00	1.5E+00	3.0E+01
	std	9.6E+00	5.1E+01	3.8E+01	1.9E+01	1.1E+01	6.9E+00	1.9E+01	3.3E+00	1.1E+01	1.4E+01	1.2E+01	3.0E-01	4.8E+00	5.9E-01	1.2E+01
	rank	12	13	15	14	7	4	10	9	5	11	6	1	3	2	8
F_{18}	mean	5.0E+03	1.1E+04	1.2E+04	3.7E+06	1.3E+04	1.4E+03	3.8E+03	1.9E+04	2.3E+03	5.9E+03	3.1E+03	1.0E+01	2.7E+02	1.9E+02	6.4E+03
	std	5.2E+03	1.3E+04	1.9E+04	5.4E+06	1.5E+04	1.5E+03	4.2E+03	9.2E+03	1.5E+03	6.5E+03	2.9E+03	1.0E+01	2.6E+02	1.7E+02	5.2E+03
	rank	8	11	12	15	13	4	7	14	5	9	6	1	3	2	10
F_{19}	mean	7.2E+01	3.1E+01	5.5E+01	1.4E+03	4.7E+01	7.5E+00	1.4E+02	4.7E+02	3.1E+01	9.7E+02	8.8E+01	3.0E-01	2.8E+01	1.6E+01	1.1E+02
	std	1.0E+02	3.7E+01	4.9E+01	1.7E+03	8.3E+01	9.2E+00	2.6E+02	5.2E+02	6.6E+01	1.3E+03	1.3E+02	7.0E-01	3.7E+01	3.4E+01	7.5E+01
	rank	9	6	8	15	7	2	12	13	5	14	10	1	4	3	11
F_{20}	mean	4.4E+01	6.5E+01	1.4E+02	6.0E+01	1.4E+01	1.1E+00	2.9E+01	1.9E+01	1.2E+01	2.3E+01	7.2E+00	1.1E-01	1.0E-01	1.6E-01	1.2E+01
	std	3.7E+01	6.7E+01	6.7E+01	1.9E+01	3.1E+01	1.3E+00	3.5E+01	5.2E+00	1.0E+01	1.2E+01	8.6E+00	1.5E-01	3.1E-01	3.6E-01	1.0E+01
	rank	12	14	15	13	8	4	11	9	6	10	5	2	1	3	7
F_{21}	mean	1.4E+02	2.2E+02	1.9E+02	1.9E+02	1.7E+02	1.5E+02	1.8E+02	1.9E+02	1.2E+02	1.7E+02	1.5E+02	1.7E+02	1.2E+02	1.2E+02	1.9E+02
	std	5.3E+01	7.4E+00	6.3E+01	4.1E+01	5.0E+01	5.6E+01	4.9E+01	4.4E+01	4.6E+01	6.0E+01	5.5E+01	5.0E+01	7.3E+00	9.6E+00	5.0E+01
	rank	4	15	14	13	9	5	10	12	3	8	6	7	2	1	11
F_{22}	mean	1.0E+02	1.5E+02	9.9E+01	5.4E+02	1.0E+02	7.8E+01	1.0E+02	1.0E+02	1.0E+02	1.1E+02	8.5E+01	9.6E+01	9.4E+01	9.6E+01	1.0E+02
	std	6.5E-01	2.1E+02	2.2E+01	2.3E+02	3.6E-01	3.4E+01	7.4E-01	1.5E+00	1.1E+00	4.6E+00	3.4E+01	1.9E+01	1.6E+01	1.2E+01	4.7E-01
	rank	12	14	6	15	7	1	10	9	11	13	2	5	3	4	8
F_{23}	mean	3.1E+02	3.3E+02	3.6E+02	4.3E+02	3.1E+02	3.1E+02	3.1E+02	3.2E+02	3.1E+02	3.3E+02	3.1E+02	3.1E+02	3.1E+02	3.0E+02	3.2E+02
	std	4.5E+00	8.9E+00	2.4E+01	3.0E+01	2.4E+00	2.2E+00	6.7E+00	4.8E+00	3.2E+00	1.3E+01	2.0E+00	2.0E+00	9.1E+00	5.1E+01	6.1E+00
	rank	3	12	14	15	5	4	9	10	6	13	8	2	7	1	11
F_{24}	mean	1.8E+02	3.5E+02	3.7E+02	3.9E+02	2.8E+02	2.5E+02	3.0E+02	3.4E+02	2.0E+02	3.3E+02	2.8E+02	3.2E+02	1.5E+02	1.3E+02	3.5E+02
	std	1.1E+02	6.0E+01	7.3E+01	7.6E+01	1.0E+02	1.2E+02	8.7E+01	4.6E+00	1.2E+02	1.0E+02	1.0E+02	5.2E+01	3.0E+01	2.3E+01	5.4E+00
	rank	3	12	14	15	6	5	8	11	4	10	7	9	2	1	13
F_{25}	mean	4.1E+02	4.5E+02	4.2E+02	7.6E+02	4.2E+02	4.0E+02	4.2E+02	4.1E+02	3.9E+02	4.3E+02	4.0E+02	4.2E+02	4.2E+02	4.2E+02	4.1E+02
	std	2.0E+01	3.8E+01	2.4E+01	1.2E+02	2.4E+01	1.0E+01	2.4E+01	2.2E+01	6.9E+01	2.3E+01	4.9E-01	2.4E+01	1.7E+01	1.8E+01	1.7E+01
	rank	5	14	12	15	9	3	11	6	1	13	2	10	8	7	4
F_{26}	mean	2.6E+02	6.7E+02	3.4E+02	1.1E+03	3.1E+02	2.3E+02	2.7E+02	3.6E+02	2.8E+02	3.1E+02	2.5E+02	3.0E+02	1.3E+02	1.8E+02	3.2E+02
	std	7.5E+01	4.7E+02	1.4E+02	1.7E+02	4.1E+01	1.2E+02	7.5E+01	9.0E+01	6.9E+01	1.3E+02	7.6E+01	0.0E+00	9.9E+01	9.6E+01	7.2E+01
	rank	5	14	12	15	10	3	6	13	7	9	4	8	1	2	11
F_{27}	mean	3.9E+02	4.0E+02	4.4E+02	5.8E+02	3.7E+02	3.9E+02	3.9E+02	4.7E+02	3.9E+02	3.9E+02	3.7E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	1.3E+01	3.2E+00	3.4E+01	4.6E+01	6.7E-01	3.3E+00	1.4E+00	4.1E+01	2.8E+00	2.7E+01	4.2E-01	1.7E-01	3.9E+00	2.3E+00	2.4E+00
	rank	9	12	13	15	2	10	11	14	7	5	1	6	4	3	8
F_{28}	mean	4.2E+02	5.4E+02	5.2E+02	9.4E+02	4.1E+02	3.1E+02	5.4E+02	4.7E+02	3.1E+02	3.8E+02	4.0E+02	4.1E+02	3.6E+02	3.9E+02	4.6E+02
	std	1.5E+02	9.7E+01	1.2E+02	1.1E+02	6.3E+01	6.3E+01	1.3E+02	1.2E+00	6.3E+01	6.2E+01	6.4E+01	1.5E+02	8.6E+01	4.9E+01	9.9E+01
	rank	8	12	11	14	6	1	13	10	1	3	5	7	2	4	9
F_{29}	mean	2.7E+02	2.8E+02	3.3E+02	3.9E+02	2.5E+02	2.5E+02	2.8E+02	2.8E+02	2.6E+02	3.1E+02	2.6E+02	2.4E+02	2.8E+02	2.7E+02	2.7E+02
	std	1.5E+01	3.8E+01	4.6E+01	5.5E+01	8.9E+00	1.0E+01	4.4E+01	1.2E+01	1.3E+01	3.8E+01	1.4E+01	9.1E+00	1.7E+01	1.3E+01	1.8E+01
	rank	7	12	14	15	2	3	9	10	4	13	5	1	11	8	6
F_{30}	mean	6.5E+04	3.5E+05	3.3E+05	8.5E+06	3.6E+02	2.9E+03	1.2E+05	7.3E+02	2.3E+03	2.0E+03	7.0E+02	1.2E+05	9.7E+03	1.2E+04	5.9E+03
	std	2.8E+05	4.6E+05	6.3E+05	7.3E+06	1.5E+02	3.1E+03	3.0E+05	6.4E+02	8.7E+02	1.5E+03	4.6E+02	3.0E+05	9.1E+03	1.3E+04	3.8E+03
	rank	10	14	13	15	1	6	12	3	5	4	2	11	8	9	7
Mean rank		7.93	12.45	11.48	14.52	5.97	3.93	8.45	9.97	5.14	10.00	5.66	3.31	5.45	5.10	8.03

Final rank	8	14	13	15	7	2	10	11	4	12	6	1	5	3	9
Algorithms	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 30-dimensional CEC'17 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	1.8E+03	4.0E+09	4.5E+08	1.9E+10	3.3E+03	3.4E+03	3.9E+03	1.2E+05	2.9E+03	3.3E+03	9.7E+02	2.3E-14	7.2E+02	5.9E+02	3.8E+03
	std	1.8E+03	4.2E+09	5.0E+08	3.7E+09	3.1E+03	2.9E+03	4.6E+03	3.4E+05	3.8E+03	3.3E+03	2.1E+03	1.3E-14	1.1E+03	8.5E+02	5.7E+03
	rank	5	14	13	15	7	9	11	12	6	8	4	1	3	2	10
F_3	mean	3.2E+00	2.1E+04	2.8E+03	7.2E+04	5.6E-04	6.6E-13	7.3E-09	1.9E+05	7.9E-02	1.6E-06	2.7E+03	1.5E-05	4.4E+04	3.5E+04	1.8E+03
	std	8.5E+00	1.9E+04	1.1E+03	1.4E+04	1.4E-03	4.0E-13	2.0E-08	7.2E+04	2.8E-01	2.8E-06	6.1E+02	4.6E-05	2.1E+04	1.7E+04	1.2E+03
	rank	7	11	10	14	5	1	2	15	6	3	9	4	13	12	8
F_4	mean	9.0E+01	2.7E+02	1.6E+02	4.3E+03	3.5E+01	1.0E+01	1.7E+01	2.6E+01	8.2E+01	5.7E+01	2.5E+01	5.2E+01	7.5E+01	7.3E+01	8.6E+01
	std	7.4E+00	2.5E+02	4.7E+01	9.8E+02	2.5E+01	1.9E+01	2.7E+01	2.2E+00	1.5E+01	3.0E+01	1.9E+00	2.2E+01	6.5E+00	1.0E+01	1.0E+01
	rank	12	14	13	15	5	1	2	4	10	7	3	6	9	8	11
F_5	mean	3.0E+01	1.4E+02	2.0E+02	2.5E+02	3.4E+01	4.6E+01	5.3E+01	1.6E+02	4.6E+01	1.3E+02	6.1E+01	2.3E+01	6.8E+01	6.1E+01	9.2E+01
	std	7.0E+00	2.9E+01	3.0E+01	2.9E+01	8.7E+00	1.1E+01	1.9E+01	1.3E+01	1.4E+01	2.1E+01	1.6E+01	5.3E+00	1.8E+01	1.1E+01	2.2E+01
	rank	2	12	14	15	3	5	6	13	4	11	7	1	9	8	10
F_6	mean	1.1E+00	9.8E+00	5.5E+01	5.1E+01	3.9E-03	1.9E-04	8.1E-02	2.6E-07	2.4E-04	5.6E+00	2.0E-01	4.2E-03	1.8E-03	5.4E-13	5.8E-01
	std	4.0E-01	6.0E+00	7.4E+00	9.7E+00	3.4E-03	6.1E-04	1.1E-01	7.6E-08	1.2E-03	2.7E+00	1.8E-01	6.2E-03	1.0E-02	5.2E-13	8.0E-01
	rank	11	13	15	14	6	3	8	2	4	12	9	7	5	1	10
F_7	mean	7.5E+01	1.6E+02	3.0E+02	7.1E+02	7.1E+01	8.1E+01	1.0E+02	2.0E+02	9.2E+01	2.5E+02	1.3E+02	5.7E+01	1.1E+02	1.0E+02	1.1E+02
	std	1.2E+01	4.4E+01	6.8E+01	1.1E+02	1.1E+01	1.4E+01	1.7E+01	1.6E+01	2.0E+01	4.7E+01	2.4E+01	6.4E+00	1.7E+01	1.6E+01	1.9E+01
	rank	3	11	14	15	2	4	6	12	5	13	10	1	8	7	9
F_8	mean	3.1E+01	1.3E+02	1.5E+02	2.4E+02	3.6E+01	4.9E+01	5.2E+01	1.5E+02	5.2E+01	1.1E+02	5.8E+01	2.7E+01	7.0E+01	7.5E+01	9.3E+01
	std	8.6E+00	3.3E+01	2.7E+01	1.7E+01	1.0E+01	1.2E+01	1.5E+01	1.4E+01	1.5E+01	1.9E+01	1.4E+01	6.0E+00	1.4E+01	1.5E+01	1.6E+01
	rank	2	12	13	15	3	4	6	14	5	11	7	1	8	9	10
F_9	mean	3.3E+00	2.4E+03	3.4E+03	3.8E+03	7.1E+00	2.3E+01	1.8E+01	1.5E-12	5.5E+00	1.8E+03	2.4E+01	3.1E+00	1.2E+02	7.5E+01	2.1E+02
	std	2.6E+00	1.6E+03	6.7E+02	1.4E+03	7.2E+00	1.8E+01	1.0E+01	2.3E-12	7.9E+00	6.2E+02	1.2E+01	3.4E+00	2.9E+02	8.0E+01	1.9E+02
	rank	3	13	14	15	5	7	6	1	4	12	8	2	10	9	11
F_{10}	mean	2.5E+03	3.7E+03	4.8E+03	6.7E+03	2.6E+03	2.2E+03	3.3E+03	7.8E+03	2.6E+03	3.2E+03	3.1E+03	1.8E+03	3.6E+03	3.7E+03	3.5E+03
	std	4.8E+02	4.7E+02	6.0E+02	3.7E+02	6.0E+02	4.0E+02	8.8E+02	2.9E+02	5.6E+02	4.6E+02	5.2E+02	2.7E+02	3.8E+02	4.6E+02	4.9E+02
	rank	3	12	13	14	5	2	8	15	4	7	6	1	10	11	9
F_{11}	mean	7.8E+01	5.4E+02	2.4E+02	2.6E+03	4.3E+01	8.3E+01	8.4E+01	5.3E+01	6.6E+01	1.1E+02	4.8E+01	9.1E+01	9.6E+01	8.8E+01	8.0E+01
	std	2.9E+01	9.9E+02	5.9E+01	1.0E+03	2.2E+01	3.1E+01	2.5E+01	1.5E+01	3.3E+01	3.1E+01	1.8E+01	3.2E+01	5.4E+01	4.8E+01	4.3E+01
	rank	5	14	13	15	1	7	8	3	4	12	2	10	11	9	6
F_{12}	mean	3.8E+04	8.8E+07	1.1E+07	3.8E+09	6.0E+04	1.8E+04	2.1E+04	8.9E+06	1.9E+04	6.4E+04	2.3E+05	2.1E+03	6.0E+05	4.6E+05	8.0E+05
	std	4.8E+04	1.6E+08	1.1E+07	9.8E+08	3.8E+04	1.1E+04	1.1E+04	6.0E+06	8.8E+03	3.5E+04	1.6E+05	1.5E+03	4.0E+05	2.9E+05	2.0E+06
	rank	5	14	13	15	6	2	4	12	3	7	8	1	10	9	11
F_{13}	mean	1.2E+04	3.6E+05	2.6E+06	8.4E+08	2.3E+04	2.9E+03	9.4E+03	6.7E+05	8.6E+03	1.1E+04	1.8E+03	3.4E+02	5.8E+03	4.3E+03	3.2E+04
	std	1.2E+04	1.1E+06	1.3E+07	5.4E+08	2.8E+04	2.6E+03	7.8E+03	7.1E+05	8.7E+03	8.5E+03	2.0E+03	3.7E+02	4.7E+03	4.2E+03	2.8E+04
	rank	9	12	14	15	10	3	7	13	6	8	2	1	5	4	11
F_{14}	mean	1.5E+03	6.5E+04	2.6E+04	4.0E+05	5.5E+03	2.2E+03	3.1E+03	3.8E+04	4.0E+03	4.8E+03	5.5E+03	1.2E+02	3.2E+04	3.8E+04	9.8E+03
	std	1.2E+03	9.8E+04	6.3E+04	2.2E+05	5.8E+03	2.5E+03	2.3E+03	2.2E+04	3.0E+03	3.9E+03	3.8E+03	4.4E+01	2.4E+04	3.6E+04	7.5E+03
	rank	2	14	10	15	7	3	4	12	5	6	8	1	11	13	9
F_{15}	mean	1.4E+03	3.3E+04	6.2E+03	1.5E+05	4.9E+03	6.6E+02	6.1E+03	1.9E+05	3.2E+03	3.8E+03	5.8E+02	1.7E+02	1.5E+02	1.6E+02	1.1E+04
	std	1.7E+03	3.8E+04	1.1E+04	9.1E+04	6.0E+03	4.1E+02	7.7E+03	1.7E+05	3.7E+03	5.0E+03	7.2E+02	1.0E+02	4.6E+01	5.3E+01	1.3E+04
	rank	6	13	11	14	9	5	10	15	7	8	4	3	1	2	12
F_{16}	mean	4.9E+02	1.1E+03	1.4E+03	2.6E+03	3.9E+02	5.3E+02	6.5E+02	1.5E+03	6.0E+02	9.4E+02	4.2E+02	4.5E+02	5.9E+02	5.6E+02	7.5E+02
	std	1.8E+02	2.6E+02	2.8E+02	3.0E+02	1.6E+02	1.6E+02	2.6E+02	1.4E+02	2.7E+02	2.2E+02	2.2E+02	1.7E+02	1.7E+02	1.7E+02	2.1E+02
	rank	4	12	13	15	1	5	9	14	8	11	2	3	7	6	10
F_{17}	mean	1.6E+02	6.3E+02	7.4E+02	8.8E+02	1.8E+02	1.4E+02	1.6E+02	4.6E+02	1.0E+02	4.1E+02	1.4E+02	1.1E+02	1.4E+02	1.5E+02	3.0E+02
	std	6.2E+01	2.6E+02	3.3E+02	1.8E+02	1.1E+02	8.9E+01	1.0E+02	1.2E+02	7.8E+01	1.4E+02	7.3E+01	5.6E+01	9.1E+01	7.5E+01	1.1E+02
	rank	7	13	14	15	9	3	8	12	1	11	5	2	4	6	10
F_{18}	mean	6.4E+04	5.0E+05	9.5E+04	3.8E+06	1.5E+05	7.1E+04	7.5E+04	2.1E+06	8.8E+04	9.1E+04	1.6E+05	4.6E+02	9.8E+04	1.1E+05	1.4E+05
	std	2.4E+04	6.8E+05	3.4E+05	4.2E+06	1.3E+05	6.6E+04	3.6E+04	9.3E+05	6.7E+04	7.1E+04	1.0E+05	6.9E+02	4.7E+04	5.0E+04	8.2E+04
	rank	2	13	7	15	11	3	4	14	5	6	12	1	8	9	10
F_{19}	mean	4.2E+03	1.9E+05	3.2E+03	1.7E+06	5.9E+03	4.5E+02	7.1E+03	3.1E+04	2.4E+03	7.3E+03	7.3E+02	9.7E+01	9.7E+01	1.1E+02	9.2E+03
	std	4.2E+03	6.8E+05	4.8E+03	1.2E+06	7.1E+03	9.9E+02	8.2E+03	2.8E+04	3.0E+03	6.0E+03	1.2E+03	5.8E+01	1.0E+02	8.7E+01	9.8E+03
	rank	8	14	7	15	9	4	10	13	6	11	5	1	2	3	12
F_{20}	mean	2.6E+02	4.8E+02	7.5E+02	6.5E+02	1.5E+02	2.0E+02	2.0E+02	4.0E+02	1.7E+02	3.1E+02	1.6E+02	1.4E+02	2.0E+02	2.1E+02	3.3E+02
	std	2.7E+01	1.8E+02	1.8E+02	1.1E+02	6.5E+01	8.0E+01	7.9E+01	1.3E+02	6.7E+01	1.4E+02	6.6E+01	5.7E+01	7.6E+01	8.9E+01	1.3E+02
	rank	9	13	15	14	2	7	6	12	4	10	3	1	5	8	11
F_{21}	mean	2.3E+02	3.4E+02	4.1E+02	4.8E+02	2.3E+02	2.5E+02	2.6E+02	3.7E+02	2.5E+02	3.1E+02	2.6E+02	2.3E+02	2.7E+02	2.6E+02	2.9E+02
	std	8.8E+00	3.3E+01	4.7E+01	3.3E+01	1.0E+01	1.5E+01	1.9E+01	1.3E+01	1.4E+01	3.3E+01	1.5E+01	7.6E+00	2.9E+01	1.8E+01	1.8E+01
	rank	2	12	14	15	3	4	7	13	5	11	6	1	9	8	10
F_{22}	mean	1.0E+02	3.8E+03	3.0E+03	4.7E+03	1.0E+02	1.9E+02	4.0E+02	7.6E+03	1.0E+02	1.2E+03	1.0E+02	3.5E+02	4.2E+02	3.4E+02	1.9E+03
	std	1.2E+00	1.2E+03	2.6E+03	2.1E+03	1.2E+00	3.9E+02	9.3E+02	4.4E+02	9.8E-01	1.8E+03	2.1E-09	7.5E+02	9.3E+02	7.4E+02	1.8E+03
	rank	4	13	12	14	3	5	8	15	2	10	1	7	9	6	11
F_{23}	mean	3.9E+02	4.9E+02	7.9E+02	9.3E+02	3.9E+02	4.0E+02	4.2E+02	5.1E+02	4.0E+02	6.2E+02	4.3E+02	3.8E+02	4.2E+02	4.3E+02	4.7E+02
	std	1.3E+01	3.7E+01	1.4E+02	8.0E+01	1.2E+01	1.2E+01	2.3E+01	1.8E+01	1.5E+01	6.1E+01	1.5E+01	8.6E+00	1.4E+01	1.5E+01	2.7E+01
	rank	3	11	14	15	2	4	6	12	5	13	8	1	7	9	10
F_{24}	mean	4.6E+02	5.9E+02	7.9E+02	1.3E+03	4.6E+02	4.5E+02	4.8E+02	6.1E+02	4.7E+02	7.5E+02	5.2E+02	4.5E+02	5.2E+02	5.4E+02	5.5E+02
	std	1.6E+01	5.1E+01	9.2E+01	1.3E+02	7.4E+00	5.8E+01	1.3E+01	8.6E+00	1.8E+01	9.0E+01	2.8E+01	7.6E+00	3.7E+01	2.5E+01	2.2E+01
	rank	4	11	14	15	3	2	6	12	5	13	7	1	8	9	10
F_{25}	mean	3.9E+02	4.6E+02	4.7E+02	9.7E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.9E+02	3.8E+02	3.9E+02	3.8E+02	3.8E+02	3.9E+02
	std	2.0E+00	7.7E+01	3.6E+01	1.4E+02	4.0E-01	1.1E+00	2.4E+00	6.6E-01	8.8E-01	2.5E+01	7.2E-01	4.0E-01	4.5E+00	1.3E+00	1.4E+00
	rank	11	13	14	15	2	10	9	1	7	12	3	6	5	4	8
F_{26}	mean	4.1E+02	2.7E+03	3.5E+03	5.7E+03	1.2E+03	7.7E+02	9.9E+02	2.3E+03	8.8E+02	2.8E+03	1.3E+03	1.3E+03	1.7E+03	1.4E+03	2.2E+03
	std	3.5E+02	4.7E+02	1.8E+03	7.4E+02	4.9E+02	6.7E+02	7.9E+02	1.4E+02	6.7E+02	1.8E+03	6.8E+02	1.0E+02	6.7E+02	7.2E+02	2.6E+02
	rank	1	12	14	15	5	2	4	11	3	13	6	7	9	8	10
F_{27}	mean	5.1E+02	5.4E+02	7.3E+02	1.4E+03	4.9E+02	5.2E+02	5.3E+02	5.0E+02	5.2E+02	4.7E+02	5.0E+02	5.1E+02	4.6E+02	4.6E+02	5.3E+02
	std	1.1E+01	1.3E+01	1.7E+02	2.4E+02	2.5E+01	7.5E+00	1.6E+01	4.3E-05	8.3E+00	9.0E+00	1.4E+01	7.3E+00	8.2E+00	4.5E+00	1.5E+01
	rank	8	13	14	15	4	10	11	6	9	3	5	7	2	1	12
F_{28}	mean	3.9E+02	7.9E+02	5.0E+02	2.0E+03	4.0E+02	3.3E+02	3.4E+02	5.0E+02	3.5E+02	3.3E+02	4.4E+02	3.4E+02	4.5E+02	4.5E+02	4.6E+02
	std	4.5E+01	4.0E+02	3.6E+01	3.4E+02	4.7E+01	4.8E+01	5.5E+01	7.0E-05	5.9E+01	5.5E+01	3.6E+01	6.6E+01	1.5E+01	1.9E+01	5.8E+01
	rank	6	14	13	15	7	1	4	12	5	2	8	3	10	9	11
F_{29}	mean	5.8E+02	9.9E+02	1.6E+03	2.5E+03	4.7E+02	5.4E+02	6.0E+02	8.3E+02	5.1E+02	7.6E+02	5.1E+02	5.3E+02	5.8E+02	5.9E+02	9.3E+02
	std	7.1E+01	2.4E+02	2.7E+02	4.2E+02	6.7E+01	9.3E+01	9.8E+01	1.9E+02	7.0E+01	2.0E+02	1.1E+02	7.4E+01	8.9E+01	6.5E+01	1.6E+02
	rank	6	13	14	15	1	5	9	11	2	10	3	4	7	8	12
F_{30}	mean	3.9E+03	2.1E+05	5.8E+04	7.5E+07	3.3E+03	4.1E+03	4.2E+03	1.9E+04	4.8E+03	1.9E+03	8.4E+02	2.3E+03	2.7E+03	2.4E+03	3.5E+04
	std	7.2E+02	4.0E+05	6.4E+04	7.2E+07	4.1E+03	8.9E+02	1.3E+03	1.8E+04	2.3E+03	1.5E+03	6.2E+02	3.1E+02	1.3E+03	1.4E+03	4.9E+04
	rank	7	14	13	15	6	8	9	11	10	2	1	3	5	4	12
Mean rank		5.41	12.76	12.83	14.79	4.90	4.45	6.83	10.76	5.17	9.07	5.59	3.03	7.24	6.97	10.21

Final rank	5	13	14	15	3	2	7	12	4	10	6	1	9	8	11
Algorithms	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

Statistical results on 50-dimensional CEC'17 problems for AHPSO vs comparison algorithms

F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO
F_1	mean	2.4E+03	1.5E+10	2.7E+09	2.0E+10	1.2E+04	3.1E+03	3.7E+03	1.0E+06	3.8E+03	2.0E+03	1.4E+03	9.4E-11	1.4E+03	9.7E+02	2.9E+07
	std	2.9E+03	9.8E+09	1.1E+09	3.5E+09	1.3E+04	3.4E+03	3.7E+03	1.3E+06	4.9E+03	1.9E+03	1.8E+03	3.1E-10	1.3E+03	1.0E+03	1.6E+08
	rank	6	14	13	15	10	7	8	11	9	5	4	1	3	2	12
F_3	mean	4.8E+02	8.6E+04	1.4E+04	1.6E+05	3.9E+02	3.1E-06	1.2E+02	5.5E+05	7.6E+02	4.2E-01	2.1E+04	1.4E+04	1.7E+05	1.7E+05	3.5E+04
	std	2.9E+02	2.8E+04	4.3E+03	3.3E+04	2.6E+02	6.9E-06	1.5E+02	1.5E+05	6.0E+02	5.0E-01	3.6E+03	2.0E+04	3.4E+04	3.3E+04	8.5E+03
	rank	5	11	8	12	4	1	3	15	6	2	9	7	14	13	10
F_4	mean	1.4E+02	1.2E+03	5.1E+02	4.2E+03	4.1E+01	2.7E+01	6.3E+01	4.6E+01	1.1E+02	6.8E+01	4.5E+01	3.2E+01	9.9E+01	1.0E+02	1.3E+02
	std	4.3E+01	1.2E+03	1.5E+02	8.6E+02	9.0E+00	3.6E+01	4.3E+01	5.0E-01	4.4E+01	4.2E+01	1.4E+00	3.3E+01	2.8E+01	3.3E+01	4.4E+01
	rank	12	14	13	15	3	1	6	5	10	7	4	2	8	9	11
F_5	mean	5.6E+01	3.0E+02	3.8E+02	4.7E+02	7.6E+01	1.1E+02	1.2E+02	3.6E+02	1.0E+02	2.5E+02	1.4E+02	6.2E+01	2.1E+02	2.2E+02	2.0E+02
	std	1.7E+01	6.2E+01	4.1E+01	3.2E+01	1.4E+01	2.0E+01	1.5E+01	1.7E+01	1.8E+01	3.6E+01	3.7E+01	1.2E+01	4.1E+01	3.9E+01	3.2E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	10	8
F_6	mean	1.7E+00	2.4E+01	6.7E+01	6.5E+01	3.5E-01	3.5E-04	8.7E-01	2.7E-05	4.0E-03	9.5E+00	1.9E+00	1.0E-01	1.3E-02	3.1E-03	1.2E+01
	std	3.9E-01	1.0E+01	7.4E+00	1.3E+01	5.1E-01	1.3E-03	7.4E-01	4.9E-06	7.3E-03	3.9E+00	6.5E-01	1.1E-01	4.4E-02	9.7E-03	5.2E+00
	rank	9	13	15	14	7	2	8	1	4	11	10	6	5	3	12
F_7	mean	1.3E+02	4.7E+02	7.7E+02	1.2E+03	1.3E+02	1.6E+02	1.9E+02	4.1E+02	2.1E+02	5.1E+02	2.4E+02	1.2E+02	2.6E+02	2.8E+02	2.6E+02
	std	1.9E+01	1.4E+02	1.1E+02	1.9E+02	2.3E+01	2.4E+01	2.8E+01	1.5E+01	3.1E+01	8.4E+01	4.4E+01	1.5E+01	3.3E+01	3.3E+01	4.1E+01
	rank	3	12	14	15	2	4	5	11	6	13	7	1	8	10	9
F_8	mean	5.3E+01	3.2E+02	4.0E+02	4.9E+02	7.2E+01	1.2E+02	1.2E+02	3.6E+02	1.0E+02	2.6E+02	1.5E+02	6.0E+01	2.2E+02	2.2E+02	2.2E+02
	std	1.2E+01	6.5E+01	5.0E+01	4.2E+01	1.3E+01	2.1E+01	2.9E+01	2.0E+01	2.5E+01	3.1E+01	3.1E+01	1.1E+01	4.2E+01	3.1E+01	3.3E+01
	rank	1	12	14	15	3	5	6	13	4	11	7	2	9	8	10
F_9	mean	2.0E+01	7.3E+03	1.4E+04	1.7E+04	3.1E+01	1.8E+02	2.1E+02	7.3E-05	1.5E+02	8.5E+03	2.0E+02	3.4E+01	3.0E+03	3.7E+03	1.8E+03
	std	8.6E+00	3.5E+03	2.3E+03	5.1E+03	2.2E+01	9.1E+01	1.8E+02	2.0E-04	9.4E+01	1.6E+03	6.6E+01	1.9E+01	1.8E+03	2.8E+03	8.1E+02
	rank	2	12	14	15	3	6	8	1	5	13	7	4	10	11	9
F_{10}	mean	4.6E+03	7.4E+03	8.6E+03	1.3E+04	5.1E+03	4.2E+03	5.5E+03	1.4E+04	5.0E+03	5.6E+03	6.2E+03	3.6E+03	8.2E+03	8.6E+03	6.6E+03
	std	8.1E+02	8.2E+02	8.5E+02	4.1E+02	8.8E+02	4.6E+02	7.2E+02	3.8E+02	5.8E+02	6.8E+02	8.5E+02	3.9E+02	7.4E+02	7.9E+02	7.9E+02
	rank	3	10	13	14	5	2	6	15	4	7	8	1	11	12	9
F_{11}	mean	1.2E+02	2.4E+03	6.2E+02	8.8E+03	1.2E+02	1.5E+02	1.8E+02	1.9E+02	1.5E+02	1.6E+02	1.4E+02	2.0E+02	2.5E+02	2.4E+02	2.2E+02
	std	2.4E+01	2.4E+03	1.4E+02	6.3E+03	5.1E+01	4.0E+01	3.9E+01	2.2E+01	3.8E+01	3.3E+01	3.6E+01	5.9E+01	1.3E+02	1.3E+02	8.7E+01
	rank	2	14	13	15	1	5	7	8	4	6	3	9	12	11	10
F_{12}	mean	5.6E+05	8.8E+08	6.0E+08	6.8E+09	3.3E+05	9.1E+04	1.1E+05	1.1E+08	3.9E+05	4.7E+05	1.8E+06	8.0E+03	1.6E+06	1.7E+06	6.9E+06
	std	3.5E+05	1.5E+09	7.1E+08	2.0E+09	1.6E+05	4.4E+04	5.0E+04	3.8E+07	2.2E+05	3.1E+05	9.1E+05	5.8E+03	5.5E+05	6.0E+05	1.5E+07
	rank	7	14	13	15	4	2	3	12	5	6	10	1	8	9	11
F_{13}	mean	2.8E+03	3.9E+07	6.4E+07	5.1E+08	5.8E+03	3.1E+03	4.1E+03	1.8E+06	3.5E+03	5.5E+03	1.1E+03	2.0E+03	2.1E+03	1.7E+03	1.4E+06
	std	1.5E+03	1.2E+08	2.4E+08	3.2E+08	8.9E+03	3.1E+03	4.8E+03	1.8E+06	4.8E+03	3.0E+03	7.6E+02	3.5E+03	1.4E+03	8.5E+02	5.7E+06
	rank	5	13	14	15	10	6	8	12	7	9	1	3	4	2	11
F_{14}	mean	2.3E+04	1.0E+05	5.4E+05	3.7E+06	3.9E+04	1.2E+04	1.5E+04	5.9E+05	2.7E+04	2.4E+04	9.5E+04	3.1E+02	5.2E+05	7.0E+05	8.3E+04
	std	1.6E+04	1.2E+05	2.3E+06	3.0E+06	2.8E+04	9.4E+03	9.7E+03	2.3E+05	2.1E+04	1.6E+04	4.7E+04	7.7E+01	3.9E+05	8.6E+05	6.1E+04
	rank	4	10	12	15	7	2	3	13	6	5	9	1	11	14	8
F_{15}	mean	4.9E+03	4.8E+06	1.7E+04	9.7E+05	7.5E+03	8.6E+02	5.6E+03	8.7E+05	3.0E+03	1.1E+04	2.1E+03	4.5E+02	3.5E+03	2.8E+03	1.8E+04
	std	3.3E+03	1.8E+07	1.1E+04	8.5E+05	8.1E+03	8.8E+02	6.0E+03	6.5E+05	3.0E+03	1.2E+04	2.8E+03	1.7E+02	3.2E+03	3.2E+03	1.0E+04
	rank	7	15	11	14	9	2	8	13	5	10	3	1	6	4	12
F_{16}	mean	6.5E+02	2.3E+03	2.4E+03	3.4E+03	8.4E+02	1.1E+03	1.2E+03	3.5E+03	1.2E+03	1.6E+03	9.1E+02	9.6E+02	1.1E+03	1.1E+03	1.7E+03
	std	2.6E+02	3.9E+02	4.8E+02	4.1E+02	3.7E+02	2.8E+02	2.5E+02	2.4E+02	2.6E+02	5.7E+02	2.8E+02	2.5E+02	1.8E+02	2.1E+02	2.7E+02
	rank	1	12	13	14	2	7	8	15	9	10	3	4	6	5	11
F_{17}	mean	6.7E+02	1.7E+03	1.8E+03	2.0E+03	7.0E+02	9.3E+02	1.1E+03	2.0E+03	9.2E+02	1.4E+03	9.0E+02	7.3E+02	8.2E+02	8.3E+02	1.3E+03
	std	2.0E+02	3.4E+02	3.0E+02	2.8E+02	1.8E+02	2.2E+02	3.1E+02	1.8E+02	2.5E+02	2.3E+02	2.2E+02	1.8E+02	1.5E+02	1.5E+02	3.5E+02
	rank	1	12	13	15	2	8	9	14	7	11	6	3	4	5	10
F_{18}	mean	5.8E+04	3.1E+06	3.1E+05	2.2E+07	1.7E+05	3.3E+04	4.4E+04	9.0E+06	8.2E+04	8.9E+04	7.6E+05	6.4E+02	8.4E+05	9.2E+05	4.4E+05
	std	2.5E+04	6.5E+06	3.0E+05	1.2E+07	1.4E+05	2.4E+04	2.9E+04	2.9E+06	3.2E+04	4.1E+04	4.2E+05	5.9E+02	6.0E+05	6.6E+05	7.9E+05
	rank	4	13	8	15	7	2	3	14	5	6	10	1	11	12	9
F_{19}	mean	1.5E+04	6.7E+06	1.4E+05	9.5E+05	1.8E+04	8.5E+02	1.6E+04	1.1E+05	7.9E+03	1.4E+04	2.9E+03	1.8E+02	7.1E+03	5.9E+03	2.5E+04
	std	5.3E+03	3.3E+07	2.8E+05	8.9E+05	1.2E+04	1.0E+03	1.1E+04	1.2E+05	7.6E+03	7.0E+03	2.9E+03	5.5E+01	4.3E+03	3.6E+03	2.6E+04
	rank	8	15	13	14	10	2	9	12	6	7	3	1	5	4	11
F_{20}	mean	4.5E+02	1.3E+03	1.4E+03	1.6E+03	3.9E+02	5.5E+02	7.7E+02	2.0E+03	5.6E+02	8.8E+02	5.8E+02	5.0E+02	6.1E+02	6.0E+02	8.8E+02
	std	1.5E+02	2.8E+02	3.7E+02	2.5E+02	1.9E+02	2.2E+02	2.1E+02	1.9E+02	2.5E+02	2.9E+02	2.1E+02	2.1E+02	1.6E+02	1.5E+02	1.8E+02
	rank	2	12	13	14	1	4	9	15	5	11	6	3	8	7	10
F_{21}	mean	2.6E+02	5.1E+02	6.6E+02	7.1E+02	2.8E+02	3.2E+02	3.1E+02	5.7E+02	3.0E+02	5.0E+02	3.5E+02	2.6E+02	4.0E+02	4.0E+02	4.1E+02
	std	1.2E+01	6.1E+01	6.9E+01	4.5E+01	1.5E+01	2.6E+01	2.7E+01	2.0E+01	1.9E+01	3.3E+01	4.8E+01	1.2E+01	3.1E+01	2.3E+01	4.1E+01
	rank	2	12	14	15	3	6	5	13	4	11	7	1	8	9	10
F_{22}	mean	4.3E+02	7.7E+03	9.8E+03	1.3E+04	3.9E+03	3.8E+03	5.4E+03	1.5E+04	4.4E+03	5.9E+03	5.3E+03	3.8E+03	8.5E+03	8.5E+03	7.3E+03
	std	1.3E+03	9.1E+02	1.1E+03	4.1E+02	2.3E+03	2.0E+03	2.1E+03	4.6E+02	2.5E+03	2.6E+03	2.8E+03	1.7E+03	2.3E+03	2.7E+03	5.4E+02
	rank	1	10	13	14	4	3	7	15	5	8	6	2	11	12	9
F_{23}	mean	5.1E+02	8.2E+02	1.4E+03	1.3E+03	5.2E+02	5.5E+02	5.7E+02	7.5E+02	5.5E+02	9.3E+02	5.9E+02	5.0E+02	6.5E+02	6.7E+02	7.2E+02
	std	2.5E+01	7.2E+01	2.0E+02	1.4E+02	1.8E+01	3.2E+01	3.2E+01	1.6E+01	2.8E+01	9.8E+01	4.0E+01	2.0E+01	4.2E+01	4.6E+01	3.8E+01
	rank	2	12	15	14	3	4	6	11	5	13	7	1	8	9	10
F_{24}	mean	5.8E+02	8.5E+02	1.3E+03	1.6E+03	5.8E+02	6.1E+02	6.4E+02	8.9E+02	6.2E+02	1.3E+03	7.7E+02	5.6E+02	8.2E+02	8.2E+02	7.3E+02
	std	2.3E+01	8.3E+01	1.8E+02	9.4E+01	2.8E+01	9.8E+01	3.9E+01	2.4E+01	2.9E+01	1.9E+02	4.5E+01	1.1E+01	5.3E+01	6.7E+01	4.7E+01
	rank	3	11	14	15	2	4	6	12	5	13	8	1	9	10	7
F_{25}	mean	5.0E+02	9.9E+02	8.3E+02	2.4E+03	4.4E+02	5.2E+02	5.3E+02	4.3E+02	5.3E+02	4.5E+02	4.3E+02	5.4E+02	4.9E+02	4.7E+02	5.3E+02
	std	2.6E+01	4.8E+02	9.4E+01	3.1E+02	1.1E+01	3.0E+01	3.4E+01	1.3E-01	4.0E+01	2.0E+01	1.5E-01	4.0E+01	3.5E+01	3.0E+01	3.3E+01
	rank	7	14	13	15	3	8	10	1	11	4	2	12	6	5	9
F_{26}	mean	3.0E+02	4.9E+03	8.2E+03	7.9E+03	1.8E+03	1.9E+03	2.1E+03	4.1E+03	2.3E+03	6.3E+03	2.7E+03	1.9E+03	3.7E+03	3.6E+03	3.9E+03
	std	4.6E-03	7.2E+02	2.8E+03	1.6E+03	6.5E+02	1.1E+03	1.1E+03	1.9E+02	5.8E+02	2.4E+03	6.8E+02	2.2E+02	5.6E+02	5.1E+02	4.0E+02
	rank	1	12	15	14	2	3	5	11	6	13	7	4	9	8	10
F_{27}	mean	5.8E+02	8.3E+02	1.3E+03	2.4E+03	5.0E+02	6.8E+02	7.1E+02	5.0E+02	6.7E+02	4.6E+02	5.0E+02	5.9E+02	4.7E+02	4.7E+02	8.4E+02
	std	4.3E+01	1.1E+02	3.5E+02	3.6E+02	1.3E+01	5.1E+01	5.8E+01	4.0E-05	6.6E+01	1.5E+01	2.0E-04	4.0E+01	1.3E+01	1.1E+01	9.7E+01
	rank	7	12	14	15	4	10	11	6	9	1	5	8	3	2	13
F_{28}	mean	4.9E+02	3.8E+03	8.2E+02	3.4E+03	4.5E+02	5.0E+02	4.9E+02	5.0E+02	4.8E+02	4.5E+02	4.5E+02	4.9E+02	4.9E+02	4.8E+02	8.6E+02
	std	2.0E+01	1.7E+03	2.0E+02	4.9E+02	5.8E+00	2.2E+01	2.0E+01	5.4E-05	2.3E+01	1.2E+01	1.3E+01	1.9E+01	7.2E+00	8.3E+00	9.0E+02
	rank	7	15	12	14	1	10	9	11	4	3	2	8	6	5	13
F_{29}	mean	8.7E+02	1.8E+03	3.5E+03	4.4E+03	8.1E+02	8.2E+02	9.4E+02	2.2E+03	7.6E+02	1.4E+03	8.5E+02	6.8E+02	9.1E+02	9.0E+02	1.8E+03
	std	1.8E+02	4.4E+02	7.1E+02	6.6E+02	2.4E+02	1.8E+02	2.5E+02	2.1E+02	2.4E+02	3.7E+02	2.2E+02	1.2E+02	1.4E+02	1.5E+02	3.4E+02
	rank	6	11	14	15	3	4	9	13	2	10	5	1	8	7	12
F_{30}	mean	8.0E+05	2.8E+07	7.5E+06	2.1E+08	3.7E+03	7.4E+05	8.5E+05	3.5E+06	8.3E+05	3.5E+03	1.1E+03	7.6E+05	2.3E+03	2.1E+03	4.1E+06
	std	9.7E+04	8.3E+07	5.5E+06	3.6E+07	4.4E+03	8.8E+04	1.1E+05	2.3E+06	8.0E+04	2.5E+03	6.0E+02	1.7E+05	1.2E+03	7.0E+02	2.7E+06
	rank	8	14	13	15	5	6	10	11	9	4	1	7	3	2	12

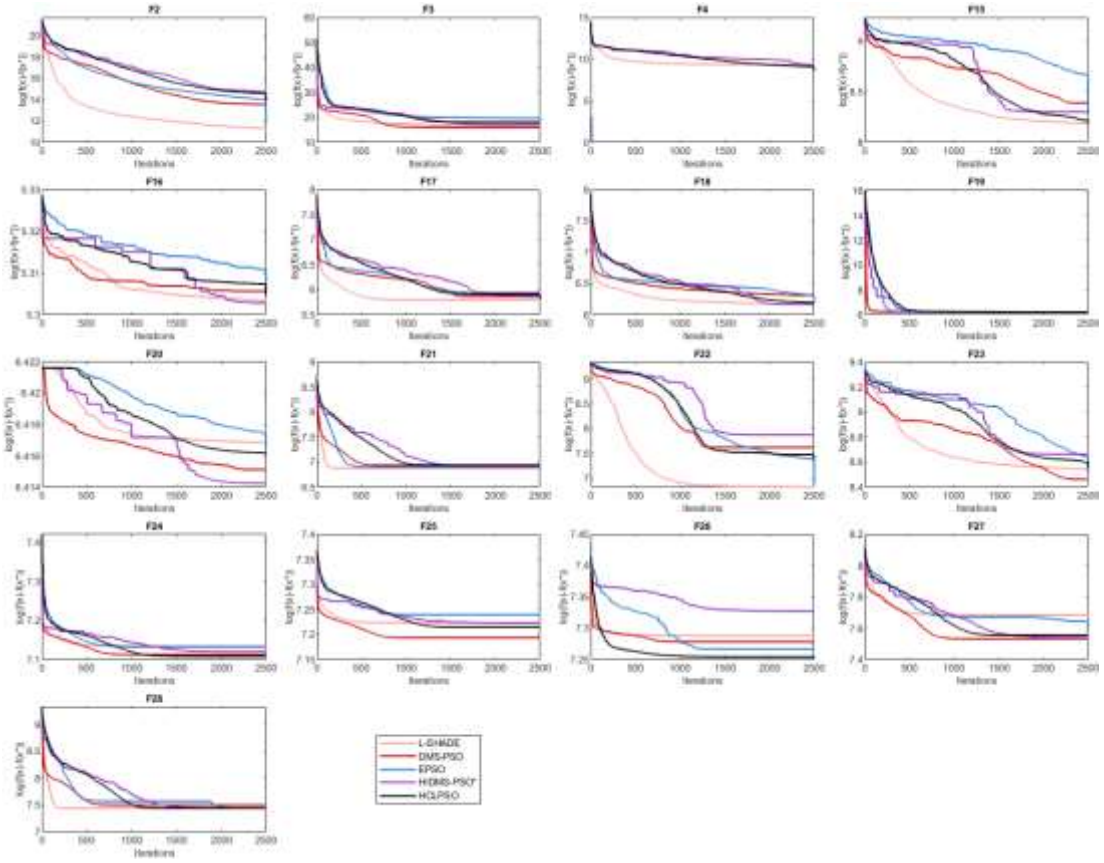
Mean rank	4.38	12.52	13.03	14.55	4.24	4.52	6.93	10.93	5.90	8.31	5.76	3.38	7.69	7.59	10.28
Final rank	3	13	14	15	2	4	7	12	6	10	5	1	9	8	11
Algorithms	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO

Statistical results on 100-dimensional CEC'17 problems for AHPSO vs comparison algorithms

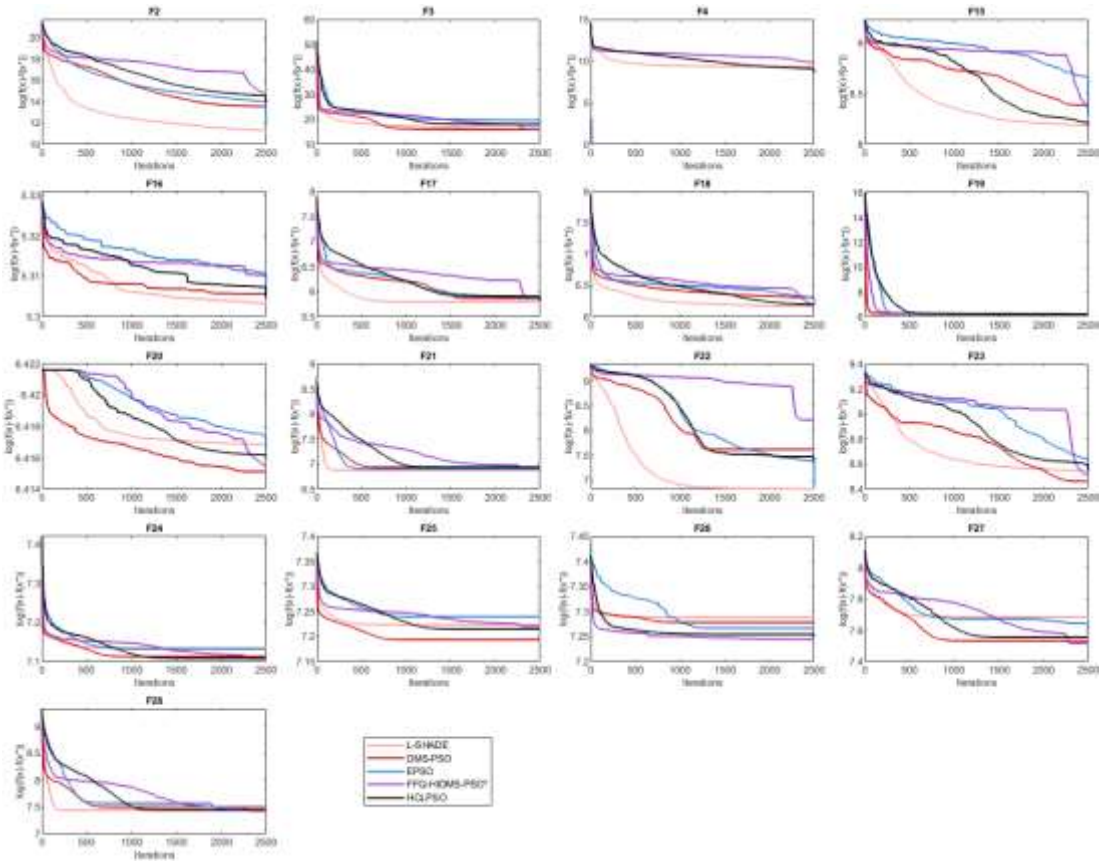
F	Criteria	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSOTVAC	LPSO	L-SHADE	MaPSO	MiPSO	UPSO
F_1	mean	5.5E+05	6.3E+10	1.3E+10	1.2E+10	8.6E+03	1.2E+04	7.5E+03	3.9E+06	8.5E+03	4.6E+03	3.5E+03	1.1E-09	3.1E+03	3.8E+03	1.5E+04
	std	9.2E+05	2.6E+10	4.4E+09	2.3E+09	1.1E+04	1.3E+04	9.4E+03	4.4E+06	1.1E+04	4.9E+03	3.5E+03	1.2E-09	2.1E+03	3.9E+03	2.1E+04
	rank	11	15	14	13	8	9	6	12	7	5	3	1	2	4	10
F_3	mean	2.1E+04	4.7E+05	7.1E+04	4.3E+05	3.6E+04	1.4E+02	2.9E+04	1.6E+06	2.3E+04	5.2E+03	1.7E+05	1.3E+05	4.8E+05	5.0E+05	2.5E+05
	std	3.8E+03	7.6E+04	1.7E+04	4.8E+04	1.0E+04	1.9E+02	9.5E+03	6.7E+05	7.3E+03	2.4E+03	1.8E+04	1.1E+05	4.4E+04	4.6E+04	3.2E+04
	rank	3	12	7	11	6	1	5	15	4	2	9	8	13	14	10
F_4	mean	2.6E+02	7.4E+03	2.0E+03	3.2E+03	1.1E+02	1.6E+02	2.2E+02	9.7E+01	2.2E+02	2.0E+02	9.6E+01	1.3E+02	2.6E+02	2.5E+02	3.6E+02
	std	2.6E+01	4.7E+03	4.5E+02	3.5E+02	3.0E+01	4.4E+01	4.1E+01	1.1E+00	3.6E+01	4.0E+01	1.2E+00	4.7E+01	4.5E+01	5.5E+01	8.9E+01
	rank	10	15	13	14	3	5	8	2	7	6	1	4	11	9	12
F_5	mean	1.4E+02	8.8E+02	1.1E+03	1.1E+03	1.8E+02	3.3E+02	3.1E+02	9.5E+02	2.9E+02	6.8E+02	5.2E+02	2.2E+02	8.6E+02	8.5E+02	5.6E+02
	std	2.0E+01	1.3E+02	7.0E+01	9.3E+01	2.9E+01	6.0E+01	5.0E+01	3.0E+01	4.6E+01	6.1E+01	1.1E+02	2.4E+01	8.5E+01	5.8E+01	5.1E+01
	rank	1	12	14	15	2	6	5	13	4	9	7	3	11	10	8
F_6	mean	3.3E+00	5.2E+01	8.0E+01	7.0E+01	2.8E+00	1.1E-04	1.0E+01	8.2E-02	4.5E-01	1.4E+01	8.5E+00	1.6E+00	4.2E-02	4.2E-02	4.0E+01
	std	6.5E-01	7.7E+00	2.9E+00	1.4E+01	2.1E+00	3.4E-04	3.7E+00	2.2E-02	5.6E-01	3.6E+00	1.6E+00	1.2E+00	4.4E-02	7.1E-02	7.3E+00
	rank	8	13	15	14	7	1	10	4	5	11	9	6	2	3	12
F_7	mean	2.9E+02	2.4E+03	2.4E+03	2.2E+03	3.5E+02	5.0E+02	5.8E+02	1.1E+03	6.9E+02	1.6E+03	5.9E+02	4.2E+02	9.3E+02	9.7E+02	8.1E+02
	std	3.8E+01	7.0E+02	2.4E+02	2.3E+02	7.0E+01	8.2E+01	4.9E+01	2.5E+01	1.1E+02	1.7E+02	9.0E+01	4.0E+01	6.9E+01	6.9E+01	1.1E+02
	rank	1	14	15	13	2	4	5	11	7	12	6	3	9	10	8
F_8	mean	1.3E+02	9.0E+02	1.1E+03	1.1E+03	1.8E+02	3.4E+02	3.2E+02	9.5E+02	2.9E+02	7.6E+02	5.5E+02	2.3E+02	8.3E+02	8.5E+02	5.5E+02
	std	2.0E+01	1.2E+02	7.3E+01	6.4E+01	3.8E+01	4.6E+01	6.3E+01	3.2E+01	2.8E+01	8.1E+01	1.1E+02	3.4E+01	5.1E+01	8.0E+01	7.0E+01
	rank	1	12	15	14	2	6	5	13	4	9	7	3	10	11	8
F_9	mean	9.4E+01	3.1E+04	3.8E+04	5.2E+04	1.9E+02	2.3E+03	1.7E+03	4.5E+02	2.7E+03	2.0E+04	1.5E+03	1.9E+03	4.1E+04	3.9E+04	1.1E+04
	std	2.4E+01	9.7E+03	4.3E+03	1.1E+04	6.5E+01	9.3E+02	7.6E+02	2.5E+02	9.1E+02	2.7E+03	4.9E+02	7.4E+02	9.2E+03	9.7E+03	4.8E+03
	rank	1	11	12	15	2	7	5	3	8	10	4	6	14	13	9
F_{10}	mean	1.1E+04	1.6E+04	2.1E+04	2.9E+04	1.3E+04	1.1E+04	1.3E+04	3.2E+04	1.2E+04	1.4E+04	1.8E+04	1.0E+04	2.6E+04	2.5E+04	1.5E+04
	std	1.0E+03	2.0E+03	1.4E+03	1.3E+03	1.7E+03	7.6E+02	1.3E+03	5.6E+02	1.4E+03	1.1E+03	1.6E+03	4.9E+02	9.1E+02	1.4E+03	1.2E+03
	rank	3	9	11	14	6	2	5	15	4	7	10	1	13	12	8
F_{11}	mean	9.6E+02	4.6E+04	9.5E+03	1.3E+05	7.0E+02	8.3E+02	8.8E+02	4.5E+05	7.6E+02	1.0E+03	1.2E+03	1.2E+03	1.7E+04	1.2E+04	1.4E+03
	std	1.0E+02	3.8E+04	2.1E+03	4.2E+04	3.1E+02	2.1E+02	1.5E+02	1.7E+05	2.0E+02	1.9E+02	2.4E+02	2.8E+02	1.6E+04	1.5E+04	3.1E+02
	rank	5	13	10	14	1	3	4	15	2	6	8	7	12	11	9
F_{12}	mean	1.7E+06	1.3E+10	4.7E+09	2.1E+09	1.1E+06	2.7E+05	3.6E+05	2.0E+09	8.2E+05	9.4E+05	1.4E+07	7.8E+04	1.2E+07	1.1E+07	1.5E+08
	std	8.0E+05	1.0E+10	2.6E+09	6.2E+08	6.3E+05	1.1E+05	1.7E+05	6.3E+08	3.1E+05	4.3E+05	7.0E+06	7.9E+04	3.3E+06	2.8E+06	2.3E+08
	rank	7	15	14	13	6	2	3	12	4	5	10	1	9	8	11
F_{13}	mean	6.3E+03	9.1E+08	3.9E+08	1.2E+06	6.4E+03	1.0E+03	3.8E+03	2.0E+06	3.8E+03	8.7E+03	2.9E+03	5.3E+03	4.2E+03	3.7E+03	5.1E+04
	std	2.6E+03	8.8E+08	2.9E+08	7.6E+05	7.5E+03	3.2E+02	3.3E+03	1.7E+06	4.2E+03	5.2E+03	3.4E+03	6.2E+03	1.6E+03	1.3E+03	3.0E+04
	rank	8	15	14	12	9	1	5	13	4	10	2	7	6	3	11
F_{14}	mean	7.8E+04	4.5E+06	9.9E+05	9.7E+06	1.8E+05	5.3E+04	5.2E+04	6.9E+06	1.7E+05	9.8E+04	7.6E+05	4.5E+03	3.5E+06	2.3E+06	5.1E+05
	std	2.1E+04	7.5E+06	6.2E+05	3.7E+06	1.0E+05	2.9E+04	3.1E+04	2.4E+06	1.0E+05	3.4E+04	3.8E+05	1.6E+04	3.9E+06	1.2E+06	2.9E+05
	rank	4	13	10	15	7	3	2	14	6	5	9	1	12	11	8
F_{15}	mean	1.8E+03	5.3E+08	1.2E+07	3.0E+04	6.6E+03	6.6E+02	1.7E+03	2.1E+06	2.4E+03	1.7E+03	1.4E+03	4.2E+02	1.0E+03	9.6E+02	2.3E+04
	std	1.5E+03	7.4E+08	8.6E+06	7.4E+03	1.3E+04	2.6E+02	1.6E+03	3.1E+06	2.9E+03	1.9E+03	6.6E+02	8.4E+01	3.4E+02	4.4E+02	1.3E+04
	rank	8	15	14	12	10	2	6	13	9	7	5	1	4	3	11
F_{16}	mean	1.9E+03	5.5E+03	7.9E+03	8.5E+03	2.6E+03	3.0E+03	3.0E+03	9.8E+03	3.2E+03	4.4E+03	3.2E+03	2.8E+03	3.1E+03	3.1E+03	4.0E+03
	std	6.2E+02	7.6E+02	1.2E+03	9.0E+02	5.1E+02	4.8E+02	5.6E+02	3.9E+02	5.0E+02	8.1E+02	7.6E+02	3.1E+02	3.7E+02	3.7E+02	3.8E+02
	rank	1	12	13	14	2	4	5	15	9	11	8	3	6	7	10
F_{17}	mean	1.7E+03	4.8E+03	5.5E+03	5.3E+03	2.6E+03	2.5E+03	2.9E+03	6.3E+03	2.8E+03	3.6E+03	3.1E+03	2.2E+03	2.5E+03	2.4E+03	3.7E+03
	std	4.1E+02	5.8E+02	9.3E+02	3.7E+02	5.5E+02	3.1E+02	4.6E+02	3.1E+02	4.0E+02	6.0E+02	4.0E+02	2.9E+02	2.9E+02	3.7E+02	6.1E+02
	rank	1	12	14	13	6	4	8	15	7	10	9	2	5	3	11
F_{18}	mean	2.5E+05	3.2E+06	1.7E+06	9.9E+06	3.6E+05	1.0E+05	1.7E+05	4.0E+07	3.2E+05	1.6E+05	2.7E+06	1.0E+05	3.8E+06	4.1E+06	1.1E+06
	std	7.1E+04	3.0E+06	1.2E+06	6.0E+06	1.1E+05	3.6E+04	6.5E+04	9.1E+06	1.3E+05	6.5E+04	8.5E+05	6.6E+04	2.7E+06	2.2E+06	8.6E+05
	rank	5	11	9	14	7	1	4	15	6	3	10	2	12	13	8
F_{19}	mean	1.6E+03	4.7E+08	4.0E+07	1.6E+05	3.8E+03	4.3E+02	1.9E+03	7.1E+05	1.7E+03	2.4E+03	1.2E+03	2.5E+03	9.2E+02	1.5E+03	2.2E+05
	std	1.3E+03	8.7E+08	8.5E+07	9.5E+04	6.0E+03	3.6E+02	2.4E+03	8.3E+05	1.4E+03	4.3E+03	1.0E+03	4.4E+03	9.4E+02	1.1E+03	4.3E+05
	rank	5	15	14	11	10	1	7	13	6	8	3	9	2	4	12
F_{20}	mean	1.8E+03	3.4E+03	3.7E+03	4.8E+03	1.9E+03	2.5E+03	2.8E+03	5.7E+03	2.4E+03	2.8E+03	2.8E+03	2.0E+03	2.3E+03	2.4E+03	3.2E+03
	std	3.5E+02	6.0E+02	4.4E+02	4.3E+02	2.6E+02	3.9E+02	5.9E+02	2.6E+02	5.2E+02	5.2E+02	3.7E+02	2.8E+02	3.0E+02	3.9E+02	3.1E+02
	rank	1	12	13	14	2	7	8	15	5	10	9	3	4	6	11
F_{21}	mean	3.8E+02	1.2E+03	1.8E+03	1.5E+03	4.2E+02	5.9E+02	5.8E+02	1.2E+03	5.5E+02	1.1E+03	7.4E+02	4.6E+02	1.0E+03	1.0E+03	8.0E+02
	std	1.4E+01	1.4E+02	1.8E+02	7.2E+01	2.4E+01	5.2E+01	6.8E+01	2.7E+01	5.2E+01	9.1E+01	1.3E+02	3.3E+01	4.9E+01	5.7E+01	7.9E+01
	rank	1	13	15	14	2	6	5	12	4	11	7	3	9	10	8
F_{22}	mean	1.5E+03	1.7E+04	2.3E+04	3.0E+04	1.2E+04	1.2E+04	1.3E+04	3.3E+04	1.3E+04	1.6E+04	1.8E+04	1.1E+04	2.7E+04	2.7E+04	1.7E+04
	std	4.2E+03	1.3E+03	1.7E+03	2.2E+03	3.1E+03	5.9E+02	4.7E+03	6.8E+02	1.2E+03	1.5E+03	2.3E+03	7.1E+02	1.0E+03	9.2E+02	1.1E+03
	rank	1	9	11	14	4	3	5	15	6	7	10	2	13	12	8
F_{23}	mean	7.6E+02	1.5E+03	3.1E+03	2.0E+03	7.4E+02	8.0E+02	1.0E+03	1.5E+03	9.0E+02	1.7E+03	8.9E+02	7.2E+02	1.0E+03	1.0E+03	1.2E+03
	std	4.3E+01	1.2E+02	4.1E+02	1.8E+02	3.4E+01	5.1E+01	8.8E+01	3.9E+01	6.4E+01	1.7E+02	3.8E+01	3.6E+01	7.0E+01	7.8E+01	1.1E+02
	rank	3	11	15	14	2	4	7	12	6	13	5	1	8	9	10
F_{24}	mean	1.1E+03	2.0E+03	4.3E+03	2.8E+03	1.1E+03	1.3E+03	1.4E+03	1.9E+03	1.5E+03	3.3E+03	1.4E+03	1.2E+03	1.6E+03	1.6E+03	1.6E+03
	std	4.4E+01	1.6E+02	4.0E+02	1.7E+02	2.3E+01	4.1E+01	7.9E+01	3.1E+01	1.2E+02	4.0E+02	1.4E+02	4.3E+01	6.7E+01	6.3E+01	1.4E+02
	rank	2	12	15	13	1	4	5	11	7	14	6	3	9	8	10
F_{25}	mean	8.1E+02	7.5E+03	2.2E+03	2.2E+03	7.4E+02	7.7E+02	6.6E+04	8.1E+02	7.6E+02	7.8E+02	7.5E+02	7.6E+02	8.6E+02	8.5E+02	8.3E+02
	std	5.4E+01	3.4E+03	3.7E+02	2.0E+02	3.5E+01	6.5E+01	6.6E+04	2.1E+01	7.4E+01	8.3E+01	3.6E+01	8.1E+01	2.7E+01	4.4E+01	7.7E+01
	rank	7	14	12	13	1	5	15	8	4	6	2	3	11	10	9
F_{26}	mean	2.1E+03	1.6E+04	2.8E+04	1.9E+04	5.5E+03	7.4E+03	6.6E+04	1.4E+04	7.8E+03	1.9E+04	9.1E+03	6.0E+03	1.2E+04	1.3E+04	1.1E+04
	std	2.0E+03	1.5E+03	5.7E+03	3.5E+03	2.9E+02	1.8E+03	6.6E+04	3.0E+02	6.3E+02	2.1E+03	1.2E+03	5.1E+02	1.0E+03	8.4E+02	1.0E+03
	rank	1	11	14	12	2	4	15	10	5	13	6	3	8	9	7
F_{27}	mean	7.2E+02	1.1E+03	2.0E+03	2.2E+03	5.0E+02	6.6E+04	6.6E+04	5.0E+02	9.0E+02	1.1E+03	5.0E+02	7.6E+02	7.4E+02	7.0E+02	1.2E+03
	std	6.3E+01	1.3E+02	4.8E+02	1.7E+02	4.9E+00	6.6E+04	6.6E+04	4.0E-05	8.2E+01	3.4E+02	1.4E-04	6.1E+01	9.8E+01	1.2E+02	1.7E+02
	rank	5	9	12	13	1	14	14	3	8	10	2	7	6	4	11
F_{28}	mean	5.8E+02	1.5E+04	2.1E+03	4.3E+03	4.9E+02	5.5E+02	6.6E+04	5.0E+02	5.6E+02	5.2E+02	4.9E+02	5.4E+02	6.0E+02	5.9E+02	2.8E+03
	std	1.4E+01	1.1E+03	5.8E+02	5.9E+02	9.7E+00	3.1E+01	6.6E+04	5.2E-05	3.6E+01	2.0E+01	2.8E+00	3.9E+01	2.5E+01	2.1E+01	3.8E+03
	rank	8	14	11	13	2	6	15	3	7	4	1	5	10	9	12
F_{29}	mean	2.4E+03	5.1E+03	1.0E+04	8.1E+03	2.4E+03	3.2E+03	6.6E+04	7.7E+03	3.5E+03	3.8E+03	2.7E+03	2.5E+03	4.0E+03	3.7E+03	4.6E+03
	std	4.4E+02	7.3E+02	1.9E+03	8.3E+02	4.3E+02	3.6E+02	6.6E+04	5.3E+02	5.6E+02	5.3E+02	4.1E+02	3.7E+02	5.5E+02	4.4E+02	5.6E+02
	rank	1	11	14	13	2	5	15	12	6	8	4	3	9	7	10
F_{30}	mean	1.2E+04	5.2E+08	3.4E+08	8.9E+07	7.0E+03	6.6E+04	6.6E+04	4.2E+05	6.3E+03	5.3E+03	1.2E+03	3.1E+03	4.1E+03	4.1E+03	9.6E+05
	std	9.8E+03	7.9E+08	6.3E+08	3.5E+07	9.6E+03	6.6E+04	6.6E+04	5.0E+05	2.6E+03	4.2E+03	3.0E+02	4.5E+02	2.5E+03	2.5E+03	1.3E+06

rank	8	13	12	11	7	14	14	9	6	5	1	2	4	3	10
Mean rank	4.17	12.52	12.83	13.28	4.21	4.62	7.45	11.14	5.76	8.17	5.97	3.55	8.41	8.14	9.72
Final rank	2	13	14	15	3	4	7	12	5	9	6	1	10	8	11
Algorithms	AHPSO	BBPSO	BRPSO	CLPSO	DMS-PSO	EPSO	FDR	FIPS	HCLPSO	HPSO-TVAC	LPSO	L-SHADE	MaPSO	MIPSO	UPSO

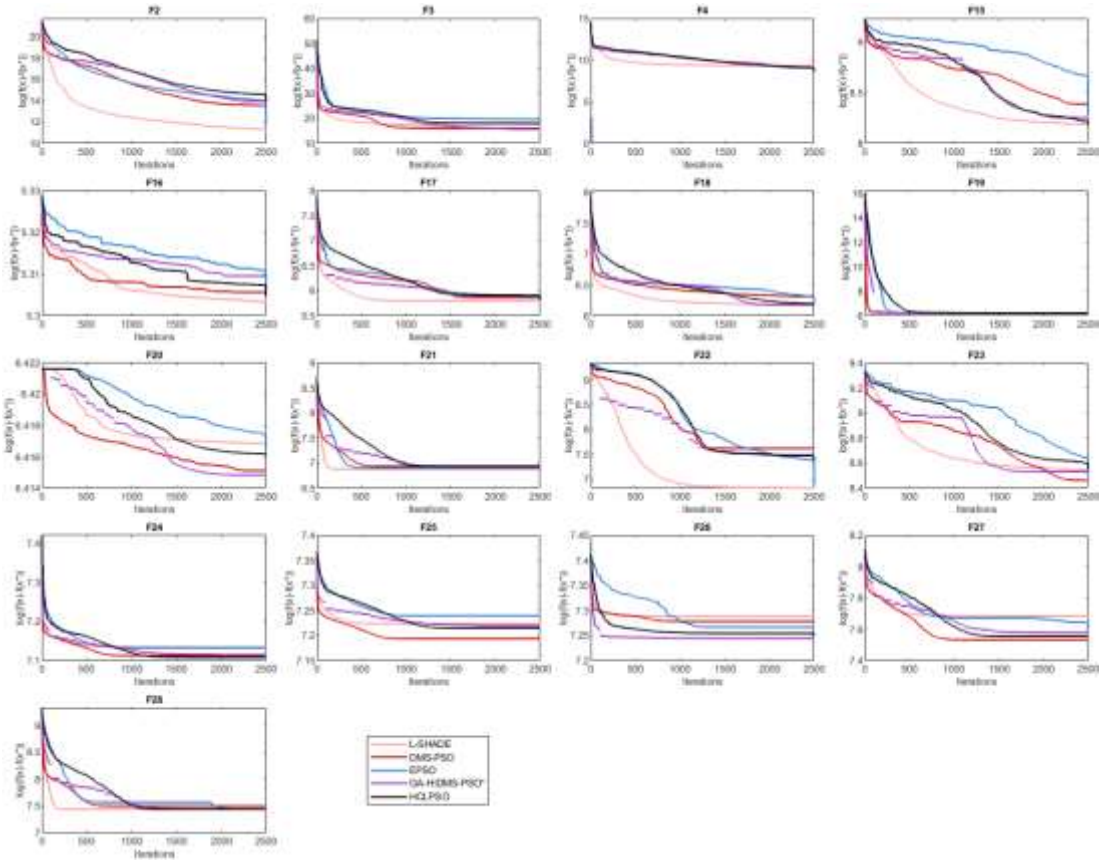
Appendix 3: Convergence Characteristic Graphs for CEC'13 Benchmark Functions



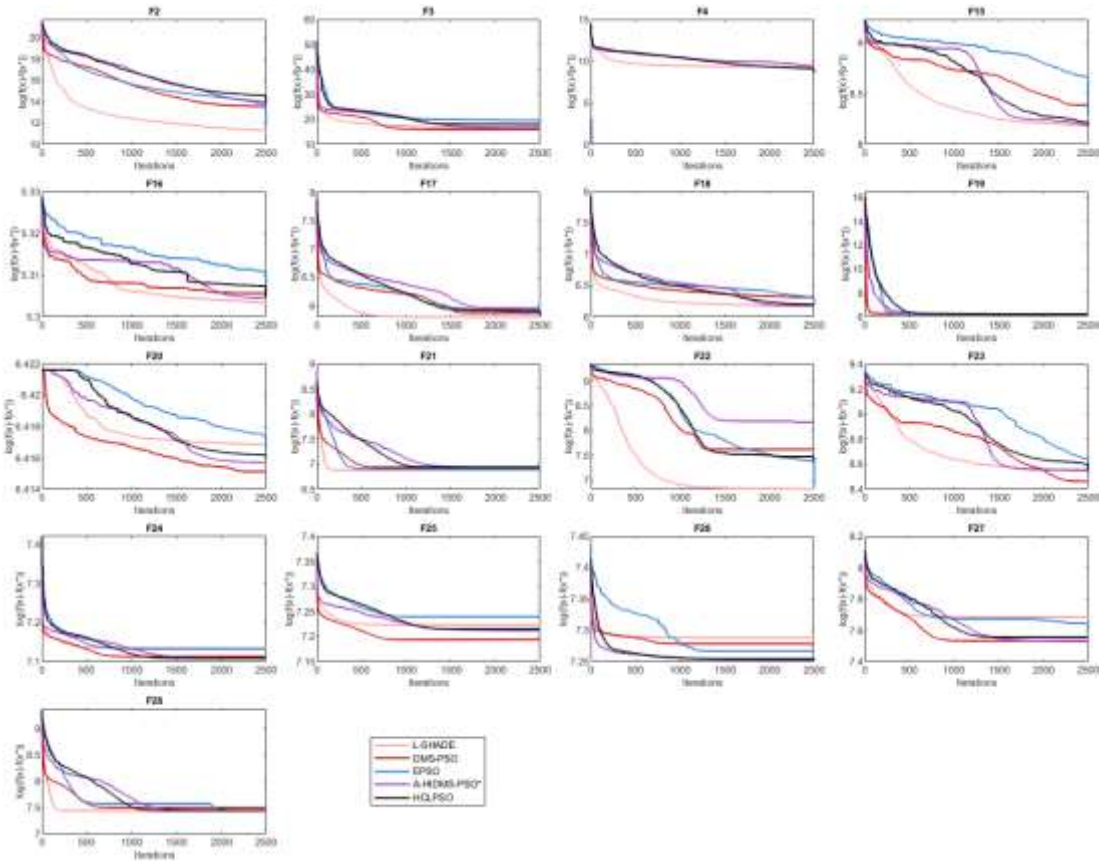
Convergence Rates of HIDMS-PSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



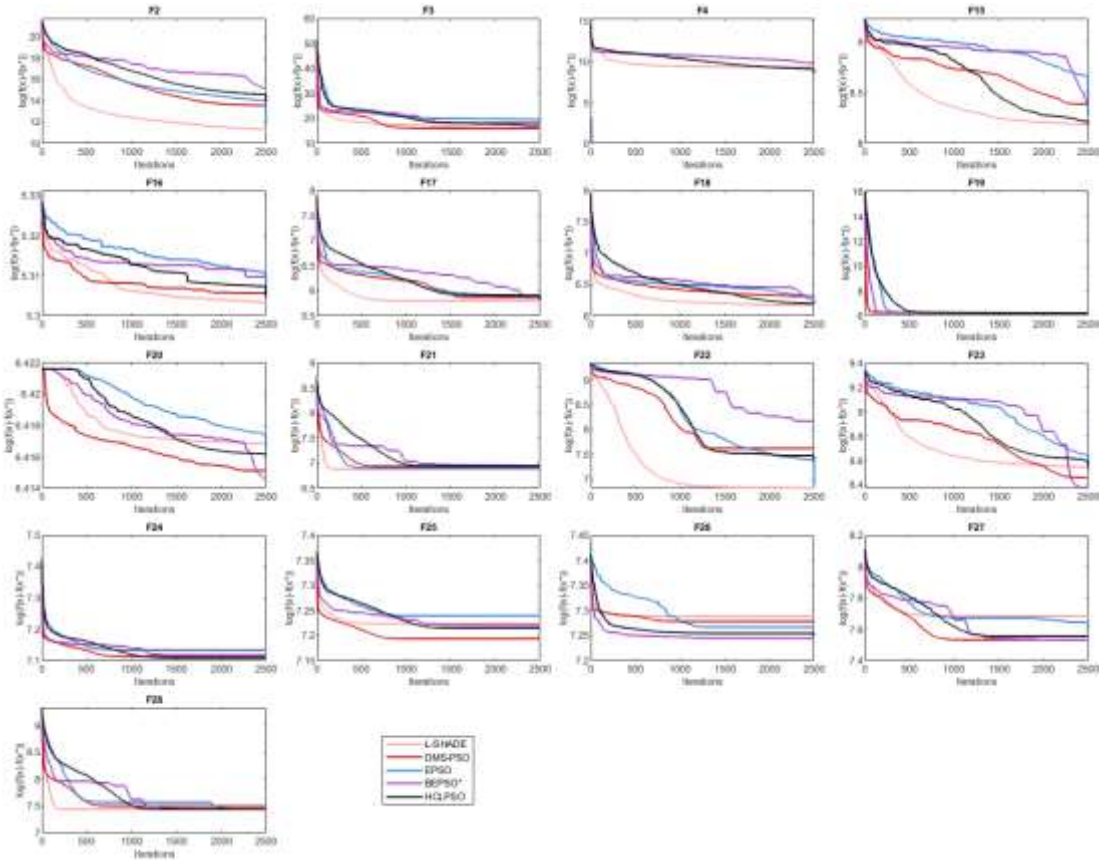
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



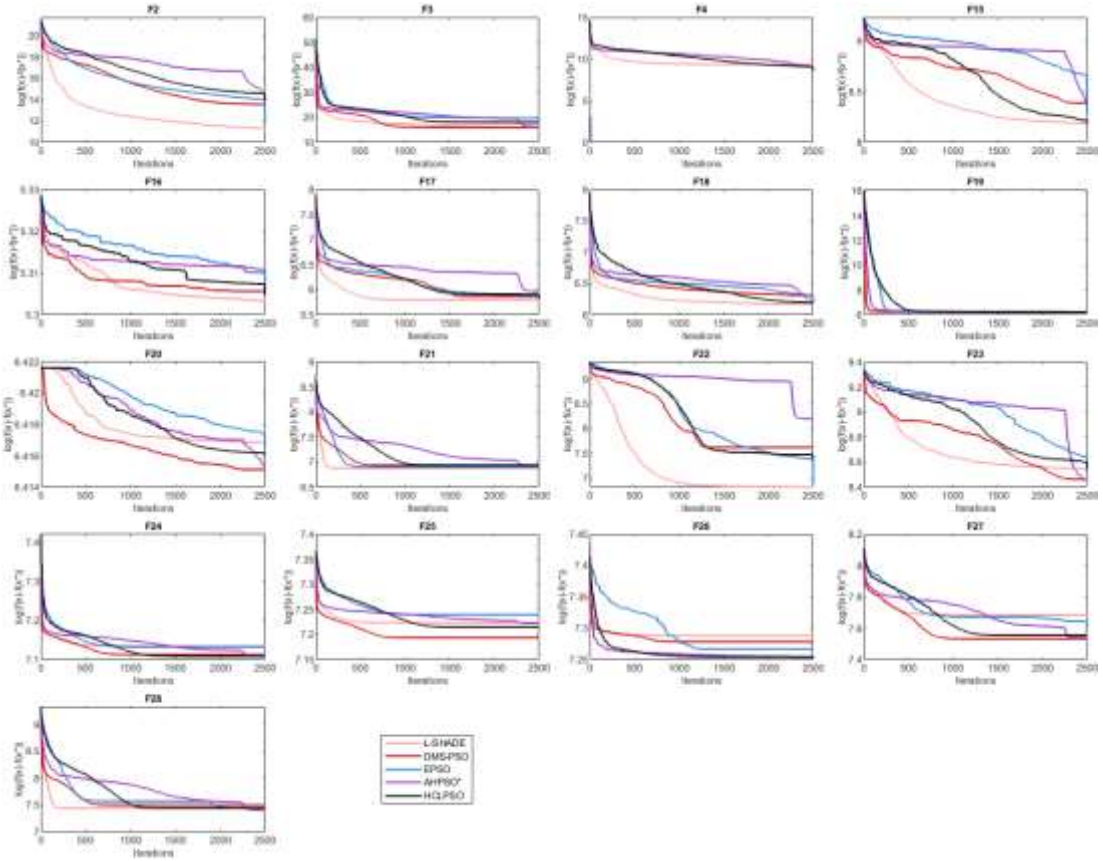
Convergence Rates of GA-HIDMS-PSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



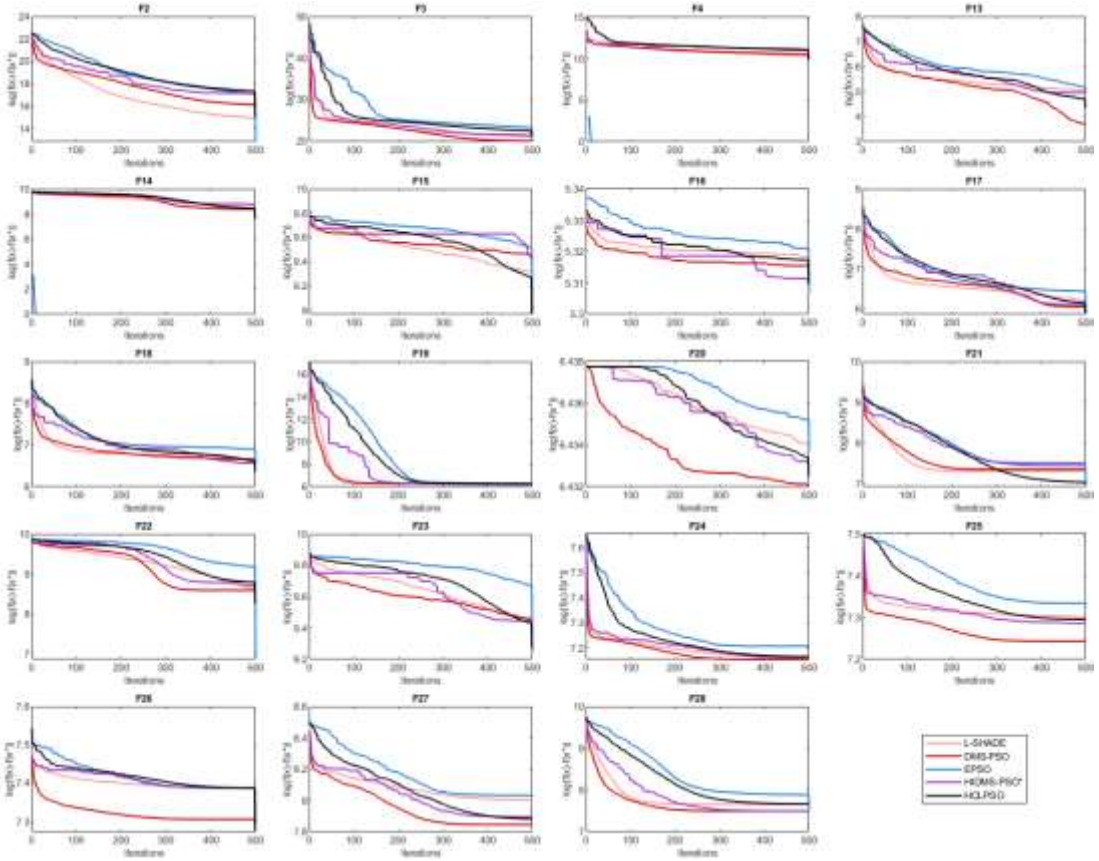
Convergence Rates of A-HIDMS-PSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



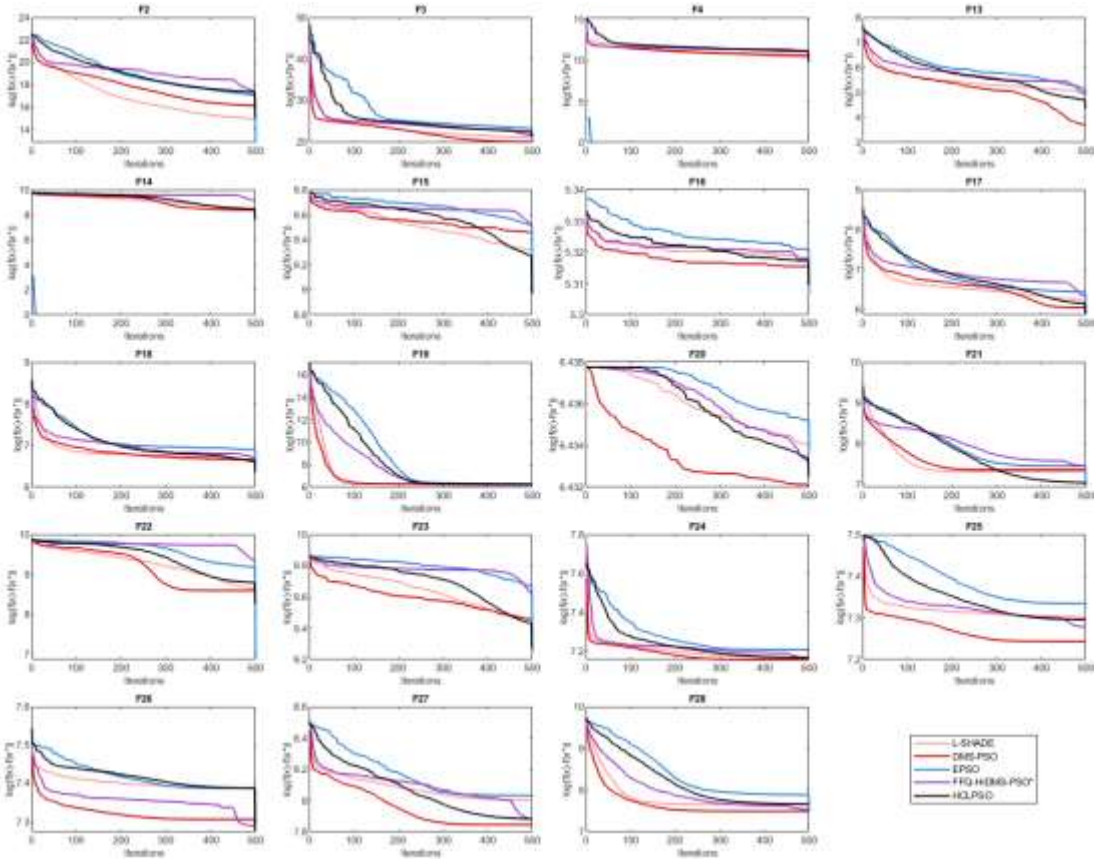
Convergence Rates of BEPSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



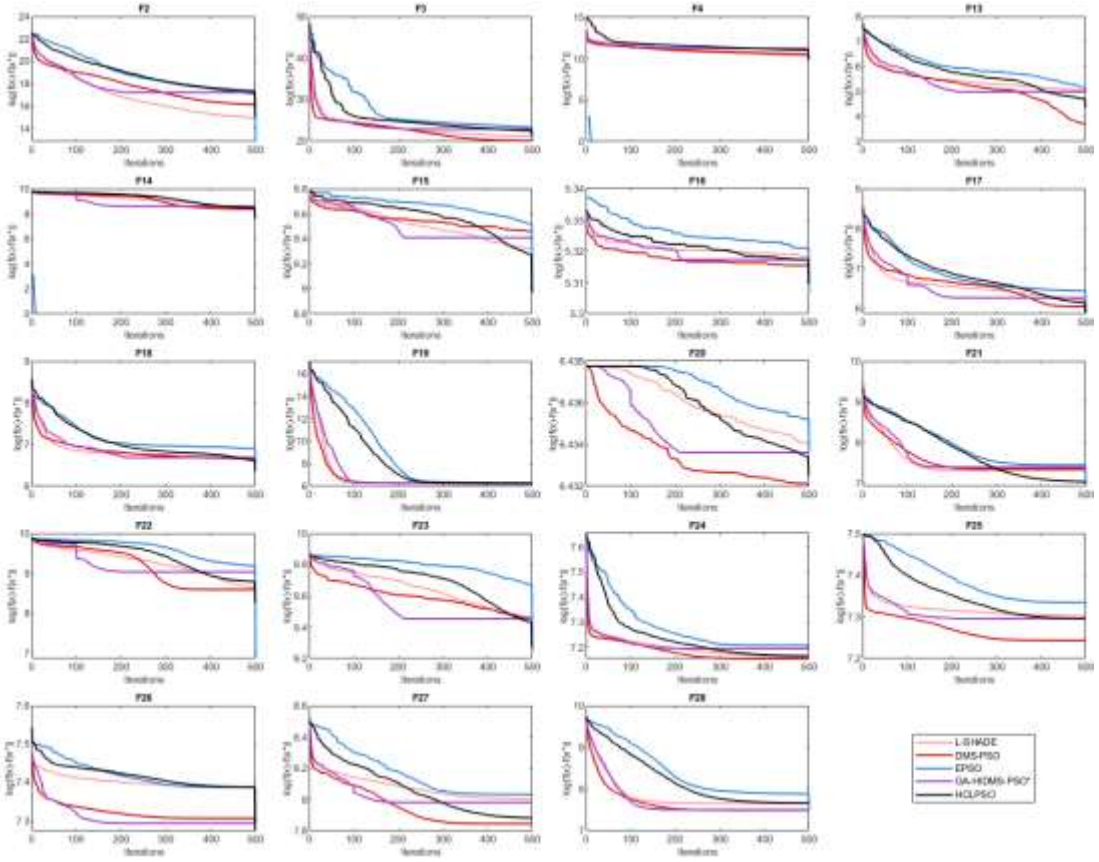
Convergence Rates of AHPSO Algorithm for 30-dimensional CEC'13 Benchmark Functions



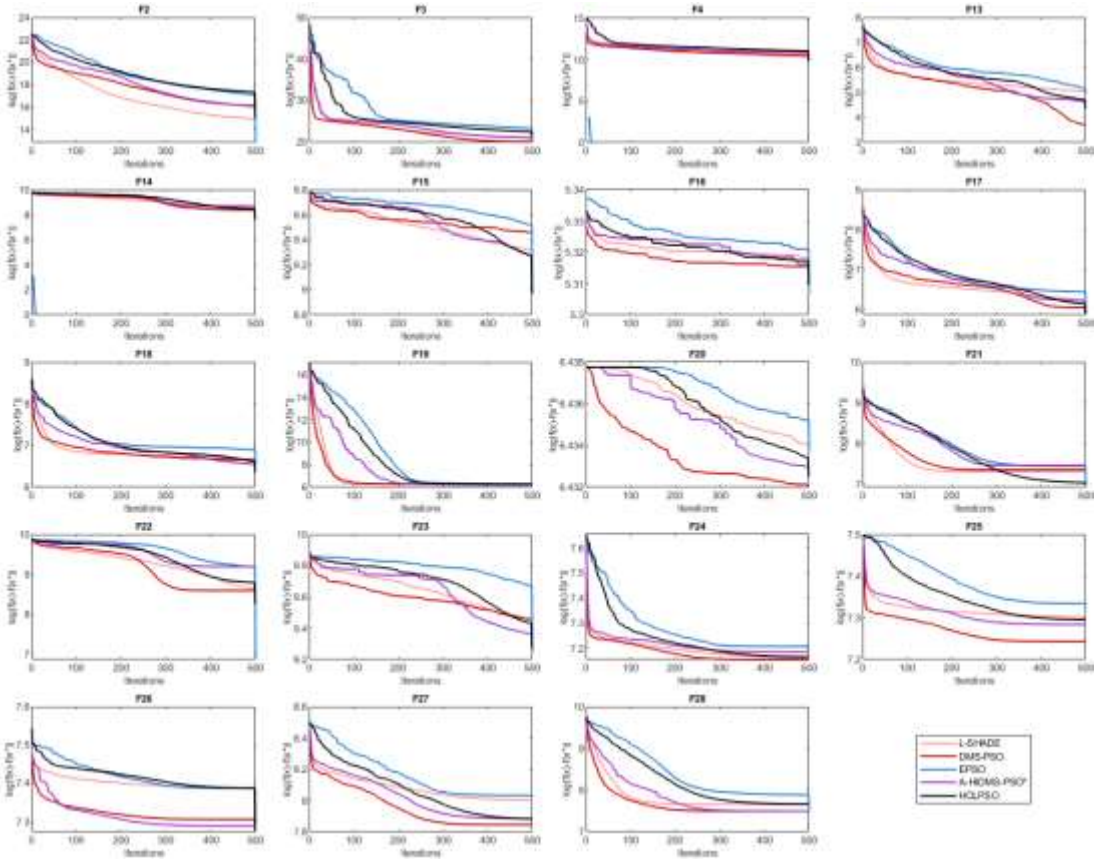
Convergence Rates of HIDMS-PSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



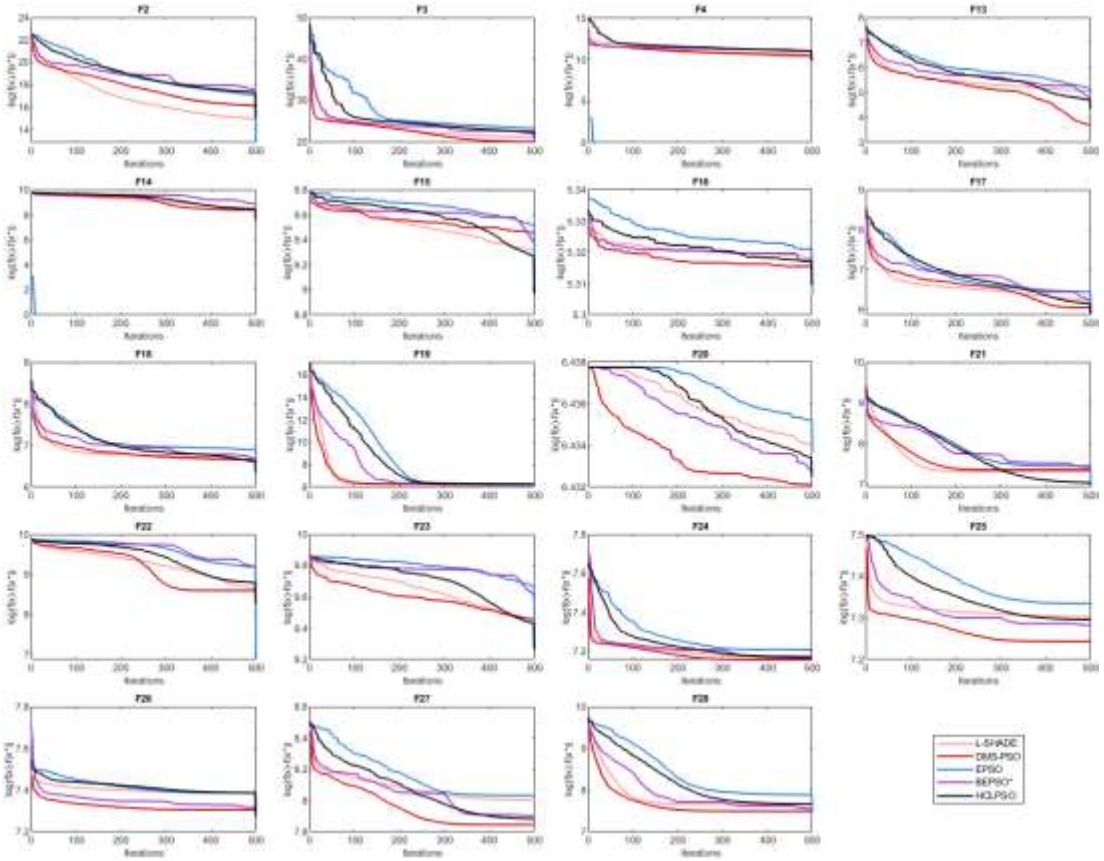
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



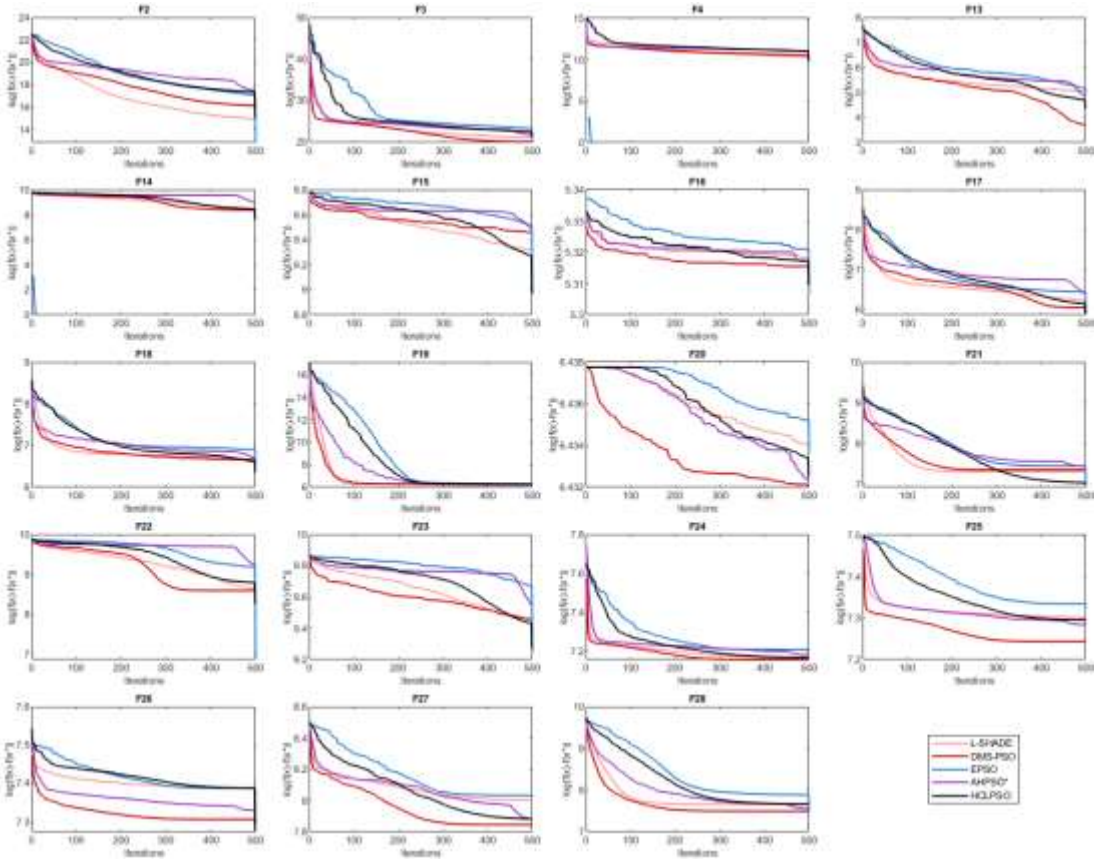
Convergence Rates of GA-HIDMS-PSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



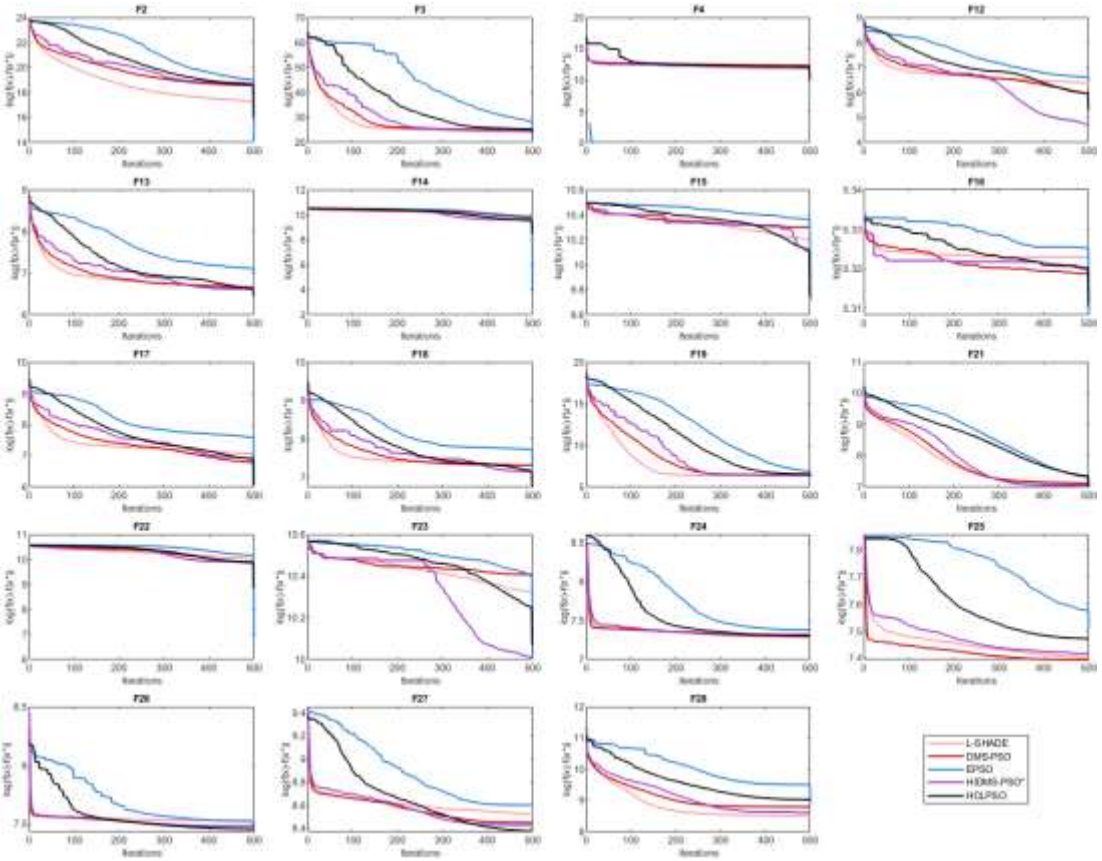
Convergence Rates of A-HIDMS-PSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



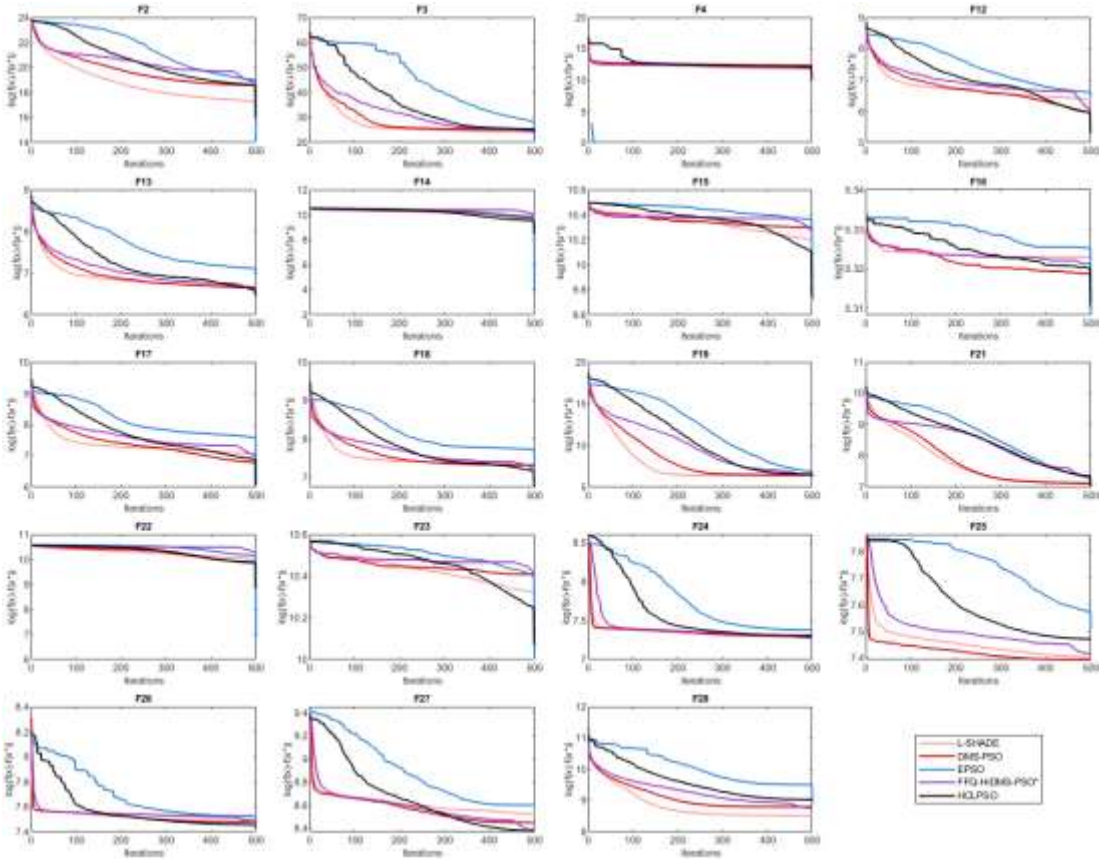
Convergence Rates of BEPSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



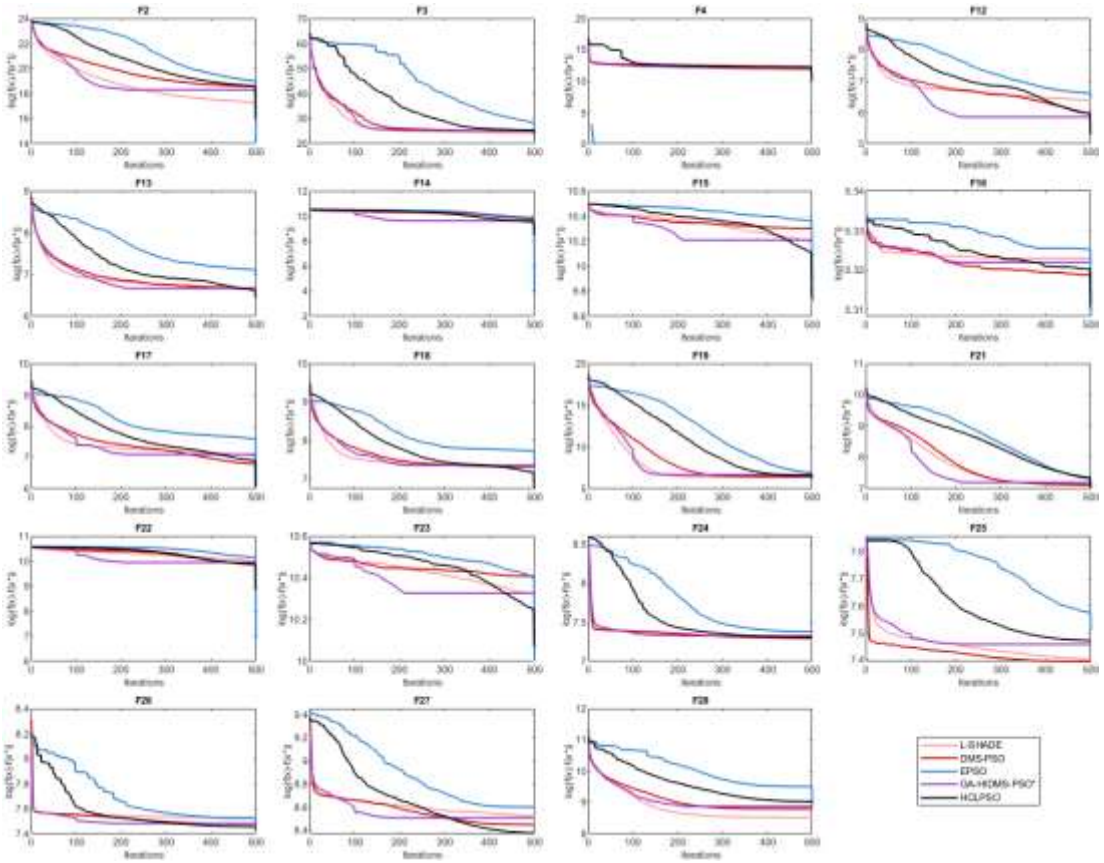
Convergence Rates of AHPSO Algorithm for 50-dimensional CEC'13 Benchmark Functions



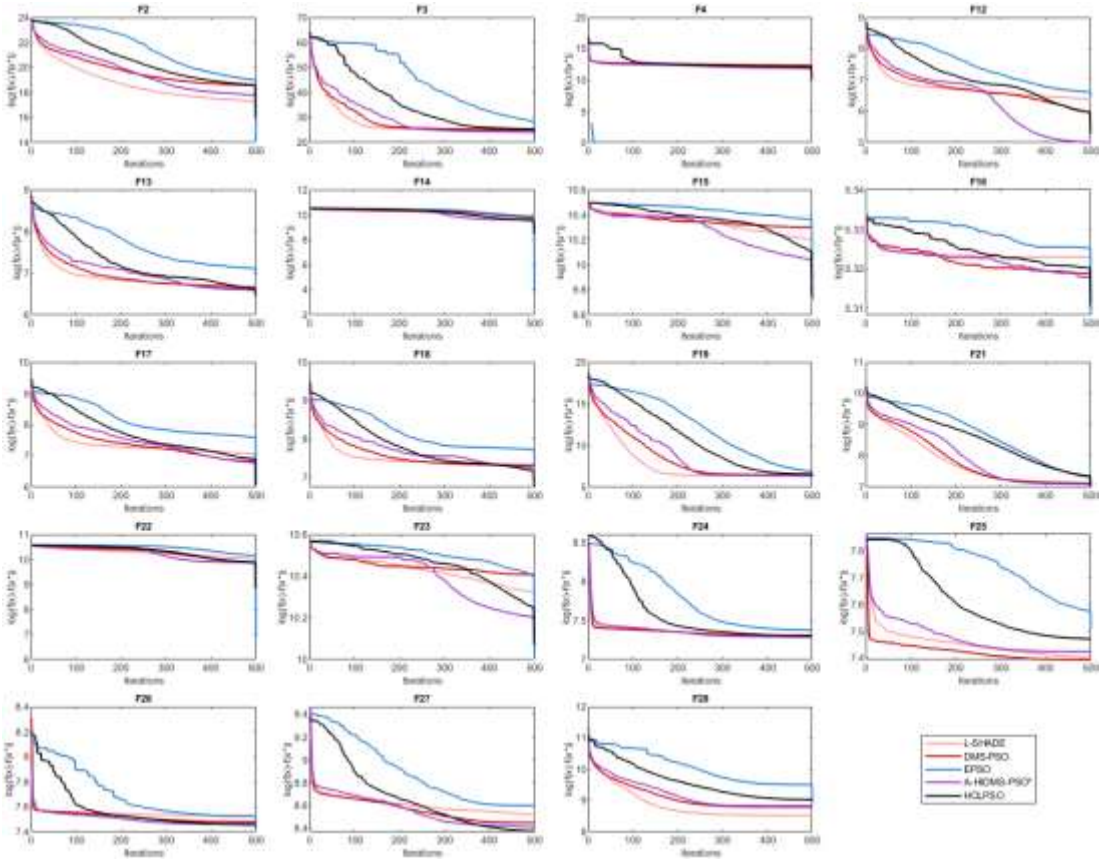
Convergence Rates of HIDMS-PSO Algorithm for 100-dimensional CEC'13 Benchmark Functions



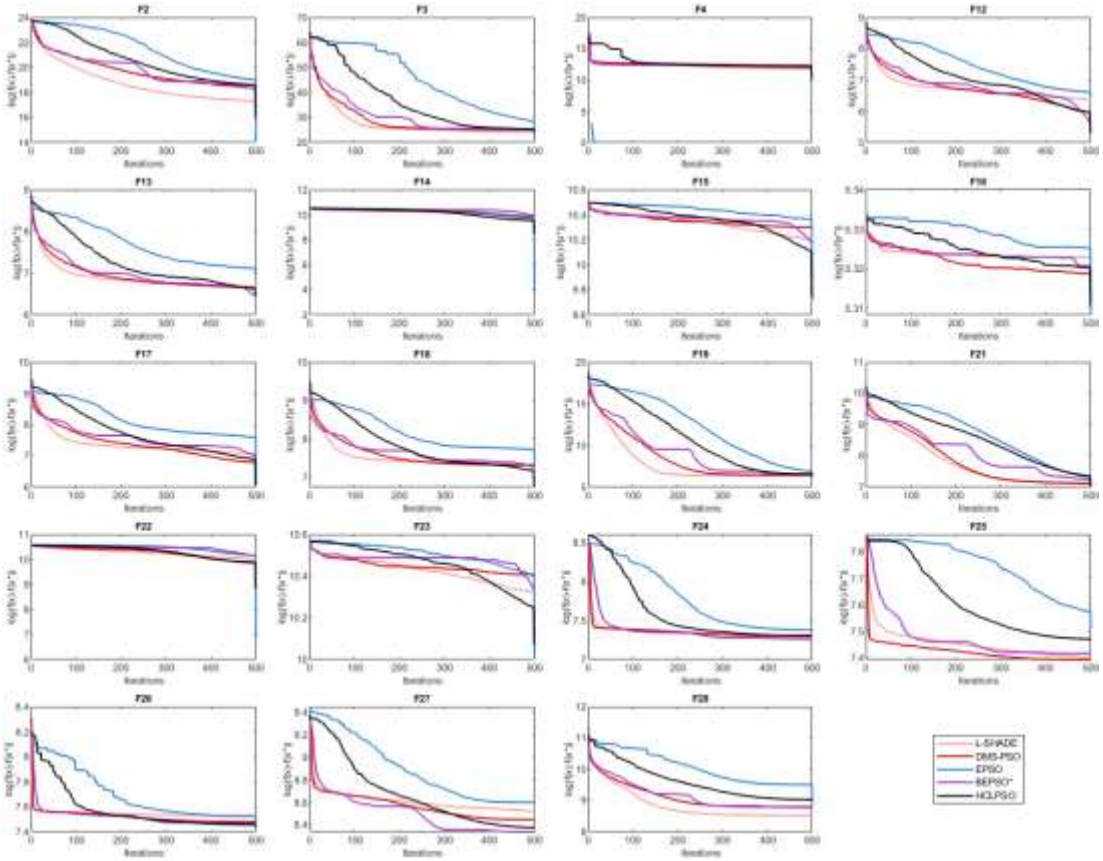
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 100-dimensional CEC'13 Benchmark Functions



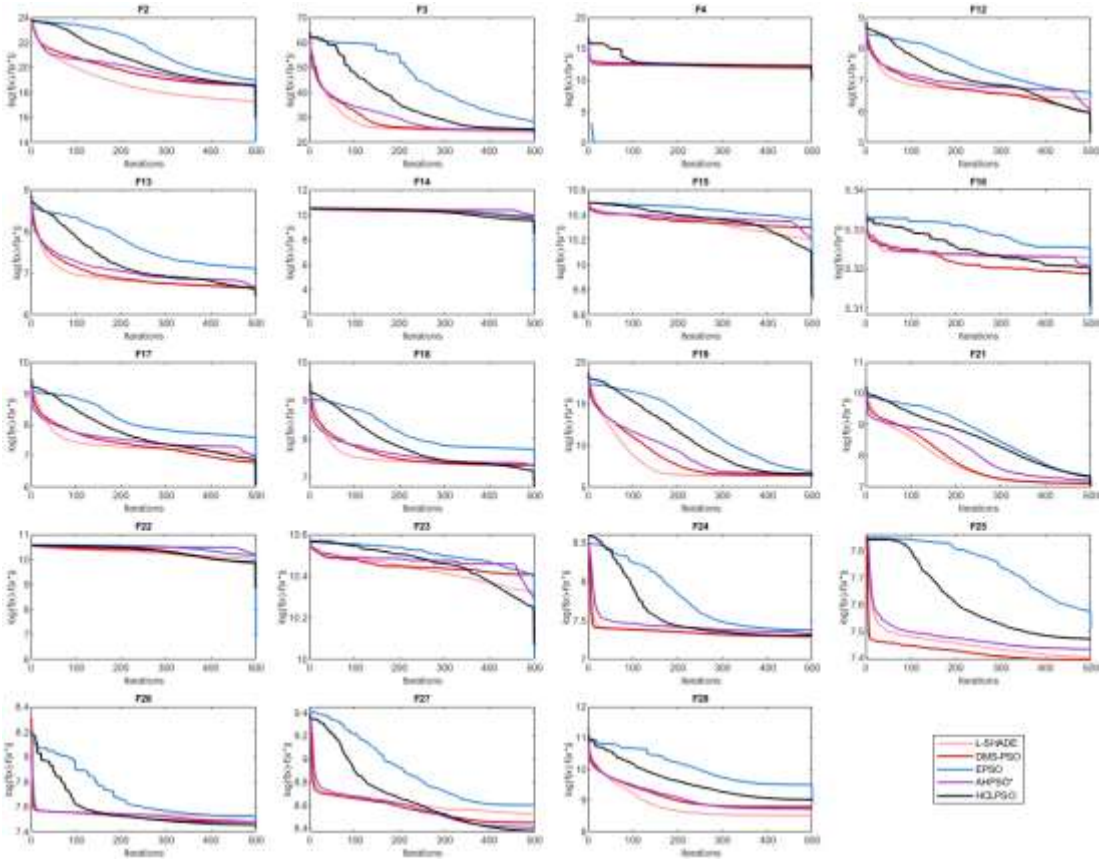
Convergence Rates of GA-HIDMS-PSO Algorithm for 100-dimensional CEC'13 Benchmark Functions



Convergence Rates of A-HIDMS-PSO Algorithm for 100-dimensional CEC'13 Benchmark Functions

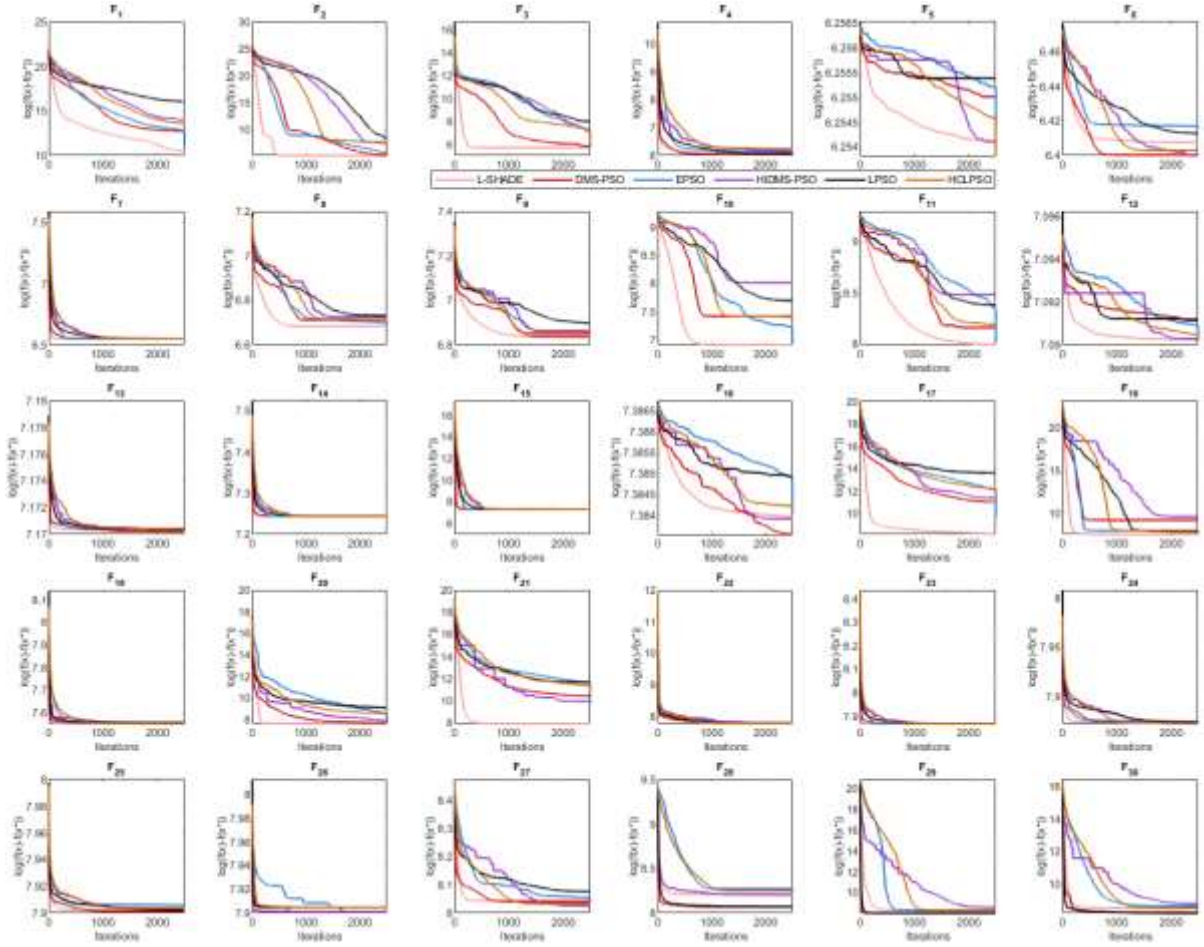


Convergence Rates of BEPSO Algorithm for 100-dimensional CEC'13 Benchmark Functions

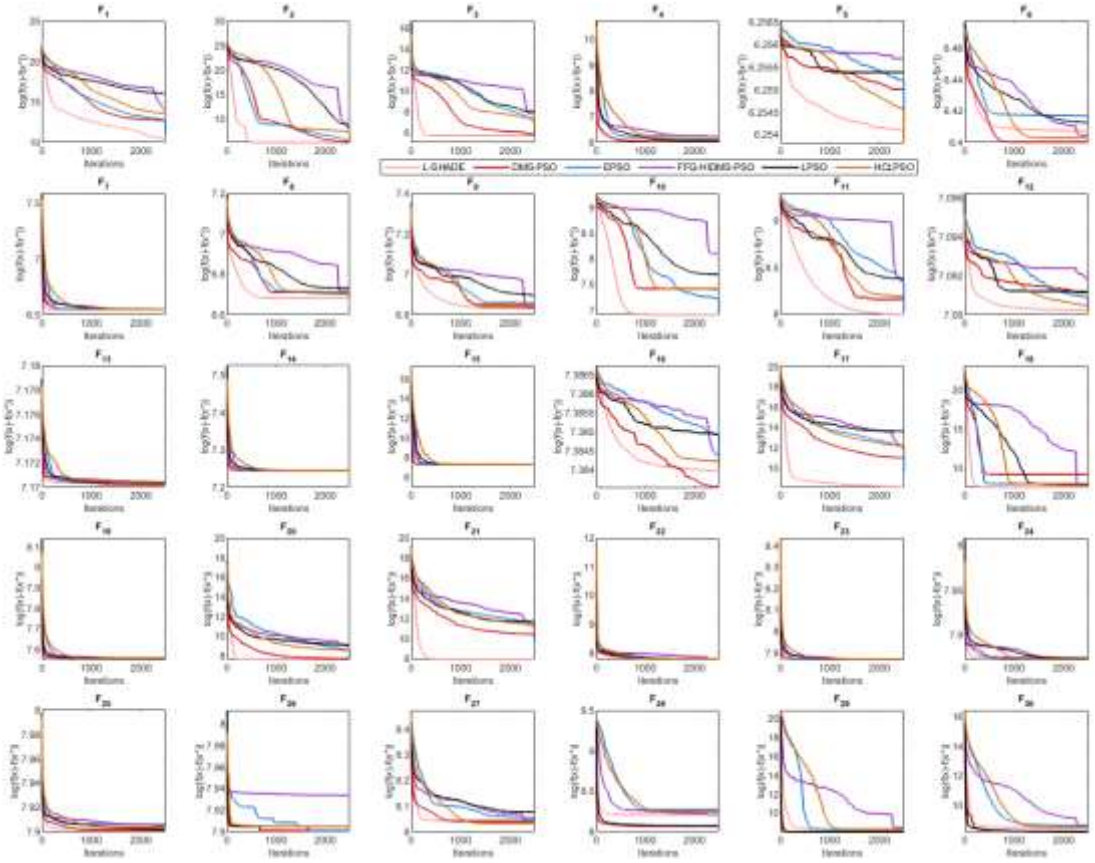


Convergence Rates of AHPSO Algorithm for 100-dimensional CEC'13 Benchmark Functions

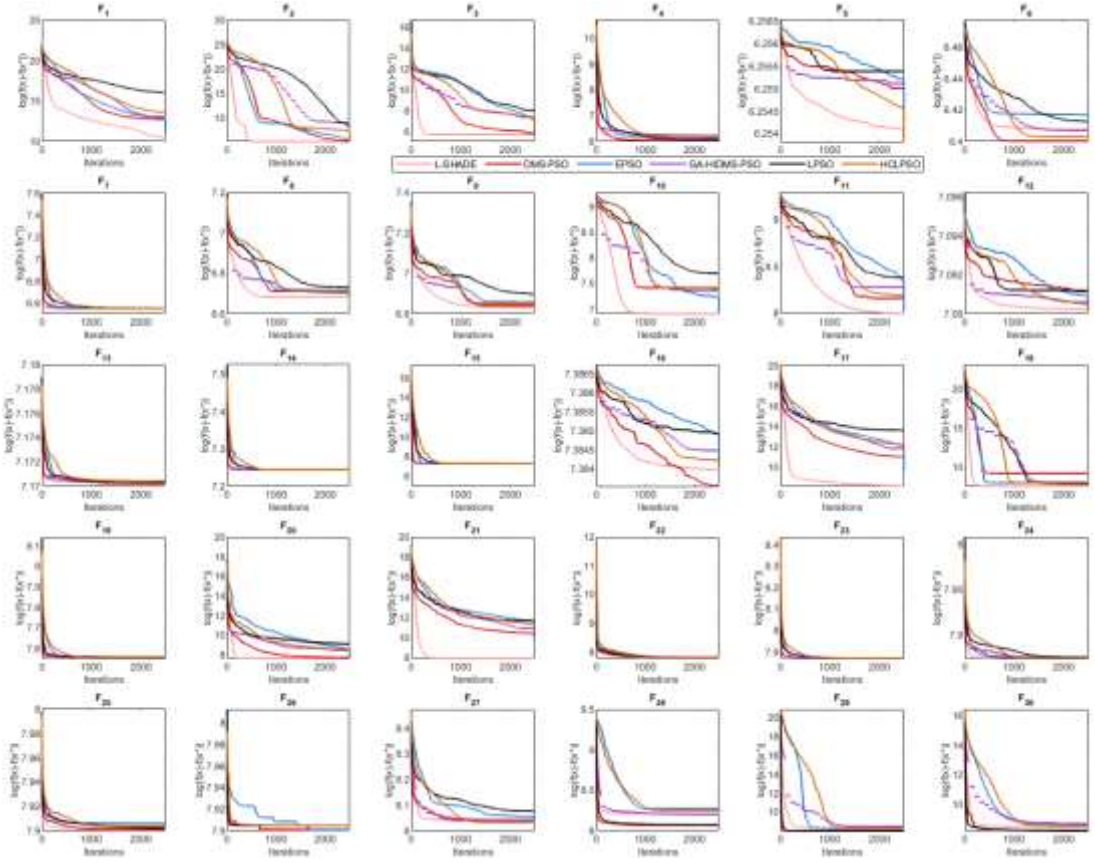
Appendix 4: Convergence Characteristic Graphs for CEC'14 Benchmark Functions



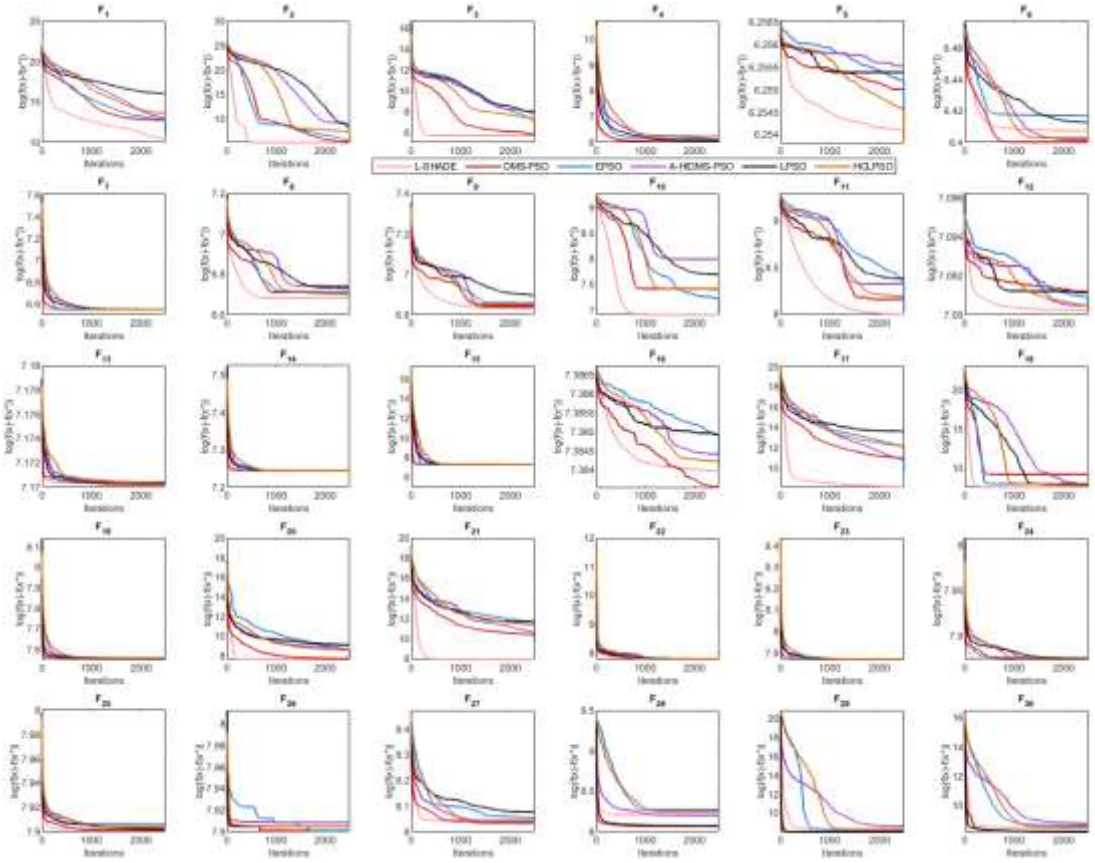
Convergence Rates of HIDMS-PSO Algorithm for 30-dimensional CEC'14 Benchmark Functions



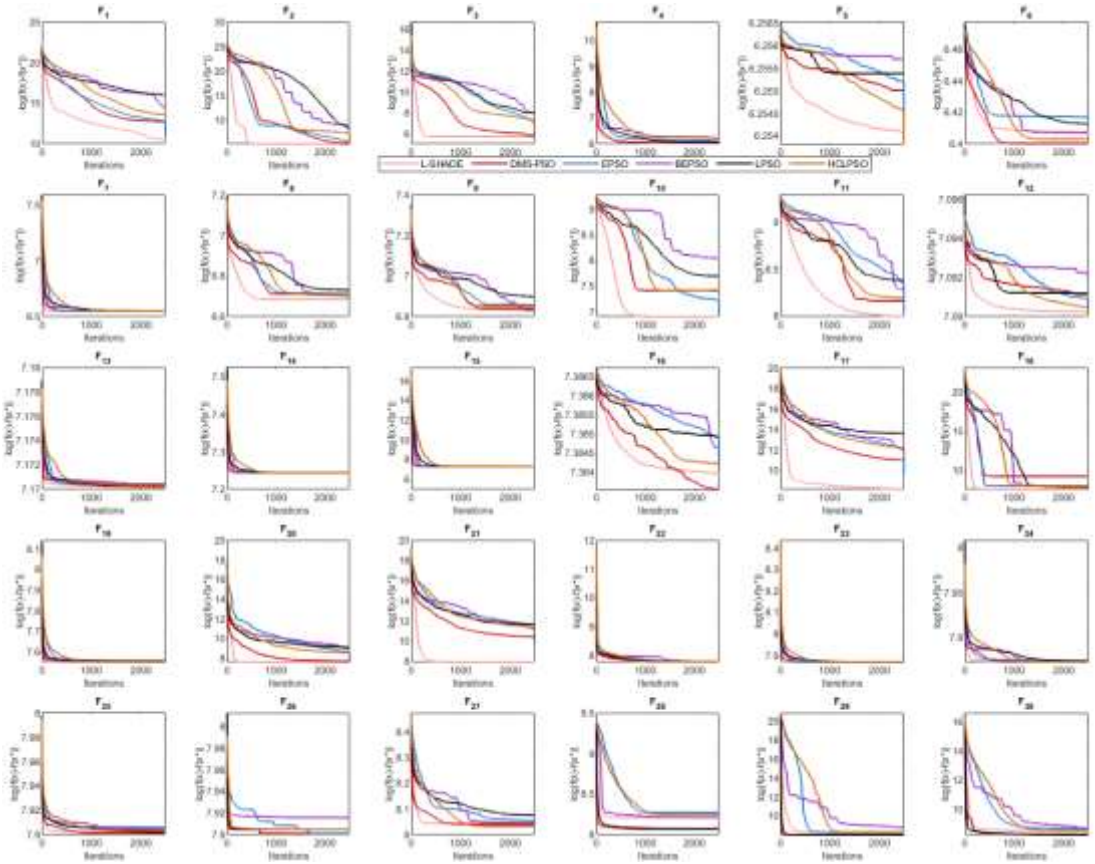
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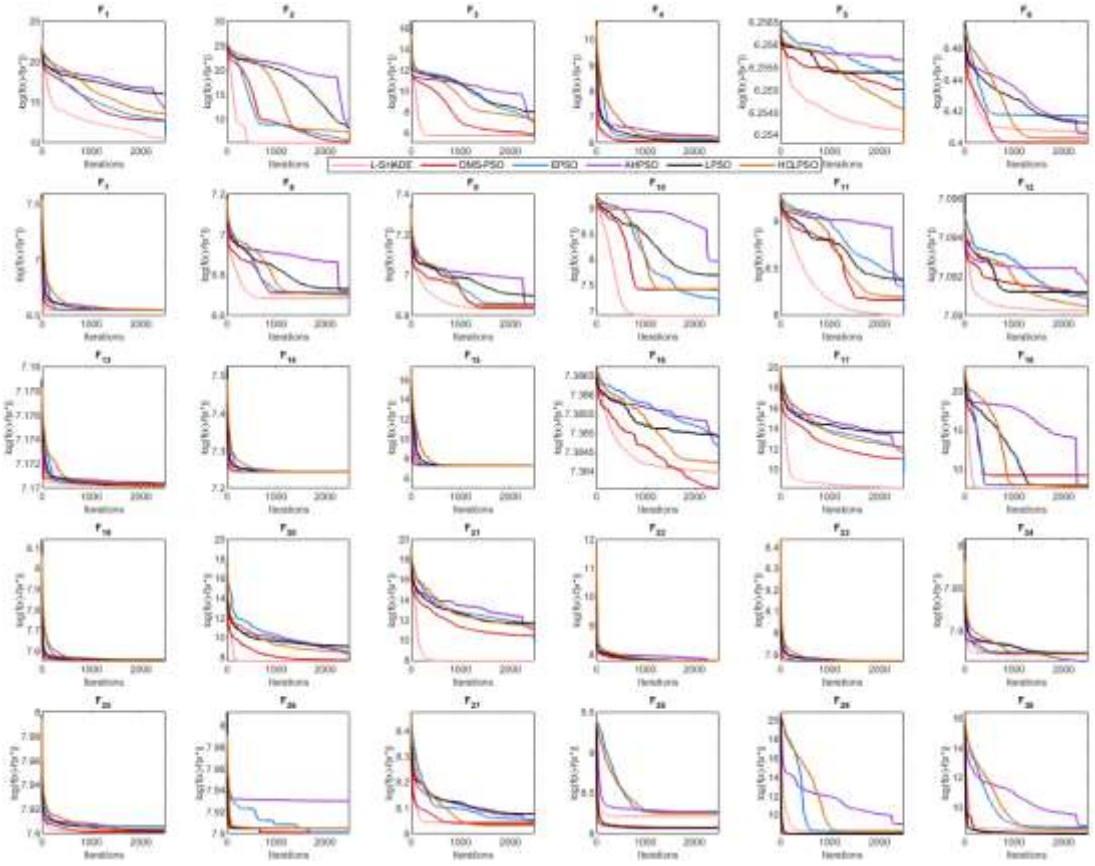
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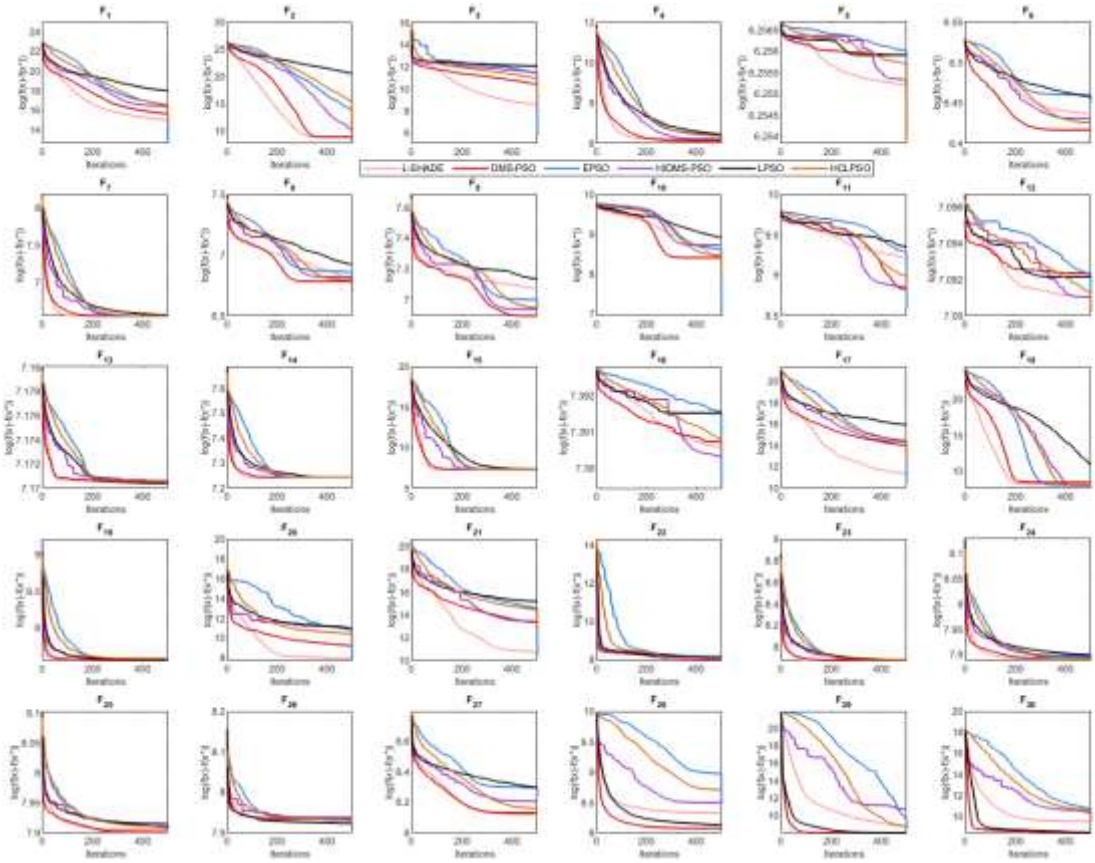
Convergence Rates of A-HIDMS-PSO Algorithm for 30-dimensional CEC'14 Benchmark Functions



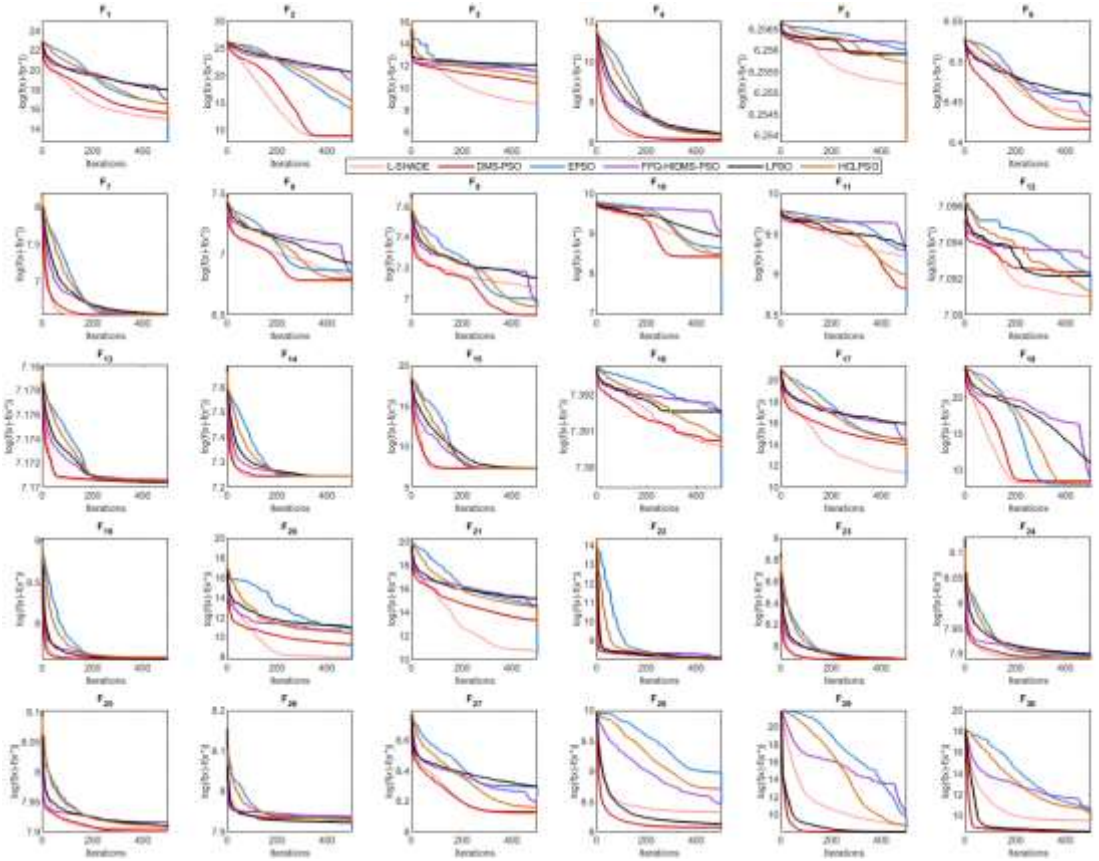
Convergence Rates of BEPSO Algorithm for 30-dimensional CEC'14 Benchmark Functions



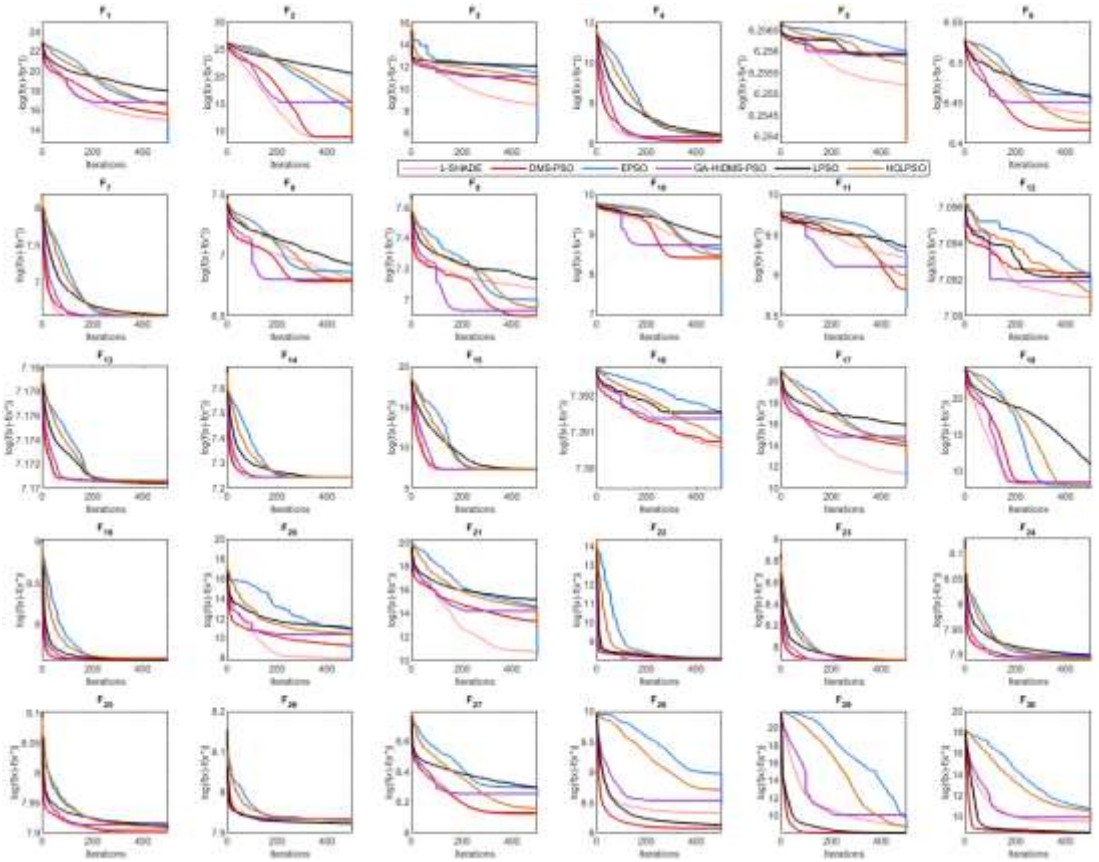
Convergence Rates of AHPSO Algorithm for 30-dimensional CEC'14 Benchmark Functions



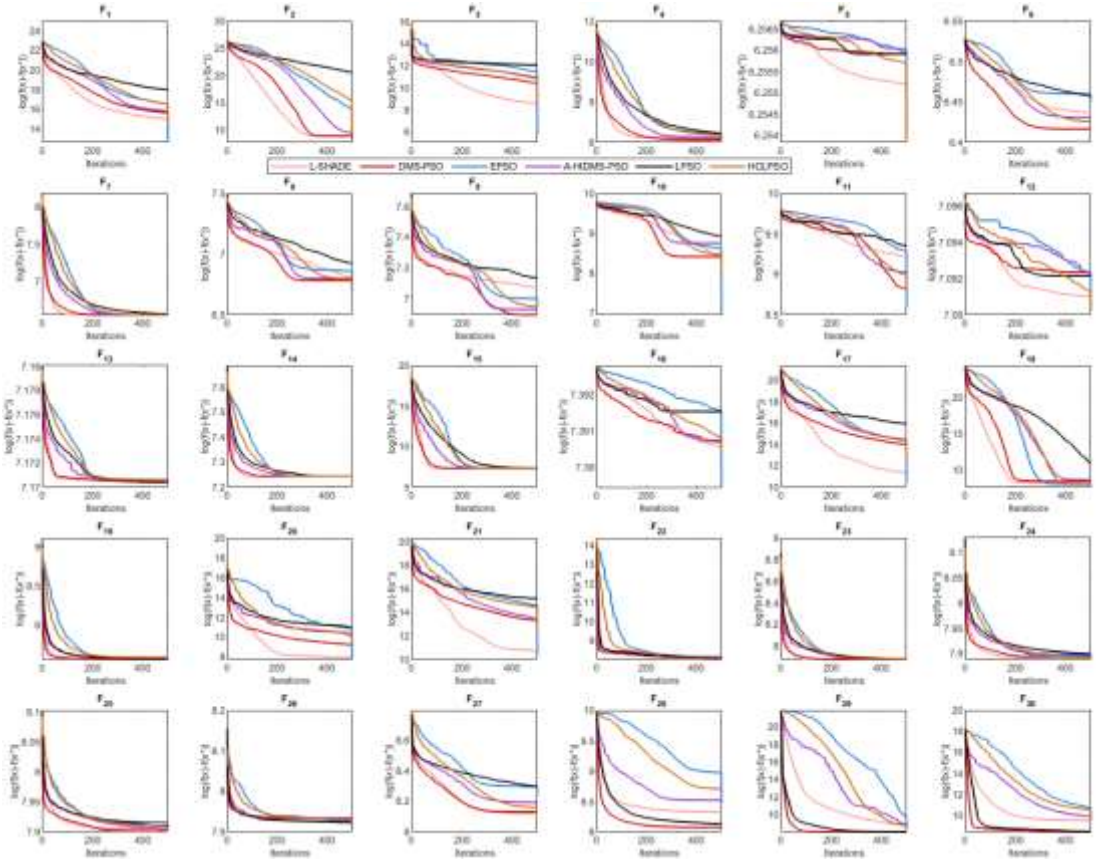
Convergence Rates of HIDMS-PSO Algorithm for 50-dimensional CEC'14 Benchmark Functions



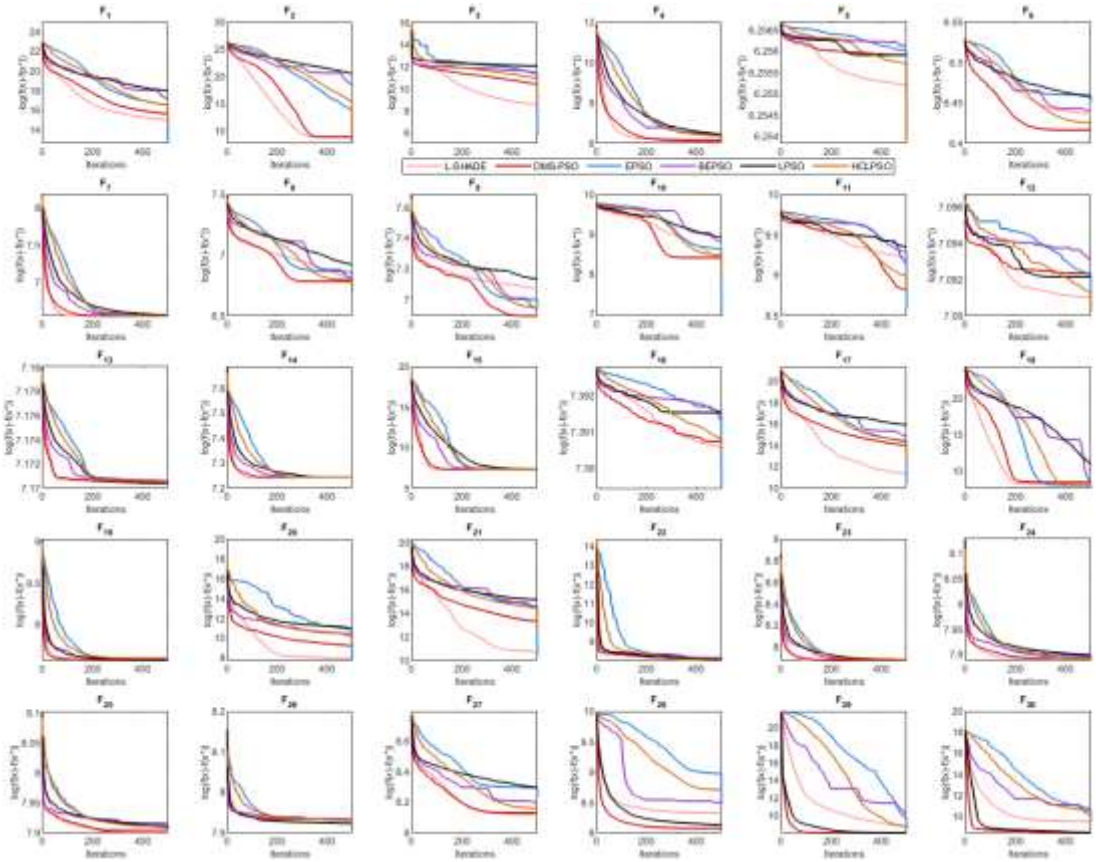
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 50-dimensional CEC'14 Benchmark Functions



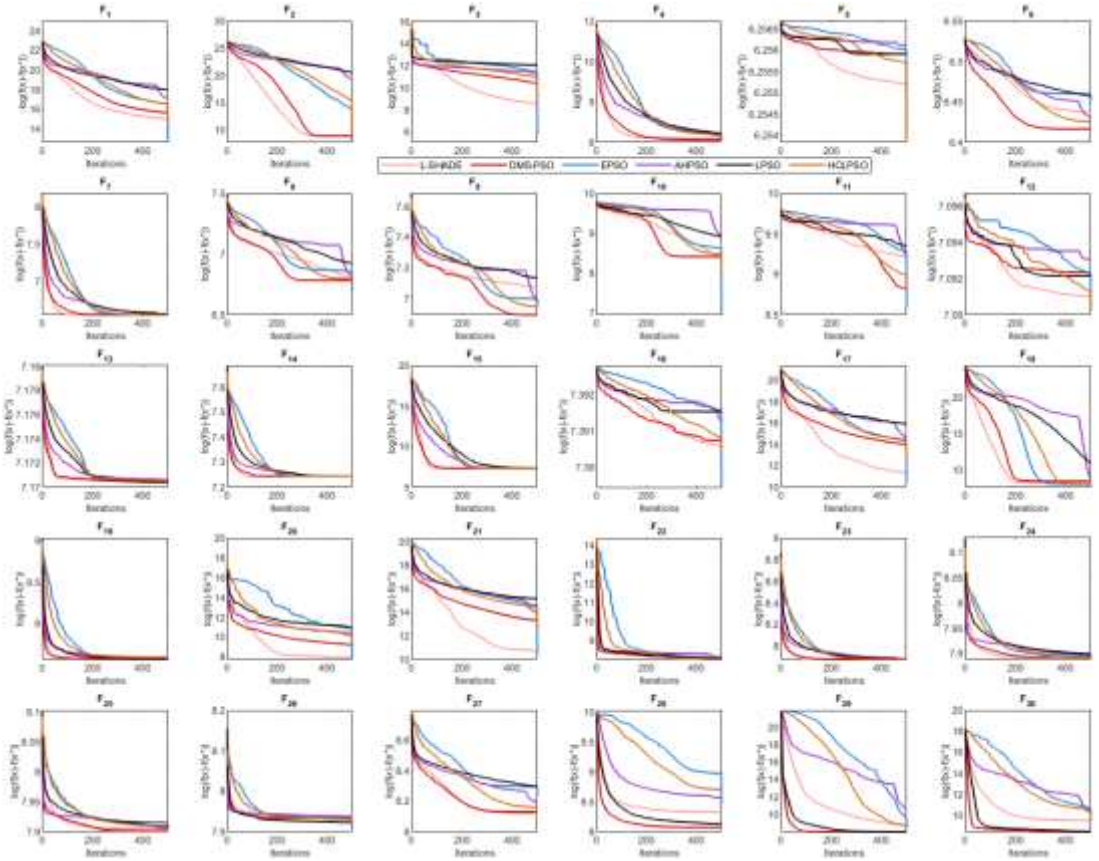
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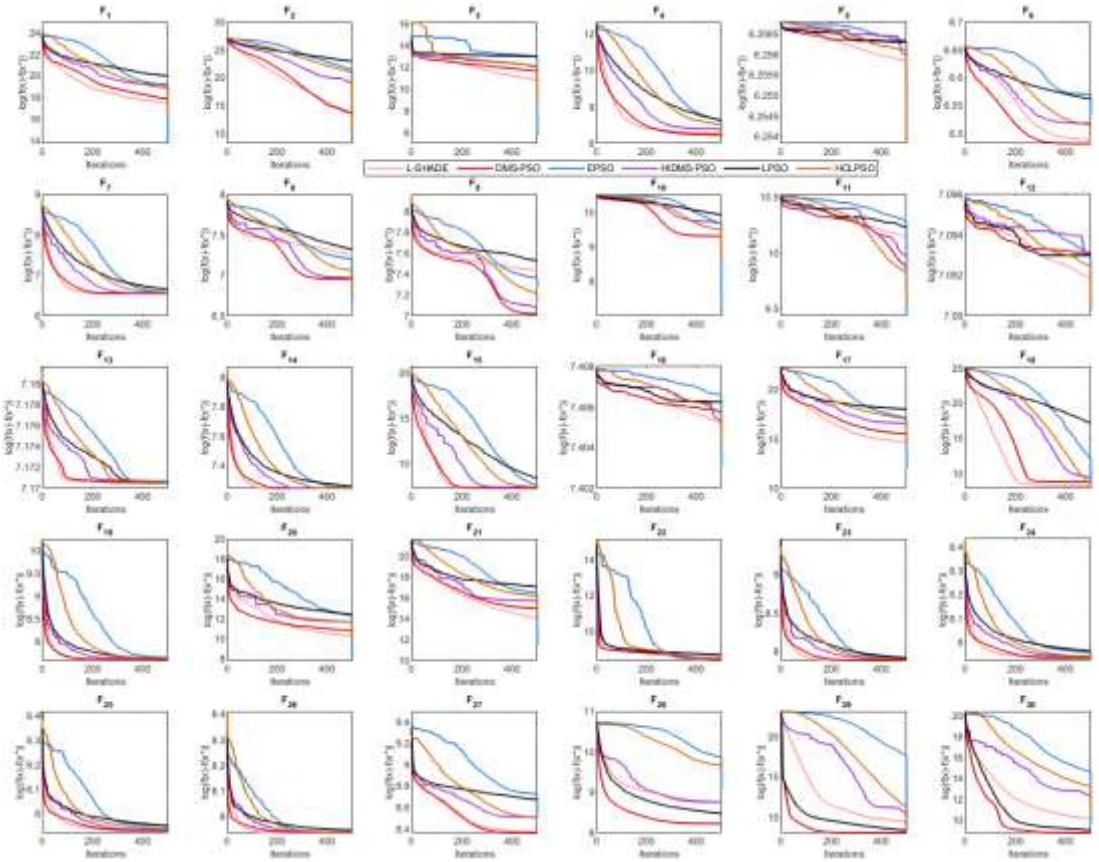
Convergence Rates of A-HIDMS-PSO Algorithm for 50-dimensional CEC'14 Benchmark Functions



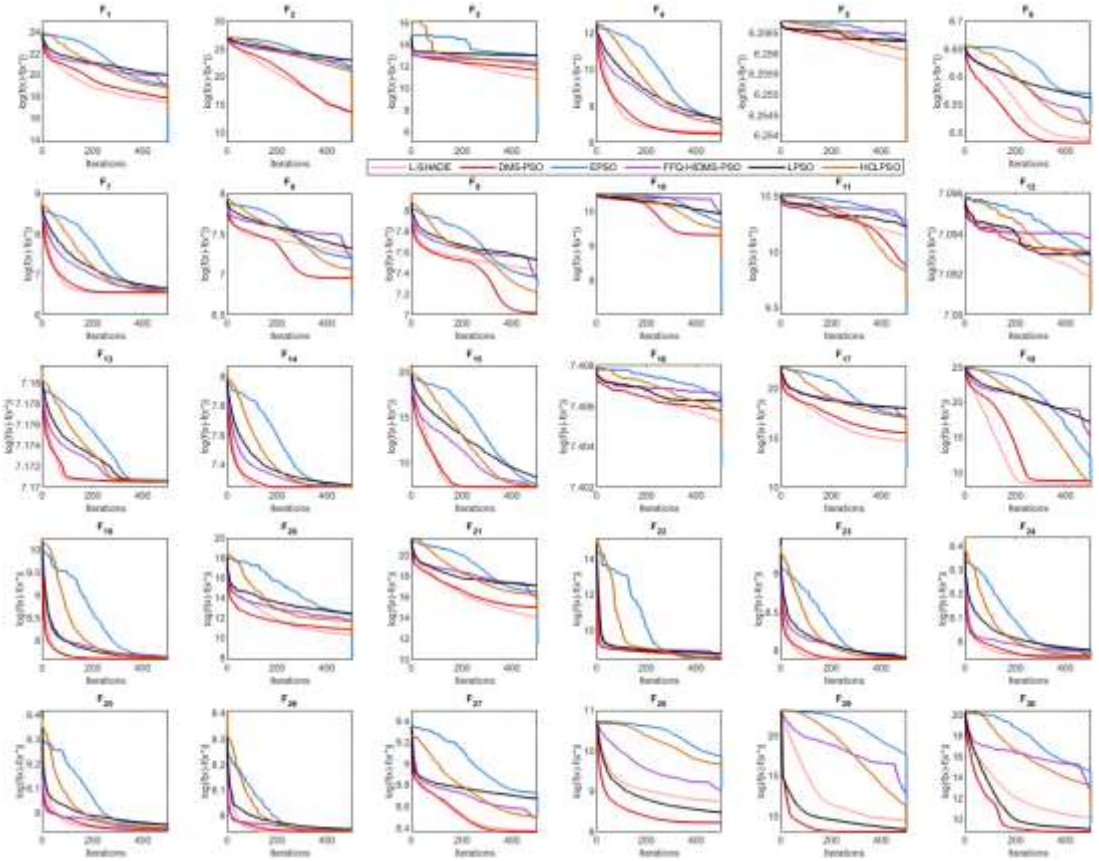
Convergence Rates of BEPSO Algorithm for 50-dimensional CEC'14 Benchmark Functions



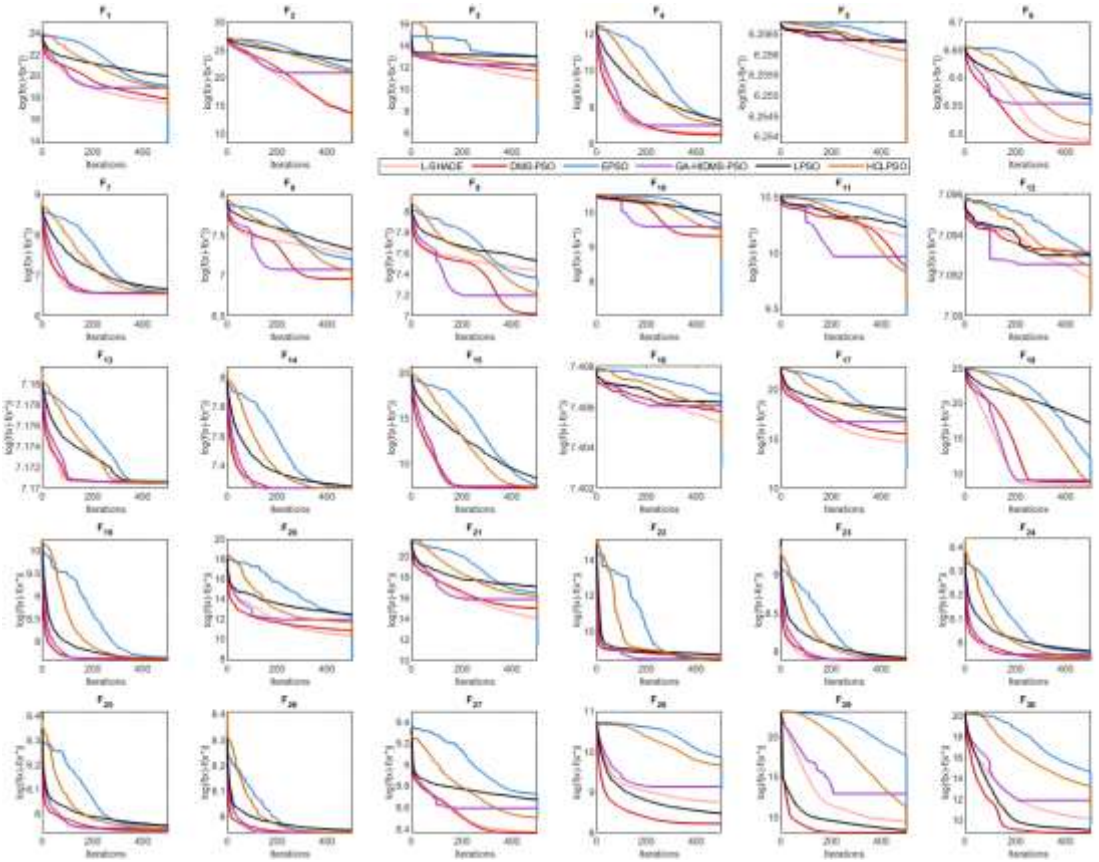
Convergence Rates of AHPSO Algorithm for 50-dimensional CEC'14 Benchmark Functions



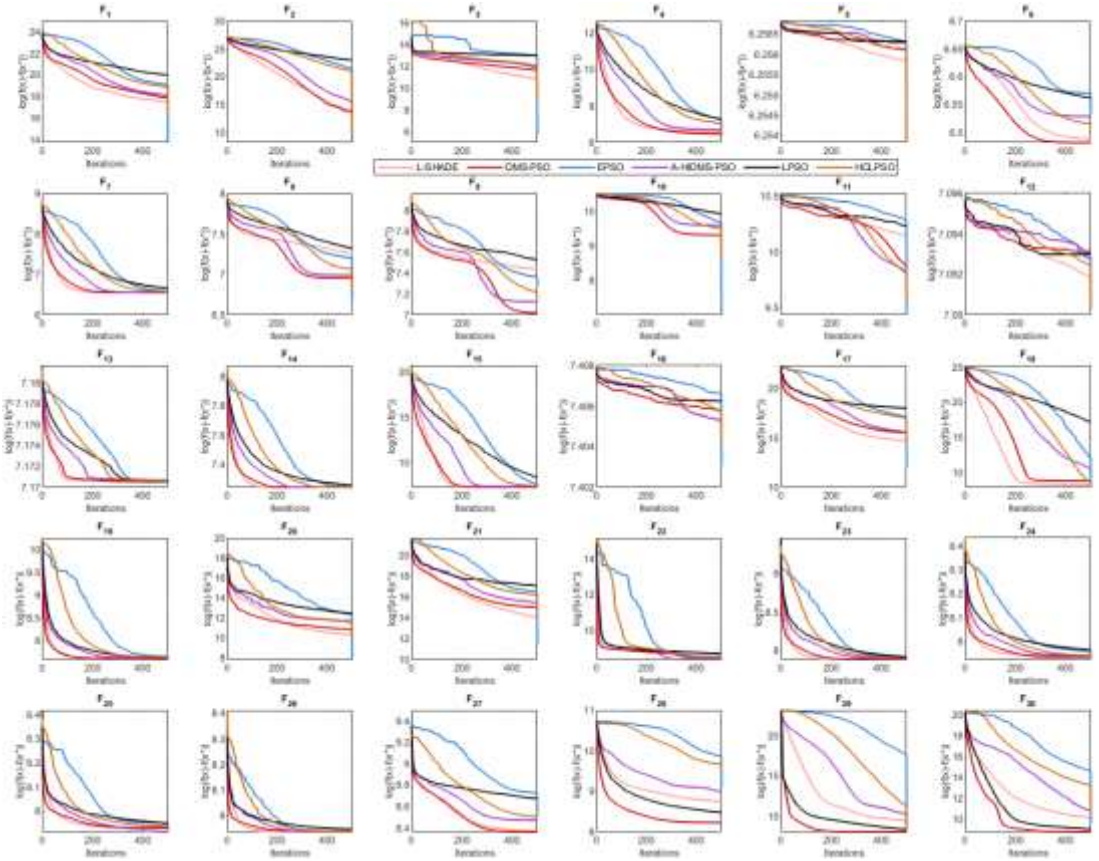
Convergence Rates of HIDMS-PSO Algorithm for 100-dimensional CEC'14 Benchmark Functions



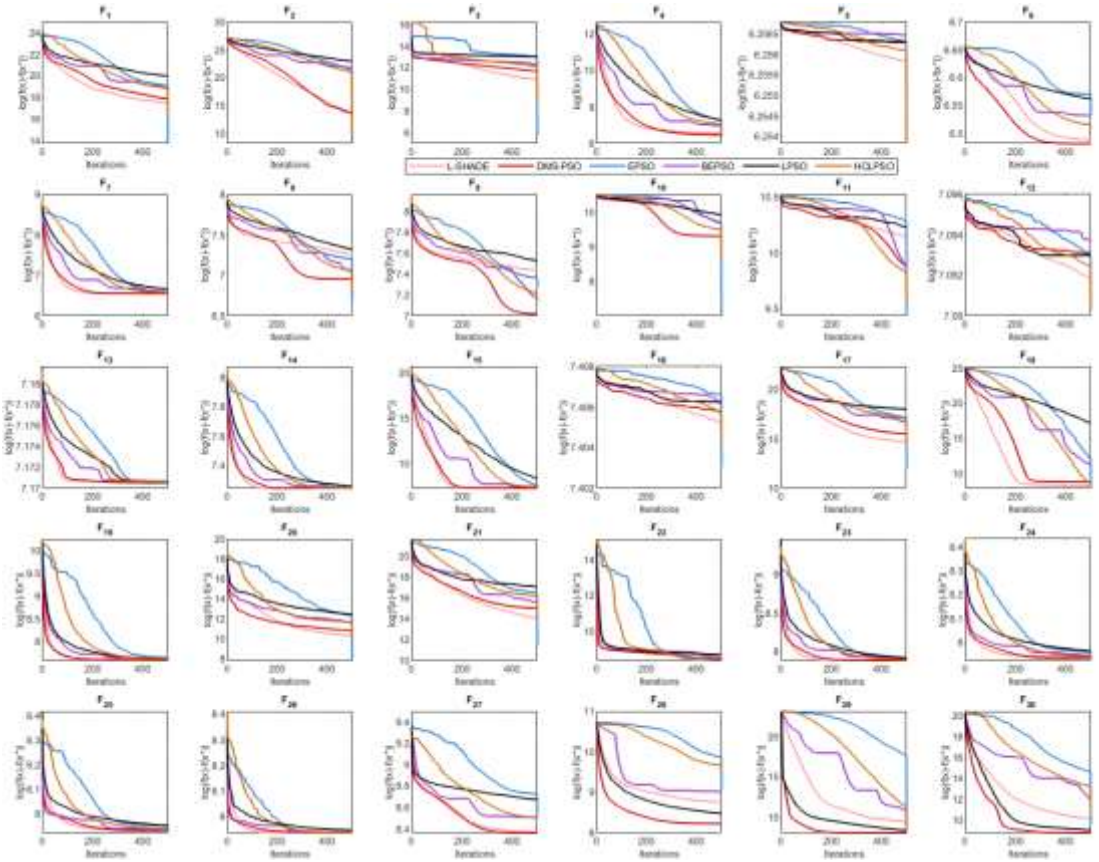
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 100-dimensional CEC'14 Benchmark Functions



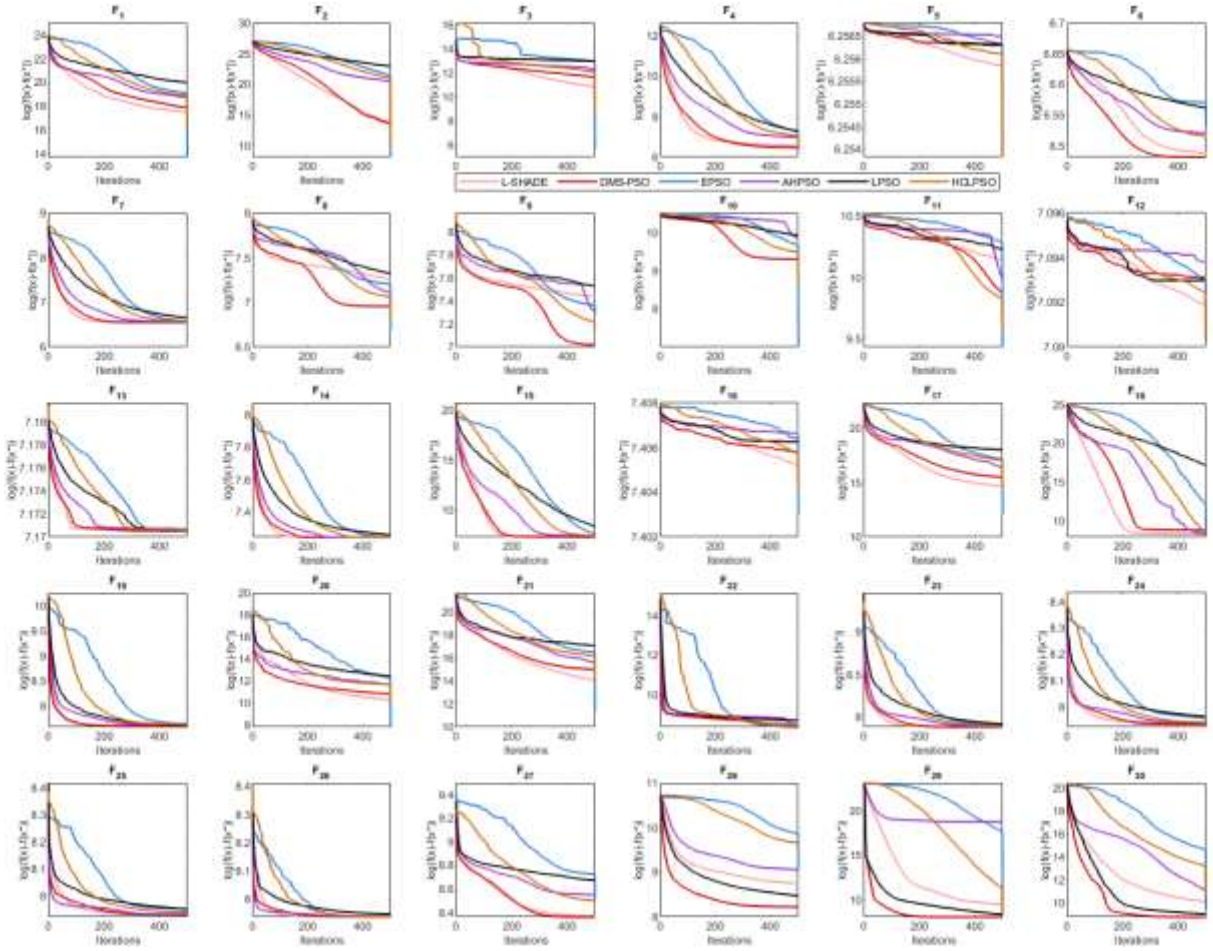
Convergence Rates of GA-HIDMS-PSO Algorithm for 100-dimensional CEC'14 Benchmark Functions



Convergence Rates of A-HIDMS-PSO Algorithm for 100-dimensional CEC'14 Benchmark Functions

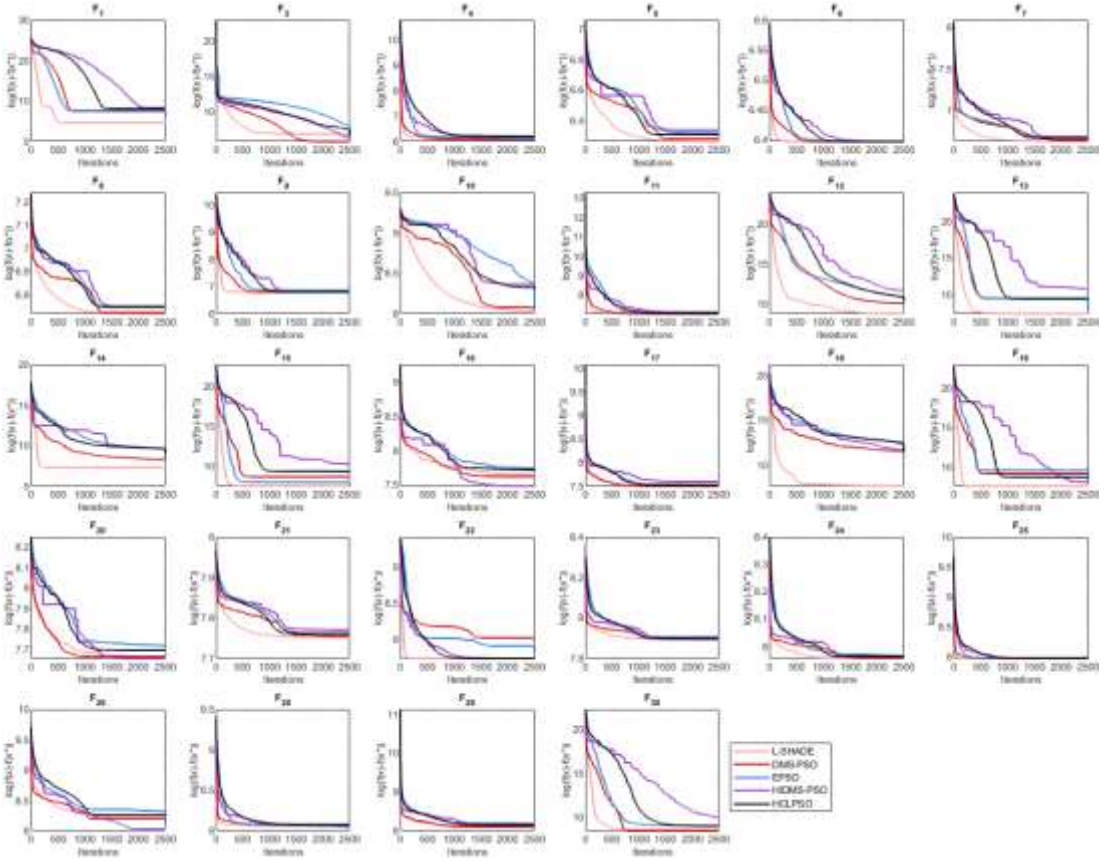


Convergence Rates of BEPSO Algorithm for 100-dimensional CEC'14 Benchmark Functions

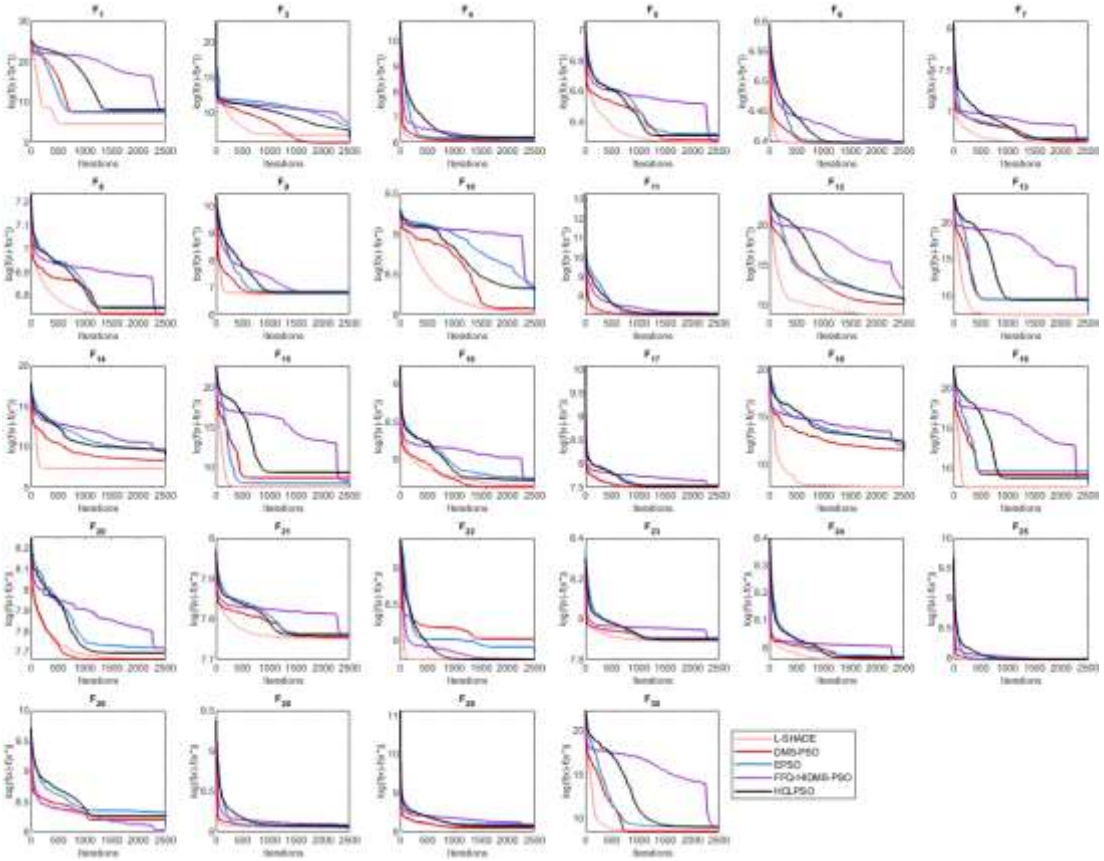


Convergence Rates of AHPSO Algorithm for 100-dimensional CEC'14 Benchmark Functions

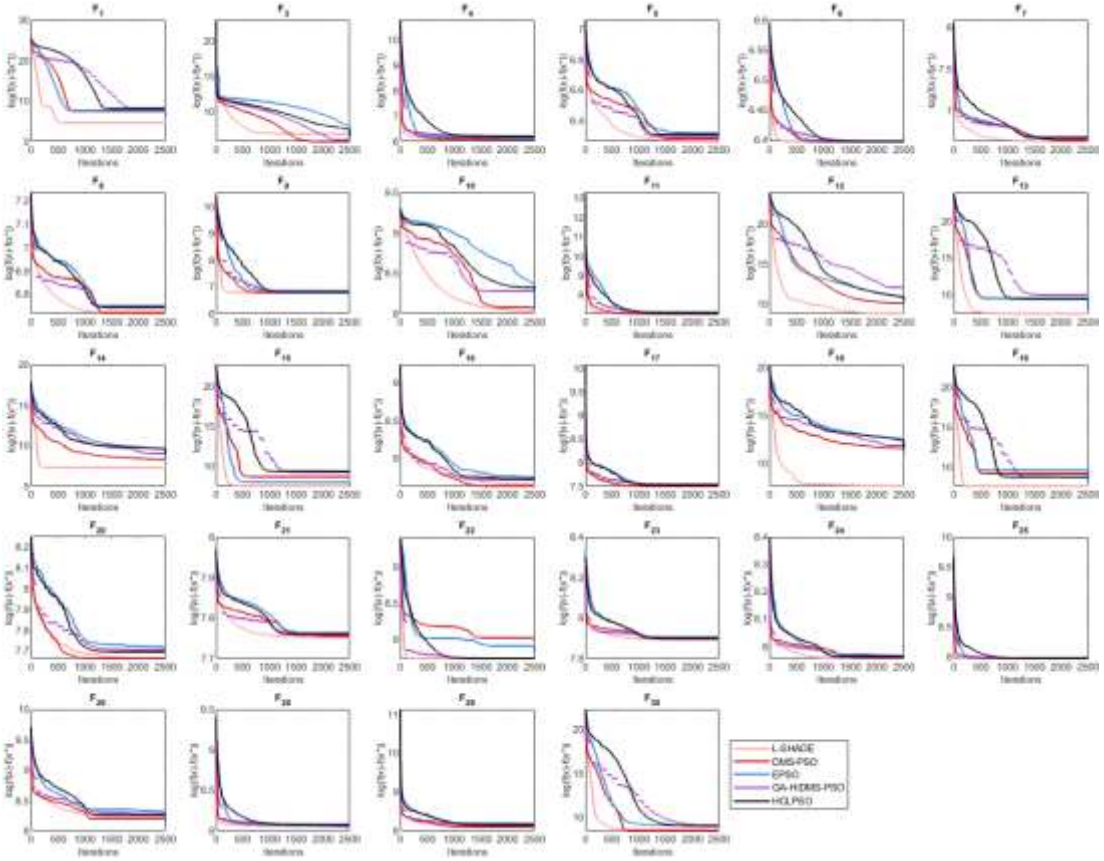
Appendix 5: Convergence Characteristic Graphs for CEC'17 Benchmark Functions



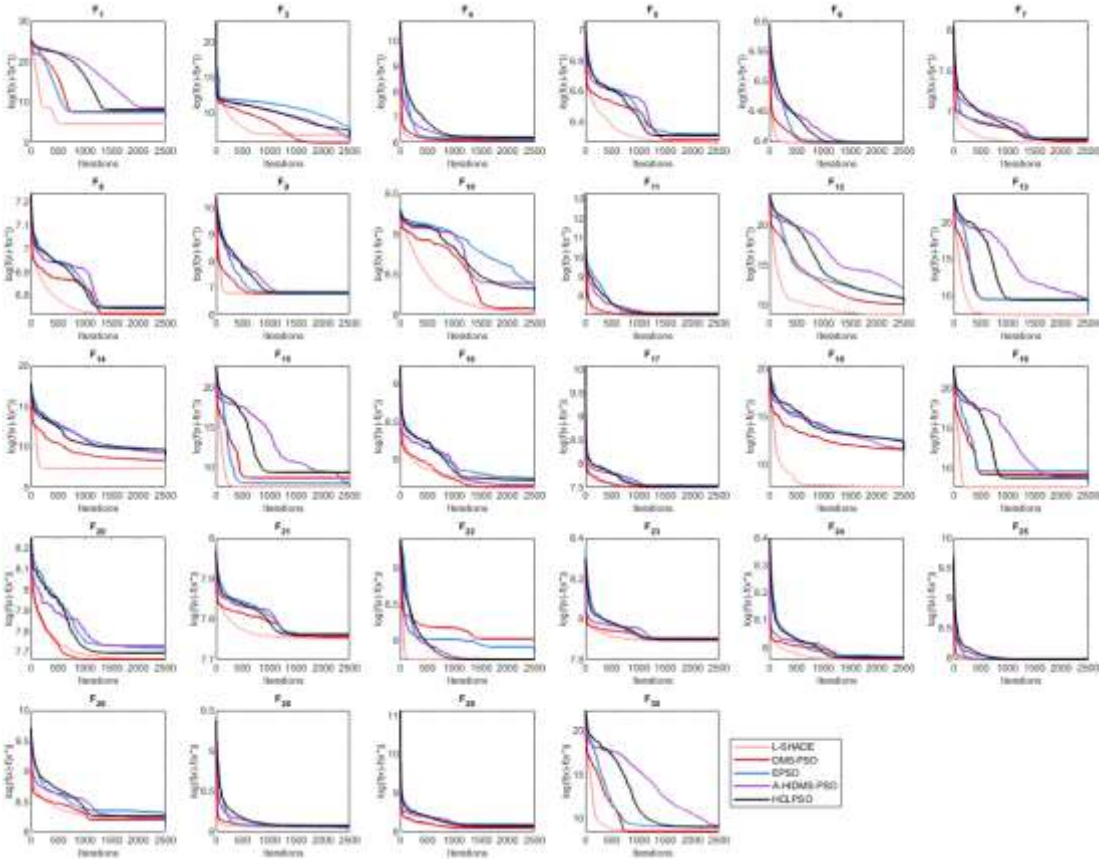
Convergence Rates of HIDMS-PSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



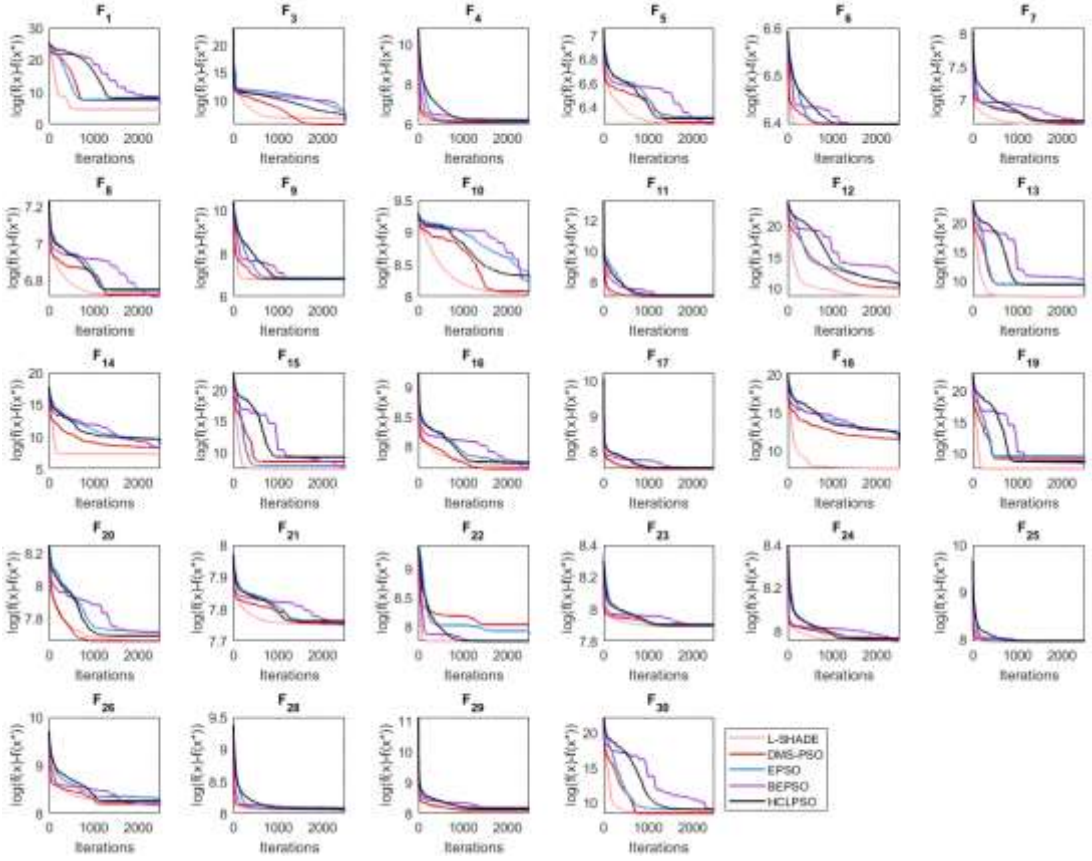
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



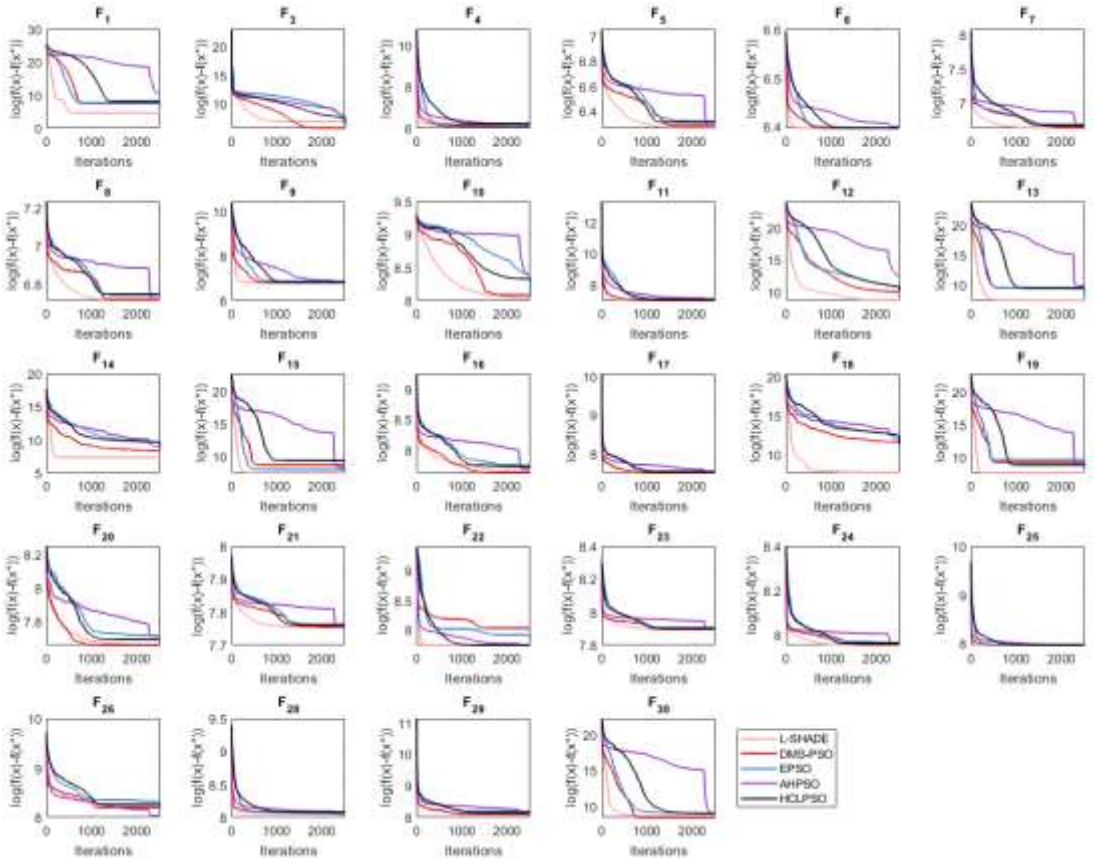
Convergence Rates of GA-HIDMS-PSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



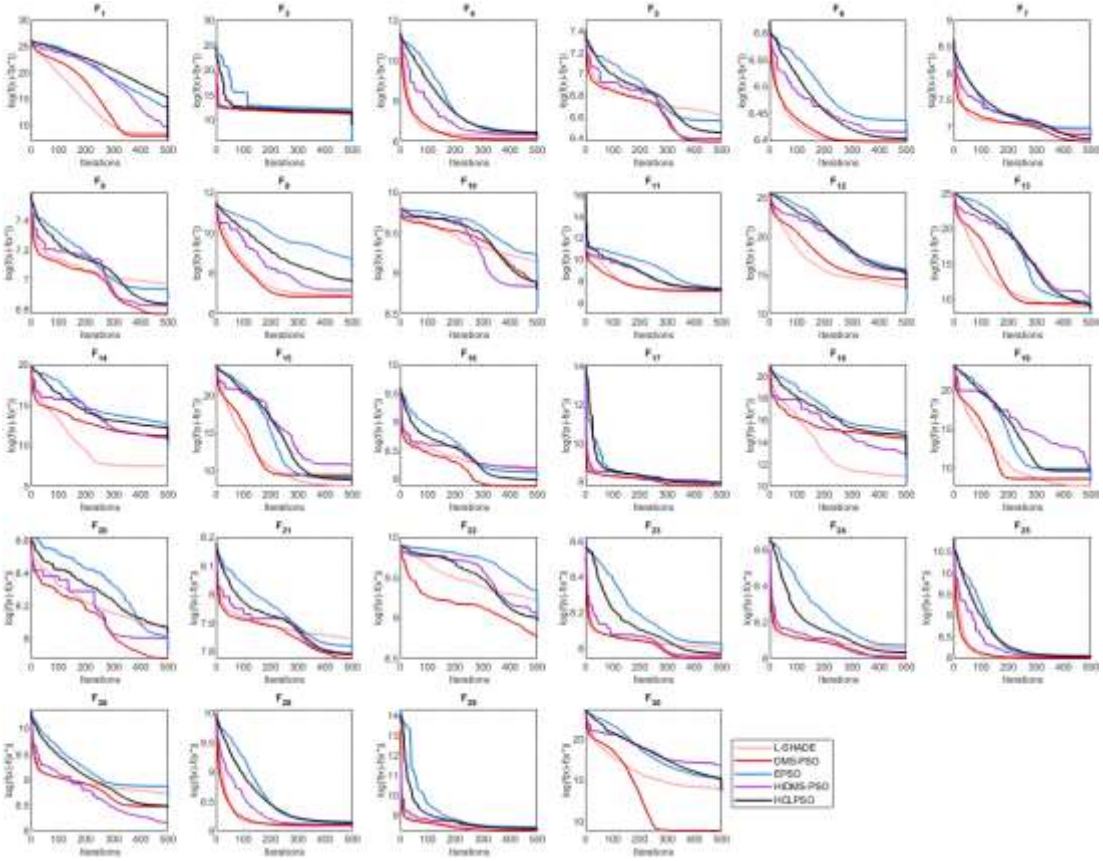
Convergence Rates of A-HIDMS-PSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



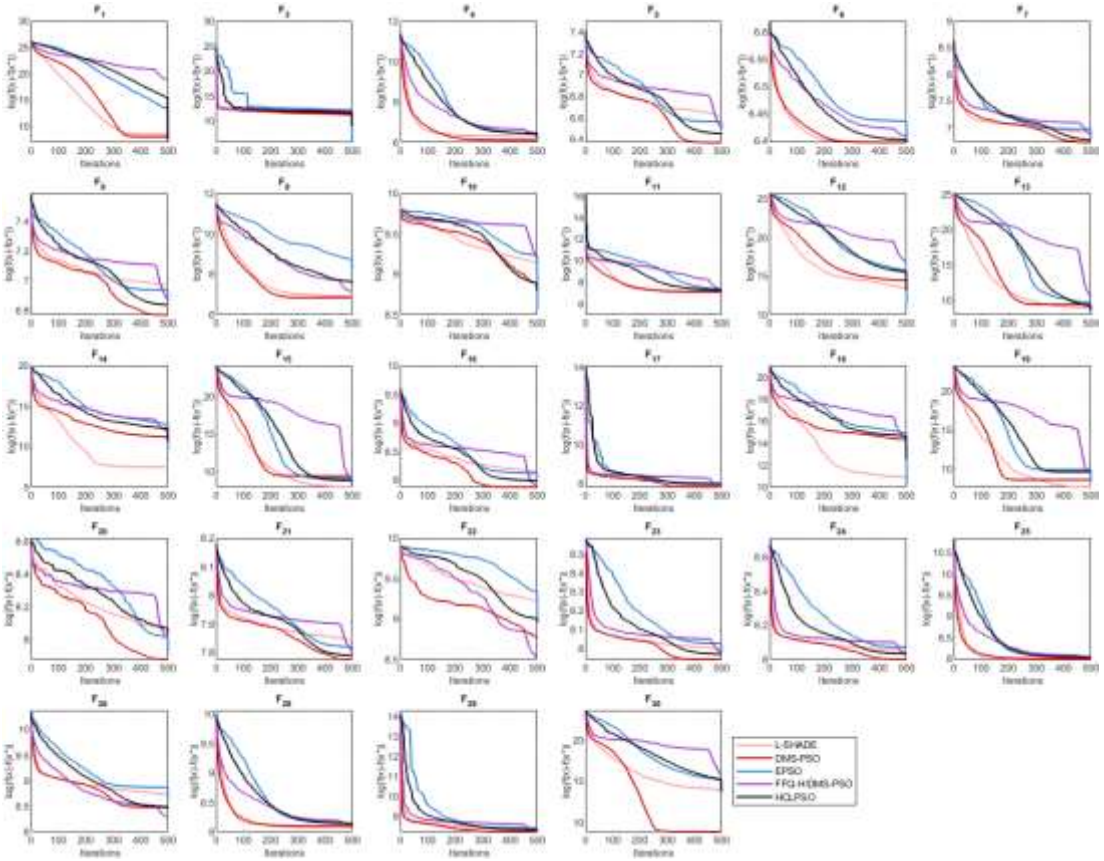
Convergence Rates of BEPSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



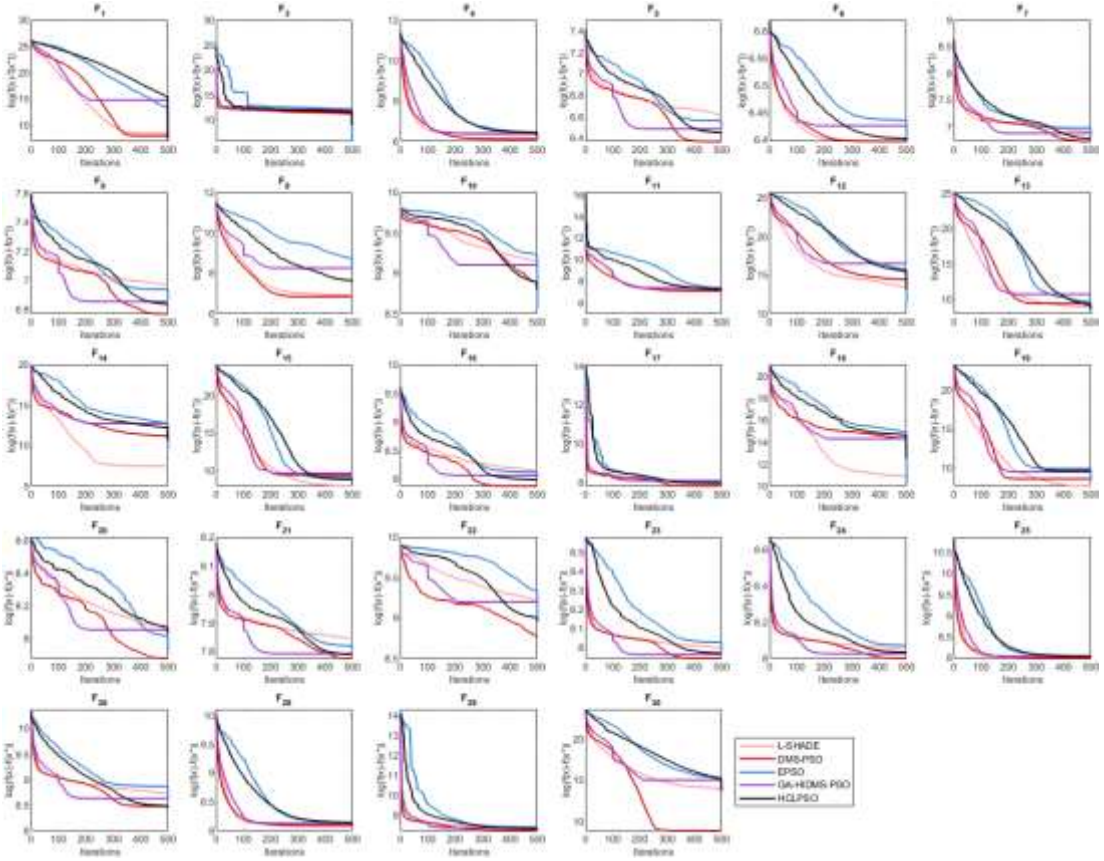
Convergence Rates of AHPSO Algorithm for 30-dimensional CEC'17 Benchmark Functions



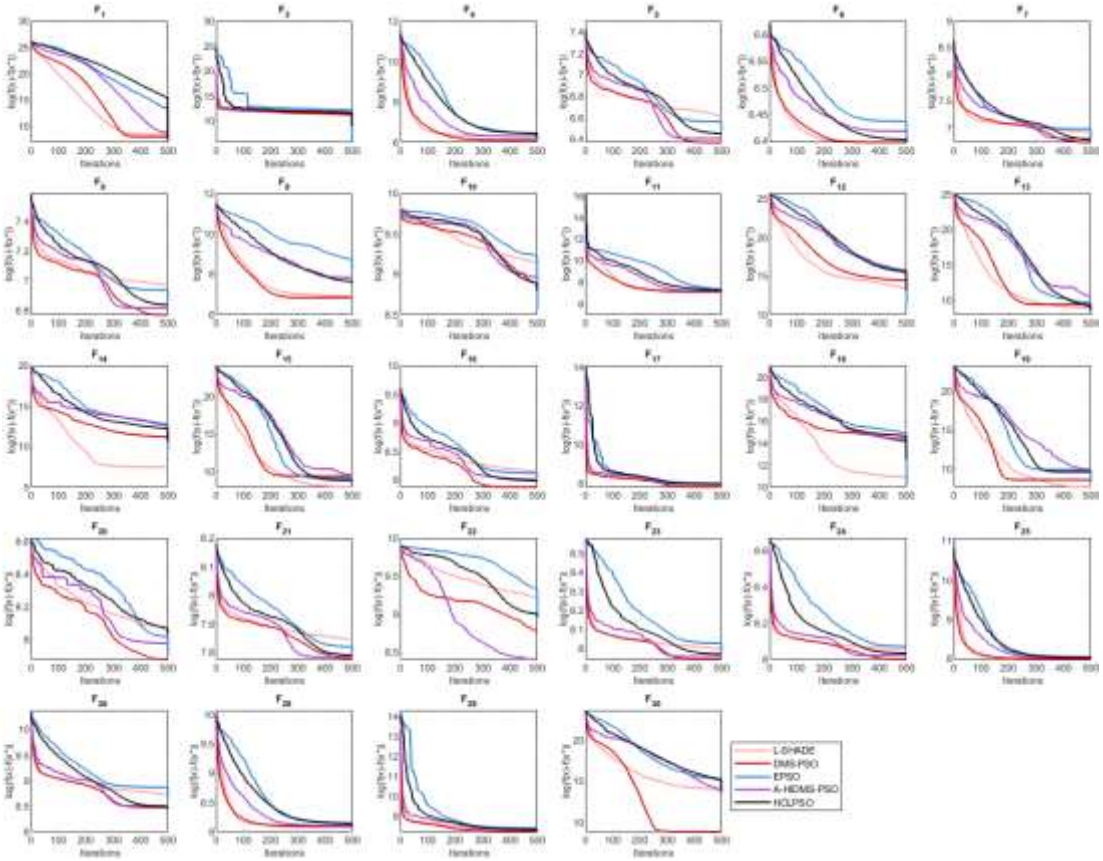
Convergence Rates of HIDMS-PSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



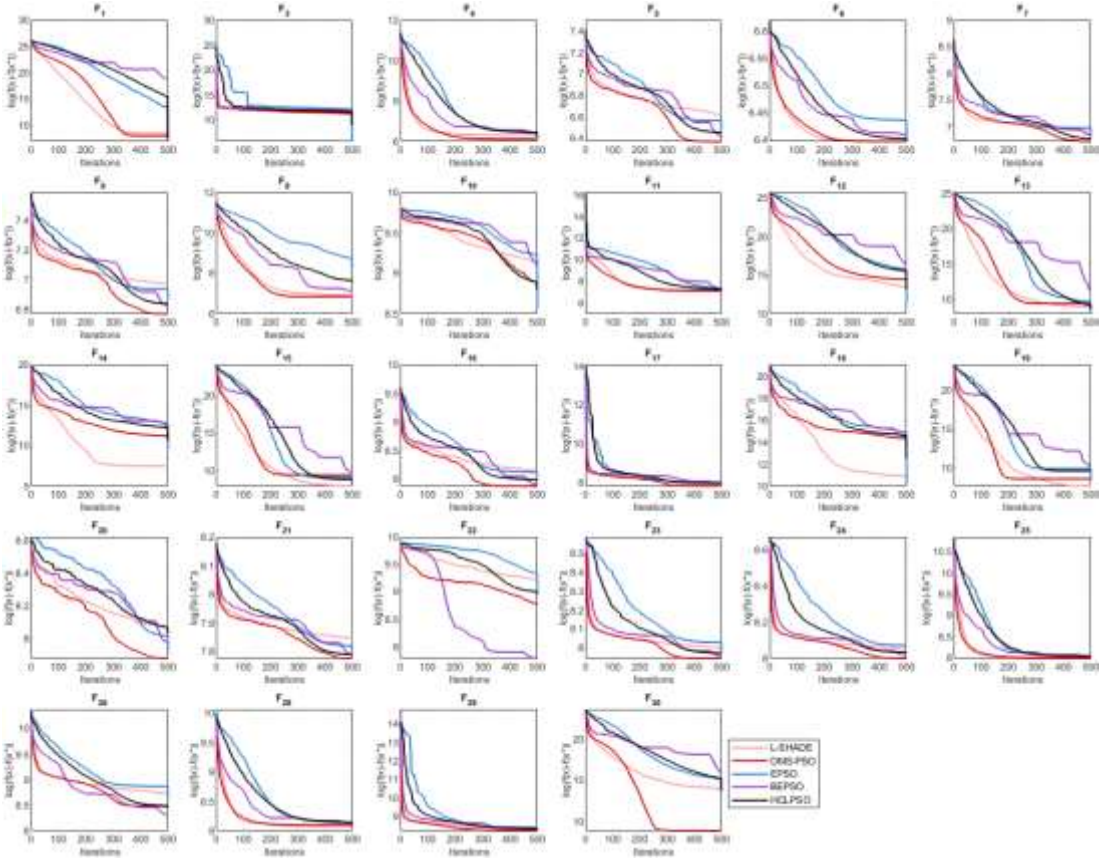
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



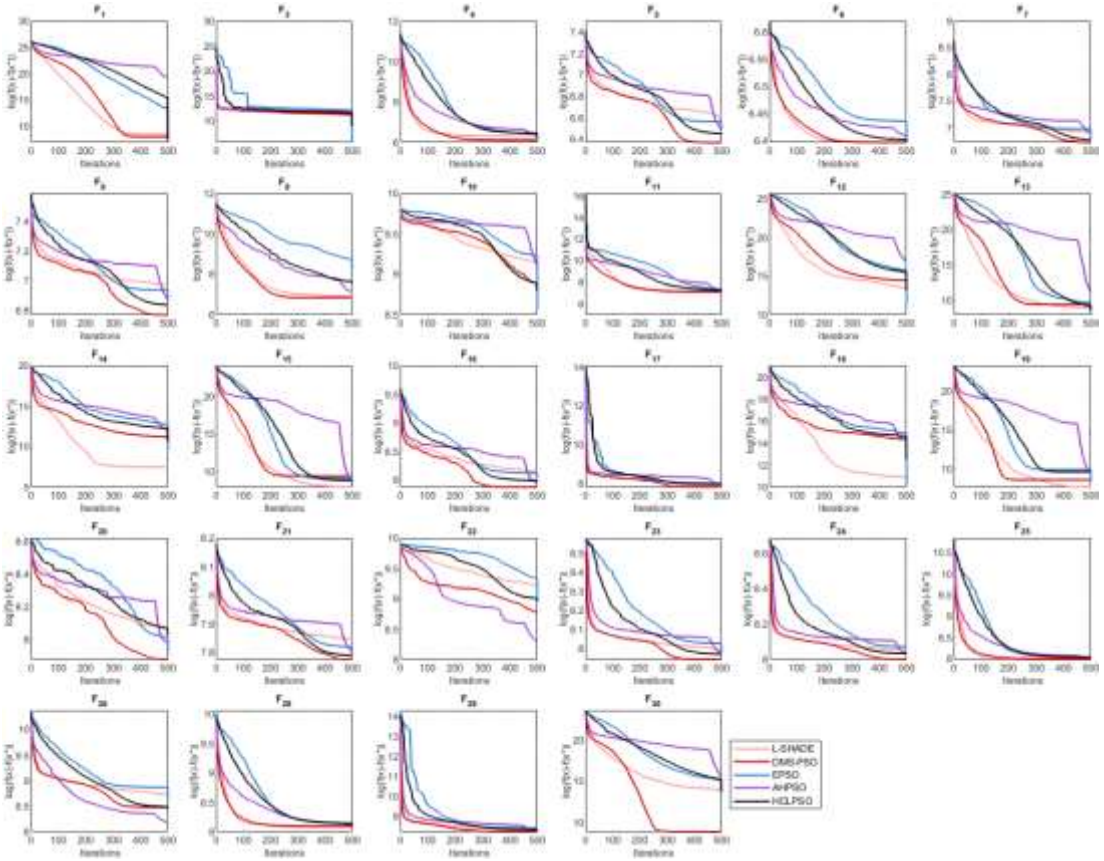
Convergence Rates of GA-HIDMS-PSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



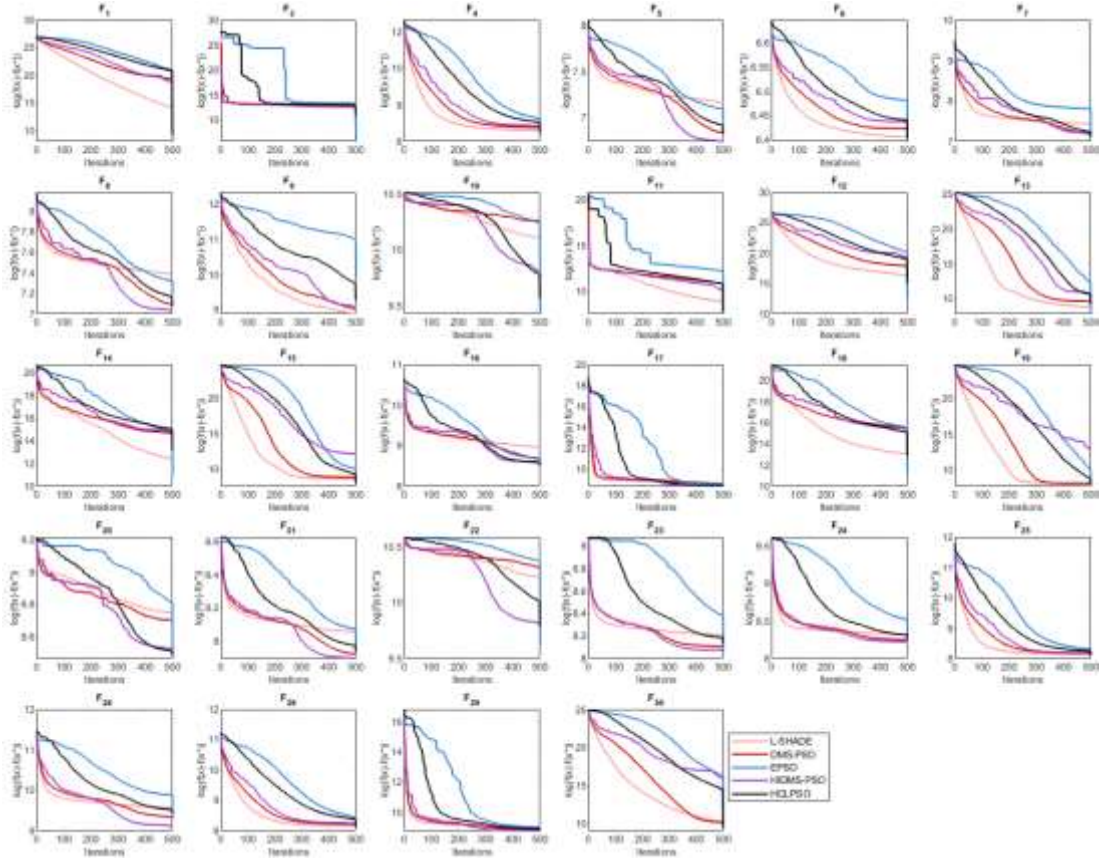
Convergence Rates of A-HIDMS-PSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



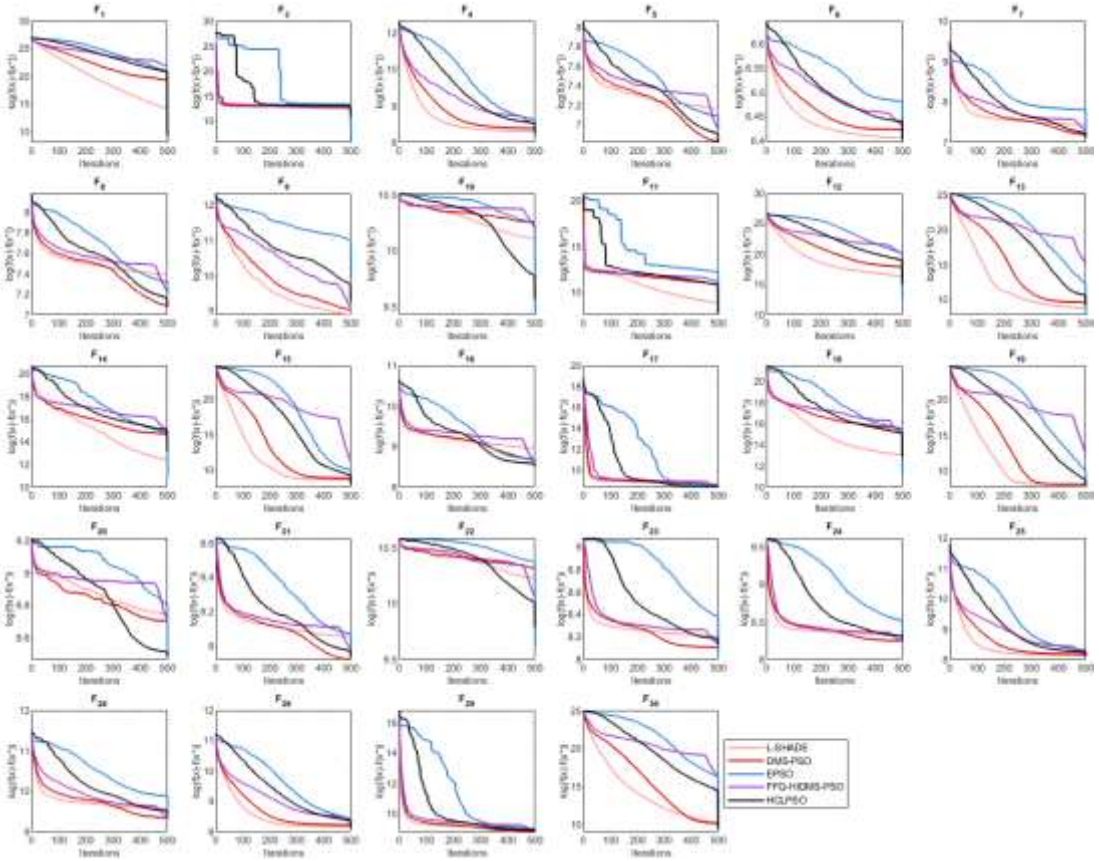
Convergence Rates of BEPSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



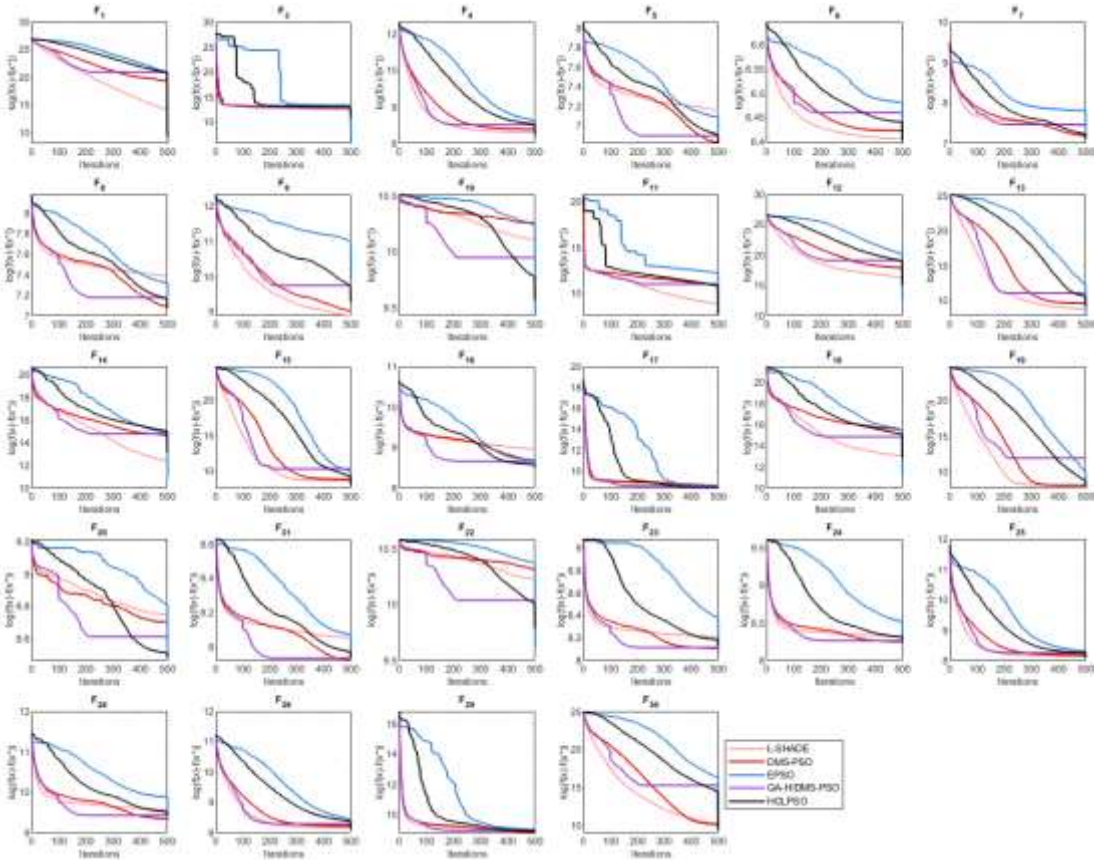
Convergence Rates of AHPSO Algorithm for 50-dimensional CEC'17 Benchmark Functions



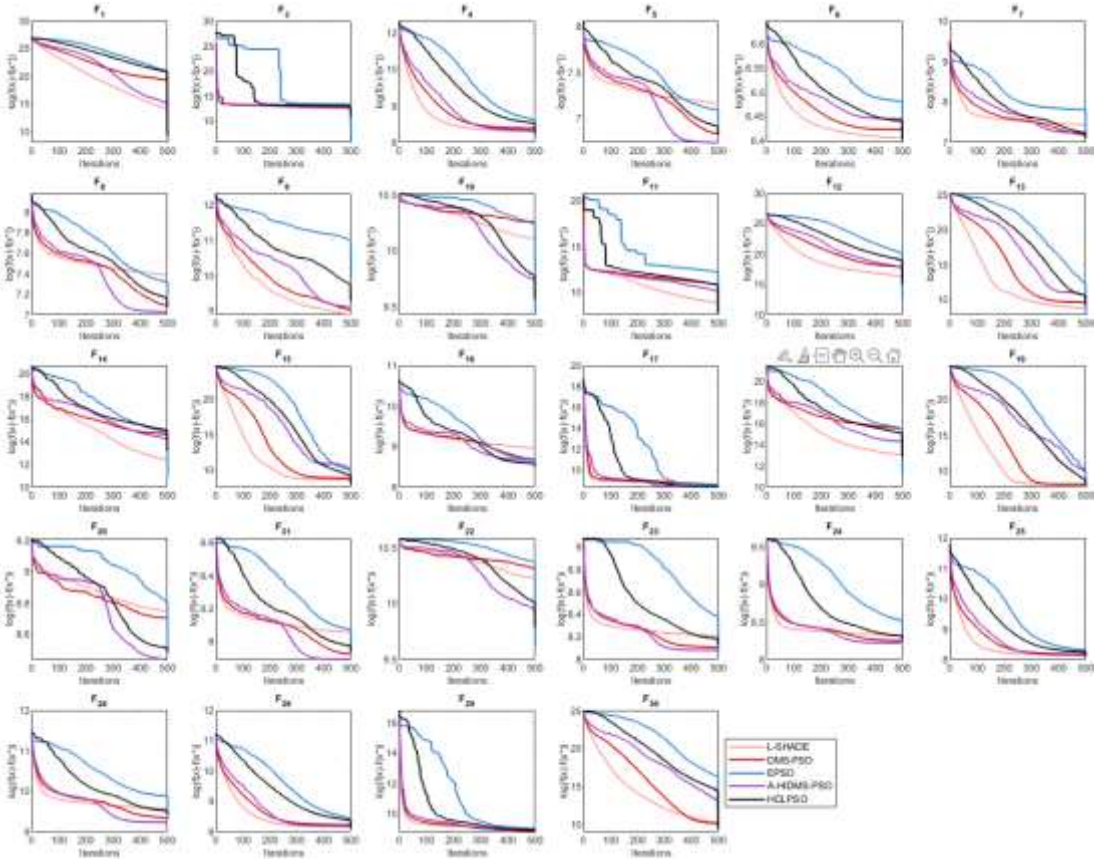
Convergence Rates of HIDMS-PSO Algorithm for 100-dimensional CEC'17 Benchmark Functions



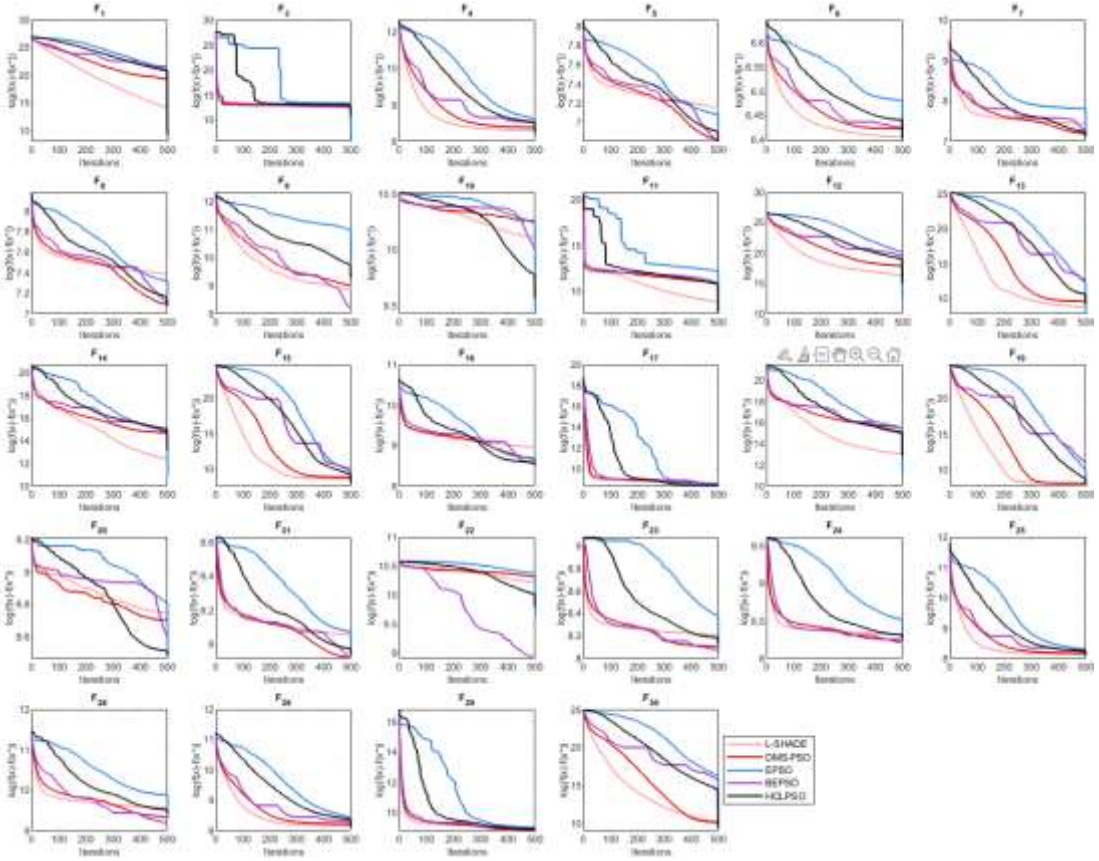
Convergence Rates of FFQ-HIDMS-PSO Algorithm for 100-dimensional CEC'17 Benchmark Functions



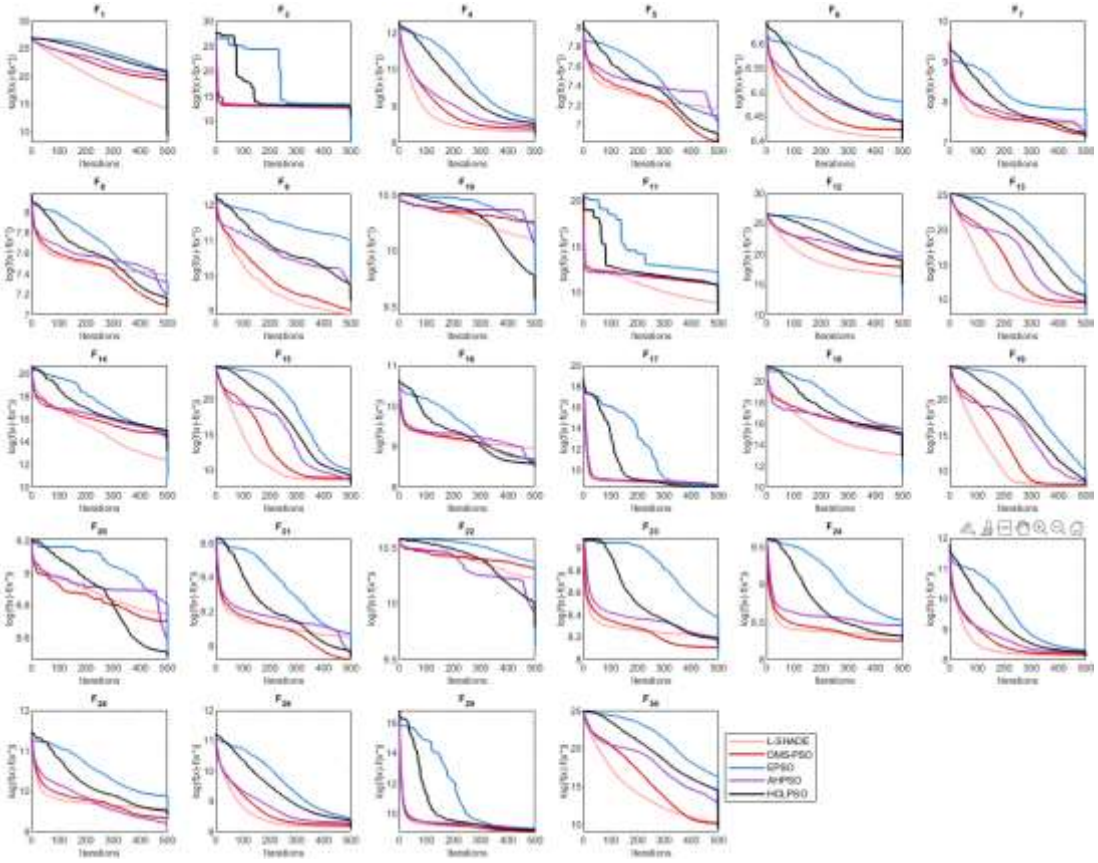
Convergence Rates of GA-HIDMS-PSO Algorithm for 100-dimensional CEC'17 Benchmark Functions



Convergence Rates of A-HIDMS-PSO Algorithm for 100-dimensional CEC'17 Benchmark Functions



Convergence Rates of BEPSO Algorithm for 100-dimensional CEC'17 Benchmark Functions



Convergence Rates of AHPSO Algorithm for 100-dimensional CEC'17 Benchmark Functions

Appendix 6: Experimental Results on the Non-Convex Constrained Real-world Problems

Statistical results for the optimal operation of alkylation unit problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	6.27E+24	1.29E+24	1.16E+25	4.81E+25	1.70E+22
BBPSO	5.16E+17	-4148.47	2.31E+18	1.03E+19	-4529.09
FIPS	-4316.51	-4450.41	436.7362	-2732.29	-4528.97
LPSO	4.64E+09	-3738.31	2.08E+10	9.29E+10	-4495.42
L-SHADE	3.67E+13	-33.2251	1.64E+14	7.33E+14	-142.719
HCLPSO	8.12E+12	307.4123	3.63E+13	1.62E+14	-4491.26
CLPSO	1.12E+19	1.29E+18	3.71E+19	1.67E+20	949.2223
DMS-PSO	5.17E+17	512.9879	2.31E+18	1.03E+19	-4732.82
UPSO	-4497.64	-4490.99	13.19961	-4484.01	-4523.07
EPSO	-4454.44	-4517.48	200.3013	-3868.71	-4528.99
FDR	-2878.45	-4451.97	2319.508	937.1978	-4451.97
AHPSO*	-892.755	442.3442	1966.3	620.7985	-4522.55
BEPSO*	518.7737	507.1039	94.49383	654.861	334.0016
A-HIDMS-PSO*	5.65E+13	615.76	2.53E+14	1.13E+15	-4431.63
HIDMS-PSO*	540.9794	659.391	188.4482	659.391	-7.85354
FFQ-HIDMS-PSO*	613.3909	631.1754	127.5347	768.6918	133.0777
GA-HIDMS-PSO*	382.0639	541.836	1031.788	855.0688	-3961.44

Statistical results for the process synthesis problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.19E+00	2.17E+00	1.57E-01	2.69E+00	2.00E+00
BBPSO	2.00E+00	2	4.56E-16	2.00E+00	2
FIPS	2	2	4.56E-16	2	2
LPSO	2	2	4.56E-16	2	2
L-SHADE	2	2	4.56E-16	2	2
HCLPSO	2	2	4.56E-16	2	2
QCLPSO	2.00E+00	2.00E+00	4.56E-16	2.00E+00	2
DMS-PSO	2	2	4.56E-16	2.00E+00	2
UPSO	2	2	4.56E-16	2	2
EPSO	2	2	4.56E-16	2	2
FDR	2	2	4.56E-16	2	2
AHPSO*	2	2	4.56E-16	2	2
BEPSO*	2	2	4.56E-16	2	2
A-HIDMS-PSO*	2	2	4.56E-16	2.00E+00	2
HIDMS-PSO*	2	2	4.56E-16	2	2
FFQ-HIDMS-PSO*	2	2	4.56E-16	2	2
GA-HIDMS-PSO*	2	2	4.56E-16	2	2

Statistical results for the process synthesis and design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.69E+12	5.81E+12	5.16E+12	2.19E+13	3.70E+09
BBPSO	3.00E+00	2.669648	5.54E-01	3.93E+00	2.576142
FIPS	3.130603	2.926487	4.18E-01	3.834381	2.559068
LPSO	2.578592	2.569605	2.07E-02	2.627196	2.558258
L-SHADE	2.74446	2.763017	5.01E-02	2.77877	2.599424
HCLPSO	3.038468	2.926123	3.61E-01	3.875742	2.58391
CLPSO	2.62E+00	2.60E+00	6.33E-02	2.78E+00	2.558451
DMS-PSO	3.03E+00	2.886341	4.39E-01	3.88E+00	2.584513
UPSO	2.619183	2.643098	3.31E-02	2.643098	2.562715
EPSO	3.02214	2.926123	2.55E-01	3.926123	2.926123
FDR	2.686719	2.607514	1.30E-01	2.926113	2.606593
AHPSO*	2.687836	2.688721	5.83E-02	2.815312	2.584609
BEPSO*	2.633792	2.617948	4.20E-02	2.728812	2.572587
A-HIDMS-PSO*	2.679452	2.656999	6.18E-02	2.83E+00	2.611488
HIDMS-PSO*	2.752055	2.80718	1.11E-01	2.926123	2.586104
FFQ-HIDMS-PSO*	2.856213	2.721941	6.63E-01	5.65738	2.566378
GA-HIDMS-PSO*	2.733275	2.758233	8.67E-02	2.816942	2.589616

Statistical results for the process flow sheeting problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	1.61E+13	1.68E+13	1.12E+13	3.36E+13	3.63E+10
BBPSO	1.19E+00	1.25	9.13E-02	1.35E+00	1.076543
FIPS	1.085308	1.076543	3.88E-02	1.25	1.076543
LPSO	1.145926	1.076543	8.72E-02	1.25	1.076543
L-SHADE	1.12858	1.076543	8.16E-02	1.25	1.076543
HCLPSO	1.111235	1.076543	7.12E-02	1.25	1.076543
CLPSO	1.08E+00	1.08E+00	1.30E-04	1.08E+00	1.076543
DMS-PSO	1.15E+00	1.076543	8.85E-02	1.25E+00	1.076543
UPSO	1.093889	1.076543	5.34E-02	1.25	1.076543
EPSO	1.076543	1.076543	0.00E+00	1.076543	1.076543
FDR	1.223981	1.25	6.35E-02	1.25	1.076543
AHPSO*	1.102813	1.076543	6.34E-02	1.25	1.076543
BEPSO*	1.076543	1.076543	6.66E-07	1.076546	1.076543
A-HIDMS-PSO*	1.076543	1.076543	5.24E-16	1.08E+00	1.076543
HIDMS-PSO*	1.076543	1.076543	7.61E-16	1.076543	1.076543
FFQ-HIDMS-PSO*	1.076543	1.076543	0.00E+00	1.076543	1.076543
GA-HIDMS-PSO*	1.076543	1.076543	2.49E-15	1.076543	1.076543

Statistical results for the two-reactor problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	8.93E+16	7.22E+16	8.31E+16	3.35E+17	1.36E+16
BBPSO	6.00E+14	1.00E+15	5.03E+14	1.00E+15	106.773
FIPS	4.2E+14	6.62E+13	4.92E+14	1.01E+15	10773966
LPSO	5.5E+14	1.00E+15	5.10E+14	1.00E+15	99.24188
L-SHADE	1.00E+15	1.00E+15	3.78E+10	1.00E+15	1.00E+15
HCLPSO	4.5E+14	123.6076	5.10E+14	1.00E+15	106.4628
CLPSO	9.57E+14	1.00E+15	1.60E+14	1.03E+15	4.74E+14
DMS-PSO	1.00E+15	1.00E+15	2.03E+01	1.00E+15	1.00E+15
UPSO	124.9299	133.1286	1.28E+01	133.4324	100.1506
EPSO	121.7768	120.4118	5.18E+00	133.4324	109.9801
FDR	1.00E+15	1.00E+15	1.21E+01	1.00E+15	1.00E+15
AHPSO*	1.00E+15	1.00E+15	7.26E+00	1.00E+15	1.00E+15
BEPSO*	1.00E+15	1.00E+15	6.83E+00	1.00E+15	1.00E+15
A-HIDMS-PSO*	9.5E+14	1.00E+15	2.24E+14	1.00E+15	99.806
HIDMS-PSO*	8.5E+14	1.00E+15	3.66E+14	1.00E+15	110.7766
FFQ-HIDMS-PSO*	9.5E+14	1.00E+15	2.24E+14	1.00E+15	99.77743
GA-HIDMS-PSO*	1.00E+15	1.00E+15	2.65E+03	1.00E+15	1.00E+15

Statistical results for the process synthesis problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	7.62E+22	2.60E+22	1.02E+23	3.64E+23	1.44E+20
BBPSO	4.16E+00	3.94E+00	1.16E+00	6.56E+00	2.924831
FIPS	4.291874	3.917903	1.25E+00	6.56E+00	2.924835
LPSO	6.988422	6.50E+00	3.62E+00	1.33E+01	3.896224
L-SHADE	2.96E+00	2.92E+00	6.40E-02	3.08E+00	2.92E+00
HCLPSO	3.738248	3.896224	7.91E-01	6.50E+00	2.924831
CLPSO	4.75E+19	2.71E+17	1.54E+20	6.71E+20	3.142466
DMS-PSO	7.91E+00	6.56E+00	2.97E+00	1.43E+01	3.94E+00
UPSO	8.312741	10.74685	3.48E+00	13.30685	3.896224
EPSO	5.848743	6.499582	1.16E+00	6.499582	3.896224
FDR	7.09E+00	6.56E+00	2.34E+00	1.43E+01	3.90E+00
AHPSO*	2.93E+00	2.93E+00	2.30E-04	2.93E+00	2.92E+00
BEPSO*	3.17E+00	2.92E+00	4.30E-01	3.90E+00	2.92E+00
A-HIDMS-PSO*	2.925996	2.92E+00	4.94E-03	2.95E+00	2.924833
HIDMS-PSO*	2.924846	2.92E+00	3.25E-05	2.92E+00	2.924831
FFQ-HIDMS-PSO*	2.962918	2.92E+00	1.54E-01	3.62E+00	2.924832
GA-HIDMS-PSO*	2.94E+00	2.95E+00	1.11E-02	2.95E+00	2.92E+00

Statistical results for the process design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	9.38E+14	4.14E+14	1.06E+15	2.99E+15	2.93E+04
BBPSO	2.69E+04	2.69E+04	3.73E-12	2.69E+04	26887.42
FIPS	26887.42	26887.42	3.73E-12	2.69E+04	26887.42
LPSO	26887.42	2.69E+04	3.73E-12	2.69E+04	26887.42
L-SHADE	2.69E+04	2.69E+04	3.73E-12	2.69E+04	2.69E+04
HCLPSO	26887.42	26887.42	3.73E-12	2.69E+04	26887.42
CLPSO	2.45E+04	2.43E+04	4.05E+02	2.60E+04	24310.77
DMS-PSO	2.69E+04	2.69E+04	3.73E-12	2.69E+04	2.69E+04
UPSO	26887.42	26887.42	3.73E-12	26887.42	26887.42
EPSO	26887.42	26887.42	3.73E-12	26887.42	26887.42
FDR	2.69E+04	2.69E+04	3.73E-12	2.69E+04	2.69E+04
AHPSO*	2.69E+04	2.69E+04	4.00E-12	2.69E+04	2.69E+04
BEPSO*	2.69E+04	2.69E+04	3.73E-12	2.69E+04	2.69E+04
A-HIDMS-PSO*	26887.42	2.69E+04	3.73E-12	2.69E+04	26887.42
HIDMS-PSO*	26887.42	2.69E+04	3.73E-12	2.69E+04	26887.42
FFQ-HIDMS-PSO*	26887.42	2.69E+04	3.73E-12	2.69E+04	26887.42
GA-HIDMS-PSO*	2.69E+04	2.69E+04	3.73E-12	2.69E+04	2.69E+04

Statistical results for the multi-product batch plant problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.86E+19	1.22E+15	7.53E+19	3.13E+20	138053.75
BBPSO	60250.98	59038.47	6066.25	81694.88	53652.92
FIPS	53673.74	53652.68	84.77	54030.40	53640.00
LPSO	58104.90	58523.31	1529.21	58853.66	53640.64
L-SHADE	58505.45	58505.45	0.00	58505.45	58505.45
HCLPSO	58398.37	58527.82	1148.96	59263.60	53640.46
CLPSO	2.231E+16	1.481E+05	9.827E+16	4.398E+17	101552.96
DMS-PSO	58793.19	58550.27	768.4382	61939.07	58481.53
UPSO	55591.36	53643.70	2447.50	58586.14	53643.70
EPSO	53649.76	53651.88	5.77	53665.98	53639.01
FDR	58572.91	58545.13	81.14	58767.68	58490.61
AHPSO*	58547.24	58533.10	61.51	58784.34	58506.28
BEPSO*	58536.40	58517.37	49.45	58714.57	58505.59
A-HIDMS-PSO*	58541.94	58506.79	154.40	59190.15	58468.48
HIDMS-PSO*	58507.29	58508.95	8.56	58513.62	58471.92
FFQ-HIDMS-PSO*	58520.70	58517.59	13.78	58552.73	58506.26
GA-HIDMS-PSO*	58523.41	58520.32	27.83	58620.52	58491.94

Statistical results for the weight minimisation of a speed reducer problem

	Mean	Median	STD	Max	Min

Statistical results for the optimisation design of industrial refrigeration system problem

	Mean	Median	STD	Max	Min

HPSO-TVAC	1.66E+14	1.41E+14	1.63E+14	6.18E+14	3326.08
BBPSO	2995.72	2994.42	3.99	3007.39	2994.42
FIPS	2994.89	2994.42	2.09	3003.76	2994.42
LPSO	3037.60	3033.70	12.58	3056.00	2994.42
L-SHADE	2994.42	2994.42	0.00	2994.42	2994.42
HCLPSO	2994.42	2994.42	0.00	2994.42	2994.42
CLPSO	3.010E+03	2.998E+03	4.963E+01	3.220E+03	2994.91
DMS-PSO	3035.853	3033.702	46.79705	3215.547	2994.424
UPSO	3027.81	3033.70	14.39	3033.70	2994.42
EPSO	2994.42	2994.42	0.00	2994.42	2994.42
FDR	3013.98	2994.42	47.68	3202.58	2994.42
AHPSO*	2994.42	2994.42	0.00	2994.42	2994.42
BEPSO*	2994.42	2994.42	0.00	2994.42	2994.42
A-HIDMS-PSO*	2994.42	2994.42	0.00	2994.42	2994.42
HIDMS-PSO*	2994.42	2994.42	0.00	2994.42	2994.42
FFQ-HIDMS-PSO*	3014.06	3014.06	20.15	3033.70	2994.42
GA-HIDMS-PSO*	2994.42	2994.42	0.00	2994.42	2994.42

HPSO-TVAC	1.1658E+18	1.0751E+18	8.6027E+17	2.3229E+18	5.4232E+16
BBPSO	8.2524E+14	4.4710E-01	2.8330E+15	1.2751E+16	0.03
FIPS	5.1158E+13	1.3508E+01	1.0490E+14	2.5565E+14	0.05
LPSO	2.8052E+00	3.4764E-01	8.9958E+00	4.0408E+01	0.03
L-SHADE	9.3848E+13	3.2213E-02	2.8886E+14	9.3848E+14	0.03
HCLPSO	4.4940E-01	4.5173E-02	1.1202E+00	4.5912E+00	0.03
CLPSO	2.5547E+13	8.3351E+11	4.5271E+13	1.3944E+14	5.06
DMS-PSO	9.38E+13	3.496312	2.89E+14	9.38E+14	0.04
UPSO	4.5857E-02	5.1356E-02	8.0688E-03	5.1356E-02	0.03
EPSO	3.3011E-02	3.2213E-02	2.8900E-03	4.4884E-02	0.03
FDR	4.6924E+13	2.5948E+00	2.0985E+14	9.3848E+14	0.05
AHPSO*	3.8727E-02	3.7999E-02	6.3742E-03	6.3041E-02	0.03
BEPSO*	4.2726E-02	4.3163E-02	8.1177E-03	5.7231E-02	0.03
A-HIDMS-PSO*	4.6924E+13	3.2213E-02	2.0985E+14	9.3848E+14	0.03
HIDMS-PSO*	9.3848E+13	3.2213E-02	2.8886E+14	9.3848E+14	0.03
FFQ-HIDMS-PSO*	6.5694E+14	9.3848E+14	4.4124E+14	9.3848E+14	0.03
GA-HIDMS-PSO*	1.4077E+14	1.1424E-01	3.4381E+14	9.3848E+14	0.03

Statistical results for the tension/compression spring design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	1.9336E+14	1.1955E-01	3.4518E+14	9.9602E+14	0.038
BBPSO	1.3122E-02	1.2823E-02	1.1118E-03	1.7773E-02	0.01
FIPS	1.2707E-02	1.2708E-02	1.3612E-05	1.2719E-02	0.01
LPSO	1.2677E-02	1.2672E-02	1.3445E-05	1.2719E-02	0.01
L-SHADE	1.2665E-02	1.2665E-02	2.5515E-07	1.2666E-02	0.01
HCLPSO	1.2691E-02	1.2681E-02	1.9453E-05	1.2719E-02	0.01
CLPSO	1.3143E-02	1.3040E-02	4.5964E-04	1.4243E-02	0.01
DMS-PSO	1.2922E-02	1.2861E-02	2.6078E-04	1.3602E-02	0.01
UPSO	1.2676E-02	1.2678E-02	8.9150E-06	1.2707E-02	0.01
EPSO	1.2696E-02	1.2715E-02	2.5881E-05	1.2719E-02	0.01
FDR	1.2674E-02	1.2666E-02	1.7986E-05	1.2719E-02	0.01
AHPSO*	1.2686E-02	1.2677E-02	2.2683E-05	1.2730E-02	0.01
BEPSO*	1.2685E-02	1.2676E-02	2.1009E-05	1.2737E-02	0.01
A-HIDMS-PSO*	1.2682E-02	1.2683E-02	1.0478E-05	1.2719E-02	0.01
HIDMS-PSO*	1.2675E-02	1.2668E-02	2.4286E-05	1.2772E-02	0.01
FFQ-HIDMS-PSO*	1.2720E-02	1.2715E-02	4.2821E-05	1.2780E-02	0.01
GA-HIDMS-PSO*	1.2693E-02	1.2687E-02	2.2984E-05	1.2719E-02	0.01

Statistical results for the pressure vessel design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	3.7126E+04	3.1419E+04	1.9182E+04	8.0437E+04	1.0873E+04
BBPSO	6.5266E+03	6.3297E+03	5.7256E+02	7.5445E+03	6059.71
FIPS	6.0993E+03	6.0597E+03	1.0129E+02	6.4129E+03	6059.71
LPSO	6.0986E+03	6.0597E+03	9.4099E+01	6.3709E+03	6059.71
L-SHADE	6.1321E+03	6.0846E+03	1.2317E+02	6.3708E+03	6059.71
HCLPSO	6.0828E+03	6.0905E+03	1.3689E+01	6.0905E+03	6059.71
CLPSO	6.6182E+03	6.4446E+03	7.2066E+02	9.1678E+03	6074.58
DMS-PSO	6358.69	6370.787	341.3471	7332.842	6059.71
UPSO	6.0908E+03	6.0597E+03	9.5744E+01	6.3708E+03	6059.71
EPSO	6.0597E+03	6.0597E+03	1.8662E-12	6.0597E+03	6059.71
FDR	6.3508E+03	6.3708E+03	2.6276E+02	7.3328E+03	6090.53
AHPSO*	6.3604E+03	6.3708E+03	9.4271E+01	6.4101E+03	6090.53
BEPSO*	6.2760E+03	6.3708E+03	1.4096E+02	6.4101E+03	6090.53
A-HIDMS-PSO*	6.1326E+03	6.0905E+03	1.0267E+02	6.3708E+03	6090.53
HIDMS-PSO*	6.0705E+03	6.0597E+03	1.5078E+01	6.0905E+03	6059.71
FFQ-HIDMS-PSO*	6.1326E+03	6.0905E+03	1.0267E+02	6.3708E+03	6090.53
GA-HIDMS-PSO*	6.0813E+03	6.0905E+03	1.4455E+01	6.0905E+03	6059.71

Statistical results for the welded beam design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.5398E+00	6.0052E+00	1.6598E+00	8.8266E+00	2.2728E+00
BBPSO	1.7090E+00	1.6710E+00	6.2657E-02	1.8167E+00	1.67
FIPS	1.6702E+00	1.6702E+00	7.2961E-13	1.6702E+00	1.67
LPSO	1.6702E+00	1.6702E+00	4.2129E-09	1.6702E+00	1.67
L-SHADE	1.6703E+00	1.6702E+00	4.0789E-04	1.6720E+00	1.67
HCLPSO	1.6702E+00	1.6702E+00	3.8476E-07	1.6702E+00	1.67
CLPSO	1.8996E+00	1.9058E+00	2.1654E-01	2.3072E+00	1.67
DMS-PSO	1.6849E+00	1.6702E+00	4.5090E-02	1.8167E+00	1.67
UPSO	1.6702E+00	1.6702E+00	7.0186E-12	1.6702E+00	1.67
EPSO	1.6702E+00	1.6702E+00	3.9649E-13	1.6702E+00	1.67
FDR	1.6775E+00	1.6702E+00	3.2757E-02	1.8167E+00	1.67
AHPSO*	1.6703E+00	1.6702E+00	1.6141E-04	1.6708E+00	1.67
BEPSO*	1.6702E+00	1.6702E+00	1.1624E-05	1.6703E+00	1.67
A-HIDMS-PSO*	1.6702E+00	1.6702E+00	5.6895E-05	1.6705E+00	1.67
HIDMS-PSO*	1.6702E+00	1.6702E+00	2.6236E-06	1.6702E+00	1.67
FFQ-HIDMS-PSO*	1.6702E+00	1.6702E+00	3.9651E-06	1.6702E+00	1.67
GA-HIDMS-PSO*	1.6702E+00	1.6702E+00	9.0608E-07	1.6702E+00	1.67

Statistical results for the three-bar truss design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.7710E+02	2.7899E+02	7.9775E+00	2.9200E+02	264.60
BBPSO	2.6390E+02	2.6390E+02	1.5291E-05	2.6390E+02	263.90
FIPS	2.6390E+02	2.6390E+02	4.9493E-05	2.6390E+02	263.90
LPSO	2.6390E+02	2.6390E+02	2.0477E-05	2.6390E+02	263.90
L-SHADE	2.6390E+02	2.6390E+02	1.4191E-04	2.6390E+02	263.90
HCLPSO	2.6390E+02	2.6390E+02	2.1823E-04	2.6390E+02	263.90
CLPSO	2.6390E+02	2.6390E+02	1.6809E-02	2.6397E+02	263.90
DMS-PSO	2.6390E+02	2.6390E+02	3.2479E-05	2.6390E+02	263.90
UPSO	2.6390E+02	2.6390E+02	2.0233E-05	2.6390E+02	263.90
EPSO	2.6390E+02	2.6390E+02	1.0330E-05	2.6390E+02	263.90
FDR	2.6390E+02	2.6390E+02	5.0321E-05	2.6390E+02	263.90
AHPSO*	2.6390E+02	2.6390E+02	3.9401E-04	2.6390E+02	263.90
BEPSO*	2.6390E+02	2.6390E+02	2.7383E-05	2.6390E+02	263.90
A-HIDMS-PSO*	2.6390E+02	2.6390E+02	4.0469E-05	2.6390E+02	263.90
HIDMS-PSO*	2.6390E+02	2.6390E+02	1.3628E-05	2.6390E+02	263.90
FFQ-HIDMS-PSO*	2.6390E+02	2.6390E+02	3.6828E-05	2.6390E+02	263.90
GA-HIDMS-PSO*	2.6390E+02	2.6390E+02	6.2295E-06	2.6390E+02	263.90

Statistical results for the multiple disk clutch brake design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.3600E-01	5.4453E-01	1.1545E-01	8.2536E-01	0.3263
BBPSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
FIPS	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
LPSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
L-SHADE	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
HCLPSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
CLPSO	2.3663E-01	2.3597E-01	2.0667E-03	2.4442E-01	0.24
DMS-PSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.2352
UPSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
EPSO	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
FDR	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
AHPSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
BEPSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
A-HIDMS-PSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
HIDMS-PSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
FFQ-HIDMS-PSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24
GA-HIDMS-PSO*	2.3524E-01	2.3524E-01	1.1391E-16	2.3524E-01	0.24

Statistical results for the step-cone pulley problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.8764E+19	5.9655E+19	2.7905E+19	1.1022E+20	3.2697E+18
BBPSO	8.8021E+09	1.8256E+01	3.9364E+10	1.7604E+11	16.30
FIPS	8.8021E+09	1.7136E+01	3.9364E+10	1.7604E+11	17.14
LPSO	1.8200E+01	1.8256E+01	2.5036E-01	1.8256E+01	17.14
L-SHADE	1.6090E+01	1.6090E+01	8.1505E-16	1.6090E+01	16.09
HCLPSO	1.7611E+01	1.7136E+01	5.5242E-01	1.8256E+01	17.14
CLPSO	2.2472E+09	6.1386E+07	4.6106E+09	1.6422E+10	1629.15
DMS-PSO	1.8822E+11	1.8256E+01	4.9319E+11	1.6182E+12	18.25
UPSO	1.7750E+01	1.7693E+01	3.5899E-01	1.8256E+01	17.14
EPSO	1.7136E+01	1.7136E+01	1.5119E-03	1.7136E+01	17.13
FDR	1.8052E+01	1.8106E+01	3.2026E-01	1.8256E+01	17.14
AHPSO*	1.7326E+01	1.7134E+01	5.3798E-01	1.8256E+01	16.30
BEPSO*	1.7306E+01	1.7134E+01	3.9592E-01	1.8250E+01	17.05
A-HIDMS-PSO*	1.7136E+01	1.7136E+01	1.1736E-04	1.7137E+01	17.14
HIDMS-PSO*	1.7136E+01	1.7136E+01	1.6126E-07	1.7136E+01	17.14
FFQ-HIDMS-PSO*	1.7286E+01	1.7131E+01	3.8570E-01	1.8256E+01	17.11
GA-HIDMS-PSO*	1.7088E+01	1.7118E+01	9.8114E-02	1.7136E+01	16.77

Statistical results for the hydro-static thrust bearing design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	1.1564E+22	1.1878E+20	3.7202E+22	1.6706E+23	5.7113E+03
BBPSO	2.1533E+03	2.0745E+03	3.8607E+02	3.0484E+03	1756.30
FIPS	2.6152E+03	2.6076E+03	4.7624E+02	3.5405E+03	2043.31
LPSO	1.9454E+03	1.9173E+03	1.1558E+02	2.2660E+03	1774.20
L-SHADE	1.7849E+03	1.6161E+03	5.6899E+02	4.0191E+03	1616.12
HCLPSO	1.9645E+03	1.9598E+03	2.0378E+02	2.4605E+03	1743.01
CLPSO	3.2657E+03	3.0401E+03	7.0573E+02	4.5903E+03	2388.39
DMS-PSO	2.2220E+03	2.1547E+03	3.3851E+02	3.0393E+03	1731.36
UPSO	2.0164E+03	1.7846E+03	3.4442E+02	2.9853E+03	1784.57
EPSO	1.8115E+03	1.7996E+03	8.7870E+01	2.1016E+03	1693.29
FDR	2.2422E+03	2.3316E+03	1.4026E+02	2.3316E+03	1899.52
AHPSO*	1.7451E+03	1.7338E+03	4.0283E+01	1.8198E+03	1686.34

Statistical results for the 10-bar truss design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	6.2714E+09	8.0820E+02	2.8047E+10	1.2543E+11	6.6789E+02
BBPSO	5.2787E+02	5.2576E+02	3.1504E+00	5.3248E+02	524.71
FIPS	5.2549E+02	5.2491E+02	1.1873E+00	5.2909E+02	524.54
LPSO	5.2503E+02	5.2486E+02	3.7694E-01	5.2572E+02	524.60
L-SHADE	5.2537E+02	5.2445E+02	2.2457E+00	5.3059E+02	524.45
HCLPSO	5.2483E+02	5.2466E+02	3.2399E-01	5.2552E+02	524.53
CLPSO	7.2987E+02	7.6657E+02	1.1466E+02	8.5506E+02	542.72
DMS-PSO	5.2944E+02	5.2559E+02	7.6189E+00	5.5189E+02	524.50
UPSO	5.2530E+02	5.2489E+02	7.3744E-01	5.2754E+02	524.80
EPSO	5.2855E+02	5.3073E+02	2.8990E+00	5.3073E+02	524.58
FDR	5.2484E+02	5.2468E+02	3.3388E-01	5.2599E+02	524.53
AHPSO*	5.2482E+02	5.2475E+02	2.2272E-01	5.2545E+02	524.54

BEPSO*	1.9621E+03	1.9719E+03	8.3513E+01	2.0795E+03	1797.91
A-HIDMS-PSO*	1.9346E+03	1.9152E+03	1.0453E+02	2.3071E+03	1843.81
HIDMS-PSO*	1.8519E+03	1.8596E+03	5.5574E+01	1.9527E+03	1674.81
FFQ-HIDMS-PSO*	1.9591E+03	1.9836E+03	6.9020E+01	2.0668E+03	1821.59
GA-HIDMS-PSO*	1.9296E+03	1.9275E+03	8.8698E+01	2.1285E+03	1838.25

BEPSO*	5.2463E+02	5.2462E+02	8.7821E-02	5.2492E+02	524.54
A-HIDMS-PSO*	5.2486E+02	5.2489E+02	1.5598E-01	5.2530E+02	524.52
HIDMS-PSO*	5.2457E+02	5.2453E+02	7.6746E-02	5.2482E+02	524.51
FFQ-HIDMS-PSO*	5.2468E+02	5.2468E+02	1.1869E-01	5.2494E+02	524.48
GA-HIDMS-PSO*	5.2476E+02	5.2464E+02	1.9893E-01	5.2504E+02	524.55

Statistical results for the rolling element bearing problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.3214E+16	2.1248E+16	1.9312E+16	7.4245E+16	3.2607E+14
BBPSO	1.4632E+04	1.4614E+04	3.5332E+01	1.4701E+04	14614.14
FIPS	1.4614E+04	1.4614E+04	7.4650E-12	1.4614E+04	14614.14
LPSO	1.4644E+04	1.4614E+04	4.2410E+01	1.4701E+04	14614.14
L-SHADE	1.4614E+04	1.4614E+04	7.4650E-12	1.4614E+04	14614.14
HCLPSO	1.4627E+04	1.4614E+04	3.1749E+01	1.4701E+04	14614.14
CLPSO	7.1677E+03	7.1375E+03	8.8702E+01	7.3943E+03	7060.81
DMS-PSO	1.4723E+04	1.4701E+04	1.8562E+02	1.5497E+04	14614.14
UPSO	1.4623E+04	1.4614E+04	2.6675E+01	1.4701E+04	14614.14
EPSO	1.4614E+04	1.4614E+04	7.4650E-12	1.4614E+04	14614.14
FDR	1.4650E+04	1.4614E+04	4.3004E+01	1.4701E+04	14614.14
AHPSO*	1.4670E+04	1.4701E+04	4.2410E+01	1.4701E+04	14614.14
BEPSO*	1.4618E+04	1.4614E+04	1.9379E+01	1.4701E+04	14614.14
A-HIDMS-PSO*	1.4614E+04	1.4614E+04	5.4089E-12	1.4614E+04	14614.14
HIDMS-PSO*	1.4614E+04	1.4614E+04	6.4783E-12	1.4614E+04	14614.14
FFQ-HIDMS-PSO*	1.4614E+04	1.4614E+04	6.5319E-12	1.4614E+04	14614.14
GA-HIDMS-PSO*	1.4614E+04	1.4614E+04	6.1614E-12	1.4614E+04	14614.14

Statistical results for the gas transmission compressor design (GTCD) problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	8.5146E+06	8.1052E+06	1.6784E+06	1.1385E+07	5.0718E+06
BBPSO	2.9677E+06	2.9649E+06	8.6361E+03	2.9930E+06	2964895.42
FIPS	2.9649E+06	2.9649E+06	2.0147E+00	2.9649E+06	2964895.54
LPSO	2.9649E+06	2.9649E+06	7.5385E+00	2.9649E+06	2964895.43
L-SHADE	2.9649E+06	2.9649E+06	9.5552E-10	2.9649E+06	2964895.42
HCLPSO	3.0289E+06	2.9649E+06	8.9552E+04	3.1479E+06	2964895.69
CLPSO	3.2044E+06	3.0138E+06	3.8122E+05	4.3044E+06	2965733.79
DMS-PSO	2.9740E+06	2.9649E+06	4.0921E+04	3.1479E+06	2964775
UPSO	2.9649E+06	2.9649E+06	1.7697E+00	2.9649E+06	2964895.42
EPSO	2.9649E+06	2.9649E+06	3.5303E-01	2.9649E+06	2964895.50
FDR	2.9649E+06	2.9649E+06	7.5097E+01	2.9651E+06	2964895.63
AHPSO*	2.9653E+06	2.9650E+06	5.9291E+02	2.9672E+06	2964895.44
BEPSO*	2.9649E+06	2.9649E+06	2.2865E+01	2.9650E+06	2964895.42
A-HIDMS-PSO*	2.9651E+06	2.9649E+06	4.2167E+02	2.9662E+06	2964901.15
HIDMS-PSO*	2.9649E+06	2.9649E+06	4.3437E+01	2.9651E+06	2964895.43
FFQ-HIDMS-PSO*	2.9649E+06	2.9649E+06	2.9573E+00	2.9649E+06	2964895.42
GA-HIDMS-PSO*	2.9650E+06	2.9649E+06	1.5319E+02	2.9655E+06	2964895.49

Statistical results for the gear train design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.8244E-03	1.4083E-03	3.2978E-03	1.2393E-02	8.2306E-06
BBPSO	7.3420E-26	0.0000E+00	3.2825E-25	1.4680E-24	0.00
FIPS	7.8210E-17	6.2207E-17	9.9930E-17	3.9050E-16	0.00
LPSO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00
L-SHADE	7.9343E-16	0.0000E+00	3.5205E-15	1.5750E-14	0.00
HCLPSO	2.0062E-26	0.0000E+00	8.1324E-26	3.6373E-25	0.00
CLPSO	8.4855E-18	1.0273E-22	2.9770E-17	1.3228E-16	0.00
DMS-PSO	7.70E-35	0.00E+00	2.37E-34	7.70E-34	0.00
UPSO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.00
EPSO	4.7027E-24	7.7037E-34	1.2858E-23	5.1924E-23	0.00
FDR	3.8519E-35	0.0000E+00	1.7226E-34	7.7037E-34	0.00
AHPSO*	5.3367E-18	0.0000E+00	2.3495E-17	1.0515E-16	0.00
BEPSO*	4.7462E-18	4.3565E-28	1.3881E-17	4.6499E-17	0.00
A-HIDMS-PSO*	5.0506E-28	8.3966E-28	4.2047E-28	8.3966E-28	0.00
HIDMS-PSO*	3.7335E-28	4.9304E-32	1.6073E-27	7.1993E-27	0.00
FFQ-HIDMS-PSO*	1.3873E-15	2.0905E-23	4.6821E-15	2.0141E-14	0.00
GA-HIDMS-PSO*	3.1932E-29	0.0000E+00	8.1462E-29	3.6148E-28	0.00

Statistical results for the Himmelblau's function

	Mean	Median	STD	Max	Min
HPSO-TVAC	-2.8595E+04	-2.8730E+04	7.1322E+02	-2.6985E+04	#########
BBPSO	-3.0666E+04	-3.0666E+04	8.5522E-12	-3.0666E+04	-30665.54
FIPS	-3.0666E+04	-3.0666E+04	1.8662E-11	-3.0666E+04	-30665.54
LPSO	-3.0666E+04	-3.0666E+04	1.8662E-11	-3.0666E+04	-30665.54
L-SHADE	-3.0666E+04	-3.0666E+04	1.8662E-11	-3.0666E+04	-30665.54
HCLPSO	-3.0666E+04	-3.0666E+04	8.3461E-12	-3.0666E+04	-30665.54
CLPSO	-3.0973E+04	-3.1074E+04	2.5052E+02	-3.0540E+04	-31230.17
DMS-PSO	-3.0666E+04	-3.0666E+04	1.8662E-11	-3.0666E+04	-30665.54
UPSO	-3.0666E+04	-3.0666E+04	1.8867E-11	-3.0666E+04	-30665.54
EPSO	-3.0666E+04	-3.0666E+04	1.9069E-11	-3.0666E+04	-30665.54
FDR	-3.0666E+04	-3.0666E+04	4.7213E-12	-3.0666E+04	-30665.54
AHPSO*	-3.0666E+04	-3.0666E+04	2.7054E-04	-3.0666E+04	-30665.54
BEPSO*	-3.0666E+04	-3.0666E+04	1.3010E-11	-3.0666E+04	-30665.54
A-HIDMS-PSO*	-3.0666E+04	-3.0666E+04	6.8824E-12	-3.0666E+04	-30665.54
HIDMS-PSO*	-3.0666E+04	-3.0666E+04	7.0325E-12	-3.0666E+04	-30665.54
FFQ-HIDMS-PSO*	-3.0666E+04	-3.0666E+04	2.9897E-10	-3.0666E+04	-30665.54
GA-HIDMS-PSO*	-3.0666E+04	-3.0666E+04	5.1449E-12	-3.0666E+04	-30665.54

Statistical results for the SOPWM for 3-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.8439E+19	6.1026E+19	1.4903E+19	7.3983E+19	1.98E+19
BBPSO	1.2912E+17	1.3075E+00	5.7743E+17	2.5824E+18	0.416706
FIPS	2.5782E+15	5.8037E+11	1.0969E+16	4.9155E+16	0.622581
LPSO	6.0765E-01	5.9259E-01	2.2972E-01	9.3897E-01	0.277091
L-SHADE	4.6674E-01	4.7078E-01	1.2515E-01	7.9659E-01	0.241258
HCLPSO	9.2473E-01	9.1254E-01	4.1949E-01	2.1969E+00	0.43437
CLPSO	5.1289E+18	2.8116E+18	5.7327E+18	1.9343E+19	4.15E+08
DMS-PSO	7.0231E+16	1.3114E+00	3.1408E+17	1.4046E+18	0.50433
UPSO	6.6482E-01	6.2191E-01	2.6493E-01	1.7381E+00	0.422874
EPSO	4.0500E+17	9.3678E-01	1.8112E+18	8.1000E+18	0.936777
FDR	1.0755E+00	1.2550E+00	3.2628E-01	1.3897E+00	0.399775
AHPSO*	9.3739E-01	9.3678E-01	2.4768E-03	9.4790E-01	0.936777
BEPSO*	8.4925E-01	7.4621E-01	3.5559E-01	1.7672E+00	0.282168
A-HIDMS-PSO*	7.5950E-01	5.8497E-01	2.6447E-01	1.4136E+00	0.584973
HIDMS-PSO*	9.2648E-01	1.1557E+00	2.8413E-01	1.1567E+00	0.408087
FFQ-HIDMS-PSO*	3.2214E+04	9.3678E-01	1.3875E+05	6.2136E+05	0.281599
GA-HIDMS-PSO*	0.9641127	0.74450771	0.5109445	1.88000046	0.264958

Statistical results for the SOPWM for 5-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.8730E+19	5.6808E+19	8.0738E+18	7.1943E+19	4.55E+19
BBPSO	4.7614E+17	5.6313E-01	1.8046E+18	8.1000E+18	0.272421
FIPS	8.4256E+14	1.7858E+14	1.6725E+15	6.9843E+15	4.39E+09
LPSO	1.0569E+13	4.5355E-01	4.7265E+13	2.1137E+14	0.167505
L-SHADE	4.5891E-01	5.1969E-01	2.2679E-01	8.2965E-01	0.163296
HCLPSO	1.7782E+13	8.4786E-01	5.3268E+13	2.3534E+14	0.277096
CLPSO	2.2329E+18	8.2043E+17	4.6207E+18	2.0071E+19	1.001427
DMS-PSO	1.8931E+13	1.0102E+00	3.5440E+13	1.1188E+14	0.197715
UPSO	2.0077E+12	3.2686E-01	8.9789E+12	4.0155E+13	0.164292
EPSO	1.0405E+01	3.7046E-01	1.2688E+01	2.5551E+01	0.277096
FDR	1.2926E+02	2.3394E-01	5.7636E+02	2.5780E+03	0.233937
AHPSO*	2.7705E-01	2.7710E-01	7.3989E-05	2.7710E-01	0.276915
BEPSO*	3.9733E-01	3.5220E-01	1.4065E-01	8.4119E-01	0.209267
A-HIDMS-PSO*	3.2768E+14	4.0960E+14	1.6810E+14	4.0960E+14	0.486343
HIDMS-PSO*	5.8924E-01	6.1110E-01	1.7715E-01	1.1365E+00	0.326086
FFQ-HIDMS-PSO*	3.1008E+04	9.5427E-01	8.6856E+04	3.8647E+05	0.523945
GA-HIDMS-PSO*	0.83966032	0.84197518	0.2913592	1.13590489	0.289211

Statistical results for the SOPWM for 7-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.8961E+19	6.0207E+19	1.4889E+19	8.6830E+19	2.72E+19
BBPSO	2.1458E+18	1.5270E+00	3.5596E+18	8.1000E+18	0.281944
FIPS	1.6374E+15	4.4418E+13	5.5691E+15	2.5004E+16	148.5797
LPSO	6.4000E+11	4.1596E-01	1.9699E+12	6.4000E+12	0.117799
L-SHADE	7.7410E-01	6.9779E-01	2.9332E-01	1.2544E+00	0.298854
HCLPSO	8.4640E+13	7.1890E-01	2.6052E+14	8.4640E+14	0.280385
CLPSO	6.5271E+18	9.6808E+17	1.0758E+19	3.6642E+19	5.50E+15
DMS-PSO	4.2830E+16	8.8858E+13	9.4344E+16	2.6618E+17	0.34341
UPSO	5.2942E-01	4.7165E-01	2.1024E-01	1.0368E+00	0.272901
EPSO	4.7627E-01	4.1729E-01	1.6554E-01	8.3819E-01	0.208766
FDR	1.6472E+13	1.5810E+13	2.2474E+13	8.8666E+13	0.324362
AHPSO*	8.1616E+14	8.4640E+14	4.3178E+13	8.4640E+14	7.48E+14
BEPSO*	1.0763E+13	1.1619E+00	3.3131E+13	1.3278E+14	0.365424
A-HIDMS-PSO*	3.8247E-01	4.1729E-01	6.4801E-02	4.1759E-01	0.17832
HIDMS-PSO*	5.8929E-01	5.1791E-01	1.7355E-01	1.0062E+00	0.285022
FFQ-HIDMS-PSO*	2.5116E+04	2.4103E-01	1.1232E+05	5.0232E+05	0.103612
GA-HIDMS-PSO*	0.53844331	0.56732168	0.1484818	0.80369784	0.199384

Statistical results for the SOPWM for 9-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	9.2495E+19	9.0921E+19	1.6151E+19	1.3632E+20	6.72E+19
BBPSO	9.2871E+17	3.7697E+01	2.4671E+18	8.1000E+18	0.297486
FIPS	3.4396E+16	6.5839E+15	8.3188E+16	3.4446E+17	3.35E+11
LPSO	5.4733E+14	3.9778E-01	1.1283E+15	3.5910E+15	0.154517
L-SHADE	6.9860E-01	5.8845E-01	2.0507E-01	1.0809E+00	0.310661
HCLPSO	5.9200E+14	7.7853E-01	2.6107E+15	1.1683E+16	0.349851
CLPSO	1.4327E+19	9.1586E+18	1.3937E+19	4.1997E+19	1.88E+17
DMS-PSO	7.2145E+17	7.0743E+16	1.0681E+18	2.4923E+18	0.234515
UPSO	2.6004E+13	2.4457E-01	1.1590E+14	5.1840E+14	0.191183
EPSO	3.3760E+13	5.2604E-01	1.1658E+14	5.1840E+14	0.432793
FDR	1.7943E+14	1.1701E-01	4.4077E+14	1.2805E+15	0.117011
AHPSO*	7.7760E+13	4.3279E-01	1.8991E+14	5.1840E+14	0.432765
BEPSO*	7.9019E+12	6.4130E-01	3.3799E+13	1.5136E+14	0.151147
A-HIDMS-PSO*	2.7440E+13	5.2916E-01	3.8366E+13	7.8400E+13	0.504006
HIDMS-PSO*	4.1772E-01	4.0346E-01	1.1666E-01	7.5401E-01	0.218742
FFQ-HIDMS-PSO*	1.9600E+13	1.4559E+03	3.4830E+13	7.8400E+13	0.475219
GA-HIDMS-PSO*	1.9079E+14	0.41207814	8.532E+14	3.82E+15	0.30021

Statistical results for the SOPWM for 11-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	9.2841E+19	9.4750E+19	1.6574E+19	1.1908E+20	5.88E+19
BBPSO	2.6570E+18	1.0312E+18	3.3788E+18	8.1000E+18	0.194253
FIPS	4.3402E+16	1.6472E+16	6.6545E+16	2.7506E+17	2.05E+15
LPSO	1.3523E+15	7.1243E+14	1.6455E+15	7.1102E+15	1.11E+14
L-SHADE	1.3277E+12	3.1082E-01	3.2374E+12	1.2969E+13	0.300916
HCLPSO	4.1404E+17	9.3029E+13	1.8095E+18	8.1000E+18	0.3126
CLPSO	9.8537E+18	5.3230E+18	1.1905E+19	4.5891E+19	2.28E+17
DMS-PSO	5.2762E+16	4.1766E+15	1.7503E+17	7.9273E+17	2.34E+09
UPSO	1.1203E+16	2.6377E+14	3.8776E+16	1.6836E+17	0.428105
EPSO	1.6240E+17	2.9201E+17	1.4714E+17	2.9201E+17	0.194253
FDR	2.5448E+15	6.4828E+13	5.2494E+15	1.5596E+16	2.48E+13
AHPSO*	1.9441E-01	1.9425E-01	6.7241E-04	1.9726E-01	0.194253
BEPSO*	4.7912E+14	7.9469E+13	1.0326E+15	4.3996E+15	1.1E+13
A-HIDMS-PSO*	2.7772E+15	2.7772E+15	7.9095E+08	2.7772E+15	2.78E+15
HIDMS-PSO*	5.5380E+14	6.0952E+14	2.0399E+14	1.1942E+15	2.52E+14
FFQ-HIDMS-PSO*	2.7772E+15	2.7772E+15	1.1615E+08	2.7772E+15	2.78E+15
GA-HIDMS-PSO*	1.70E+15	1.35E+15	1.04E+15	2.78E+15	2.05E+14

Statistical results for the SOPWM for13-level Inverters problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	9.7776E+19	9.5541E+19	2.4057E+19	1.3536E+20	5.02E+19
BBPSO	3.8979E+17	1.2712E+00	8.1891E+17	2.3828E+18	0.095164
FIPS	1.7681E+16	3.8471E+15	3.0293E+16	1.1398E+17	1.96E+13
LPSO	5.6182E+14	4.4004E-01	1.5966E+15	6.1465E+15	0.161191
L-SHADE	2.6585E-01	2.2789E-01	1.0765E-01	5.3685E-01	0.086513
HCLPSO	3.5700E+15	3.1745E-01	9.9907E+15	4.2802E+16	0.094981
CLPSO	8.3979E+18	3.4795E+18	1.1040E+19	3.8785E+19	8.14E+17
DMS-PSO	1.5312E+17	2.8138E+13	4.0310E+17	1.6266E+18	0.173508
UPSO	9.7322E+14	4.2928E-01	4.3512E+15	1.9459E+16	0.137725
EPSO	3.0223E+13	2.0346E-01	1.3516E+14	6.0445E+14	0.095164
FDR	4.7316E+14	6.9739E-01	1.2531E+15	5.0184E+15	0.427713
AHPSO*	9.4987E-02	9.5164E-02	7.9227E-04	9.5167E-02	0.091621
BEPSO*	6.8408E+13	2.5955E-01	2.5674E+14	1.1369E+15	0.091897
A-HIDMS-PSO*	8.4640E+14	8.4640E+14	3.2742E+07	8.4640E+14	8.46E+14
HIDMS-PSO*	1.3431E+14	2.2050E+14	1.0864E+14	2.2050E+14	0.264073
FFQ-HIDMS-PSO*	8.4640E+14	8.4640E+14	5.8385E+07	8.4640E+14	8.46E+14
GA-HIDMS-PSO*	2.12E+14	2.41E-01	3.76E+14	8.46E+14	0.213335

Statistical results for the tension/compression spring design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	1.93E+14	1.20E-01	3.45E+14	9.96E+14	3.82E-02
BBPSO	1.31E-02	1.28E-02	1.11E-03	1.78E-02	1.27E-02
FIPS	1.27E-02	1.27E-02	1.36E-05	1.27E-02	1.27E-02
LPSO	1.27E-02	1.27E-02	1.34E-05	1.27E-02	1.27E-02
L-SHADE	1.27E-02	1.27E-02	2.55E-07	1.27E-02	1.27E-02
HCLPSO	1.27E-02	1.27E-02	1.95E-05	1.27E-02	1.27E-02
CLPSO	1.30E-02	1.29E-02	2.28E-04	1.35E-02	1.27E-02
DMS-PSO	1.29E-02	1.28E-02	3.37E-04	1.39E-02	1.27E-02
UPSO	1.27E-02	1.27E-02	8.92E-06	1.27E-02	1.27E-02
EPSO	1.27E-02	1.27E-02	2.59E-05	1.27E-02	1.27E-02
FDR	1.27E-02	1.27E-02	1.80E-05	1.27E-02	1.27E-02
AHPSO*	1.27E-02	1.27E-02	2.27E-05	1.27E-02	1.27E-02
BEPSO*	1.27E-02	1.27E-02	2.10E-05	1.27E-02	1.27E-02
A-HIDMS-PSO*	1.27E-02	1.27E-02	1.05E-05	1.27E-02	1.27E-02
HIDMS-PSO*	1.27E-02	1.27E-02	2.43E-05	1.28E-02	1.27E-02
FFQ-HIDMS-PSO*	1.27E-02	1.27E-02	4.28E-05	1.28E-02	1.27E-02
GA-HIDMS-PSO*	1.27E-02	1.27E-02	2.30E-05	1.27E-02	1.27E-02

Statistical results for the pressure vessel design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	3.71E+04	3.14E+04	1.92E+04	8.04E+04	1.09E+04
BBPSO	6.53E+03	6.33E+03	5.73E+02	7.54E+03	6.06E+03
FIPS	6.10E+03	6.06E+03	1.01E+02	6.41E+03	6.06E+03
LPSO	6.10E+03	6.06E+03	9.41E+01	6.37E+03	6.06E+03
L-SHADE	6.13E+03	6.08E+03	1.23E+02	6.37E+03	6.06E+03
HCLPSO	6.08E+03	6.09E+03	1.37E+01	6.09E+03	6.06E+03
CLPSO	6.24E+03	6.15E+03	2.31E+02	7.00E+03	6.06E+03
DMS-PSO	6.36E+03	6.37E+03	3.41E+02	7.33E+03	6.06E+03
UPSO	6.09E+03	6.06E+03	9.57E+01	6.37E+03	6.06E+03
EPSO	6.06E+03	6.06E+03	1.87E-12	6.06E+03	6.06E+03
FDR	6.35E+03	6.37E+03	2.63E+02	7.33E+03	6.09E+03
AHPSO*	6.36E+03	6.37E+03	9.43E+01	6.41E+03	6.09E+03
BEPSO*	6.28E+03	6.37E+03	1.41E+02	6.41E+03	6.09E+03
A-HIDMS-PSO*	6.13E+03	6.09E+03	1.03E+02	6.37E+03	6.09E+03
HIDMS-PSO*	6.07E+03	6.06E+03	1.51E+01	6.09E+03	6.06E+03
FFQ-HIDMS-PSO*	6.13E+03	6.09E+03	1.03E+02	6.37E+03	6.09E+03
GA-HIDMS-PSO*	6.08E+03	6.09E+03	1.45E+01	6.09E+03	6.06E+03

Statistical results for the weight minimisation of a speed reducer problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	1.66E+14	1.41E+14	1.63E+14	6.18E+14	3.33E+03
BBPSO	3.00E+03	2.99E+03	3.99E+00	3.01E+03	2.99E+03
FIPS	2.99E+03	2.99E+03	2.09E+00	3.00E+03	2.99E+03
LPSO	3.04E+03	3.03E+03	1.26E+01	3.06E+03	2.99E+03
L-SHADE	2.99E+03	2.99E+03	4.67E-13	2.99E+03	2.99E+03
HCLPSO	2.99E+03	2.99E+03	4.67E-13	2.99E+03	2.99E+03
CLPSO	3.01E+03	3.00E+03	1.85E+01	3.04E+03	2.99E+03
DMS-PSO	3.04E+03	3.03E+03	4.68E+01	3.22E+03	2.99E+03
UPSO	3.03E+03	3.03E+03	1.44E+01	3.03E+03	2.99E+03
EPSO	2.99E+03	2.99E+03	4.67E-13	2.99E+03	2.99E+03
FDR	3.01E+03	2.99E+03	4.77E+01	3.20E+03	2.99E+03
AHPSO*	2.99E+03	2.99E+03	4.61E-12	2.99E+03	2.99E+03
BEPSO*	2.99E+03	2.99E+03	5.81E-13	2.99E+03	2.99E+03
A-HIDMS-PSO*	2.99E+03	2.99E+03	1.57E-11	2.99E+03	2.99E+03
HIDMS-PSO*	2.99E+03	2.99E+03	5.36E-12	2.99E+03	2.99E+03
FFQ-HIDMS-PSO*	3.01E+03	3.01E+03	2.01E+01	3.03E+03	2.99E+03
GA-HIDMS-PSO*	2.99E+03	2.99E+03	2.98E-11	2.99E+03	2.99E+03

Statistical results for the three-bar truss design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	2.77E+02	2.79E+02	7.98E+00	2.92E+02	2.65E+02
BBPSO	2.64E+02	2.64E+02	1.53E-05	2.64E+02	2.64E+02
FIPS	2.64E+02	2.64E+02	4.95E-05	2.64E+02	2.64E+02
LPSO	2.64E+02	2.64E+02	2.05E-05	2.64E+02	2.64E+02
L-SHADE	2.64E+02	2.64E+02	1.42E-04	2.64E+02	2.64E+02
HCLPSO	2.64E+02	2.64E+02	2.18E-04	2.64E+02	2.64E+02
CLPSO	2.64E+02	2.64E+02	8.87E-03	2.64E+02	2.64E+02
DMS-PSO	2.64E+02	2.64E+02	1.40E-04	2.64E+02	2.64E+02
UPSO	2.64E+02	2.64E+02	2.02E-05	2.64E+02	2.64E+02
EPSO	2.64E+02	2.64E+02	1.03E-05	2.64E+02	2.64E+02
FDR	2.64E+02	2.64E+02	5.03E-05	2.64E+02	2.64E+02
AHPSO*	2.64E+02	2.64E+02	3.94E-04	2.64E+02	2.64E+02
BEPSO*	2.64E+02	2.64E+02	2.74E-05	2.64E+02	2.64E+02
A-HIDMS-PSO*	2.64E+02	2.64E+02	4.05E-05	2.64E+02	2.64E+02
HIDMS-PSO*	2.64E+02	2.64E+02	1.36E-05	2.64E+02	2.64E+02
FFQ-HIDMS-PSO*	2.64E+02	2.64E+02	3.68E-05	2.64E+02	2.64E+02
GA-HIDMS-PSO*	2.64E+02	2.64E+02	6.23E-06	2.64E+02	2.64E+02

Statistical results for the welded beam design problem

	Mean	Median	STD	Max	Min
HPSO-TVAC	5.54E+00	6.01E+00	1.66E+00	8.83E+00	2.27E+00
BBPSO	1.71E+00	1.67E+00	6.27E-02	1.82E+00	1.67E+00
FIPS	1.67E+00	1.67E+00	7.30E-13	1.67E+00	1.67E+00
LPSO	1.67E+00	1.67E+00	4.21E-09	1.67E+00	1.67E+00
L-SHADE	1.67E+00	1.67E+00	4.08E-04	1.67E+00	1.67E+00
HCLPSO	1.67E+00	1.67E+00	3.85E-07	1.67E+00	1.67E+00
CLPSO	1.84E+00	1.83E+00	9.45E-02	2.07E+00	1.68E+00
DMS-PSO	1.68E+00	1.67E+00	4.51E-02	1.82E+00	1.67E+00
UPSO	1.67E+00	1.67E+00	7.02E-12	1.67E+00	1.67E+00
EPSO	1.67E+00	1.67E+00	3.96E-13	1.67E+00	1.67E+00
FDR	1.68E+00	1.67E+00	3.28E-02	1.82E+00	1.67E+00
AHPSO*	1.67E+00	1.67E+00	1.61E-04	1.67E+00	1.67E+00
BEPSO*	1.67E+00	1.67E+00	1.16E-05	1.67E+00	1.67E+00
A-HIDMS-PSO*	1.67E+00	1.67E+00	5.69E-05	1.67E+00	1.67E+00
HIDMS-PSO*	1.67E+00	1.67E+00	2.62E-06	1.67E+00	1.67E+00
FFQ-HIDMS-PSO*	1.67E+00	1.67E+00	3.97E-06	1.67E+00	1.67E+00
GA-HIDMS-PSO*	1.67E+00	1.67E+00	9.06E-07	1.67E+00	1.67E+00

Appendix 7: Run Time Comparison Results

Average execution time (in seconds) recorded for 10-dimensional CEC'17 problems

	HPSO-TVAC	BBPSO	FIPS	LPSO	L-SHADE	BRPSO	HCLPSO	CLPSO	DMS-PSO	MaPSO	MIPSO	UPSO	EPSO	FDR	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO
F1	0.22	0.67	0.07	0.07	0.43	0.86	0.55	0.29	0.20	0.23	0.86	0.36	1.04	1.10	0.47	4.27	0.65	0.81	1.17	1.39
F3	0.22	0.66	0.07	0.07	0.40	0.83	0.60	0.29	0.19	0.28	0.82	0.36	1.09	1.10	0.48	3.76	0.67	0.79	1.16	1.36
F4	0.22	0.65	0.07	0.07	0.41	0.85	0.53	0.28	0.20	0.23	0.79	0.36	0.93	1.08	0.48	4.46	0.70	0.79	1.16	1.41
F5	0.24	0.68	0.10	0.10	0.48	0.87	0.66	0.32	0.30	0.26	0.90	0.38	1.21	1.08	0.51	2.39	0.72	0.79	1.19	1.42
F6	0.35	0.79	0.20	0.20	0.54	0.99	0.72	0.42	0.72	0.34	0.97	0.49	1.15	1.24	0.60	4.44	0.84	0.91	1.29	1.55
F7	0.25	0.69	0.10	0.10	0.47	0.89	0.64	0.34	0.33	0.25	0.85	0.39	1.12	1.14	0.52	1.70	0.74	0.83	1.19	1.45
F8	0.24	0.68	0.10	0.10	0.47	0.87	0.63	0.32	0.31	0.26	0.88	0.37	1.14	1.11	0.51	2.54	0.73	0.81	1.19	1.44
F9	0.26	0.70	0.10	0.10	0.45	0.87	0.63	0.34	0.33	0.29	0.89	0.40	1.13	1.10	0.52	4.12	0.73	0.83	1.19	1.45
F10	0.26	0.72	0.13	0.13	0.50	0.87	0.68	0.34	0.41	0.27	0.86	0.42	1.30	1.12	0.54	2.76	0.76	0.84	1.22	1.47
F11	0.24	0.69	0.08	0.08	0.46	0.84	0.57	0.30	0.26	0.23	0.86	0.38	1.01	1.11	0.50	3.45	0.71	0.79	1.17	1.43
F12	0.24	0.69	0.09	0.09	0.45	0.86	0.61	0.31	0.28	0.22	0.86	0.38	1.06	1.12	0.51	2.98	0.72	0.80	1.15	1.44
F13	0.25	0.71	0.09	0.09	0.45	0.88	0.64	0.32	0.29	0.25	0.87	0.38	1.35	1.13	0.51	1.65	0.72	0.80	1.16	1.44
F14	0.26	0.71	0.11	0.11	0.45	0.93	0.65	0.33	0.36	0.27	0.92	0.39	1.20	1.14	0.53	1.76	0.74	0.77	1.18	1.45
F15	0.23	0.68	0.08	0.08	0.45	0.88	0.61	0.30	0.25	0.22	0.87	0.37	1.20	1.12	0.50	1.71	0.71	0.78	1.16	1.42
F16	0.25	0.71	0.10	0.10	0.48	0.90	0.60	0.32	0.31	0.27	0.90	0.38	1.04	1.13	0.52	3.09	0.73	0.76	1.17	1.43
F17	0.31	0.76	0.17	0.16	0.52	0.98	0.70	0.40	0.57	0.30	0.93	0.45	1.25	1.22	0.59	2.41	0.79	0.81	1.24	1.51
F18	0.24	0.69	0.09	0.09	0.45	0.91	0.66	0.32	0.30	0.26	0.85	0.38	1.38	1.13	0.51	1.50	0.73	0.74	1.17	1.40
F19	0.78	1.22	0.63	0.63	0.99	1.45	1.19	0.87	2.49	0.82	1.40	0.91	1.77	1.64	1.07	2.01	1.23	1.26	1.71	2.12
F20	0.32	0.75	0.17	0.17	0.53	0.96	0.71	0.40	0.58	0.35	0.95	0.46	1.23	1.18	0.60	3.12	0.75	0.90	1.24	1.53
F21	0.33	0.77	0.18	0.18	0.54	0.99	0.73	0.41	0.63	0.38	0.96	0.46	1.23	1.18	0.60	3.38	0.75	0.90	1.25	1.53
F22	0.38	0.81	0.23	0.22	0.55	0.99	0.77	0.46	0.80	0.40	1.03	0.51	1.24	1.24	0.65	4.37	0.79	0.93	1.29	1.57
F23	0.40	0.83	0.24	0.24	0.61	1.02	0.81	0.48	0.89	0.39	0.99	0.53	1.50	1.22	0.67	3.30	0.83	0.97	1.32	1.58
F24	0.41	0.83	0.26	0.26	0.66	1.02	0.83	0.50	0.99	0.43	1.04	0.54	1.40	1.25	0.69	3.03	0.88	1.00	1.33	1.61
F25	0.36	0.80	0.20	0.20	0.56	0.96	0.76	0.44	0.72	0.37	0.98	0.48	1.20	1.22	0.63	4.04	0.83	0.93	1.27	1.53
F26	0.45	0.88	0.29	0.29	0.64	1.05	0.83	0.52	1.08	0.46	1.05	0.58	1.29	1.25	0.71	4.78	0.91	1.02	1.36	1.64
F27	0.47	0.90	0.32	0.31	0.68	1.10	0.93	0.54	1.15	0.50	1.05	0.60	1.53	1.19	0.74	4.76	0.93	1.05	1.38	1.67
F28	0.42	0.86	0.26	0.26	0.63	1.06	0.83	0.49	0.99	0.43	0.99	0.57	1.31	1.15	0.69	4.46	0.89	0.98	1.34	1.61
F29	0.40	0.83	0.26	0.25	0.64	1.04	0.81	0.48	0.93	0.46	0.94	0.56	1.38	1.21	0.68	2.21	0.90	0.97	1.33	1.60
F30	0.87	1.33	0.72	0.72	1.07	1.47	1.26	0.95	2.84	0.85	1.36	1.03	1.80	1.46	1.16	3.77	1.37	1.41	1.80	2.17

Average execution time (in seconds) recorded for 100-dimensional CEC'17 problems

	HPSO-TVAC	BBPSO	FIPS	LPSO	L-SHADE	BRPSO	HCLPSO	CLPSO	DMS-PSO	MaPSO	MIPSO	UPSO	EPSO	FDR	AHPSO	BEPSO	HIDMS-PSO	FFQ-HIDMS-PSO	A-HIDMS-PSO	GA-HIDMS-PSO
F1	1.04	1.43	1.02	0.99	1.43	1.90	1.62	1.99	3.86	1.80	2.43	1.49	2.95	5.73	1.47	24.49	4.06	1.65	4.09	2.25
F3	1.01	1.38	1.02	0.99	1.43	1.91	1.80	1.96	4.00	1.28	1.73	1.51	4.25	5.67	1.46	3.69	4.06	1.64	4.01	2.24
F4	1.04	1.35	1.04	1.00	1.45	1.92	1.69	1.95	4.06	1.85	2.47	1.51	3.34	5.57	1.48	24.51	4.09	1.66	4.11	2.39
F5	1.27	1.57	1.33	1.30	1.72	2.18	1.90	2.26	5.09	2.16	2.83	1.80	3.36	5.96	1.77	8.93	4.36	1.95	4.39	2.76
F6	2.47	2.78	2.35	2.42	2.91	3.36	3.17	3.40	9.87	3.25	3.87	2.98	4.56	7.08	2.92	22.40	5.56	3.07	5.69	4.20
F7	1.32	1.71	1.30	1.31	1.73	2.22	1.88	2.31	5.02	2.22	2.75	1.82	3.15	6.17	1.77	9.68	4.40	1.96	4.56	2.81
F8	1.24	1.71	1.29	1.33	1.76	2.21	1.95	2.34	5.03	2.40	2.75	1.83	3.53	5.97	1.78	9.33	4.42	1.98	4.50	2.80
F9	1.28	1.78	1.30	1.31	1.80	2.26	2.07	2.32	5.49	2.19	2.43	1.86	3.73	6.08	1.78	19.61	4.41	1.96	4.59	2.84
F10	1.52	2.05	1.72	1.63	2.10	2.47	2.37	2.60	6.59	2.25	2.99	2.16	5.82	6.22	2.07	5.57	4.71	2.28	4.92	3.16
F11	1.18	1.60	1.18	1.13	1.59	2.04	1.85	2.09	4.49	2.07	2.60	1.66	3.62	5.79	1.59	6.11	4.20	1.81	4.43	2.61
F12	1.38	1.78	1.42	1.36	1.82	2.25	2.08	2.31	5.52	2.14	2.83	1.88	3.75	5.87	1.80	25.02	4.44	2.02	4.66	2.87
F13	1.19	1.58	1.20	1.15	1.59	2.06	1.83	2.09	4.60	2.05	2.56	1.68	3.43	5.73	1.60	22.84	4.23	1.81	4.43	2.63
F14	1.65	2.04	1.63	1.59	2.03	2.49	2.37	2.52	6.53	2.56	3.08	2.12	5.98	5.85	2.05	16.37	4.69	2.25	4.87	3.15
F15	1.16	1.54	1.15	1.11	1.52	2.01	1.83	2.05	4.44	2.02	2.56	1.65	3.44	5.69	1.56	23.04	4.17	1.67	4.35	2.56
F16	1.28	1.68	1.34	1.27	1.73	2.15	2.00	2.20	5.13	2.26	2.70	1.81	3.60	5.82	1.72	12.22	4.33	1.83	4.54	2.72
F17	2.01	2.37	2.04	1.96	2.43	2.88	2.65	2.91	7.89	2.88	3.38	2.51	4.11	6.46	2.43	11.67	5.06	2.51	5.22	3.55
F18	1.31	1.70	1.29	1.25	1.70	2.15	1.99	2.19	5.13	2.20	2.70	1.79	4.41	5.79	1.70	15.00	4.34	1.82	4.53	2.71
F19	6.69	7.03	6.78	6.57	6.94	7.55	7.32	7.55	26.49	7.39	8.15	7.10	8.96	10.20	7.12	29.49	9.76	7.30	9.84	9.31
F20	2.21	2.63	2.24	2.16	2.64	3.06	2.94	3.19	8.33	3.25	3.57	2.69	7.12	6.58	2.62	5.84	5.26	2.81	5.49	3.66
F21	3.31	3.66	3.36	3.26	3.72	4.25	3.93	4.37	12.84	4.26	4.80	3.80	5.68	7.62	3.80	12.16	6.36	3.92	6.53	5.10
F22	3.68	4.05	3.81	3.68	4.25	4.60	4.53	4.76	14.54	4.36	4.66	4.27	7.54	8.27	4.15	10.67	6.77	4.30	6.94	5.65
F23	4.36	4.77	4.46	4.31	4.90	5.31	5.06	5.44	16.27	5.17	5.72	4.88	7.05	8.10	4.84	21.26	7.41	4.98	7.62	6.38
F24	4.69	5.10	4.72	4.61	5.09	5.64	5.35	5.66	17.89	5.39	6.06	5.16	7.38	8.24	5.16	20.64	7.76	5.31	7.81	6.81
F25	4.55	4.93	4.50	4.38	4.86	5.41	5.18	5.45	16.63	5.38	5.92	5.00	6.85	8.96	4.94	28.97	7.48	5.08	7.49	6.56
F26	5.47	5.81	5.53	5.39	5.86	6.40	6.06	6.59	20.01	6.62	6.97	6.02	8.01	9.18	5.92	18.50	8.38	5.98	8.54	7.67
F27	6.44	6.52	6.17	6.36	6.83	7.36	7.15	7.40	25.60	7.24	7.69	6.90	9.05	8.97	6.90	30.56	9.26	7.05	9.61	8.84
F28	5.68	5.93	5.34	5.47	6.02	6.56	6.27	6.54	22.08	6.18	6.92	6.12	8.46	8.95	6.06	30.46	8.49	6.21	8.85	7.85
F29	3.85	4.13	3.80	3.77	4.32	4.70	4.53	4.79	14.81	4.46	5.15	4.31	6.55	7.90	4.24	19.15	6.75	4.43	7.01	5.76
F30	8.50	8.36	8.37	8.45	8.92	8.82	9.36	9.42	33.17	9.36	9.67	9.00	12.64	9.09	8.94	31.26	11.45	8.97	11.56	11.28